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Spontaneous cooperation for public goods

Jeroen Bruggeman [a], Rudolf Sprik [b], and Rick Quax [c]

[a]Department of Sociology, University of Amsterdam, Amsterdam, Netherlands; [b]Van Der Waals-Zeeman Institute, Institute of Physics, University of Amsterdam, Amsterdam, Netherlands; [c]Informatics Institute, Faculty of Science, University of Amsterdam, Amsterdam, Netherlands

ABSTRACT

Cooperation for public goods poses a dilemma, where individuals are tempted to free ride on others' contributions. Classic solutions involve monitoring, reputation maintenance and costly incentives, but there are important collective actions based on simple and cheap cues only, for example, unplanned protests and revolts. This can be explained by an Ising model with the assumption that individuals in uncertain situations tend to conform to the local majority in their network. Among initial defectors, noise such as rumors or opponents' provocations causes some of them to cooperate accidentally. At a critical level of noise, these cooperators trigger a cascade of cooperation. We find an analytic relationship between the phase transition and the asymmetry of the Ising model, which in turn reflects the asymmetry of cooperation and defection. This study thereby shows that in principle, the dilemma of cooperation can be solved by nothing more than a portion of random noise, without rational decision-making.

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People may want to realize or preserve public goods, for example, democracy and clean air, but because contributors are disadvantaged in the face of free riders, there is a dilemma (Gavrilets, 2015; Hardin, 1968; Olson, 1965; Ostrom, 2009). Solutions typically require efforts of the participants to monitor one another (Rustagi, Engel, & Kosfeld, 2010) and spread information (gossip) (Nowak & Sigmund, 2005) through their network reliably that establishes reputations (Panchanathan & Boyd, 2004), upon which some of them have to deliver individual rewards or (threats of) punishments (Fehr & Fischbacher, 2003), under pro-social norms to preclude arbitrariness. These provisions are not always (sufficiently) available, though, whereas in certain situations, participants still manage to self-organize cooperation, even without leaders. Cases in point are impromptu help at disasters, non-organized revolts against political regimes (Lohmann, 1994; Tilly, 2002; Tufekci, 2017) and spontaneous street fights between groups of young men.

These examples have in common a high uncertainty of outcomes, and unknown benefits and costs. Rational decision-making is therefore not feasible. Participants who identify with their group or its goal (Van Stekelenburg & Klandermans, 2013), and thereby feel group solidarity (Durkheim, 1912), use the heuristic of *conformism* to the majority of their network neighbors (Wu, Li, Zhang, Cressman, & Tao, 2014), which can be based on no more than visual information. Human ancestors lived in groups for millions of years (Shultz, Opie, & Atkinson, 2011) and in all likelihood, solidarity and conformism are both cultural and genetic (Boyd, 2018). On an evolutionary timescale, conformism must have been beneficial on average when future benefits and costs were unknown (Van den Berg & Wenseleers, 2018). To explain cooperation under conformism, we use an Ising model (Weidlich, 1971; Galam, Gefen, & Shapir, 1982; Jones, 1985; Stauff, 2008; Castellano, Fortunato, &

CONTACT Jeroen Bruggeman j.p.bruggeman@uva.nl Department of Sociology, University of Amsterdam, Nieuwe Achtergracht 166, Amsterdam 1018 WV, Netherlands

Loreto, 2009). This model recovers the critical mass that makes cooperation self-reinforcing but without the rationality assumptions of critical mass theory (Marwell & Oliver, 1993). Other applications of the Ising model to a range of social science problems are reviewed by Castellano *cum suis* (2009).

1. Model

A group g of individuals who share an interest in a public good is modeled as a network with weighted and usually asymmetric ties A_{ij} denoting i paying attention to j. There is no assumption that i and j know each other before they meet at the site where collective action might take place. Consistent with other models of social influence (Friedkin & Johnsen, 2011), the adjacency matrix is row-normalized, yielding cell values $a_{ij} = A_{ij}/\sum_j A_{ij}$, hence $\sum_j a_{ij} = 1$.

Individuals have two behavioral options, defect (D) and cooperate (C), $C > D > 0$, and all defect at the start. The average degree of cooperation among n individuals is described by an order parameter $M = 1/n\sum_{i=1}^{n} S_i$, where the behavioral variable S_i can take the value $S_i = C$ or $S_i = -D$, for example $S = \{1, -1/2\}$. Everybody gets an equal share of the public good but cooperators incur a cost. A widely used definition of payoffs for cooperators Π_C^g and defectors Π_D^g in a group g is the following (Perc et al., 2017),

$$\Pi_C^g = r(N_C + 1)/n - 1,$$

$$\Pi_D^g = rN_C/n, \tag{1}$$

with $r > 1$ an enhancement, or synergy, factor of cooperation, and N_C the number of cooperators when the focal player decides. In our case,

$$P_C^g = r(M + C/n) - C,$$

$$P_D^g = r(M - D/n), \tag{2}$$

which is identical to Eq. 1 in standard game theory when $C = 1$ and $D = 0$. Our payoffs can be negative but that does not matter because they are used only comparatively. Other, for example non-linear, payoff functions (Marwell & Oliver, 1993) may also be used. The key point, however, is that under high uncertainty, participants do not maximize their payoff (directly) but align with others instead, thereby forming a collective lever that can increase their payoff while avoiding exploitation. Behavior and network ties are expressed in the conventional, but here asymmetric, Ising model

$$H = -\sum_{i \neq j}^{n} a_{ij} S_i S_j. \tag{3}$$

Solving the model boils down to minimizing H, where H/n can be interpreted as average dissatisfaction. Minimizing can be done computationally with a Metropolis algorithm (Barrat, Barthelemy, & Vespignani, 2008), where individuals decide sequentially as in many network models, or through a mean-field analysis, elaborated below.

High-uncertainty situations to which the model applies are characterized by *turmoil*, T or temperature in the original model. It causes arousal, measurable as heart rates (Konvalinka et al., 2011) and produces noise (Lewenstein, Nowak, & Latané, 1992) in individuals' information about the situation, which in turn becomes partly false, ambiguous, exaggerated or objectively irrelevant. Turmoil and its noise may consist of rumors, fire, provocations and violence. Some social movements produce turmoil by themselves, for instance an increasingly frequent posting of online messages (Johnson et al., 2016). Arousal and noise entail "trembling hands" (Dion & Axelrod, 1988) as game theorists say, which means a chance that some individuals accidentally change their behavior. The model is to show that few accidental cooperators entail a cascade of cooperation. An example of turmoil and its ramifications is the self-immolation of a street vendor in December 2010, which, in the given circumstances, set off the Tunisian revolution. Other examples are the revolts in East Germany (Lohmann, 1994) and Romania in 1989 and in Egypt and Syria in 2011 (Hussain & Howard, 2013), where protesters were agitated by rumors about the events in neighboring countries. Autocratic rulers try to prevent revolts by suppressing turmoil, for instance by tightening media control.

Noise is different from a stable bias, for instance the ideology of an autocratic regime, which entails revolts against it less often than a weakened regime or stumbling opponents in street fights. Opponents' weakness gives off noisy signals that they might be overcome, which readily entail collective actions against them (Collins, 2008; Goldstone, 2001; Skocpol, 1979). Whereas responses to noise are typically spontaneous, collective responses to stable signals, biased or not, tend to be mounted by organized groups with norms, incentives and all that (Goldstone, 2001; Tilly, 2002; Tufekci, 2017). Combinations of signal and noise also occur, of course, which can result in, for example, an organized peaceful demonstration to suddenly turn violent. Our focus is on spontaneous cooperation.

2. Results

Our general result is that within finite time and at low turmoil, cooperation does not get off the ground, but it does emerge at a critical level T_c. This pattern is illustrated by the red line in Figure 1(b) along the direction of the arrows. The figure was obtained with a mean-field approach, but numerical simulations with the Metropolis algorithm show up the same pattern, with lower T_c for small networks (Figure 2). Other topological variations, of density, clustering and degree distribution, are elaborated elsewhere (Bruggeman & Sprik, 2020). Moreover, if a certain cluster is exposed to locally higher turmoil, cooperation emerges there. The model thus shows that at a critical level of turmoil, few accidental cooperators can trigger a cascade of cooperation. If T keeps increasing way beyond T_c, cooperators co-exist with increasing numbers of defectors, until the two behaviors become equally frequent. If in actuality cooperation then collapses completely is an issue for further study. Otherwise, cooperation ends when the public good is achieved, the participants run out of steam, or others intervene.

Alternatively, if participants get to understand an enduring situation, their uncertainty will reduce and they may start acting strategically, which requires pro-social norms to prevent. If the participants then develop such norms prescribing rewards and punishments, these norms can be easily modeled as field(s) by adding term(s) $-h\sum S_i$ to the Hamiltonian (Eq.3). Consequently, cooperation emerges without a phase transition. The actual maintenance of these norms, however, will entail additional costs over and above the contributions, whereas spontaneous cooperation is relatively cheap.

2.1. *Comparison with the symmetric Ising model*

Rewriting the asymmetric Ising model in a symmetric form enables a direct comparison with results in the literature for symmetric models and a generalization to arbitrary values of $S = \{C, -D\}$. A model with asymmetric values can be reformulated as a symmetric model with an offset, or bias, $S_0 = (C - D)/2$ and an increment $\Delta = (C + D)/2$ by the mapping

$$S = \{C, -D\} \rightarrow \{S_0 + \Delta, S_0 - \Delta\} \tag{4}$$

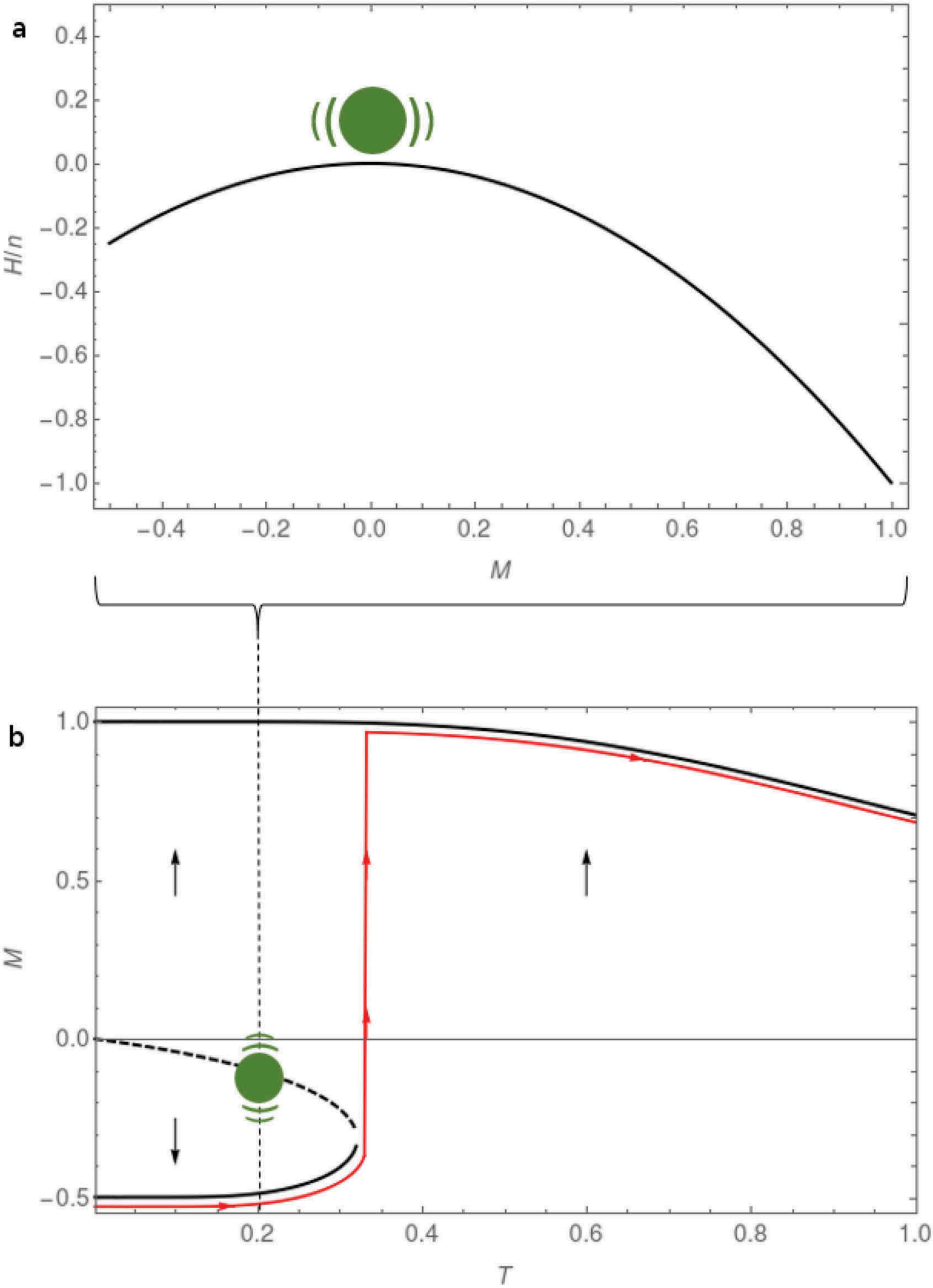

Figure 1. Cooperation for public goods. (a) Mean dissatisfaction H/n and level of cooperation M. When all defect, H/n is at a local minimum, on the left, but to proceed to the global minimum where all cooperate, on the right, participants are hindered by a hill. (b) Mean-field analysis with $S = \{1, -1/2\}$ shows below T_c one stable state with mostly cooperators, at the top, and another stable state in finite time with mostly defectors, at the bottom. A metastable state in between, indicated by the dotted line, corresponds to the hilltop in Figure 1(a). Above T_c only one state remains, where with increasing T, cooperators are joined by increasing numbers of defectors.

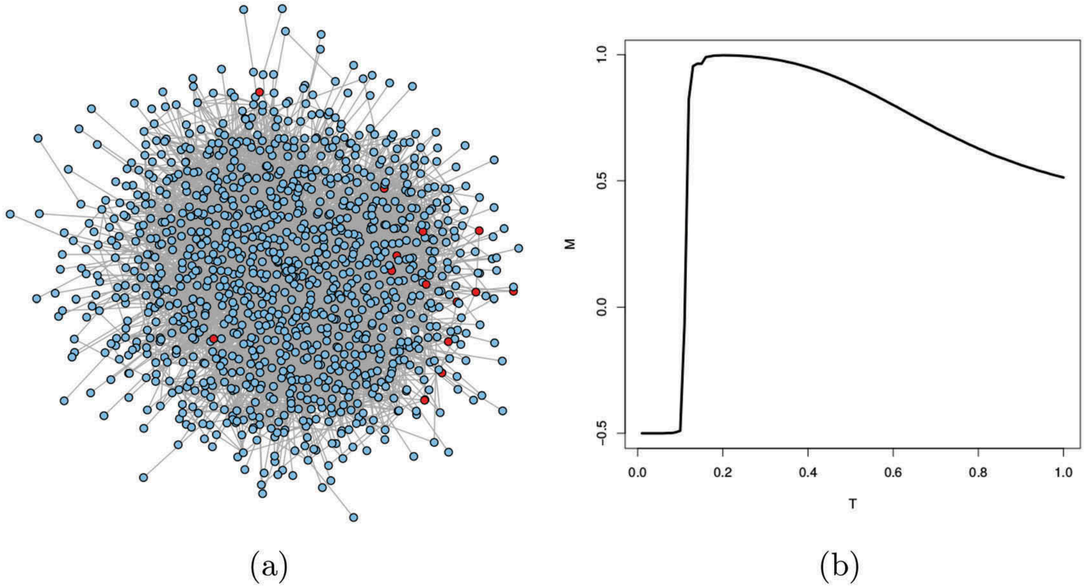

Figure 2. Numerical simulation on a university e-mail network with $n = 1133$ and clustering $C = 0.254$; data from Guimerà (2003). (a) the network at a near-critical turmoil level of $T = 0.101$; $S = \{1, -1/2\}$. Some nodes start cooperating (red) whereas most still defect (blue). (b) T_c is smaller than in the mean-field approximation, but the overall pattern is qualitatively the same (compare to Fig. 1 b).

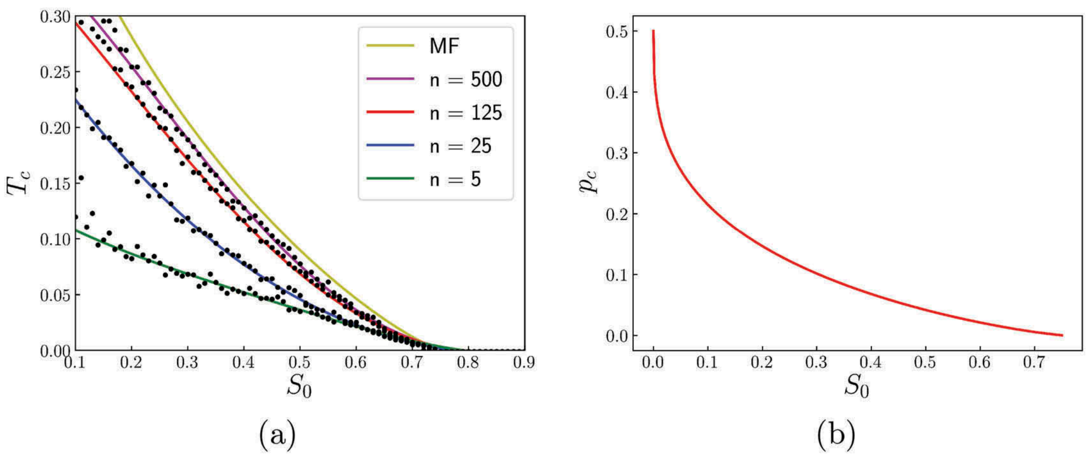

Figure 3. Consequences of shifting S_0 with the mean-field approach (MF), keeping $\Delta = 0.75$. (a) With increasing S_0, less agitation is necessary to turn defectors into cooperators. For comparison, numerical simulations on several random networks with density = 0.8 are shown as well. (b) The proportion of defectors p_c at T_c decreases with increasing S_0.

Accordingly, the values chosen in Figure 1(b), $S = \{1, -1/2\}$, imply $\Delta = 0.75$ and $S_0 = 0.25$. For given Δ, increasing S_0 means increasing interest in the public good, or, in line with the literature on protests, increasing grievances (Van Stekelenburg & Klandermans, 2013).

Substitution of S_0 and $\hat{S}_i$ chosen from $\{\Delta, -\Delta\}$ in H yields

$$H = -\sum_{ij} a_{ij}(S_0 + \hat{S}_i)(S_0 + \hat{S}_j). \tag{5}$$

Expanding H in orders of S_0 yields

$$-\sum_{ij} a_{ij}\hat{S}_i\hat{S}_j - S_0\left(\sum_{ij} a_{ij}\hat{S}_i + \sum_{ij} a_{ij}\hat{S}_j\right) - S_0^2\sum_{ij} a_{ij}. \tag{6}$$

The first term in the expansion $H_{sym} = -\sum_{ij} a_{ij}\hat{S}_i\hat{S}_j$ is a symmetric model with the same adjacency matrix as the original asymmetric model. The second term $H_{loc} = -S_0(\sum_{ij} a_{ij}\hat{S}_i + \sum_{ij} a_{ij}\hat{S}_j)$ is proportional to S_0 and can be interpreted as a local field that modifies H_{sym}. The contribution of this local field can be expressed in terms of row and column sums of a_{ij} as

$$H_{loc} = -S_0\sum_i\left(\sum_j a_{ij} + \sum_j a_{ji}\right)\hat{S}_i. \tag{7}$$

For row-normalized adjacency matrices, with $\sum_j a_{ij} = 1$ for all rows i, H_{loc} becomes

$$H_{loc}^{row} = -S_0\sum_i\left(\sum_j a_{ij}\right)\hat{S}_i - S_0\sum_i \hat{S}_i, \tag{8}$$

where the first term is a local field varying for each $\hat{S}_i$, and the second term is a homogeneous external field independent of a_{ij}. The third term in the expansion of H is independent of the values of $\hat{S}$ and is a constant depending on a_{ij} only. Hence, it does not play a role in the minimization of H. For a connected network with row-normalization, the last expression can be further simplified to

$$H_{loc}^{row} = -2S_0\sum_i \hat{S}_i. \tag{9}$$

The asymmetry in S is then equivalent to a symmetric system with an external field $2S_0$. In another paper, we simulate the effect of different S_0 values in different network clusters on the tipping point (Bruggeman & Sprik, 2020).

2.2. *Mean-field analysis*

The expected value of M as a function of T can be obtained by assuming that the network is very large and by abstracting away from its topology; in the language of thermodynamics, by approximating the interaction energy by the energy of one spin (here, behavior) in the mean-field of its neighbors (Barrat et al., 2008), $M = \langle S\rangle$. The value of M can now be expressed in closed form in terms of the probabilities given by the exponential of the Hamiltonian energy and T as

$$M = \frac{(S_0-\Delta)e^{\frac{(S_0-\Delta)M}{T}} + (S_0+\Delta)e^{\frac{(S_0+\Delta)M}{T}}}{e^{\frac{(S_0-\Delta)M}{T}} + e^{\frac{(S_0+\Delta)M}{T}}}. \tag{10}$$

This reduces to an implicit relation,

$$\frac{M}{\Delta} = \frac{S_0}{\Delta} + \tanh\left(\frac{\Delta^2}{T}\frac{M}{\Delta}\right), \tag{11}$$

where only dimensionless ratios of M, S_0 and T with Δ remain in the expression. The mean degree $\langle k\rangle$, defined for binary ties, does not occur in it because the adjacency matrix is row-normalized and the mean weighted outdegree $\langle k_w\rangle = 1$.

By analyzing the intersection of the line defined by $M/\Delta - S_0/\Delta$ and the tanh term on the right-hand side of Eq. 11, the possible values for M at a given T can be found. For $T > T_c$ there is one stable high T solution and for $T < T_c$ there is one stable solution of (nearly) full cooperation, another solution that is stable in finite time with (nearly) full defection, and one unstable solution. At $T = T_c$

the two stable solutions merge and the intersecting line coincides with the tangent line touching the tanh function; see Figure 1(b). At that point, a closed relation for T_c in terms of S_0 and Δ can be found,

$$\frac{S_0}{\Delta} = \frac{\sqrt{\frac{\gamma-1}{\gamma+1}}(\gamma+1)\gamma - \cosh^{-1}(\gamma)}{\gamma^2}, \tag{12}$$

where $\gamma = \sqrt{\frac{\Delta^2}{T_c}}$. Eq. 12 is used in Figure 3(a). It shows that if S_0 increases while keeping Δ constant, less agitation is required to motivate defectors to cooperate. When Δ decreases to $\Delta = S_0$, defection loses its appeal. The figure also shows that numerical simulations yield very similar results for large networks but diverge for small ones. This also holds true for T_c in Figure 1, which is lower for smaller networks (not shown).

From the mean-field approximation follows the proportion of defectors p_c and cooperators $1 - p_c$ at given S_0 and pertaining T_c, after a time long enough for the system to settle down. The proportion of cooperators $1 - p_c$ at T_c is the critical mass (Marwell & Oliver, 1993), and can be inferred from the value of M_c at the phase transition,

$$M_c = p_c(S_0 - \Delta) + (1 - p_c)(S_0 + \Delta). \tag{13}$$

The mean-field analysis of T_c yields

$$\frac{M_c}{\Delta} = \frac{-\cosh^{-1}(\gamma)}{\gamma^2}. \tag{14}$$

Note that the $\cosh^{-1}$ function in Eq. 14 only yields a result when $\gamma > 1$, and sets a limit to T_c for given Δ. For the choice $\Delta = 0.75$, the maximum value of $T_c = \Delta^2 = 0.565$. Solving for p_c yields

$$p_c = \frac{1}{2} - \frac{S_0}{2\Delta} + \frac{1}{2}\frac{\cosh^{-1}(\gamma)}{\gamma^2}, \tag{15}$$

used for Figure 3(b). It shows that the proportion of defectors p_c at T_c decreases with increasing S_0. In contrast to critical mass theory, however, the Ising model has no assumptions about initiative takers or leaders who win over the rest, rational decision-making (Marwell & Oliver, 1993), or learning that would require fairly stable feedback (Macy, 1991).

3. Discussion and conclusion

We have shown that under high uncertainty, the dilemma of collective action can be solved by nothing more than a portion of random noise. The asymmetric Ising model does not require any knowledge or accurate expectations of the participants, and only depends on conformism, which can be empirically observed in synchronous motion, gestures or shouting (McNeill, 1995; Jones, 2013). In particular, it has no assumptions about actors' rationality, in contrast to critical mass theory, whereas it supports that theory's key findings of the critical mass and the tipping point. Simulations add to our mean-field result that turmoil-driven cooperation is most likely in small groups, where cooperation starts at relatively low levels of noise.

Shortly before we finished this manuscript, two other papers appeared where an Ising model was used to solve this dilemma (Adami & Hintze, 2018; Sarkar & Benjamin, 2019), but their symmetric model requires complex quantum physics to define payoffs, in contrast to our simple definition. Along with empirical testing, perhaps also on other species, a future direction might be to explicitly model noisy information transmission on the group's network (Quax, Apolloni, & Sloot, 2013), too.

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ORCID

Jeroen Bruggeman http://orcid.org/0000-0002-0422-1842
Rudolf Sprik http://orcid.org/0000-0002-1573-1071
Rick Quax http://orcid.org/0000-0002-0299-0074

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