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Feshbach resonances in 40K

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Appendix B

Hyperfine structure

The fine structure of an alkali atom is determined by the coupling of the outer electron's spin **S** with its orbital angular momentum **L** to the total angular momentum of the electron†

$$\mathbf{J} = \mathbf{S} + \mathbf{L}.$$

The **L**–**S** coupling leads in alkalis to the D1 and D2 line with $J = 1/2$ and $J = 3/2$ respectively. For ^{40}K the fine structure splitting is 1.7 THz (see Tables A.1 and A.2), all additional perturbations due to the hyperfine interaction and external magnetic fields can be treated separately for each **J** when they are small compared to the fine structure splitting. The interaction between the angular momentum of the nucleus **I** and the electron **J** couples to the total angular momentum of the atom

$$\mathbf{F} = \mathbf{I} + \mathbf{J}$$

and results in the hyperfine splitting. All the angular momentum operators $\mathbf{F}, \mathbf{I}, \mathbf{J}$ have corresponding quantum numbers F, I, J which obey the triangular relation

$$|I - J| \leq F \leq I + J,$$

†In atoms with more than one valence electron the coupling can differ from the described **L**–**S** coupling. In that case **j**–**j** coupling occurs or mixtures of both **j**–**j** and **L**–**S** coupling, depending on the energy scales.

Property	**Symbol**	**Value**	**Ref.**
Mass	m	39.96399848(21) u	[NIS10]
Nuclear spin	I	4	
Number of Neutrons	N	21	
Atomic number	Z	19	
Natural abundance		0.000117(1) %	[NIS10]
Isotope lifetime	$\tau_{40\mathrm{K}}$	1.248×10^9 y	[NND11]

Table B.1: Physical properties of ^{40}K. The mass is given in unified atomic mass units ($1\,\mathrm{u} = 1.660538921 \times 10^{-27}\,\mathrm{kg}$).

State	Property	Symbol	Value [MHz]	Ref.
$^2S_{1/2}$	magnetic dipole constant	$a_{\rm hf}$	$h\times -285.7308(24)$	[Ari77]
$^2P_{1/2}$	magnetic dipole constant	$a_{\rm hf}$	$h\times -34.523(25)$	[Fal06]
$^2P_{3/2}$	magnetic dipole constant	$a_{\rm hf}$	$h\times -7.585(10)$	[Fal06]
$^2P_{3/2}$	electric quadrupole constant	$b_{\rm hf}$	$h\times -3.445(90)$	[Fal06]

Table B.2: Hyperfine structure coefficients for ^{40}K.

so there are $(2J+1)$ possible values for F when $J < I$. The angular momentum operators obey the relation

$$\mathbf{I}\cdot\mathbf{J} = \frac{1}{2}(\mathbf{F}^2 - \mathbf{I}^2 - \mathbf{J}^2).$$

The hyperfine interaction is described by the Hamiltonian

$$\mathbf{H}_{\rm hf} = \frac{1}{\hbar^2}\left(a_{\rm hf}\mathbf{I}\cdot\mathbf{J} + b_{\rm hf}\frac{3(\mathbf{I}\cdot\mathbf{J})^2 + \frac{3}{2}\mathbf{I}\cdot\mathbf{J} - \mathbf{I}^2\mathbf{J}^2}{2I(2I-1)J(2J-1)}\right), \tag{B.1}$$

using the magnetic dipole constant $a_{\rm hf}$ and the electric quadrupole constant $b_{\rm hf}$. The quadrupole term only exists for states with $J > 1/2$, as derived in [MK85]. The values for $a_{\rm hf}$ and $b_{\rm hf}$ are shown in Table B.2.

B.1 Hyperfine splitting with an external magnetic field

The hyperfine interaction in presence of an external magnetic field B is described by

$$\mathbf{H}^B_{\rm hf} = \mathbf{H}_{\rm hf} + \mathbf{H}_{\rm Z}, \tag{B.2}$$

where $\mathbf{H}_{\rm Z}$ is the Zeeman interaction

$$\mathbf{H}_{\rm Z} = \frac{\mu_{\rm B}}{\hbar}(g_J\mathbf{J} + g_I\mathbf{I})\cdot\mathbf{B}, \tag{B.3}$$

with the Landé g-factor of the electron g_J, the gyromagnetic factor of the nucleus g_I and the Bohr magneton $\mu_{\rm B}$. Here the sign convention is[‡]:

$$\boldsymbol{\mu}_I = -g_I\mu_{\rm B}\frac{\mathbf{I}}{\hbar} \qquad \text{and} \qquad \boldsymbol{\mu}_J = -g_J\mu_{\rm B}\frac{\mathbf{J}}{\hbar}. \tag{B.4}$$

The values for the g-factors are in Table B.3. The level structure of the hyperfine states for ^{40}K is shown for the ground state $|^2S_{1/2}\rangle$ in Fig. B.1 and for the excited state $|^2P_{3/2}\rangle$ in Fig. B.2.

In practice we solve the field dependence and energy splitting of the hyperfine states numerically, however in the special case of $F = I \pm 1/2$ ($J = 1/2$) the Breit-Rabi formula [Bre31, Oh08] provides an analytical expression for the eigenvalues of $\mathbf{H}^B_{\rm hf}$ for the Zeeman states with quantum number m_F:

‡The sign convention for $\boldsymbol{\mu}_I$ is chosen as in [Ari77].

State	Property	Symbol	Value	Ref.
All states	total nuclear g-factor	g_I	0.000176490(34)	[Ari77][‡]
$^2S_{1/2}$	total electronic g-factor	g_J	2.00229421 (24)	[Ari77]
$^2P_{1/2}$	total electronic g-factor	g_J	2/3	
$^2P_{3/2}$	total electronic g-factor	g_J	4/3	

Table B.3: Electronic and gyromagnetic factors for ^{40}K.

$$E(F = I \pm 1/2, m_F) = -\frac{a_{\mathrm{hf}}}{4} + m_F g_I \mu_{\mathrm{B}} B \pm \frac{\Delta E_{\mathrm{hf}}}{2}\sqrt{1 + \frac{4m_F}{2I+1}x + x^2} \tag{B.5}$$

using the abbreviation

$$x = \frac{(g_J - g_I)\mu_{\mathrm{B}}}{\Delta E_{\mathrm{hf}}} B$$

and the hyperfine splitting energy

$$\Delta E_{\mathrm{hf}} = a_{\mathrm{hf}}\left(I + \frac{1}{2}\right).$$

We employ this analytical expression for the calibration of the magnetic field described in Sec. 4.5.

B.2 Limit of high and low magnetic fields

For low magnetic fields B the **I**–**J** coupling is valid and the total angular momentum **F** precesses around the direction of the magnetic field. The hyperfine energy for states with $J = 1/2$ is then well described by the linear Zeeman effect:

$$E_{\mathrm{hf}}^{B,\mathrm{low}} = m_F g_F \mu_B B + \Delta E_{\mathrm{hf}}^0 \tag{B.6}$$

using the hyperfine splitting at zero field

$$\Delta E_{\mathrm{hf}}^0 = \frac{a_{\mathrm{hf}}}{2}[F(F+1) - I(I+1) - J(J+1)]$$

and

$$g_F = g_J \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)} + g_I \frac{F(F+1) + I(I+1) - J(J+1)}{2F(F+1)}.$$

The values for g_F for the different manifolds are shown in Table B.4. In high magnetic field the **I**–**J** coupling is lifted and both angular momenta precess independently around the direction of the magnetic field. In this so called Paschen-Back regime, the hyperfine energy of a state with quantum number m_I, m_J and $J = 1/2$ is approximated by:

$$\Delta E_{\mathrm{hf}}^{B,\mathrm{high}} = m_J g_J \mu_B B + a_{\mathrm{hf}} m_I m_J. \tag{B.7}$$

State	Value
$^2S_{1/2}, F = 9/2$	0.222634
$^2S_{1/2}, F = 7/2$	−0.222281
$^2P_{1/2}, F = 9/2$	0.074231
$^2P_{1/2}, F = 7/2$	−0.073878
$^2P_{3/2}, F = 11/2$	0.363765
$^2P_{3/2}, F = 9/2$	0.229102
$^2P_{3/2}, F = 7/2$	−0.020985
$^2P_{3/2}, F = 5/2$	−0.571176

Table B.4: Landé g_F factors for ^{40}K.

The hyperfine field B_{hf} is a characteristic crossover field. It is defined as the magnetic field where the energy of the states in the low-field approximation equals the energy in the high-field approximation. For $J = 1/2$ it is [Leg01]

$$B_{\text{hf}} = \frac{a_{\text{hf}}(I + 1/2)}{(g_J - g_I)\mu_{\text{B}}} \approx \frac{a_{\text{hf}}(I + 1/2)}{2\mu_{\text{B}}}.$$

The hyperfine field for the ground state manifold $^2S_{1/2}$ of ^{40}K is $B_{\text{hf}} = 459\,\text{G}$. The low-field approximation is valid to describe the cold atoms in the MOT and the magnetic trap, as the magnetic fields used are much lower than B_{hf}.

B.3 Magnetic trapping potential

Neutral atoms are trapped magnetically due to the Zeeman effect: an applied magnetic field $\mathbf{B}$ shifts the eigenenergies of an atom proportionally to the magnetic field value $|B|$. The applied field results in a magnetic moment μ which is aligned with the external field. The magnetic potential is

$$U(B) = -\mu \cdot \mathbf{B} = m_F g_F \mu_B B, \tag{B.8}$$

States where $m_F g_F > 0$ are trapped, and states where $m_F g_F < 0$ are expelled from a magnetic gradient as described in Sec. 3.5.1. The exact trapping potential depends on the geometry of the magnetic field, a more detailed discussion about magnetic trapping and trap geometries can be found in [Ber87, Ket92, Met99, Ket99, For07].

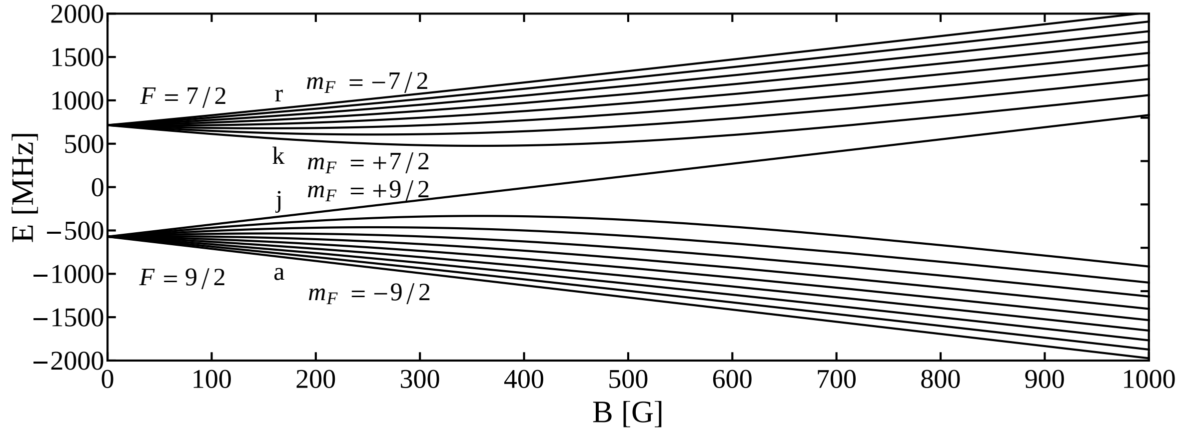

Figure B.1: The hyperfine structure of the ground state $|^2S_{1/2}\rangle$ of ^{40}K. The states are labelled with the low field quantum numbers $|F, m_F\rangle$ and with a to r with rising energy. In the lower hyperfine manifold (F = 9/2), the states f to j are low-field seeking at low magnetic field. In the upper hyperfine manifold (F = 7/2) the states o to r are low-field seeking. The hyperfine structure is inverted unlike in most other alkalis.

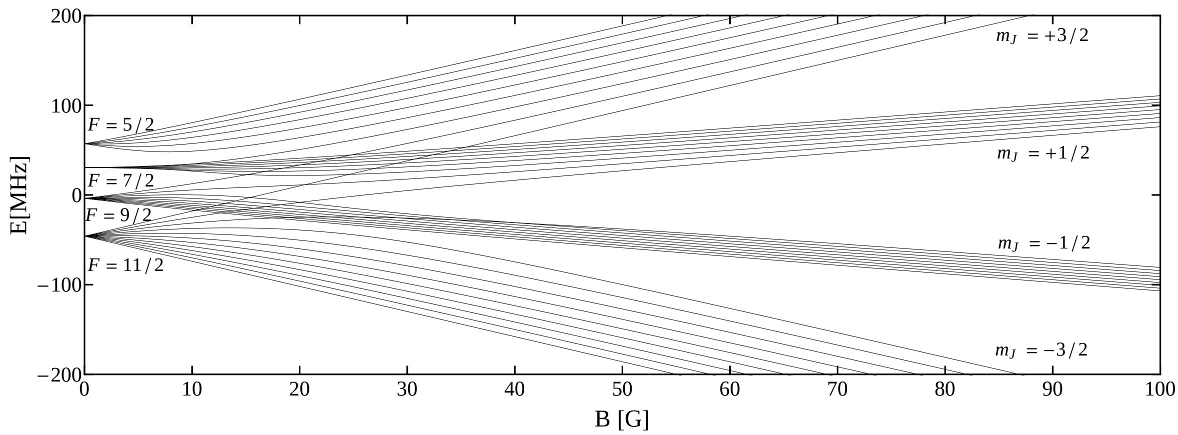

Figure B.2: The hyperfine splitting of the excited state $|^2P_{3/2}\rangle$. The states are labelled with the high-field quantum numbers.