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DECOMPOSING GENERALIZED QUANTIFIERS

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Abstract. This note explains the circumstances under which a type $\langle 1 \rangle$ quantifier can be *decomposed* into a type $\langle 1, 1 \rangle$ quantifier and a set, by fixing the first argument of the former to the latter. The motivation comes from the semantics of Noun Phrases (also called Determiner Phrases) in natural languages, but in this article, I focus on the logical facts. However, my examples are taken among quantifiers appearing in natural languages, and at the end, I sketch two more principled linguistic applications.

1. Introduction. The motivation for the results in this note comes from linguistics: Determiners and other phrases with a similar function in natural languages denote type $\langle 1, 1 \rangle$ (generalized) quantifiers, and noun phrases (NPs or DPs) denote type $\langle 1 \rangle$ quantifiers, often but not always obtained by restricting or *freezing* the noun argument of a $\langle 1, 1 \rangle$ quantifier. I study the ‘inverse’ of freezing: *decomposition* of a type $\langle 1 \rangle$ quantifier (when possible) by means of a set and a type $\langle 1, 1 \rangle$ quantifier.

In this article, I focus on the logical facts: characterizations of classes of decomposable quantifiers are provided and questions of uniqueness of the decomposition addressed. The results are not difficult, but they yield a fairly clear picture of the situation and can be directly applied to certain issues in semantics. Two such applications—to reciprocals and to possessives—are sketched in the final section; the latter are dealt with extensively in Peters & Westerståhl (in preparation).

Section 2 gives background on (generalized) quantifiers and their properties, section 3 provides the relevant notions of freezing and decomposition, section 4 presents and proves the results, and section 5 presents linguistic applications. Some observations on decomposition and freezing were made in Peters & Westerståhl (2006, chap. 4.5.5); here, they are generalized and put in a coherent picture.

2. Background on quantifiers. To make the presentation more self-contained, central notions and definitions are provided in this section. Notation is as in Peters & Westerståhl (2006), where (much) more background information on quantifiers in language and logic can be found.

2.1. Quantifiers of types $\langle 1, 1 \rangle$ and $\langle 1 \rangle$. Except for one example in section 5, we only deal with quantifiers of these two types.

DEFINITION 2.1. A (generalized) quantifier of type $\langle 1, 1 \rangle$ Q associates with each universe M a binary relation Q_M between subsets of M . In the type $\langle 1 \rangle$ case, Q_M is a unary relation.

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Logicians are more used to see Q a class of structures. This is essentially just a notational difference; in the type $\langle 1, 1 \rangle$ case:

$$(M, A, B) \in Q \iff Q_M(A, B).$$

Type $\langle 1, 1 \rangle$ quantifiers interpreting English determiner phrases are conveniently named by those phrases, as in the examples below. For all M and all $A, B \subseteq M$,

$$all_M(A, B) \iff A \subseteq B$$

$$(all_{ei})_M(A, B) \iff \emptyset \neq A \subseteq B$$

$$no_M(A, B) \iff A \cap B = \emptyset$$

$$at\ least\ two_M(A, B) \iff |A \cap B| \geq 2$$

$$exactly\ five_M(A, B) \iff |A \cap B| = 5$$

$$all\ but\ three_M(A, B) \iff |A - B| = 3$$

$$more\ than\ two\ thirds\ of\ the_M(A, B) \iff |A \cap B| > 2/3 \cdot |A|$$

$$most_M(A, B) \iff |A \cap B| > |A - B|$$

$$the\ ten_M(A, B) \iff |A| = 10 \text{ and } A \subseteq B$$

$$John's_M(A, B) \iff \emptyset \neq A \cap \{a : \text{John 'possesses' } a\} \subseteq B$$

$$no\ except\ John_M(A, B) \iff A \cap B = \{j\}$$

$$infinitely\ many_M(A, B) \iff A \cap B \text{ is infinite}$$

$$an\ even\ number\ of_M(A, B) \iff |A \cap B| \text{ is even.}$$

all_{ei} is 'all with existential import': It is the quantifier that Aristotle (as well as many modern linguists, but few modern logicians) took 'every' to mean. Of course, lots of type $\langle 1, 1 \rangle$ quantifiers are much more remotely related to natural language:

$$sqrt_M(A, B) \iff |A \cap B| > \sqrt{|A|}$$

$$I_M(A, B) \iff |A| = |B| \quad (\text{the H\"artig quantifier}).$$

As to type $\langle 1 \rangle$ quantifiers, all but the last two of the following are familiar from logic:

$$\forall_M(B) \iff B = M$$

$$\exists_M(B) \iff B \neq \emptyset$$

$$(\exists_{=n})_M(B) \iff |B| = n$$

$$(I_a)_M(B) \iff a \in B \quad (\text{a Montagovian individual})$$

$$Q_M^R(B) \iff |B| > |M - B| \quad (\text{the Rescher quantifier})$$

$$(Q_0)_M(B) \iff B \text{ is infinite}$$

$$(C^{pl,u})_M(B) \iff \emptyset \neq C \subseteq B$$

$$(C^{pl,e})_M(B) \iff C \cap B \neq \emptyset.$$

For each M , $(I_a)_M$ is the principal filter on $P(M)$ generated by a ; it is also what Montague used to interpret *proper names*, so that the sentences:

(1) John smokes.

(2) Some students smoke.

are interpreted on the same principle, as:

$$[[NP]]_M([[smoke]]),$$

where M is the discourse universe, and $[[NP]] = some^{\llbracket student \rrbracket}$ (defined in section 3) in (2) and $= I_j$ in (1).

The quantifiers $C^{pl,u}$ and $C^{pl,e}$ can be used to obtain the universal and existential interpretation of *bare plurals* along the same lines, as in, respectively,

- (3) Firemen are brave.
- (4) Firemen are available.

It should be emphasized that quantifiers are *global*: With each universe M , they associate a second-order relation (a local quantifier) over M . This is faithful to the meaning of the corresponding phrases in natural languages. To know what ‘some’ means is not to know different things for different universes of discourse; it is to know just one thing, namely, the quantifier *some*.¹ In the case of Montagovian individuals, this means that if $j \notin M$, $(I_j)_M(B)$ is always false, which seems perfectly reasonable.

2.2. Basic properties of quantifiers. A crucial feature of quantifiers involved in natural language semantics, and in logic too for that matter, is that they *mean the same thing on every universe*. The following property comes very close to capturing this intuitive idea.

DEFINITION 2.2. Q satisfies extension (EXT) iff whenever $A, B \subseteq M \subseteq M'$, we have $Q_M(A, B) \Leftrightarrow Q_{M'}(A, B)$. Similarly for the type $\langle 1 \rangle$ case; EXT makes sense for quantifiers of any type.

Thus, extending the universe (but not the arguments of the quantifier) has no effect. It is easy to define quantifiers that mean different things on different universes (e.g., one meaning *some* on universes with fewer than 10 elements, and *all* on larger universes), but these do not appear in natural language semantics. All type $\langle 1, 1 \rangle$ quantifiers mentioned above and all type $\langle 1 \rangle$ quantifiers, except \forall and Q^R , are EXT. In general, it seems that all type $\langle 1 \rangle$ natural language quantifiers are EXT, except some of those that explicitly refer to the universe, for example, by means of the predicate *thing*: *John, firemen, three cats, all but five students, some things, at most three things* are all EXT, whereas *everything, most things* are not.

For EXT quantifiers, mention of the universe M can be omitted; this is done whenever possible below.

DEFINITION 2.3. Q is closed under isomorphism (ISOM) iff the corresponding class of structures is closed under isomorphism. In the type $\langle 1, 1 \rangle$ case, this is equivalent to the requirement that $Q_M(A, B)$ only depends on the cardinals of $A - B$, $A \cap B$, $B - A$, and $M - (A \cup B)$.

Many but not all natural language quantifiers are ISOM; the exceptions mentioned above are those explicitly mentioning some individual or property, like *Mary, all except John, every student's*, and so forth. For logicians, it is natural to treat these individuals or properties as arguments too. For example, one could think of the quantifier at work in *every student's* as having type $\langle 1, 1, 1 \rangle$: *every C's A is B*. Similarly, one could think of *Mary's* as taking two sets and one individual as arguments. These quantifiers—we could call them *fully abstracted*—would be ISOM, but the problem is that they have the wrong category. *Every student's* and *Mary's* are in important ways similar to *some* or *most*: They take a noun argument to form an NP, just like other determiners, so from a linguistic point of

¹ In the present context, ‘meaning’ stands for extension; intensions, possible worlds, and so forth, will not be relevant.

view, they should have type $\langle 1, 1 \rangle$. Similarly, NPs formed by a determiner and a noun should have type $\langle 1 \rangle$, not type $\langle 1, 1 \rangle$.

The natural way to deal with this issue is to have a *background model* \mathcal{M}_0 fix the interpretation of the relevant proper names or nouns. Provided the fully abstracted quantifiers are EXT, there is a *unique* way to treat expressions like *every student's*, *Mary's*, *John*, *three cats*, and so forth, as quantifiers of the desired types ($\langle 1, 1 \rangle$ or $\langle 1 \rangle$), relative to \mathcal{M}_0 (Peters & Westerståhl, 2006, chap. 3.5). Naturally, however, ISOM is lost.

ISOM and EXT apply to arbitrary quantifiers. The next property, on the other hand, concerns only quantifiers having a *restriction argument* (usually a set argument), in particular, those of type $\langle 1, 1 \rangle$.

DEFINITION 2.4. A type $\langle 1, 1 \rangle$ quantifier Q is conservative (CONSERV) iff for all M and all $A, B \subseteq M$, $Q_M(A, B) \Leftrightarrow Q_M(A, A \cap B)$.

All determiner interpretations are CONSERV and EXT. This fact is responsible for much of the special behavior of natural language quantification, and in particular for all the results in the present note. For such quantifiers, $Q_M(A, B)$ depends only on the sets $A - B$ and $A \cap B$. If Q in addition is ISOM, only the cardinality of those two sets matter.

Boolean operations on quantifiers (of the same type) are defined in the obvious way. In addition to ordinary negation, sometimes called *outer negation*, there is also an *inner negation* for quantifiers of these types; define, in the type $\langle 1 \rangle$ case, the quantifier $Q\neg$ by:

$$(Q\neg)_M(B) \Leftrightarrow Q_M(M - B).$$

In the type $\langle 1, 1 \rangle$ case, we replace the second argument by its complement.

Finally, we shall need suitable notions of *triviality*.

DEFINITION 2.5. A local quantifier Q_M is trivial iff it holds either of all subsets of M or of no such subsets. Q is trivial iff each Q_M is trivial.

Below, we sometimes restrict attention to *finite* universes; this is indicated by writing FIN.

3. Freezing and decomposition.

3.1. Freezing. The first argument of a type $\langle 1, 1 \rangle$ quantifier denoted by a determiner is called the *noun* or *restriction argument*, and the idea of freezing is simply to fix that argument to some given set C . However, the result should be a global type $\langle 1 \rangle$ quantifier, defined on every universe M , whether M contains C or not. This is done as follows.

DEFINITION 3.1. If Q is any type $\langle 1, 1 \rangle$ quantifier and C any set, define the type $\langle 1 \rangle$ quantifier Q^C , the freezing of Q to C , by:

$$(5) (Q^C)_M(B) \Leftrightarrow Q_{M \cup C}(C, B),$$

for every M and every $B \subseteq M$.

Thus, the universe is expanded, if necessary, so that it includes C . Though the idea of freezing is familiar, the definitions in the literature are often vague as to how to read Q^C on an arbitrary universe. One could, instead, keep M and cut down C to $C \cap M$, or one could simply stipulate that $(Q^C)_M(B)$ is false when $C \not\subseteq M$. Note that all three alternatives are equivalent when $C \subseteq M$. It is argued in Peters & Westerståhl (2006) that (5) is the *correct* definition, mainly because it preserves EXT:

FACT 3.2. If Q is EXT, so is Q^C .

Proof. If $B \subseteq M \subseteq M'$: $(Q^C)_M(B) \Leftrightarrow Q_{CUM}(C, B) \Leftrightarrow Q_{CUM'}(C, B) \text{ (EXT)} \Leftrightarrow (Q^C)_{M'}(B)$. \square

The other definitions suggested above do not preserve EXT. In addition, it can be seen that the facts about freezing and decomposition to be established below, as well as a number of other similar results, depend on defining Q^C in the correct way, that is, as in (5).

3.2. Decomposition.

DEFINITION 3.3.

- (i) A type $\langle 1 \rangle$ quantifier Q is decomposable iff there is a set C and a CONSERV and EXT type $\langle 1, 1 \rangle$ quantifier Q_1 such that $Q = Q_1^C$.
- (ii) If Q_1 is also ISOM, Q is said to be ISOM decomposable.

Since Q_1 is required to be CONSERV and EXT, decomposition is an inverse to freezing in a precise sense explained in Peters & Westerståhl (2006, chap. 4, fact 4). Note that without any requirements on Q_1 , the notion of (ISOM) decomposability would be empty.

FACT 3.4. For every type $\langle 1 \rangle$ quantifier Q , there is a set C and a type $\langle 1, 1 \rangle$ quantifier Q_1 such that $Q = Q_1^C$.

Proof. Define:

$$(Q_1)_M(A, B) \iff A = \emptyset \ \& \ Q_M(B).$$

Then, $(Q_1)_M^\emptyset(B) \Leftrightarrow (Q_1)_M(\emptyset, B) \Leftrightarrow Q_M(B)$, so $Q = (Q_1)_M^\emptyset$. Note that Q_1 is not CONSERV, but it is EXT (respectively, ISOM) if Q is EXT (ISOM). \square

Here are some familiar examples of ISOM decomposable quantifiers:

- (6) $I_a = all^{[a]} = (all_{ei})^{[a]} = (the_{sg})^{[a]} = some^{[a]}$.
- (7) $I_a \wedge I_b = all^{[a,b]} = (all_{ei})^{[a,b]} = (the_{pl})^{[a,b]} \ (a \neq b)$.
- (8) $I_a \vee I_b = some^{[a,b]}$.
- (9) $C^{pl,u} = (all_{ei})^C$.
- (10) $C^{pl,e} = some^C$.

Examples of decomposable but not ISOM decomposable quantifiers and of EXT but not decomposable type $\langle 1 \rangle$ quantifiers are given below.

A decomposable quantifier is only ‘active’ on the set to which the underlying type $\langle 1, 1 \rangle$ quantifier is frozen. The following definition and result make this claim precise.

DEFINITION 3.5. Let Q_1, Q_2 be CONSERV and EXT type $\langle 1, 1 \rangle$ quantifiers.

- (i) Q_1 and Q_2 agree on C iff for all sets B , $Q_1(C, B) \Leftrightarrow Q_2(C, B)$.
- (ii) Q_1 is ISOM on k (where k is any cardinal), iff for all M and $A, B \subseteq M$, and all M' and $A', B' \subseteq M'$, if $|A| = |A'| = k$, $|A \cap B| = |A' \cap B'|$, and $|A - B| = |A' - B'|$, then $(Q_1)_M(A, B) \Leftrightarrow (Q_1)_{M'}(A', B')$.

FACT 3.6. For CONSERV and EXT Q_1 and Q_2 :

- (a) If $Q = Q_1^C$ and Q_1 and Q_2 agree on C , then $Q = Q_2^C$.
- (b) If Q_1 is ISOM, then Q_1 is ISOM on k for every k .
- (c) If $Q = Q_1^C$ where Q_1 is ISOM on $|C|$, then Q is ISOM decomposable.

Proof. Parts (a) and (b) are immediate. As to (c), if Q , Q_1 , and C are as assumed, define Q_3 by:

$$(Q_3)_M(A, B) \iff \begin{cases} (Q_1)_M(A, B) & \text{if } |A| = |C| \\ A \cap B \neq \emptyset \text{ (say)} & \text{otherwise.} \end{cases}$$

Clearly, Q_3 is CONSERV and EXT. Distinguishing the cases $|A| = |C|$ and $|A| \neq |C|$, we see that Q_3 is also ISOM (in the former case, since Q_1 is ISOM on $|C|$, and in the latter case, since Q_3 then amounts to *some*). In addition, Q_1 and Q_3 agree on C , so it follows from part (a) that $Q = Q_3^C$. \square

Fact 3.6 has the following immediate consequence.

COROLLARY 3.7. *Q is ISOM decomposable if and only if $Q = Q_1^C$ for some C and some CONSERV and EXT Q_1 , which is ISOM on $|C|$.*

4. Results.

4.1. Global quantifiers living on sets. We are looking for some property of type $\langle 1 \rangle$ quantifiers, which gives a necessary and sufficient condition for decomposability. By Fact 3.2, one necessary condition is EXT; \forall and the Rescher quantifier (*everything* and *most things*) are not decomposable already because they are not EXT. It turns out that the right place to start is the *live-on* property introduced in Barwise & Cooper (1981). Conservativity is crucial to most facts about global quantifiers in natural language, but Barwise and Cooper, who were among the first to realize its importance, thought of it as a property of *local* quantifiers.

DEFINITION 4.1. *Let C be any set. A local type $\langle 1 \rangle$ quantifier Q_M lives on C iff for all $B \subseteq M$, $Q_M(B) \iff Q_M(C \cap B)$.*

The connection with conservativity (which is immediate if the right notion of freezing has been chosen) is the following.

FACT 4.2. *A type $\langle 1, 1 \rangle$ quantifier Q is CONSERV iff for each set C and each universe M , $(Q^C)_M$ lives on C .*

Here are some easily verified facts about the live-on property (cf. Peters & Westerståhl, 2006, chap. 3, lemma 1):

FACT 4.3.

- (a) Q_M always lives on M , but need not live on any proper subset of M .
- (b) Q_M lives on \emptyset iff it is trivial.
- (c) If Q_M lives on C and D , it lives on $C \cap D$.
- (d) If Q_M lives on C and $C \subseteq D$, Q_M lives on D .

We did not require that $C \subseteq M$ in the definition of ‘ Q_M lives on C ’, but the aforementioned fact shows that Q_M lives on C if and only if Q_M lives on $M \cap C$. Indeed, it shows that:

$$\{X \subseteq M : Q_M \text{ lives on } X\}$$

is a filter on $P(M)$, which is proper iff Q_M is nontrivial. If M is finite, it follows that Q_M lives on:

$$W_{Q_M} = \bigcap \{X \subseteq M : Q_M \text{ lives on } X\},$$

which then generates the filter.

We now extend the live-on notion to global quantifiers as follows.

DEFINITION 4.4. *Let C be a set. A (global) type $\langle 1 \rangle$ quantifier Q lives on C iff for all M , Q_M lives on C .*

Fact 4.3, parts (b)–(d), carry over directly to the extended notion.

FACT 4.5.

- (a) Q lives on \emptyset iff it is trivial.
- (b) If Q lives on C and D , it lives on $C \cap D$.
- (c) If Q lives on C and $C \subseteq D$, Q lives on D .

If Q lives on some set, we let:

$$W_Q = \bigcap \{X : Q \text{ lives on } X\}.$$

FACT 4.6. (FIN) *If Q lives on some set, it lives on W_Q . That is, W_Q is then the smallest set that Q lives on.*

Proof. We are assuming that Q lives on some finite set C . Then, clearly,

$$W_Q = \bigcap \{X \cap C : Q \text{ lives on } X\}.$$

But on the right-hand side, we have an intersection of finitely many sets. Thus, by Fact 4.5, part (b), Q lives on W_Q .² □

The next fact relates W_Q to W_{Q_M} .

FACT 4.7. (FIN) *If Q lives on some set then, for all M , $W_{Q_M} \subseteq W_Q$.*

Proof. It is enough to show that if Q lives on a set C , then for all M , $W_{Q_M} \subseteq C$. But Q_M lives on C by definition and hence on $M \cap C$. And W_{Q_M} is the smallest subset of M that Q_M lives on, so $W_{Q_M} \subseteq M \cap C$. Thus, $W_{Q_M} \subseteq C$. □

4.2. Decomposition and the live-on property. Even though the live-on property for global quantifiers draws on Barwise and Cooper’s local version, it is a very different property. The local version essentially codifies conservativity, and so is to be expected in all natural language contexts. Also, *every* local quantifier lives on some set (its universe). Global quantifiers, on the other hand, ‘normally’ do not live on any sets at all. For example, a nontrivial and ISOM type $\langle 1 \rangle$ quantifier lives on no set (Fact 4.11). In fact, the ones that do live on some set are essentially the decomposable quantifiers, as we now see.

² This fails without the restriction to finite sets. Let Q be *infinitely many numbers*, that is, $Q = \text{infinitely many}^N$, where $N = \{0, 1, 2, \dots\}$. Q lives not only on N but also on $N - \{0\}$, $N - \{0, 1\}$, \dots . So there is no smallest set on which it lives, and W_Q as defined above is empty. Note that if Q lives on some set C , W_Q is well defined: It is a subset of C defined by a first-order set-theoretic formula. The notation W_Q was used in a slightly different sense in Peters & Westerståhl (2006, chap. 4.6), namely, for certain Q such that W_{Q_M} is independent of M (when Q_M is nontrivial). But when this is the case, that notion coincides with the one defined here.

The next lemma is immediate (either directly from definitions or from Facts 4.2 and 4.5, part (c)).

LEMMA 4.8. *If Q is CONSERV, Q^C lives on C and all its supersets.*

We obtain the following characterization.

THEOREM 4.9. *Q is decomposable if and only if it is EXT and lives on some set.*

Proof. Suppose Q is decomposable as Q_1^C . Then, Q is EXT, and by Lemma 4.8, Q lives on C .

In the other direction, suppose Q is EXT and lives on C . Define Q_1 by:

$$(Q_1)_M(A, B) \iff A = C \ \& \ Q_M(A \cap B).$$

Q_1 is CONSERV by definition, and EXT since Q is EXT. We have, for all M and all $B \subseteq M$,

$$\begin{aligned} (Q_1^C)_M(B) &\iff (Q_1)_{M \cup C}(C, B) \\ &\iff Q_{M \cup C}(C \cap B) \quad (\text{by definition}) \\ &\iff Q_M(C \cap B) \quad (\text{by EXT}) \\ &\iff Q_M(B) \quad (\text{since } Q_M \text{ lives on } C). \end{aligned}$$

That is, $Q = Q_1^C$. □

COROLLARY 4.10.

- (a) *If Q is decomposable with C as the underlying set, then for every $D \supseteq C$, Q is decomposable with D as the underlying set.*
- (b) *The class of decomposable type $\langle 1 \rangle$ quantifiers is closed under Boolean operations, including inner negation and dual.*

Proof. Part (a) follows from the theorem and Lemma 4.8. As to part (b), the claims about negations (and hence duals) follow from the easily verified observations that for CONSERV and EXT Q_1 ,

$$(11) \quad \neg(Q_1^C) = (\neg Q_1)^C.$$

and

$$(12) \quad (Q_1^C)\neg = (Q_1\neg)^C.$$

Now suppose $Q = Q_1^{C_1}$ and $Q' = Q_2^{C_2}$, for CONSERV and EXT Q_1 and Q_2 . By Lemma 4.8, both Q and Q' live on $C_1 \cup C_2$. But then, it readily follows that $Q \wedge Q'$ and $Q \vee Q'$ also live on $C_1 \cup C_2$. Since they are also EXT, they are decomposable, by the theorem. □

We can use the easy direction of the characterization to show that various type $\langle 1 \rangle$ quantifiers, including many denoted by noun phrases in natural languages, are *not* decomposable. To begin, *something* and *nothing* (\exists and $\neg\exists$) are EXT but not decomposable. More generally, the quantifiers *at least n things* ($\exists_{\geq n}$), for $n \geq 1$, and all nontrivial Boolean combinations of these, including inner negations and duals, are not decomposable. These include cases like *everything* (\forall) and *all but three things* ($(\exists_{\geq 3} \wedge \exists_{\leq 3})\neg$), which are not even EXT, and so *a fortiori* not decomposable, but also EXT noun phrase denotations like *between three and five things* ($\exists_{\geq 3} \wedge \exists_{\leq 5}$), not to mention *an even number of things*, and so forth. All this follows from the simple fact that these quantifiers are (nontrivial and) ISOM.

FACT 4.11. *If a type $\langle 1 \rangle$ quantifier is nontrivial and ISOM, it is not decomposable.*

Proof. Suppose Q is nontrivial and ISOM. By Fact 3.2, we may also suppose that Q is EXT.

Case 1: $\neg Q(\emptyset)$. By nontriviality, there is some M and some $B \subseteq M$ such that $Q_M(B)$. By EXT, it follows that $Q_B(B)$. Now suppose that Q lives on some set C . Take B' such that $|B'| = |B|$ and $C \cap B' = \emptyset$. It follows, since Q is ISOM, that $Q_{B'}(B')$ and hence, since $Q_{B'}$ lives on C , that $Q_{B'}(C \cap B')$. But this contradicts the assumption of the case. Thus, Q does not live on any set and is therefore not decomposable.

Case 2: $Q(\emptyset)$. Then, the previous argument shows that $Q' = \neg Q$ is not decomposable. But then, by (11), neither is Q . □

What about EXT but not ISOM quantifiers? Among noun phrase denotations, we find the following. Define, for any set D , the quantifier *only D* by:

$$(13) \text{ (only } D)_M(B) \iff \emptyset \neq B \subseteq D.$$

These are at work in the interpretation of sentences like:

- (14) Only John left the party. ($D = \{j\}$).
- (15) Only firemen wear helmets.

FACT 4.12. *Quantifiers of the form only D are EXT, but not decomposable when $D \neq \emptyset$.*

Proof. That EXT holds is clear from the definition of *only D*. Suppose *only D* were decomposable and hence lived on some set C . Take any $a \notin C \cup D$. Since $(\text{only } D)(D)$ holds ($D \neq \emptyset$), we obtain $(\text{only } D)(C \cap D)$ and therefore $(\text{only } D)(C \cap (D \cup \{a\}))$, so $(\text{only } D)(D \cup \{a\})$, which contradicts (13). □

This underscores the familiar observation in linguistics that, first appearances notwithstanding, *only* is in fact not an English determiner (e.g. Peters & Westerståhl, 2006, p. 139, fn. 15). If it were a determiner, one would expect *only firemen* to be interpreted as a frozen type $\langle 1, 1 \rangle$ quantifier, but Fact 4.12 says it cannot be.

4.3. Characterizing ISOM decomposability. We have seen several examples of ISOM decomposable quantifiers, as well as EXT but nondecomposable quantifiers, but what about quantifiers that are decomposable but not ISOM decomposable? A host of such quantifiers can be identified using the characterization of ISOM decomposability in this subsection.

DEFINITION 4.13. *A type $\langle 1 \rangle$ quantifier Q does not distinguish subsets of C of the same size iff whenever $B_1, B_2 \subseteq C \cap M$ are such that $|B_1| = |B_2|$ and $|C - B_1| = |C - B_2|$, $Q_M(B_1) \iff Q_M(B_2)$.*

THEOREM 4.14. *Q is ISOM decomposable if and only if Q is EXT and lives on some set C such that Q does not distinguish subsets of C of the same size.*

Proof. If $Q = Q_1^C$ for some CONSERV, EXT, and ISOM Q_1 , then Q lives on C , and it is immediate that Q does not distinguish subsets of C of the same size.

In the other direction, suppose Q is EXT and lives on a set C such that Q does not distinguish subsets of C of the same size. Define Q_1 by:

$$(Q_1)_M(A, B) \iff |A| = |B| \ \& \ \exists X \subseteq C (Q_M(X) \ \& \ |X| = |A \cap B| \ \& \ |C - X| = |A - B|).$$

Q_1 is CONSERV by definition, and EXT since Q is EXT. Also, Q_1 is clearly ISOM on $|C|$. Moreover, we have, for any B ,

$$\begin{aligned} Q_1(C, B) &\Leftrightarrow \exists X \subseteq C (Q_M(X) \ \& \ |X| = |C \cap B| \ \& \ |C - X| = |C - B|) \\ &\quad \text{(by definition)} \\ &\Leftrightarrow Q(C \cap B) \quad \text{(by assumption)} \\ &\Leftrightarrow Q(B) \quad \text{(since } Q \text{ lives on } C). \end{aligned}$$

So $Q = Q_1^C$, and Q is ISOM decomposable by Fact 3.6, part (c). □

Consider $Q = I_j \vee (I_m \wedge I_s)$, which interprets the noun phrase *John, or Mary and Sue*, as in:

(16) John, or Mary and Sue, will come to the party.

The atoms are ISOM decomposable, as we have seen, and it follows from Corollary 4.10, part (b), that Q is decomposable. Suppose Q were ISOM decomposable as Q_1^D . We have $Q_1(D, \{j\})$, hence $Q_1(D, D \cap \{j\})$ by CONSERV, so $j \in D$ since $\neg Q_1(D, \emptyset)$. A similar argument shows that either $m \in D$ or $s \in D$. Thus, since $Q(\{j\})$ but neither $Q(\{m\})$ nor $Q(\{s\})$ holds, we have contradicted Theorem 4.14.³

Here is a generalization of this observation.

FACT 4.15. *The quantifier some men or all women is decomposable but not ISOM decomposable. More generally, the quantifier $Q = some^D \vee all^E$ is decomposable but not ISOM decomposable, if $D \neq \emptyset$, $|E| > 1$, and $D \cap E = \emptyset$.*

Proof. (Note that $I_j \vee (I_m \wedge I_s)$ is the special case when $D = \{j\}$ and $E = \{m, s\}$.) The claim about decomposability follows from Corollary 4.10, part (b). Next, by our assumptions, we have for all $a \in D$, $Q(\{a\})$, whereas for all $b \in E$, $\neg Q(\{b\})$. Thus, by Theorem 4.14, it suffices to show that if $Q = Q_1^C$, we have $C \cap D \neq \emptyset$ and $C \cap E \neq \emptyset$. But this follows from the conservativity of Q_1 : Since $Q_1(C, D)$, we have $Q_1(C, C \cap D)$ and hence $C \cap D \neq \emptyset$ since $\neg Q_1(C, \emptyset)$, and similarly for E . □

4.4. Uniqueness. Suppose Q is (ISOM) decomposable. When is the decomposition unique, that is, when can we recover the underlying type $\langle 1, 1 \rangle$ quantifier and the underlying set? The first part of the question is easy. We already saw that in many cases, the underlying quantifier is not unique: $I_a = all^{\{a\}} = (all_{ei})^{\{a\}} = (the_{sg})^{\{a\}} = some^{\{a\}}$, and so forth. In fact, it can *never* be recovered.

FACT 4.16. *For every decomposable quantifier Q , one can find Q_1, Q_2 and C such that $Q_1 \neq Q_2$ and $Q = Q_1^C = Q_2^C$.*

Proof. This is more or less immediate from Fact 3.6: If $Q = Q_1^C$, just let Q_2 differ from Q_1 on some D with $|D| \neq |C|$, but be the same as Q_1 otherwise. □

The second part of the question is more interesting. From Corollary 4.10, part (a), we know that it is only in the case of ISOM decomposability that we have any chance to recover the underlying set.

³ But in this case, the conclusion follows directly from the fact that Q_1 is ISOM: If $m \in D$, $Q_1(D, \{j\})$ implies $Q_1(D, \{m\})$ by ISOM.

LEMMA 4.17. (FIN) *Suppose Q is nontrivial and decomposable as Q_1^C , where Q_1 is ISOM on $|C|$. Suppose further that Q lives on some set D . Then, $C \subseteq D$.*

Proof. Suppose first that $\neg Q(\emptyset)$ holds. Let B_0 be a set of the *smallest size* such that $Q(B_0)$. (Since Q is EXT by assumption, we can leave out the universe as usual.) Such a set exists by nontriviality. By assumption, $B_0 \neq \emptyset$. Moreover, by the conservativity of Q_1 (or the fact that Q_M lives on C), we may assume that $B_0 \subseteq C$.

Suppose $a \in C - D$. Take $B \subseteq C$ such that $a \in B$ and $|B| = |B_0|$. Since Q_1 is ISOM on $|C|$, C is finite, and $Q_1(C, B_0)$ holds, it follows that $Q_1(C, B)$ and thus $Q(B)$. Since Q lives on D , we get $Q(D \cap B)$ and thus $Q(D \cap (B - \{a\}))$ since $a \notin D$. But then, $Q(B - \{a\})$, again because Q lives on D . Since B is finite, this contradicts the assumption that it was of the smallest size such that $Q(B)$.

Thus, $C \subseteq D$. If instead $Q(\emptyset)$ holds, We apply the same reasoning, but this time to $\neg Q$, which also satisfies the assumptions in the lemma. □

THEOREM 4.18. (FIN) *If Q is nontrivial and ISOM decomposable, its underlying set is uniquely determined. More exactly, there is a unique set C such that for some Q_1 which is CONSERV, EXT, and ISOM (on $|C|$), $Q = Q_1^C$.*

Proof. Suppose $Q = Q_1^C = Q_2^D$, where Q_1, Q_2 are CONSERV and EXT, and Q_1 is ISOM on $|C|$, whereas Q_2 is ISOM on $|D|$. By Lemma 4.8, Q lives on D . Thus, by Lemma 4.17, $C \subseteq D$. A symmetric argument shows that $D \subseteq C$, so $C = D$. The corresponding claim of uniqueness when Q_1, Q_2 are ISOM follows by Fact 3.6, part (b). □

Let

$$Un(Q, Y) \iff Q = Q_1^Y \text{ for some CONSERV, EXT, and ISOM } Q_1.$$

By the theorem, assuming FIN, the relation Un is single-valued for nontrivial Q , so we can define a function U from type (1) quantifiers to sets by:

$$U(Q) = \begin{cases} Y & \text{if } Q \text{ is nontrivial and } Un(Q, Y) \\ \emptyset & \text{otherwise.} \end{cases}$$

U , restricted to ISOM decomposable quantifiers, can also be defined in terms of Q alone. Recall the definition of W_Q in section 4.1.

COROLLARY 4.19. (FIN) *If Q is ISOM decomposable, then $U(Q) = W_Q$.*

Proof. If Q is trivial, $U(Q) = W_Q = \emptyset$, by Fact 4.5, part (a). Suppose $Q = Q_1^C$ for some CONSERV, EXT, and ISOM Q_1 , and Q is nontrivial. Then, Q lives on C , and $C = U(Q)$ by the theorem, so, by Fact 4.6, $W_Q \subseteq U(Q)$. Also, since Q lives on W_Q , it follows by Lemma 4.17 that $U(Q) \subseteq W_Q$. So in both cases, $U(Q) = W_Q$. □

Putting together all the above, we obtain a final characterization of ISOM decomposability.

COROLLARY 4.20. (FIN) *Q is ISOM decomposable if and only if Q is EXT, W_Q exists, and Q does not distinguish subsets of W_Q of the same size.*

The restriction to finite universes is essential for the results in this subsection. For example, the quantifier mentioned in footnote 2, $Q = \textit{infinitely many numbers}$, is nontrivial and ISOM decomposable, $W_Q = \emptyset$, Q does not live on any finite set, and for each n , $Q = (\textit{infinitely many})^{N - \{0,1,\dots,n\}}$ (where $N = \{0, 1, 2, \dots\}$).

4.5. Some instructive examples. Suppose $Q = Q_1^C$, where Q_1 is *not* ISOM. Can we conclude anything about the ISOM decomposability of Q ? No: Even if Q is ISOM decomposable, it is equal to Q_1^C for infinitely many C and infinitely many non-ISOM Q_1 . We give some examples to illustrate that natural quantifiers of the form Q_1^C with non-ISOM Q_1 are sometimes ISOM decomposable and sometimes not.

In section 2.1, we defined simple possessive quantifiers like *John's* by:

$$\text{John's}_M(A, B) \iff \emptyset \neq A \cap R_j \subseteq B,$$

(where $R_j = \{b : R(j, b)\}$, and $j = \text{John}$). This is a special case of:

$$Q_2 \text{ of John's}_M(A, B) \iff \emptyset \neq A \cap R_j \ \& \ Q_2(A \cap R_j, B).$$

Compare sentences like:

- (17) John's bikes were (all) stolen.
- (18) Most of Mary's friends are here.
- (19) All but two of Henry's job applications failed.

Here, $Q_2 = \text{all}$, most , and all but two , respectively. None of the possessive type $\langle 1, 1 \rangle$ quantifiers in (17)–(19) is ISOM, but we still have the following.

FACT 4.21. *If Q_2 is CONSERV, EXT, and ISOM, quantifiers of the form $Q = (Q_2 \text{ of John's})^C$ are ISOM decomposable. Moreover (FIN), $W_Q = C \cap R_j$.*

Proof. Q is clearly EXT. We have:

$$(20) \ Q(B) \iff \emptyset \neq C \cap R_j \ \& \ Q_2(C \cap R_j, B).$$

If $C \cap R_j = \emptyset$, then Q is trivial and hence trivially ISOM decomposable, and $W_Q = \emptyset$ (Lemma 4.5, part (a)). If $C \cap R_j \neq \emptyset$, then, by (20), $Q = (Q_2)^{C \cap R_j}$ and hence is ISOM decomposable by the assumption on Q_2 . Also, if FIN holds, we see from Corollary 4.19 that $W_Q = C \cap R_j$. \square

Thus, the smallest set that, say, *John's^{books}* lives on is not the set of books in the discourse universe, but the set of books owned by John (if $R = \text{owns}$) (cf. Corollary 4.20).

What about other frozen possessive quantifiers? I will not try to answer this question in any generality here but only give a few more illustrative examples. Consider the determiner *some student's*. This has (at least) two readings, a *universal* and an *existential* one, illustrated by:

- (21) Some student's tennis rackets were stolen.
- (22) Some student's books were left in the classroom.

A plausible reading of (21) is that some students had all their tennis rackets stolen, whereas (22) can be taken to mean that some, but not necessarily all, of the students' books had been left in the room. The two readings are given by, respectively,

- (23) *(some D's)^u*(A, B) $\iff \exists a \in D (\emptyset \neq A \cap R_a \subseteq B)$.
- (24) *(some D's)^e*(A, B) $\iff \exists a \in D (\emptyset \neq A \cap R_a \cap B)$.

FACT 4.22. *$Q = ((\text{some D's})^e)^C$ is ISOM decomposable, for all D , C , and R .*

Proof. We use Theorem 4.14. Let:

$$(25) \ C_0 = C \cap \bigcup_{a \in D} R_a.$$

It is not hard to verify that:

(26) Q lives on C_0 .

Also, it follows from the definition of $(\text{some } D's)^e$ that:

(27) If $\emptyset \neq B \subseteq C_0$, then $Q(B)$ holds.

This entails that Q does not distinguish subsets of C_0 of the same size. Since Q is clearly EXT, it follows from Theorem 4.14 that Q is ISOM decomposable. \square

Although I will not prove it here, it is a general fact about possessive quantifiers frozen to a set C that they live on the set C_0 defined in (25). In particular, $((\text{some } D's)^u)^C$ does so, and we see that (with $D = \{j\}$) the quantifiers $(Q_2 \text{ of John's})^C$ do so too. In this latter case, we have $W_Q = C_0 = C \cap R_j$, but in general, W_Q can be a proper subset of C_0 . To give an example, suppose $D = \{s_1, s_2\}$, $C = \{b_1, b_2\}$, and $R = \{(s_1, b_1), (s_2, b_1), (s_2, b_2)\}$. Then, $C_0 = C$, and one easily verifies that with $Q = ((\text{some } D's)^u)^C$, we have $\neg Q(\emptyset)$, $Q(\{b_1\})$, $\neg Q(\{b_2\})$, and $Q(\{b_1, b_2\})$. From this, one sees that:

$$W_Q = \{b_1\}.$$

For this particular choice of C, D, R , we also have that $((\text{some } D's)^u)^C$ is ISOM decomposable. This is a consequence of Corollary 4.20.

FACT 4.23. (FIN) *If Q is EXT, and W_Q exists and has at most one element, Q is ISOM decomposable.*

Proof. Then, Q trivially does not distinguish subsets of W_Q of the same size! \square

However, a similar example shows that a slightly different possessive quantifier need not be ISOM decomposable. Consider:

(28) (Exactly) two students' dorm rooms were burglarized.

One plausible interpretation is:

(29) $\text{two } D's(A, B) \iff |D \cap \{a : \emptyset \neq A \cap R_a \subseteq B\}| = 2.$

Let $D = \{s_1, s_2, s_3\}$, $C = \{b_1, b_2\}$, and $R = \{(s_1, b_1), (s_2, b_1), (s_3, b_2)\}$. Again $C_0 = C$, and one sees that with $Q = (\text{two } D's)^C$, we have $\neg Q(\emptyset)$, $Q(\{b_1\})$, $\neg Q(\{b_2\})$, and $\neg Q(\{b_1, b_2\})$. For example, in the last case, the number of D s such that every C they are related to by R belongs to $\{b_1, b_2\}$ is 3, not 2. But then, Q lives neither on $\{b_1\}$ nor on $\{b_2\}$, although it does live on $C_0 = \{b_1, b_2\}$. So $W_Q = \{b_1, b_2\}$, and it follows that Q does distinguish some subsets of W_Q of the same cardinality (namely, $\{b_1\}$ and $\{b_2\}$). We have shown the following.

FACT 4.24. *There are D, C , and R such that $(\text{two } D's)^C$ is not ISOM decomposable.*

We see that the ISOM decomposability (or not) of frozen possessive quantifiers sometimes depends on the choice of the 'possessor' relation. For example, we can make the following observation.⁴

FACT 4.25. *If R is a (partial) function, then for all D and C , $Q = ((\text{some } D's)^u)^C$ is ISOM decomposable.*

⁴ Thanks to Christian Bennet for the suggestion.

Proof. That R is a partial function means that for all a , R_a has at most one element. From this, and the fact that Q is *monotone* ($Q(B)$ and $B \subseteq B'$ implies $Q(B')$), one verifies that, with C_0 as in (25),

$$(30) \text{ for } B \subseteq C_0: Q(B) \iff B \neq \emptyset \ \& \ \forall b \in B \ Q(\{b\}).$$

It follows that for all non-empty subsets B of C_0 we have $Q(B)$. So in particular, Q does not distinguish subsets of C_0 of the same size. Since Q lives on C_0 and is EXT, Q is ISOM decomposable, by Theorem 4.14. \square

However, most ‘possessor’ relations, like real ownership, are not partial functions (one normally owns many different things).

Note finally that although the proof of Fact 4.24 makes use of Corollary 4.20, it identifies a finite set as the smallest set the quantifier lives on. This is independent of the size of the universe; a finite counterexample to ISOM decomposability is given, and this fact, in contrast with positive results of ISOM decomposability using Corollary 4.20, like Fact 4.23, does not rely on FIN. The same holds for our final example.

FACT 4.26. *There are D , C , and R such that $Q = ((\text{some } D's)^u)^C$ is not ISOM decomposable.*

Proof. Let $D = \{s_1, s_2, s_3\}$, $C = \{b_1, b_2, b_3, b_4\}$, and (draw a diagram!) $R = \{(s_1, b_1), (s_2, b_1), (s_2, b_2), (s_3, b_3), (s_3, b_4)\}$. As to subsets of $C_0 = C$, we then have:

- (i) Q holds of $\{b_1\}$ and its supersets and of $\{b_3, b_4\}$ and its supersets.
- (ii) Q does not hold of: \emptyset , $\{b_2\}$, $\{b_3\}$, or $\{b_4\}$, nor of $\{b_2, b_3\}$ or $\{b_2, b_4\}$.

Using the monotonicity of Q , and going through the various cases, one can verify that:

$$(31) \ Q \text{ lives on } \{b_1, b_3, b_4\},$$

and then that

$$(32) \ W_Q = \{b_1, b_3, b_4\}.$$

So Q distinguishes subsets of W_Q of the same size (e.g., $\{b_1\}$ and $\{b_3\}$) and hence is not ISOM decomposable. \square

Observe also that:

$$((\text{some } D's)^u)^C = \bigvee_{a \in D} (\text{all of } a's)^C,$$

so we have another illustration of the fact that ISOM decomposability is not closed under disjunction (cf. Fact 4.15).

5. Applications. Noun phrases abound in most languages, as immediate constituents of other phrases: sentences, verb phrases, prepositional phrases, and so forth. In a compositional semantics, the semantic value of the larger phrase is determined by the values of the immediate constituents (and the ‘mode of composition’). Suppose the value of the NP is of the form Q^C , but calculating the value of the larger phrase requires access to C and/or Q . Then, we have at least a *prima facie* problem for compositionality since the phrases whose values are Q and C are not *immediate* constituents of the larger phrase. Does this situation actually occur? We look at two cases.

5.1. Reciprocals. Consider the sentences⁵:

(33) Most of the pirates stared at each other in surprise.

(34) Most of the boys in my class know each other.

In examples like this, *each other* can be construed as a type $\langle 1, 2 \rangle$ quantifier EO , where $EO(A, R)$ says roughly that all individuals in A ‘ R each other’. But this can mean different things. For example, in (34), we seem to have:

$$EO_1(A, R) \iff \forall a, b \in A (a \neq b \Rightarrow R(a, b)).$$

(33), on the other hand, rather uses EO_2 :

$$EO_2(A, R) \iff \forall a \in A \exists b \in A (a \neq b \& R(a, b)).$$

(It is difficult to stare at more than one person at a time.)

The meaning of the whole sentence can then be obtained as a ‘Ramsey lift’ along the following scheme:

$$(35) \text{Ram}_i(Q)(C, R) \iff \exists X \subseteq C (Q^C(X) \& EO_i(X, R)).$$

That is, if Q is a type $\langle 1, 1 \rangle$ quantifier like *most* (one usually assumes that the quantifier is increasing in the right argument, like *most*, or *John’s*), (35) gives a type $\langle 1, 2 \rangle$ quantifier $\text{Ram}_i(Q)$ suitable for interpreting certain reciprocal sentences.

The most natural compositional analysis of sentences like (33) and (34) would seem to go like this:

[Det C] [R each other]

$$Q^C \quad \lambda X EO(X, R).$$

But to get the meaning of the sentence, according to (35), at the last step, we in general need access not only to Q^C but also to C . Thus, Theorem 4.18 applies, when Q^C is ISOM decomposable (like *most pirates* or *John’s companions*) and only finite universes are considered. Since $C = U(Q^C) = W_{Q^C}$, this theorem guarantees the compositionality of the corresponding reciprocal constructions.⁶

5.2. Possessives. Next, consider:

(36) Most planets’ rings are made of ice.

Here too we have an NP, *most planets*, whose interpretation is of the form Q^C and which is naturally seen as an immediate constituent of the phrase *most planets’* (which can be taken to be a determiner). But in this case, access to C does not seem to be enough. This is because *most* in (36) does not really quantify over all the planets, but only over those that have rings. Planets without rings are irrelevant to the truth value of (36). The phenomenon was called *narrowing* in Barker (1995) (from which the example is taken).

⁵ See Dalrymple *et al.* (1998) for a treatment of the semantics of reciprocals. (33) is a variant of a sentence discussed there.

⁶ One may note that since Q is CONSERV,

$$\text{Ram}_1(Q)(C, R) \iff \exists X (Q^C(X) \& EO_1(X, R)).$$

So in this case, access to C is in fact not needed. This is because EO_1 is defined by a universal condition. But EO_2 is not, and C cannot be similarly eliminated for $\text{Ram}_2(Q)$.

Thus, we need access to $Q^{C'}$, for some $C' \subseteq C$. But, as we have seen, this is in general not possible. One cannot recover Q from Q^C and therefore not $Q^{C'}$ either, when $C' \neq C$, even if C' is known. In Peters and Westerståhl (in preparation), we analyze the situation and conclude that this appears to constitute a serious problem for the compositionality of the semantics for possessives.

We also note a related problem for possessive semantics. If one construes the semantics so that not Q^C but both Q and C are taken as arguments when calculating the meaning of possessive phrases, one has an additional problem besides compositionality: What to do when the NP is not quantified, as in (37) and (38)?

(37) Mary's grant applications were successful.

(38) John or Mary's grant applications were successful.

The obvious answer is: decompose! But one problem is that such decomposition is not unique. For example, $I_m = all^{(m)} = (all_{ei})^{(m)}$, and the truth conditions vary slightly with the decomposition chosen.⁷ Another issue is that phrases of the form [NP 's] are sometimes problematic. Consider:

(39) (?) John, or Mary and Sue's grant applications were successful.

(40) (?) Some man or every woman's grant applications were successful.

Many speakers find (39) and (40) meaningless or at least very strange.⁸ Our logical results may apply here too. For example, we note that the NPs in (39) and (40) are not ISOM decomposable (Fact 4.15), in contrast with those in (37) and (38). These issues too are discussed further in Peters & Westerståhl (in preparation).

In conclusion, the ubiquity of linguistic constructions involving phrases whose interpretations are type ⟨1⟩ quantifiers, decomposable or not, makes it likely that the simple logical facts about decomposition established in this note are relevant to various issues in natural language semantics.

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⁷ On our account, if one uses $all^{(m)}$, (37) becomes true iff Mary did not make any grant applications or all her applications were successful. With $(all_{ei})^{(m)}$, it is—more reasonably—true iff she made at least one grant application and all her applications were successful.

⁸ Note, *first*, that we are of course disregarding readings of (40), which say that either some man was successful or every woman's grant application was successful, and, *second*, we realize that there is a tendency to read (40) as the unproblematic:

(i) Some man's or every woman's grant applications were successful.

But it could still be maintained that this is just a way to charitably make sense of or 'save' the deviant (40).

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BIBLIOGRAPHY

- Barker, C. (1995). *Possessive Descriptions*. Stanford, CA: CSLI Publications.
- Barwise, J., & Cooper, R. (1981). Generalized quantifiers and natural language. *Linguistics and Philosophy*, **4**, 159–219.
- Dalrymple, M., Kanazawa, M., Kim, Y., Mchombo, S., & Peters, S. (1998). Reciprocal expressions and the concept of reciprocity. *Linguistics & Philosophy*, **21**, 159–210.
- Peters, S., & Westerståhl, D. (2006). *Quantifiers in Language and Logic*. Oxford, UK: Oxford University Press.
- Peters, S., & Westerståhl, D. (in preparation). The semantics of possessive constructions.

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