



## UvA-DARE (Digital Academic Repository)

### Hopf Symmetry and its breaking; Braid Statistics and Confinement in Planar Physics

Slingerland, J.K.

**Publication date**  
2002

[Link to publication](#)

**Citation for published version (APA):**

Slingerland, J. K. (2002). *Hopf Symmetry and its breaking; Braid Statistics and Confinement in Planar Physics*.

**General rights**

It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

**Disclaimer/Complaints regulations**

If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: <https://uba.uva.nl/en/contact>, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.

# Bibliography

- [1] K. von Klitzing, G. Dorda, and M. Pepper. New method for high-accuracy determination of the fine-structure constant based on quantized Hall resistance. *Phys. Rev. Lett.*, 45:494–497, 1980.
- [2] D. C. Tsui, H. L. Stormer, and A. C. Gossard. Two-dimensional magnetotransport in the extreme quantum limit. *Phys. Rev. Lett.*, 48:1559–1562, 1982.
- [3] J.P. Eisenstein and H.L. Stormer. The fractional quantum Hall effect. *Science*, 248(4962):1510–1516, 1990.
- [4] S. Montgomery. *Hopf algebras and their actions on rings*. Number 82 in CBMS regional conference series in mathematics. published for the CBMS by the AMS, 1993.
- [5] E. Abe. *Hopf Algebras*. Cambridge University Press, Cambridge, 1980. original Japanese version published by Iwanami Shoten, Tokyo, 1977.
- [6] M.E. Sweedler. *Hopf Algebras*. Benjamin, New York, 1969.
- [7] V. Chari and A. Pressley. *A guide to quantum groups*. Cambridge University Press, 1994.
- [8] S. Majid. *Foundations of quantum group theory*. Cambridge University Press, 1995.
- [9] J. M. Leinaas and J. Myrheim. On the theory of identical particles. *Nuovo Cim.*, B37:1–23, 1977.
- [10] A. N. Kirillov. Clebsch-Gordan quantum coefficients. *Zap. Nauch. semin. LOMI*, 9:67–84, 1988. English translation in *J. Soviet Math.* 53 (1991), nr. 3, 264–276.
- [11] L.L. Vaksman.  $q$ -analogues of Clebsch-Gordan coefficients, and the algebra of functions on the quantum group  $su(2)$ . *Dokl. Akad. Nauk. SSSR*, 306(2):269–271, 1989. English translation in *Soviet Math. Dokl.* 39 (1989), no. 3, 467–470.
- [12] A. N. Kirillov and N. Y. Reshetikhin. Representations of the algebra  $U_q(sl(2))$ ,  $q$ -orthogonal polynomials and invariants of links. In V. G. Kac, editor, *Infinite dimensional Lie algebras and groups, Proceedings of the conference held at CIRM, Luminy, Marseille*, pages 285–339, Singapore, 1988. World Scientific.
- [13] F. A. Bais and N. M. Muller. Topological field theory and the quantum double of  $SU(2)$ . *Nucl. Phys.*, B530:349–400, 1998. hep-th/9804130.
- [14] F. A. Bais, N. M. Muller, and B. J. Schroers. Quantum group symmetry and particle scattering in  $(2+1)$ -dimensional quantum gravity, 2002. hep-th/0205021.
- [15] F. Di Francesco, P. Mathieu, and D. Sénéchal. *Conformal field theory*. Springer, 1997.
- [16] N. Read and E. Rezayi. Beyond paired quantum Hall states: Parafermions and incompressible states in the first excited Landau level. *Phys. Rev.*, B59(12):8084–8092, 1999. cond-mat/9809384.
- [17] C. Nayak and F. Wilczek.  $2n$  quasiholes realize  $2^n-1$ -dimensional spinor braiding statistics in paired quantum Hall states. *Nucl. Phys.*, B479:529–553, 1996. cond-mat/9605145.
- [18] G. Moore and N. Read. Nonabelions in the fractional quantum Hall effect. *Nucl. Phys.*, B360:362–396, 1991.
- [19] E. Ardonne and K. Schoutens. A new class of non-Abelian spin-singlet quantum Hall states. *Phys. Rev. Lett.*, 82(25):5096–5099, 1999. cond-mat/9811352.

- [20] E. Ardonne, F.J.M. van Lankvelt, A.W.W. Ludwig, and K. Schoutens. Separation of spin and charge in paired spin-singlet quantum Hall states. *Phys. Rev. B*, 65:041305(R), 2002. 0102072.
- [21] M. Greiter, X. G. Wen, and F. Wilczek. Paired Hall state at half filling. *Phys. Rev. Lett.*, 66(24):3205–3208, 1991.
- [22] M. Greiter, X. G. Wen, and F. Wilczek. On paired Hall states. *Nucl. Phys.*, B374:567, 1992.
- [23] R.L. Willet, J.P. Eisenstein, Störmer H. L., Tsui. D. C., A. C. Gossard, and J. H. English. Observation of an even-denominator quantum number in the fractional quantum Hall effect. *Phys. Rev. Lett.*, 59, 1987.
- [24] W. Pan, J.-S. Xia, V. Shvarts, E.A. Adams, H.L. Stormer, D.C. Tsui, L.N. Pfeiffer, K.W. Baldwin, and K.W. West. Exact quantization of the even-denominator fractional quantum hall state at  $\nu = \frac{5}{2}$  Landau level filling factor. *Phys. Rev. B*, 83(17):3530–3533, 1999. cond-mat/9907356.
- [25] W. Pan, E.A., H.L. Stormer, D.C. Tsui, L.N. Pfeiffer, K.W. Baldwin, and K.W. West. Experimental evidence for a spin-polarized ground state in the  $\nu = \frac{5}{2}$  fractional quantum hall effect. *Solid State Communications*, 119:641–645, 2001. cond-mat/0103144.
- [26] R.H. Morf. Transition from quantum Hall to compressible states in the second Landau level: new light on the  $\nu = \frac{5}{2}$  enigma. *Phys. Rev. Lett.*, 80:1505, 1998. cond-mat/9809024.
- [27] F.D.M. Haldane and E. H. Rezayi. Incompressible paired Hall state, stripe order, and the composite fermion liquid phase in half-filled Landau levels. *Phys. Rev. Lett.*, 84(20):4685–4688, 2000. cond-mat/9906137.
- [28] N.K. Wilkin, J.M.F. Gunn, and R.A. Smith. Do attractive bosons condense? *Phys. Rev. Lett.*, 80(11):2265–2268, 1998. cond-mat/9705050.
- [29] N.K. Wilkin and J.M.F. Gunn. Condensation of ‘composite bosons’ in a rotating BEC. *Phys. Rev. Lett.*, 84(1):6–9, 2000. cond-mat/9906282.
- [30] N.R. Cooper, N.K. Wilkin, and J.M.F. Gunn. Quantum phases of vortices in rotating Bose-Einstein condensates. *Phys. Rev. Lett.*, 87:120405, 2001. cond-mat/0107005.
- [31] V. Gurarie and E. Rezayi. Parafermion statistics and quasihole excitations for the generalizations of the paired quantum Hall states. *Phys. Rev.*, B61(8):5473–5482, 2000. cond-mat/9812288.
- [32] E. Ardonne, N. Read, E. Rezayi, and K. Schoutens. Non-Abelian spin-singlet quantum Hall states: wave functions and quasihole state counting. *Nucl. Phys.*, B607:549–576, 2001. cond-mat/0104250.
- [33] T.L. Ho. Broken symmetry of two-component  $\nu = \frac{1}{2}$  quantum hall states. *Phys. Rev. Lett.*, 75:1186–1189, 1995.
- [34] A. Cappelli, L.S. Georgiev, and I.T. Todorov. Parafermion Hall states from coset projections of Abelian conformal theories. *Nucl. Phys.*, B599:499–530, 2001. hep-th/0009229.
- [35] D.C. Cabra, A. Lopez, and G.L. Rossini. Transition from Abelian to non-Abelian quantum Hall states. *Eur. Phys. J.*, B19:21–24, 2001. cond-mat/0006328.
- [36] J. K. Slingerland and F. A. Bais. Quantum groups and nonabelian braiding in quantum Hall systems. *Nucl. Phys.*, B612:229–290, 2001. cond-mat/0104035.
- [37] R.E. Prange and S.M. Girvin, editors. *The quantum Hall effect*. Graduate texts in contemporary physics. Springer-Verlag, New York, 1987.
- [38] S. Das Sarma and A. Piczuk, editors. *Perspectives in quantum Hall effects, novel quantum liquids in low-dimensional semiconductor structures*. John Wiley and Sons, New York, 1997.
- [39] Z.F. Ezawa, editor. *Quantum Hall effects, field theoretical approach and related topics*. World Scientific, Singapore, 2000.
- [40] L.D. Landau and E.M. Lifshitz. *Quantum mechanics, non-relativistic theory*, volume 3 of *Course of theoretical physics*. Pergamon Press, London-Paris, 1958.

- [41] R. B. Laughlin. Quantized Hall conductivity in two dimensions. *Phys. Rev.*, B23:5632–5733, 1981.
- [42] B. I. Halperin. Quantized Hall conductance, current carrying edge states, and the existence of extended states in a two-dimensional disordered potential. *Phys. Rev.*, B25:2185–2190, 1982.
- [43] R. B. Laughlin. Anomalous quantum Hall effect: An incompressible quantum fluid with fractionally charged excitations. *Phys. Rev. Lett.*, 50:1395, 1983.
- [44] L. Saminadayar, D. C. Glattli, Y. Jin, and B. Etienne. Observation of the  $e/3$  fractionally charged Laughlin quasiparticle. *Phys. Rev. Lett.*, 79:2526–2529, 1997. cond-mat/9706307.
- [45] R. de Picciotto, M. Reznikov, M. Heiblum, V. Umansky, G. Bunin, and D. Mahaluand. Direct observation of a fractional charge. *Nature*, 389:162–164, 1997. cond-mat/9707289.
- [46] M. Reznikov, R. de Picciotto, T. G. Griffiths, M. Heiblum, and V. Umansky. Observation of quasiparticles with one-fifth of an electron's charge. *Nature*, 399:238–241, 1999. cond-mat/9901150.
- [47] D. Arovas, J. R. Schrieffer, and F. Wilczek. Fractional statistics and the quantum Hall effect. *Phys. Rev. Lett.*, 53:722–723, 1984.
- [48] J. K. Jain. Composite fermion approach for the fractional quantum Hall effect. *Phys. Rev. Lett.*, 63:199–202, 1989.
- [49] F. D. M. Haldane. Fractional quantization of the Hall effect: A hierarchy of incompressible quantum fluid states. *Phys. Rev. Lett.*, 51:605–608, 1983.
- [50] B. I. Halperin. Statistics of quasiparticles and the hierarchy of fractional quantized Hall states. *Phys. Rev. Lett.*, 52:1583–1586, 1984.
- [51] S. C. Zhang, T. H. Hansson, and S. Kivelson. An effective field theory model for the fractional quantum Hall effect. *Phys. Rev. Lett.*, 62:82–85, 1988.
- [52] A. Lopez and E. Fradkin. Fractional quantum Hall-effect and Chern-Simons gauge theories. *Phys. Rev.*, B44(10):5246–5262, 1991.
- [53] J. P. Eisenstein, K. B. Cooper, L. N. Pfeiffer, and K. W. West. Insulating and fractional quantum Hall states in the first excited Landau level. *Phys. Rev. Lett.*, 88(7):076801, 2002. cond-mat/0110477.
- [54] J. Frohlich, B. Pedrini, C. Schweigert, and J. Walcher. Universality in quantum Hall systems: Coset construction of incompressible states. *J. Stat. Phys.*, 103(3-4):527–567, 2000. cond-mat/0002330.
- [55] E. Witten. Quantum field theory and the Jones polynomial. *Commun. Math. Phys.*, 121:351, 1989.
- [56] Gurarie V. and C. Nayak. A plasma analogy and Berry matrices for non-Abelian quantum Hall states. *Nucl. Phys. B*, 506(3):685–694, 1997. cond-mat/9706227.
- [57] A.B. Zamolodchikov and V.A. Fateev. Nonlocal (parafermion) currents in two-dimensional conformal quantum field theory and self-dual critical points in  $\mathbb{Z}_n$ -symmetric statistical systems. *Sov. Phys. JETP*, 62(2):215–225, 1985.
- [58] D. Gepner and Z. Qiu. Modular invariant partition functions for parafermionic field theories. *Nucl. Phys.*, B285:423–453, 1987.
- [59] P. Jacob and P. Mathieu. Parafermionic character formulae, 2000. hep-th/0006233.
- [60] F. A. Bais, P. Bouwknegt, M. SurrIDGE, and K. Schoutens. Extensions of the Virasoro algebra constructed from Kac-Moody algebras using higher order Casimir invariants. *Nucl. Phys.*, B304:348–370, 1988.
- [61] F. A. Bais, P. Bouwknegt, M. SurrIDGE, and K. Schoutens. Coset construction for extended Virasoro algebras. *Nucl. Phys.*, B304:371–391, 1988.

## Bibliography

---

- [62] P. Griffin and D. Nemeschansky. Bosonization of  $\mathbb{Z}_n$  parafermions. *Nucl. Phys.*, B323:545–571, 1989.
- [63] D. Nemeschansky. Feigin-Fuchs representation of string functions. *Nucl. Phys.*, B363:665–678, 1991.
- [64] J. Fuchs and D. Gepner. On the connection between WZW and free field theories. *Nucl. Phys.*, B294:30–42, 1987.
- [65] D. Gepner. Field identification in coset conformal field theories. *Phys. Lett. B*, 222(2):207–212, 1989.
- [66] J. Fuchs and C. Schweigert. *Symmetries, Lie algebras and representations, a graduate course for physicists*. Cambridge monographs on mathematical physics. Cambridge university press, 1997.
- [67] M. Abramowitz and I.A. Stegun, editors. *Handbook of mathematical functions*. Dover Publications, fifth edition, 1968.
- [68] L. Alvarez-Gaumé, C. Gómez, and G Sierra. Topics in conformal field theory. In L. Brink, D. Friedan, and A. M. Polyakov, editors, *The physics and mathematics of strings, memorial volume for Vadim Knizhnik*, pages 16–184, Singapore, 1990. World Scientific.
- [69] G. Keller. Fusion rules of  $U_q(sl(2, \mathbb{C}))$ ,  $q^m = 1$ . *Lett. Math. Phys.*, 21:273–286, 1990.
- [70] V. Pasquier and H. Saleur. Common structures between finite systems and conformal field theories through quantum groups. *Nucl. Phys.*, B330:523–556, 1990.
- [71] C. Gomez, M. Ruiz-Altaba, and G. Sierra. *Quantum groups in two-dimensional physics*. Cambridge monographs on mathematical physics. Cambridge university press, 1996.
- [72] V. F. R. Jones. A polynomial invariant for knots via von neumann algebras. *Bull. Am. Math. Soc.*, 12:103–111, 1985.
- [73] I.I. Kachurik and A.U. Klimyk. On Racah coefficients of the quantum algebra  $U_q(su(2))$ . *J. Phys. A: Math. Gen.*, 23:2717–2728, 1990.
- [74] D. A. Varshalovich, A. N. Moskalev, and V. K. Khershonskii. *Quantum theory of angular momentum*. World Scientific, 1988.
- [75] P. Furlan, A. Ch. Ganchev, and V. B. Petkova. Fusion matrices and  $C < 1$  (quasi)local conformal theories. *Int. J. Mod. Phys.*, A5:2721–2736, 1990.
- [76] G. Mack and V. Schomerus. Quasi-Hopf quantum symmetry in quantum theory. *Nucl. Phys.*, B370:185, 1992.
- [77] V. G. Drinfel'd. Quasi-Hopf algebras and Knizhnik-Zamolodchikov equations. In *Problems of modern quantum field theory, proc. Alushta*, pages 1–13, Berlin, 1989. Springer.
- [78] H. Wenzl. Hecke algebras of type  $A_n$  and subfactors. *Invent. Math.*, 92(2):349–383, 1988.
- [79] A. Tsuchiya and Y. Kanie. Vertex operators in conformal field theory on  $\mathbb{P}^1$  and monodromy representations of the braid group. *Lett. Math. Phys.*, 13:303, 1987.
- [80] A. Tsuchiya and Y. Kanie. Vertex operators in conformal field theory on  $\mathbb{P}^1$  and monodromy representations of braid group. In M. Jimbo, T. Miwa, and A. Tsuchiya, editors, *Conformal field theory and solvable lattice models*, volume 16 of *Advanced studies in pure mathematics*, pages 297–372. Academic Press, 1988. Erratum in "Integrable Systems in quantum field theory and statistical mechanics," *Advanced studies in pure mathematics* 19 (1989), 675–682.
- [81] G. Moore and N. Seiberg. Polynomial equations for rational conformal field theories. *Phys. Lett. B*, 212:451–460, 1988.
- [82] G. Moore and N. Seiberg. Classical and quantum conformal field theory. *Commun. Math. Phys.*, 123:177–254, 1989.

- [83] V. Schomerus. Construction of field algebras with quantum symmetry from local observables. *Commun. Math. Phys.*, 169:193, 1995.
- [84] C. Gomez and G Sierra. Quantum group meaning of the Coulomb gas. *Phys. Lett.*, 240B:149, 1990.
- [85] C. Gomez and G. Sierra. The quantum group symmetry of rational conformal field theories. *Nucl. Phys.*, B352:791–828, 1991.
- [86] P. Bouwknegt, J. McCarthy, and K. Pilch. Quantum group structure in the Fock space resolutions of  $\widehat{sl}(n)$  representations. *Commun. Math. Phys.*, 131:125, 1991.
- [87] J. Fuchs. *Affine Lie algebras and quantum groups*. Cambridge monographs on mathematical physics. Cambridge university press, 1992.
- [88] L. Alvarez-Gaumé, C. Gómez, and G. Sierra. Quantum group interpretation of some conformal field theories. *Phys. Lett. B*, 220:142–152, 1989.
- [89] P. Freyd, D. Yetter, J. Hoste, W.B.R. Lickorish, K. Millett, and A. Ocneanu. A new polynomial invariant of knots and links. *Bull. Am. Math. Soc.*, 12:239–246, 1985.
- [90] P.N. Hoefsmit. *Representations of Hecke algebras of finite groups with BN pairs of classical type*. PhD thesis, University of British Columbia, 1974.
- [91] H. Boerner. *Representations of groups with special consideration for the needs of modern physics*. North Holland, second edition, 1969.
- [92] D.A. Ivanov. Non-Abelian statistics of half-quantum vortices in p-wave superconductors. *Phys. Rev. Lett.*, 86(2):268–271, 2001. cond-mat/0005069.
- [93] D. Gepner. New conformal field theories associated with Lie algebras and their partition functions. *Nucl. Phys.*, B290:10–24, 1987.
- [94] B. Ponsot. Monodromy of solutions of the Knizhnik-Zamolodchikov equation:  $SL(2,C)/SU(2)$  WZNW model, 2002. hep-th/0204085.
- [95] N. Read. Nonabelian braid statistics versus projective permutation statistics, 2002. hep-th/0201240.
- [96] M. Freedman, M. Larsen, and Z. Wang. A modular functor which is universal for quantum computation. quant-ph/0001108.
- [97] G. 't Hooft. On the phase transition towards permanent quark confinement. *Nucl. Phys.*, B138:1–, 1978.
- [98] S. Mandelstam. Vortices and quark confinement in nonabelian gauge theories. *Phys. Rept.*, 23:245–249, 1976.
- [99] F. A. Bais. Flux metamorphosis. *Nucl. Phys.*, B170:32, 1980.
- [100] F. A. Bais, P. van Driel, and M. de Wild Propitius. Quantum symmetries in discrete gauge theories. *Phys. Lett.*, B280:63–70, 1992.
- [101] F. A. Bais, P. van Driel, and M. de Wild Propitius. Anyons in discrete gauge theories with Chern-Simons terms. *Nucl. Phys.*, B393:547–570, 1993.
- [102] F. A. Bais, A. Morozov, and M. de Wild Propitius. Charge screening in the Higgs phase of Chern-Simons electrodynamics. *Phys. Rev. Lett.*, 71:2383–2386, 1993.
- [103] F. A. Bais and M. de Wild Propitius. Quantum groups in the Higgs phase. *Theor. Math. Phys.*, 98:357–367, 1994.
- [104] M. de Wild Propitius and F. A. Bais. Discrete gauge theories. In G. Semenoff and L. Vinet, editors, *Particles and Fields*, CRM Series in Mathematical Physics, pages 353–439, New York, 1998. Springer Verlag.

## Bibliography

---

- [105] L. M. Krauss and F. Wilczek. Discrete gauge symmetry in continuum theories. *Phys. Rev. Lett.*, 62:1221, 1989.
- [106] J. Preskill and L. M. Krauss. Local discrete symmetry and quantum mechanical hair. *Nucl. Phys.*, B341:50–100, 1990.
- [107] S.L. Woronowicz. Compact matrix pseudogroups. *Commun. Math. Phys.*, 111:613–665, 1987.
- [108] T. H. Koornwinder, B. J. Schroers, J. K. Slingerland, and F. A. Bais. Fourier transform and the Verlinde formula for the quantum double of a finite group. *J. Phys.*, A32:8539–8549, 1999.
- [109] R. Dijkgraaf, V. Pasquier, and P. Roche. Quasi Hopf algebras, group cohomology and orbifold models. *Nucl. Phys. B (Proc. Suppl.)*, 18B:60–72, 1990.
- [110] G. Lusztig. Leading coefficients of character values of Hecke algebras. *Proceedings of symposia in pure mathematics*, 47:235–262, 1987.
- [111] T.H. Koornwinder, F.A. Bais, and N.M. Muller. Tensor product representations of the quantum double of a compact group. *Commun. Math. Phys.*, 198:157–186, 1998.
- [112] J. Packer. Transformation group  $C^*$ -algebras: a selective survey. In R.S. Doran, editor,  *$C^*$ -algebras: 1943–1993 (San Antonio, TX, 1993)*, volume 167 of *Contemp. Math.*, pages 182–217, Providence, RI, 1994. Amer. Math. Soc.
- [113] T.H. Koornwinder and N.M. Muller. The quantum double of a (locally) compact group. *J. Lie Theory*, 7:33–52, 1997.
- [114] M. de Wild Propitius. *Topological interactions in broken gauge theories*. PhD thesis, University of Amsterdam, 1995.
- [115] W.D. Nichols and M. B. Zoeller. A Hopf algebra freeness theorem. *Amer J. Math.*, 111:381–385, 1989.
- [116] D. Nikshych.  $K_0$ -rings and twisting of finite dimensional semisimple Hopf algebras. *Commun. Algebra*, 26:321–342, 1998.
- [117] W.D. Nichols and M. B. Richmond (Zoeller). The Grothendieck group of a Hopf algebra. *J. Pure Appl. Algebra*, 106:297–306, 1996.
- [118] Larson R.G. and D.E. Radford. Finite-dimensional cosemisimple Hopf algebras in characteristic 0 are semisimple. *J. Algebra*, 117:267–269, 1988.
- [119] G. James and M. Liebeck. *Representations and characters of groups*. Cambridge University Press, Cambridge, 1993.
- [120] H.-J. Schneider. Normal basis and transitivity of crossed products for Hopf algebras. *J. Algebra*, 152:289–312, 1992.
- [121] R.J. Blattner, M. Cohen, and S. Montgomery. Cross products and inner actions of Hopf algebras. *Trans. AMS*, 298:671–711, 1986.
- [122] Y. Doi and M. Takeuchi. Cleft comodule algebras for a bialgebra. *Commun. Alg.*, 14:801–818, 1986.
- [123] S. Majid. More examples of bicrossproduct and double crossproduct Hopf algebras. *Israel J. Math.*, 72:133–148, 1990.
- [124] Andruskiewitsch N. and Devoto J. Extensions of Hopf algebras. *Algebra i Analiz*, 7:22–61, 1995.
- [125] Andruskiewitsch N. Notes on extensions of Hopf algebras. *Can. J. Math.*, 48:3–42, 1996.
- [126] G. 't Hooft. A property of electric and magnetic flux in non-Abelian gauge theories. *Nucl. Phys.*, B153:141–160, 1979.