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AdS boundary conditions and the Topologically Massive Gravity/CFT correspondence

Kostas Skenderis[a,b], Marika Taylor[a] and Balt C. van Rees[a]

[a] *Institute for Theoretical Physics, University of Amsterdam,
Valckenierstraat 65, 1018 XE Amsterdam, The Netherlands*

[b] *Korteweg-de Vries Institute for Mathematics,
Science Park 904, 1098 XH Amsterdam, The Netherlands*

Abstract.
     The AdS/CFT correspondence provides a new perspective on recurrent questions in General Relativity such as the allowed boundary conditions at infinity and the definition of gravitational conserved charges. Here we review the main insights obtained in this direction over the last decade and apply the new techniques to Topologically Massive Gravity. We show that this theory is dual to a non-unitary CFT for any value of its parameter μ and becomes a Logarithmic CFT at $\mu = 1$.

INTRODUCTION

Three dimensional gravity offers an interesting arena to investigate both the quantization of gravitational theories and holography. Since Einstein gravity in three dimensions does not have propagating degrees of freedom it is not a good toy model for higher dimensional gravitational theories. Adding higher derivative terms gives propagating degrees of freedom but the theory generically then contains ghost-like excitations. In recent times there has been renewed interest in topologically massive gravity with a negative cosmological constant in three dimensions: [1, 2]

$$S = \int d^3x \left(\sqrt{-g}(R - 2\Lambda) + \frac{1}{2\mu}(\Gamma d\Gamma + \frac{2}{3}\Gamma^3) \right) \tag{1}$$

This theory admits asymptotically AdS solutions and has been used as an arena to explore holography. It has also been conjectured to be free of instability problems for $\mu = 1$. At $\mu \neq 1$ the perturbative massive modes around the AdS background have negative energy and the theory is unstable, but it was claimed in [3] that at $\mu = 1$ there are no negative energy modes and the theory is stable. The corresponding dual two dimensional field theory was conjectured to contain only a right moving sector, and thus to be a chiral conformal field theory.

This claim proved controversial as other authors found non-chiral modes and instabilities at $\mu = 1$ [4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. The unstable modes have fall-off conditions which are different from those that the metric satisfies in pure three-dimensional Einstein gravity, the so-called Brown-Henneaux boundary conditions [14] for asymptot-

ically AdS spacetimes. The main issue is then the question: *What are the allowed fall-off conditions for the fields at infinity?*

The traditional point of view regarding fall-off conditions goes back (at least) to the work of Regge and Teitelboim [15] and can be summarized as follows:

1. Select physically "reasonable" fall-off conditions such that relevant solutions, for example black holes, satisfy them.
2. Check that conserved charges are finite with this choice.

One may then consider different fall-off conditions as defining different theories.

The AdS/CFT correspondence however provides a **new perspective** which leads to a comprehensive answer to such questions. The focus of this article will be to explain the new insights and methodology originating from AdS/CFT and their applications to topologically massive gravity. More details can be found in the article [16].

ADS/CFT: BASICS

An asymptotically *AdS* spacetime has a conformal boundary at which boundary conditions for all bulk fields need to be defined. In the framework of the AdS/CFT correspondence the bulk fields $\phi_{(0)}^I$ parametrizing these boundary conditions at conformal infinity are identified with sources that couple to operators O^I of the dual CFT. The defining relation of the AdS/CFT correspondence is that the on-shell action, $S_{onshell}[\phi_{(0)}]$, is the generating functional of CFT correlation functions:

$$\langle O \rangle \sim \frac{\delta S_{onshell}[\phi_{(0)}]}{\delta \phi_{(0)}}, \quad \langle O(x)O(y) \rangle \sim \frac{\delta^2 S_{onshell}[\phi_{(0)}]}{\delta \phi_{(0)}(x)\delta \phi_{(0)}(y)}, \tag{2}$$

etc. These identifications lead to new intuition about the boundary conditions. In quantum field theory the sources that couple to operators are unconstrained, because one functionally differentiates with respect to them. This implies that one should be able to formulate the bulk/boundary problem by specifying arbitrary functions/tensors as boundary conditions for bulk fields.

Let us consider the case where the bulk field of interest is the metric. In the physics literature, prior to the AdS/CFT correspondence, there were a number of works discussing Asymptotically AdS spacetimes, for example [17, 18, 19]. In these works the metric approaches that of AdS at conformal infinity; the spacetimes are asymptotically AdS.

For AdS/CFT however such boundary conditions do not suffice: one needs more general boundary conditions. In particular, the boundary conditions must be parametrized by an unconstrained metric, since this metric should act as a source for the energy momentum tensor T_{ij} of the dual CFT. Fortunately this more general set-up has been developed in the mathematics literature [20]. The corresponding spacetimes are called Asymptotically locally AdS spacetimes (AlAdS).

An AlAdS spacetime always admits the following metric in a finite neighborhood of the conformal boundary, located at $r = 0$:

$$ds^2 = \frac{dr^2}{r^2} + \frac{1}{r^2}g_{ij}(x,r)dx^i dx^j \tag{3}$$

where

$$\lim_{r \to 0} g_{ij}(x,r) = g_{(0)ij}(x) \tag{4}$$

is an arbitrary non-degenerate metric. The coordinates used here are Gaussian normal coordinates centered at the conformal boundary.

Let us emphasize that the only requirement put on $g_{ij}(x,r)$ a priori is that it should have a non-degenerate limit as $r \to 0$. The precise form of $g_{ij}(x,r)$ is determined by solving the bulk field equations asymptotically. This problem reduces to solving algebraic equations, so the **most general** asymptotic solution can be readily found for any given bulk theory that admits AlAdS solutions.

For Einstein gravity in $(d+1)$ dimensions, the relevant expansion is [20, 21]:

$$g_{ij}(x,r) = g_{(0)ij}(x) + r^2 g_{(2)ij} + \cdots + r^d(g_{(d)ij} + h_{(d)ij}\log(r^2)) + \cdots \tag{5}$$

Here the coefficients $(g_{(2)ij}, \cdots, h_{(d)ij})$ are locally determined in terms of $g_{(0)}$. $g_{(d)ij}$ is only partially determined by asymptotics: this coefficient is related via AdS/CFT to the 1-point function of T_{ij} and thus to bulk conserved charges. The trace and divergence of $g_{(d)ij}$ are determined and relate to dilatation and diffeomorphism Ward identities. The logarithmic coefficient $h_{(d)}$ is non-zero when d is even and is greater than two, and it is related to the Weyl anomaly of the boundary theory [22]

$$h_{(d)} \sim \frac{\delta}{\delta g_{(0)}} \int (conformal\ anomaly). \tag{6}$$

Note that in the specific case of pure Einstein gravity in three bulk dimensions

$$g_{ij}(x,r) = g_{(0)ij}(x) + r^2 g_{(2)ij} + \cdots \tag{7}$$

In this case, $h_{(2)}$ actually vanishes because the integral of the conformal anomaly is a topological quantity (the Euler number).

The precise form of this expansion is specific to Einstein gravity. Coupling to matter changes the coefficients. For example, coupling Einstein gravity to a free massless scalar induces a logarithmic term in the expansion, i.e. $h_{(2)} \neq 0$ in this theory [22]. There is also an example of 3d gravity coupled to scalars with $\log^2$ terms in the asymptotic expansion, see [23], appendix E. Even the power of the leading order correction can change, for example it can be r rather than r^2 [24].

Note that the Brown-Henneaux boundary conditions are as in (7) with the additional restriction $g_{(0)ij}(x) = \delta_{ij}$ (in the Euclidean), i.e. the metric is asymptotically AdS_3. Moreover, these boundary conditions are often quoted as:

$$g_{ij}(x,r) = \delta_{ij} + \mathcal{O}(r^2), \tag{8}$$

i.e. it is assumed that the fall-off of the subleading terms is polynomial rather than logarithmic. It is important to emphasize that in the AdS/CFT correspondence such boundary conditions are not sufficiently general. For example, the Brown-Henneaux boundary conditions are violated whenever one wishes to consider the CFT in a non-trivial background, or when one wishes to compute correlation functions of the stress energy tensor. Logarithmic terms generically arise in the expansion of the subleading terms and are related to matter and gravitational conformal anomalies.

Conserved charges

This is an another area where the AdS/CFT duality provides a new and systematic approach [25]. In quantum field theory the energy is computed using the energy momentum tensor,

$$E = \langle H \rangle = \int d^{d-1}x \langle T_{00} \rangle \tag{9}$$

Generically this expression needs renormalization due to UV infinities.

In the AdS/CFT correspondence

$$\langle T_{ij} \rangle = \frac{\delta S_{onshell}[g_{(0)}]}{\delta g_{(0)}^{ij}} \tag{10}$$

This expression is also formally infinite, due to the infinite volume of spacetime (IR divergences) and needs *holographic renormalization* [26].

One can holographically renormalize the theory by adding local boundary covariant counterterms [21, 27, 28] and thus obtain a finite 1-point function for T_{ij} for a general AlAdS spacetime [22]

$$\langle T_{ij} \rangle \sim g_{(d)ij} + X_{ij}[g_{(0)}] \tag{11}$$

with $X_{ij}[g_{(0)}]$ a known local function of $g_{(0)}$. One can furthermore prove rigorously from first principles (e.g. using Noether's method or Wald's covariant phase space methods) that the holographic charges are the correct gravitational conserved charges [29]. Note that the proofs given in [29] apply equally well to cases where there are logarithmic terms in the asymptotic expansions.

Summary

In summary, the holographic methodology that replaces previous approaches is:

1. Derive the most general solution of the bulk equations with general Dirichlet boundary conditions for all fields.
2. General results guarantee that the conserved charges are well-defined and can be obtained from the holographic 1-point functions.

The holographic framework allows one to go further and obtain new information by computing two and higher point functions.

APPLICATION TO TMG

Topologically massive gravity is obtained by adding to 3d Einstein gravity the gravitational Chern-Simons term, see equation (1). The equations of motion are:

$$R_{\kappa\lambda} + 2g_{\kappa\lambda} + \frac{1}{\mu}\varepsilon_\kappa{}^{\rho\sigma}\nabla_\rho R_{\sigma\lambda} + \kappa \leftrightarrow \lambda = 0, \tag{12}$$

and these admit asymptotically *AdS* solutions, for example the BTZ black hole, as well as perturbative massive modes when one expands around *AdS*. When $\mu \neq 1$ however, the massive modes have negative energy and the theory is known to be unstable.

In exploring holography for TMG Strominger et al [3] claimed that the dual 2d CFT is chiral at $\mu = 1$ in the following sense:

1. There are no left moving modes in the bulk satisfying Brown-Henneaux boundary conditions.
2. The left moving central charge c_L of the CFT is zero at $\mu = 1$.
3. There are no negative energy modes and the theory is therefore stable at $\mu = 1$.

A holographic correspondence between TMG and a chiral CFT was proposed.

However, the non-chiral mode of topologically massive gravity found in [7] has the asymptotic form

$$g_{ij}(x,r) = \delta_{ij} + r^2(g_{(2)ij} + \log(r^2)h_{(2)ij}) + \cdots \tag{13}$$

which differs from the Brown-Henneaux boundary conditions because of the $h_{(2)}$ logarithmic term. A discussion followed as to whether such boundary conditions could be consistent and subsequently it was proven by [30] that conserved charges are indeed finite with such boundary conditions. As mentioned above, the fact that the charges are finite is unsurprising since the general proof given in [29], although strictly speaking not applicable for TMG, encompasses cases with logarithmic fall-off behaviors.

From the perspective of AdS/CFT:

1. a subleading log is not surprising, as the subleading coefficients *routinely change and involve logs* as one changes the bulk action;
2. the form of the asymptotic expansion should not be fixed by hand but should rather be derived by solving the bulk equations asymptotically.

We will return to the most general asymptotic solution of TMG shortly.

HOLOGRAPHY FOR TMG AND LCFT

We now move to apply holographic methodology to the topologically massive gravity. Let us first consider the theory at $\mu = 1$. There is an important new element compared to earlier holographic literature: the field equations are third order in derivatives, so there are two independent boundary data: one can fix the metric and a certain component of the extrinsic curvature. The boundary metric $g_{(0)ij}$ is the source for the energy momentum tensor T_{ij}. The boundary field $b_{(0)ij}$ parametrizing the boundary behavior of the extrinsic curvature is a source for a new operator t_{ij}.

We need one further ingredient. It turns out that t_{ij} is obtained as a limit of an irrelevant operator. In CFT, when one couples an irrelevant operator, this generates severe UV divergences and the theory is not conformal in the UV. In gravity, a source for an irrelevant operator introduces severe IR divergences and the solution is not asymptotically AdS [22]. In both cases, one bypasses the problems by treating the source perturbatively and thus we will work to first order in $b_{(0)}$, which suffices for

the computation of correlation functions that involve at most two insertions of t_{ij}. In particular, we can compute all 2-point functions.

The most general asymptotically locally AdS solution (i.e. with non-degenerate conformal boundary) of the TMG equations of motion, with terms linear in the source $b_{(0)}$ for the irrelevant operator also included, is then:

$$ds^2 = \frac{dr^2}{r^2} + \frac{1}{r^2} g_{ij}(x,r) dx^i dx^j \tag{14}$$

with

$$g_{ij}(x,r) = b_{(0)ij} \log r^2 + g_{(0)ij} + r^2 (g_{(2)ij} + b_{(2)ij} \log r^2) + \cdots \tag{15}$$

Only $b_{(0)\bar{z}\bar{z}}$ is non-zero and is the source for the new operator t_{zz}. The subleading coefficients $g_{(2)}$ and $b_{(2)}$ are constrained partially by the asymptotic analysis, with the constraints as usual relating to Ward identities.

The (finite) holographic 1-point functions can be computed in complete generality:

$$\langle T_{ij} \rangle = \frac{1}{4G_N} \Big(g_{(2)ij} + \frac{1}{2} R[g_{(0)}] g_{(0)ij} \tag{16}$$

$$- \frac{1}{2} \Big(\varepsilon_i{}^k g_{(2)kj} + (i \leftrightarrow j) \Big) - 2b_{(2)ij} + \frac{1}{2} A_{ij}[g_{(0)ij}] \Big)$$

$$\langle t_{zz} \rangle = \frac{1}{2G_N} (g_{(2)zz} + b_{(2)zz})$$

T_{ij} satisfies the expected anomalous CFT Ward identities:

$$\langle T_i^i \rangle = \frac{1}{4G_N} \Big(\frac{1}{2} R[g_{(0)}] + \frac{1}{2} A_i^i[g_{(0)}] \Big) \tag{17}$$

$$\nabla^j \langle T_{ij} \rangle = \frac{1}{4G_N} \Big(\frac{1}{4} \varepsilon_{ij} \nabla^j R[g_{(0)}] + \frac{1}{2} \nabla^j A_{ij}[g_{(0)}] \Big)$$

The right hand side of the second equation contains the expected consistent (non-covariant) diffeomorphism anomaly. The improved energy momentum tensor, $\hat{T}_{ij} = T_{ij} - \frac{1}{8G_N} A_{ij}$ has instead a covariant diffeomorphism anomaly [31]. From the trace Ward identity one can extract the sum of left and right central changes, $c_L + c_R$.

The energy momentum tensor T_{ij} can be used to obtain the conserved charges. For example one can compute the conserved charges for the BTZ black hole:

$$ds^2 = \frac{dr^2}{r^2} - \left[\frac{1}{r^2} - \frac{1}{2}(r_+^2 + r_-^2) + \frac{1}{4}(r_+^2 - r_-^2)^2 r^2 \right] dt^2 \tag{18}$$

$$+ \left[\frac{1}{r^2} + \frac{1}{2}(r_+^2 + r_-^2) + \frac{1}{4}(r_+^2 - r_-^2)^2 r^2 \right] d\phi^2 + 2r_+ r_- dt d\phi.$$

The stress energy tensor becomes chiral at $\mu = 1$,

$$T_{\bar{z}\bar{z}} = \frac{2}{G_N}(r_+ + r_-)^2, \quad T_{zz} = 0 \tag{19}$$

and the conserved charges are

$$M = -\int d\phi\, T_t^t = \frac{\pi}{4G_N}(r_+ + r_-)^2$$

$$J = -\int d\phi\, T_\phi^t = M \tag{20}$$

Note that $J = M$ even away from extremality, i.e. for $r_+ \neq |r_-|$.

Given the general expressions for the 1-point functions, we can use the general solution of the linearized equations of motion about AdS to extract the following non-zero 2-point functions:

$$\langle t_{zz}(z,\bar{z}) t_{zz}(0) \rangle = \frac{(3/G_N) \log |z|^2}{z^4}, \tag{21}$$

$$\langle t_{zz}(z,\bar{z}) T_{zz}(0) \rangle = \frac{(-3/G_N)}{2z^4},$$

$$\langle T_{\bar{z}\bar{z}}(z,\bar{z}) T_{\bar{z}\bar{z}}(0) \rangle = \frac{(3/G_N)}{2\bar{z}^4},$$

These are precisely the non-zero 2-point functions of a Logarithmic CFT with central charges:

$$c_L = 0, \qquad c_R = \frac{3}{G_N}. \tag{22}$$

In the left moving sector the operators (t_{zz}, T_{zz}) form a logarithmic pair with non-diagonalizable two point functions and "new anomaly" parameter

$$b = -\frac{3}{G_N} \tag{23}$$

such that

$$\langle t_{zz}(z,\bar{z}) T_{zz}(0) \rangle = \frac{b}{2z^4}, \tag{24}$$

characterizing the LCFT.

We also analyzed the theory in the neighborhood of $\mu = 1$. Letting $\mu = 2\lambda + 1$, near $\lambda = 0$, the general solution to the linearized equations of motion is expanded near the boundary as

$$h_{ij} = h_{(-2\lambda)ij} r^{-2\lambda} + h_{(0)ij} + h_{(2)ij} r^2 + \ldots, \tag{25}$$

where $h_{(0)ij}$ is the usual source for the energy-momentum tensor and $h_{(-2\lambda)ij}$ is traceless and chiral and acts as a source for an irrelevant operator X_{ij}.

The nonvanishing two-point functions are:

$$\langle T_{\bar{z}\bar{z}}(z,\bar{z}) T_{\bar{z}\bar{z}}(0) \rangle = \frac{3}{2G_N} \frac{\lambda+1}{2\lambda+1} \frac{1}{\bar{z}^4}$$

$$\langle T_{zz}(z,\bar{z}) T_{zz}(0) \rangle = \frac{3}{2G_N} \frac{\lambda}{2\lambda+1} \frac{1}{z^4}$$

$$\langle X_{zz}(z,\bar{z}) X_{zz}(0) \rangle = -\frac{1}{2G_N} \frac{\lambda(\lambda+1)(2\lambda+3)}{2\lambda+1} \frac{1}{z^{2\lambda+4}\bar{z}^{2\lambda}}.$$

From these expressions we see that

$$(c_L, c_R) = \frac{3}{2G_N}\left(1 - \frac{1}{\mu}, 1 + \frac{1}{\mu}\right) \tag{26}$$

whilst X has weights $(h_L, h_R) = (2 + \lambda, \lambda)$.

These correlation functions smoothly reduce to those at $\mu \to 1$; the operator t_{zz} is given by

$$t_{zz} = -\frac{1}{\lambda}(X_{zz} - T_{zz}) \tag{27}$$

and we recover the value of b given previously. In fact, our discussion mirrors the degeneration of a CFT to a logarithmic CFT as $c \to 0$ discussed by [32]. As here their logarithmic partner of the stress energy tensor originates from another primary whose dimension approaches $(2, 0)$ in the $c \to 0$ limit. There are other ways to take a $c \to 0$ limit (avoiding "catastrophe", and demanding that the OPE remains well defined), but it is this approach which is realized holographically.

From the form of the 2-point functions one finds that the CFT contains a state $|X\rangle$ of negative norm and $\langle X|H|X\rangle < 0$ in that state. This is the counterpart of the bulk instability due to negative energy of massive gravitons.

CONCLUSIONS

Topologically massive gravity at $\mu = 1$ is dual to a logarithmic CFT and therefore it is not unitary. Away from the "chiral point" the theory contains states of negative norm. One may try to restrict to the right-moving sector of the theory, which could yield a consistent chiral subsector. Arguments for such a truncation at the classical level were given in [33]. From the current perspective a necessary requirement for such a truncation would be that the logarithmic operator t is not generated in the OPE of the right-moving operators; in particular, the three point function $\langle t\bar{T}\bar{T}\rangle$ must vanish. This indeed holds for certain LCFTs, in particular for those discussed in [32], and it would be interesting to compute this 3-point function holographically for TMG at $\mu = 1$.

We should emphasize however that the existence of such a truncation only shows that a set of operators of the LCFT (in this case the right moving stress energy tensor) form a closed subsector, not that this subsector has a dual of its own. To give an example in a more familiar setting let us consider $N = 4$ SYM in four dimensions and the dual string theory on $AdS_5 \times S^5$. There are a number of consistent truncations of the bulk theory. For example, it is generally believed (and it has been proven for certain subsectors) that the maximally supersymmetric $SO(6)$ gauged supergravity in five dimensions is a consistent truncation of type IIB supergravity on S^5. The existence of this consistent truncation however does not imply that there is a new duality: the dual theory is always $N = 4$ SYM and the consistent truncation only implies that certain operators (those in the stress energy supermultiplet in this example) are closed under OPE's in the large N limit.

Finally, the following argument suggests that difficulties are generic in formulating a duality between a unitary CFT and a bulk theory that only involves three dimensional

gravity, such as TMG, instead of a string theory that at low energies reduces to the gravitational theory. In the AdS/CFT correspondence we expect to have a bulk field for every boundary gauge invariant operator. The existence of black holes in these theories implies that the dual theory has a very large number of operators to account for the entropy of the black hole. For each of those operators the bulk theory should have a corresponding bulk field. Pure gravity however only contains the metric so it can only describe the stress energy tensor holographically. If we allow for higher derivative terms, as in the case of TMG, which are treated exactly (rather than perturbatively, as they would be in a string theory set up) one can incorporate a few more gauge invariant operators but then the theory generically becomes non-unitary. In all cases the bulk description is missing the fields that would provide the sources for the operators dual to the black hole microstates. Instead, in a string theory set up these operators would be dual to corresponding string states.

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REFERENCES

1. Deser, S., Jackiw, R., and Templeton, S., *Ann. Phys.*, **140**, 372–411 (1982).
2. Deser, S., Jackiw, R., and Templeton, S., *Phys. Rev. Lett.*, **48**, 975–978 (1982).
3. Li, W., Song, W., and Strominger, A., *JHEP*, **04**, 082 (2008).
4. Ayon-Beato, E., and Hassaine, M., *Ann. Phys.*, **317**, 175–181 (2005).
5. Ayon-Beato, E., and Hassaine, M., *Phys. Rev.*, **D73**, 104001 (2006).
6. Carlip, S., Deser, S., Waldron, A., and Wise, D. K., *Class. Quant. Grav.*, **26**, 075008 (2009).
7. Grumiller, D., and Johansson, N., *JHEP*, **07**, 134 (2008).
8. Park, M.-i., *JHEP*, **09**, 084 (2008).
9. Grumiller, D., Jackiw, R., and Johansson, N. (2008).
10. Carlip, S., Deser, S., Waldron, A., and Wise, D. K., *Phys. Lett.*, **B666**, 272–276 (2008).
11. Carlip, S., *JHEP*, **10**, 078 (2008).
12. Giribet, G., Kleban, M., and Porrati, M., *JHEP*, **10**, 045 (2008).
13. Blagojevic, M., and Cvetkovic, B. (2008).
14. Brown, J. D., and Henneaux, M., *Commun. Math. Phys.*, **104**, 207–226 (1986).
15. Regge, T., and Teitelboim, C., *Ann. Phys.*, **88**, 286 (1974).
16. Skenderis, K., Taylor, M., and van Rees, B. C., *JHEP*, **09**, 045 (2009).
17. Abbott, L. F., and Deser, S., *Nucl. Phys.*, **B195**, 76 (1982).
18. Ashtekar, A., and Magnon, A., *Class. Quant. Grav.*, **1**, L39–L44 (1984).
19. Henneaux, M., and Teitelboim, C., *Commun. Math. Phys.*, **98**, 391–424 (1985).
20. Fefferman, C., and Graham, C., *Elie Cartan et les Mathématiques d'aujourd'hui (Asterisque 95)* (1985).
21. Henningson, M., and Skenderis, K., *JHEP*, **07**, 023 (1998).
22. de Haro, S., Solodukhin, S. N., and Skenderis, K., *Commun. Math. Phys.*, **217**, 595–622 (2001).
23. Kanitscheider, I., Skenderis, K., and Taylor, M., *JHEP*, **04**, 023 (2007).
24. Berg, M., and Samtleben, H., *JHEP*, **05**, 006 (2002).

25. Skenderis, K., *Int. J. Mod. Phys.*, **A16**, 740–749 (2001).
26. Skenderis, K., *Class. Quant. Grav.*, **19**, 5849–5876 (2002).
27. Henningson, M., and Skenderis, K., *Fortsch. Phys.*, **48**, 125–128 (2000).
28. Balasubramanian, V., and Kraus, P., *Commun. Math. Phys.*, **208**, 413–428 (1999).
29. Papadimitriou, I., and Skenderis, K., *JHEP*, **08**, 004 (2005).
30. Henneaux, M., Martinez, C., and Troncoso, R. (2009).
31. Bardeen, W. A., and Zumino, B., *Nucl. Phys.*, **B244**, 421 (1984).
32. Kogan, I. I., and Nichols, A. (2002).
33. Maloney, A., Song, W., and Strominger, A. (2009).