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Gauge field production in supergravity inflation: Local non-Gaussianity and primordial black holes

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When inflation is driven by a pseudoscalar field χ coupled to vectors as $\frac{\alpha}{4}\chi F\tilde{F}$, this coupling may lead to a copious production of gauge quanta, which in turns induces non-Gaussian and non-scale-invariant corrections to curvature perturbations. We point out that this mechanism is generically at work in a broad class of inflationary models in supergravity, hence providing them with a rich set of observational predictions. When the gauge fields are massless, significant effects on cosmic microwave background scales emerge only for relatively large α. We show that in this regime, the curvature perturbations produced at the last stages of inflation have a relatively large amplitude that is of the order of the upper bound set by the possible production of primordial black holes by non-Gaussian perturbations. On the other hand, within the supergravity framework described in our paper, the gauge fields can often acquire a mass through a coupling to additional light scalar fields. Perturbations of these fields modulate the duration of inflation, which serves as a source for non-Gaussian perturbations of the metric. In this regime, the bounds from primordial black holes are parametrically satisfied and non-Gaussianity of the local type can be generated at the observationally interesting level $f_{\rm NL} \sim \mathcal{O}(10)$.

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I. INTRODUCTION

In a recent series of papers [1–3] a new broad class of models of chaotic inflation in supergravity (SUGRA) has been developed. These models generalize the simplest model of this type proposed long ago in Ref. [4]; see also Refs. [5–20] for a partial list of other closely related publications.

The new class of models [1–3] describes two scalar fields, S and Φ, with the superpotential

$$W = Sf(\Phi), \tag{1}$$

where $f(\Phi)$ is a real holomorphic function such that $\bar{f}(\bar{\Phi}) = f(\Phi)$. Any function which can be represented by Taylor series with real coefficients has this property. The Kähler potential can be chosen to have the functional form

$$K = K((\Phi - \bar{\Phi})^2, S\bar{S}). \tag{2}$$

In this case, the Kähler potential does not depend on $\phi = \sqrt{2}\,\mathrm{Re}\Phi$. Under certain conditions on the Kähler potential, inflation occurs along the direction $S = \mathrm{Im}\Phi = 0$, and the field ϕ plays the role of the inflaton field with the potential

$$V(\phi) = |f(\phi/\sqrt{2})|^2. \tag{3}$$

All scalar fields have canonical kinetic terms along the inflationary trajectory $S = \mathrm{Im}\Phi = 0$.

This class of models can be further extended [3,11] to incorporate a KKLT-type construction with strong moduli stabilization [21], which may have interesting phenomenological consequences and may provide a simple solution of the cosmological moduli and gravitino problems [22,23].

The generality of the functional form of the inflationary potential $V(\phi)$ allows one to describe *any* combination of the parameters n_s and r. Thus, this rather simple class of models may describe *any* set of observational data which can be expressed in terms of these two parameters by an appropriate choice of the function $f(\Phi)$ in the superpotential. Meanwhile the choice of the Kähler potential controls masses of the fields orthogonal to the inflationary trajectory [1–3]. Reheating in this scenario requires considering the scalar-vector coupling $\sim\phi F_{\mu\nu}F^{\mu\nu}$ [3,24]. If not only the inflaton but some other scalar field has a mass much smaller than H during inflation, one may use it as a curvaton field [25] for the generation of non-Gaussian perturbations in this class of models [26].

In this paper, we will study an alternative formulation of this class of models, with the Kähler potential

$$K = K((\Phi + \bar{\Phi})^2, S\bar{S}). \tag{4}$$

The simplest version of models of that type, with $W = mS\Phi$ and the Kähler potential $\mathcal{K} = S\bar{S} + \frac{1}{2}(\Phi + \bar{\Phi})^2$, was first proposed in Ref. [4]. In this class of models, the Kähler potential does not depend on $\chi = \sqrt{2}\,\mathrm{Im}\Phi$, which plays the role of the inflaton field with the potential

$$V(\chi) = |f(\chi/\sqrt{2})|^2. \tag{5}$$

The description of inflation in the models (2) and (4) coincides with each other, up to a trivial replacement $\phi \to \chi$, as long as vector fields are not involved in the process.

The difference appears when one notices that in the model (4) the inflaton field is a pseudoscalar, which can have a coupling to vector fields

$$\frac{\alpha}{4}\chi F_{\mu\nu}\tilde{F}^{\mu\nu}, \tag{6}$$

where $\tilde{F}^{\mu\nu} \equiv \epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ and α is a dimensionful constant. This coupling is expected to be present since it is compatible with all the symmetries, including a shift symmetry in χ.

The study of the phenomenological effects of such a coupling during inflation has received a lot of attention lately [27–33]. In particular, it has been shown in Refs. [30,31] that, if the constant α is large enough, such a coupling can lead to a copious production of gauge fields due to the time dependence of χ. Through their *inverse decay* into inflaton perturbations, these gauge fields yield an additional contribution to the scalar power spectrum, which is both non-Gaussian and violates scale invariance. In this way it is possible to obtain non-Gaussian and non-scale-invariant effects that can be observed by the Planck satellite and has not yet been ruled out by WMAP, although the parameter space corresponding to such a signal is relatively small [33]. In addition, gauge fields source tensor modes and lead to a stochastic gravity-wave signal that could be detected at interferometers, such as Advanced LIGO or Virgo [32,34] (see also [35]).

Since the new class of inflationary models in supergravity needs a coupling between the inflaton and gauge fields to have successful reheating, we have to consistently take into account the violations of Gaussianity and scale invariance induced by the inverse decay mechanism. This is the topic of Sec. II.

A potential threat in this model is the overproduction of primordial black holes. As we will see in Sec. III, at very small scales—far beyond what is observable by the cosmic microwave background (CMB)—the produced gauge quanta largely increase the curvature power spectrum. At some point, various forms of backreaction stops this growth, but by then the power spectrum has reached $\Delta_\zeta^2 \sim \mathcal{O}(10^{-3})$. At such high values, a statistical fluctuation might locally increase the density so that primordial black holes are formed. In this way the nondetection of primordial black holes puts an observational upper bound on the power spectrum [36–41], which we discuss in Sec. IV. Our estimates for the late-time power spectrum land a factor of six above this bound [compare, e.g., Eq. (33) with Eq. (39)]. Since we expect our estimate to be reliable up to factors of order one, we cannot definitively claim that the inverse decay mechanism and its interesting phenomenology is incompatible with current data, but our result on the production of primordial black holes highlights a clear tension.

In Sec. V we describe an alternative mechanism of the generation of non-Gaussian perturbations, proposed in Ref. [33]. This mechanism requires the introduction of a light charged field h with mass $m_h \ll H$, where H is the Hubble constant during inflation. Inflationary perturbations of this field generate a slightly inhomogeneous distribution of a classical scalar field $h(x)$. This field induces the vector field mass due to the Higgs effect.

As a result, the vector field mass $\sim eh(x)$ takes different values, controlled by fluctuations of the field h. In the parts of the universe where the value of the vector field mass is small, the vector field fluctuations are easily produced since the gauge mass quenches the tachyonic instability. This in turns leads to a longer stage of inflation because of the additional friction generated by the gauge fields. Meanwhile, in the parts of the universe where the fluctuations of the light scalar field h make this field large, the vector field mass becomes larger and inflation is shorter due to the lack of backreaction. As a result, fluctuations of the light scalar field h lead to fluctuations of the total number of e-foldings δN, i.e., to adiabatic perturbations of the metric. We will show that this effect may generate significant primordial local non-Gaussianity. Also, in the regime of parameters relevant for this scenario the primordial black hole bounds are satisfied parametrically.

To implement this mechanism in our supergravity-based inflationary scenario, one should find a way to guarantee the smallness of the mass of the field h during inflation. We will describe a model where the mass squared of this field during inflation is equal to $m_h^2 = \gamma H^2$, where γ can be made small by a proper choice of the Kähler potential.

In Sec. VI we study the evolution of the light field h during inflation in our scenario, which is similar to the evolution of the curvaton field σ in Ref. [26], so for simplicity we will continue calling this field the curvaton. One can use the results of Ref. [26] for the description of its evolution. However, in the original model of Ref. [26], just as in any other curvaton model [25], adiabatic perturbations of the metric are generated by perturbations of the field σ after a complicated sequence of reheating, expansion of the universe, and the subsequent decay of the curvaton field. In our scenario, adiabatic perturbations are produced due to the modulation of the duration of inflation by the perturbations of the field h. As we will demonstrate, this mechanism can easily produce local non-Gaussianity in the potentially interesting range $f_{\rm NL}$ from $\mathcal{O}(10)$ to $\mathcal{O}(100)$, even if the coupling constant α is not very large.

Finally, in Sec. VII, we find that typical values of the coupling constant α considered in this work lead to a relatively high perturbative reheating temperature $T \sim 10^{10}$ GeV. This should be read as a lower limit, since the already copious nonperturbative production of gauge fields during inflation could lead to an even higher reheating temperature. This could lead to the cosmological gravitino problem [42], but in the class of models with strong moduli stabilization and gravitino mass $\mathcal{O}(100)$ TeV this problem does not appear [23].

II. CMB SCALES: VIOLATIONS OF GAUSSIANITY AND SCALE INVARIANCE

Recently there has been a lot of interest in the effect of gauge field production in axion inflation [27–33]. In this section we summarize the main points.

Consider a pseudoscalar inflaton with a potential suitable for inflation. The symmetries of the theory allow for a coupling $\chi F_{\mu\nu}\tilde{F}^{\mu\nu}$ to some U(1) gauge sector. This coupling is essential for reheating in the supergravity models we discussed in Sec. I. We will therefore consider the following bosonic part of the action:[1]

$$S = -\int d^4x\sqrt{-g}\left[\frac{1}{2}(\partial\chi)^2 + \frac{1}{4}F^2 + \frac{\alpha}{4}\chi F\tilde{F} + V(\chi)\right].$$

Since all relevant effects arise from the couplings above we can safely neglect the gravitational interaction between perturbations and work with an unperturbed Friedmann-Lemaître-Robertson Walker metric.[2] We organize the perturbation theory based on the equations that we are able to solve. Consider two classical[3] fields $\vec{A}(x,t)$ and $\chi(t)$ that solve these two coupled differential equations,

$$\ddot{\chi} + 3H\dot{\chi} + \frac{\partial V}{\partial\chi} = \alpha\langle\vec{E}\cdot\vec{B}\rangle, \tag{7}$$

$$\vec{A}'' - \nabla^2\vec{A} - \alpha\chi'\nabla\times\vec{A} = 0, \tag{8}$$

where $\vec{E} \equiv -\dot{\vec{A}}/a$, $\vec{B} \equiv a^{-2}\nabla\times\vec{A}$ and $\vec{E}\cdot\vec{B} = -F\tilde{F}/4$ are computed from $\vec{A}$.

Now let us look at the action expanded around χ and $\vec{A}$, i.e., $S[\chi + \delta\chi, \vec{A} + \delta\vec{A}]$. By organizing the result at various orders in $\delta\chi$ and $\delta\vec{A}$, one finds

$$\begin{aligned} S = \text{const} - &\int d^4x\sqrt{-g}(\delta\chi)\alpha[\langle\vec{E}\cdot\vec{B}\rangle - \vec{E}\cdot\vec{B}] \\ &- \int d^4x\sqrt{-g}\left[\frac{1}{2}(\partial\delta\chi)^2 + \frac{1}{2}\frac{\partial^2 V}{\partial\chi^2}(\delta\chi)^2 + \frac{1}{4}(\delta F)^2 \right. \\ &\left. + \frac{\alpha}{4}\chi\delta F\delta\tilde{F} + \frac{\alpha}{2}\delta\chi\delta F\tilde{F}\right] \\ &- \int d^4x\sqrt{-g}\left[\frac{\alpha}{4}\delta\chi\delta F\delta\tilde{F} + \frac{1}{6}(\delta\chi)^3\frac{\partial^3 V}{\partial\chi^3}\right], \end{aligned} \tag{9}$$

where again the classical background fields χ and $\vec{A}$ solve Eqs. (7) and (8). Notice that there is a "tadpole" for $\delta\chi$ due to the fact that at the background level we solved an inhomogeneous equation for $\vec{A}$ but just a homogeneous one for χ. From this term one also sees that $\delta\chi$ will source δA^0, and hence it will modify the constraint. The equations of motion in Coulomb gauge $\partial_i A^i = 0$ are

$$\begin{aligned} a\ddot{\delta A}_i - \frac{\partial_k^2(\delta A_i)}{a} + aH\dot{\delta A}_i - \alpha\dot{\chi}\nabla\times(\delta\vec{A}) \\ = \alpha\dot{\delta\chi}\nabla\times\vec{A} - \alpha(\nabla\delta\chi)\times\dot{\vec{A}} - \partial_t(a\partial_i(\delta A^0)), \end{aligned} \tag{10}$$

$$\ddot{(\delta\chi)} + 3H\dot{\delta\chi} - \nabla^2\delta\chi + \frac{\partial^2 V}{\partial\chi^2}\delta\chi = \frac{\alpha}{4}(\langle F\tilde{F}\rangle - F\tilde{F} - 2\delta F\tilde{F}), \tag{11}$$

$$a\partial_i\partial_i(\delta A)^0 = -\alpha\nabla(\delta\chi)\cdot\nabla\times\vec{A}. \tag{12}$$

The solution for the constraint equation for δA^0 is

$$\delta A^0(x,t) = a^{-1}\int d^3y\,\frac{\alpha\nabla(\delta\chi)\cdot\nabla\times\vec{A}}{4\pi|x-y|}. \tag{13}$$

Unfortunately, this coupled system of equations is hard to solve. Hence, Refs. [30,31] made the approximation of neglecting all terms quadratic or higher in $\delta\chi$, δA, and A, which yields

$$a\ddot{\delta A}_i - \frac{\partial_k^2\delta A_i}{a} + aH\dot{\delta A}_i - \alpha\dot{\chi}\nabla\times\delta\vec{A} = 0 \tag{14}$$

$$\ddot{\delta\chi} + 3H\dot{\delta\chi} - \nabla^2\delta\chi + \frac{\partial^2 V}{\partial\chi^2}\delta\chi = \alpha(\langle\vec{E}\cdot\vec{B}\rangle - \vec{E}\cdot\vec{B}). \tag{15}$$

This is a good approximation as long as $F\tilde{F}$ (or equivalently $\langle\vec{E}\cdot\vec{B}\rangle$) is not too large [a more quantitative condition is given in Eq. (29)], which is the regime we will discuss in this section. Note from Eqs. (8) and (14) that in this approximation there is no way to tell A apart from δA. In the next section we will see that, since $\langle\vec{E}\cdot\vec{B}\rangle$ grows with time, towards the end of inflation this description in not valid anymore, and one has to take backreaction into account.

Solving the equation of motion (8) one finds a tachyonic enhancement of the gauge fields. For the growing mode of one of the two polarizations of the gauge field we get

$$A = \frac{1}{\sqrt{2k}}e^{\pi\xi/2}W_{-i\xi,1/2}(2ik\tau). \tag{16}$$

Here $W_{\lambda,\mu}(x)$ denotes the Whittaker function, and ξ is defined as[4]

[1]Notice that in the existing literature, such a coupling is usually associated with the interaction of the axion field with vector fields, with a coupling $-\frac{\alpha}{4f}$. In our approach it is not necessary to associate the pseudoscalar field with the axion field with the radius of the potential $\sim f$, so we normalize the coupling in terms of the reduced Planck mass M_p, which we then set to one, and consider the interaction term $-\frac{\alpha}{4}\chi F_{\mu\nu}\tilde{F}^{\mu\nu}$.

[2]We are neglecting vector and tensor modes and the slow-roll suppressed interactions coming from the solution of the constraints on the lapse and the shift.

[3]Here we are assuming that the occupation number of the relevant gauge modes is large enough that one can approximate the resulting electromagnetic field with a classical one. This assumption is implicit in all other approaches so far.

[4]Note that we have some minus signs that are different from Ref. [30], but this is a matter of conventions. We will work with a model that has $\dot{\chi} < 0$ during inflation and define ξ to be positive. The sign of $\langle\vec{E}\cdot\vec{B}\rangle$ is always opposite to the sign of $\dot{\chi}$. Therefore the physical effect of the tachyonic enhancement is always that inflation is prolonged. To be precise: when $\dot{\chi}$ is negative, the growing field is actually the opposite polarization, i.e., A_-, which makes $\langle\vec{E}\cdot\vec{B}\rangle > 0$ (see, for example, Eq. (8) in Ref. [28]).

$$\xi \equiv -\frac{\dot{\chi}\alpha}{2H}. \tag{17}$$

As one can see, the relation between the coupling constant α and the value of ξ 60 e-foldings before the end of inflation is model dependent, but for our model there is an approximate relation that is valid for the parameters that we are going to explore,

$$\alpha \sim 15\xi. \tag{18}$$

For $\xi > 1$ the new coupling therefore leads to the generation of perturbations of the vector fields around horizon scales. The produced gauge fields then change the dynamics of χ and H. The cosmological homogeneous Klein-Gordon equation and the Friedmann equation get extra contributions from the gauge fields and can now be written as

$$\ddot{\chi} + 3H\dot{\chi} + \frac{\partial V}{\partial \chi} = \alpha\langle \vec{E}\cdot\vec{B}\rangle, \tag{19}$$

$$3H^2 = \frac{1}{2}\dot{\chi}^2 + V + \frac{1}{2}\langle \vec{E}^2 + \vec{B}^2\rangle. \tag{20}$$

They are computed as

$$\langle \vec{E}\cdot\vec{B}\rangle = \frac{1}{4\pi^2 a^4}\int_0^\infty dk k^3 \frac{\partial}{\partial\tau}|A|^2, \tag{21}$$

$$\left\langle \frac{\vec{E}^2 + \vec{B}^2}{2}\right\rangle = \frac{1}{4\pi^2 a^4}\int_0^\infty dk k^2[|A'|^2 + k^2|A|^2]. \tag{22}$$

After renormalization, one can reduce the integration interval to the region $\frac{1}{8\xi} < \frac{k}{aH} < 2\xi$, which is where the enhancement in the (derivative of the) gauge field takes place.

From the homogeneous Klein-Gordon equation (15) one reads off that the influence of the produced gauge fields on the homogeneous dynamics of χ and H can be safely neglected as long as

$$\frac{\alpha\langle \vec{E}\cdot\vec{B}\rangle}{3H\dot{\chi}} \ll 1, \qquad \frac{\frac{1}{2}\langle \vec{E}^2 + \vec{B}^2\rangle}{3H^2} \ll 1. \tag{23}$$

Of these two conditions the first one is always the most stringent. When it stops to hold, the backreaction on the homogeneous evolution becomes important and the evolution of χ and H will be slowed down, which makes inflation last longer. We will see in the next section that the backreaction on the inhomogeneous equation for $\delta\chi$ happens even earlier. In this section we focus on the regime in which all of these effects are negligible, which, e.g., for a quadratic potential corresponds roughly to $\xi \lesssim 4$. This is appropriate for the description of CMB scales.

Now we move to the power spectrum. The copiously generated gauge fields may, by inverse decay, produce additional perturbations of the inflaton field $\delta\chi$, proportional to the square of the vector field perturbations. As was shown in Refs. [30,31], this can be described (up to backreaction effects to be described in the next section) by using Eq. (15). The inclusion of the source term leads to an extra contribution to the power spectrum of the curvature perturbation on uniform density hypersurfaces $\zeta = -\frac{H}{\dot{\chi}}\delta\chi$, which has been computed in Refs. [30,31] (we present a quick estimate in Appendix B),

$$\Delta^2_\zeta(k) = \Delta^2_{\zeta,\mathrm{sr}}(k)(1 + \Delta^2_{\zeta,\mathrm{sr}}(k) f_2(\xi) e^{4\pi\xi}), \tag{24}$$

where $f_2(\xi)$ was defined in Refs. [30,31] and can be computed numerically [a useful large ξ approximation is given in Eq. (B14)], and

$$\Delta^2_{\zeta,\mathrm{sr}}(k) = \left(\frac{H^2}{2\pi|\dot{\chi}|}\right)^2 \tag{25}$$

is the amplitude of the vacuum inflationary perturbations as in standard slow-roll inflaton. WMAP [43] has measured $\Delta^2_{\zeta,\mathrm{sr}}(k_\star) = 2.43\times 10^{-9}$, where $k_\star = 0.002\ \mathrm{Mpc}^{-1}$ is the pivot scale that we will take to correspond with $N = 60$ e-foldings before the end of inflation. The second term in Eq. (24) violates scale invariance (and Gaussianity, as we will see below) since it comes schematically from A^2, i.e., the square of a Gaussian which grows with time as in Eq. (16).

We move to the bispectrum. The produced gauge fields lead to equilateral non-Gaussianity in the CMB [30,31],

$$f_{\mathrm{NL}} = \frac{\Delta^6_\zeta(k)}{\Delta^4_{\zeta,\mathrm{sr}}(k)} e^{6\pi\xi} f_3(\xi), \tag{26}$$

where $f_3(\xi)$ is another function defined in Refs. [30,31], which can be computed numerically [see Eq. (D7) for a useful approximation]. The amount of non-Gaussianity, therefore, depends exponentially on ξ. Between $\xi = 0$ and $\xi = 3$ it grows from $\mathcal{O}(1)$ to $\mathcal{O}(10^4)$ and most of the growth takes place in a small interval around $\xi \simeq 2.5$.

The analysis of Ref. [33] showed that the bounds coming from the power spectrum (especially from WMAP plus ACT, because of the violation of scale invariance) and from the bispectrum (from WMAP) are compatible, with the former being typically slightly more stringent. Specifying a confidence region in ξ requires assuming some prior for this parameter. The physically best-motivated prior is log-flat in ξ reflecting the fact that the scale of the dimension-five coupling $\chi F\tilde{F}$ could be anywhere (with strong indications that it should be below the Planck scale [44]). In this case at 95% C.L. one finds $\xi \lesssim 2.2$. A flat prior on ξ leads to $\xi \lesssim 2.4$.

III. VERY SMALL SCALES: STRONG BACKREACTION

In this section we want to estimate the power spectrum and bispectrum towards the end of inflation, i.e., on scales that are too small to be observed in the CMB. The only observational handle available in this regime is the nondetection of primordial black holes, which puts an upper bound on the power spectrum [36–41].

To make these estimates it is essential to recognize that many of the formulas described in the previous section and given in the literature about inverse decay are valid only in the regime in which the backreaction on the inhomogeneous equation for $\delta\chi$ is small [see Eq. (29)]. As we show in the following, the scales relevant for the production of primordial black holes leave the horizon when backreaction is large. The authors of Ref. [45] did not account for backreaction and therefore their conclusion that gauge field production during inflation leads to black hole production might be premature.

For concreteness, we will consider a quadratic potential $V(\chi) = \frac{1}{2}m^2\chi^2$, with the mass chosen such that at the pivot scale $k_\star$ (that we take to correspond with $N = 60$) we get $\Delta_\zeta^2(k_\star) = 2.43 \times 10^{-9}$.

Let us first look at the dynamics of χ and H. As we already discussed, when enough gauge field quanta have been produced, the conditions in Eq. (23) stop to hold (the inequality for $\langle \vec{E}\cdot\vec{B}\rangle$ is violated first) and χ and H are slowed down. As a result, inflation lasts longer. Let us check this. The behavior of χ, H, and ξ as functions of N (the number of e-folds left to the end of inflation) follows from simultaneously solving Eqs. (17), (19), and (20). In Figs. 1 and 2 we have plotted the solutions for $\chi(N)$ and $H(N)$, with and without backreaction taken into account. For $\xi(N = 60) = 2.2$, the effect of backreaction becomes 10% around $N = 11$.

Now let us consider perturbations. Of course they will be affected by the backreaction on the homogeneous dynamics χ and H that we described above, but there is more. Let us consider Eqs. (10)–(12). In the last section we solved for A in a *homogeneous* background and used that result [Eq. (16)] to compute the source term in the equation for χ perturbations. But as A and $\delta\chi$ grow larger toward the end of inflation (both of them grow as $e^{2\pi\xi}$) the source on the right-hand side of Eq. (10) cannot be neglected anymore. If we were able to solve this equation, we would find that $\vec{E}\cdot\vec{B}$ now depends on the perturbation $\delta\chi$. By expanding $\vec{E}\cdot\vec{B}$—which is the source term in Eq. (11)—in powers of $\delta\chi$ we find several new terms, including additional friction and a modified speed of sound. In Refs. [28,32] it was proposed that one can estimate these effects in the regime of strong backreaction by just considering the additional friction term $\delta\dot{\chi}$. The equation of motion for the perturbation $\delta\chi$ becomes

$$\delta\ddot{\chi} + 3\beta H\delta\dot{\chi} - \frac{\nabla^2}{a^2}\delta\chi + \frac{\partial^2 V}{\partial\chi^2}\delta\chi = \alpha[\vec{E}\cdot\vec{B} - \langle\vec{E}\cdot\vec{B}\rangle], \quad (27)$$

with the additional friction term

$$\beta \equiv 1 - 2\pi\xi\alpha\frac{\langle\vec{E}\cdot\vec{B}\rangle}{3H\dot{\chi}}. \quad (28)$$

Here the new term in β is caused by the dependence of $\langle\vec{E}\cdot\vec{B}\rangle$ on $\dot{\chi}$ (via its dependence on ξ). The behavior of β has been plotted in Fig. 3. It is always positive.[5]

The new source of backreaction can be neglected as long as

$$2\pi\xi\alpha\frac{\langle\vec{E}\cdot\vec{B}\rangle}{3H\dot{\chi}} \ll 1. \quad (29)$$

Note [from a comparison with Eq. (23)] that the factor of $2\pi\xi$ causes the backreaction on the power spectrum to become significant before the backreaction on H and χ does. For $\xi(N = 60) = 2.2$ we find that backreaction becomes of order 10% ($\beta = 1.1$) at $N = 22$.

The modified equation of motion (27) suggests that (as was already noted in Ref. [32]; see also Appendix B) we can estimate

$$\delta\chi \approx \frac{\alpha(\vec{E}\cdot\vec{B} - \langle\vec{E}\cdot\vec{B}\rangle)}{3\beta H^2}, \quad (30)$$

which leads to the power spectrum

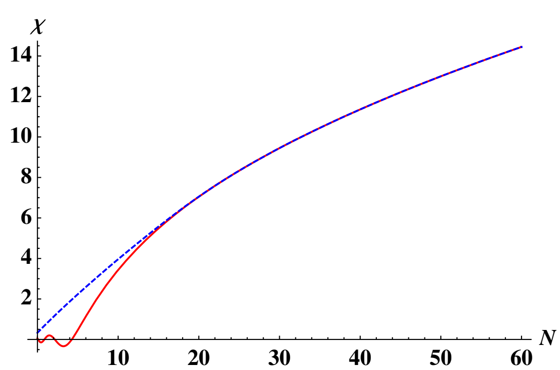

FIG. 1 (color online). The evolution of the inflaton field χ, as a function of the number of e-folds N left to the end of inflation (time is moving to the left) for $\xi[N = 60] = 2.2$. The result in dashed blue does take backreaction from the sources in Eqs. (19) and (20) into account; the result in red does not. It is clear that backreaction prolongs inflation.

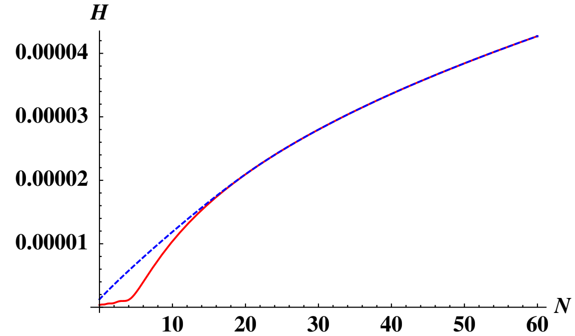

FIG. 2 (color online). The evolution of the Hubble scale H as a function of N for $\xi[N = 60] = 2.2$. Again the dashed blue line is the result corrected for backreaction from the sources in Eqs. (19) and (20).

[5]We work with negative $\dot{\chi}$, which yields positive $\langle\vec{E}\cdot\vec{B}\rangle$, while working with $\dot{\chi} > 0$ gives $\langle\vec{E}\cdot\vec{B}\rangle < 0$.

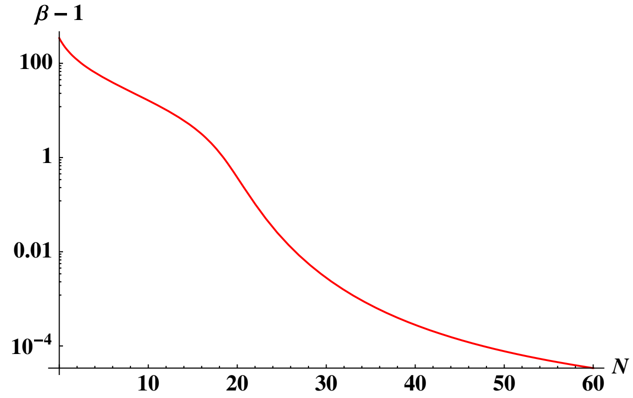

FIG. 3 (color online). Evolution of $(\beta - 1)$ as function of N, for $\xi(N = 60) = 2.2$.

$$\Delta^2_\zeta(k) \simeq \langle\zeta(x)^2\rangle \simeq \left(\frac{\alpha\langle\vec{E}\cdot\vec{B}\rangle}{3\beta H\dot{\chi}}\right)^2. \tag{31}$$

This estimate turns out to be particularly good in the regime in which we can check it, i.e., when $\xi \lesssim 4$, that is, when the backreaction is negligible and we can compare with Eq. (24) (see Appendix B). This gives us confidence to also use it in the strong-backreaction regime. It is easy to see that when the backreaction becomes large, the second term in Eq. (28) dominates, and we end up with

$$\Delta^2_\zeta(k) \simeq \left(\frac{1}{2\pi\xi}\right)^2. \tag{32}$$

The estimate (31) for the power spectrum has been plotted in Fig. 4 together with the formula (24), valid only when the backreaction is negligible. Indeed, in the regime of strong backreaction the power spectrum asymptotes the estimate in Eq. (32). At the end of inflation we have $\xi \simeq 6.7$ [for $\xi(N = 60) = 2.2$], which gives

$$\Delta^2_\zeta(k) \simeq 7.5 \times 10^{-4}. \tag{33}$$

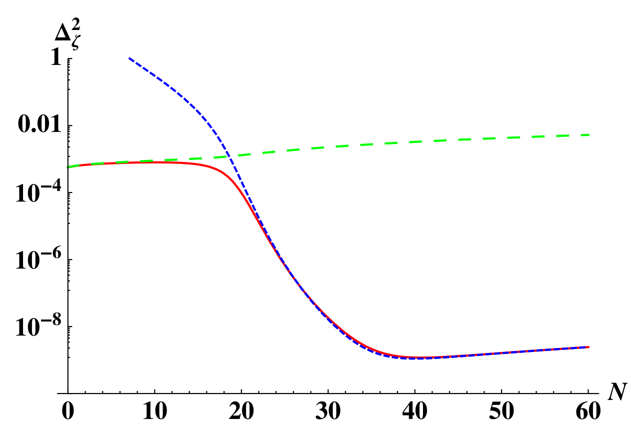

FIG. 4 (color online). Evolution of the power spectrum as a function of N, for $\xi(N = 60) = 2.2$. The expression (24) that does not take backreaction into account is in tinily dashed blue. In solid red is the estimate (31). When backreaction becomes significant this estimate coincides with the late-time estimate $(2\pi\xi[N])^{-2}$, in largely dashed green.

IV. BOUNDS FROM PRIMORDIAL BLACK HOLES

Now let us try to compare this with the existing bounds on the power spectrum coming from the nondetection of primordial black holes. These will form if at horizon reentry (i.e., smoothing ζ on scales of order H) we have $\zeta > \zeta_c$, with $\zeta_c \sim 1$ denoting the critical value leading to black hole formation. If one assumes that ζ follows a Gaussian distribution (with $\langle\zeta\rangle = 0$) one can express the probability of having $\zeta > \zeta_c$ in terms of the variance $\langle\zeta^2\rangle$ by analyzing the Gaussian probability distribution function. This probability corresponds to the fraction of space b that can collapse to form horizon-sized black holes. Hawking evaporation and present-day gravitational effects constrain this fraction b. Typically one finds b in the range $(10^{-28}$–$10^{-5})$, with the strongest bounds coming from CMB anisotropies [37] (spectral distortion and photodissociation of deuterium lead to a bound $b \lesssim 10^{-20}$, as for example in Ref. [36]). Setting $\zeta_c = 1$ gives for the upper bound on the power spectrum [40]

$$\Delta^2_{\zeta,c}(k) \simeq \langle\zeta(x)^2\rangle \simeq 0.008\text{–}0.05. \tag{34}$$

Here the lower bound corresponds to $b = 10^{-28}$ and the upper bound to $b = 10^{-5}$.

However, in our case ζ does not follow a Gaussian distribution. Instead we have (see Appendix B)

$$\zeta = -\frac{\alpha(\vec{E}\cdot\vec{B} - \langle\vec{E}\cdot\vec{B}\rangle)}{3\beta H\dot{\chi}}. \tag{35}$$

The stochastic properties of the vector field A are close to those in a free theory, i.e., it has Gaussian perturbations around $\langle A\rangle = 0$. As a consequence we can write[6]

$$\zeta = g^2 - \langle g^2\rangle, \tag{36}$$

with g a Gaussian distributed field. This model was studied in Ref. [40] and we follow that derivation (see also Refs. [38,39]). The probability distribution function of ζ follows from setting $P(\zeta)d\zeta = P(g)dg$, and takes the form

$$P(\zeta) = \frac{1}{\sqrt{2\pi(\zeta + \sigma^2)}\sigma} e^{-\frac{\zeta+\sigma^2}{2\sigma^2}}, \tag{37}$$

with $\sigma^2 \equiv \langle g^2\rangle$. For a given value of b we can again infer σ^2. Setting $t \equiv \frac{\zeta}{\sigma^2} + 1$ (and $t_c \equiv \frac{\zeta_c}{\sigma^2} + 1$) we have $d\zeta = \sigma^2 dt$, which gives

$$b = \int_{\zeta_c}^\infty P(\zeta)d\zeta = \int_{t_c}^\infty \frac{e^{-\frac{t}{2}}}{\sqrt{2\pi t}}dt = \mathrm{Erfc}\left(\sqrt{\frac{t_c}{2}}\right), \tag{38}$$

where $\mathrm{Erfc}(x) \equiv 1 - \mathrm{Erf}(x)$ is the complementary error function. Taking again b in the range 10^{-28}–10^{-5} one

[6]Here we can safely neglect the linear term, which is just the standard vacuum slow-roll contribution to ζ. See also our estimate for $f_{\rm NL}$ at small scales in Appendix D.

gets a tighter upper bound on the power spectrum than in the Gaussian case,[7]

$$\Delta^2_{\zeta,c}(k) \simeq \langle \zeta(x)^2 \rangle = 2\langle g^2 \rangle^2 \simeq 1.3\times 10^{-4}\text{–}5.8\times 10^{-3}. \quad (39)$$

Now let us estimate what value of b is relevant for our investigation.

At the end of inflation, the total mass concentrated in the volume associated with perturbations leaving the horizon N e-foldings before the end of inflation with the Hubble constant H can be estimated by

$$M_N \simeq \frac{4}{3}\pi\rho r^3 \simeq \frac{4\pi M_p^2}{H} e^{3N}, \quad (40)$$

where we reinserted the reduced Planck mass M_p, which is set to one in the rest of the paper, and H is calculated at the end of inflation. In order to study the subsequent evolution of matter in the comoving volume corresponding to this part of the universe, one should distinguish between two specific possibilities depending on the dynamics of reheating after inflation, discussed in Sec. VII.

If reheating is not very efficient, then the universe for a long time remains dominated by scalar field oscillations, with the average equation of state $p = 0$. In this case, the total mass in the comoving volume does not change, and therefore at the moment when the black hole forms, its mass $M_{\rm BH}$ is equal to M_N evaluated in Eq. (40). For the parameters of our model, this gives an estimate of (see Appendix E for details)

$$M_{\rm BH} \simeq 10 e^{3N}\ {\rm g}. \quad (41)$$

On the other hand, if reheating is efficient, then the post-inflationary universe is populated by ultrarelativistic particles and the energy density in the comoving volume scales inversely to the expansion of the universe. In this case, the black hole mass can be estimated as (see Appendix E)

$$M_{\rm BH} \simeq 10 e^{2N}\ {\rm g}. \quad (42)$$

In our estimates of the black hole production we will assume the latter possibility, though in general one may have a sequence of the first and the second regime. The final conclusion will only mildly depend on the choice between these two possibilities.

Now, the bounds on b in terms of the would-be black hole mass $M_{\rm BH}$ were given in Ref. [36] and updated in Ref. [37]. Here we follow the result in Ref. [37].[8] Using Eq. (38) and our estimates of the black hole mass as a function of N, we can translate this into a bound on the power spectrum as a function of N. The result is shown in Fig. 5.

[7]A similar but less precise estimate was made in Ref. [46].

[8]However, we do not take the constraints for $M_{\rm BH} < 10^8$ g into account, as these are either very model dependent, or assume that black hole evaporation leaves stable relics.

Our estimate (33) *violates this bound* for all $N \lesssim 20$ by a factor of about six. Since we have made some approximations both in deriving the late-time power spectrum and in deriving its observational upper bound, our estimate could well be off by some order-one factor and therefore we cannot draw a definitive conclusion. It is clear though that the parameter values giving rise to an observable but not yet ruled out violation of scale invariance and non-Gaussianity in the CMB window produce a late-time power spectrum that comes at least very close to the primordial black hole bound. A more precise computation is needed to establish whether or not this bound is actually violated.

However, if such a computation revealed that primordial black holes do indeed constrain these models, that would yield a much stronger bound on ξ as the ones coming from non-Gaussianity and the violation of scale invariance. Since we have seen that the power spectrum has a late-time asymptotic of $(2\pi\xi[N])^{-1}$, this problem persists for a wide range of values of ξ.

For all values of ξ, our estimate for the power spectrum sharply increases before the end of inflation, the closer to the end the smaller ξ is. However, if we disregard black hole bounds for $M_{\rm BH} \lesssim 10^8$ g, which rely on uncertain model-dependent assumptions, there are no black hole bounds for $N \lesssim 8$. From Fig. 5 we then see that we get

$$\xi(N_{\rm CMB}) \lesssim 1.5 \quad (43)$$

for the bound on ξ at CMB scales from primordial black hole production. In terms of the coupling constant α, this bound implies the constraint

$$\alpha \lesssim 23. \quad (44)$$

This bound is derived using Eq. (42), i.e., radiation domination right after the end of inflation. This assumption

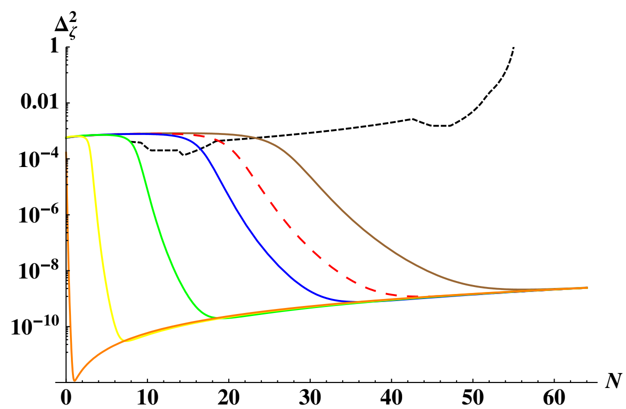

FIG. 5 (color online). Evolution of our estimate for the power spectrum as a function of N. The lines shown in the figure, in order from right to left: In dashed red is the result for $\xi[N = 64] = 2.2$. Other lines are for $\xi[N = 64] = 2.5$ (solid brown), $\xi[N = 64] = 2$ (solid blue), $\xi[N = 64] = 1.5$ (solid green), $\xi[N = 64] = 1$ (solid yellow), and $\xi[N = 64] = 0.5$ (solid orange). The black hole bound is the upper line in dashed black.

fixes the expansion history of the universe and therefore specifies $N_{\mathrm{CMB}} \simeq 64$, for the N corresponding to CMB scales (see Appendix E for a derivation). This is required for consistency but changes the numerics very little. Therefore in all other sections we still use $N_{\mathrm{CMB}} = 60$.

For the matter-domination regime, the black hole masses would be greater for a given N [see Eq. (41)], and therefore we would have a slightly stronger constraint on ξ and α. We find $\xi \lesssim 1.3$, which corresponds to $\alpha \lesssim 20$. Instead of concentrating on it, we will now investigate the model where non-Gaussian perturbations may be generated for much smaller ξ and α, without leading to the primordial black hole problem.

V. LOCAL NON-GAUSSIANITY FROM HEAVY VECTOR FIELDS

Now let us turn to a scenario—described in Ref. [33]—in which the produced gauge fields are massive. The production of gauge quanta decreases with the mass of the gauge fields: for $m_A \sim \xi H$ all production is killed. In this scenario, the gauge fields get their mass via the Higgs mechanism. Fluctuations in the Higgs field h lead to fluctuations in m_A, which in turn generate fluctuations in the amount of produced gauge quanta, and therefore in the amount of extra friction in the dynamics of χ and H. In the end, one has perturbations in ΔN, namely the number of extra e-folds of inflation due to gauge field production. This leads to a non-Gaussian signal in the CMB of the local type [33]. Using the δN formalism one finds

$$f_{\mathrm{NL}}^{\mathrm{local}} \sim 10^2 \left(\frac{\Delta N_{\max}^{3/4} e}{\xi 10^{-3}}\right)^4 \left(\frac{m_A}{\xi H}\right)^2. \tag{45}$$

Here $\Delta N_{\max}$ is the increase of the duration of inflation for the case where the vector fields are massless, h is the Higgs-like field responsible for the mass of the gauge field, e is its U(1) charge, $m_A = eh$, and we assumed a quadratic inflaton potential, so that $H = \frac{m\chi}{\sqrt{6}}$.

For a complete description we refer the reader to the original reference [33], Sec. VII. Here we only want to stress that this scenario can also work for $\xi \sim 1$. Then it will surely satisfy the bounds from primordial black holes.

Note that the classical field $h(x)$, which gives the vector field mass eh, can be produced either due to the tachyonic mass of the field h at $h = 0$, as in the standard Higgs model, or due to accumulation of long-wavelength inflationary perturbations of the field h. In both cases, the mechanism of Ref. [33] requires that the mass of the field h during inflation should be smaller than the Hubble constant. As a result, even if one assumes that the field has the standard Higgs potential, the value of the field during inflation does not correspond to the position of the minimum of the potential. Instead of that, the field takes different values in different exponentially large parts of the universe. The value of $f_{\mathrm{NL}}^{\mathrm{local}}$ in this scenario will depend on a typical local value of the field h, which can be determined by the stochastic approach to the investigation of curvaton fluctuations [26].

For simplicity, and to make a clear link to the investigation performed in Ref. [26], we will call the light field h the curvaton, but one should remember that the mechanism of conversion of perturbations of the curvaton field to adiabatic perturbations is different, involving a complicated dynamical processes during reheating. In our case, fluctuations of the field h lead to fluctuations δN during inflation, and thus to a direct production of adiabatic perturbations of the metric.

This scenario can work only if we have a charged scalar field with a mass much smaller than H. At first glance, one could achieve it by assuming that the relatively light field S plays the role of the Higgs field. However, the superpotential $W = mS\Phi$ would break gauge invariance unless we assume that the field Φ is also charged. This would be inconsistent with the postulated functional form of the Kähler potential.[9] Therefore we must add to our model at least one charged scalar field Q.

Fortunately, one can easily do this. Just like in the simplest supersymmetric (SUSY) version of the Abelian scalar electrodynamics, one should consider the charged field Q without any superpotential associated with it. In the global SUSY limit, the simplest version of this theory with vanishing FI coefficient would contain a D-term potential $V_D = \frac{g^2}{2}(\bar{Q}Q)^2$, but it would not induce any mass of the field Q.

However, in supergravity the radial component $h/\sqrt{2}$ of the scalar field Q does acquire mass, depending on the choice of the Kähler potential. (The complex phase of the field $Q = \frac{h}{\sqrt{2}} e^{i\theta}$ is eliminated due to the Higgs effect.) We will consider the following addition to the Kähler potential of our model:

$$\Delta K = Q\bar{Q} + \kappa Q\bar{Q}S\bar{S}. \tag{46}$$

Terms of similar functional form were included in many versions of our inflationary scenario to stabilize the inflaton trajectory. One can easily find that the resulting mass squared of the field h during inflation is given by

$$m_h^2 = 3H^2(1 - \kappa). \tag{47}$$

Thus in the absence of the term $\kappa Q\bar{Q}S\bar{S}$ the field h would be too heavy, but by considering models with $\gamma \equiv 3(1 - \kappa) \ll 1$ one can have a consistent theory of a light charged scalar field with mass squared γH^2 with $\gamma \ll 1$, as required.

Of course, this requires fine-tuning, but this is just a price which one should be prepared to pay for the description of non-Gaussian inflationary perturbations. We will

[9] We are grateful to the referee for attracting our attention to this issue.

study observational consequences of this model in the next section.

VI. STOCHASTIC APPROACH

In this section we want to find out how fluctuations in the curvaton field h lead to a variable gauge field mass, and therefore to a non-Gaussian signal in the CMB. We will begin our study with an investigation of the behavior of the distribution of the fluctuations in h, following Ref. [26]. During inflation, the long-wavelength distribution of this field generated at the early stages of inflation behaves as a nearly homogeneous classical field, which satisfies the equation

$$3H\dot{h} + V_h = 0, \tag{48}$$

which can be also written as

$$\frac{dh^2}{dt} = -\frac{2V_h h}{3H}. \tag{49}$$

However, during each time interval H^{-1} new fluctuations of the scalar field are generated, with an average amplitude squared of[10]

$$\langle \delta h^2 \rangle = \frac{H^2}{2\pi^2}. \tag{50}$$

The wavelength of these fluctuations is rapidly stretched by inflation. This effect increases the average squared value of the classical field h in a process similar to Brownian motion. As a result, the square of the field h at any given point with inflationary fluctuations taken into account changes—on average—with the speed, which differs from the predictions of the classical equation of motion by $\frac{H^3}{4\pi^2}$,

$$\frac{dh^2}{dt} = -\frac{2V_h h}{3H} + \frac{H^3}{2\pi^2}. \tag{51}$$

Using $3H\dot{\chi} = -V_\chi$, one can rewrite this equation as

$$\frac{dh^2}{d\chi} = \frac{2V_h h}{V_\chi} - \frac{V^2}{6\pi^2 V_\chi}. \tag{52}$$

By solving this equation with different boundary conditions, one can find the most probable value of the locally homogeneous field h.

Now we will consider the case when the mass of the curvaton field is given by

$$m_h^2 = \gamma H^2 = \frac{\gamma m^2 \chi^2}{6}, \tag{53}$$

with $\gamma \ll 1$. This corresponds to the total potential

$$V(\chi, h) = \frac{m^2}{2}\chi^2 + \frac{\gamma}{2}H^2 h^2. \tag{54}$$

We assume that $h \ll 1$, and therefore one can estimate $H^2 \approx \frac{m^2}{6}\chi^2$. In this case, Eq. (52) becomes

$$\frac{dh^2}{d\chi} = \frac{2\gamma\chi h^2}{6} - \frac{m^2\chi^3}{24\pi^2}. \tag{55}$$

This equation has a family of different solutions,

$$h^2 = \frac{3m^2}{4\pi^2\gamma^2}\left(1 + \gamma\frac{\chi^2}{6}\right) + Ae^{\gamma\chi^2/6}, \tag{56}$$

where A is a constant which could be either positive or negative, depending on initial conditions. During inflation these solutions converge to a simple attractor solution,

$$h = \frac{\sqrt{3}m}{2\gamma\pi}\sqrt{1 + \frac{\gamma\chi^2}{6}}. \tag{57}$$

We are interested in using this formula to estimate the size of the non-Gaussianity, which is produced by the conversion of perturbations in h into curvature perturbations when the backreaction from gauge fields on the homogeneous evolution becomes substantial, i.e., close to the end of inflation. Hence we should take $\chi \sim 1$ in Eq. (57). For $\gamma \ll 1$, this solution approaches a constant $h = \frac{\sqrt{3}m}{2\gamma\pi}$ during the last stages of inflation. Note that this *a posteriori* justifies the assumption that $h \ll 1$, as long as $\gamma \gg 10^{-6}$.

To give a particular numerical example, we will use Eq. (45) for the case $\xi = 0.5$. A numerical analysis shows that in this case $\Delta N_{\max} \sim 0.044$, and therefore

$$f_{\rm NL}^{\rm local} \sim 2.5 \times 10^{11} e^6 \gamma^{-2}\chi^{-2} \tag{58}$$

at the end of inflation with $\gamma\chi^2/6 \ll 1$.

All our approximations should work fine if the mass of the vector field is much smaller than H, which leads to a constraint $e \ll \gamma\chi$.

Consider for example $\gamma = 0.1$ and $\chi = 1$, which corresponds to the very end of inflation. [We should stress that it would not be consistent to take χ much larger than $\mathcal{O}(1)$ in Planck units since that is its value when curvature perturbations are generated in our scenario. Moreover, the main contribution to $\Delta N_{\max}$ is given by the last part of the inflationary trajectory where $\chi = \mathcal{O}(1)$.] In this case,

$$f_{\rm NL}^{\rm local} \sim 2.5 \times 10^{13} e^6. \tag{59}$$

To have non-Gaussian perturbations with $f_{\rm NL}^{\rm local} = O(10)$ one should take $e \approx 10^{-2}$.

[10]For a real massless field we would get $\langle \delta h^2 \rangle = \frac{H^2}{4\pi^2}$. An extra coefficient of 2 appears in Eq. (50) because the field Q is complex, so its absolute value changes faster because of independent fluctuations of its two components. One could argue that in the unitary gauge we only have one scalar degree of freedom. However, unitary gauge is problematic in the description of Brownian motion and cosmic string formation in the Higgs model. We present the results which should be valid in the regime of a small gauge coupling constant e. Our main conclusions are unaffected by this factor-of-2 issue.

VII. GAUGE FIELD PRODUCTION IN SUGRA INFLATION: REHEATING

We have found that the coupling $\chi F\tilde{F}$ needed for reheating in (the pseudoscalar variant of) the new class of SUGRA inflation models proposed in Refs. [1–3] can as well yield an observable non-Gaussian signal. It only remains to be seen what the effects of the typically needed values for ξ are for the reheating in the combined model.

In Ref. [3] the reheating temperature T_R for the decay of a scalar inflaton field to two photons due to the interaction $\frac{\alpha}{4}\phi F^2$ was estimated as

$$T_R \approx \sqrt{2}\alpha \times 10^9 \text{ GeV}. \tag{60}$$

A similar estimate is valid in our case. One may also represent it in an equivalent way using the relation $\frac{\alpha}{4} = -\frac{\xi H}{2\dot{\chi}}$ and an expression for the slow-roll parameter $\epsilon = \frac{\dot{\chi}^2}{2H^2}$,

$$T_R \approx \frac{2\xi}{\sqrt{\epsilon}} \times 10^9 \text{ GeV}. \tag{61}$$

As long as one can describe reheating as a particle-by-particle decay, reheating in inflationary models of this type does not depend much on whether the inflaton field is a scalar or a pseudoscalar. In both types of models, one may consider interactions with $\alpha \ll 1$, which results in a reheating temperature $T_R \lesssim 10^8$ GeV. This solves the cosmological gravitino problem for gravitino in the typical mass range $m_{3/2} \lesssim 1$ TeV.

However, for $\alpha \gtrsim 1$, which is required for the production of non-Gaussianity in the models based on the pseudoscalar inflaton, the estimate described above gives $T_R > 10^9$ GeV. It is good for the theory of leptogenesis, but it could be bad from the point of view of the gravitino problem. Moreover, for $\alpha \gtrsim 1$ an entirely different mechanism of reheating is operating. At the end of inflation, when the time-dependent parameter ξ grows and becomes large, a significant fraction of the energy of the inflaton field gradually becomes converted to the energy of the vector field (see Fig. 6). This is a very efficient mechanism, which may lead to a very rapid thermalization of energy in the hidden sector. This may exacerbate the gravitino problem. Fortunately, this problem does not appear for the superheavy gravitino with mass $m_{3/2} \gtrsim 10^2$ TeV. Such gravitinos appear in many versions of the models of mini-split supersymmetry, which became quite popular during the last few years; see Refs. [23,47] and references therein.

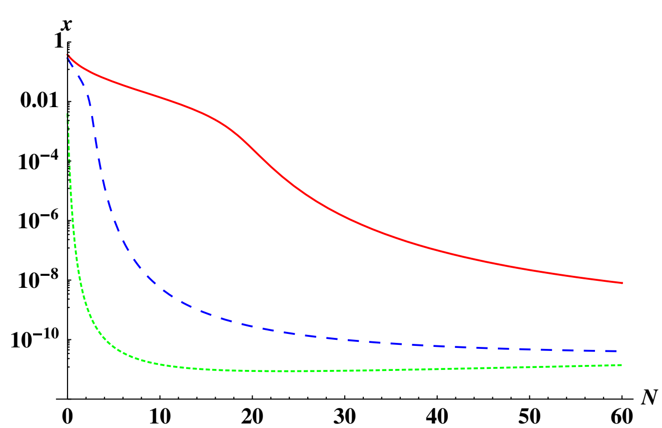

FIG. 6 (color online). Evolution of the normalized energy of the vector field, $x \equiv \frac{1}{2}(E^2 + B^2)/3H^2$ as a function of N, for $\xi[N = 60] = 2.2$ (solid red), $\xi[N = 60] = 1.0$ (largely dashed blue) and $\xi[N = 60] = 0.5$ (tinily dashed green).

VIII. CONCLUSIONS

The new class of chaotic inflation models in supergravity needs a gauge-gauge-inflaton coupling for reheating. The inclusion of this coupling can produce gauge fields and can provide a Planck observable, but they have not yet ruled out a non-Gaussian signal in the CMB.

In this article we have studied two possible realizations of this scenario. Taking the parameter $\xi \simeq 2.2$–2.5 ($\alpha \simeq 32$–37) produces a large amount of gauge quanta, that by inverse decay give rise to an equilateral non-Gaussianity in the CMB, as studied in Refs. [30,31]. However, we have estimated that towards the end of inflation the power spectrum grows so much that the model may be ruled out because it overproduces primordial black holes. As our order-one estimate lands within a factor of six from the critical black hole bound on the power spectrum (with the non-Gaussian nature of the signal taken into account), we need a more precise computation to draw a definitive conclusion.

In the second scenario, where the produced gauge fields are massive due to the Higgs effect in the presence of a light curvaton-type field, one can take a smaller value for ξ, of order 0.5–1, corresponding to an α from 8–15. Then the model is free of black hole trouble. In this case, fluctuations in the curvaton field modulate the duration of inflation and can give rise to adiabatic non-Gaussian perturbations of the local type with $f_{\mathrm{NL}} \sim \mathcal{O}(10)$. For smaller values of α, we return to the standard chaotic inflation scenario with Gaussian adiabatic perturbations [48].

ACKNOWLEDGMENTS

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APPENDIX A: VARIANCE OF $\vec{E} \cdot \vec{B}$

The variance of $\vec{E} \cdot \vec{B}$ is defined as

$$\sigma^2 \equiv \langle (\vec{E} \cdot \vec{B})^2 \rangle - \langle \vec{E} \cdot \vec{B} \rangle^2 \tag{A1}$$

$$= \langle E_i E_j \rangle \langle B_i B_j \rangle + \langle E_i B_j \rangle \langle B_i E_j \rangle. \tag{A2}$$

We find

$$\langle E_iE_j\rangle\langle B_iB_j\rangle = \frac{1}{a^8}\int\frac{dkdq}{(2\pi)^6}|A'(k)|^2|A(q)|^2q^4k^2\int d^2\Omega_k d^2\Omega_q \epsilon_i(k)\epsilon_i(q)\epsilon_j^*(k)\epsilon_j^*(q),$$
$$\langle E_iB_j\rangle\langle B_iE_j\rangle = \frac{1}{a^8}\int\frac{dkdq}{(2\pi)^6}A(k)A'^*(k)A'(q)A^*(q)q^3k^3\int d^2\Omega_k d^2\Omega_q \epsilon_i(k)\epsilon_i(q)\epsilon_j^*(k)\epsilon_j^*(q). \quad (A3)$$

Here we use the polarization tensor conventions given in Ref. [31],

$$\vec{k}\cdot\vec{\epsilon}_\pm(\vec{k}) = 0, \qquad \vec{k}\times\vec{\epsilon}_\pm(\vec{k}) = \mp ik\vec{\epsilon}_\pm(\vec{k}), \qquad \vec{\epsilon}_\pm(-\vec{k}) = \vec{\epsilon}_\pm(\vec{k})^\star, \quad (A4)$$

which are normalized via $\vec{\epsilon}_\lambda(\vec{k})^\star\cdot\vec{\epsilon}_{\lambda'}(\vec{k}) = \delta_{\lambda\lambda'}$. Given our conventions (see Footnote 4), here we are dealing with $\vec{\epsilon}_-$.

The angular integral gives $(4\pi)^2/3$, i.e., a third of the whole sphere. The integrals over the modulus are similar to the one in Ref. [31] and are computed in the same way,

$$I_2 = \frac{1}{a^4}\int\frac{dk}{(2\pi)^3}|A'(k)|^2k^2 \simeq 2.2\times10^{-5}\frac{H^4}{\xi^3}e^{2\pi\xi}, \quad (A5)$$

$$I_3 = \frac{1}{a^4}\int\frac{dk}{(2\pi)^3}\frac{\partial_\tau}{2}|A(k)|^2k^3 \simeq 1.9\times10^{-5}\frac{H^4}{\xi^4}e^{2\pi\xi}, \quad (A6)$$

$$I_4 = \frac{1}{a^4}\int\frac{dk}{(2\pi)^3}|A(k)|^2k^4 \simeq 1.9\times10^{-5}\frac{H^4}{\xi^5}e^{2\pi\xi}. \quad (A7)$$

Putting things together, one finds

$$\sigma = \sqrt{\frac{(4\pi)^2}{3}(I_3^2 + I_2I_4)} \quad (A8)$$

$$= 2.0\times10^{-4}\frac{H^4}{\xi^4}e^{2\pi\xi} \simeq \langle\vec{E}\cdot\vec{B}\rangle. \quad (A9)$$

APPENDIX B: POWER SPECTRUM ESTIMATE

In Refs. [30,31] the power spectrum (24) was obtained by the Green's function method. In Ref. [32] a quick estimate was introduced to compute the power spectrum in the case of large backreaction ($\beta \gg 1$). Here we want to review and further explore this estimate, showing how it leads to Eq. (31) and also how, in the case of negligible backreaction, it approximates the precise result (24) within a factor of two.

The full equation of motion for the perturbation $\delta\chi$ is (in real space)

$$\delta\ddot{\chi} + 3\beta H\delta\dot{\chi} - \frac{\nabla^2}{a^2}\delta\chi + m^2\delta\chi = \alpha[\vec{E}\cdot\vec{B} - \langle\vec{E}\cdot\vec{B}\rangle], \quad (B1)$$

with

$$\beta \equiv 1 - 2\pi\xi\alpha\frac{\langle\vec{E}\cdot\vec{B}\rangle}{3H\dot{\chi}}. \quad (B2)$$

Near the horizon crossing we can estimate $\partial \sim H$. Since we have (near the horizon crossing) $H^2 = \frac{k^2}{a^2}$, the first term cancels the third one. The second term can be approximated as $3\beta H^2\delta\chi$. The last term on the left-hand side is just a slow-roll correction and can be discarded. This directly gives

$$\delta\chi \approx \frac{\alpha(\vec{E}\cdot\vec{B} - \langle\vec{E}\cdot\vec{B}\rangle)}{3\beta H^2}, \quad (B3)$$

and therefore we have

$$\zeta \equiv -\frac{H}{\dot{\chi}}\delta\chi \approx -\frac{\alpha(\vec{E}\cdot\vec{B} - \langle\vec{E}\cdot\vec{B}\rangle)}{3\beta H\dot{\chi}}. \quad (B4)$$

For the position-space two-point function of ζ we immediately get

$$\langle\zeta(x)^2\rangle \equiv \frac{H^2}{\dot{\chi}^2}\langle\delta\chi^2\rangle \approx \frac{H^2}{\dot{\chi}^2}\left(\frac{\alpha\sigma}{3\beta H^2}\right)^2 = \left(\frac{\alpha\langle\vec{E}\cdot\vec{B}\rangle}{3\beta H\dot{\chi}}\right)^2, \quad (B5)$$

with σ the variance computed in the previous subsection.

To compare the position-space power spectrum with the momentum-space power spectrum we use

$$\langle\zeta(\vec{k})\zeta(\vec{k}')\rangle \equiv (2\pi)^3\delta^3(\vec{k}+\vec{k}')P(k),$$
$$P(k) \equiv \frac{2\pi^2\Delta_\zeta^2(k)}{k^3}, \quad (B6)$$
$$\langle\zeta(x)^2\rangle = \int d\ln k\,\Delta_\zeta^2(k) \simeq \mathcal{O}(1)\Delta_\zeta^2(k).$$

This gives the result (31),

$$\Delta_\zeta^2(k) \simeq \langle\zeta(x)^2\rangle = \left(\frac{\alpha\langle\vec{E}\cdot\vec{B}\rangle}{3\beta H\dot{\chi}}\right)^2. \quad (B7)$$

This expression has been plotted in Fig. 4.

Now, when backreaction is strong we can approximate $\beta \approx -2\pi\xi\alpha\frac{\langle\vec{E}\cdot\vec{B}\rangle}{3H\dot{\chi}}$, which immediately gives the approximation (32)

$$\Delta_\zeta^2(k) = \frac{1}{(2\pi\xi)^2}. \quad (B8)$$

We can also make an approximation for the case where $\beta \approx 1$ (negligible backreaction) and compare the result with the precise result (24), just to see how well this whole approximation works. For $\beta = 1$ we have

$$\Delta_\zeta^2(k) = \left(\frac{\alpha\langle\vec{E}\cdot\vec{B}\rangle}{3H\dot{\chi}}\right)^2. \quad (B9)$$

Upon using the estimate for $\langle \vec{E}\cdot\vec{B}\rangle$ found in Refs. [30,31],

$$\langle \vec{E}\cdot\vec{B}\rangle \approx 2.4\times 10^{-4}\frac{H^4}{\xi^4}e^{2\pi\xi}, \tag{B10}$$

and

$$\alpha \equiv -\frac{2H\xi}{\dot{\chi}}, \tag{B11}$$

we find

$$\begin{aligned}\Delta_\zeta^2(k) &= \frac{4H^2\xi^2}{\dot{\chi}^2}\times 5.76\times 10^{-8}\times\frac{H^8}{\xi^8}e^{4\pi\xi}\times\frac{1}{9H^2\dot{\chi}^2}\\ &= 2.56\times 10^{-8}\times\frac{H^8}{\dot{\chi}^4}\times\frac{e^{4\pi\xi}}{\xi^6}\\ &= 2.56\times 10^{-8}\times\left(\frac{H^2}{2\pi\dot{\chi}}\right)^4\times(2\pi)^4\times\frac{e^{4\pi\xi}}{\xi^6}\\ &= 4.0\times 10^{-5}\times\Delta_{\zeta,\mathrm{sr}}^4(k)\times\frac{e^{4\pi\xi}}{\xi^6}.\end{aligned} \tag{B12}$$

This can be compared with the more precise result computed in Refs. [30,31] that uses the Green's function approach,

$$\Delta_\zeta^2(k) = \Delta_{\zeta,\mathrm{sr}}^4(k)\times f_2(\xi)\times e^{4\pi\xi} \tag{B13}$$

$$\simeq \Delta_{\zeta,\mathrm{sr}}^4(k)\frac{7.5\times 10^{-5}}{\xi^6}\times e^{4\pi\xi}, \tag{B14}$$

where in the second line we used the large ξ limit for f_2. We infer that this quick estimate is off by a factor less than two.

Actually, for some ξ the estimate comes even closer than this ratio $\frac{7.5}{4}$. Let us examine the situation at $\xi = 3$ [which, for $\xi(N=60)=2.2$), corresponds to $N\approx 35$]. Above, we approximated the numerical function $f_2(\xi)$ by $\frac{7.5\times 10^{-5}}{\xi^6}$, which yields an overestimate by a factor of 1.3. On the other hand, we also approximated the numerically found result for $\langle \vec{E}\cdot\vec{B}\rangle$ by the estimate (B10) (which is an underestimate), which for $\xi = 3$ only captures a fraction of 0.73 of the true $\langle \vec{E}\cdot\vec{B}\rangle$. Putting everything together one finds that, at $\xi = 3$ ($N = 35$), our estimate (B7) with β set to 1 overestimates the precisely computed numerical result (B13) by a factor of

$$\frac{4}{7.5}\times\frac{1.3}{(0.73)^2}\approx 1.3. \tag{B15}$$

At $\xi = 2.2$ ($N = 60$) we find that our estimate (B7) overestimates the precisely computed result by a factor of 2.5.

Now one might introduce a fudge factor such that at some preferred value for ξ our approximation precisely matches the numerically computed result. However, we have just seen that the inclusion of such a fudge factor will induce only a small shift in our estimate that we only trust up to corrections of order one. Besides, the fudge factor would always be arbitrary, as it depends on the preferred value of ξ that makes both signals match. Therefore it seems safe to neglect it altogether. In Fig. 7 we have for once plotted how the total power spectrum (including the standard slow-roll contribution) would shift from such a correction. In the rest of the paper we work with our uncorrected estimate for the power spectrum.

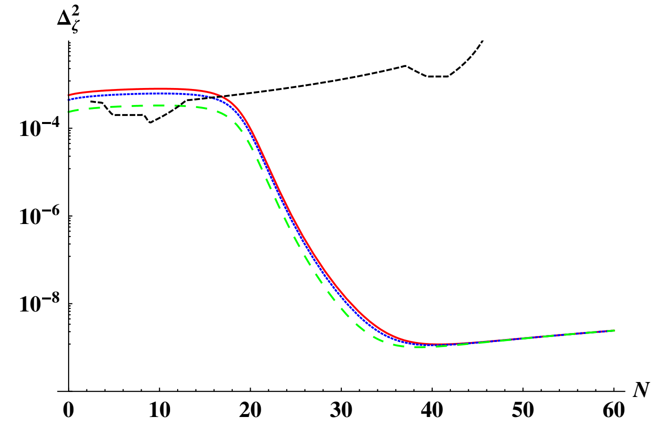

FIG. 7 (color online). Evolution of the power spectrum as in Fig. 4, still for $\xi[N=60]=2.2$. The red solid line is our estimate. The blue, tinily dashed line is our estimate corrected with a fudge factor of 1.3. The green, largely dashed line is our estimate corrected with a fudge factor of 2.5. All signals remain within an order-one factor from the black hole bounds in dashed black.

N. B. This estimate involves only the gauge field contribution to the power spectrum. Apart from that there is always the standard slow-roll component $\Delta_{\zeta,\mathrm{sr}}^2(k)$. This is the dominant contribution on CMB scales. That is why any estimate of the total power spectrum matches the precise result so well on CMB scales, whatever order-one fudge factor one chooses.

APPENDIX C: SKEWNESS OF $\vec{E}\cdot\vec{B}$

We want to compute

$$\begin{aligned}\tau^3 &\equiv \langle(\vec{E}\cdot\vec{B}-\langle\vec{E}\cdot\vec{B}\rangle)^3\rangle\\ &= \langle(\vec{E}\cdot\vec{B})^3\rangle - 3\langle\vec{E}\cdot\vec{B}\rangle\sigma^2 - \langle\vec{E}\cdot\vec{B}\rangle^3\\ &\simeq \langle(\vec{E}\cdot\vec{B})^3\rangle_c + 3\langle\vec{E}\cdot\vec{B}\rangle^3,\end{aligned} \tag{C1}$$

where we used $\langle(\vec{E}\cdot\vec{B})^2\rangle \simeq 2\langle\vec{E}\cdot\vec{B}\rangle^2$ from the previous section, and in the last step we recognized that there are $1+3\times 2 = 7$ nonconnected diagrams in $\langle(\vec{E}\cdot\vec{B})^3\rangle$, each one equal to $\langle\vec{E}\cdot\vec{B}\rangle^3$. Using Wick's theorem we find many terms. All of them have the same angular integral,

$$\int d^2\Omega_{k_1}d^2\Omega_{k_2}d^2\Omega_{k_3}\epsilon_i(k_1)\epsilon_i(k_2)\epsilon_j^*(k_1)\epsilon_j(k_3)\epsilon_j^*(k_2)\epsilon_j^*(k_3) = \frac{2\pi^5}{3}. \tag{C2}$$

Counting all the possible pairwise contractions, one finds

$$\langle(\vec{E}\cdot\vec{B})^3\rangle_c = -\frac{2\pi^5}{3}(2I_3^3 + I_2I_3I_4) = \left[-2.4\times 10^{-4}\frac{H^4}{\xi^4}e^{2\pi\xi}\right]^3 \simeq -\langle\vec{E}\cdot\vec{B}\rangle^3, \tag{C3}$$

and therefore

$$\tau^3 \simeq 2\langle\vec{E}\cdot\vec{B}\rangle^3. \tag{C4}$$

APPENDIX D: BISPECTRUM AND f_{NL} ESTIMATE

The position-space three-point function of ζ can be directly generalized from (B5),

$$\langle\zeta(x)^3\rangle \equiv -\frac{H^3}{\dot{\chi}^3}\langle\delta\chi^3\rangle \approx -\frac{H^3}{\dot{\chi}^3}\left(\frac{\alpha\tau}{3\beta H^2}\right)^3 = -2\left(\frac{\alpha\langle\vec{E}\cdot\vec{B}\rangle}{3\beta H\dot{\chi}}\right)^3, \tag{D1}$$

where we used the definition of the skewness h^3 [Eq. (C1)] and its estimate (C4). $\langle\zeta(x)^3\rangle$ is positive. (Again, we work with a negative $\dot{\chi}$, which yields positive $\langle\vec{E}\cdot\vec{B}\rangle$, while working with $\dot{\chi} > 0$ would give $\langle\vec{E}\cdot\vec{B}\rangle < 0$.)

Let us first analyze this result in the regime where the backreaction is negligible, i.e., $\beta = 1$. Using Eqs. (B10) and (B11) we get

$$\langle\zeta(\vec{x})^3\rangle \simeq 2\frac{8}{27}(2.4\times 10^{-4})^3\frac{H^{12}e^{6\pi\xi}}{\xi^9\dot{\chi}^6} \simeq 8.2\times 10^{-12}\frac{H^{12}e^{6\pi\xi}}{\xi^9\dot{\chi}^6}. \tag{D2}$$

Now we want to compare this with the momentum-space bispectrum $B(k)$, defined via

$$\langle\zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle \equiv (2\pi)^3\delta^3(k_1+k_2+k_3)B(\vec{k}_1,\vec{k}_2,\vec{k}_3), \tag{D3}$$

for which we can write

$$\langle\zeta(\vec{x})^3\rangle = \int\frac{d^3k_1}{(2\pi)^3}\int\frac{d^3k_2}{(2\pi)^3}B(\vec{k}_1,\vec{k}_2,-\vec{k}_1-\vec{k}_2). \tag{D4}$$

When non-Gaussianity is large mostly on equilateral triangles, the integral is supported in the region $k_2 \simeq k_1$ and $\theta_{12} \simeq \pi/3$. Hence we estimate

$$\langle\zeta(\vec{x})^3\rangle = \int d\log k\frac{8\pi^2}{(2\pi)^6}k^6B_{\mathrm{eq}}(k) \simeq \frac{8\pi^2}{(2\pi)^6}k^6B_{\mathrm{eq}}(k)\mathcal{O}(1), \tag{D5}$$

where $B_{\mathrm{eq}}(k)$ is the bispectrum evaluated on equilateral triangles. Now we can compare our estimate (D2) with the precisely computed result using the Green's function approach, which we take from result (2.8) of Ref. [33],

$$B_{\mathrm{eq}}(k) = \frac{1}{(2\pi)^3}\langle\zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle \simeq \frac{3\times3\times2.8\times10^{-7}}{10(2\pi)^2}\frac{H^{12}e^{6\pi\xi}}{\xi^9\dot{\phi}^6}\frac{1}{k^6}, \tag{D6}$$

where we have used the large-ξ estimate

$$f_3(\xi) = \frac{2.8\times10^{-7}}{\xi^9}. \tag{D7}$$

This last result leads to

$$\langle\zeta(\vec{x})^3\rangle \simeq \frac{8\pi^2}{(2\pi)^6}k^6B_{\mathrm{eq}}(k) \simeq 8.2\times10^{-12}\frac{H^{12}e^{6\pi\xi}}{\xi^9\dot{\phi}^6}, \tag{D8}$$

which agrees (surprisingly) well with Eq. (D2).

In the regime of strong backreaction we can write $\beta \approx -2\pi\xi\alpha\frac{\langle\vec{E}\cdot\vec{B}\rangle}{3H\dot{\chi}}$ and the estimate (D1) directly gives the generalization of (B8),

$$\langle\zeta(\vec{x})^3\rangle \simeq \frac{1}{4\pi^3\xi^3}. \tag{D9}$$

Finally, we want to convert these results into a value for f_{NL}. We take f_{NL} to be defined via

$$\langle\zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle = (2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)(2\pi)^4\frac{3}{10}f_{\mathrm{NL}}\Delta_\zeta^4(k)\frac{\sum_i k_i^3}{\Pi_i k_i^3}. \tag{D10}$$

This gives

$$f_{\mathrm{NL}} = B(\vec{k}_1,\vec{k}_2,\vec{k}_3)\frac{10}{3}\frac{1}{(2\pi)^4}\frac{1}{\Delta_\zeta^4(k)}\frac{\Pi_i k_i^3}{\sum_i k_i^3}, \tag{D11}$$

which for the equilateral case becomes

$$f_{\mathrm{NL}}^{\mathrm{eq}} = B_{\mathrm{eq}}(\vec{k})\frac{10}{3}\frac{1}{(2\pi)^4}\frac{1}{\Delta_\zeta^4(k)}\frac{k^9}{3k^3} = \frac{(2\pi)^6}{8\pi^2}\frac{1}{k^6}\langle\zeta(\vec{x})^3\rangle\times\frac{10}{3}\frac{1}{(2\pi)^4}\frac{1}{\Delta_\zeta^4(k)}\frac{k^9}{3k^3} = \frac{10}{9}\frac{(2\pi)^2}{8\pi^2}\frac{\langle\zeta(\vec{x})^3\rangle}{\Delta_\zeta^4(k)}. \tag{D12}$$

In the regime of negligible backreaction we can then take our estimate (D2) and conclude that

$$f_{\mathrm{NL}}^{\mathrm{eq}} = \frac{2.8\times10^{-7}}{\xi^9}\frac{e^{6\pi\xi}\Delta_{\zeta,\mathrm{sr}}^6(k)}{\Delta_\zeta^4(k)}. \tag{D13}$$

This again matches the result obtained in Refs. [30,31] by a more precise computation. (Of course, after finding that the expressions for $\langle\zeta(\vec{x})^3\rangle$ match so well, this is only a consistency check.)

In the regime of strong backreaction, finally, we need to insert Eq. (D9) into Eq. (D12). Using our power spectrum estimate (B8) we find

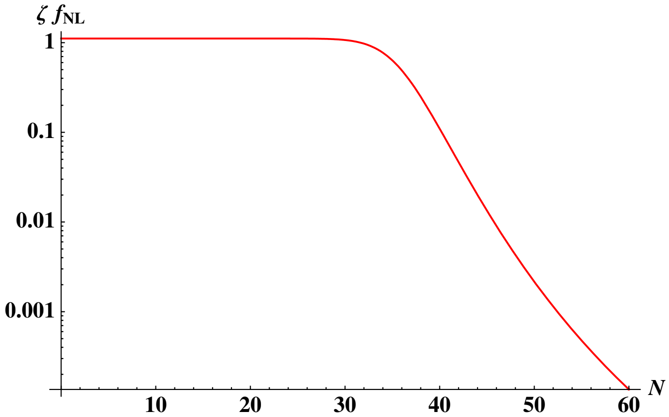

FIG. 8 (color online). Evolution of $f_{\rm NL} \times \zeta$ as a function of N, for $\xi[N = 60] = 2.2$.

$$f_{\rm NL}^{\rm eq} = \frac{10}{9}\frac{(2\pi)^2}{8\pi^2}\frac{(2\pi\xi)^4}{4\pi^3\xi^3} = \frac{10}{9}2\pi\xi \simeq 42, \tag{D14}$$

where we have used the fact that towards the end of inflation we have $\xi \simeq 6$.

Notwithstanding the precise match between Eqs. (D2) and (D8), there is a still an order-one factor between the estimate for the three-point function (and for $f_{\rm NL}$) and its precisely computed numerical value. Again, to arrive at Eq. (D2) we have used the estimate (B10) for $\langle \vec{E} \cdot \vec{B} \rangle$, and to arrive at Eq. (D8) we have inserted the large-ξ approximation $\frac{2.8 \cdot 10^{-7}}{\xi^9}$ for $f_3(\xi)$. When using precise numerical prescriptions rather than estimates for $\langle \vec{E} \cdot \vec{B} \rangle$ and $f_3(\xi)$ we find that our estimates overshoots the precisely computed $f_{\rm NL}$ by a factor of 9.5 at $\xi = 2.2$ ($N = 60$), and by a factor of 3.8 at $\xi = 3$ ($N \approx 35$).

Again, we will not bother introducing a fudge factor to close this gap at some preferred value of ξ. Anyway, when the backreaction is large $f_{\rm NL}$ is no longer a suitable indicator for the amount of non-Gaussianity. In Fig. 8 we plot our estimate for a more meaningful quantity: the skewness, which is equivalent to $f_{\rm NL}\zeta$. When backreaction becomes important, it saturates at a value of about one, which *a posteriori* justifies our approach (36).

APPENDIX E: BLACK HOLE MASSES

In this Appendix we give some details about the derivation of Eq. (42) for the black hole mass and about the total number of e-foldings enforced by a specific expansion history.

Suppose that the universe is radiation dominated right after the end of inflation. Then the expansion proceeds as $a \sim (t/t_0)^{1/2}$, so $H(t) = \frac{1}{2t}$. This regime starts at t_0, which is not the time since the beginning of the Big Bang, but simply the constant $t_0 = \frac{1}{2H}$, where H is the Hubble constant at the end of inflation. We distinguish it from the decreasing $H(t) = \frac{1}{2t}$. The wavelength $l_{t_0} = H^{-1}e^N$ grows as $l_t = H^{-1}(t/t_0)^{1/2}e^N = H^{-1}(2Ht)^{1/2}e^N$. The horizon size $1/H(t) = 2t$ grows and becomes equal to l_t (and black holes form) at

$$2t = H^{-1}(2Ht)^{1/2}e^N,$$

i.e., at

$$(2Ht)^{1/2} = (t/t_0)^{1/2} = e^N.$$

In other words, the black holes form after the universe expands by a factor e^N since the end of inflation. The initial energy stored inside the volume $H^{-1}e^N$ was $M_N \simeq 10e^{3N}$ g, but during this extra expansion it scales down (redshifts) by the factor e^N, so it becomes

$$M_{\rm BH} \simeq 10e^{2N}\ {\rm g}.$$

It should be stressed that specifying the energy density at the end of inflation and at reheating directly determines the number of e-foldings corresponding to any scale (and in particular CMB scales) according to [49]

$$N(k) = 62 - \log\frac{k}{a_0H_0} - \log\frac{10^{16}\ {\rm GeV}}{V_*^{1/4}} + \log\frac{V_*^{1/4}}{V_{\rm end}^{1/4}} - \frac{1}{3}\log\frac{V_{\rm end}^{1/4}}{\rho_{\rm reh}^{1/4}}, \tag{E1}$$

where V_* is the energy density during inflation when the mode k left the horizon, $V_{\rm end}$ is the energy density at the end of inflation, $\rho_{\rm reh}$ is the energy density at reheating and the subscript 0 refers to today's value. Taking for example $\rho_{\rm reh} = V_{\rm end} = m^2M_p^2/2$ and $V_k = m^2 15^2 M_p^2/2$ with $m = 6 \times 10^6 M_p$ gives $N_{\rm CMB} = N(a_0H_0) \simeq 64$. We use this value in our discussion of primordial black holes, but since the difference between 60 and 64 changes very little in our numerics, for simplicity we use $N_{\rm CMB} = 60$ in the rest of the paper.

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