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Critical temperatures of A1 and A2 phases of a superfluid Fermi-gas in a magnetic field

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Abstract

We investigate the phase diagram for a superfluid Fermi-gas in a magnetic field and calculate the A1 and A2 critical temperatures including their field and pressure dependence. The results are in qualitative agreement with experiments on the phase diagram of superfluid ^{3}He.

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1. Introduction

In our previous papers [1–3] we calculated from first principles the strong-coupling corrections to the Ginzburg–Landau free-energy functional in a superfluid Fermi-gas with repulsion. On the basis of these corrections we determined all the global minima of the G–L free-energy and constructed the phase diagram. Our calculations showed [1,2] that in the absence of a magnetic field the phase diagram consists only of BW- and ABM-phases ruling out a possibility of exotic phases raised in some papers [4–7] in the recent literature. In strong magnetic fields exceeding the paramagnetic limit ($H > H_p = T_C/\mu_B$) the BW-phase is totally suppressed. The calculations of the field dependence of strong-coupling terms [3] have shown that the phase diagram in this case consists only of the A1- and ABM-phases again in agreement with the original experiments in ^{3}He [8,9]. The ABM-phase in a magnetic field is hereafter called the A2-phase.

2. Theory

In the present Letter we carry out a detailed analysis of the A1 and A2 critical temperatures in the presence of a magnetic field. Here T^{A1} is the transition temperature from the normal-phase to the A1-phase and T^{A2} is the transition temperature from the A1-phase to the A2-phase. To be specific we calculate the ratio $(T^{A1} - T_C)/(T_C - T^{A2})$ as a function of a coupling constant $\lambda = 2ap_F/\pi$ (a is a scattering length, p_F is the Fermi-momentum) and the spin polarization η (in which $\eta = (N_\uparrow - N_\downarrow)/(N_\uparrow + N_\downarrow)$). Note that the critical temperature T^{A1} is completely determined by the normal properties of the system. An exact analytical expression in leading order in the coupling constant for T^{A1} as a function of η and λ was for the first time

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obtained in Ref. [10]. The results show a strongly non-monotonic behavior of T^{A1} with a large initial increase, a well-pronounced maximum at $\eta = 0.48$ and a decrease (reentrant behavior) for larger η. The formula for T^{A1} reads

$$T^{A1} = T_C \exp\left(\frac{\phi(\eta)}{\lambda^2}\right), \tag{1}$$

where $T_C = \epsilon_F \exp(-13/\lambda^2)$ is the critical temperature in zero magnetic field, and ϵ_F is the Fermi-energy. The function $\phi(\eta)$ has been given explicitly in Ref. [10]. For small polarizations $\phi(\eta)$ can be expanded and is given by

$$\phi(\eta) = A\eta + B\eta^2 + O(\eta^3), \tag{2}$$

where $A = 31.5$, $B = 37.2[\ln(1/\eta) - 2.9]$ and $O(\eta^3)$ denotes terms of order η^3 with logarithmic corrections. Note that the theory of Ref. [10] leads to a logarithmical term in B in contrast to Fermi-liquid type of theories [11]. As a result, in the region where $\phi(\eta)/\lambda^2 \ll 1$,

$$T^{A1} = T_C\left(1 + \frac{A\eta}{\lambda^2} + \frac{\tilde{B}\eta^2}{\lambda^2}\right), \tag{3}$$

where $\tilde{B} = B + \frac{1}{2}A^2/\lambda^2$. The situation with T^{A2} is different. The normal properties of the system determine T^{A2}_{bare}. This temperature enters in the coefficient $a_\downarrow = N(0)\ln(T/T^{A2}_{\text{bare}})$ in front of the quadratic term $a_\downarrow \Delta^2_{\downarrow\downarrow}$ corresponding to the ordering of pairs with down spins in the G–L free energy. For small polarizations η it is equal to

$$T^{A2}_{\text{bare}} = T_C\left(1 - \frac{A\eta}{\lambda^2} + \frac{\tilde{B}\eta^2}{\lambda^2}\right), \tag{4}$$

with the same coefficients A and $\tilde{B}$ as in Eq. (3). However, the actual critical temperature T^{A2} differs from T^{A2}_{bare} due to the influence of the ordering of the A1-phase. In fact the G–L functional contains besides the standard quartic terms $b_\uparrow \Delta^4_{\uparrow\uparrow}$ and $b_\downarrow \Delta^4_{\downarrow\downarrow}$ also a strong-coupling term $(\alpha + \gamma)\Delta^2_{\uparrow\uparrow}\Delta^2_{\downarrow\downarrow}$, see Ref. [3]. This term describes the interaction between the up-up (A1) and the down–down (A2) Cooper-pairs. The actual T^{A2} is determined from the condition that the down–down superfluid gap $\Delta_{\downarrow\downarrow}$ is zero at $T = T^{A2}$,

$$\begin{aligned} 0 = \Delta^2_{\downarrow\downarrow} &= -\frac{a_\downarrow}{4b} - \frac{\alpha+\gamma}{4b}\Delta^2_{\uparrow\uparrow} \\ &= -\frac{a_\downarrow}{4b} + \frac{\alpha+\gamma}{16b^2}a_\uparrow, \end{aligned} \tag{5}$$

where $a_\uparrow = N(0)\ln(T/T^{A1})$, $b = \frac{1}{2}(\beta_2+\beta_4) = \frac{1}{2}\beta_{24}$, $\alpha+\gamma = 2\beta_2+2\beta_4+4\beta_5 = 2\beta_{245}+2\beta_5$, and $\beta_2, \beta_4, \beta_5$ are the standard fourth order coefficients related to the invariants of the G–L expansion [12]. For the case of a superfluid Fermi-gas with repulsion these coefficients in leading order in T/ε_F and up to cubic terms in the coupling constant were calculated in Refs. [2,3]. From (5) it immediately follows that

$$\frac{T^{A2}_{\text{bare}} - T^{A2}}{T_C} = \frac{T^{A1} - T^{A2}_{\text{bare}}}{T_C}\,\frac{-\frac{1}{2}r_{SC}}{1+\frac{1}{2}r_{SC}}, \tag{6}$$

where $r_{SC} = -2(\beta_{245}+\beta_5)/\beta_{24}$. As a result

$$\begin{aligned} t^{\downarrow\downarrow} &= \frac{T_C - T^{A2}}{T_C} \\ &= \frac{A\eta}{\lambda^2}(1 - \tilde{r}_{SC}) - \frac{\tilde{B}\eta^2}{\lambda^2}, \end{aligned} \tag{7}$$

where

$$\tilde{r}_{SC} = \frac{r_{SC}}{1+\frac{1}{2}r_{SC}}, \tag{8}$$

in which r_{SC} as a function of λ according to the calculations of Ref. [2], cf. also Ref. [3], is given by

$$r_{SC} = 69\lambda^2(1 + 2.2\lambda)\exp(-13/\lambda^2). \tag{9}$$

From (3) we obtain that

$$t^{\uparrow\uparrow} = \frac{T^{A1} - T_C}{T_C} = \frac{A\eta}{\lambda^2} + \frac{\tilde{B}\eta^2}{\lambda^2}. \tag{10}$$

This yields

$$\begin{aligned} \frac{t^{\uparrow\uparrow}}{t^{\downarrow\downarrow}} &= \frac{T^{A1} - T_C}{T_C - T^{A2}} = \frac{1 + r_{\text{field}}}{1 - \tilde{r}_{SC} - r_{\text{field}}} \\ &\simeq 1 + 2r_{\text{field}} + \tilde{r}_{SC}, \end{aligned} \tag{11}$$

where $r_{\text{field}} = (\tilde{B}/A)\eta$. The width of the A1-phase is given by

$$T^{A1} - T^{A2} = T_C(t^{\uparrow\uparrow} + t^{\downarrow\downarrow}) = T_C\frac{2A\eta}{\lambda^2}(1 - \tfrac{1}{2}r_{SC}). \tag{12}$$

Note that formulae (6)–(12) are valid only for $\eta \ll \lambda^2/A$ implying that r_{SC} in (8) can be calculated at the

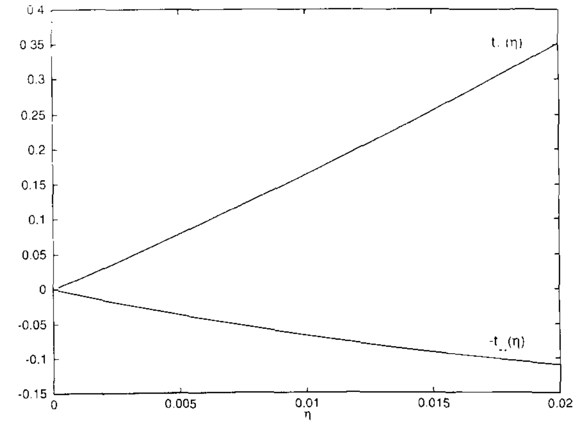

Fig. 1. Reduced critical temperatures $t^{\uparrow\uparrow}$ and $t^{\downarrow\downarrow}$ as a function of the spin polarization η at the value $\lambda = 1.45$ (corresponding to the melting pressure of ^{3}He).

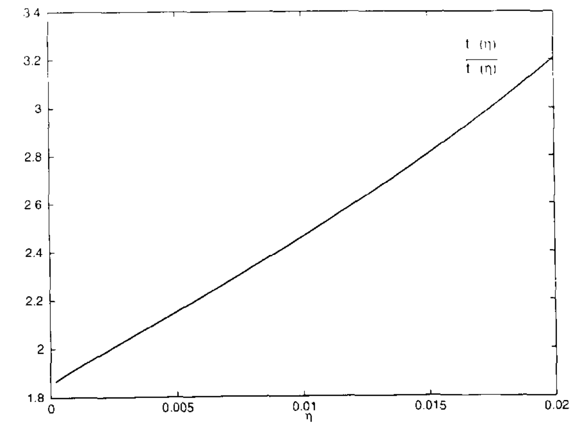

Fig. 2. Asymmetry parameter $t^{\uparrow\uparrow}/t^{\downarrow\downarrow}$ as a function of the spin polarization η at the value $\lambda = 1.45$.

small field limit. The qualitative dependences of $t^{\uparrow\uparrow}$, $t^{\downarrow\downarrow}$ and the asymmetry parameter $t^{\uparrow\uparrow}/t^{\downarrow\downarrow}$ upon polarization are presented in Figs. 1, 2. For physically reasonable values of λ we have $\tilde{B} > 0$ leading to a positive curvature of the functions $t^{\uparrow\uparrow}(\eta)$ and $t^{\downarrow\downarrow}(\eta)$ for polarizations η satisfying the above condition. Note that the exact results without using the expansion of the exponent of Ref. [10] show that the curvature of the function $t^{\uparrow\uparrow}(\eta)$ changes sign for polarizations $\eta > 0.2$ and arbitrary values of λ. However these polarizations are outside the validity of our theory.

3. An estimate for ^{3}He

Strictly speaking the theory presented here is valid for small λ. This means that it is exact in the case of diluted ^{3}He–^{4}He mixtures. For pure ^{3}He λ changes from 1.3 at $p = 0$ bar to 1.45 at melting pressure $p =$ 34 bar [2]. In this case our theory can serve only as a qualitative estimate. However even for these values of λ we can obtain a qualitative agreement with experiments on the phase diagram of real ^{3}He. In fact our theory can help to distinguish between two experimental results [13,14] devoted to the pressure dependence of the ratio $t^{\uparrow\uparrow}/t^{\downarrow\downarrow}$. For $1.3 < \lambda < 1.45$ both r_{field} and r_{SC} are positive. That is why the ratio $t^{\uparrow\uparrow}/t^{\downarrow\downarrow}$ is larger than one for all pressures, which is in favor of the results of Israelson et al. [13] but disagrees with the results of Sagan et al. [14]. A better quantitative agreement with experiments, confirming also Ref. [15], may be obtained taking into account fourth order coefficients in the coupling constant λ which we are planning to evaluate in the near future. The strong-coupling corrections r_{SC} are increasing with pressure, also in agreement with experiment. For $\lambda = 1.3$ (at $p = 0$ bar) we have $\tilde{r}_{\text{SC}} = 0.19$. Note that the denominator in (8) is not important for small pressures. For melting pressure $p = 34$ bar ($\lambda = 1.45$) $\tilde{r}_{\text{SC}} = 0.75$ and we still have $T^{\text{A2}} < T_{\text{C}}$ due to the denominator in (8). As a result, the width of the A1-phase ($T^{\text{A1}} - T^{\text{A2}}$) increases with pressure in agreement with Ref. [13]. At melting pressure

$$T^{\text{A1}} - T^{\text{A2}} = T_{\text{C}} \frac{2A\eta}{\lambda^2} 0.6 = 45\eta \text{ [mK]}. \qquad (13)$$

For small polarizations η is linear in magnetic field leading to a linear dependence of ($T^{\text{A1}} - T^{\text{A2}}$) on H. At $p = 34$ bar, $\eta = 0.004H$ [T] and Eq. (12) gives an estimate of the width of the A1-phase which is two times as large as the experimental one [13]. This discrepancy possibly will be lifted by including third order corrections in the exponential in the formula for T_{C}. Note that in the case of ^{3}He our theory is valid for $\eta \ll 0.06$. For typical magnetic fields $H = 5$ T ($\eta = 0.02$) the field corrections at $p = 0$ bar, $r_{\text{field}} =$ 0.21 are of the same order of magnitude as the corresponding strong-coupling corrections. At $p = 34$ bar, however, $r_{\text{field}} = 0.17$ and strong-coupling corrections are dominant.

4. Conclusion

Concluding we would like to emphasize once more that we completed the phase diagram of a superfluid Fermi-gas in a strong magnetic field by calculating the pressure and magnetic field dependence of the A1 and A2 critical temperatures from first principles on the basis of the theory for superfluid Fermi-gas with repulsion. The strong-coupling corrections to the A2 critical temperature makes the phase-diagram asymmetric ($T^{A1} - T_C > T_C - T^{A2}$) already at very small magnetic fields, while field corrections lead to the deviations from linear behavior in field of the A1 and A2 temperatures. The calculated asymmetry is in qualitative agreement with experimental results of Ref. [13] but not with those of Ref. [14].

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