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Krzhizhanovskaya, V.V.; Melnikova, N.B.

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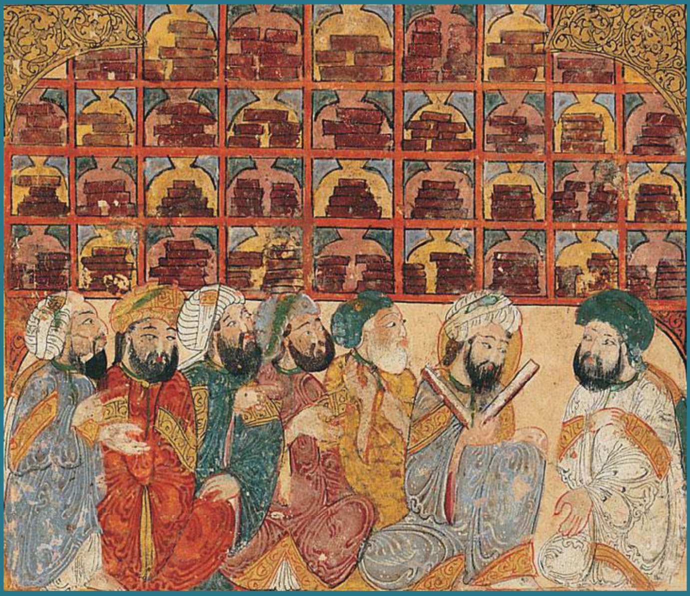

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Modeling Earthen Dikes: Sensitivity Analysis and Calibration of Soil Properties Based on Sensor Data

V.V. Krzhizhanovskaya[1,2], **N.B. Melnikova**[1,2]
[1] *National Research University ITMO, Russia*
[2] *University of Amsterdam, The Netherlands*

emails: Valeria.Krzhizhanovskaya@gmail.com

Abstract

A mathematical model of earthen dike behavior under dynamic hydraulic load describes a coupled fluid-structure interaction system. Transient flow through porous media is modeled by Richards equation with Van Genuchten model for water retention in partially saturated soil around the phreatic surface. Structural stability analysis is based on Drucker-Prager elastic perfectly plastic material model. We show the results of sensitivity analysis of porous flow dynamics to soil permeability; and calibration of soil properties based on sensor data from a real sea dike.

Key words: dike, stability, flow in porous media, Richards equation, sensitivity analysis, calibration, soil permeability

MSC2000: 76S05

1 Introduction

Regular floods pose a serious threat to human life, valuable property and city infrastructure. Many international projects are aimed at the development of flood protection systems [1], including the *UrbanFlood* European project [2] that unites the work on monitoring dikes with sensor techniques [3], physical study of dike failure mechanisms [4], and software development for dike stability analysis [5], [6], simulation of dike breaching, flood, and city evacuation [7], [8], [9].

Most of the flood defenses are based on earthen dikes and levees. Their stability is of utmost importance for flood prevention. One of the goals of the *UrbanFlood* project was the development of the *Virtual Dike* computational model for advanced research into dike stability and failure mechanisms [6], and for training the artificial intelligence module on simulated dike instabilities [5].

In numerical analysis of stability of an earthen dike under dynamically changing hydraulic loads, water pressure inside the dike contributes to the total stresses in the dike. The dynamics of pore pressure can provide important engineering information on the safety of the dike. Abnormal distribution of pore pressures may indicate the onset of dike macro-instability or piping.

Sea tides or changing river levels influence the position of phreatic surface. Moving water table creates the zones with partially saturated soils. Resistance of porous media to the flow is modeled by Darcy's law suitable for low velocities [10]. A problem of unconfined porous flow dynamics can be modelled either by Darcy's equation on a moving mesh, adjusting the domain boundary position to coincide with the surface of zero pore pressure [11], or by using stationary mesh and solving Richards equation with non-linear rheological properties of the media depending on the effective water content. These non-linear properties can be modeled by classical models of Van Genuchten [13] or Brookes and Corey model [14], as well as by some approximations [12] simplified for faster numerical convergence. In this work we used Richards equation with Van Genuchten model, in order to perform simulations on a fixed mesh.

A real dike is a heterogeneous porous media with unknown distribution of soil parameters. This paper presents numerical analysis of sensitivity of the porous flow simulation to the variation of soil permeability. Calibration of soil properties for the tidal groundwater flow is often performed by tidal method [15], [16], based on one-dimensional analytical models of half-infinite or finite aquifers. The method is suitable for aquifers with low elevation of phreatic surface with respect to the average depth. A more accurate way that works well for high amplitude of water level variation is direct numerical simulation. In present work, both analytical and numerical approaches have been tested and compared.

Sensitivity analysis and calibration of soil properties have been performed for the LiveDike, a sea dike in Groningen, the Netherlands, equipped with a number of pore pressure sensors installed in four cross-sections (Figure 1). The height of the dike is 9 m, the width is about 60 m, the length is about 300 m.

The remainder of this paper is organized as follows: Section 2 describes the mathematical model of dike behavior under dynamic hydraulic load; Section 3 gives model implementation details; Section 4 presents the results of sensitivity analysis and calibration of soil properties; and Section 5 concludes the paper.

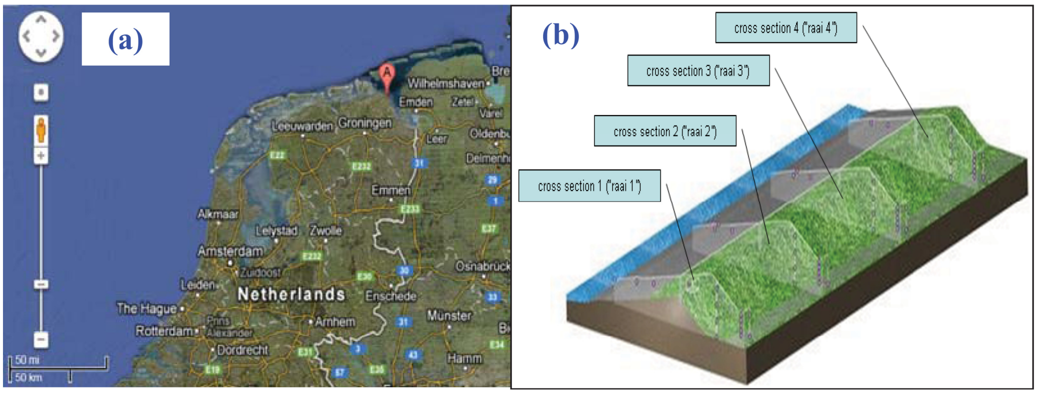


Figure 1. (a) Location of the LiveDike, sea dike near Groningen, The Netherlands; (b) LiveDike cross-sections with marked sensor locations

2 Mathematical model of dike behavior under hydraulic load

Dike stability analysis requires solving a coupled fluid-structure interaction problem that was described in detail in [6]. Due to the page limit, here we describe only the fluid sub-model. It calculates the contribution of water pressure inside the dike to the total load calculated in the structural dynamics sub-model. The problem of flow through porous media is described by the pressure-based form of Richards equation [10], taking into account wetting and drying of the area above phreatic surface:

$$(C+\theta_e S)\frac{\partial p}{\partial t}+\nabla\cdot\left[-\frac{K_S}{\mu}k_r\nabla(p+\rho g y)\right]=-\frac{\partial \varepsilon}{\partial t}, \tag{1}$$

where $C=\partial\theta/\partial p$ is specific moisture capacity, [1/Pa]; $\theta_e=\theta_e(p)$ is effective water content; S is storage coefficient, [1/Pa]; p is gauge pressure (relative to atmospheric pressure), [Pa]; t is time, [s]; K_S is permeability of saturated media, [m^2]; $k_r=k_r(p)$ is relative permeability of unsaturated soil; μ is dynamic viscosity of water, [Pa·s]; g is standard gravity, [m/s^2]; ρ is water density [kg/m^3]; y is a coordinate in vertical direction, [m].

The right-hand-side term $\partial\varepsilon/\partial t$ is a rate of volume deformation of soil skeleton, calculated by the structural sub-model. In this study, we assume that soil skeleton does not undergo large volume deformations during tidal load and therefore the RHS term is zero. This assumption makes our model one-way coupled, with an independent fluid sub-model.

In saturated soil C=0, θ_e=1, k_r=1, and p≥0. In unsaturated zone, pressures are negative and properties of unsaturated soil (specific moisture capacity and relative permeability) are defined by van Genuchten equations as follows [13]:

$$C = \begin{cases} \dfrac{am}{1-m}(\theta_s - \theta_r)\theta_e^{1/m}(1-\theta_e^{1/m})^m, & p < 0, \\ 0, \quad p \ge 0 \end{cases} \tag{2}$$

$$k_r = \begin{cases} \theta_e^l\left[1-(1-\theta_e^{1/m})^m\right]^2, & p < 0 \\ 1, \quad p \ge 0 \end{cases}, \tag{3}$$

where θ_s is saturated water content; θ_r is residual water content; and θ_e is effective water content calculated by

$$\theta_e = \begin{cases} \left[1+\left(\alpha\dfrac{-p}{\rho g}\right)^n\right]^{-m}, & p < 0 \\ 1, \quad p \ge 0 \end{cases}, \tag{4}$$

a, n, m=1-1/*n, l* are Van Genuchten parameters specific for each type of soil.

Equation (1) with soil properties defined by (2)-(4) is solved with boundary conditions of two possible types: pressure specified at the boundary *S1* or normal flow velocity specified at the boundary *S2*:

$$p\big|_{S1} = p_s, \quad V_n\big|_{S_2} = \underline{n}\cdot[-K_S k_r \nabla(p+\rho g y)], \tag{5}$$

where $\underline{n}$ is a vector normal to the boundary surface. On impermeable walls V_n=0; for rainfall infiltration $V_n = V_n(t)$.

3 Model implementation

3.1. Computational domain and boundary conditions

Simulation domain represents a cross-section of the dike (Figure 2a). Boundary conditions for the flow sub-model are the following:

At the sea side of the dike (black line on the left of Figure 2a), sea tides are modeled by a harmonic function for pressure dynamics:

$$p = \rho g[h_{amp} \cdot \sin(\omega t) - y], \tag{6}$$

where $h_{amp} = 1.5$ m is the amplitude of tidal oscillation of sea water level; $\omega = 2\pi / T$ is radial frequency of the tidal cycle; tidal period $T = 12$ hr 25 min; y [m] is a vertical coordinate (y=0 is a reference water level).

At the land side of the dike (cyan line on the right of Figure 2a) water level stays at the "NAP" level, a reference sea level in the Netherlands:

$$p = -\rho g y \, . \tag{7}$$

Zero flux boundary condition $\partial p / \partial n = 0$ is imposed at the remaining boundaries (magenta lines in Figure 2a).

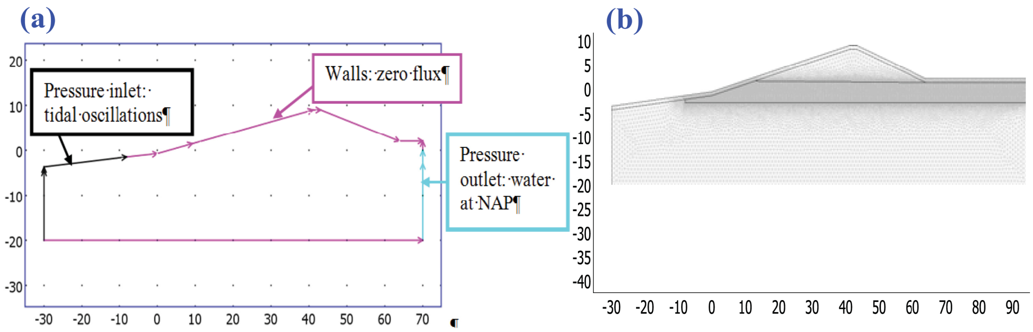


Figure 2. (a) Simulation domain and boundary conditions. "NAP" is a reference sea level in the Netherlands (corresponds to y=0).
(b) Finite element mesh with refinement area around the phreatic line.

3.2. Computational mesh and implementation details

Partial differential equations (1) are solved by the finite element method: the computational domain is discretized into finite elements and original equations are reduced to a system of ordinary differential equations solved by implicit time integration scheme.

We used the six-node triangular finite elements with second-order approximation. The mesh was refined in the zone around the phreatic line (see Figure 2b), where flow parameter gradients can be very high. Convergence of the finite element solution on a number of meshes with increasing density and higher order of approximation has been tested. The coarsest mesh that provided the error of less than 0.01% was chosen: 15000 elements of second order approximation.

Computational time strongly depends on the amount of non-linearities in the model. The van Genuchten model for unsaturated soil is highly non-linear, and this fact significantly affects the rate of convergence for load regimes with zone saturation and drainage. For a period of 1 day (24 hours), porous flow simulation takes up to 1 hour. Convergence rate can be increased by choosing a less non-linear approximation of water retention curve. Structural sub-model takes about 10 minutes to simulate tidal oscillations of 1 day physical time. Memory demand does not exceed 1 Gb for a 2D problem.

4 Simulation results

4.1. Sensitivity analysis

Sensitivity analysis has been performed to study the influence of soil permeability on pore pressure dynamics. Two main parameters can be identified to characterize the dike response to the tidal water load:

1. amplitude of pore pressure oscillations and
2. phase shift between the tidal load and pore pressure oscillations.

2D homogeneous computational domain has been considered. Geometric prototype of the domain is a LiveDike cross-section. The boundary conditions have been described in Section 3: harmonic tidal pressure head (eq. 6) is applied at the seaside; and constant pressure head (eq. 7) is applied at the landside.

A number of porous flow simulations have been performed, with values of saturated permeability K_s in the range of 10^{-7}-10^{-11} m^2. Storage coefficient S was constant and equal to 10^{-7} Pa^{-1}. Distribution of pore pressure amplitudes in a horizontal slice of the dike (at the level $y = -6$ m) is presented in Figure 3a.

For relatively high values of permeability (K = $10^{-7} \div 10^{-9}$ m^2) pressure amplitude is linear (like in 1D analytical models), with a very small non-linear tail close to the sea-side (left) slope. Non-linear part corresponds to the zone where the flow is

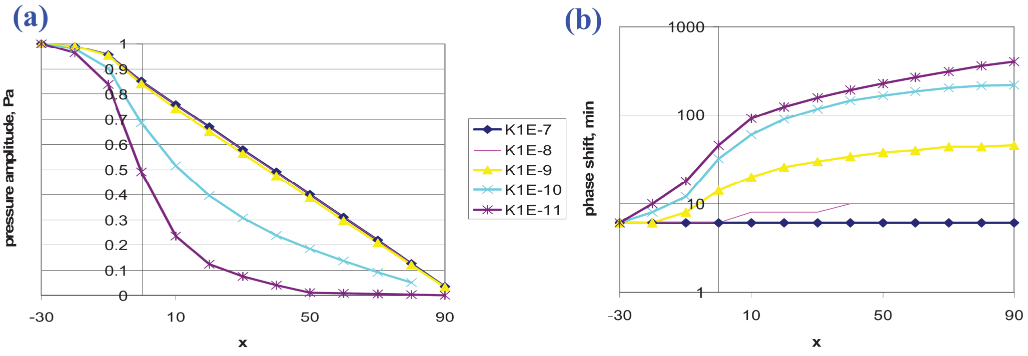


Figure 3. (a) pressure amplitude distribution along the dike.
(b) phase shift distribution along the dike (in logarithmic scale).
Data shown in a horizontal slice $y = -6$ m.

essentially 2D and can not be considered one-dimensional: at x<=10 m, water inlet is located both at the vertical boundary and on the under-water slope of the dike (see Figure 2a). At x>=10 m the flow can be treated as one-dimensional, and pressure amplitude distribution corresponds to the analytical solution (not shown in the paper due to the page limit).

For permeability $K=10^{-10}$ m^2 and lower, significant non-linearity appears. The analytical model also predicts non-linear profiles of pressure distribution, however the absolute values of pore pressure do not agree with the finite element simulation. For example at $K=10^{-10}$ m^2 analytical pressure amplitude in point x=50 m is 0.5 Pa, while simulated amplitude is 0.2 Pa. Real pore pressure amplitude measured by the sensor "E4" located at x=50 m (see Figure 5) is 0.25 Pa. This corresponds to the permeability value $K = 3\cdot10^{-10}$ m^2 (Figure 4a).

Figure 3a clearly shows that pressure amplitude for highly permeable media (like (coarse sands) is insensitive to the actual value of permeability (lines for $K=10^{-7}$, $K=10^{-8}$ $K=10^{-9}$ coincide). The distribution is defined only by the boundary conditions. To the contrary, the phase shift is sensitive to the value of permeability (Figure 3b), therefore phase shift can be calibrated by choosing appropriate *Ks* to match sensor data: Phase shift between the tide and real sensor "E4" is 30 min, which corresponds to the permeability value $Ks = 10^{-9}$ m^2 (see Figure 4b). Calibration of the LiveDike soil parameters based on this sensitivity analysis is described in the following subsection.

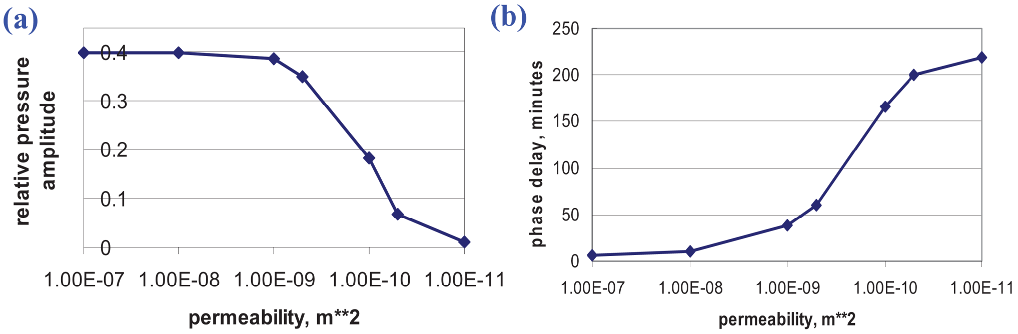
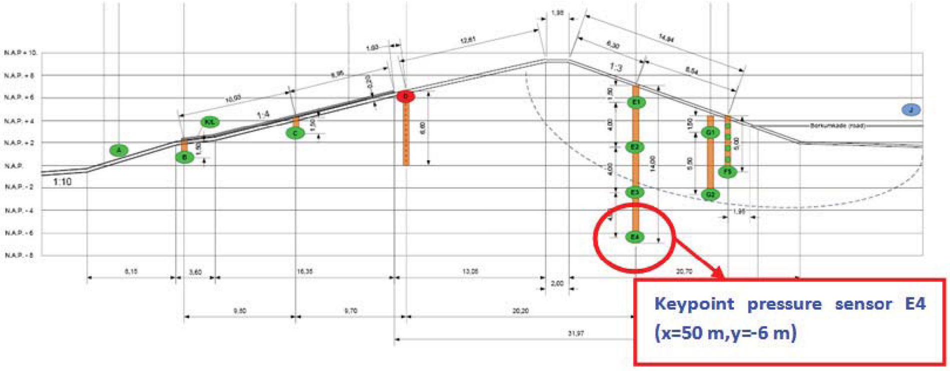


Figure 4. Influence of soil permeability on (a) pressure amplitude and (b) phase shift in sensor location (50m; -6m)

4.2. Calibration of soil properties

The most informative sensor in the LiveDike is "E4" located in saturated zone (coordinates x=50 m, y=-6 m), see Figure 5, Figure 6. In addition, there are two more pressure sensors located in saturated zone: "E3" (also at x=50 m) and "G2" (x=72 m). E3 pressure oscillations are in phase with E4, with relative pressure amplitude 20% from tidal amplitude. G2 relative pressure amplitude is only 7%

and that indicates that G2 is installed in a zone with low permeability. The distance between G2 and E3 is only 12 m, but pressure amplitude drops significantly. G2 sensor data allows to locate the outlet vertical boundary of simulation domain close to G2, and to specify a constant water level at zero meters from average sea level.



Keypoint pressure sensor E4 (x=50 m,y=-6 m)

Figure 5. Cross-section of the LiveDike with sensors shown

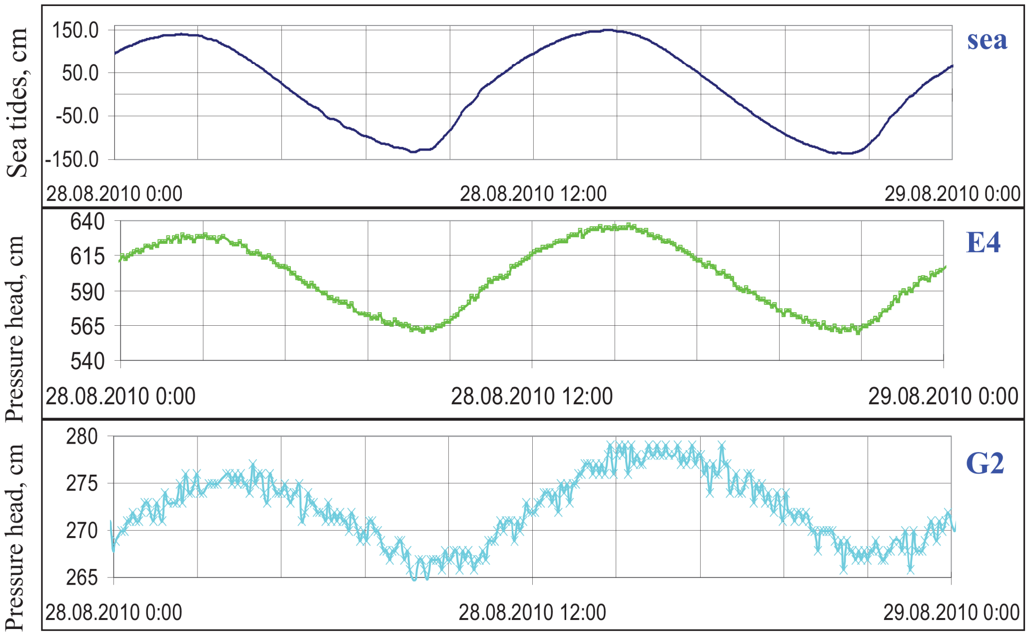


Figure 6. Top: Tidal sea level measured in LiveDike.
Middle: pore pressure dynamics from sensor "E4" (x=50 m).
Bottom: pore pressure dynamics from sensor "G2" (x=72 m).

Pressure amplitude in E4 sensor has been calibrated by tuning appropriate location of the outlet boundary (x=70 m), where specified water table is zero meters from reference level. Phase shift for E4 has been calibrated by choosing *Ks*=$0.9\cdot10^{-9}$ m^2 and *S*=10^{-7} Pa^{-1} in the zone x<=50 m between the sea and the sensor. The simulation results then perfectly match real sensor data, both for the "training" period shown in Figure 7a and for a long-term simulation shown in Figure 7b. For x>50 m soil parameters were found to be *Ks*=10^{-12} m^2, *S*=10^{-7} Pa^{-1} to provide low pressure amplitude in sensor G2.

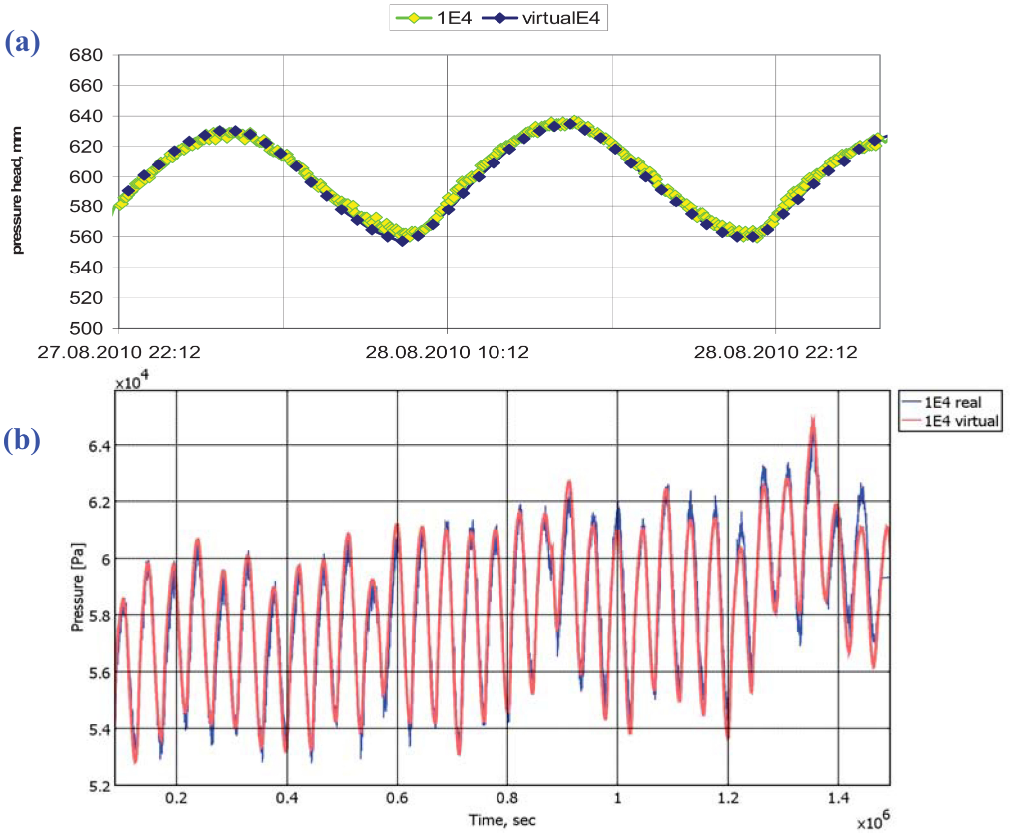


Figure 7. Pore pressure dynamics in sensor E4 with calibrated soil properties. (a) comparison of real sensor data (light green) with simulation results (black) for a "training period"; (b) the same, for a period of 16 days.

5 Conclusions and future work

A mathematical model of earthen dike behavior under dynamic hydraulic load has been developed. Transient flow through porous media was modeled by Richards equation with Van Genuchten model for water retention in partially saturated soil.

Sensitivity analysis of porous flow dynamics to soil permeability showed that:

1. Distribution of pore pressure amplitudes across the dike (in horizontal direction from the sea) is close to linear for highly permeable soils (like coarse sands) and is significantly non-linear for non-permeable soils, such as clays.
2. Pressure amplitude for coarse media ($Ks \sim 10^{-9} \div 10^{-7}$ m^2 and $S=10^{-7}$ Pa^{-1}) is insensitive to the value of permeability, and is defined only by boundary conditions.
3. The phase shift is always sensitive to the value of permeability and can be calibrated by choosing appropriate saturated permeability to match sensor data.
4. The results of analytical one-dimensional model qualitatively agree to the finite element simulation results, but the actual parameter values differ.

Soil properties have been calibrated based on the sensor data from a real sea dike. Simulation results with calibrated soil parameters match well experimental data, not only on the "training set" but also for a much longer period of time.

This calibrated and validated model is now being integrated in the UrbanFlood early warning system, where the simulation can be run with a real-time input from water level sensors or with predicted high water levels due to the upcoming storm surge or river flood. In the first case, comparison of simulated pore pressures with real data can indicate a change in soil properties or in dike operational conditions (e.g. failure of a drainage pump). In the second case, simulation can predict the structural stability of the dike and indicate the "weak" spots in the dikes that require attention of dike managers and city authorities.

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