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Reconsidering the 'Ingredients' of Explicit Knowledge*

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Abstract

This paper reviews alternative formal definitions of the concept of *explicit knowledge*, with the goal of unravelling the central notions that different logical approaches employ. The meanings of these notions are classified, explaining how their differences might change the logical outcome. Then, the paper proposes an abstract framework where the most suitable notions come together, with the aim of shedding some light on their underlying theoretical foundations, and also of clarifying their relationship.
Keywords: Knowledge, Awareness, Justification, Actions.

1 Introduction

It is well-known that, in classic epistemic logic (*EL* from now on) (Hintikka 1962), the agents' knowledge is closed under logical consequence. This property, useful in some applications, is nevertheless an unrealistic assumption when modelling 'real' agents;[1] after all, the purpose of disciplines as Mathematics and Computer Science is to fill in the logical consequences of what we already know. One of the most prominent ideas for 'solving' this logical omniscience problem has been to acknowledge that there are different notions of knowledge (or, more precisely, there are different logical accounts of the notion of *information*; van Benthem and Martínez 2008). From this perspective, *"the K operator really just describes implicit semantic information of the agent, which definitely has the preceding closure property. The point is rather that closure need not hold for a related, but different intuitive notion [of] explicit [...] knowledge [...], in some suitable sense to be defined"* (van Benthem and Velázquez-Quesada 2010, Page 6).

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[1]Or even computational ones, which might lack the required resources (space, time).

Although this notion of *explicit knowledge* is intuitively clear, different proposals have provided different definitions. For example, while some authors take it as a primitive notion (e.g., Levesque 1984), others define it as the implicit knowledge the agent is *aware of* (Fagin and Halpern 1988), some others define it in terms of a notion of *awareness that* (Velázquez-Quesada 2014), and further ones require an appropriate *justification* (Artëmov and Kuznets 2009). All in all, in different approaches, the concept of *explicit knowledge* is built-up from different 'ingredients'.

This paper starts by reviewing some of the most prominent proposals discussing explicit knowledge (namely, the ones concerning the concepts of *awareness of* and *awareness that*, with the latter related to that of *justification*), identifying and debating the involved notions (Section 2). Then, it proposes an abstract framework where those concepts considered to be most suitable come together. By creating a common setting for the discussed approaches, the proposal sheds some light on the theoretical foundations that underlie them, and thus clarifies their relationship (Section 3). Moreover, it also brings to light the different *epistemic actions* that are considered crucial in each case (Section 4).[2]

2 Main epistemic concepts: the ingredients

An informal and intuitive approach to the notion of "knowledge" will automatically give rise to what is called "explicit knowledge" in formal *EL* developments. "Explicit" will then always refer to the information the agent actually has and is able to access and perform decisions with. In this sense "explicit" stands here for "real" or "actual".

The background intuition that supports the introduction of this notion is the so called "problem of logical omniscience" which therefore involves the concept of "agents with limited reasoning abilities". Here arises the need for modelling a type of knowledge that is not idealized and is applicable to those real agents (represented by human beings and computing machines).

This concept is different from the idealized knowledge that can be found in standard *EL*; which may be better called "implicit". Hence, "implicit knowledge" will always refer to some idealization of knowledge (and correspond to a logical construct).

The distinction between implicit and explicit information is not new. We find one of the first proposals in Levesque (1984); and another relevant one, in Fagin and Halpern (1988), that includes the agent's awareness in their system. We will now dig a bit deeper into these two proposals concerning explicit knowledge in order to find some of the mentioned 'ingredients'.

Deductive system by Konolige (1986) and Levesque (1984): in this approach (Figure 1), explicit knowledge is defined as the primitive knowledge the agent has. Implicit knowledge is then what follows (deductively) from what is explicitly known. Such interpretation assumes that the only action available to the agent is *deductive inference*. It is precisely the possibility of performing this inference what creates the implicit knowledge set.

[2]For space reasons, this proposal only discusses the concepts involved in the definition of explicit knowledge (as well as their relationship). A formal counterpart, proposing a semantic structure and a formal language for representing all of them, is left for future work.

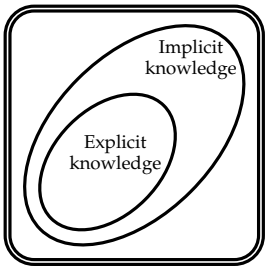


Figure 1: Implicit knowledge and explicit knowledge in Konolige (1984) (diagram from Konolige 1986).

In the diagram we have a rectangle representing all true propositions and two ellipses representing knowledge. The small one represents those propositions the agent really knows; the large one refers to those propositions she can get to know by deduction.

Awareness Logic by Fagin and Halpern (1988): in their initial proposal of "Awareness Logic" (*AL* from now on), explicit knowledge is defined as the implicit knowledge the agent is aware of. The agent's awareness represents the information she entertains, irrespectively of its condition or the agent's inclination towards it. Note also that what is here called "implicit knowledge" is not what follows from the agent's explicit knowledge, as it was in the previous approach, but rather what the agent would know explicitly if she were aware of every formula that is true in all her epistemic possibilities. The authors take the implicit knowledge from standard *EL* as their starting point. By adding the agent's awareness, which acts as a filter, they obtain explicit knowledge.[3]

In Figure 2 below, we see how implicit knowledge intersects with awareness and gives rise to explicit knowledge.

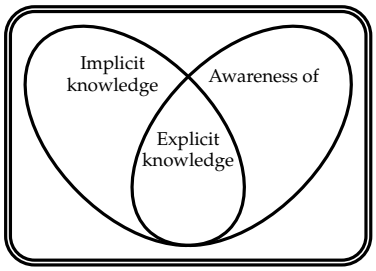


Figure 2: Implicit knowledge, awareness and explicit knowledge in Fagin and Halpern (1988) (diagram from Konolige 1986).

The concept of "awareness" deserves special mention, since it is a very polysemic term. The authors are (and we will be) always using "awareness" in an epistemic sense, as opposed to a moral sense. But there is still the need for specifying the different uses that the literature on *AL*, and its developments, have made of the term. For doing so, we want to allude to a very useful distinction: *awareness of* vs. *awareness that*.

In Dretske (1993), the author distinguishes "awareness of things" (aware *of* *X*) from "awareness of facts" (aware *that* *X* is the case). We could say that, for

[3] A similar strategy for defining explicit knowledge is used in epistemic justification logic (Artemov 2008, Renne 2012), where explicit beliefs are defined as those implicit beliefs the agent has a justification for.

the author, awareness of facts stands for the awareness of those thoughts (or believed propositions) we can form and ascribe a truth value to; awareness of things would be somehow the very fact of perceiving and forming our mental content, irrespectively of its truth.

Although the difference he proposes stems from a philosophical (and not a logical) point of view, we find it very clarifying. Applying this distinction will result in different interpretations of the awareness operator. In addition, it shows different views of its connection to the implicit knowledge operator when defining explicit knowledge.

Keeping this in mind, we can establish that the awareness Fagin and Halpern are dealing with in their proposal corresponds to Dretske's awareness of things, since they emphasize the fact that awareness can include any type of information. In fact, the two primitive concepts in *AL*, implicit knowledge and awareness, are completely independent of each other.

We could also conclude that in *AL*, explicit knowledge is in some sense analogous to Dretske's awareness of facts, since it corresponds exactly to that part of awareness that intersects with implicit knowledge, and hence is formed only by true propositions the agent is aware of.

Other interpretations of explicit knowledge: different philosophical and logical approaches define what explicit knowledge is and how it can be modelled. But to conclude this part, we want to shed light on one specific sense that can be found in the seminal work of *EL*: Hintikka (1962).

Sillari (2008) calls our attention to an interesting distinction that Hintikka makes in his work: the difference between a weak and a strong sense of "knowing". On the one hand, knowing something *weakly* refers to the fact of entertaining this information and knowing it to be true (analogous to the intuitive sense of explicit knowledge). On the other hand, the *strong* sense of knowing requires not only to be informed about it, but also to have a justification for it.

It is this strong sense of knowing that awakens our interest from both a theoretical and a logical perspective (the weak sense will not be discussed here). We have mentioned that explicit knowledge corresponds to the intuitive sense of "knowing". We could then stretch this informal reasoning and say that for an agent to really explicitly know φ she needs not only to be informed about its truth and be aware of it, but also be able to provide a justification (understanding "justification" as an answer to 'why does she know what she knows?').

This more o less intuitive understanding of real explicit knowledge will be the core of our proposal in the next section. While providing a redefinition of explicit knowledge, we will also classify the other ingredients that have been mentioned above and establish a conceptual framework that allows us to introduce the dynamic actions that transform information.

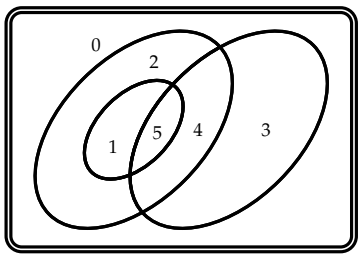
3 Combining the ingredients

The approaches whose main ideas are depicted in the diagrams of Figures 1 and 2 are not necessarily in conflict with each other. They do look at the logical omniscience problem from different perspectives: one considering agents that

might not have 'instantaneous' reasoning abilities (Konolige 1984), and another considering agents that might not be fully attentive (Fagin and Halpern 1988). Still, one can combine both ideas in order to create a coherent picture in which there might be different reasons for an agent to not have at her immediate disposal all information she might be able to get.

For this, we take the diagram in Figure 1, which simply distinguishes between the agent's explicit knowledge and its logical consequences (implicit knowledge), as our starting point.[4] Then, we can make a further distinction by bringing awareness into the picture. In doing so, the new ellipse representing awareness will overlap the previous two areas creating two new divisions: the explicit and implicit knowledge the agent is aware of.

In our view, awareness acts as a flashlight that 'illuminates' certain area of the agent's information, making it readily available (reachable) in the sense that the agent can talk about it.[5]



0.- Information
1.- Explicit not in working memory (**K**)
2.- Implicit not in working memory ($\Box$)
3.- Awareness of (**A**)
4.- Reachable aware knowledge ($\Box^{\mathbf{A}}$)
5.- Real explicit knowledge ($\mathbf{K}^{\mathbf{A}}$)

Figure 3: Combined proposal of implicit knowledge, explicit knowledge and awareness.

The diagram depicted above captures the ingredients that have been highlighted in the previous section; it also shows the different types of information a more real agent may have. Let us now devote some words to each one of those zones.

0 - Information: this area corresponds to those propositions outside the agent's current focus. From a theoretical point of view, we could interpret information in a wide sense, standing not only for propositions but also for beliefs, doubts, procedures, etc. But from a formal point of view, we will have to stick only to propositions, as it is done in most logical systems.

1 - Explicit not in working memory: in Figure 1, this area (together with 5) corresponds to explicit knowledge. In our diagram, "explicit" stands for the information the agent really knows or has known. Since we distinguish now between what is or not in her working memory (awareness), in this zone 1 we find only those propositions the agent knows for sure, but does not currently have in her working memory. These propositions may be reached by the action of *becoming aware*.

[4] An analysis similar to the one that follows can also be performed by starting from the diagram in Figure 2 and splitting its *implicit knowledge* area into two, distinguishing in this way what the agent has already derived and what requires further deductive reasoning.

[5] Note: this does not mean that the agent gets to know all the information in the illuminated area; it only makes it part of the current 'topic of conversation', so now the agent can use it (if she happens to know whether it is true or not) or wonder about its truth-value (otherwise).

2 - Implicit not in working memory: this zone (together with 4) corresponds to what was called "implicit knowledge" in Figure 1. In this setting, implicit refers to those propositions the agent could reach by deduction (action of *deductive inference*), after becoming aware of some other proposition that permits this inference.

3 - Awareness: the complete ellipse formed by areas 3, 4 and 5 would correspond to the awareness depicted in Figure 2. This part numbered with 3 stands for what we called "awareness of". Here we find everything the agent is currently considering or the questions she is thinking over, but whose truth values cannot be reached by the action of *deductive inference*.

4 - Reachable aware knowledge: this area corresponds to those propositions the agent is already *aware of*, but does not know "really explicitly", since she has not performed the needed inference steps that can provide her with a *justification*. It is a reachable or potential knowledge, in the sense that it is only one deductive step away from being really explicit, that is, from belonging to zone 5.

5 - Real explicit knowledge: lastly, we arrive at the part that has more properties attached to it: the *real explicit knowledge*. As advanced in previous sections, what we refer to here is what Hintikka called "strongly knowing". For the agent to really explicitly know something, she needs to be *aware that* the proposition is true (and therefore aware of) and have a *justification* for it. This justification may come from a deductive inference performed with information she already knew explicitly in this real sense, or by the action of observation, meaning, she was informed about it from an infallible source.

The fact that our real explicit knowledge calls for a justification relies on the intutive claim that 'there is no unjustified knowledge'. But then, this requires a wide-ranged definition of justification that includes not only 'deductive proofs' but also observations. In a more fine-grained approach, we could classify the justifications, and hence the resulting explicit knowledge, depending on the dynamic action(s) that provided the justification.

In the next section, we will discuss some of the dynamic actions that the agent can perform to move information from one zone to another. These actions will highlight not only the dynamics of information, but also the theoretical benefits we obtain when employing this combined diagram.

4 Epistemic actions

The discussion so far has rotated around the 'ingredients' of explicit knowledge: for the agent to really explicitly know a given φ, she must be aware of it and she must have some form of justification for it. This way, we can distinguish between explicit knowledge, what the agent actually has, and different forms of 'obtainable' knowledge, what she might eventually get.

While recognising the different ingredients is useful to create a hierarchy of epistemic notions (some of them without idealised closure properties), doing so also highlights the fact that a given 'piece of information' can be moved across the different hierarchy layers by 'adding' or 'removing' the adequate

ingredient. In other words, the agent can move a piece of information across different zones by performing the appropriate *epistemic action*.[6]

Recognising that epistemic actions are involved is crucial. First, providing a list of extra requirements for something to be called explicit knowledge might make the agent non-omniscient, but providing the epistemic actions that allow her to fulfil such requirements guarantees that she will not be defective or ignorant (in other words, she will be *rational*; cf. Duc 1997). Second, introducing the actions that lead to 'omniscient' states may demystify them. In Conan Doyle's detective stories, the explanation offered at the end turns Holmes' 'magical powers' into a sequence of observations and deductive acts, making the whole procedure "elementary, my dear Watson".

The diagrams in Figures 1 and 2 already hint each at an epistemic action that captures the crucial idea behind the respective proposal (*deductive inference* for the first; *becoming aware* for the second). Both actions still make sense in this proposal's framework, as the following paragraphs describe. Still, its diagram (Figure 3) also allows a systematic analysis that reveals other epistemic actions which are also meaningful in this more general setting.

Deductive inference: this is the crucial action in Konolige (1984) (Figure 1), where implicit knowledge becomes explicit after an act of deductive inference. In our proposal (Figure 3), deductive inference is the action that allows pieces of information to move from region 4 ($\Box^{\mathbf{A}}$: the information the agent is aware of and that follows deductively from what she really explicitly knows) to region 5 ($\mathbf{K}^{\mathbf{A}}$: the information she really explicitly knows).[7]

Becoming aware: this is the crucial action that arises when considering *AL* (Figure 2), where explicit knowledge is the implicit knowledge the agent is aware of. In our proposal (Figure 3), becoming aware stands for different transitions. First, the agent can become *aware of* certain piece of information about which she did not know anything before. This, which can be understood as becoming aware of a brand new possibility, corresponds to a transition from region 0 to region 3 ($\mathbf{A}$). But the agent can also become aware of possibilities she does not know anything about, but whose truth can be inferred from her real explicit knowledge; this corresponds to a transition from region 2 ($\Box$) to region 4 ($\Box^{\mathbf{A}}$). Finally, she can also become aware of information she already recognised as true, but did not have 'in her working memory' at the time (e.g., a modal logician who, while watching a football match, is questioned about the finite model property of modal logic); this corresponds to a transition from region 1 ($\mathbf{K}$) to region 5 ($\mathbf{K}^{\mathbf{A}}$).[8]

Observation: this action, reflecting intuitively the act of receiving external information, is already meaningful in the individual settings of both discussed papers.[9] It is nowadays called a *public announcement* but, in a single-agent setting, it can be better understood as an act of individual *observation*. By means of it, the agent can turn really explicit (i.e., take to region 5, $\mathbf{K}^{\mathbf{A}}$) not

[6] From this perspective, the different forms of 'obtainable knowledge' can be classified in terms of *the actions* the agent needs to perform in order to make the given piece of information really explicit.

[7] Formal epistemic-logic accounts of such action can be found in Duc (1997), Ågotnes and Alechina (2007), Jago (2009), Velázquez-Quesada (2013).

[8] Acts of *becoming aware* have been formally represented and studied in, e.g., Hill (2010), van Benthem and Velázquez-Quesada (2010), van Ditmarsch and French (2011).

[9] It was first formally discussed in Plaza (1989), Gerbrandy and Groeneveld (1997).

only those pieces of information she entertained but did not know anything about (region 3, $\mathbf{A}$), but also those she did not entertain and could not have deduced on her own (region 0), and even those she was unaware of but still could have reached by means of deductive inference (region 2, $\Box$).[10] In fact, an act of observation can move a piece of information from *any* region to region 5 ($\mathbf{K^A}$).[11]

Further epistemic actions: of course, not all epistemic actions need to lead to a more optimal informational state, and the diagram also makes this clear. Just as one can find actions describing transitions that go 'to the center' of the diagram, it is also possible to find actions that go 'to the corners'. For example, the agent will drop certain piece of information from her 'working memory' if she *becomes unaware* of it.[12] Depending on what further requirements this piece satisfied, this corresponds to either a transition from region 5 ($\mathbf{K^A}$) to region 1 ($\mathbf{K}$), one from 4 ($\Box\mathbf{^A}$) to 2 ($\Box$), or one from 3 ($\mathbf{A}$) to 0. Another relevant epistemic action is that of *forgetting*[13] which, depending on how the agent acquired the to-be-forgotten information, might take it from region 5 ($\mathbf{K^A}$) to region 4 ($\Box\mathbf{^A}$) (she forgets her sister's age, but she can still infer it from her knowledge of her birthday's year), or from region 5 ($\mathbf{K^A}$) to region 3 ($\mathbf{A}$) (she forgets somebody's age, and she does not have enough information to deduce it).

Finally, note how some actions might have 'side effects'. For example, in an appropriate initial state, an observation of $p \wedge q$ can move this formula from region 0 to region 5 ($\mathbf{K^A}$). But this action makes the agent aware of both p and q, and therefore makes any of those formulas 'reachable' via deductive inference (they go from region 0 to region 4 –$\Box\mathbf{^A}$–).

5 Summary and further work

This paper deals with the concept of *explicit knowledge*, commonly used when dealing with non-ideal agents, and corresponding intuitively to what the agent 'currently has'. In its first part it recalls two general strategies for defining this concept, unravelling and exploring the central notions each one employs, and recalling briefly related proposals. In essence, while the first takes explicit knowledge as the *primitive* notion, defining then implicit knowledge as the logical consequences of its explicit counterpart, the second *defines* explicit knowledge as the implicit knowledge the agent is aware of. Therefore, while the first avoids logical omniscience by limiting the agent's (deductive) inferential abilities, the second does it by limiting the concepts she entertains at a given time (i.e., by limiting her current language). This shows how, even though both strategies deal with the same intuitive concept, they follow 'orthogonal' directions, producing then different kinds of agents. Indeed, while agents of the first type have a full language (awareness) but limited reasoning abilities, agents of the second have full inferential abilities within their restricted language (awareness).

[10] Note how the last two possibilities highlight the fact that, by observing a given φ (and thus getting to know that it is the case), the agent also becomes aware of it.

[11] Thus, for example, a 'lazy' agent might know really explicitly both p and $p \to q$, and yet get to know q not by inferring it but rather by observing it.

[12] See, e.g., van Benthem and Velázquez-Quesada (2010), van Ditmarsch et al. (2012).

[13] See, e.g., van Ditmarsch et al. (2009), Fernández-Duque et al. (2015).

In its second part, the paper merges the key ideas behind the discussed frameworks. The resulting setting allows for finer epistemic concepts, as it combines the two 'ways' in which the agent might 'miss' something. Moreover; by doing it, it also highlights the different epistemic *actions* that are involved. Indeed, the combined setting distinguishes not only between the 'truths' the agent has acknowledged and are currently in her 'working memory' ($\mathbf{K}^{\mathbf{A}}$; region 5) and those she has acknowledged but are 'out of the topic of conversation' ($\mathbf{K}$; region 1), but also between those she does not know but might get to know by means of deductive inference ($\Box^{\mathbf{A}}$; region 4) and those she does not know but might get to know by means of a raise in awareness *and then* a deductive inference ($\Box$; region 2). Finally, it also distinguishes between those truths the agent cannot reach by deductive inference but are still currently entertained ($\mathbf{A}$; region 3) and those that are out of deductive reach and also not being currently 'discussed' (region 0). Thus, on the one hand, the setting has a single notion of 'real' explicit knowledge, corresponding to what the agent has 'in her hand' at the given moment and thus does not require any further epistemic action to be available. On the other hand, it has several notions of obtainable knowledge, each one of them corresponding roughly to the different *sequence of actions* that are needed to turn them explicit.

This paper's goal has been to clarify the concept of explicit knowledge, and the presented discussion of different frameworks dealing with it is a first step. Still, there is further work to be done. One of the most appealing task is to propose a formal framework in which all these epistemic notions have a place. An already explored possibility is the use of the awareness logic setting to represent both the agent's language and what she has acknowledged as true (Grossi and Velázquez-Quesada 2015); this uses implicit knowledge, awareness and 'acknowledgement' as the primitive concepts, defining the rest in terms of them, and follows the *dynamic epistemic logic* framework (van Ditmarsch et al. 2008, van Benthem 2011) for dealing with the involved actions. Still, there are other possibilities. For example, one can use *neighbourhood models* (Scott 1970, Montague 1970), under which the primitive concept would be that of explicit knowledge, with its implicit counterpart being definable as the fixed point of a 'deductive closure' operation (Velázquez-Quesada 2013). One can also follow different directions for modelling awareness: the syntactic approach of Fagin and Halpern (1988) is one possibility, but one can also use semantic tools, as the relational approach used for representing a notion of 'issues being discussed' in logics for questions (van Benthem and Minică 2012, Baltag et al. 2017).

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