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de Bakker, B.V.

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# Absence of barriers in dynamical triangulation

Bas V. de Bakker<sup>1</sup>

*Institute for Theoretical Physics, University of Amsterdam, Valckenierstraat 65, 1018 XE Amsterdam, The Netherlands*

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## Abstract

Due to the unrecognizability of certain manifolds there must exist pairs of triangulations of these manifolds that can only be reached from each other by going through an intermediate state that is very large. This might reduce the reliability of dynamical triangulation, because there will be states that will not be reached in practice. We investigate this problem numerically for the manifold  $S^5$ , which is known to be unrecognizable, but see no sign of these unreachable states.

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## 1. Introduction

Although the basis of it was laid in 1982 [1], the dynamical triangulation model of quantum gravity has in the last few years received a lot of interest. See e.g. [2,3]. In this model, the path integral over euclidean metrics is defined by a sum over simplicial complexes. This sum can then be approximated using Monte Carlo techniques, where a computer program generates appropriately weighted configurations of simplices.

To generate all these configurations we need an algorithm that is ergodic, i.e. a set of moves that can transform any triangulation into any other triangulation with the same topology. A well known set of moves that satisfy this condition are the so-called  $(k, l)$  moves, whose ergodicity was shown in [4].

## 2. Noncomputability

Unfortunately, the number of moves we need to get from one configuration to another can be very large.

To be more precise, the following theorem holds: if the manifold under consideration is unrecognizable, then for any set of local moves the number of moves needed to get from one configuration of  $N$  simplices to another such configuration is not bounded by a computable function of  $N$ . This was shown in Ref. [5]. We will explain some of the terms in this theorem in a way that is not mathematically precise, but hopefully intuitively clear. See [5] for details.

A manifold is unrecognizable if, given a triangulation  $A$  of this manifold, there does not exist an algorithm that, given as input an arbitrary triangulation  $B$ , can decide whether  $A$  and  $B$  are homeomorphic. The definition of unrecognizability is not important for the rest of this article, it is only important to know that for some manifolds the above theorem holds. Certain four dimensional manifolds are unrecognizable, but for the sphere  $S^4$ , which is usually used in dynamical triangulation, this is not known. It is known, however, that the five dimensional sphere  $S^5$  is unrecognizable.

Local moves are moves that involve a number of simplices that is bounded by a constant, in other words a number that does not grow with the volume of the

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<sup>1</sup> E-mail: bas@phys.uva.nl.

configuration.

A computable function is a function from  $\mathbb{N}$  to  $\mathbb{N}$  that can be computed by a large enough computer. Although the computable functions are only an infinitesimally small fraction of all the functions from  $\mathbb{N}$  to  $\mathbb{N}$ , most functions one can think of are computable. A fast-growing example of a computable function would be  $N!! \cdots !$  with  $N$  factorial signs.

The above theorem might seem a terrible obstacle for numerical simulation, but the theorem says nothing about the behaviour of the number of moves needed to generate any particular size of configuration. In fact, take any function  $g(n)$  with the property that it is not bounded by a computable function. Replacing any finite number of values of this function will result in another function  $g'(n)$  that is also not bounded by a computable function.

### 3. Barriers

From the theorem stated above it follows that for an unrecognizable manifold the maximum size  $N_{\text{int}}(N)$  of the intermediate configurations needed to interpolate between any two configurations of size  $N$  is also not bounded by a computable function of  $N$ . If  $N_{\text{int}}(N)$  did have such a bound, a bound on the number of possible configurations of size less than or equal to  $N_{\text{int}}(N)$  would be a bound on the number of moves needed, which would violate the theorem. A simple computable bound on the number of configurations of size  $N$  is  $((d+1)N)!$ , where  $d$  is the dimension of the simplices.

It was pointed out in [6] that this means that for such a manifold there must exist barriers of very high sizes between certain points in configuration space. Although the situation is not clear from the theorem, it seems natural that these barriers occur at all scales. We can then apply the following method, which was formulated in [6]. We start from an initial configuration with minimum size. For  $S^4$  and  $S^5$ , there is a unique configuration of minimum size with 6 and 7 simplices respectively. We increase the volume to some large number and let the system evolve for a while, which might take it over a large barrier. Next, we rapidly decrease the volume, hoping to trap the configuration on the other side of this barrier.

We can check whether this has happened by trying

to decrease the volume even more. If this brings us back to the initial configuration, we have gone full circle and cannot have been trapped at the other side of such a barrier. Conversely, if we get stuck we are apparently in a metastable state, i.e. at a point in configuration space where the volume has a local minimum.

This was tried in [6] for  $S^4$ , but no metastable states were found. To judge the significance of this, it is useful to investigate the situation for a manifold which is known to be unrecognizable. It is rather difficult to construct a four dimensional manifold for which this is known, but if we go to five dimensions this is easy, because already the sphere  $S^5$  is not recognizable.

### 4. Results

Because my program for dynamical triangulation was written for any dimension, it was not difficult to investigate  $S^5$ . The description by Catterall in [7] of his dynamical triangulation program for arbitrary dimension turned out to be a very close description of mine, presumably because both were based on ideas put forward in [8,9]. The Regge-Einstein action in the five dimensional model is

$$S = \kappa_5 N_5 - \kappa_3 N_3 \quad (1)$$

where  $N_i$  is the number of simplices of dimension  $i$ . This is not the most general action linear in  $N_i$  in five dimensions as this would take three parameters, but for the purposes of this paper this is not relevant.

I generated 26, 24 and 8 configurations at  $N_5 = 8000, 16000$  and  $32000$  simplices respectively. These were recorded each 1000 sweeps, starting already after the first 1000 sweeps, where a sweep is  $N_5$  accepted moves. All configurations were made at curvature coupling  $\kappa_3 = 0$ , making each configuration contribute equally to the partition function, in other words making them appear equally likely in the simulation. Looking at the number of hinges  $N_3$ , the system seemed to be thermalized after about 6000 sweeps, irrespective of the volume.

The critical value of  $\kappa_5$  (the bare cosmological constant) below which the volume diverges was measured as explained in [10,11]. It turned out to be 0.8252(4), 0.8366(5) and 0.8446(8) for the three volumes used. The last error cannot be trusted, because of the low statistics at the largest volume.

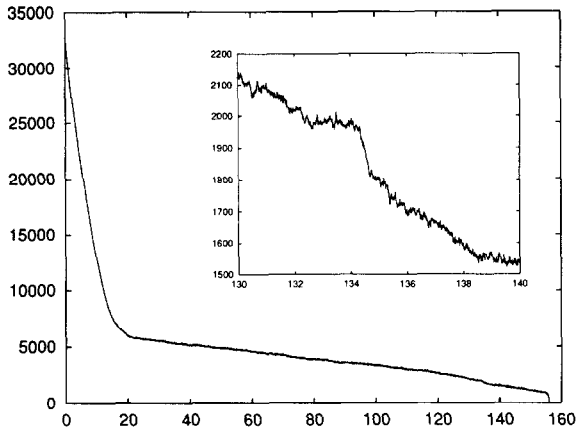


Fig. 1. A typical cooling run starting at  $N_5 = 32000$ , using  $\kappa_5 = 2$ . The horizontal units are 1000 accepted moves. The vertical axis is the number of 5-simplices. The inset is a blowup of a small part of the curve.

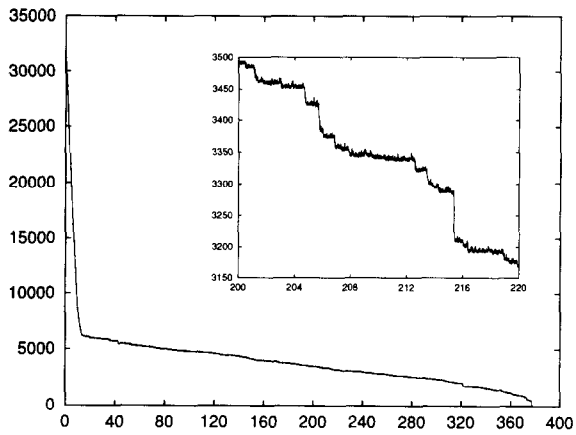


Fig. 2. As Fig. 1, but with  $\kappa_5 = 3$ .

Starting with these configurations, the volume was decreased by setting  $\kappa_5$  to a fixed number larger than the critical value. We call this process cooling, because it attempts to reach a configuration of minimum volume and thereby minimum action.

For each configuration we cooled four times with  $\kappa_5 = 2$  and two times with  $\kappa_5 = 3$ . For both values of  $\kappa_5$  used one of the runs is shown in Figs. 1 and 2. In the insets we can see the typical volume fluctuations that occurred. These were of the order 30 at  $\kappa_5 = 2$  and 6 at  $\kappa_5 = 3$ . The volume would first decrease very quickly until it reached roughly a quarter of the starting value and then started to decrease much slower. In all cases the initial configuration of 7 simplices was reached.

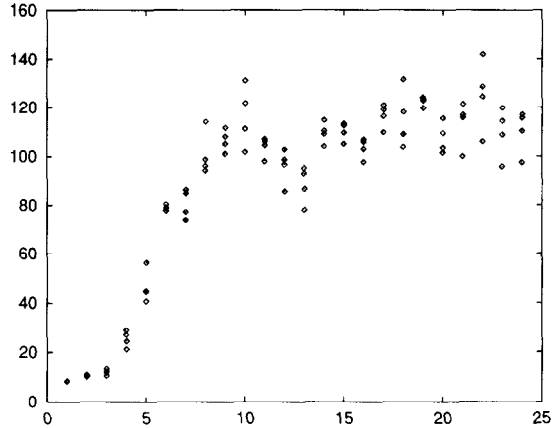


Fig. 3. Number of moves needed for cooling at  $\kappa_5 = 2$  as a function of the number of sweeps at the large volume of 16000 simplices. Horizontal units are 1000 sweeps, vertical units are 1000 accepted moves.

We also tried to use  $\kappa_5 = 4$ . The same behaviour of fast and slow cooling was seen, but the latter was so slow that due to CPU constraints these had to be stopped before either a stable situation or the minimal volume was reached.

There is an important difference between four and five dimensions. In four dimensions there is a move that leaves the volume constant. Therefore the system can evolve at constant volume. In five dimensions this is not possible, because all moves change the volume. In this case the volume has to fluctuate for the configuration to change. This is why much larger values of the cosmological constant (such as were used in [6]) would effectively freeze the system.

Initially, before the system was thermalized, there was a strong positive correlation between the time used to evolve the system at large volume and the time needed to cool the system back to the initial configuration. The rates of fast and slow cooling did not change, but the volume at which the slow cooling set in became larger. Some of these times are shown in Fig. 3 for the case of 16000 simplices. This trend does not continue and the cooling times seem to converge.

## 5. Discussion

So, contrary to expectation, no metastable states were seen. Small volume fluctuations were necessary, but these gave no indication of the high barriers ex-

pected.

It is not clear why we don't see any metastable states. There are several possibilities. First the barriers might be much larger than 32000 and we need to go to extremely large volumes before cooling. Second, there might be no barriers much larger than the volume for the volumes we looked at and the size of the intermediate configurations needed still grows very slowly for the volumes considered, even though this size is not bounded by a computable function. Third, the metastable regions in configuration space might be very small and the chance that we see one is therefore also very small.

It was speculated in [6] that the absence of visible metastable states might indicate that  $S^4$  is indeed recognizable. The results shown in this paper for  $S^5$  (which is known to be unrecognizable) indicate that, unfortunately, the results for  $S^4$  say nothing about its recognizability. This, of course, in no way invalidates the conclusion of [6] that the number of unreachable configurations of  $S^4$  seems to be very small.

Recognizability is not the only thing that matters. Even if  $S^4$  is recognizable, this says little about the actual number of  $(k, l)$  moves needed to interpolate between configurations, except that this number might be bounded by a computable function.

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