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Arsiwalla, X.D.; Verlinde, E.P.

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Xerxes D. Arsiwalla[*] and Erik P. Verlinde[†]

Institute for Theoretical Physics, University of Amsterdam, Valckenierstraat 65, 1018 XE Amsterdam, The Netherlands
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We study the problem of spatially stabilizing four dimensional extremal black holes in background electric/magnetic fields. Whilst looking for stationary stable solutions describing black holes placed in external fields we find that taking a continuum limit of Denef *et al.*'s multicenter supersymmetric black hole solutions provides a supergravity description of such backgrounds within which a black hole can be trapped within a confined volume. This construction is realized by solving for a levitating black hole over a magnetic dipole base. We comment on how such a construction is akin to a mechanical levitron.

I. INTRODUCTION

Being inspired by ongoing interest in questions concerning black hole production; in this paper we address the following curiosity: after detection, how does one stabilize such a black hole using say external fields? This would in fact serve as a black hole analog of a particle-trap, or rather as we shall see below, of a classical levitron. However unlike familiar subatomic particle traps or even Millikan's well-known oil drop experiment [1], including the effects of general relativity indeed gives rise to interesting modifications of the above stabilization mechanisms, which were based purely on Newtonian gravity. We shall describe how this idea can in fact be materialized by writing down solutions for black holes levitating in external electromagnetic as well as gravitational fields.

For the purpose of this article, we consider four dimensional extremal black hole solutions to minimal $\mathcal{N} = 2$ SUGRA ([2–4]). Furthermore, let us confine these configurations to only include an electric and/or magnetic charge q, p respectively. These extremal black holes are known to satisfy the BPS constraint. The most general metric ansatz consistent with supersymmetry can then be written as

$$ds^2 = -\frac{\pi}{S(\vec{x})}(dt + \omega_i dx^i)^2 + \frac{S(\vec{x})}{\pi}dx^i dx^i \quad \text{with}$$

$$S(\vec{x})/\pi = \mathcal{P}^2(\vec{x}) + \mathcal{Q}^2(\vec{x}) \quad \text{and} \tag{1}$$

$$\mathcal{A} = 2\pi\mathcal{Q}(\vec{x})(dt + \omega_i dx^i) + \Theta$$

is the four dimensional gauge field. $\mathcal{P}(\vec{x})$, $\mathcal{Q}(\vec{x})$ are harmonic functions associated to charges p and q accordingly. Θ is the Dirac part of the vector potential satisfying $d\Theta = *d\mathcal{P}(\vec{x})$ with the Hodge star $*$ defined on $\mathbb{R}^3$. For a single spherically symmetric black hole in vacuum, it holds that $\vec{\omega} = 0$. However, for our considerations here, we shall be looking for solutions when the black hole is placed in external electric and magnetic fields. There now exists a nonzero Poynting vector corresponding to a rotating ge-

ometry. We first look for levitating solutions in constant background fields. It turns out these are inadequate for stabilization in all three directions. Then we look for more nontrivial backgrounds, which are obtained by extracting a continuum limit of Denef *et al.*'s [2–4] multicenter supergravity solutions. We find that turning on dipole fields already achieves the desired result.

II. BLACK HOLE LEVITATION IN CONSTANT EXTERNAL FIELDS

Given the metric ansatz in Eq. (1), we begin by looking for stationary solutions of a black hole placed in constant electric, magnetic and gravitational fields. In order to achieve this we have to specify explicit harmonic functions describing this configuration, then compute the off-diagonal elements $\vec{\omega}$ and solve the associated integrability equations. We claim that the desired harmonic functions describing this configuration are

$$\mathcal{P}(\vec{x}) = u + \frac{p}{|\vec{x} - \vec{l}|} + Bz$$

$$\mathcal{Q}(\vec{x}) = v + \frac{q}{|\vec{x} - \vec{l}|} + Ez \tag{2}$$

where B and E are constant magnetic, respectively, electric fields oriented along the z-direction and z denotes the z-coordinate. $\vec{l}$ marks the position of the black hole's horizon, which we determine via integrability conditions. u, v are constants. In principle, we can absorb u and v via a shift in the z-coordinate. This point will be made clear when we solve for $\vec{l}$. The Bz and Ez in Eq. (2) are linear terms that satisfy Laplace's equation and can be recognized as the usual electro/magneto-static potentials associated to constant fields. Note that extremality implies the above linear terms also source constant gravitational fields.

A nice way to motivate the expressions for $\mathcal{P}(\vec{x})$ and $\mathcal{Q}(\vec{x})$ is to extract them via a special limit of Denef *et al.*'s multicenter solutions [2–4]. More specifically, let us consider the two-center solution. This is a regular BPS solution of four dimensional $\mathcal{N} = 2$ supergravity. It is stationary but nonstatic and hence caries an intrinsic angular momen-

[*]X.D.Arsiwalla@uva.nl
[†]E.P.Verlinde@uva.nl

tum. Moreover the black holes comprising this bound state possess mutually nonlocal charges. Let us denote the corresponding two charge vectors as $\mathbf{\Gamma} = (p, q)$ and $\tilde{\mathbf{\Gamma}} = (\tilde{p}, \tilde{q})$. The idea is now to carry the charge $\tilde{\mathbf{\Gamma}}$ all the way to infinity while scaling $(\tilde{p}, \tilde{q})$ and the radial coordinate of the charges in such a way that the magnitudes of the electric/magnetic fields themselves are held fixed. Applying this limit to the expressions for electro/magneto-static fields of point charges indeed leaves us with constant fields oriented opposite to the direction of the source charges $\tilde{\mathbf{\Gamma}}$. Without loss of generality, the z-axis can then be chosen to point in the direction of the sources. Integrating these fields along the line element, precisely yields the linear potential terms in Eq. (2).

In fact we may also use this limiting two-center system to captures other features of our original configuration of a black hole in constant external fields. Following [2–4], we can determine the off-diagonal terms in the metric using

$$\nabla \times \vec{\omega} = \mathcal{P}(\vec{x})\nabla\mathcal{Q}(\vec{x}) - \mathcal{Q}(\vec{x})\nabla\mathcal{P}(\vec{x}). \tag{3}$$

Below we shall solve $\vec{\omega}$ for a class of nonstatic solutions. Furthermore operating the gradient on both sides of Eq. (3) leads to the following integrability equation

$$\mathcal{P}(\vec{x})\nabla^2\mathcal{Q}(\vec{x}) - \mathcal{Q}(\vec{x})\nabla^2\mathcal{P}(\vec{x}) = 0 \tag{4}$$

which we evaluate at $\vec{x} = \vec{l}$ to get

$$l = \frac{qu - pv}{pE - qB}. \tag{5}$$

This gives us the position of the black hole. Here $\vec{l} = (0, 0, l)$ can be chosen on grounds of symmetry. One can also perform a shift of coordinates, so as to place the black hole at the origin. This can be achieved by setting constants $u = v = 0$. Note however that $(pE - qB) \neq 0$ is required in order to preserve mutual nonlocality.

Equation (3) can be conveniently solved using spherical coordinates (r, θ, ϕ). And that leads to a system of coupled differential equations

$$(\nabla \times \vec{\omega})_r = \frac{2\cos\theta(pE - qB)}{r}$$
$$(\nabla \times \vec{\omega})_\theta = -\frac{\sin\theta(pE - qB)}{r} \tag{6}$$

while $(\nabla \times \vec{\omega})_\phi = 0$ due to ϕ-independence on the right-hand side. Our objective is now to seek out a nontrivial solution which confers to the description of a black hole rotating in the presence of external electromagnetic fields. We find that there exists such a simple solution with azimuthal symmetry

$$\omega_\phi = \sin\theta(pE - qB) \tag{7}$$

while $\omega_r = \omega_\theta = 0$. For completeness let us also mention that the solution presented in Eq. (7) is certainly not the most general. For instance, we also find that solutions with harmonic variations such as $\frac{\partial\omega_\theta}{\partial\phi} = \cos\phi$ also exist and very

likely one may well find a more general class of these. But we shall not require that for our purposes.

The solution above allows us to levitate a black hole at a fixed height on the xy-plane owing to the balancing act between gravitational attraction and electro/magneto-static repulsion. However it is not stable in all three directions and can move about the surface of the plane. To localize the black hole in all three directions we need a more complicated background field where the black hole can be held at a local minimum of an effective potential. This we do in the next few sections.

III. CONTINUUM LIMIT OF MULTICENTER SOLUTIONS

In this section we start looking for extremal stationary solutions to Einstein-Maxwell gravity that admit backgrounds with multipole electromagnetic fields. As before, we work with four dimensional gravity with just one gauge field. Generalizations to $n - 1$ vector fields or inclusion of other charges such as D0 and/or D6 in Type II A are rather straightforward. Let us now see how taking a continuum limit of Denef *et al.*'s multicenter solutions yields the desired backgrounds. In order to write down harmonic functions for such a smeared distribution of black holes, we define density functions $\rho_e(\vec{x}')$, $\rho_m(\vec{x}')$ via

$$\int_V \rho_e(\vec{x}')d\tau' = Q \quad \text{and} \quad \int_V \rho_m(\vec{x}')d\tau' = P, \tag{8}$$

where $d\tau'$ is a volume element within a compact support V, that covers the distribution. In the continuum limit, harmonic functions for multiple black holes take the form

$$Q(\vec{x}) = v + \int_V \frac{\rho_e(\vec{x}')}{|\vec{x} - \vec{x}'|}d\tau'$$
$$\mathcal{P}(\vec{x}) = u + \int_V \frac{\rho_m(\vec{x}')}{|\vec{x} - \vec{x}'|}d\tau'. \tag{9}$$

To these harmonics one may also add linear terms Ez and Bz corresponding to constant fields, whenever required. From a computational point of view, the real utility of the above-mentioned smeared distributions shows up in their respective multipole expansions. Expressing this in the regime that $|\vec{x}| \gg |\vec{x}'|$ holds, we have

$$Q(\vec{x}) = v + \frac{Q}{|\vec{x}|} + \frac{x_i\Delta_e^i}{|\vec{x}|^3} + \frac{1}{2}\frac{x_ix_jT_e^{ij}}{|\vec{x}|^5} + \cdots\cdots$$
$$\mathcal{P}(\vec{x}) = u + \frac{P}{|\vec{x}|} + \frac{x_i\Delta_m^i}{|\vec{x}|^3} + \frac{1}{2}\frac{x_ix_jT_m^{ij}}{|\vec{x}|^5} + \cdots\cdots, \tag{10}$$

where Q, P are electric, respectively, magnetic monopole moments; $\mathbf{\Delta_e}$, $\mathbf{\Delta_m}$ are electric and magnetic dipole moment vectors; and $\mathbf{T_e}$, $\mathbf{T_m}$ are, respectively, electric and magnetic quadrupole moment tensors—all defined in the usual way. We employ boldface characters to denote vectors as well as tensors. The "$\cdots\cdots$" in Eq. (10) denote terms with higher order moments. When $|\vec{x}| \gg |\vec{x}'|$, the

series is convergent and these functions can be used to describe supergravity solutions associated to any specific multimoment source, provided all lower moments vanish for that distribution. As an illustrative example, we analyze the solution for a charge distribution with dipole order corrections.

First let us check that the functions in Eq. (9) yield meaningful expressions for continuum black hole configurations. Evaluating Eq. (4) for these harmonics gives

$$\rho_e(\vec{x})\mathcal{P}(\vec{x}) - \rho_m(\vec{x})\mathcal{Q}(\vec{x}) = 0. \tag{11}$$

Outside the support V, this expression vanishes identically; whereas points within the support region ought to satisfy

$$u\rho_e(\vec{x}) + \rho_e(\vec{x}) \int_V \frac{\rho_m(\vec{x}')}{|\vec{x} - \vec{x}'|} d\tau' - v\rho_m(\vec{x})$$

$$- \rho_m(\vec{x}) \int_V \frac{\rho_e(\vec{x}')}{|\vec{x} - \vec{x}'|} d\tau' = 0. \tag{12}$$

After performing the relevant integrals, the above expression can be evaluated for all points $\vec{x} \in V$, and that defines the locus of solutions for the black hole distribution. In following sections, we will solve this condition for specific distribution functions. At the moment though, as a consistency check, let us confirm that, analogous to any multicenter configuration, asymptotically the above continuum configurations also behave like a single-center black hole with total charge P and Q. This can be done by seeing how the constants u and v (which themselves are asymptotically defined) relate to the total monopole charges Q and P, and if this relation is the same as that obtained for a single-center black hole with the same monopole charges. In order to do this we simply integrate both sides of Eq. (12) over all $\vec{x} \in V$. This yields

$$uQ - vP = 0 \tag{13}$$

which is precisely what one obtains for a single-center solution with charges Q and P; thereby confirming the asymptotic dependence of u and v for an arbitrary continuum configuration having fixed total (monopole) charges Q and P.

Having checked consistency of integrability conditions, we next compute the off-diagonal elements $\vec{\omega}$ in the metric via

$$\nabla \times \vec{\omega} = -\mathcal{P}(\vec{x})\mathbf{E}(\vec{x}) + \mathcal{Q}(\vec{x})\mathbf{B}(\vec{x}) \tag{14}$$

where $\mathbf{E}(\vec{x})$ and $\mathbf{B}(\vec{x})$ refer to exact electric and magnetic fields corresponding to distributions $\rho_e(\vec{x})$ and $\rho_m(\vec{x})$ respectively. In this sense the continuum limit described here is much simpler than a finite N many body black hole system for which integrability equations turn out to be quite hard to solve in full generality.

For our objectives, it will suffice to solve Eq. (14) using its multipole expansion. As an illustration, we consider a smeared distribution where the monopole contributions to $\vec{\omega}$ get magnetic dipole corrections coming from $\mathbf{\Delta_m}$, which

is aligned along the z-axis. In spherical coordinates, Eq. (14) takes the form

$$(\nabla \times \vec{\omega})_r = \frac{2v\Delta_m \cos\theta}{r^3} + \frac{Q\Delta_m \cos\theta}{r^4}$$

$$(\nabla \times \vec{\omega})_\theta = \frac{v\Delta_m \sin\theta}{r^3} + \frac{Q\Delta_m \sin\theta}{r^4} \tag{15}$$

while $(\nabla \times \vec{\omega})_\phi = 0$ due to symmetry in the ϕ-direction. Note that while writing down Eq. (15), we make use of the integrability constraint Eq. (13) (inserting it into Eq. (14)). As before, we seek solutions characterized by azimuthal symmetry. The ensuing result is

$$\omega_\phi = \frac{v\Delta_m \sin\theta}{r^2} + \frac{Q\Delta_m \sin\theta}{2r^3} \tag{16}$$

and $\omega_r = \omega_\theta = 0$. At large distances away from the smeared sources, Eq. (16) gives dipole corrections to leading order contributions in the metric. In fact these constitute subleading contributions to the geometry. It is these multipole corrections that distinguish a true one-centered black hole from a multicenter distribution of black holes, when viewed at asymptotic infinity. For a pure one-center solution, $\vec{\omega}$ identically vanishes. While for the multicenter case, it is nontrivial but quite difficult to compute for any given discrete configuration. The continuum limit, on the other hand, facilitates viable computations, at least order by order in a multipole series expansion.

IV. TOWARDS A BLACK HOLE LEVITRON

We are now ready to combine results of the last two sections to construct stable levitating black hole solutions and realize a Levitron-like construction. We perturb the constant background fields of Sec. II with a magnetic dipole field and over this perturbed background solve for a black hole held at a fixed height. The dipole fields are produced by the smeared distribution discussed in Sec. III. For simplicity we consider a black hole with only electric charge q (a dyonic generalization is also straightforward). This construction is captured by the following harmonics

$$\mathcal{Q}(\vec{x}) = v + \frac{q}{|\vec{x} - \vec{l}|} + Ez$$

$$\mathcal{P}(\vec{x}) = u + \frac{\Delta_m \cos\theta}{|\vec{x}|^2} + Bz. \tag{17}$$

The dipole moment is aligned parallel to the z-axis and carries a magnitude Δ_m. While θ is a coordinate denoting the angle that the position vector $\vec{x}$ makes with the z-axis. Below we shall see, how solving integrability conditions for these harmonics constrains allowed solutions for $|\vec{l}|$ and θ, where a black hole with charge q is held stable in the vicinity of a continuum distribution with dipole charge Δ_m.

For the rest of the computation however, it will suffice to turn off the constant fields E and B. This is because a dipole background will turn out to be sufficient hold the black

hole at a fixed height and keep it stable in all three directions. Superposing constant fields do not affect stability of the solution but ultimately we will need the constant fields for giving an interpretation of black hole levitation in a constant gravitational field (as would be the case if we were ever to trap a small black hole in a laboratory somewhere on Earth).

Continuing with the calculation, the position of the black hole $\vec{l}$ is determined by evaluating Eq. (4) at the location of the pole $\vec{x} = \vec{l}$ using harmonics in Eq. (17) with $E = B = 0$. This gives

$$|\vec{l}| = \sqrt{\frac{-\Delta_m \cos\theta}{u}} \qquad (18)$$

This gives us a locus of solutions $|\vec{l}|$, θ for the black hole configuration described in Eq. (17) (with $E = B = 0$). Before discussing further reality constraints on these solutions, let us also evaluate the integrability equation at the other pole $\vec{x} = 0$. This then determines the constant v as

$$v = -\frac{q}{|\vec{l}|} \qquad (19)$$

Note that physical solutions only exist $l(\equiv |\vec{l}|)$ real and non-negative and this restricts the values that the angle θ can assume. For instance, let us first consider the case when $u > 0$. Then θ can attain values only from 0 to $\frac{\pi}{2}$ provided the dipole is directed along the negative z-axis, while the ϕ coordinate remains unconstrained. On the other hand, for a dipole pointing in the positive z-direction, the angle θ can only span the range $\frac{\pi}{2}$ to π (as shown in Fig. 1). In the other
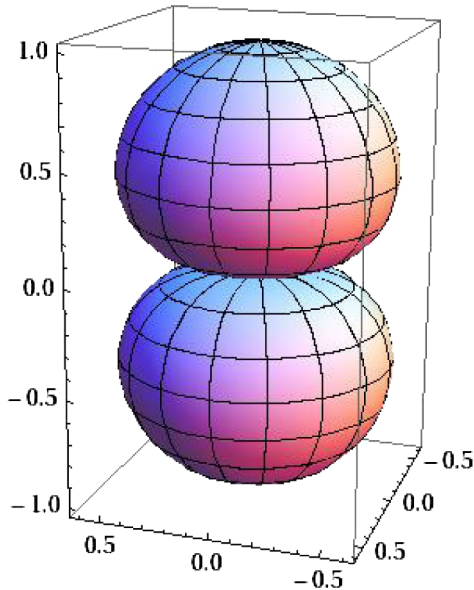

FIG. 1 (color online). Here we make a 3D plot of Eq. (18) for the solution space of $\vec{l}$ for positive as well as negative dipole orientations. Points on the upper globular surface correspond to (l, θ, ϕ) for $\Delta_m < 0$ and $u > 0$. Points on the lower globular surface correspond to those with $\Delta_m > 0$ when $u > 0$.

case, when $u < 0$, then the signs appropriately reverse, namely, when the dipole is directed along the negative z-axis, then θ goes from $\frac{\pi}{2}$ to π; whereas with a dipole along the positive z-orientation, θ spans values from 0 to $\frac{\pi}{2}$. The solution space of the black hole is now confined to a restricted parameter space. More precisely these are circular orbits corresponding to given values of θ on an equipotential surface of a dipole field. And in turn each orbit refers to a solution with a specified radial distance l. We plot the solution space for physical values of (l, θ, ϕ) in Fig. 1. The dipole surface in the figure represents locations where a single black hole with a point charge can be stabilized in the gravitational and magnetic field of a continuum black hole distribution centered around the origin and carrying a magnetic dipole moment.

In Fig. 1, we have plotted Eq. (18). At $\theta = 0$ the black hole sits at a fixed height on the z-axis; at $\theta = \frac{\pi}{2}$ it falls into the origin; while the case $0 < \theta < \frac{\pi}{2}$ corresponds to the black hole being located anywhere on a circular orbit centered at height $l\cos\theta$ and having radius $l\sin\theta$. Solutions on the positive z-axis correspond to the case when $\Delta_m < 0$ (for $u > 0$), while those on the negative axis refer to $\Delta_m > 0$. For each value of θ in Eq. (18) there exists a solution for $\vec{\omega}$. At $\theta = 0$ the solution space is just a single point and that is when the black hole achieves stability in all three directions at a fixed height on the z-axis.

For completeness we first compute $\vec{\omega}$ when the black hole is still sitting at the origin, that is when $\vec{l} = 0$. After that we shall determine the modification in $\vec{\omega}$ required to achieve stable levitation at a fixed height on the z-axis. In fact the solution at $\vec{l} = 0$. can simply be borrowed from our calculation in Eq. (16) once we make the substitutions $Q \rightarrow q$ and $P \rightarrow 0$.

On the other hand, when the black hole is made to levitate at a fixed height l on the z-axis we have to solve the following system of equations

$$
\begin{aligned}
(\nabla \times \vec{\omega})_r &= -\frac{qu(r - l\cos\theta)}{(r^2 + l^2 - 2rl\cos\theta)^{3/2}} - \frac{2q\Delta_m \cos\theta}{lr^3} \\
&\quad - \frac{q\Delta_m \cos\theta(r - l\cos\theta)}{r^2(r^2 + l^2 - 2rl\cos\theta)^{3/2}} \\
&\quad + \frac{2q\Delta_m \cos\theta}{r^3(r^2 + l^2 - 2rl\cos\theta)^{1/2}} \\
(\nabla \times \vec{\omega})_\theta &= -\frac{qul\sin\theta}{(r^2 + l^2 - 2rl\cos\theta)^{3/2}} - \frac{q\Delta_m \sin\theta}{lr^3} \\
&\quad - \frac{ql\Delta_m \sin\theta \cos\theta}{r^2(r^2 + l^2 - 2rl\cos\theta)^{3/2}} \\
&\quad + \frac{q\Delta_m \sin\theta}{r^3(r^2 + l^2 - 2rl\cos\theta)^{1/2}} \qquad (20)
\end{aligned}
$$

and again $(\nabla \times \vec{\omega})_\phi = 0$. Also $\vec{l} = (0, 0, l)$. This now becomes fairly more complicated compared to the non-

levitating case. The modification in the metric reflects a modification to the geometry of the system. If we restrict to azimuthally symmetric cases, we find that Eq. (20) has a solution only for small heights of levitation, that is when $l \ll r$. This can be understood in the following way. In this setup the system consists of the black hole plus the source of the dipole field. Let us call the latter the base. The levitating we are looking for requires that the base be rigid against the gravitational pull of the black hole, that is the center of mass of the whole system be as close to the base as possible. For very large charges, corresponding to large values of l, a stable symmetric levitating solution does not seem to exist (we see this from numerical checks). In that case more complicated nonsymmetric solutions may be sought for, but we would hardly call those levitating.

Narrowing down to our regime of interest, we expand around $l \ll r$ and solve Eq. (20) order by order in l. Truncating up to second order terms we get

$$
\begin{aligned}
\omega_\phi = & -\frac{qu(1-\cos\theta)}{r\sin\theta} - \frac{q\Delta_m \sin\theta}{lr^2} + \frac{q\Delta_m \sin\theta}{2r^3} \\
& - \left\{ \frac{qu\sin\theta}{r^2} \right\} \cdot l + \left\{ -\frac{3qu\cos\theta\sin\theta}{2r^3} \right. \\
& \left. - \frac{q\Delta_m(1+3\cos^2\theta)\sin\theta}{8r^5} \right\} \cdot l^2 + \mathcal{O}(l^3)
\end{aligned}
\tag{21}
$$

while $\omega_r = \omega_\theta = 0$. This solution enables us to write down the full metric for a stationary system of a black hole levitating in equilibrium above a magnetic dipole field. Also this calculation easily extends to the case of a dyonic black hole.

Comparison to a levitron

We now compare the levitation of black holes discussed above with that of a levitron [5]. The latter is a spin stabilized magnetic levitation device first invented by Roy Harrigan [6]. It basically consists of a permanent base magnet above which a spinning top with a magnetic dipole moment levitates midair and is stable in all three directions. This gives rise to an apparent paradox due to Earnshaw's theorem [7] which states that no stationary configuration composed of electric/magnetic charges and masses can be held in stable equilibrium purely by static forces. And the reason for this is simply that all static potentials satisfy Laplace's equation whose solutions only exhibit saddles at critical points : there are neither any maxima nor minima. It was Sir Michael Berry's [8] (see also [9]) remarkable insight invoking adiabatic averaging that helped resolve the apparent paradox. He showed that a slow precession mode (when averaged over the fast rotation mode) was responsible for creating an effective stationary potential with a stable minimum. This is the same principle used in neutron traps as well as other particles carrying magnetic dipole moment.

A natural question which arises is whether our black hole construction also mimics the physics of the levitron and how it overcomes Earnshaw's theorem. The latter it already seems to evade since it is based on Einstein's gravity rather than Newton's. However the gravitational interpretation of our Black Hole levitron's balancing mechanism admittedly requires further investigation. Nevertheless a naive classical intuition can be obtained from the fact that a nonvanishing Poynting vector gives rise to a rotating black hole geometry and in turn a rotating electric distribution induces a magnetic field that repels the base magnet. It is the $\vec{\omega}$ in the metric that is responsible for inducing this balancing force. On the other hand the gauge theoretic interpretation of this multicenter balancing has been better understood in terms of Denef's quiver quantum mechanics [10] wherein the distance between centers is determined via an effective potential whose minima determine the stability loci $\vec{l}$.

V. CONCLUSIONS AND DISCUSSION

As has been extensively discussed in the literature, a very important application of supersymmetric multicenter black hole solutions is for the problem of microstate counting [11]. However even for the simplest configurations with more than two centers, solving integrability constraints to determine the full metric becomes a highly formidable task. Our initial motivation for this work was to investigate whether analytic results could still be found in some interesting limiting cases of these geometries. Indeed we find that such a limit exists in the form of a large n number of centers. In this work we have constructed a continuum distribution of black holes and solved integrability conditions towards obtaining the metric. Upon this continuum system we have performed a multipole expansion to find smeared black hole geometries with multipole moments.

As a fun application of these continuum solutions, we use these to address the problem of black hole stabilization in external fields. For this we construct a levitating black hole solution. This black hole levitron stabilizes a test extremal black hole at a fixed location in the electromagnetic and gravitational field produced by the continuous distribution. Our solution is inclusive of the black hole's backreaction on the continuous distribution. In this work we started off by using Denef *et al.*'s multicenter supersymmetric solutions, which by themselves are stable, stationary BPS solutions with nonlocal charges. Our harmonic functions and integrability conditions can all be retrieved as special limits of the discrete multicenter case. Therefore our levitating solutions also describe stable, stationary supersymmetric configurations. This black hole construction very much resembles a mechanical levitron, though it is different in that it includes general relativistic considerations, but on the other hand, it is also restricted to stationary solutions. For a more general class

of time-dependent, precessing multiblack hole solutions (when posterity finally discovers these), it would be an interesting problem to give a proof of the general relativistic analog of Berry's stabilization mechanism. It would also be of practical relevance to construct solutions for nonextremal black hole levitrons.

Other interesting directions might be investigating further applications of the continuum limit of multicenter black hole solutions. Compared to discrete-centered configurations, the smeared distribution lends itself to more viable computations. One may ask what role these distributions play in microstate counting of black hole geometries.

[1] R. A. Millikan, Phys. Rev. **2**, 109 (1913).
[2] F. Denef, J. High Energy Phys. 08 (2000) 050.
[3] F. Denef, arXiv:hep-th/0010222.
[4] B. Bates and F. Denef, arXiv:hep-th/0304094.
[5] E. W. Hones and W. H. Hones, U.S. Patent No. 5 404 062, 1995.
[6] R. M. Harrigan, U.S. Patent No. 4 382 245, 1983.
[7] S. Earnshaw, Trans. Cambridge Philos. Soc. **7**, 97 (1842).
[8] M. V. Berry, Proc. R. Soc. A **452**, 1207 (1996).
[9] M. D. Simon, L. O. Heflinger, and S. L. Ridgway, Am. J. Phys. **65**, 286 (1997).
[10] F. Denef, J. High Energy Phys. 10 (2002) 023.
[11] F. Denef and G. W. Moore, arXiv:hep-th/0702146.