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Andreev conductance of a domain wall

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At low temperatures the transport through a superconductor-ferromagnet (SF) tunnel interface is due to the tunneling of electrons in pairs. The exchange field of a single domain ferromagnet aligns electron spins and suppresses the two-electron tunneling. The presence of the domain walls at the SF interface strongly enhances the subgap current. The Andreev conductance is proved to be proportional to the total length of domain walls at the SF interface when the ferromagnet is fully polarized.

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Superconductivity and ferromagnetism are two competing phenomena: while the first prefers antiparallel spin orientation of electrons in Cooper pairs, the second forces the spins to be aligned in parallel. Their coexistence in one and the same material or their interaction in spatially separated materials leads to a number of new interesting phenomena,[1] for example, π-state of superconductor-ferromagnet-superconductor (SFS) Josephson junctions,[2–4] highly nonmonotonic dependence of the critical temperature T_c of a SF bilayer as a function of the ferromagnet thickness,[5] and so on. Investigations of SF structures are often based on a bare assumption that the ferromagnet consists of a single domain or that the domain structure is not important. However, this approximation is not always valid.[6–10] For instance, recently it has been demonstrated that the domain structure of the ferromagnet modifies T_c of strongly coupled thin SF bilayers; in addition, vortices may appear in the superconducting film and significantly modify the lateral conductance of the bilayers.[10]

This paper is largely concerned with the influence of the ferromagnetic domain structure on the Andreev conductance of SF junctions. First, let us consider the SF junction with a single-domain ferromagnet. When the voltage V between the superconductor and the ferromagnet is smaller than the superconducting gap Δ, an electron exchange between the superconductor and the ferromagnet is provided by the Andreev processes.[11] The processes involve transfer of two electrons with the opposite spins from the ferromagnet into the superconductor or vice versa. The Andreev conductance is proportional therefore to the product $\nu_\parallel \nu_\parallel$ of the minor $\nu_\parallel$ and major $\nu_\parallel$ band densities of states in the ferromagnet. Thus, if in the ferromagnet the majority of electron spins are polarized along the direction of the magnetization subgap, electron transport through the SF junction is suppressed.

If the ferromagnet consists of several domains, domain walls separate the regions with the different directions of magnetization. If a domain wall is located near the SF interface, electrons with the opposite spins involved into the Andreev processes originate from the adjacent domains. This effect leads to the finite value of the Andreev conductance at any polarization of the ferromagnet. In the case of the fully polarized ferromagnet, as we derived, the Andreev conductance of the SF junction is proportional to the total length $L_D^{(\mathrm{tot})}$ of the domain walls located at the SF boundary and is given by

$$G_A = \frac{\hbar}{\pi e^2}\frac{g_N^2}{\nu_S \Delta} L_D^{(\mathrm{tot})} F\left(\frac{4\delta}{\pi \xi_0}\right), \tag{1}$$

where ν_S is the density of states in the superconductor, g_N stands for the normal conductance of the SF junction per unit area, and δ is the width of the domain wall (see Fig. 1). The coherence length of the superconductor ξ_0 is equal to $v_F/\pi\Delta$ in the *clean* case (elastic mean free path $l_{el} \gg v_F/\pi\Delta$) and equals $\sqrt{8D/\pi^2\Delta}$ in the *dirty* case ($l_{el} \ll v_F/\pi\Delta$). Here v_F denotes the Fermi velocity and D stands for the diffusion coefficient. The function $F(z)$ is different for dirty and clean superconductors, but in the both cases it has the following asymptotic behavior:

$$F(z) = \begin{cases} 1, & z \ll 1 \\ \dfrac{\pi}{4z}, & z \gg 1. \end{cases} \tag{2}$$

Result (1) holds if the superconductor and the ferromagnet are weakly coupled. The condition allows to neglect the exchange field induced in the superconductor due to the proximity effect. The magnetization of a domain produces the vector potential $A_{\mathrm{eff}} = H_{\mathrm{ex}} d$ and, consequently, the supercurrent at the superconductor near the SF interface. Here d is the characteristic size of a domain and H_{ex} is the exchange field. The influence of the supercurrent on the subgap electron transport through the SF junction can be neglected if the condition $eA_{\mathrm{eff}} d/hc \ll 1$ is fulfilled, the latter being typical.

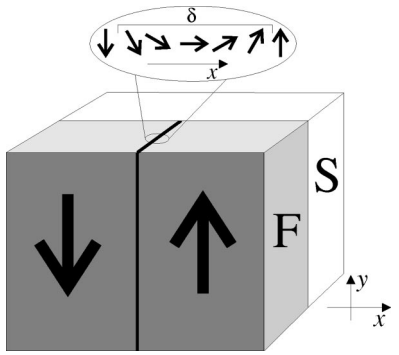

FIG. 1. The superconductor-ferromagnet junction. The domain wall.

Also we imply that the typical size of a domain is much larger than the width of a domain wall, $d \gg \delta$. We leave the more complicated general case for future investigation.

The model. The Hamiltonian describing a system of a superconductor weakly coupled to a ferromagnet is

$$H = H_S + H_F + H_{\rm int}, \tag{3}$$

where $H_S = \Sigma_{p,\sigma} E_{p\sigma} c^\dagger_{p,\sigma} c_{p,\sigma} + \Sigma_p \{\Delta c^\dagger_{p,\uparrow} c^\dagger_{-p,\downarrow} + \text{H.c.}\}$ is the BCS Hamiltonian of the superconductor, $H_F = \Sigma_{k,\sigma} \varepsilon_{k,\sigma} a^\dagger_{k,\sigma} a_{k,\sigma}$ is the Hamiltonian of the ferromagnet, and $H_{\rm int} = \Sigma_{k,p,\sigma} \{a^\dagger_{k,\sigma} t_{k,\sigma;p,\sigma} c_{p,\sigma} + \text{H.c.}\}$. Here $a_{k,\sigma}$ corresponds to the ferromagnet whereas $c_{p,\sigma}$ refers to the superconductor. Labels k and p stand for electron momenta and $\sigma = \pm 1$ denotes spin degree of freedom.

The current flow through the SF junction can be described in terms of the tunneling rates $\Gamma_A^{S \leftarrow F}(V)$ and $\Gamma_A^{S \rightarrow F}(V)$. The first one has the meaning of the probability per second for the Cooper pair creation in the superconductor from two electrons with the opposite spins in the ferromagnet and vice versa for $\Gamma_A^{S \rightarrow F}(V)$. If the voltage between the superconductor and the ferromagnet is less than the superconducting gap, $|eV| < \Delta$, the current equals

$$I(V) = e\{\Gamma_A^{S \leftarrow F}(V) - \Gamma_A^{S \rightarrow F}(V)\}. \tag{4}$$

Using the Fermi golden rule the rates can be found in the second order in the tunneling amplitude $t_{k,\sigma;p,\sigma}$. Following the approach developed in Ref. 13, we finally obtain

$$\Gamma_A^{S \leftarrow F}(V) = 4\pi^3 \int d\xi n_F(\xi - eV) n_F(-\xi - eV) \times \frac{\Delta^2}{[\Delta^2 - \xi^2]} \sum_\sigma \tilde{\Xi}_\sigma(2\sqrt{\Delta^2 - \xi^2}), \tag{5}$$

where $n_F(\xi)$ is the Fermi function. Hereafter we take $\hbar = 1$. The rate $\Gamma_A^{S \rightarrow F}(V)$ can be obtained from Eq. (5) by the substitution of $(1 - n_F)$ for n_F. The kernel $\tilde{\Xi}_\sigma(s) \equiv \int_0^\infty dt \Xi_\sigma(t) e^{-st}$ is the Laplace transform of $\Xi_\sigma(t)$. It can be expressed through the classical probability $P(X_1, \hat{p}_1; X_2, \hat{p}_2, t)$ meaning that an electron with the momentum directed along $\hat{p}_1$ initially located at the point X_1 on the SF boundary arrives (through the superconductor) at time t to the point X_2 at the SF boundary with the momentum directed along $\hat{p}_2$. So

$$\Xi_\sigma(t) = \frac{1}{8\pi^3 e^4 \nu_S} \int d\hat{p}_{1,2} \int dX_{1,2} P(X_1, \hat{p}_1; X_2, \hat{p}_2, t) \times \left\{ G(X_1, \hat{p}_1, \sigma) G(X_2, \hat{p}_2, \sigma) \sin^2\left(\frac{\theta(X_1, X_2)}{2}\right) + G(X_1, \hat{p}_1, \sigma) G(X_2, \hat{p}_2, -\sigma) \cos^2\left(\frac{\theta(X_1, X_2)}{2}\right) \right\}. \tag{6}$$

Here the spatial integration is performed over the surface of the SF junction. We choose the spin quantization axis in the direction of the local magnetization. The quasiclassical probabilities $G(X, \hat{p}, \sigma)$ that an electron with spin polarization σ tunnels from the ferromagnet to the superconductor are normalized in such way that the junction normal conductance per unit area $g_\sigma(X)$ and the total normal conductance G_N are determined as[13,14]

$$g_\sigma(X) = \int d\hat{p} G(X, \hat{p}, \sigma), \quad G_N = \int dX \sum_\sigma g_\sigma(X). \tag{7}$$

Then the normal conductance per unit area, discussed above, is defined as $g_N = G_N / \mathcal{A}$, where $\mathcal{A}$ is the surface area of the SF interface. Symbol $\theta(X_1, X_2)$ is the angle between the magnetizations of the ferromagnet at points X_1 and X_2 near the SF boundary. Equations (4)–(6) describe the subgap current through a SF junction with general domain structure of the ferromagnet. In the limiting case of weak spin polarization ($\nu_\parallel / \nu_\parallel \to 1$) the equations describe the contribution to the subgap conductivity of a normal metal-superconductor junction due to the interference in the superconductor.[13] For the SF junction with the fully polarized single-domain ferromagnet, the subgap current vanishes according to Eqs. (4)–(6). However, inelastic processes provide small but nonlinear contribution to the subgap current which is asymmetric with respect to the sign of the bias voltage.[12]

When the applied voltage is small $|eV| \ll \Delta$, the current is proportional to the voltage, $I(V) = G_A V$. At temperatures small compared to the critical temperature of the superconductor and the voltage $T < \min\{T_c, |eV|\}$, Eqs. (4)–(5) reduce to

$$G_A = 8\pi^3 e^2 \sum_\sigma \tilde{\Xi}_\sigma(2\Delta). \tag{8}$$

Deriving Eq. (5) and (8), we neglected the contributions due to the interference[13] in the ferromagnet. In NS junctions (in particular, when N is dirty) this term gives a large contribution to the conductance.[13] However the exchange field of the ferromagnet can diminish it. When the ferromagnet is fully polarized ($\nu_\parallel = 0$), this interference contribution vanishes for any domain structure of the ferromagnet since it contains a product $\nu_\parallel(X) \nu_\parallel(X) = 0$. Following the procedure described in Refs. 13,17, we find that the interference in the ferromagnet results in the contribution to Γ_A proportional to $\int d\xi n_F(\xi - eV) n_F(-\xi - eV) \Xi^{(F)}(2\xi)$, $|eV| < \Delta$, where $\Xi^{(F)}(2\xi)$ is defined in a similar way as Ξ in Eq. (6). The characteristic length scale in the ferromagnet is $\xi_F = \sqrt{D/E_{\rm ex}}$ in the dirty case and $E_{\rm ex}/v_F$ in the clean case.[5] If $\xi_F \ll \delta$, the Fermi energy $E_F \gg E_{\rm ex}$ and the ferromagnet is quasiballistic, the kernel of $\Xi^{(F)}$ is proportional to $\sin(p_\uparrow |X_1 - X_2|) \sin(p_\downarrow |X_1 - X_2|) / p_\uparrow p_\downarrow |X_1 - X_2|^2$, where $p_{\uparrow(\downarrow)} = \sqrt{2m(E_F \pm E_{\rm ex})}$. The kernel oscillates at the length scale ξ_F and its integral over the SF surface vanishes. Similar considerations show that the interference contributions to the current are small if the ferromagnet is dirty. If $\xi_F \gg \delta$ and $E_{\rm ex} \ll E_F$, using the procedure developed in Refs. 13,17, it is possible to show that $\Xi^{(F)}(\xi)$ has a δ peak at $\xi = 0$. Then, it follows that the interference contribution to the conductance is small. In the intermediate regime $\xi_F \sim \delta$ our results are not

valid; this is a more complicated case since the spin-flip processes in the ferromagnet[15] should be taken into account and the interference in the ferromagnet may become important. We mention, however, that this is not the case for the experiments similar to those presented in Ref. 10.

Andreev conductance of a single-domain wall. The most interesting case is the fully polarized ferromagnet because the Andreev conductance of the SF junction is completely determined by the contribution of domain walls. First, we consider the SF junction with the ferromagnet consisting of two domains as shown in Fig. 1. If we choose the frame of reference according to Fig. 1, the angle of magnetization rotates as follows:[16]

$$\theta(x,-\infty)=\arccos\tanh\frac{x}{\delta}, \tag{9}$$

$$\theta(x_1,x_2)=\theta(x_1,-\infty)-\theta(x_2,-\infty). \tag{10}$$

The classical probability $P(X_1,\hat{p}_1;X_2,\hat{p}_2,t)$ is different in the dirty and clean superconductors. We consider these cases separately.

Provided the superconductor is dirty, we can neglect the momentum dependence of the classical probability $P(X_1,\hat{p}_1;X_2,\hat{p}_2,t)$

$$P(X_1,X_2,t)=\frac{2}{(4\pi Dt)^{3/2}}\exp\left[-\frac{(X_1-X_2)^2}{4Dt}\right], \tag{11}$$

where a factor of 2 appears because the superconductor occupies a half-space. Now we can integrate over the momentum directions $\hat{p}_i$ in Eq. (6). Supposing that $g_\sigma(X)$ is a slowly varying function of X on the length scale $\max\{\xi_0,\delta\}$, we can perform the integrations over the SF interface in Eq. (6) and obtain

$$\tilde{\Xi}_\sigma(s)=\frac{L_D^{(\mathrm{tot})}}{4\pi^4e^4\nu_s s}g_\sigma(g_\sigma-g_{-\sigma})F_d\left(\delta\sqrt{\frac{s}{D}}\right)+\frac{\mathcal{A}}{8\pi^3e^4\nu_s\sqrt{sD}}g_\sigma g_{-\sigma}. \tag{12}$$

Here the function $F_d(z)$ is defined as

$$F_d(z)=\int_0^\infty dx K_0(x)x\tanh\left(\frac{x}{2z}\right), \tag{13}$$

where $K_0(x)$ is the modified Bessel function of the second kind. With the help of Eqs. (8) and (12) we find that the Andreev conductance of the SF junction can be written as $G_A=G_A^{(0)}+G_A^{(D)}$, where the surface and domain-wall contributions are given by

$$G_A^{(0)}=\frac{4\mathcal{A}}{\pi e^2\nu_s\Delta\xi_0}g_\uparrow g_\downarrow, \tag{14}$$

$$G_A^{(D)}=\frac{L_D^{(\mathrm{tot})}}{\pi e^2\nu_s\Delta}(g_\uparrow-g_\downarrow)^2F_d\left(\frac{4\delta}{\pi\xi_0}\right). \tag{15}$$

The surface contribution $G_A^{(0)}$ is suppressed in the case of the fully polarized ferromagnet, $g_\uparrow\gg g_\downarrow$, and the domain-wall contribution $G_A^{(D)}$ is the only one that survives. In the most interesting cases the function $F_d(z)$ has the following asymptotic behavior

$$F_d(z)=\begin{cases}1+\dfrac{\pi^2z^2}{12}\left(\ln z+1+\dfrac{6}{\pi^2}\zeta'(2)\right), & z\ll 1\\ \dfrac{\pi}{4z}\left(1+\dfrac{3}{4z^2}\right), & z\gg 1,\end{cases} \tag{16}$$

where $\zeta'(z)$ is the derivative of the Riemann zeta function. Using Eqs. (15) and (16) for the case of the fully polarized ferromagnet, $g_\downarrow=0$, we obtain result (1).

In the case of the *clean* superconductor we can estimate the classical probability as

$$P(X_1,X_2,t)=\frac{2}{4\pi(X_1-X_2)^2}\delta(|X_1-X_2|-v_Ft). \tag{17}$$

This probability describes the tunneling through the disordered SF boundary. With a help of Eq. (17) one can reproduce the results of Ref. 17 concerning the conductance of a clean normal metal–superconductor–normal metal structure. In a similar way as above, we obtain

$$\tilde{\Xi}_\sigma(s)=\frac{L_D^{(\mathrm{tot})}}{4\pi^4e^4\nu_s s}g_\sigma(g_\sigma-g_{-\sigma})F_c\left(\frac{2\delta s}{v_F}\right)+\frac{\mathcal{A}}{8\pi^3e^4\nu_s v_F}g_\sigma g_{-\sigma}\left(\ln\frac{v_F}{\lambda_F s}-\gamma\right), \tag{18}$$

where $\gamma\approx 0.577$ is the Euler constant and the function $F_c(z)$ is defined as

$$F_c(z)=\int_0^\infty dx K_0(x)\ln\cosh\frac{x}{z}. \tag{19}$$

Then, the surface and domain-wall contributions to the Andreev conductance are as follows:

$$G_A^{(0)}=\frac{2\mathcal{A}}{\pi e^2\nu_s\Delta\xi_0}g_\uparrow g_\downarrow\left(\ln\frac{\xi_0}{\lambda_F}-\gamma\right), \tag{20}$$

$$G_A^{(D)}=\frac{L_D^{(\mathrm{tot})}}{\pi e^2\nu_s\Delta}(g_\uparrow-g_\downarrow)^2F_c\left(\frac{4\delta}{\pi\xi_0}\right), \tag{21}$$

where λ_F denotes the Fermi length. The function $F_c(z)$ has the following asymptotic behavior:

$$F_c(z)=\begin{cases}1-\dfrac{\pi z}{2}\ln 2, & z\ll 1\\ \dfrac{\pi}{4z}\left(1+\dfrac{3}{z^2}\right), & z\gg 1.\end{cases} \tag{22}$$

In the case of the fully polarized ferromagnet, $g_\downarrow = 0$, Eqs. (21) and (22) yield result (1).

Andreev conductance of several domain walls. Now we consider the domain structure with several domain walls touching the SF interface. If the domain walls separate domains with the opposite directions of magnetization, the Andreev conductance is a sum of contributions from each domain wall. Assuming that the characteristic domain size is much larger than the domain wall width and the magnetization rotation is given by Eq. (9), we find result (1) with $L_D^{\rm tot}$ being the total length of the domain walls at the SF interface. Usually, the domain structure at the SF interface is more complicated. Nevertheless, the Andreev conductance remains proportional to the total domain-wall length whereas the function $f(x)$ in Eq. (1) may depend on the particular domain structure.

A possible experimental setup can be prepared in a similar way as in a recent experiment.[10] To ensure the parallel magnetization of domains with respect to the SF surface and the absence of domain-induced vortices,[10] the superconducting layer should be thicker than ξ_0. The normal conductance between the superconductor and the ferromagnet should be smaller than the normal conductances of the leads, the ferromagnet, and the superconductor (then the voltage drops mainly at the SF interface). Varying the applied magnetic field, we change the number of domains in the ferromagnet. According to Eq. (1) the Andreev conductance is proportional to the number of domain walls in the ferromagnet and, consequently, to the number of domains. This can be checked experimentally by measuring the Andreev conductance as a function of the applied magnetic field.

In conclusion, we evaluated the low-voltage Andreev conductance of the SF junction when the ferromagnet, is strongly polarized and consists of several domains. The main transport mechanism under subgap conditions is the two-electron tunneling (with zero total spin of an electron pair) whereas the transfer of single electrons is strongly suppressed. The exchange field of the ferromagnet aligns electron spins and suppresses the two-electron tunneling. However, the tunneling is not suppressed near the domain walls where electrons involved come from (or come to) the adjacent domains. We found that at strong polarization of the ferromagnet the domain-wall contribution to the Andreev conductance is the largest one. We presented an approach that gives an opportunity to find the subgap current for different geometries of an experimental setup. The dynamics of domains in the magnetic field can be probed experimentally through the SF conductance measurement.

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