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Bock, W.; Smit, J.; Vink, J.C.

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Chiral gauge theories on the lattice and restoration of gauge symmetry

Wolfgang Bock[a*], Jan Smit[b] and Jeroen C. Vink[a]

[a] University of California, San Diego, Department of Physics,
9500 Gilman Drive 0319, La Jolla, CA 92093-0319, USA

[b] Institute of Theoretical Physics, University of Amsterdam,
Valckenierstraat 65, 1018 XE Amsterdam, The Netherlands

We investigate a proposal for the construction of models with chiral fermions on the lattice using staggered fermions. In this approach the gauge invariance is broken by the coupling of the staggered fermions to the gauge fields. We aim at a dynamical restoration of the gauge invariance in the full quantum model. If the gauge symmetry breaking (SB) is not too severe, this procedure could lead in the continuum limit to the desired gauge invariant chiral gauge theory.

1. Introduction

The non-perturbative formulation of chiral gauge theories is still an unsolved problem. For a review on recent attempts in this direction see ref. [1]. In this contribution we will focus on the staggered fermion approach. The basic idea here is to use the species doublers as physical degrees of freedom, rather than to try to decouple them. Staggered fermions are represented on the lattice by a one-component complex field χ_x, which describes four Dirac flavors in the continuum limit. The Dirac and flavor components of these four staggered flavors do not appear in an explicit form since they are spread out over the lattice and it is not clear a priori how to construct actions with arbitrary spin-flavor coupling. It has been shown in ref. [2] that this can be achieved by first rewriting the naive action in terms of the the 4×4 matrix fields Ψ^{df} with d and f acting as Dirac and flavor indices and then using the relation

$$\Psi_x^{df} = \frac{1}{8} \sum_b [\gamma^{x+b}]^{df} \chi_{x+b} \qquad (1)$$

to express the action in terms of the independent χ-fields. The sum in (1) is over the 16 corners of a lattice hypercube, $b_\mu = 0, 1$, and $\gamma^x \equiv \gamma_1^{x_1} \gamma_2^{x_2} \gamma_3^{x_3} \gamma_4^{x_4}$. It is evident from (1) that the 16 spin-flavor components of Ψ_x are not in-

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*Speaker at the conference

dependent. One can show however that the components of the Fourier transform $\Psi(p)$ are independent in the restricted momentum interval $-\pi/2 < p_\mu \leq +\pi/2$. The target model which we will consider in this paper is a four flavor axial-vector model with all axial charges q equal to $+1$. For the action in terms of the χ-fields we find the following expression

$$S = -\frac{1}{2} \sum_{x\mu} \left[c_{\mu x} \frac{1}{16} \sum_b \eta_{\mu x+b} \right.$$
$$\times (\overline{\chi}_{x+b} \chi_{x+b+\hat{\mu}} - \overline{\chi}_{x+b+\hat{\mu}} \chi_{x+b})$$
$$+ is_{\mu x} \frac{1}{16} \sum_{b+c=n} \eta_{5x+b} (\eta_{\mu x+c} \overline{\chi}_{x+b} \chi_{x+c+\hat{\mu}}$$
$$\left. - \eta_{\mu x+b} \overline{\chi}_{x+b+\hat{\mu}} \chi_{x+c}) \right] , \qquad (2)$$

where $c_{\mu x} = \operatorname{Re} U_{\mu x}$, $s_{\mu x} = \operatorname{Im} U_{\mu x}$, $n = (1, 1, 1, 1)$. The sign factors $\eta_{\mu x} = (-1)^{x_1 + \dots + x_{\mu-1}}$ and $\eta_{5x} = -\eta_{4x} \eta_{3x+\hat{4}} \eta_{2x+\hat{3}+\hat{4}} \eta_{1x+\hat{2}+\hat{3}+\hat{4}}$ represent here the γ_μ and γ_5. It can be shown easily that (2) reduces in the classical continuum limit to the target model. The expression (2) however lacks gauge invariance, since gauge transformations on the U-fields cannot be carried through to the χ-fields. A perturbative calculation in two dimensions has shown that the model performs well for smooth external gauge fields [3]. The important issue however is whether this remains true also when taking into account the full quan-
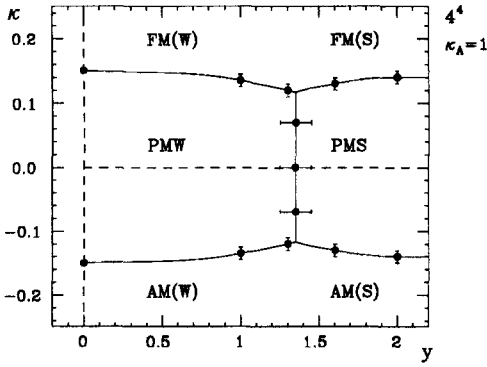
Figure 1. *Phase diagram of the ISF model.*

tum fluctuations. Based on experience with other non-gauge invariant lattice models we shall aim here for a dynamical symmetry restoration (SR), which may occur if the SB effects due the high momentum modes are not too severe.

2. Restoration of gauge symmetry

Let's start first from a generic non-gauge invariant lattice action, $S(U)$. It has been shown in ref. [2] that the partition function can be written in the following form $Z = \int DU e^{S(U_{\mu\bullet})} = \int DU DV e^{S(V_x^\dagger U_{\mu x} V_{x+\mu})}$, where the V_x is a radially frozen Higgs field. The new form of the action $S(V_x^\dagger U_{\mu x} V_{x+\hat\mu})$ is trivially invariant under the local gauge transformations $U_{\mu x} \rightarrow \Omega_x U_{\mu x} \Omega_{x+\hat\mu}^\dagger$, $V_x \rightarrow \Omega_x V_x$. The important question of symmetry restoration is whether the resulting model still can describe the physics of the underlying gauge invariant model which in our case is the chirally invariant axial-vector vector model in the continuum. Before we try to find an answer to this question for the staggered model, let us first give an example of a non-chiral lattice model where gauge symmetry gets restored. The model is given by the action

$$ S = -\frac{1}{2} \sum_{x\mu} \left\{ \overline{\psi}_x' \gamma_\mu (U_{\mu x} P_L + U_{\mu x}^* P_R) \psi_{x+\hat\mu}' - \right. $$

$$ \left. \overline{\psi}_{x+\hat\mu}' \gamma_\mu (U_{\mu x}^* P_L + U_{\mu x} P_R) \psi_x' \right\} - y \sum_x \overline{\psi}_x' \psi_x' , \quad (3) $$

which for $y = 0$ reduces in the classical continuum limit to a gauge invariant $8(q = +1) + 8(q = -1)$ axial-vector model. This model we shall regard now as our target model. The mass term in (3) breaks gauge invariance. The action, which results after integrating over the gauge fields in the partition function is given by (3) with $U_{\mu x} \rightarrow V_x^* U_{\mu x} V_{x+\hat\mu}$. The ψ'-fields in this action are screened form the gauge fields by the V-fields and are therefore *neutral* with respect to the U(1) gauge transformations. To see how the symmetry gets restored it is useful to express the action in terms of a *charged* fermion field ψ which is related to the ψ'-fields by a gauge transformation, $\psi_x' = (V_x^* P_L + V_x P_R)\psi_x$. The ψ-action is identical with (3) (with $\psi' \rightarrow \psi$), except that the bare mass term turns into a Yukawa-term $-y \sum_x \overline{\psi}_x (V_x^2 P_L + V_x^{*2} P_R)\psi_x$ with a charge two Higgs field V_x^2. We have studied this Yukawa-model in the global symmetry limit, $U_{\mu x} = 1$, and with the term $2\kappa \sum_{x\mu} \text{Re}\,(V_x^* V_{x+\hat\mu})$ added to the action. The model we have studied numerically is a $4(q = +1) + 4(q = -1)$ model which results after a mirror fermion doubling needed for the Hybrid Monte Carlo algorithm and a standard fermion number reduction. This model we shall call the invariant staggered fermion (ISF) model, to emphasize that the ψ-and ψ'-forms are related by a gauge transformation. The κ-y phase diagram of the ISF model is displayed in fig. 1. Besides the ferromagnetic (FM) and antiferromagnetic (AM) phases there are two different symmetric phases, PMW and PMS. Previous investigations have shown that the physics in the PMW (PMS) phase is described by the action in terms of the ψ (ψ')-fields and that the fermions are massless (massive) in the PMW (PMS) phase. The gauge symmetry gets restored if the coefficient y of the SB term is $< y_c(\kappa) \approx 1.4$: Anywhere in the PMW phase, away from the phase boundaries, the scalar particles decouple and even though $y > 0$, the spectrum contains only free massless fermions. The effective action in this region (with gauge fields switched on) is given by (3) with the SB mass term dropped. In contrast for $y > y_c(\kappa)$ fermions are massive and do not couple to the gauge fields. In this region we do not recover the physics of the target model.
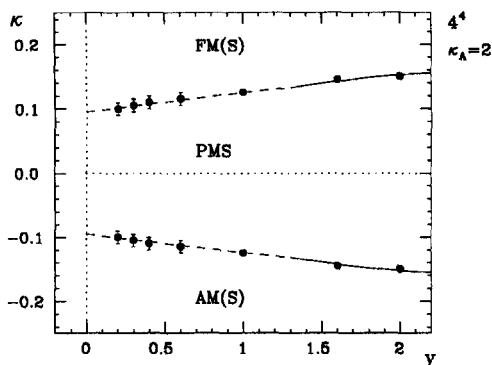
Figure 2. *Phase diagram of the NISF model.*

3. SR in the staggered model?

The gauge invariant version of the staggered fermion model with the Higgs fields included is given by (2) with $c_{\mu x} \to \mathrm{Re}\,(V_x^* U_{\mu x} V_{x+\hat{\mu}})$ and $s_{\mu x} \to \mathrm{Im}\,(V_x^* U_{\mu x} V_{x+\hat{\mu}})$. It is clear that the χ-fields are neutral with respect to the U(1) transformations which allows us also to add a term $-y \sum_x \overline{\chi}_x \chi_x$ to the action. In contrast to the ISF model we cannot relate the χ-fields by a gauge transformation to a charged fermion field. The crucial question however is whether the Higgs fields are sufficiently smooth such that this transformation could effectively take place, leading to a PMW phase at small y with a charged massless fermion (then gauge symmetry would be restored dynamically). The gauge symmetry is broken now due to the high momentum modes in the V-fields and the strength of the gauge SB cannot be easily controlled by a parameter in the action (like y in (3)). We hope however that the SB effects are not very severe and a PMW phase, where the gauge symmetry gets restored, emerges at small y. For technical reasons the numerical simulations have been performed in a vector-like $4(q = +1) + 4(q = -1)$ model which is obtained from (2) after a mirror doubling and which we shall call the non-invariant staggered fermion (NISF) model. The phase diagram of this model contains indeed at small y a phase with very bizarre properties, not at all characteristic

for a PMW phase [4]. Its existence could be ascribed to the hypercubic average in the γ_μ-term in (2). To make the model more similar to the ISF model we have dropped the $\sum_b$ in this term which does not change the classical continuum limit [4]. The κ-y phase diagram of this modified model is shown in fig. 2. It can be seen that the PMS phase now extends down to $y = 0$. Also on a larger lattice, which can accommodate a larger number of small momentum modes we found no indication for the emergence of a PMW phase. Another possibility to enhance the low momentum modes of the scalar field, is to increase the value of κ towards the FM-PM phase transition where the scalar field correlation length may become large enough that a PMW phase opens up. The FM(S)-PMS phase transition for $y \lesssim 1.3$ is however of first order and the scalar field correlation length cannot exceed a certain bound. Our runs at $\kappa \approx \kappa_c^{FM-PM}$ show indeed no qualitative difference from the results at other $\kappa < \kappa_c^{FM-PM}$. This negative result means that more sophisticated strategies have to be developed to make the SR working. As an alternative one can start also from a gauge fixed continuum model and regularize it using the lattice and staggered fermions. The model then becomes very similar to the Rome proposal, which uses Wilson fermions [5].

The numerical calculations were performed on the CRAY Y-MP4/464 at SARA, Amsterdam. This research was supported by the "Stichting voor Fundamenteel Onderzoek der Materie (FOM)", by the "Stichting Nationale Computer Faciliteiten (NCF)" and by the DOE under contract DE-FG03-90ER40546.

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