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The decay of optical emission from the γ -ray burst GRB970228

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The origin of γ -ray bursts has been one of the great unsolved mysteries in high-energy astrophysics for almost 30 years. The recent discovery of fading sources at X-ray¹ and optical^{2,3} wavelengths coincident with the location of the γ -ray burst GRB970228 therefore provides an unprecedented opportunity to probe the nature of these high-energy events. The optical counterpart appears to be a transient point source embedded in a region of extended nebulosity³⁻⁶, the latter having been tentatively identified as a high-redshift galaxy³. This would seem to favour models that place γ -ray bursts at cosmological distances, although a range of mechanisms for producing the bursts is still allowed. A crucial piece of information for distinguishing between such models is how the brightness of the optical counterpart evolves with time. Here we re-evaluate the existing photometry of the optical counterpart of GRB970228 to construct an optical light curve for the transient event. We find that between 21 hours and six days after the burst, the R-band brightness decreased by a factor of ~ 40 , with any subsequent decrease in brightness occurring at a much slower rate. As the point source faded, it also became redder. The initial behaviour of the source appears to be consistent with the 'fireball' model⁷, but the subsequent decrease in the rate of fading may prove harder to explain.

The γ -ray burst of 28 February 1997, detected⁸ with the Gamma-Ray Burst Monitor on board the BeppoSAX satellite, and located with an $\sim 3'$ radius position with the Wide Field Camera on the same satellite, was the first for which a fading X-ray¹ and optical counterpart^{2,3} were discovered.

The optical counterpart was discovered^{2,3} from a comparison of V- and I-band images taken with the William Herschel Telescope (WHT) on February 28.99 UT, and the Isaac Newton Telescope (INT; V band) and the WHT (I band) on March 8.8 UT. After the counterpart had weakened by several magnitudes, it was found to coincide with an extended object^{9,10}. In subsequent observations with the Hubble Space Telescope (HST) on 26 March and 7 April 1997, Sahu *et al.*⁴⁻⁶ found that the optical counterpart consists of a

point source and an extended ($\sim 1''$) object, offset from the point source by $\sim 0.5''$. The extension is in the same direction as that seen in the earlier ground-based observations.

In view of the importance of the optical light curve in constraining possible models of γ -ray bursts, we have re-assessed the photometric information presented by Van Paradijs *et al.*³, together with more recent information in light of the HST findings, and derived an optical light curve of the point source component of the GRB counterpart.

In Table 1 we have collected the available optical photometry on GRB970228. These results have been obtained in several passbands, primarily V, R and I (effective wavelengths $\sim 5,500 \text{ \AA}$, $\sim 6,500 \text{ \AA}$ and $\sim 8,000 \text{ \AA}$, respectively, corresponding closely to the Cousins VRI system). As the R band occupies a central position relative to most of the passbands used in the available observations, we have reduced all photometry to the R band. In the required interpolations we have assumed that the spectra of the point source and the extended emission are similar to main-sequence stars (that is, not dominated by emission lines). We have used the relation between the colour indices $V - R$ and $V - I$ given by Thé *et al.*¹¹ for late-type stars; for bluer stars we have inferred this relation from the tables given by Johnson¹² for main-sequence stars and the colour transformations to the Cousins VRI system given by Bessell¹³. For given magnitudes V and I the uncertainty in the interpolated magnitude R is determined by the uncertainty in the ratio $\alpha = (V - R)/(V - I)$, because $R = V - \alpha(V - I)$. Over the range in $(V - I)$ covered by the point source, this ratio α varies between 0.45 (at $V - I = 0.7$) and 0.55 (at $V - I = 2.0$). We have made alternative estimates of α from numerical integrations of power-law flux distributions ($F(\lambda) \propto \lambda^\beta$, with β ranging between -4 and $+2$), and of Planck functions (with temperatures between 3,000 and 10,000 K), each multiplied by V, R and I response functions¹⁴. We find that these 'theoretical' α values cluster near 0.4, that is, only slightly smaller than the values observed for stars. We conclude that if the flux distribution of the optical counterpart is smooth, the uncertainty in the interpolation of R from V and I is unlikely to exceed 0.1 mag.

From the V and I magnitudes obtained from the two HST images (Table 1; to arrive at these values we applied a colour-dependent correction term¹⁵) we infer that between the two HST observations the point source became marginally fainter; the extended source did not show a significant brightness change.

In the 28 February observation with the WHT the optical transient is much brighter than any steady counterpart, and the

Table 1 Summary of optical observations

| Date | Time (ut) | Telescope | Magnitude | Remarks |
|---------|-----------|-----------|---------------------|------------------|
| Feb. 28 | 18.4 | BUT | $R = 21.1 \pm 0.2$ | OT |
| Feb. 28 | 19.4 | RAO | $R = 20.5 \pm 0.5$ | OT |
| Feb. 28 | 23.8 | WHT | $V = 21.3 \pm 0.1$ | OT |
| Feb. 28 | 23.8 | WHT | $I = 20.6 \pm 0.1$ | OT |
| Mar. 03 | 2.4 | APO | $B = 23.3 \pm 0.5$ | OT |
| Mar. 04 | 20.7 | NOT | $V > 24.2$ | OT + ext. source |
| Mar. 06 | 7.7 | Keck | $R = 24.0$ | OT + ext. source |
| Mar. 08 | 20.7 | INT | $V > 23.6$ | OT + ext. source |
| Mar. 08 | 21.2 | WHT | $I > 22.2$ | OT + ext. source |
| Mar. 09 | 21.5 | INT | $R = 24.0 \pm 0.2$ | OT + ext. source |
| Mar. 09 | 20.5 | INT | $B = 25.4 \pm 0.4$ | OT + ext. source |
| Mar. 13 | 0.0 | NTT | $R = 24.3 \pm 0.2$ | OT + ext. source |
| Mar. 26 | 9.2 | HST | $V = 26.1 \pm 0.1$ | OT |
| Mar. 26 | 11.2 | HST | $I = 24.2 \pm 0.1$ | OT |
| Mar. 26 | 9.2 | HST | $V = 24.9 \pm 0.3$ | Ext. source |
| Mar. 26 | 11.2 | HST | $I = 24.5 \pm 0.3$ | Ext. source |
| Apr. 7 | 5.2 | HST | $V = 26.4 \pm 0.1$ | OT |
| Apr. 7 | 7.2 | HST | $I = 26.4 \pm 0.1$ | OT |
| Apr. 7 | 5.2 | HST | $V = 25.2 \pm 0.35$ | Ext. source |
| Apr. 7 | 7.2 | HST | $I = 24.3 \pm 0.35$ | Ext. source |

Abbreviations: OT, optical transient; ext. source, extended source; BUT, Bologna University Telescope; RAO, Rome Astrophysical Observatory; WHT, William Herschel Telescope; APO, Apache Point Observatory; NOT, Nordic Optical Telescope; INT, Isaac Newton Telescope; HST, Hubble Space Telescope.

latter's contribution can therefore be entirely neglected. From a re-analysis of the images we confirm the (V, I) magnitudes for the transient as given before^{2,3} (Table 1), but those for the substantially weaker nearby star are somewhat different: (V, I) = (23.0, 21.2). The latter values are in good agreement with those obtained with the HST: (V, I) = (23.0, 21.4).

Guarneri *et al.*¹⁶ found no variable objects in B- and R-band images taken with the Bologna University Telescope on February 28.76, and on March 1.81, 3.76, 4.82 and 5.86 UT. However, a subsequent analysis of these images confirms that the optical transient is present on February 28.76 UT at the position given by Groot *et al.*⁹ with $R = 21.1 \pm 0.2$ (A. Guarneri, personal communication).

Pedichini *et al.*¹⁷ confirmed the detection of the optical transient on a Schmidt CCD image taken at the Rome Astrophysical Observatory on February 28.81 UT; it was then 1.6 ± 0.5 mag brighter than the nearby late-type star (for which $R = 22.1$). The broad-band filter used in this observation had a peak at 7,000 Å, similar to that of the R band, but because its width is almost twice that of the latter the inferred $R = 20.5 \pm 0.5$ may suffer from a colour effect. But because on February 28.99 UT (at about the same brightness level) $V - I = 0.7$, this colour effect is likely to be small compared to the quoted error of 0.5 mag.

In an observation with the Apache Point Observatory on March 3.1 UT, Margon *et al.*¹⁸ detected the optical transient at $B = 23.3 \pm 0.5$. We have inferred an estimated R-band magnitude, by interpolating between the colour index $B - R = 1.4$, observed on

9 March (see Table 1), and a value for this colour index on 28 February, $B - R = 1.1$, inferred from the observed $V - I$, assuming again that the ($B - R, V - I$) relation follows that of main-sequence stars. This estimate of R involves an extrapolation rather than an interpolation (as with V and I observations); this entails an additional uncertainty, which we conservatively estimate at 0.5 mag. The corresponding R-band magnitude on March 3.1 UT then is 22.2 ± 0.7 .

In the 4 March observation with the Nordic Optical Telescope, and the 8 March observations with the WHT and the INT, the counterpart was not detected (see Table 1 for the upper limits).

In the 9 March observation with the INT and the 13 March observations with the ESO New Technology Telescope (NTT), the source is detected. In the NTT image the source appears extended; this was found¹⁰ independently from images obtained with the Keck Telescope. The direction of this offset is consistent with being caused by the faint extended emission detected with the HST observations.

We have transformed the collection of V, R and I magnitudes to one common R-band magnitude, in the following way. For near-simultaneous V and I observations we have used the ($V - I, V - R$) relation (see above) to get $R = V - (V - R)$. In case no such simultaneous observations are available, we have interpolated linearly in the relation between ($V, V - I$) obtained from the 28 February and the HST observations. We have collected these results in Table 2.

We note that the R-band magnitude for the combined point source plus extended component obtained from the HST images (Table 2) differs substantially from the value, $R = 24.9 \pm 0.3$, given by Metzger *et al.*¹⁹ on 5 and 6 April 1997. Moreover, their conclusion that the component causing the earlier (ground-based) images to appear extended has faded below detection level seems difficult to reconcile with the HST observation that both in the V and I bands the extended component is at least as bright as the point source.

We have subtracted the contribution of the extended component from the ground-based results. From the corrected HST magnitudes (see above) we infer that the R-band magnitude of the extended component (averaged over the two HST observations) is $R = 24.7 \pm 0.3$. These corrected R-band magnitudes for the point source are also given in Table 2.

The R-band light curve obtained from our analysis of the available photometry is shown in Fig. 1, in which we have plotted R versus $\log t$, where t is the time since the γ -ray burst occurred.

A single power-law decay of the flux (that is, $F_R(t) \propto t^{-\alpha}$) during the entire 38-day period does not represent the data well. The upper limit on R obtained on 4 March and the R magnitude measured on 6 March are incompatible with such decay, which in Fig. 1 would be represented by a straight line.

The initial decline, between 28 February and 6 March was much faster than that after 6 March. Before 6 March the average decay rate was $0.7^{+0.2}_{-0.1}$ mag d⁻¹, during the month following 6 March it was $0.02^{+0.02}_{-0.04}$ mag d⁻¹. Expressed in terms of a power-law exponent α , the average decay before 6 March corresponds to $\alpha = 2.1^{+0.3}_{-0.5}$, whereas after 6 March we find $\alpha < 0.35$. Expressed in terms of an exponential decay time τ of the R-band flux, we find before 6 March $\tau \approx 1.4$ d, and afterwards $\tau > 25$ d.

As the point-source component of the optical transient became fainter, it also became redder, from $V - I = 0.7$ at $V = 21.3$ (28 February) to $V - I = 1.9$ at $V = 26.1$ (26 March), and $V - I = 1.8$ at $V = 26.4$ (7 April).

The exponent of the power law describing the decay of the X-ray afterglow of GRB970228, as obtained from the results given by Costa *et al.*¹ and Yoshida *et al.*²⁰, is $\alpha_X = 1.4 \pm 0.2$. A comparison of this value with the initial power-law slope of the optical brightness decay indicates that during the first week after the γ -ray burst the optical transient decayed more rapidly than the X-ray transient.

From recent comparisons^{6,21} of the optical and X-ray observations of GRB970228 with predictions of the 'fireball' model (see, for

Table 2 The R-band light curve of GRB970228

| Date (UT) | Telescope | R (OT + ext. source) | R (OT) |
|------------|-----------|------------------------|----------------------|
| Feb. 28.76 | BUT | 21.1 ± 0.2 | 21.1 ± 0.2 |
| Feb. 28.81 | RAO | 20.5 ± 0.5 | 20.5 ± 0.5 |
| Feb. 28.99 | WHT | 20.9 ± 0.14 | 20.9 ± 0.14 |
| Mar. 03.10 | APO | 22.2 ± 0.7 | $22.3^{+0.9}_{-0.7}$ |
| Mar. 04.86 | NOT | >23.3 | >23.6 |
| Mar. 06.32 | Keck | 24.0 ± 0.2 | $24.8^{+1.2}_{-0.5}$ |
| Mar. 08.88 | INT + WHT | >22.6 | >22.8 |
| Mar. 09.90 | INT | 24.0 ± 0.2 | $24.8^{+1.2}_{-0.5}$ |
| Mar. 13.00 | NTT | 24.3 ± 0.2 | $25.6^{+2.6}_{-0.9}$ |
| Mar. 26.42 | HST | 24.1 ± 0.2 | 25.17 ± 0.13 |
| Apr. 07.23 | HST | 24.2 ± 0.15 | 25.50 ± 0.13 |

Abbreviations as Table 1.

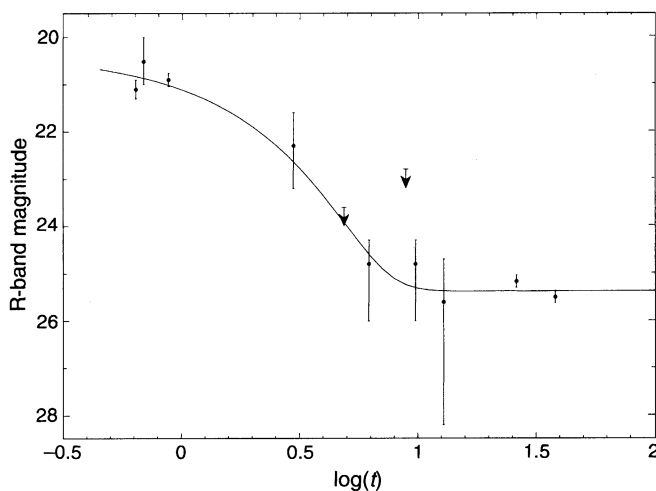


Figure 1 R-band light curve of the point source component in the optical counterpart of GRB970228. The R-band data are from Table 2, and t represents the time interval since the γ -ray burst occurred. For reference, an exponential decay with $\tau = 1.4$ days plus a constant is shown as a solid curve.

example, Meszaros⁷) it has been concluded that this model is consistent with the initial decay of the γ -ray burst afterglow. We note however, that the decay does not continue beyond 6 March. This slow-down of the brightness decrease after 6 March may be the result of changes in one or more factors determining the optical emission, such as changes in source geometry, or optical depth effects. It may signal the changeover to a different dominant energy source. If γ -ray bursts are the result of mergers of neutron star binaries (see, for example, Piran²² and references therein), decay of radioactive nuclei produced by de-neutronization of neutron-star matter²³ may play a role. □

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Pure $d_{x^2-y^2}$ order-parameter symmetry in the tetragonal superconductor $Tl_2Ba_2CuO_{6+\delta}$

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Crucial to the successful development of a theoretical model for high-temperature superconductivity is knowledge of the symmetry of the order parameter (or wavefunction) that describes the pairing of electrons in the superconducting state. Several experimental studies^{1–8} provide convincing evidence for an anisotropic order parameter, consistent with a $d_{x^2-y^2}$ symmetry. But none of these earlier experiments could rule out unambiguously an addi-

tional contribution from isotropic s -wave pairing; these experiments either involved superconductors with an orthorhombic crystal structure (for which a mixed $s + d$ state is becoming increasingly recognized as a likely consequence^{9,10}), or their interpretation required detailed modelling of the uncertain effects of disorder and defects. Here we report the results of an experiment designed to circumvent these difficulties: the material studied is the single-layer tetragonal superconductor $Tl_2Ba_2CuO_{6+\delta}$, and the experimental configuration is such that the interpretation of the results relies solely on symmetry considerations. Our results indicate that this material has pure $d_{x^2-y^2}$ pairing symmetry, so providing a starting point for understanding the more complex mixed $s + d$ state that appears to characterize other high-temperature superconductors.

An $s + d$ pair state in orthorhombic copper oxides such as $YBa_2Cu_3O_{7-\delta}$ (YBCO) is characterized by a gap function $\Delta(k)$ that transforms like $s(k_x^2 + k_y^2) + d(k_x^2 - k_y^2)$, where k_x and k_y are components of the wavevector k , and s and d are measures of the amounts of s -wave and d -wave pairing in the admixture. Node lines in the gap function ($\Delta = 0$) given by $k_y = \pm [(d + s)/(d - s)]^{1/2} k_x$ exist provided that $d/s \geq 1$. The deviation of the slope of the node lines from the diagonals $k_x = \pm k_y$ of the Brillouin zone depends on the extent of the $s + d$ admixture. Because the Pb-YBCO corner superconducting quantum interference device, SQUID, (or single Josephson junction) interference experiments rely on a sign change between the a and b faces of YBCO, they cannot distinguish pure d -wave from mixed $s + d$ pair states as long as $d/s \geq 1$. The tricrystal experiments with YBCO and $Tl_2Ba_2CuO_{6+\delta}$ (Tl2001) can, in principle, locate the nodes on the Fermi surface. However, this requires a systematic series of tricrystal experiments and a detailed model describing pair tunnelling beyond the Sigrist–Rice formula¹¹, which does not take into account interface roughness or disorder.

In the face of these difficulties, it is important to demonstrate unambiguously the existence of a pure $d_{x^2-y^2}$ wave high- T_c copper

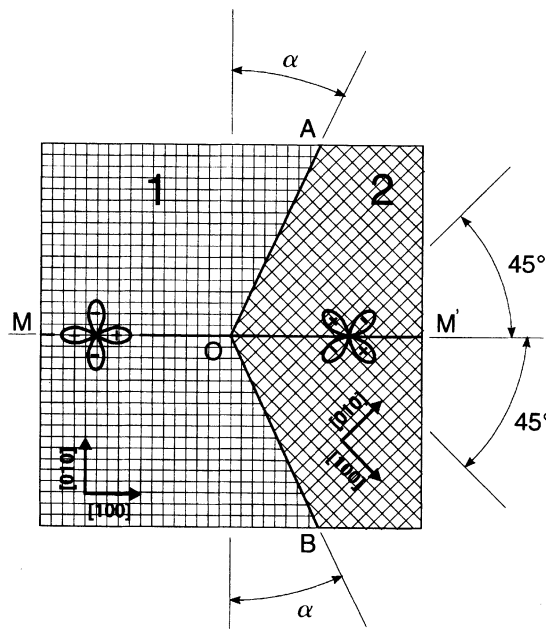


Figure 1 Schematic diagram illustrating the geometrical configuration of a tetragonal (100) $SrTiO_3$ substrate, which effectively consists of two crystals rotated about the normal to the substrate plane by $\pi/4$ with respect to each other. For a given total misorientation angle $\theta = \theta_1 + \theta_2$, the pair tunnelling current is maximized⁹ for a symmetric grain boundary $\theta_1 = \theta_2 = \pi/8$. We chose the angle $\alpha = \theta$, between the vertical ($[010]$ in grain 1) and the grain boundaries (OA and OB) as 25° . Also shown are the polar plots of the assumed $d_{x^2-y^2}$ gap functions aligned with the crystallographic axes in the substrate.