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Assenza, T.; Brock, W.A.; Hommes, C.H.

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Animal Spirits, Heterogeneous Expectations and the Amplification and Duration of Crises

Tiziana Assenza[a,b] William A. Brock[c] Cars H. Hommes[b]

[a] *ITEMQ, Catholic University of Milan, Tiziana.Assenza@unicatt.it*,

[b] *CeNDEF, University of Amsterdam, Valckenierstraat 65-67, C.H.Hommes@uva.nl*,

[c] *University of Wisconsin, Department of Economics, Madison, wbrock@ssc.wisc.edu*

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Abstract

We introduce a simple equilibrium model of a market for loans. Households lend to firms and form expectations about their loan default probability. Under heterogeneous expectations, with switching between forecasting strategies driven by reinforcement learning, even a small fraction of pessimistic traders has a large aggregate effect, causing a heterogeneous expectations risk premium, i.e. significantly higher contract rates for loans and significantly lower output. Our stylized model illustrates how animal spirits and heterogeneous expectations may lead to a confidence loss and to financial instability amplifying the magnitude of economic crises and slowing down recovery.

JEL codes: E32, D83, D84

Keywords: Heterogeneous Expectations, Crises, Animal Spirits.

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1 Introduction

In their recent book Akerlof and Shiller (2009) stressed the importance of "animal spirits" for the origin and propagation of a financial-economic crisis, for the subsequent recession and for the "exit" process from the recession. They discuss recent advances in behavioral economics in order to identify different types of "animal spirits", with "confidence" being one of the cornerstone animal spirits. Akerlof and Shiller point to an important problem facing economics: "confidence" (whatever this means) shares with "financial factors" (whatever this means) the fate of being difficult to conceptualize, model, and measure. The present paper is essentially an attempt to build a dynamic equilibrium model of agents' confidence. We introduce a simple dynamic equilibrium model for loanable funds, and show how a sudden collapse of confidence may, on the one hand, accelerate and amplify the downturn of a crisis after a negative shock, and, on the other hand, slow down the recovery from a crisis. The core ingredient of our model is the crucial role we assign to expectations' heterogeneity and, especially how the dynamics of that heterogeneity feeds into the dynamics of wages, output and the dynamics of contracting terms that the lending side of the economy imposes on the borrowing side of the economy in dynamic equilibrium. After all, it is almost a commonplace that the behavior of a variable in the aggregate - i.e. at the macroeconomic level - does not necessarily correspond to the behavior of the same variable as decided at the microeconomic level by a "representative" individual: *"Any meaningful model of the macroeconomy must analyze not only the characteristics of the individuals but also the structure of their interactions"* (Colander et al., 2008, p.237). Arrow already stressed the key role of heterogeneous expectations for modeling individual interactions: *"One of the things that microeconomics teaches you is that individuals are not alike. There is heterogeneity, and probably the most important heterogeneity here is heterogeneity of expectations. If we didn't have heterogeneity, there would be no trade. But developing an analytic model with heterogeneous agents is difficult."* (Ken Arrow, In: Colander et al., 2004, p.301).

In behavioral modeling of animal spirits and confidence, bounded rationality plays a key role. In the last two decades in macroeconomics much work has already been done on bounded rationality and adaptive learning; see e.g. Sargent, (1993) and Evans and

Honkapohja, (2001), for extensive discussions. In the adaptive learning literature, the representative agent assumption is still the workhorse of contemporary models. Moreover most attention has focussed on cases where the learning process ends with the discovery of the "true model" of the economy, thus confirming rational expectations ex post. More recently a number of macro models with heterogeneous expectations have been introduced, e.g. Brock and de Fontnouvelle (2000), Evans and Honkapohja (2003, 2006), Berardi (2007) and Assenza and Berardi (2009). We will use the heterogeneous expectations framework of Brock and Hommes (1997,1998), where agents are boundedly rational and switch between different expectations rules based upon their relative success[1]. Branch and Evans (2006), Branch and McGough (2009), Lines and Westerhoff (2010), Anufriev et al. (2012), Brazier et al. (2008) and DeGrauwe (2011) have applied this heterogeneous expectations framework in various macro-economic settings. Cornea et al. (2011) recently estimated a heterogeneous expectations model with forward looking fundamentalists versus backward looking naive expectations to US-inflation data.

There is quite some empirical evidence for the persistence of heterogeneity in expectations, both in survey data and in laboratory experiments. For example, Mankiw et al. (2003), Branch (2004) and Pfajfar and Santoro (2010) provided empirical evidence in support of heterogeneous expectations using survey data on inflation expectations. Expectations heterogeneity in experimental data is found e.g. in Hommes et al. (2005), Adam (2007), Pfajfar and Zakelj (2011), Assenza et al. (2011) and Roos and Luhan (2012); see Duffy (2008) for an overview of experimental work in macro.

In order to model the Akerlof-Shiller "animal spirits" and "confidence", we apply the Brock-Hommes heterogeneous expectations framework to a dynamic equilibrium model of loanable funds. We abstract from the complexity of the real world contract terms for a loan by using a one-dimensional proxy variable that we call the "contract rate". The reader should think of a contract rate not only as a measure of the interest rate for the loan, but more generally of "qualification adjusted contract terms" describing today's

[1]Simsek (2011) and Scheinkman and Xiong (2003) have stressed the role of overly optimistic (over confident) believers in driving bubble like phenomena in a framework where rational agents take into account the presence of overly optimistic believers. In our model most agents are boundedly rational, without perfect knowledge about the beliefs of other agents; see Hommes (2006) for an overview and extensive discussion of heterogeneous expectations and bounded rationality.

difficulties of getting a loan. A higher contract rate then does not necessarily mean a higher interest rate for the loan, but represents increasing general qualifications to obtain a loan, e.g. raising credit score qualifications, increasing down payment requirements for the loan, etc.

We borrow from recent work by Brock and Manski (2008, 2011), (B&M hereafter) to describe and conceptualize ambiguity and pessimism in a credit market economy. In particular B&M take into account the existence in credit markets of an informational problem due to partial knowledge of loan repayments, i.e. lenders do not know a priori whether a borrower will totally repay his debt or only part of it, or, in the worst case scenario, he will not repay at all. In B&M lenders must build a model of borrower behavior, which they are unable to completely specify due to lack of knowledge. We assume that most lenders lack fully rational expectations in forming expectations about the future share of loans that will be paid back. While B&M use a static model, we study the role of expectations in a dynamic equilibrium model for loanable funds driven by an exogenous stochastic process for the probability that loans will be paid back. We deviate from rational expectations by considering a model with heterogeneous, boundedly rational expectations. In particular, we replace rational expectations with other heterogeneous types of expectational schemes, including rational, naive, average, trend following and pessimistic expectations. As in Brock and Hommes (1997,1998), agents select among forecasting rules, depending upon the relative success of each rule in predicting the loan default probability.

The presence of non-rational expectations and heterogeneity will play an important role when the credit market experiences an unexpected negative shock. The main result of our paper is that heterogeneity in expectations and the presence of pessimistic agents, even when their fraction is relatively small, has a significant and persistent aggregate effect. Indeed, even a small fraction of pessimistic traders causes a *heterogeneous expectations risk premium*, i.e. significantly higher contract rates in the loan market and significantly lower wages and output. Heterogeneity in expectations affects both the magnitude of a crisis and the speed of recovery from a crisis. More precisely, heterogeneity in expectations has a significant effect upon the increase of the contract rates for

loans, a subsequent decline of lending, wages and output and slows down the recovery from an economic crisis.

The present paper is closely linked to the "confidence" "Animal Spirits" of Akerlof and Shiller (2009), because we introduce a measure of confidence, represented by the lender's expectation about the borrower's probability of success, i.e. the probability to repay the loan. In fact, we can interpret the probability of success as a measure of optimism about the share of borrowers that will be solvent. In other words the higher the expectation of the probability of success the higher the lender's confidence that tomorrow the borrower will reimburse the loan (and vice versa). We view our paper as moving a step ahead introducing the endogenous role of heterogeneous expectations in building an explicit stylized dynamic model of (part of) Akerlof and Shiller's conceptual framework of animal spirits and confidence to model economic crises. This enables us to study the way in which heterogeneity affects the path towards recovery after a negative shock to the economy. In particular we find that a snap collapse of confidence, due to an unanticipated negative shock, in the presence of heterogeneous agents, may amplify a downturn and may keep the economy in a recession phase for a longer period than in the case of a representative rational agent. To put it in another way we show how different individual expectational schemes on "confidence" impact the aggregate dynamics of output and contract terms in our model. While we emphasize the problems caused by excessively pessimistic beliefs and/or the presence of ambiguity and the aversion caused by it, we could just as easily use our model to study the opposite case of problems caused by excessively optimistic beliefs.

The paper is organized as follows. Section 2 introduces the modeling framework describing households and firms, while Section 3 presents the dynamic equilibrium. In Section 4 we consider a number of homogeneous expectations benchmarks, including naive, rational, average, trend following and pessimistic (minimum) expectations. Section 5 focuses on heterogeneous expectations and presents a number of simple 2-type examples as well as a 6-type example collecting all previous homogeneous rules. Finally, Section 6 concludes.

2 The model

This section describes the basic ingredients of our framework. We consider a market for loanable funds that is populated by households/lenders and firms/borrowers. The households' sector, which also represents the supply side of the market for loanable funds, is built by means of an overlapping generations framework in which each agent when young consumes ($c_{t,t}$) and saves earnings (s_t) from work, with wages $w_{p,t}$ and an endowment (ω_y). Savings are invested either in a safe asset or in a risky asset (productive investment). When old the agent consumes ($c_{t,t+1}$) an endowment (ω_o) and the average return on investments.

The demand side of the market for loanable funds in our economy is represented by firms that borrow a certain amount of capital (x_t) for production and remunerate work after paying back their debt. The remuneration for work is used by households to consume and to save. Savings are used to extend loans to the firms' sector.

2.1 Households

The supply side of our economy is described by means of a two-period overlapping generations structure. We assume that the young agent at date t has preferences defined over consumption when young $c_{t,t}$ and when old $c_{t,t+1}$. For the sake of convenience, we assume a logarithmic utility function. The objective function therefore is

$$u_t = \ln c_{t,t} + \ln c^e_{t,t+1}\,, \tag{2.1}$$

where $c^e_{t,t+1}$ is expected consumption when old. When young, the agent works and earns a real wage $w_{p,t}$ (i.e. wages from the productive sector), and receives an (exogenous) endowment ω_y . He invests his savings s_t partly in a safe asset, which yields a known fixed return ρ at $t+1$, and partly in a risky asset whose rate of return λ_{t+1} in period $t+1$ is uncertain. Investment in the risky asset can be conceived of as employment of resources ("capital") in the productive sector, whose output is uncertain. The expectations by the young formed at date t on the return of the risky asset at date $t+1$ are denoted by λ^e_{t+1}. When old, the agent retires and receives an (exogenous) endowment ω_o (at the

beginning of old age) and the return on asset investments. The budget constraint of the agent when young and when old respectively, therefore, are

$$c_{t,t} \leq w_t - s_t\,, \tag{2.2}$$

$$c^e_{t,t+1} \leq \omega_o + s_t[(1-\delta_t)\rho + \delta_t\lambda^e_{t+1}]\,, \tag{2.3}$$

where $w_t = w_{p,t} + \omega_y$, with $w_{p,t}$ labour income and ω_y endowment of the young. The decision problem of the young is to "optimize" (2.1) subject to (2.2) and (2.3). At date t the young agent decides real savings s_t and allocates a fraction δ_t to the "risky" asset which he anticipates to produce a real amount $s_t\delta_t\lambda^e_{t+1}$ available for consumption in $t+1$. Therefore $s_t\delta_t\lambda^e_{t+1}$ can be interpreted as expected production obtained employing $s_t\delta_t$ in the productive sector. It follows that λ^e_{t+1} can be interpreted as the expected *average productivity of capital* in this context. The amount $s_t(1-\delta_t)$ allocated at date t to the safe asset is known by the young at date t to produce $s_t(1-\delta_t)\rho$ available for consumption in period $t+1$. The expression in brackets in (2.3) i.e.,

$$\mu^e_{t+1} =: (1-\delta_t)\rho + \delta_t\lambda^e_{t+1}, \tag{2.4}$$

will be denoted as the expected *average return on investment*. Substituting the constraints into the objective function one ends up with the following maximization problem

$$\max_{s_t} \quad \ln(w_t - s_t) + \ln(\omega_0 + s_t\mu^e_{t+1}). \tag{2.5}$$

The FOC gives the following expression for savings

$$s_t = \frac{1}{2}\left(w_t - \frac{\omega_o}{\mu^e_{t,t+1}}\right). \tag{2.6}$$

Assuming, for the sake of simplicity, zero endowment when old i.e., $\omega_o = 0$, the FOC simplifies to

$$s_t = \frac{w_t}{2}. \tag{2.7}$$

Note that (2.7) says that, conditional on w_t, the demand for investment is perfectly inelastic w.r.t. known and unknown returns on assets next period.

2.2 Firms' demand for loanable funds

Following Brock and Manski (2008, 2011) we assume that borrowers get into debt in order to finance productive investments. Moreover, if returns on investments turn out to be too low, they may not be able to pay back. Therefore, we introduce a (time varying) *probability of success*, p_t and a *probability of bankruptcy* $1 - p_t$. The probability of success represents the share of firms that will be able to pay back their loans. Given the assumptions above firms choose the amount of capital x_t, borrowed from the lending side of the economy, at time t solving the maximization problem:

$$\max_{x_t} \ \{p_t(g(x_t) - r_t x_t) + (1 - p_t)(-r_t x_t)\} = \max_{x_t} \ \{p_t g(x_t) - r_t x_t\}, \tag{2.8}$$

where $r_t > 1$ is the gross "contract rate" (i.e. the "*rental rate*" on capital) and $g(x_t)$ is the production function, assumed to be strictly concave with decreasing returns to scale[2]. Here "contract rate" is a "proxy" for the general contract terms describing difficulties of getting a loan. A higher contract rate does not necessarily mean a higher interest rate for the loan, but also reflects an increase of the general qualifications to obtain a loan, such as raising credit score qualifications, increasing down payment requirements, etc. The maximization problem yields the following FOC:

$$p_t g'(x_t) = r_t \Longrightarrow x_t = x(r_t; p_t) = g'^{-1}\left(\frac{r_t}{p_t}\right). \tag{2.9}$$

Given the features of the production function $g(x_t)$, (2.9) represents a decreasing relation between the amount of capital at period t and the rental rate on capital in the same period therefore, it defines the *demand* for capital in this setting. We can define the returns to the "other factor" (i.e. labor) besides factor x as a function of the amount of factor x hired. In other words what is left over after overheads and capital are paid goes to other factors and the bulk of other factors are types of labor. Hence wages from the

[2]More precisely, we assume $g'(x_t) > 0$, $g''(x_t) < 0$ with right hand and left hand Inada conditions i.e., $g(0) = 0$, $g'(0) = \infty$, $g'(\infty) = 0$.

productive sector at time t, $w_{p,t}$, in our economy can be defined as:

$$w_{p,t} := p_{t-1}g(x_{t-1}) - r_{t-1}x_{t-1}. \tag{2.10}$$

Substituting eq. (2.9) we get

$$w_{p,t} := p_{t-1}g(x_{t-1}) - p_{t-1}g'(x_{t-1})x_{t-1}. \tag{2.11}$$

In the case of a Cobb Douglas production function $g(x_t) = x_t^\alpha$, where $0 < \alpha < 1$ represents the capital's share, (2.9) and (2.10) specialize to the *demand function* and wages given by

$$x_t = x(r_t; p_t) = \left(\frac{r_t}{p_t\alpha}\right)^{\frac{1}{\alpha-1}}, \tag{2.12}$$

$$w_{p,t} = p_{t-1}(1-\alpha)x_{t-1}^\alpha. \tag{2.13}$$

Substituting the demand for capital x_t from (2.12) into (2.13) we get the labor income in the case of Cobb Douglas production function

$$w_{p,t} = \eta(p_{t-1})^{\frac{1}{1-\alpha}} r_{t-1}^{\frac{\alpha}{\alpha-1}}, \tag{2.14}$$

where $\eta = \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)$. Since lenders get zero under bankruptcy and consequently wages for bankrupt firms are zero it follows that (2.14) represents wages paid by successful firms at time t. For later use it will also be useful to define the inverse demand function as

$$r_t = r(x_t; p_t) = \alpha p_t x_t^{\alpha-1}. \tag{2.15}$$

3 Equilibrium

In this section we will compute the equilibrium of our economy. Following Brock and Manski (2008, 2011), we indicate with $x_j(r_t)$ the j-th borrower's loan demand at a contract rate r_t. Hence for a "sample" of J firms the lender's expected *loan return* is

given by

$$\lambda^e_{t+1}(r_t) = \frac{\frac{1}{J}\sum_{j=1}^{J}\min\{\imath(j \in S_t)g(x_{j,t}), r_t x_{j,t}\}}{\frac{1}{J}\sum_{j=1}^{J} x_{j,t}}, \tag{3.1}$$

where $\imath(j \in S_t)$ is the indicator function which is unity if firm j is successful at date t and is zero otherwise. Moreover the numerator represents aggregate repayment and the denominator aggregate loan demand. We assume success is independently distributed across firms at each date t. Therefore, firm j chooses $x_{j,t}$ to satisfy:

$$x_{j,t} = \max_{x_{j,t}}\{p_{j,t}g(x_{j,t}) - r_t x_{j,t}\}. \tag{3.2}$$

provided that the maximized quantity is nonnegative, otherwise firm j shuts down and does not operate in period t, that is, it chooses $x_{j,t} = 0$.

Assume that the probability of success is the same for all firms at date t, i.e. $p_{j,t} \equiv p_t$, for all j. Then each firm solves the same maximization problem and the optimal solution is the same for all firms. Apply the Law of Large Numbers to Eq. (3.1) to obtain the "population" loan return function:

$$\lambda^e_{t+1}(r_t) = p^e_{t+1} r_t. \tag{3.3}$$

p^e_{t+1} is the expected probability of success, that is, the share of firms that is expected to be able to pay back the loan. The expected probability of success may be seen as a measure of "confidence" in our economy. Assuming risk neutrality, the *no arbitrage condition* is such that the return on the risky asset equals the return on the risk free investment i.e., $\lambda_t = \rho$. It follows that the no arbitrage value of the contract rate (r^*_t) is given by the following relation

$$r^*_t = \frac{\rho}{p^e_{t+1}}. \tag{3.4}$$

At this stage we have all the necessary ingredients to compute the equilibrium of our

model. Let us define

$$\Delta_t^*(r_t) := \bar{\imath}[r_t > \frac{\rho}{p_{t+1}^e}] := \bar{\imath}[r_t > r_t^*], \tag{3.5}$$

where the upper bar over the indicator function means that it is the set $[0,1]$ when $=$ holds instead of $>$. Hence we can define the *loan supply correspondence*, when old age endowment ω_o is zero, by

$$S_t(r_t) := \frac{w_t}{2}\bar{\imath}[r_t > r_t^* = \frac{\rho}{p_{t+1}^e}], \tag{3.6}$$

that is, when $r_t > r_t^*$ ($r_t < r_t^*$) all savings are invested into loans (the risk free asset). Note that it is the belief p_{t+1}^eformed at date t about the probability of success in $t+1$ that determines the loan supply at time t.

The demand for capital and the equilibrium value for the contract rate ($\bar{r}_t$) are determined by market clearing, i.e.

$$x(r_t; p_t) = S_t(r_t). \tag{3.7}$$

Since the supply correspondence is a (time varying) step function, there are two possibilities for the equilibrium, points A and B, as illustrated in Figure 1.

The first possibility for equilibrium (point A) is given by

$$r_A^* = \frac{\rho}{p_{t+1}^e} \tag{3.8}$$

$$x_A^* = x(r_A^*; p_t) = \left(\frac{\rho}{\alpha p_t p_{t+1}^e}\right)^{\frac{1}{\alpha-1}}, \tag{3.9}$$

arising when $x(r_A^*; p_t) < w_t/2$, where $x(\cdot)$ is the demand function (2.12).

The other possibility (point B) is given by

$$r_B^* = r(x_B^*; p_t) = \alpha p_t \left\{\frac{1}{2}\left[\omega_y + (p_{t-1})^{\frac{1}{1-\alpha}} r_{t-1}^{\frac{\alpha}{\alpha-1}}\eta\right]\right\}^{(\alpha-1)} \tag{3.10}$$

$$x_B^* = \frac{w_t}{2} = \frac{1}{2}\left[\omega_y + (p_{t-1})^{\frac{1}{1-\alpha}} r_{t-1}^{\frac{\alpha}{\alpha-1}}\eta\right] \tag{3.11}$$

and it arises when $x(r_A^*; p_t) > w_t/2$.

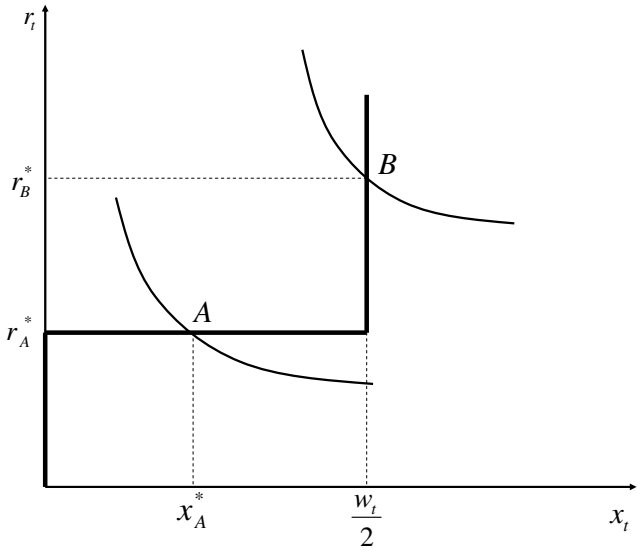


Figure 1: The loan supply correspondence (3.6) and the demand curve (2.9). Points A and B represents the two possible configurations of the temporary equilibrium allocations (x^*, r^*) depending on the time varying features of the demand and supply curves.

It is important to note the crucial role played by expectations on the firms' probability of success (p^e_{t+1}), the confidence measure in our economy. In fact, given the return on the risk free asset, the higher the expected probability of success the lower will be the non arbitrage contract rate (r^*_t) and, consequently, the higher will the the demand for capital (x^*_t). On the other hand, a low expected probability of success p^e_{t+1} causes the contract equilibrium rate r^*_A to rise sharply.

4 Homogeneous beliefs

So far we have not specified the probability of success p_t and how lenders form expectations about this probability to repay the loan. We are particularly interested in the situation where there is a "bad" exogenous shock to the economy and the probability of success suddenly drops. Instead of focussing on a single stochastic negative shock and an impulse response analysis, we assume a dynamic stochastic process for the probability of success and then study the corresponding equilibrium dynamics. We focus on the simple

case of an AR(1) process for the probability of success, given by

$$p_{t+1} = \mu + a(p_t - \mu) + \epsilon_t, \tag{4.1}$$

where μ is the long run average, a is the first order autocorrelation coefficient and ϵ_t is an IID random variable drawn from a normal distribution. Throughout the paper we fix $\mu = 0.95$, $a = 0.8$ and $\sigma = 0.01$, so that the (long run) average is 0.95 and there is some persistence in the probability of success. In all dynamic simulations in this paper, the realized random probability time series is as illustrated in Figure 2 (top panels). The success probability fluctuates between 0.899 and 0.969 over 100 periods. Between periods 20 and 30, the probability gradually declines to hit its lowest value 0.899 in period 31. We will refer to this lowest value as the "crisis" due to the exogenous shocks. Our main interest here is how confidence, that is, expectations about the probability of success, affects temporary equilibrium dynamics of contract rates, wages and output, and in particular, what happens after the exogenously generated crisis.

Before investigating the role of heterogeneous expectations, by way of comparison it is useful to consider a number of benchmark specifications of the lender's expectations in the simple case of a representative agent, i.e. we will consider some homogeneous expectations benchmarks. In addition to the standard rational expectations view, we allow for bounded rationality and consider a number of benchmark cases with a simple forecasting rule. Hey (1994) showed that in laboratory experiments where individuals forecast an exogenous stochastic AR1 time series, rational expectations is rejected in most cases and simple forecasting rules such as adaptive expectations provide a better description of individual forecasting behavior; see also Dwyer et al. (1993). In more recent learning to forecast laboratory experiments simple forecasting rules, such as naive expectations or a trend following rule, as described below, fit individual forecasting behavior quite nicely, see e.g. the survey in Hommes (2011).

4.1 Naive expectations

To get some intuition for the equilibrium dynamics, we start off with the simple case of naive expectations, where the forecast of the probability of success at period $t+1$ is given by last period's observation, i.e.,

$$p^e_{t+1} = p_t. \tag{4.2}$$

Figure 2a illustrates time series of the realized probability p_t, the naive forecast and the equilibrium contract rate r_t. Clearly the naive forecast lags realized probability by one period and the contract rate spikes in period 32, immediately after the probability of success hits its lowest value in the "crisis-period" 31 (or equivalently the probability of default hits its highest value). The dynamics of the contract rates is characterized by mean reversion to its long run equilibrium value $\bar{r} = \rho/\mu = (1.01/0.95) \approx 1.063$, where ρ is the risk free rate of return and μ is the long run mean of the AR(1) stochastic probability process. Under naive expectations, the dynamics of the contract rate r_t is thus completely driven by the exogenous probability of success, just lagging one period behind. The speed of recovery of the economy after the exogenous crisis in period 31 is the same as the speed of mean reversion of the realized probability of success, and lags only one period behind the true probability.

4.2 Rational expectations

In the case of rational expectations, lenders are assumed to have perfect knowledge about true stochastic probability process. Agents know that the probability of success follows the AR(1) process (4.1) and have perfect knowledge about its parameters. The rational forecast of the probability of success at period $t+1$ is given by

$$p^e_{t+1} = \mu + a(p_t - \mu). \tag{4.3}$$

Figure 2b illustrates time series of the realized probability p_t, the rational AR(1) forecast, and the equilibrium contract rate r_t. The rational forecast closely tracks the realized

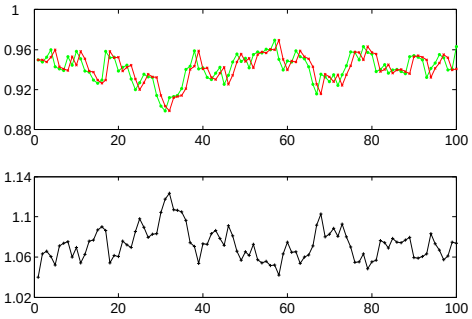
probability and the contract rate spikes in the crisis period 31 when the probability of success hits its lowest value, or equivalently when the probability of default hits its highest value. The dynamics of the contract rate under rational expectations is in fact similar to the case of naive expectations. The only difference is that there is no time lag and the peaks are somewhat less extreme, because the rational AR(1) rule correctly predicts mean reversion (on average) after an extreme observation, while naive expectations then uses the minimum (or maximum) observation. Under rational expectations, the dynamics of the contract rates is characterized by mean reversion to its long run equilibrium value $\bar{r} = \rho/\mu$, with the same speed as the true probability process and the peak in the contract rate coincides exactly with the (exogenously generated) crisis.

4.3 Average beliefs

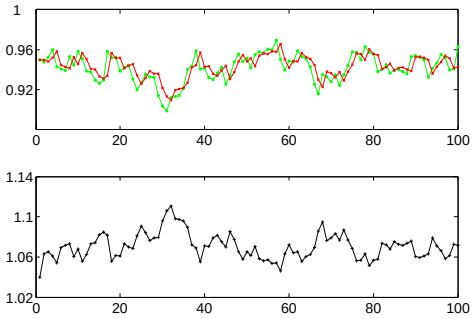
Another interesting case is when agents use long run averages in forecasting. In the case of average expectations, the forecast of the probability of success is given by the sample average of past observation, i.e.,

$$p^e_{t+1} = \frac{1}{t+1} \sum_{i=0}^{t} p_i. \tag{4.4}$$

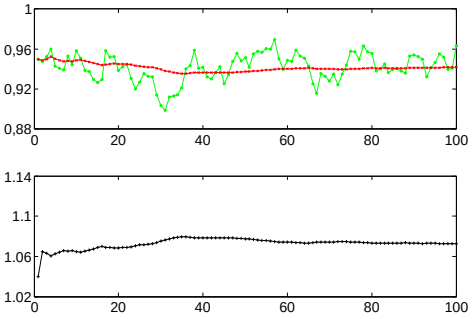
Figure 2c illustrates time series of the realized probability p_t, the average forecast and the equilibrium contract rate r_t. The average forecast adjusts slowly following realized probability and decreases gradually in the first 30 periods, until the probability of success hits its lowest value, in period 31. As a result, the contract rate slowly increases and slowly converges to its long run equilibrium level $\bar{r} = \rho/\mu \approx 1.063$. Hence, when all agents in the economy give equal weight to all past observations, the economy slowly converges to its long run equilibrium steady state.



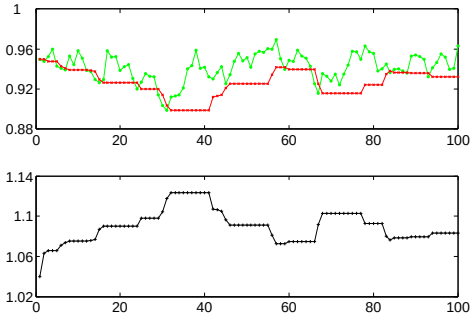
(a) Naive



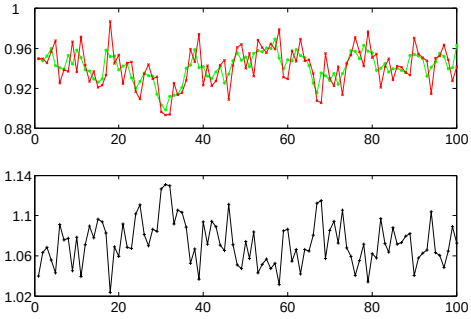
(b) Rational



(c) Average



(d) Minimum



(e) Trend Follower

Figure 2: Homogeneous expectations benchmarks: (a) naive, (b) rational (AR1), (c) average, (d) pessimistic (minimum), (e) trend follower. Top panels: realized (green) and expected (red) probability of success. Bottom panels: equilibrium contract rates r_t.

4.4 Trend following expectations

In the case of trend following expectations the forecast of the probability of success is given by a simple linear extrapolation rule

$$p^e_{t+1} = p_{t-1} + g(p_{t-1} - p_{t-2}). \tag{4.5}$$

Simple trend following rules belong to the most popular rules used in learning to forecast laboratory experiments with human subjects (e.g. Hommes, 2011) and are also popular among chartists' trading rules in financial markets and have been found in survey data (e.g. Frankel and Froot, 1990, Allen and Taylor, 1990). Figure 2e illustrates time series of the realized probability p_t, the trend follower forecast (4.5) and the equilibrium contract rate r_t. Trend followers may lead to overly pessimistic or optimistic expectations, when the trend following forecast undershoots its minimum or overshoots its maximum realized value. As a consequence, this leads to more extreme maximum values of the contract rate in periods $32-33$, immediately following the exogenously generated crisis period 31. Hence, the presence of trend followers may amplify the magnitude of a crisis.

4.5 Pessimistic expectations

Finally, consider the homogeneous benchmark case of pessimistic expectations. We model pessimistic expectations by a forecast that predicts that the probability of success remains at its lowest observed value in the last T periods, i.e.,

$$p^e_{t+1} = \min\{p_{t+1-T}, p_{t+2-T}, \cdots, p_{t-1}, p_t\}. \tag{4.6}$$

As a typical example in the simulations below we choose $T = 10$. Figure 2d illustrates time series of the realized probability p_t, the minimum forecast, together with the corresponding equilibrium contract rate r_t. The minimum forecast adjust according to the local minima of the observed probability and decreases until its lowest value in period 32 to stay there for 10 periods, after the probability of success hits its lowest value, in period 31. As a result, the contract rate increases gradually and hits its highest value in period 32 to stay there for 10 periods. Under pessimistic beliefs after each local minimum of the probability of success the contract rate spikes at a local maximum and stays there for at least $T = 10$ periods or jumps to a new (local) maximum. Hence, in a homogeneous world of pessimistic expectations crises are deep and much more persistent than the true probability process.

5 Heterogeneous beliefs

We extend our framework in order to take into account heterogeneity in agents' beliefs. In particular, we will follow Brock and Hommes (1997) to model heterogeneous expectations by a discrete choice model and evolutionary strategy selection based on their relative past performance. There is quite some empirical evidence for heterogeneity of expectations and strategy switching in various economic settings. For example, Branch (2004, 2007) estimates a simple switching model with heterogeneous expectations using exchange rate survey data, Vissing-Jorgensen (2003) presents evidence of heterogeneous beliefs of individual investors about the prospect of the stock market, and Shiller (2000) finds evidence that investor's sentiment changes over time, with both institutions and individual investors becoming more optimistic in response to recent significant increases of the stock market. Heterogeneous expectations switching models have been estimated/calibrated in various empirical applications, for example, on stock prices (e.g. Boswijk et al., 2007, Amilon, 2008, de Jong et al., 2009), exchange rates (e.g. Gilli and Winker, 2003, Westerhoff and Reitz, 2003), inflation (Cornea et al., 2012) and several commodities (e.g. gold prices Alfarano et al., 2005, and oil prices ter Ellen and Zwinkels, 2010). Anufriev and Hommes (2012) and Assenza et al. (2011) fitted a heuristics switching model to laboratory data of asset pricing and inflation/output forecasting experiments.

5.1 Heterogeneous expectations

Assume there are J types of lenders in our economy at date t. At date t, type j's forecast for period $t+1$ of the return on the risky asset is given by

$$\lambda^e_{j,t+1} = p^e_{j,t+1} r_t. \tag{5.1}$$

Hence, each forecasting rule is determined by its forecast $p^e_{j,t+1}$ of the probability of success, i.e. the probability that the firm will pay back the loan. Agents can choose between J different forecasting rules. The key idea of the switching model is that agents are boundedly rational and choose a forecasting strategy based upon its relative past

performance. Let $U_{j,t}$ be a weighted average of past squared forecasting errors of the returns, that is,

$$U_{j,t} = r_t^2 \big(p_t - p_{j,t}^e\big)^2 + \gamma\, U_{j,t-1}\,, \tag{5.2}$$

where γ is the weight given to past fitness. Let $u_{j,t}$ be the relative past squared forecasting errors of the returns of the risky asset, that is,

$$u_{j,t} = U_{j,t}/U_t^{tot}, \qquad U_t^{tot} = \sum_{j=1}^{J} U_{j,t}. \tag{5.3}$$

The fraction of the expectations rule j is updated according to a *discrete choice model with asynchronous updating* (Hommes et al., 2005; Diks and van der Weide, 2005)

$$n_{h,t} = \delta\, n_{h,t-1} + (1-\delta)\, \frac{e^{-\beta\, u_{h,t}}}{z_t}\,, \tag{5.4}$$

where $z_t = \sum_{j=1}^{J} \exp(-\beta\, u_{h,t})$ is a normalization factor. The asynchronous updating parameter $0 \leq \delta \leq 1$ reflects inertia in the choice of the heuristics [3]. In the extreme case $\delta = 1$, the initial impacts of the rules never change, no matter what their past performance was. At the other extreme, $\delta = 0$, we have the special case of synchronous updating, as in Brock and Hommes (1997), where all agents switch to better strategies in each period. In general, in each period only a fraction $1 - \delta$ of the heuristic's weight is updated according to the discrete choice model with asynchronous updating. The parameter $\beta \geq 0$ represents the intensity of choice measuring how sensitive predictor choice is to differences in heuristics' performance. In the extreme case $\beta = 0$, the relative weights of heuristics are not updated; at the other extreme $\beta = +\infty$, a fraction $1 - \delta$ of agents switch immediately to the best predictor. In the simulations of heterogeneous market equilibrium dynamics below, the parameters will be fixed at $\beta = 5$, $\delta = 0.5$ and $\gamma = 0$, but the results are fairly robust w.r.t. changes of these parameters.

[3] In recent laboratory experiments in various settings, for example in asset pricing forecasting (Anufriev and Hommes, 2012), in positive feedback (asset) and negative feedback (cobweb) markets (Anufriev et al., 2012) and in a New Keynesian macro framework (Assenza et al., 2011), it has been found that the value of the inertia parameter $\delta = 0.8$ or 0.9 is fairly high so that there is a tendency to stick to some rule before switching to another rule.

5.2 Heterogeneous market equilibrium

Under heterogeneous expectations, we define total supply of loans at date t as

$$S_t(r_t) = \frac{w_t}{2} \sum_{j=1}^{J} n_{j,t} \;\; \bar{\imath}[\lambda^e_{j,t+1}(r_t) > \rho]. \tag{5.5}$$

Recalling Eq. (3.3) we have

$$S_t(r_t) = \frac{w_t}{2} \sum_{j=1}^{J} n_{j,t} \;\; \bar{\imath}[p^e_{j,t+1} r_t > \rho], \tag{5.6}$$

where $p^e_{j,t+1}$ represents expectations of type j about the probability of success and $n_{j,t}$ represents the fraction of agents of type j at time t.

Figure 3 illustrates market equilibrium in the case of heterogeneous expectations with two types of agents ($J = 2$). Recall that, in the homogeneous case, the loan supply correspondence (3.6) is a step function (see Figure 1), with the loan supply switching from 0 to $w_t/2$ at the critical threshold $r^* = \rho/p^e_{t+1}$ determined by the expected probability of success. In the heterogeneous case with two types of expectations, $p^e_{1,t+1}$ and $p^e_{2,t+1}$, the loan supply correspondence is a 2-step function. If, for example, $p^e_{1,t+1} > p^e_{2,t+1}$ then the critical threshold levels are at $r^*_1 = \rho/p^e_{1,t+1}$, where the loan supply switches from 0 to $n_{1t}w_t/2$, and at $r^*_2 = \rho/p^e_{2,t+1}$, where the loan supply switches from $n_{1t}w_t/2$ to $w_t/2$.

5.3 Two type examples

To get some intuition for the equilibrium dynamics under heterogeneous expectations, in this section we consider three simple 2–types examples. In the first, average expectations competes against pessimistic (minimum) expectations. In the second example rational expectations, using the correct AR1 forecasting rule for the probability process, competes against pessimistic (minimum) expectations. In the third example trend follower expectations competes against pessimistic (minimum) expectations. In all three 2-type examples, for the exogenous AR1 stochastic time series of the probability of success, we use the same realizations as for the homogeneous benchmarks before, with its minimum realization in the "crisis-period" 31.

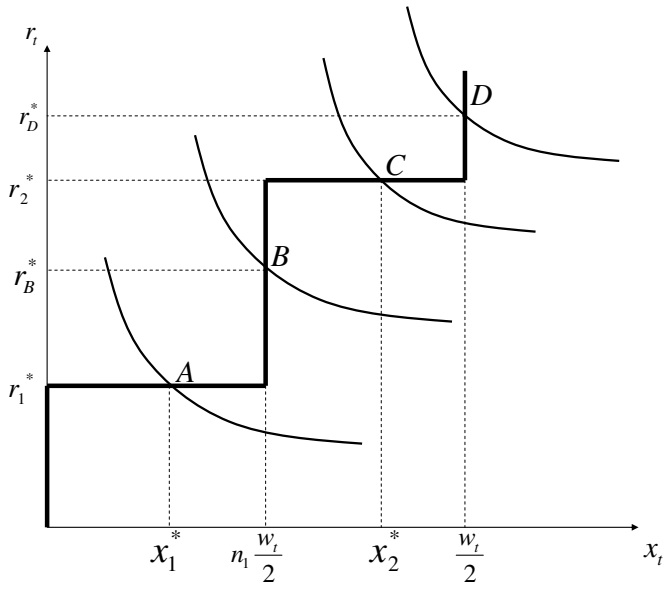


Figure 3: Temporary equilibria in a 2-type case, with four possible loan market equilibrium points, depending on the supply and demand curves. The figure illustrates the case $p^e_{1,t+1} > p^e_{2,t+1}$. The loan supply correspondence is a 2-step function with critical threshold levels at $r^* = \rho/p^e_{1,t+1}$, where the loan supply switches from 0 to $n_{1t}w_t/2$, and at $r^* = \rho/p^e_{2,t+1}$, where the loan supply switches from $n_{1t}w_t/2$ to $w_t/2$.

5.3.1 Average versus pessimistic beliefs

Figure 4 shows time series of the probability of success together with the average and pessimistic forecasts (top left panel). The fraction of pessimistic expectations fluctuates considerably over time (top right panel). The contract rate (bottom left panel) switches between persistent phases of high contract rates, when pessimistic expectations dominate, and phases of intermediate contract rates (around 6-8%), when average expectations dominate. The contract rate r_t is upward biased and is most of the time above the long run equilibrium value $\bar{r} = \rho/\mu \approx 1.063$, because neither of the two forecast rules accurately predicts the recovery of the probability of success to above average values. High contract rates occur in periods $25 - 29$ with contract rates around 10%, jumping to its highest value of around 12% in periods $32 - 36$. Output fluctuates around its average, with persistently low values after the crisis period 31. In a heterogeneous world, where none of the forecasting rules closely tracks the recovery of the economy when the probability of success recovers to high values, confidence about loan repayment is not

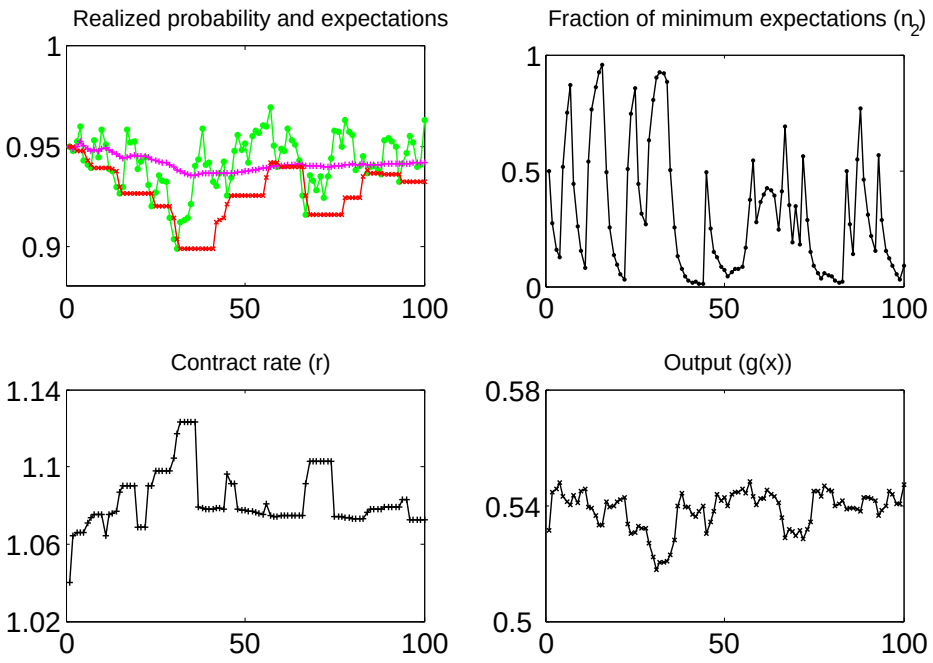


Figure 4: Two types example: average versus pessimistic (minimum) expectations. Upper left panel: realized probability of success (green), pessimistic expectations (red), average expectations (purple). Upper right panel: fraction of pessimistic believers. Lower left panel: contract rate. Lower right panel: total output.

restored and for a relatively long period contract rates remain high and output remains low.

5.3.2 Rational versus pessimistic beliefs

Figure 5 illustrates the case of rational expectations versus pessimistic beliefs. Rational agents know the true exogenous probability generating process (4.1) and therefore use the optimal, model consistent AR1 forecasting rule to predict the firms' probability of success. Notice that AR1 forecasters are not only rational forecasters, but also rational optimizers maximizing utility (2.1) under the budget constraint (2.2-2.3), given their forecast of the expected loan return $\lambda^e_{t+1} = p^e_{t+1} r_t$ in (3.3). Since the equilibrium contract rate r_t is known before making the forecasts, agents correctly take the behavior of other

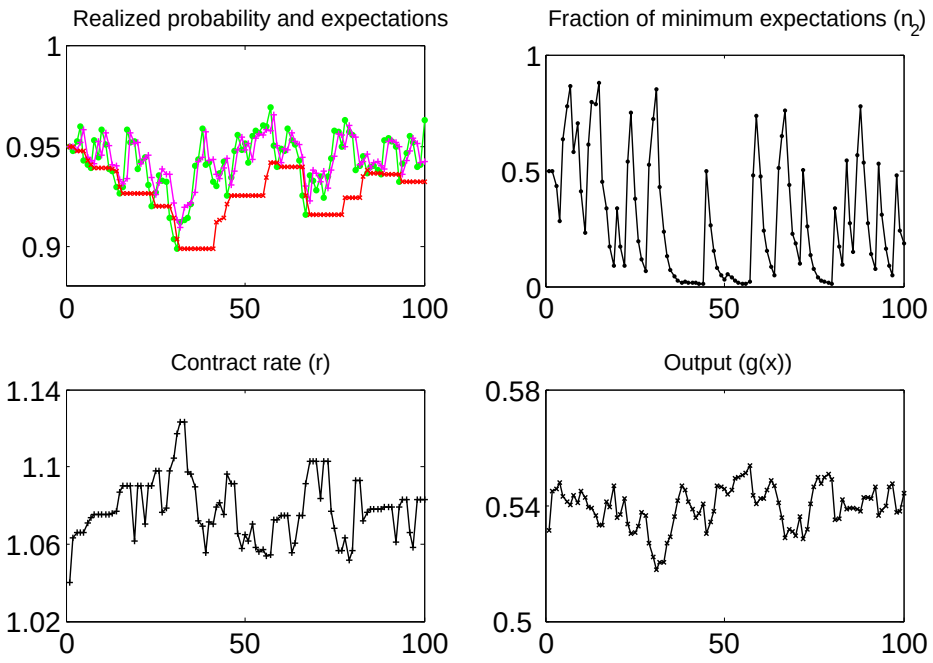


Figure 5: Two types example: rational (AR1) versus pessimistic expectations. Upper left panel: realized probability of success (green), pessimistic expectations (red), rational (AR1) expectations (purple). Upper right panel: fraction of pessimistic believers. Lower left panel: contract rate. Lower right panel: total output.

non-rational agents, who affect this equilibrium contract rate r_t, into account[4].

Figure 5 shows that the contract rate switches between persistent phases of high contract rates, when pessimistic expectations dominate, and phases of low contract rates (around 5%), when rational expectations dominate. The fraction of pessimistic and rational traders vary considerably over time (top right panel).

Persistent phases of high contract rate occur when the majority of agents switches to pessimistic expectations. In the previous 2-type example average versus pessimistic beliefs we have seen that if the probability of success recovers, average expectations drive down the contract rate somewhat but only to average values. Rational expectations more accurately track the true probability process and lead to normal or even low contract

[4]The same is true for other subjective forecasting rules, but AR1 forecasters are the only agents who are both rational optimizers and rational forecasters, while other forecasting rules are not rational forecasters as they are not model consistent with the exogenous stochastic probability process. See Sargent (1993) for a discussion of optimization and forecasting as two different aspects of rationality.

rates when the true probability recovers and attains above average values. However, this simple 2-type example also illustrates that in a heterogeneous 2-type world rational agents can not drive out pessimistic expectations and as a consequence in such a simple heterogeneous world crises i.e., periods of exceptionally high contract rates, are deeper and more persistent.

5.3.3 Trend followers versus pessimistic beliefs

Figure 6 illustrates the 2-type case of trend follower versus pessimistic (minimum) beliefs. The contract rate switches between persistent phases of high contract rates, when pessimistic expectations dominate, and phases of lower contract rates (around 6% or lower), when trend follower expectations dominate. Trend followers expectations overestimate (or underestimate) the probability of success during good (or bad) times, as the trend extrapolation rule may yield forecasts outside the range of realized values of the probability. As a consequence the presence of trend followers makes the behavior of the contract rate somewhat more extreme with even higher peaks, e.g. in periods $32 - 34$, after the crisis in period 31.

5.4 A stylized example with six belief types

In this section we present a heterogeneous expectations example with six different forecasting rules. In addition to the five rules discussed before, naive, rational, average, trend following and pessimistic (minimum) expectations, we introduce another slightly more pessimistic "worst case" expectations rule predicting that the probability of success is given by the minimum of its realization in the last T periods *and* all other forecasts in the previous period, i.e.,

$$p^e_{t+1} = \min\{p_{t+1-T}, p_{t+2-T}, \cdots, p_{t-1}, p_t, p^e_{1,t-1}, \cdots, p^e_{H,t-1}\}; \tag{5.7}$$

where $H = 5$ represents the number of different forecasting rules. As a typical example we choose $T = 10$. We will refer to this kind of beliefs as "worst case", because this forecasting rule represents agents with the lowest level of confidence, believing that the

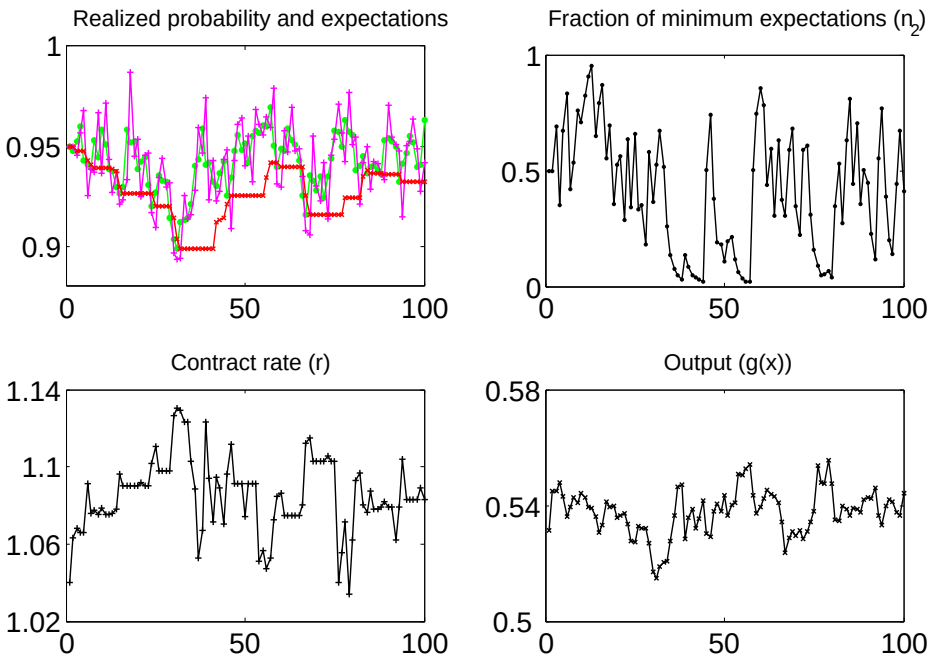


Figure 6: Two types example: trend followers versus pessimistic (minimum) expectations. Upper left panel: realized probability of success (green), pessimistic expectations (red), trend followers expectations (purple). Upper right panel: fraction of pessimistic believers. Lower left panel: contract rate. Lower right panel: total output.

probability remains at the lowest level between the last T realizations and all last period forecasts of other agents. We stress once more that our analysis focusses on problems caused by excessively pessimistic expectations, to study recovery from a crisis after a drop of confidence in the economy, but our model heterogeneous expectations switching model could be easily applied e.g. to study the role of excessively optimistic expectations in amplifying booms[5].

Figure 7 illustrates the dynamics of the 6-type case. The fractions of the six types (middle panels) show considerable fluctuations, all of them fluctuating between 0 and 0.3, with rational and naive expectations dominating (ranging from $0.15 - 0.3$), trend followers somewhere in between (ranging from $0.05 - 0.25$), average expectations wildly

[5]Heterogeneous expectations switching models have e.g. been applied and estimated to explain bubbles and crashes in the stock market (e.g. Boswijk et al., 2007, Amilon, 2008, de Jong et al., 2009, Lof, 2012) and in commodity prices (e.g. gold prices, Alfarano et al., 2005, and oil prices ter Ellen and Zwinkels, 2010) and large movements in exchange rates (e.g. Gilli and Winker, 2003, Westerhoff and Reitz, 2003) and inflation (Cornea et al., 2012).

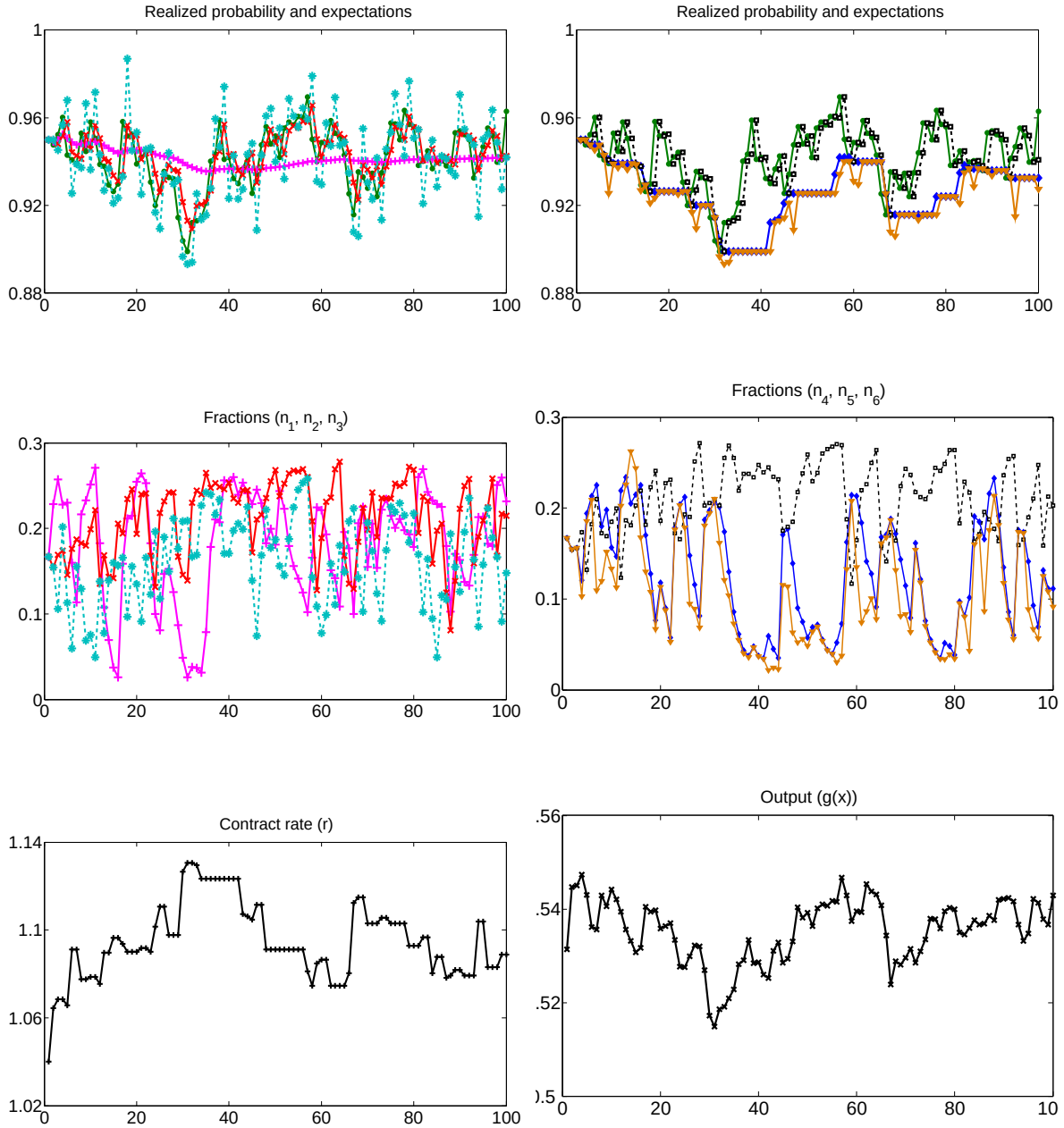


Figure 7: Heterogeneous expectations with 6 types. Upper left panel: realized probability of success (green), rational expectations (AR1) (red), average expectations (purple), trend follower expectations (cyan). Upper right panel: pessimistic (minimum) expectations (blue), naive expectations (black), worst-case expectations (yellow). Mid left panel: fractions of rational, average and trend following believers (resp. red, purple and cyan). Mid right panel: fractions of pessimistic, naive and worst-case believers (resp. blue, black and yellow). Bottom panel: contract rate (left) and output (right).

fluctuating (between 0 and 0.3 and pessimistic and worst case expectations being the minority types, but never completely driven out of the market.

The contract rate (bottom left panel) gradually increases hitting its peak around

periods $30 - 33$ and remaining persistently high between periods $34 - 42$. Overall, the contract rate is persistently higher than the long run equilibrium rate $\bar{r} = \rho/\mu = 1.063$, due to the presence of pessimistic forecasters, even when their fractions are relatively small. It is important to note that around the exogenously generated crisis of period 31, the fractions of pessimistic and worst case expectations are at a peak, both around 0.2, adding up to about 0.4, and only decrease gradually thereafter. A relatively small fraction of pessimistic traders thus has a significant impact on aggregate outcomes and contributes to a high equilibrium contract rates for more than 10 periods. The time series of output $g(x)$ is also shown (bottom right panel), with a minimum value at the exogenously generated crisis in period 31 and only slowly recovering in subsequent periods.

Figure 8 compares the 6-type heterogeneous expectations simulations with the homogeneous rational expectations benchmark. In particular, Figure 8 (top right panel) illustrates that the difference of the contract rates under boundedly rational heterogeneous expectations and homogeneous rational expectations is significantly positive over the entire sample and highly persistent. We refer to this difference $r^{HET} - r^{RE}$ as the *heterogeneous expectations risk premium* of the contract rate for loans. The average heterogeneous expectations risk premium is $\bar{r}^{HET} - \bar{r}^{RE} \approx 2.4\%$. Notice that its peak is about 6.8%, and occurs in period 39, that is, much later than the worst exogenous shock in the crisis period 31, at times when the rational forecast has already correctly predicted the mean reversion of the probability of success towards its mean, while under heterogeneous expectations the influence of a relatively small fraction of pessimistic agents on aggregate behaviour is still highly significant.

Similarly, the bottom panel of Figure 8 illustrates differences in output under heterogeneous versus homogeneous rational expectations. Under heterogeneous expectations, output is significantly lower than under rational expectations. On average, the output loss $(y^{RE} - y^{Het})/y^{RE}$ due to heterogeneous expectations is about 1.1%, with a peak of more than 3%. As for the peak in the differences in the contract rate, the biggest output loss due to heterogeneous expectations occurs in period 39, much later than the crisis period 31, and it occurs when in fact the exogenous probability of success already has

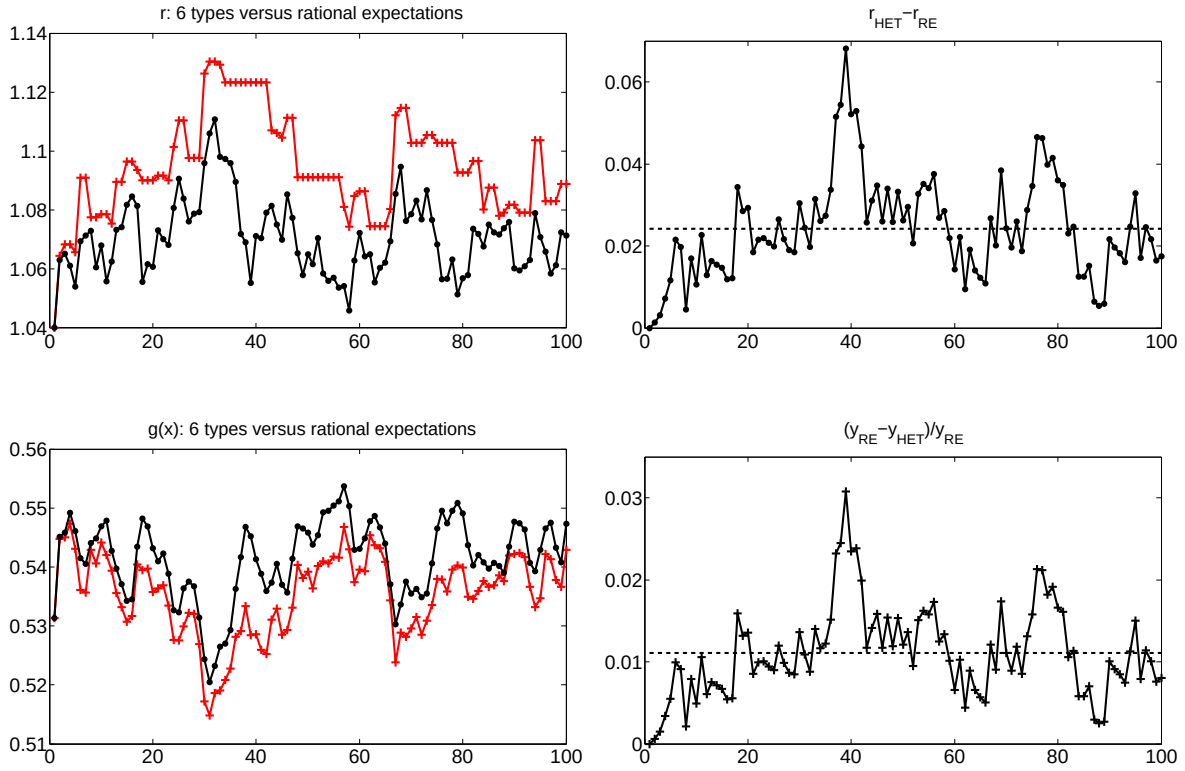


Figure 8: Differences between homogeneous rational expectations benchmark and heterogeneous expectations. Upper panel left: contract rate of 6 types benchmark (red) versus rational (AR1) benchmark (black); upper right: the heterogeneous expectations risk premium, i.e. the differences in contract rates; Bottom panel left: output for 6 types benchmark (red) versus rational (AR1) benchmark (black); bottom right: relative differences in output.

recovered to normal levels, but a drop of confidence due to boundedly rational heterogeneous expectations still affects output at the macro level significantly, even when the fractions of pessimists and worst case believers at the micro level are relatively small.

Why then are the fully rational agents, using the AR1 model consistent forecasting rule of the exogenous probability of success, not driving out all other forecasting rules, as has been suggested by the traditional rational approach, advocated e.g. by Friedman (1953) and Fama (1970)[6]? It is useful to discuss once more the main driving forces behind the simulation results of our heterogeneous expectations selection framework (5.2-5.4), which is based on Brock and Hommes (1997). There are four key elements of why non-rational forecasting rules survive in our economy with performance based

[6]We stress once more that AR1 forecasters are fully rational in our framework, as they are both rational forecasters and utility maximizers taking the behavior of other non-rational agents into account through their knowledge of the equilibrium contract rate r_t; see the discussion in Subsection 5.3.2.

strategy selection.

1. Agents choose between heterogeneous forecasting rules based upon recent forecasting performance. Their choice is *boundedly rational* in the sense that their intensity of choice to switch strategies is finite, i.e. $\beta < \infty$, implying that some agents will *not* switch to the best strategy. For $\beta < \infty$, each rule attracts some followers. When $\beta \approx 0$, the distribution of the population over the forecasting rules is flat, with fractions approximately equal. For $\beta \approx \infty$ the distribution over rules is peaked, with most agents choosing the best strategy.

2. The performance measure is a weighted average of past (relative) forecasting errors, as in (5.2). In the special case when the contract rate r_t would be constant and memory would be infinite (i.e. $\gamma = 1$), the performance measure is, up to a scaling factor, equivalent to the MSE. Therefore, in the special case of constant contract rate, infinite memory $\gamma = 1$ and infinite intensity of choice $\beta = +\infty$, in the long run the rational AR1 forecast would drive out all other forecasting rules. Hence, the rational benchmark is nested within our framework as a special case. In the more realistic case when memory is finite, i.e. $0 \leq \gamma < 1$, and when agents are boundedly rational, i.e. $\beta < \infty$, some fraction of agents will choose alternative forecasting rules. There is empirical evidence that *recent* performance is important for strategy selection. For example, evidence from empirical finance suggests that the flow in and out of mutual funds is strongly driven by the recent past performance of these funds (e.g. Sirri and Tufano, 1998, Karceski, 2002). Similarly, using data from Vanguard, Benartzi and Thaler (2007) have shown for retirement savings decisions that equity allocation of new participants rose from 58% in 1992 to 74% in 2000, following a strong rise in stock prices in the late 1990s, but dropped, back to 54% in 2002, following a strong fall in stock prices. In recent laboratory experiments with human subjects, Anufriev et al. (2012) show that individuals switch to alternative strategies which performed better in the recent past, even when such performance was driven by an exogenous random sequence and individuals had enough information about which strategy was optimal on average.

3. In the performance measure (5.2), the contract rate r_t is time varying, and equals the weight given to the most recent forecasting error of the probability of success. Hence, in times when the contract rate is high, more weight is given to recent forecasting errors. High contract rates arise in "bad times", when the exogenous probability of success is low. Exactly in these "bad times", the pessimistic forecasting rules perform relatively well and therefore attract more followers among the boundedly rational agents. Hence, especially in "bad times" pessimistic expectations will kick in more easily, in a boundedly rational heterogeneous world.

4. Our expectations selection framework (5.2-5.4) is an extension of the model with synchronous updating of Brock and Hommes (1997), allowing for *asynchronous updating* (Hommes et al., 2005b; Diks and van der Weide, 2005). The inertia parameter $0 < \delta < 1$ represents the fraction of agents sticking to their previous strategy, while in a given period only a fraction $1 - \delta$ switches strategy based on relative performance. Anufriev and Hommes (2012) fitted the heterogeneous expectations switching model with asynchronous updating to experimental data and found relative large values around $\delta = 0.8$. Consequently, once non-rational expectations rule gain some weight, e.g. when a fraction of agents becomes pessimistic in bad times, asynchronous strategy updating implies that they only disappear gradually.

These four plausible and empirically relevant elements of strategy switching cause non-rational rules to survive in the population. In particular, they cause (at least) a small fraction of agents to have pessimistic expectations. But even a relatively small fraction of pessimistic believers has a significant effect upon aggregate behaviour and causes crises to be deeper and more persistent.

6 Conclusion

This paper is an attempt to build a model of "animal spirits" and "confidence", as suggested in Akerlof and Shiller (2009). Our building block is the heterogeneous expectations switching model of Brock and Hommes (1997). We have studied an equilibrium

model for loans and compared the case of expectations heterogeneity to the standard case of homogeneous rational expectations. Heterogeneous expectations are disciplined by evolutionary selection or reinforcement learning based upon recent forecasting performance. Survey data on expectations, laboratory forecasting experiments and time series data lend empirical support to such a heterogeneous expectations hypothesis. Costless rational expectations, whose forecast uses the correct model consistent specification of the stochastic probability of success, are unable to drive out simple forecasting heuristics such as naive expectations, trend following rules and pessimistic or worst case expectations. In particular, a small fraction of pessimistic expectations survives, and even a small fraction of pessimistic believers has a large impact on aggregate macro behaviour and causes a *heterogeneous expectations risk premium* over homogeneous rational expectations to be positive, quite large and persistent. Even in the presence of costless fully rational expectations a small fraction of pessimistic agents at the micro level has an aggregate effect at the macro level and causes economic crises to be deeper and the subsequent recovery to be much slower.

In a recent survey, Brunnermeier et al. (2012) focus on financial frictions as the key mechanism causing persistence, amplification and instability at the macro economic level. While financial frictions may play an important role, our results show that persistence, amplifications and instability arise even without any financial frictions in a simple stylized equilibrium model of boundedly rational agents with heterogeneous expectations. If bounded rationality, animal spirits and expectations heterogeneity are indeed important drivers of macro economic instability amplifying economic crises and slowing down recovery, policy should not only focus on financial frictions but also on managing heterogeneous expectations, trend following behaviour and over pessimistic beliefs about the economy. Moreover, economics should pay more attention to animal spirits and expectational heterogeneity and their potentially destabilizing role and negative welfare effects in order to prevent economic losses[7].

[7]For example, Brock et al. (2009) discuss the role of financial innovation in generating financial instability. In the traditional financial economics view, under full rationality financial innovation is usually considered to be stabilizing and welfare improving. In contrast, in a simple stylized model with boundedly rational heterogeneous investors Brock et al. (2009) show that financial innovation may destabilize price fluctuations and decrease average welfare. The main reason is that, in the presence of more financial hedging instruments, investors take bigger positions (leverage) amplifying wins or losses

Off course our model is very stylized, but the same heterogeneous expectations framework can be applied to richer and more advanced models, e.g. New Keynesian macro models (e.g. DeGrauwe, 2011; Anufriev et al., 2012), including models with infinite horizon (Branch and McGough, 2009; Massaro, 2012). Future work should investigate theoretically and empirically the size and persistence of heterogeneity premia of financial contract rates and differences in real variables such as wages and output and their role in explaining economic crises.

of boundedly rational agents, thus destabilizing the market. Policy implications concerning regulating financial innovation may thus be completely opposite whether one adopts a homogeneous rational or a boundedly rational heterogeneous expectations market view.

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