UNIVERSITY OF AMSTERDAM

UvA-DARE (Digital Academic Repository)

Quasi-particles for fractional quantum Hall systems

van Elburg, R.A.J.

Publication date
2000

Link to publication

Citation for published version (APA):
van Elburg, R. A. J. (2000). *Quasi-particles for fractional quantum Hall systems*.

General rights
It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations
If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: https://uba.uva.nl/en/contact, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.

UvA-DARE is a service provided by the library of the University of Amsterdam (https://dare.uva.nl)

Download date:26 Jul 2022

5

Finite Temperature Green's Functions

5.1 Fermi-liquid to fqH-edge tunneling

Having checked that the thermodynamics of fqH-edges is correctly reproduced in the new quasi-particle language and having established some of the form factors properties we are now ready to move on and consider transport properties. Following the set-up of a number of experiments, we shall consider a situation where electrons (or holes) from a Fermi-liquid reservoir are allowed to tunnel into a $\nu = \frac{1}{p}$ fqH-edge. The DC IV-characteristic for this set-up, which were first computed by Kane and Fisher [46] (see also [82, 18]), show a cross-over from a linear (thermal) regime into a power-law behavior at high voltages and thus present a clear fingerprint of the Luttinger liquid features of the fqH-edge. The experimental results from [19] are in agreement with these predictions.

The calculations by Kane and Fisher were based on bosonization and on the Keldysh formalism for non-equilibrium transport. Our goal here is to see if we can reproduce their results in an approach directly based on the edge quasi-particle formalism. Before going into this, we would like to stress that the 'Thermodynamic Bethe Ansatz (TBA) quasi-particles' behind the approach of [18] are quite different from what we have here, the most important distinction being that the TBA quasi-particles are a combination of degrees of freedom of both sides of the tunneling barrier; they do not exist for a $\nu = \frac{1}{p}$ edge in isolation.

If the $\nu = \frac{1}{p}$ fqH-edge were to behave as a Fermi-liquid, we could calculate charge transport across a barrier using a simple (Boltzmann) kinetic equation of the form

$$I(V,T) \propto e \int_{-\infty}^{\infty} d\epsilon\, W \left\{ f_1(\epsilon - eV) F_2(\epsilon) - F_1(\epsilon - eV) f_2(\epsilon) \right\} \ . \qquad (5.1)$$

with $f(\epsilon)$ and $F(\epsilon)$ the Fermi-Dirac distributions for electrons and holes, respectively, and W the probability for an electron or hole of energy ϵ to cross the barrier and enter the edge. As is well known, this Boltzmann equation leads to an ohmic and temperature-independent current. Now that we have seen that the non-Fermi liquid features of the $\frac{1}{p}$ edge can be captured via the statistics of the edge quasi-particles we can try to write a 'Boltzmann equation' for transport to and from

fqH-edges by putting in appropriate generalizations $g(\epsilon)$ and $G(\epsilon)$ of the quantities $f_2(\epsilon)$ and $F_2(\epsilon)$, respectively. Before giving precise results we shall consider a 'naive' expression based on the intuition from the quasi-particle approach. In first approximation, the factor $g(\epsilon)$, which describes the probability for an electron to leave a $\nu = \frac{1}{p}$ edge, comprises two effects

1. a correlation effect, which can be traced to the non-trivial scaling dimension of the edge electron operator. At zero temperature, this is the so-called tunneling density of states

$$A^+(\epsilon) \propto \epsilon^{p-1} \ . \tag{5.2}$$

2. a temperature dependence related to the exclusion statistics properties of the edge electrons. As we have seen, the natural factor associated to the presence of an edge electron is the distribution function

$$\bar{n}_{g=p}(\epsilon) \ . \tag{5.3}$$

Combining these factors, we come to the naive expressions

$$\tilde{g}(\epsilon) = \epsilon^{p-1}\, \bar{n}_{g=p}(\epsilon) \ , \tag{5.4}$$

and by similar reasoning we obtain

$$\tilde{G}(\epsilon) = \epsilon^{p-1}\, e^{\beta\epsilon}\, \bar{n}_{g=p}(\epsilon) \ , \tag{5.5}$$

where the thermal factor $e^{\beta\epsilon}\, \bar{n}_p(\epsilon)$ has been dictated by the requirement of detailed balance or outside the context of tunneling by the KMS-condition.
One quickly finds that the Boltzmann equation with factors $\tilde{g}$ and $\tilde{G}$ is not exact at finite temperature. Later we shall further comment on this equation and argue that it can be viewed as part of a first stage in a systematic approach. Before we come to that, we shall in the next section present a particularly simple derivation of the exact perturbative I-V characteristics for tunneling from a Fermi-liquid to a $\nu = \frac{1}{3}$ fqH-edge. This derivation uses the idea of a kinetic equation, together with the algebraic properties of the edge electrons.

5.1.1 Kinetic Equation for Inter Edge Transport

A careful derivation, based directly on the form of the tunneling Hamiltonian

$$H_{int} \propto t \int d\epsilon \left[\Psi^\dagger_{\nu=1}(\epsilon) \Psi_{\nu=\frac{1}{3}}(\epsilon) + \text{h.c.} \right] \ , \tag{5.6}$$

where $\Psi^\dagger_\nu$ denotes the electron creation operator for a filling fraction ν fqH-edge, leads to the following kinetic equation, see e.g. [82],

$$I(V,T) \propto e\, t^2 \int_{-\infty}^{\infty} d\epsilon \left[f(\epsilon - eV) G(\epsilon) - F(\epsilon - eV) g(\epsilon) \right] \ . \tag{5.7}$$

where G, g are one particle Green's functions

$$g_V(\epsilon) = \langle \Psi^\dagger_{\nu=\frac{1}{3}}(\epsilon)\Psi_{\nu=\frac{1}{3}}(\epsilon)\rangle_{V,T} \ , \qquad G_V(\epsilon) = \langle \Psi_{\nu=\frac{1}{3}}(\epsilon)\Psi^\dagger_{\nu=\frac{1}{3}}(\epsilon)\rangle_{V,T} \tag{5.8}$$

for edge electrons in the $\nu = \frac{1}{3}$ fqH-edge, taken at $V = 0$. Note that the expression 5.7 is perturbative as it gives the lowest non-trivial order in the parameter t.

The quantities $G_V(\epsilon)$ and $g_V(\epsilon)$ can be determined by using two simple observations. The first is that of detailed balance, which can be phrased as the requirement that at zero voltage there should be no current flowing. This fixes the ratio of $G_V(\epsilon)$ and $g_V(\epsilon)$ according to

$$G_V(\epsilon) = e^{\beta(\epsilon - eV)} g_V(\epsilon) \ . \tag{5.9}$$

The second observation uses the algebraic properties of the edge electron operator, which include the anti-commutation relation

$$\left\{ \Psi^\dagger_{\nu=\frac{1}{3}}(\epsilon), \Psi_{\nu=\frac{1}{3}}(\epsilon) \right\} = \frac{2\pi}{L}\frac{1}{\rho_0}\epsilon^2 + 6\frac{E}{\rho_0} + 6\epsilon\frac{\Delta Q}{e\rho_0} \ . \tag{5.10}$$

In this formula, E is the operator for the total energy per unit length, and ΔQ is the operator for the total charge per unit length. Clearly, this anti-commutator fixes the sum $G_V(\epsilon) + g_V(\epsilon)$. The expectation values of energy and charge follow directly from our analysis in chapter 3. We find

$$\langle E \rangle = \rho_0 \left(\frac{\pi^2}{6\beta^2} + \frac{(eV)^2}{6} \right) \ , \qquad \langle \Delta Q \rangle = -e\rho_0 \frac{(eV)}{3} \tag{5.11}$$

and obtain the exact expressions

$$G_V(\epsilon) = \frac{(\epsilon - eV)^2 + \frac{\pi^2}{\beta^2}}{e^{-\beta(\epsilon - eV)} + 1} \ , \quad g_V(\epsilon) = \frac{(\epsilon - eV)^2 + \frac{\pi^2}{\beta^2}}{1 + e^{\beta(\epsilon - eV)}} \ . \tag{5.12}$$

They lead to I-V characteristics

$$I(V,T) \propto e\, t^2\, \beta^{-3} \left(\frac{\beta eV}{2\pi} + \left(\frac{\beta eV}{2\pi} \right)^3 \right) \ , \tag{5.13}$$

in agreement with the result obtained in different approaches [46, 18].

Clearly, the Green's functions 5.8 can be evaluated in other ways, for example by using a conformal transformation in the x, t domain [82]. We would like to stress that our derivation is more direct and uses nothing more than the fundamental anti-commutation relation of the edge electrons. For $\nu = \frac{1}{3}$, these are particularly simple as they derive from the so-called $N = 2$ super-conformal algebra, which has been well- studied in other contexts. For other filling fractions the fundamental anti-commutators look more complicated but are available in principle.

5.1.2 Interpretation in Terms of Exclusion Statistics

If we compare the exact kinetic equation for $\nu = \frac{1}{3}$ with a naive generalized Boltzmann equation, we see that the mistake in the latter is in the approximation of the Green's function $g(\epsilon)$ by a the product $\tilde{g}(\epsilon)$ of a tunneling density of states times a Haldane distribution for fractional statistics.
The reason why this approximation turns out to be rather poor is that the operator $N_\Psi(\epsilon) = \Psi^\dagger_{\nu=\frac{1}{3}}(\epsilon)\Psi_{\nu=\frac{1}{3}}(\epsilon)$ inside a fqH-edge is not to be viewed as a simple counting operator weighted by the appropriate power law of ϵ. This fact can be traced to the non-trivial operator terms in the r.h.s. of eq. 5.10. To further illustrate this point we evaluated the expectation value of the operator $N_\Psi(\epsilon)$ in a normalized one-electron state $|\epsilon'\rangle$

$$\langle\epsilon'|N_\Psi(\epsilon)|\epsilon'\rangle \propto \epsilon^2\delta(\epsilon-\epsilon') + 6\frac{(\epsilon'-\epsilon)(\epsilon'^2+\epsilon^2)}{\epsilon'^2}\Theta(\epsilon'-\epsilon)\ . \tag{5.14}$$

This result shows an interaction effect in the action of $N_\Psi(\epsilon)$ on a one-electron state: rather than just counting quanta of energy ϵ, the operator $N_\Psi(\epsilon)$ is sensitive to the presence of quanta at energy $\epsilon' > \epsilon$ as well. In the Green's function $g(\epsilon)$ (for $\epsilon > 0$), the first term on the r.h.s. of eq. 5.14 corresponds to $\tilde{g}(\epsilon)$, while the second term leads to the following correction term

$$g^{(1,0)}(\epsilon) - \tilde{g}(\epsilon) = 6\int_\epsilon^\infty d\epsilon'\,\frac{(\epsilon'-\epsilon)(\epsilon'^2+\epsilon^2)}{\epsilon'^2}\,\bar{n}_3(\epsilon')\ . \tag{5.15}$$

In figure 5.1 we have plotted the exact result for $g(\epsilon)$ against the approximations $\tilde{g}(\epsilon)$ and $g^{(1,0)}(\epsilon)$. Clearly, the correction term included in $g^{(1,0)}(\epsilon)$ greatly improves the accuracy of the description.
The situation here can be described as follows. As far as thermodynamics goes, the distribution functions $\bar{n}_3(\epsilon)$ and $\bar{n}_{\frac{1}{3}}(\epsilon)$ give exact results for quantities such as specific heat and conductances. However, the operators $\Psi^\dagger_{\nu=\frac{1}{3}}(\epsilon)$, $\Psi_{\nu=\frac{1}{3}}(\epsilon)$ are not one-particle operators in the usual sense, as they do not simply add or extract a single quasi-particle from a many-particle state. In edge tunneling experiments, the edge system communicates with a Fermi liquid via the operators $\Psi^\dagger_{\nu=\frac{1}{3}}(\epsilon)$ and $\Psi_{\nu=\frac{1}{3}}(\epsilon)$ and we can not avoid interaction effects. We do believe, however, that a systematic expansion based on the quasi-particle picture is possible. Evidence for this claim is the calculation above and the calculation following in the rest of this chapter.

5.2 Form Factor Expansion

The Green's function $g(\epsilon)$ can be viewed as a one-point function for the operator $N_\Psi(\epsilon) = \Psi^\dagger_{\nu=\frac{1}{p}}(\epsilon)\Psi_{\nu=\frac{1}{p}}(\epsilon)$. In the formulation on the finite system of size L, this operator is represented as $N_J(m) = aJ_{-m}J^\dagger_m$, with $\epsilon = am$, with $a = \frac{2\pi}{L\rho_0}$ the

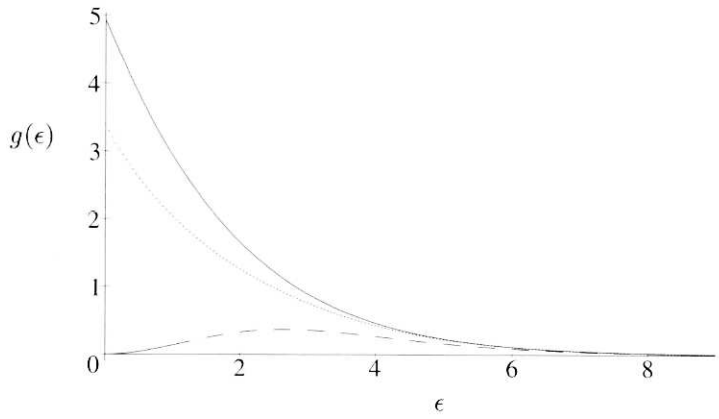

Figure 5.1: One-particle Green's function $g(\epsilon)$ in units $[\beta]$ as a function of the energie ϵ in units $[\beta^{-1}]$ for filling fraction $\nu = \frac{1}{3}$ and at zero voltage. The drawn curve is the exact result 5.12; the dashed curve is the approximation $\tilde{g}(\epsilon)$ and the dotted curve corresponds to $g^{(1,0)}(\epsilon)$.

energy level spacing in the finite size system. This one-point function is formally expressed as

$$\frac{\sum_{\text{state}\in\mathcal{H}}\langle\text{state}|N_J(m)|\text{state}\rangle\exp(-\beta E_{\text{state}})}{\sum_{\text{state}\in\mathcal{H}}\langle\text{state}|\text{state}\rangle\exp(-\beta E_{\text{state}})}. \tag{5.16}$$

The sum runs over a basis of the full Hilbert space of the edge CFT, and we can opt for the fqH quasi-particle basis discussed in chapter 4. The idea is now that the matrix elements $\langle\text{state}|N_J(m)|\text{state}\rangle$ are dominated by processes where only a few of the quasi-particles that are present in a concrete basis state $|\{m_i;n_j\}\rangle$ participate.
For the case at hand, the lowest contribution comes from 1-particle states $|(m_1)\rangle$, for which one computes the matrix element

$$D^{(1,0)}(m;m_1) = {}_N\langle(m_1)|J_{-1-m}J^\dagger_{+1+m}|(m_1)\rangle_N. \tag{5.17}$$

The expected presence of an edge electron of energy m_1 is given by the distribution function $n_p(\epsilon_1 = am_1)$. This leads to the following contribution to the Green's function

$$g^{(1,0)}(\epsilon) = a\sum_{m_1} D^{(1,0)}(m,m_1)\bar{n}_p(am_1). \tag{5.18}$$

If we now consider the matrix element of $N_J(m)$ against a two-electron state, we find (see next subsection) that it is not simply the sum of two 1-particle contributions. The left-over part is what we call the irreducible 2-electron matrix element

$$D^{(2,0)}(m;m_1,m_2) = {}_N\langle(m_1,m_2)|J_{-3-m}J^\dagger_{+3+m}|(m_2,m_1)\rangle_N$$

$$-{}_N\langle(m_1)|J_{-3-m}J^\dagger_{+3+m}|(m_1)\rangle_N - {}_N\langle(m_2)|J_{-3-m}J^\dagger_{+3+m}|(m_2)\rangle_N \ . \tag{5.19}$$

It leads to an additional contribution $g^{(2,0)}(m)$ to the Green's function

$$g^{(2,0)}(\epsilon) = a\sum_{m_1,m_2} D^{(2,0)}(m;m_1,m_2)\bar{n}_p(am_1)\bar{n}_p(am_2) \ . \tag{5.20}$$

Similarly, we define

$$D^{(1,1)}(m;m_1,n_1) = {}_N\langle(n_1,m_1)|J_{-1-m}J^\dagger_{+1+m}|(m_1,n_1)\rangle_N - {}_N\langle(m_1)|J_{-1-m}J^\dagger_{+1+m}|(m_1)\rangle_N \tag{5.21}$$

and

$$g^{(1,1)}(\epsilon) = a\sum_{m_1,n_1} D^{(1,1)}(m;m_1,n_1)\bar{n}_p(am_1)\bar{n}_{\frac{1}{p}}(an_1) \tag{5.22}$$

Continuing in this manner, we build up the following expansion

$$g = \sum_{M,N} g^{(M,N)}(\epsilon)$$

$$g^{(M,N)}(\epsilon) = a\sum_{\{m_i;n_j\}} D^{(M,N)}(m;\{m_i;n_j\})\prod_i \bar{n}_p(am_i)\prod_j \bar{n}_{\frac{1}{p}}(an_j) \tag{5.23}$$

We remark that an expansion of precisely this type has been proposed by LeClair and Mussardo [54], see also Saleur [68]. This work was done in the context of integrable qft's, that are fully characterized by a factorized S-matrix. In such a context, the irreducible form factors are constrained by the form factor axioms, and the distribution functions have their origin in a TBA procedure. Although clearly in the same spirit, the analysis that we present here is very different at the technical level. We obtain the relevant form factor by explicit computation in a theory that is regularized by the finite size of the fqH edge, and we have identified the relevant distribution functions by analyzing the state counting of the (discrete) spectrum of the finite-size system. We thus do not rely on an underlying (massless) S-matrix point of view.

5.3 Finite T Green's Function for $p = 2$

As a proto-type study for a form factor expansion based on CFT quasi-particles, we now analyze the Green's function $g(\epsilon)$, for $p = 2$ in that spirit. Obviously, an exact result is easily obtained

$$g(\epsilon) = \frac{\epsilon}{e^{\beta\epsilon} - 1} \ . \tag{5.24}$$

The Bose-Einstein denominator in this expression has its origin in the fact that the operators J, $J^\dagger$ satisfy bosonic commutation relations. In the spirit of the quasi-particle formulation, we wish to treat the J, $J^\dagger$-quanta as quasi-particles with exclusion statistics $g = 2$, and see if we can recover the Green's function $g(\epsilon)$ in such an approach.
To evaluate explicitly the leading terms in the form factor expansion 5.23 for $g(\epsilon)$, we need to evaluate the relevant irreducible matrix elements. While it is clear that these matrix elements have very special mathematical properties, we here compute them by a simple brute force computation, relying on the algebraic properties of the operators $J^\pm$, Q and $\phi^\pm$, which form a representation and a doublet representation of $SU(2)_1$,

$$\begin{aligned}
[J_s^+, J_r^-] &= s\delta_{s+r} + Q_{s+r} \\
[Q_s, Q_r] &= 2s\delta_{s+r} \\
[Q_s, \phi_r^\pm] &= \pm\phi_{s+r}^\pm \\
[Q_s, J_r^\pm] &= \pm 2J_{s+r}^\pm \\
[J_s^\mp, \phi_r^\pm] &= \phi_{s+r}^\mp \\
[J_s^\pm, \phi_r^\pm] &= 0
\end{aligned} \tag{5.25}$$

Furthermore, we use the explicit form of the two-particle states $|(m_2, m_1)\rangle$ and $|(m_1; n_1)\rangle$ at $p = 2$,

$$|(m_2, m_1)\rangle = |m_2, m_1\rangle + \frac{2}{m_2 - m_1 + 3} \sum_{l>0} |m_2 + l, m_1 - l\rangle, \tag{5.26}$$

and

$$|(m_1; n_1)\rangle = |m_1; n_1\rangle - \frac{1}{(m_1 + 2n_1 + 1)} \sum_{l>0} |m_1 + l; n_1 - l\rangle, \tag{5.27}$$

with

$$|m_2, m_1\rangle = J_{-3-m_2} J_{-1-m_1} |0\rangle \ , \qquad |m_1; n_1\rangle = J_{-1-m_1} \phi_{-\frac{1}{4}-n_1} |0\rangle \ . \tag{5.28}$$

These states have the following norms

$$N_{(m_2,m_1)} = \langle (m_1, m_2) | (m_2, m_1) \rangle = \frac{m_2 - m_1 + 1}{m_2 - m_1 + 3} (m_2 + 3)(m_1 + 1), \tag{5.29}$$

and

$$N_{(m_1;n_1)} = \langle (n_1; m_1) | (m_1; n_1) \rangle = \frac{m_1 + 2n_1 + 2)}{m_1 + 2n_1 + 1} (m_1 + 1) C_{n_1}^{(-\frac{1}{p})} \ . \tag{5.30}$$

One electron

For the (irreducible) one electron matrix element we found

$$D^{(1,0)}(m;m_1) = {}_N\langle(m_1)|J_{-1-m}J^{\dagger}_{+1+m}|(m_1)\rangle_N =$$
$$(m+1)\delta_{m,m_1} + 2\left(1-\frac{m+1}{m_1+1}\right)\Theta(m<m_1)\,. \tag{5.31}$$

Two electrons

For the irreducible two electron matrix element we find

$$\begin{aligned}
&D^{(2,0)}(m;m_2,m_1) = \\
&\quad \delta_{m-m_2}\frac{-2(m_2+3)}{m_2-m_1+3} + \delta_{m-m_1+2}\frac{-2(m_1+1)}{m_2-m_1+1} \\
&\quad + \frac{4}{(m_2-m_1+3)}\frac{1}{(m_2-m_1+1)}\frac{1}{(m_1+1)(m_2+3)} \\
&\qquad \times[\Theta(m<m_1-2)\,P(m;m_1,m_2) \\
&\qquad\quad + \Theta(m<m_2<m+m_1)\,Q(m;m_1,m_2) \\
&\qquad\quad + \Theta(m<m_2)\,R(m;m_1,m_2)]\,,
\end{aligned} \tag{5.32}$$

with

$$\begin{aligned}
&P(m;m_1,m_2) = \\
&\quad (m_2-m_1+3)(m_1-m-2)(2m_1-m_2-3) \\
&\quad + (m_1-m-2)(m_1-m-3)(-3m_2+\frac{5}{3}m_1+\frac{1}{3}m-\frac{26}{3}) \\
&\quad + (m+3)[-2(m_2-m_1+3)(2m_1-m-1) \\
&\quad -2m_1(m_1+1)+(m+3)(m_2+m_1-m+1)] \\
&Q(m;m_1,m_2) = \\
&\quad (m_1-m_2+m+1)[(m_2-m_1+3)^2+2(m_2-m_1+3)(m_1-m_2+m) \\
&\quad + \frac{2}{3}(m_1-m_2+m)(m_1-m_2+m-1)] \\
&R(m;m_1,m_2) = \\
&\quad (m_2-m)(m_1+1)(m_2-m_1+3)+\frac{1}{3}m_1(m_1+1)(m_1+3m_2-3m+2)
\end{aligned} \tag{5.33}$$

The polynomials P, Q and R enjoy special properties, which include

$$(P+Q+R)(m;m_1,m_2) = \frac{1}{3}(m_1-m_2-1)(m_1-m_2-2)(m_1-m_2-3) \ . \quad (5.34)$$

One electron and one quasi-hole

The irreducible matrix element with one electron and one hole is found to be

$$\begin{aligned} D^{(1,1)}(m;m_1,n_1) = & \\ & \delta_{m_1,m}\frac{m_1+1}{m_1+2n_1+1} \\ & +\Theta(m<m_1)\frac{1}{C_{n_1}^{(-\frac{1}{2})}(m_1+2n_1+2)(m_1+2n_1+1)(m_1+1)} \\ & \times\left[C_{n_1-m_1+m}^{(-\frac{1}{2})}S(m;m_1,n_1)+C_{n_1}^{(-\frac{1}{2})}T(m;m_1,n_1)\right] \ , \end{aligned} \quad (5.35)$$

with

$$\begin{aligned} S(m;m_1,n_1) = & \\ & (m_1+2n_1+1)^2+(m+n_1-m_1)(\frac{8}{3}-4(m_1+2n_1+2))+\frac{4}{3}(m+n_1-m_1)^2 \end{aligned}$$

$$\begin{aligned} T(m;m_1,n_1) = & \\ & 2(m_1-m)((m_1+2n_1+1)^2-1)+2(2n_1+1)(m_1-m-1) \\ & +2(\frac{2}{3}n_1+1)(2n_1+1) \ . \end{aligned} \quad (5.36)$$

5.3.1 Evaluating the Series

With the information collected in the previous subsections, we can evaluate the 1-particle and 2-particle contributions $g^{(1,0)}$, $g^{(2,0)}$ and $g^{(1,1)}$ to the Green's function $g(\epsilon)$.

The expressions 5.18, 5.20, and 5.22 for $g^{(2,0)}$ and $g^{(1,1)}$ are discrete sums, which we wish to study in the limit $a \to 0$. In this limit, one may view the expressions as Riemann sums and evaluate them using continuous integrals; however, one needs to be careful because the integrands as they stand have singularities, and the sums are not term-by-term convergent. One may check however that by carefully redistributing some of the terms, one obtains convergent sums that can be approximated by the corresponding continuous integrals. Proceeding in this manner, and using a numerical integrator, we obtained the results plotted in figure 5.3 and figure 5.2 . We observe that the form factor series converge in the following sense: while the 1-particle terms agree with the exact result for ϵ greater than about $3k_BT$, the result with 2-particle terms included has reached the exact value at ϵ greater than

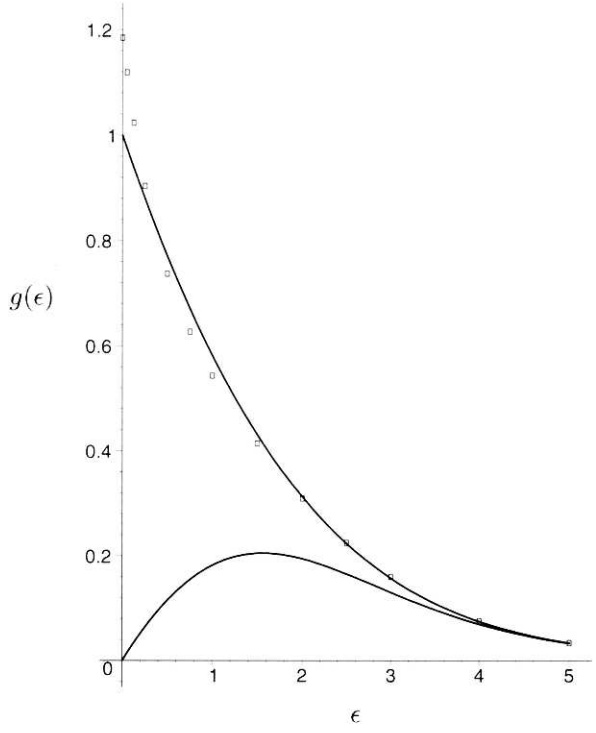

Figure 5.2: One-particle Green's function $g(\epsilon)$ in units $[\beta = \frac{1}{k_B T}]$ for filling fraction $\nu' = \frac{1}{2}$ as a function of energy in units $[\beta^{-1}]$. The upper solid curve is the exact result, the data points represent the sum of all contributions with up to two particles present. The lower solid curve represents the naive approximation discussed in the introduction of this chapter.

about $2k_B T$. For energies $\epsilon \ll k_B T$, the thermal factors do not efficiently suppress many particle contributions, and the convergence of the form factor expansion is expected to be slow.

We remark that the asymptotic behavior for $\epsilon \gg k_B T$ of the 2-particle terms is

$$g^{(2,0)}(\epsilon) \sim c_2 e^{-\beta\epsilon} \qquad g^{(1,1)}(\epsilon) \sim c_{\frac{1}{2}} e^{-\beta\epsilon} \tag{5.37}$$

with

$$c_2 = -2\int_0^\infty d\epsilon_1 \bar{n}_2(\epsilon_1) \;, \qquad c_{\frac{1}{2}} = \int_0^\infty d\tilde{\epsilon}_1 \bar{n}_{\frac{1}{2}}(\tilde{\epsilon}_1) \;, \tag{5.38}$$

Remarkably, the duality relation 3.26 between the distributions leads to the relation

$$c_2 = -c_{\frac{1}{2}} \tag{5.39}$$

meaning that the Boltzmann tails of the 2-particle terms precisely cancel. This 'conspiracy' was needed as, numerically, it is seen that the deviation between the exact curve $g(\epsilon)$ and the 1-particle term $g^{(1,0)}(\epsilon)$ is far smaller than the individual Boltzmann tails of $g^{(2,0}$ and $g^{(1,1)}$.

5.4 Conclusions

The results in this chapter show that with the fqH-basis we identified the basis in which the one-point Green's function can be approximated very efficiently using only form factors which involve a few quasi-particles. From our results in the preceding chapter we can see that in an alternative basis, like the CS-basis, the number of quasi-particles needed to describe the same form factors ranges from a few for low energy states to many for the high energy states. This shows that the fqH-basis gives the most efficient description of the degrees of freedom relevant for the description of the one point Green's functions on the fqH-edge.

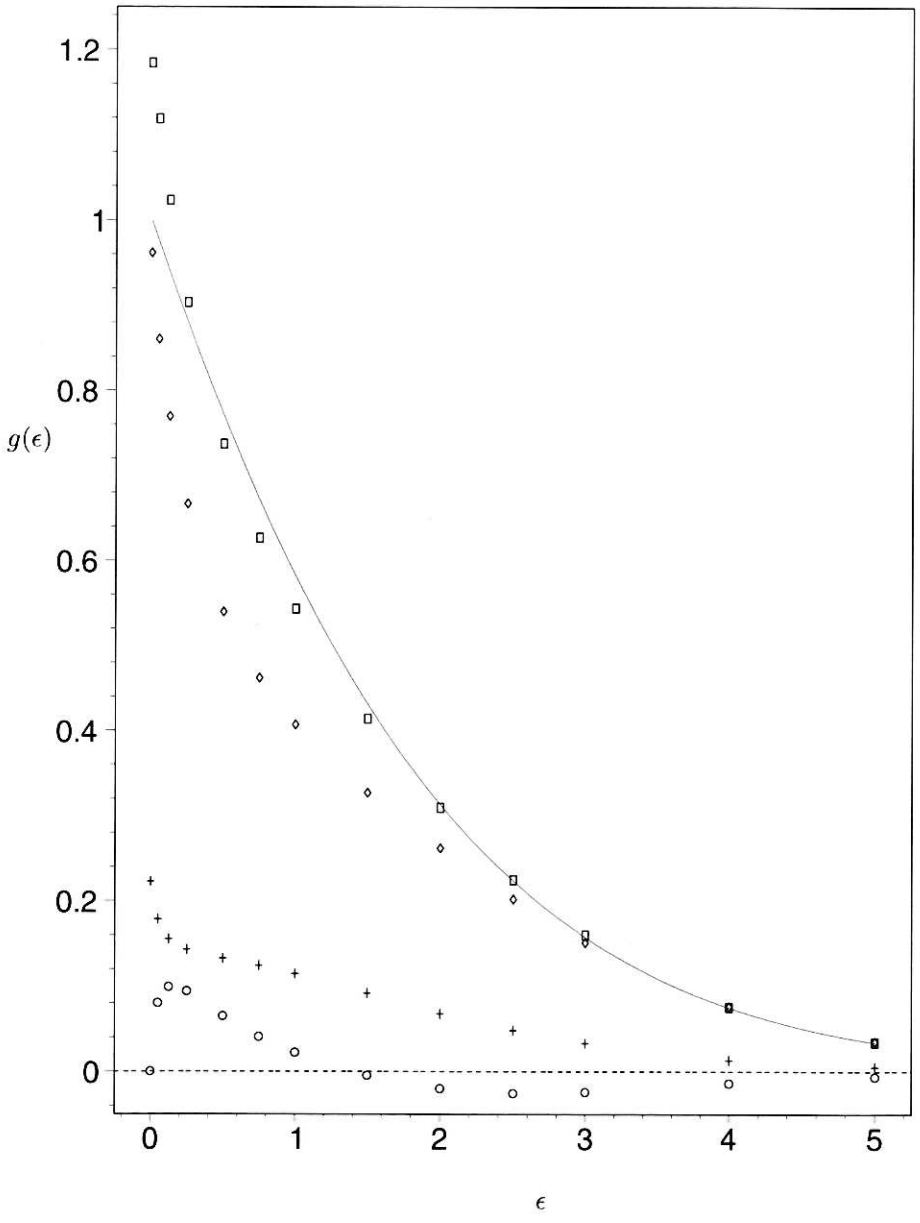

Figure 5.3: One-particle Green's function $g(\epsilon)$ for filling fraction $\nu' = \frac{1}{2}$ as a function of energy, both in units $[k_B T]$. The upper solid curve is the exact result; the data points are the numerical results for: $g^{(1,0)}$ (diamonds), $g^{(2,0)}$ (circles) and $g^{(1,1)}$ (crosses). The sum of all contributions is represented by squares.