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Consequences for Welfare and Pension Buffers under Alternative Methods of Discounting Future Pensions[*]

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Abstract

We explore numerically the behaviour of pension fund buffers, as well as the inter- and intra-generational welfare under different methods of discounting future funded pensions. Our analysis is based on an applied many-generation OLG model describing a small open economy with heterogeneous agents featuring a two-pillar pension system (with PAYG and funded tiers). We compare mark-to-market discounting against various alternatives, such as discounting against a moving average of past market curves or a constant curve. The pension buffer is kept stable by adjusting indexation and contribution rates in response to demographic, economic and financial shocks in the economy. The higher volatility of pension buffers when discounting against the market curve generates adjustments in the policy parameters that work out relatively favourably for the young generations. In particular, a reduction in the indexation of benefits early in life tends to be compensated by restoration indexation based on a larger stock of accumulated pension rights. A switch from mark-to-market discounting to alternative methods may raise aggregate welfare, although the overall improvement is small, because the switch primarily amounts to a shift in welfare from the younger to the older generations.

Keywords: funded Social Security, swap curve, indexation, pension buffer, stochastic simulations.

JEL codes: H55, I38, C61

1 Introduction

In view of the looming ageing problem and the costs of maintaining pay-as-you-go (PAYG) systems, many countries are enhancing the role of pension funding. One of the major complications in the design of a funded pension system is the measurement of the pension liabilities. The methodology is typically set by regulation, but there is a weak conceptual basis in support of any approach. The problem has two dimensions. On the one hand, cash flows may be estimated from accrued

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pension rights or from pension rights projected forward using expected wage increases. On the other hand, cash flows should be discounted using rates reflecting the systematic risk of liabilities. Unfortunately, there are no market instruments replicating such risk perfectly (De Jong, 2008). Practitioners then use fixed ad hoc rates or market-related rates generically accounting for aggregate risk. In this paper we concentrate on the second dimension of the problem (the discounting method) and assume that cash flows are estimated from the existing pension rights. Assessing the most appropriate discounting method is important to properly assess obligations (Geanakoplos and Zeldes, 2009) and the portfolio allocation of the fund's assets (Lucas and Zeldes, 2009). Further, it has a large impact on the volatility of the so-called funding ratio (the ratio of assets over liabilities).

Our study is based on simulations of a model that incorporates a pension system like that in the Netherlands, which is one of the few countries with a traditionally large role for defined benefit (DB) pension funds. Our results may also be of interest for other countries, such as the US, where most public pension funds are currently of DB type (Munnell et al., 2008). Until some years ago, future pension benefits in the Netherlands used to be discounted against a fixed interest rate. However, after the introduction of the new Pension Law and its supervisory framework ("Financieel Toetsingskader" – FTK) pension funds are obliged to discount those expected future outlays using the term structure determined by the market on a continuous basis. The particular term structure that is used is the swap curve. The motivation of this shift in regulation is to provide a closer link between the risk of the liabilities and the volatility of the discount rate. As emphasized by Brown and Wilcox (2009), DB pensions offer retirees a safe stream of income in the sense that there are strong contractual and legal protections against default on promised benefits. However, fund participants share with fund managers considerable risk arising from uncertainty about future salaries, demographics and returns from the financial market. Hence, a market-related measure should reflect this background uncertainty.

The recent crisis has clearly shown the complications created by this way of measuring pension fund liabilities. Asset markets have lost a lot of their value thereby also reducing the value of the investments of the pension funds, while interest rates, and the swap curve in particular, recently reached unprecedentedly low levels, thereby boosting the liabilities as the discounting of future pension outlays takes place at a lower rate. When the funding ratio falls below 125%, the pension fund will have to devise a long-term restoration plan, while in the case of a fall below 105% ("underfunding") it has to submit a short-run restoration plan to the supervisor (the Dutch Central Bank – DNB). The plan should present a realistic set of measures to escape the state of underfunding in three or five years. That is the situation a large number of Dutch pension funds find themselves now in. Potential measures include the abolition or reduction of indexation of the pensions (future pension rights are defined in nominal amounts), increases in employers' and workers' contributions and, in the worst case, a reduction in pension rights. However, there is much evidence to support the hypothesis that the current picture of the pension funds' situation is too gloomy and, hence, that the drastic measures required to restore the funding ratio are too harsh. Asset values seem exceptionally low due to the limited market liquidity and the sharp increase in risk aversion, while interest rates seem exceptionally low due to the generic flight to quality that has taken place recently. Moreover, the swap yield, which normally exceeds the yield on high-quality public debt, has recently been below this yield, possibly caused by the fact that pension funds, in their attempts to hedge against a further fall in the interest rate, buy fixed streams of interest income while selling a variable stream of interest income. This drives down the swap curve.

Hence, the danger with the mark-to-market valuation approach is that movements in asset prices and interest rates that are not driven by fundamentals and thus likely to reverse in the future lead to costly, but unnecessary, policy adjustments by pension funds that merely increase

the uncertainty about future pensions and that may hurt specific groups that are unable to respond to reductions in their expected pensions (for example, by working longer). Ideally, the way future pension benefits are discounted should only change in response to structural movements in the yield curve.

In this paper we explore the behaviour of pension buffers, as well as inter-generational and intra-generational welfare, for different ways of discounting the future pension expenditures. The analysis is based on stochastic simulations using a realistic calibration of the Dutch pension system and a full set of demographic, financial and economic shocks with distributions obtained through the estimation of a VAR system for the US and a model for the US swap curve. We compare mark-to-market discounting against various alternatives. The analysis yields some useful insights. As expected, the volatility of the pension buffers is highest under discounting against the market swap curve and lowest under discounting against the average swap curve or against a constant and flat discount term structure. A high volatility of the funding ratio implies more frequent adjustment of the policy parameters (in particular, indexation rates and, if necessary, contribution rates). The first adjustment typically involves a reduction of indexation; this situation is seen more favourably by the younger generations, as missed indexation is expected to be restored over the long run, when pension rights have become larger and social security accounts for most of an individual's resources. In contrast, older generations prefer alternative methods of discounting as they provide more stable social security benefits. Aggregate welfare is only marginally affected by the way in which future pension benefits are discounted. However, there is a large majority in favour of any method alternative to mark-to-market discounting against the market curve.

The remainder of the paper is structured as follows. Section 2 presents the model, of which the calibration is discussed in 3. Section 4 discusses the baseline results, as well as the results of some variations on the baseline. Section 5 concludes the main body of the paper. Finally, the appendix in Section 6 provides further details on the pension fund's policy and the estimation of the underlying models used in the stochastic simulation.

2   The model

A period in our model corresponds to one year. There are a number of D cohorts alive in any given period t. Each cohort j ($= 1, ..., D$) consists of $N_{j,t}$ individuals at time t, who are distributed in I equally-sized skill groups, $i = 1, ..., I$. A higher value of i denotes a higher skill level. The skill level of a person determines his income, given his age and the macroeconomic circumstances. All individuals within a given skill group earn the same income. Index $j = 1, ..., D$ indicates the age of the cohort, computed as the amount of time since entry into the labour force.

2.1   Cohorts and demography

We assume that each individual born in period $t - j + 1$ (that is, the person has age zero at the start of $t - j + 1$ and age one at the end of that period) has an exogenous marginal probability $\psi_{j,t-j+1} \in [0, 1]$ of reaching age j (at the end of period t) conditional on having reached age $j - 1$. For example, $\psi_{j,t-j+1} = 1$ means that an individual alive at age $j-1$ at the end of period $t-1$ will be alive with certainty at age j at the end of period t. Similarly, $\psi_{j,t-j+1} = 0$ implies that anyone alive at age $j - 1$ at the end of period $t - 1$ will surely die before the end of period t. Specifically, we assume that $\psi_{j,t-j+1} = 0$ for any $j \geq D + 1$. To be precise, individuals can die only at the start of a period, so that the survival of that moment implies that the person reaches the end of the period and receives an income and consumes during that period. We further assume that the

cohort of newborn agents in period t is $1 + n_t$ times larger than the cohort of newborn agents in period $t - 1$:

$$N_{1,t} = (1 + n_t) N_{1,t-1}. \tag{1}$$

In general, we denote with $N_{j,t}$ the size of cohort j at time t. This size depends on the history of past survival probabilities. Indeed, for $j = 2, ..., D$:

$$N_{j,t} = N_{j-1,t-1} \psi_{j,t-j+1}. \tag{}$$

2.2 Individuals

Individuals in the same cohort can only differ in terms of their income. Each individual in a given cohort belongs to some skill group i, with $i = 1, ..., I$. We assume that individuals remain in the same skill group over their entire life. Individuals work until the exogenous retirement age R and live for at most D years. During their working life $(j = 1, ..., R)$, they receive a labour income $y_{i,j,t}$ defined as follows:

$$y_{i,j,t} = e_i s_j z_t, \tag{2}$$

where e_i, $i = 1, ..., I$ is an efficiency index (linked to the skill level of class i), s_j, $j = 1, ..., R$, a seniority index (for given skill level income varies with age) and z_t is an exogenous income process:

$$z_t = (1 + g_t) z_{t-1}, \tag{3}$$

where g_t is the exogenous *nominal* growth rate of the process and $z_0 = 1$.

Average income across workers is defined as:

$$y_t = \frac{\sum_{j=1}^{R} \frac{N_{j,t}}{I} \sum_{i=1}^{I} y_{i,j,t}}{\sum_{j=1}^{R} N_{j,t}}. \tag{4}$$

If all workers have identical productivity (i.e. $e_1 = ... = e_I = s_1 = ... = s_R = 1$), then $y_t = z_t$. We make a distinction between y_t and z_t because the relative sizes of the cohorts may change over time, implying that the ratio y_t/z_t will fluctuate over time.

2.3 Social security and accidental bequests

Social security is based on a two-pillar system. The first pillar is a pay-as-you-go (PAYG) defined benefit (DB) program which pays a flat benefit to every retiree. It is organised by the government, which sets the contribution rate to ensure that the first pillar is balanced on a period-by-period basis. The second pillar is funded and may either be organised by the government or by the private sector. In reality, in the Netherlands some of the parameters of the second pillar are set by the government, while other parameters are set by the pension fund itself. Since we do not explicitly model the objectives of the different policymakers we do not need to make specific assumptions about who sets which parameters. Finally, the government redistributes the accidental bequests left by those who die.

2.3.1 The first pillar of the social security system

Each period, an individual of working age pays a mandatory contribution $p_{i,j,t}^F$ to the first pillar of the social security system. This contribution depends on the size of income $y_{i,j,t}$ relative to the thresholds $\delta^l y_t$ and $\delta^u y_t$:

$$
p_{i,j,t}^F = \left\{
\begin{array}{ll}
0 & \text{if } y_{i,j,t} < \delta^l y_t \\
\theta_t^F \left(y_{i,j,t} - \delta^l y_t \right) & \text{if } y_{i,j,t} \in \left[\delta^l y_t, \delta^u y_t \right] \\
\theta_t^F \left(\delta^u y_t - \delta^l y_t \right) & \text{if } y_{i,j,t} > \delta^u y_t
\end{array}
\right\}, \quad j \leq R,
\tag{5}
$$

where δ^l, δ^u and θ_t^F are policy parameters. In period t the benefit received by an individual retiree is a fraction ρ^F of the average income in the economy:

$$
b_t^F = \rho^F y_t.
\tag{6}
$$

Given the benefit formula in equation (6), each period the contribution rate θ_t^F adjusts such that aggregate contributions into the first pillar P_t^F equal aggregate first-pillar benefits B_t^F paid out to the retired:

$$
P_t^F = B_t^F,
\tag{7}
$$

where

$$
P_t^F = \sum_{j=1}^{R} \frac{N_{j,t}}{I} \sum_{i=1}^{I} p_{i,j,t}^F,
$$

and

$$
B_t^F = \sum_{j=R+1}^{D} \frac{N_{j,t}}{I} \sum_{i=1}^{I} b_t^F = b_t^F \sum_{j=R+1}^{D} N_{j,t}.
$$

2.3.2 The second pillar of the social security system

The second pillar consists of a DB funded program. Each period, an individual of working age also pays a mandatory contribution $p_{i,j,t}^S$ to this second pillar if his income exceeds the franchise income level. Specifically, $p_{i,j,t}^S$ is a fraction of the labour income in excess of the franchise:

$$
p_{i,j,t}^S = \theta_t^S \max \left\{ 0, y_{i,j,t} - \lambda y_t \right\}, \quad j \leq R,
\tag{8}
$$

where θ_t^S is a policy parameter.

A cohort entering retirement at age $R + 1$ receives a benefit proportional to its average wage over the years worked. Period t benefits for an individual in skill group i of cohort j are given by:

$$
b_{i,j,t}^S = M_{i,j,t}, \qquad j > R,
\tag{9}
$$

where the accumulated "stock of nominal rights" $M_{i,j,t}$ at the end of period t evolves as:

$$
M_{i,j,t} = \left\{
\begin{array}{ll}
(1-m_t) \left\{
\begin{array}{c}
\left[1 + \iota_t \left(\frac{1+g_t}{1+\pi_t} - 1 \right) \right] (1 + \kappa_t \pi_t) M_{i,j-1,t-1} \\
+ \mu \max \left[0, y_{i,j,t} - \lambda y_t \right]
\end{array}
\right\} & j \leq R \\
(1-m_t) \left[1 + \iota_t \left(\frac{1+g_t}{1+\pi_t} - 1 \right) \right] (1 + \kappa_t \pi_t) M_{i,j-1,t-1} & j \geq R + 1
\end{array}
\right\},
\tag{10}
$$

where the coefficients μ and λ denote the annual accrual rate and franchise, respectively, as shares of average income. The productivity indexation parameter ι_t and the price indexation parameter κ_t

capture the degree of indexation of nominal rights to real income growth, $\frac{1+g_t}{1+\pi_t} - 1$, and inflation, π_t, respectively. Indexation aims at following nominal wage growth. However, actual indexation may depend on the financial position of the pension fund, as explained in more detail below. Further, m_t captures a proportional reduction in nominal rights that may be applied when the pension buffer is so low that restoration using standard instruments is no longer possible (see below). We assume that $m_t > 0$ only when $\iota_t = \kappa_t = 0$. That is, nominal rights are only reduced when the reduction in indexation has been "exhausted". Each individual enters the labour market with zero nominal claims. Hence, $M_{i,0,t-j} = 0$, where $M_{i,0,t-j}$ are nominal claims at the end of period $t - j$ or beginning of period $t - j + 1$ when the generation enters the labour market at age 0.

For a given accrual rate μ and franchise λ, each period the pension fund chooses the contribution rate θ_t^S and the indexation parameters $\{\iota_t, \kappa_t\}$ in the benefit formula in equations (9)-(10). The choice of these policy parameters will depend on the level of the nominal funding ratio F_t, which is the ratio between the pension fund's assets, A_t, and its liabilities, L_t:

$$F_t = \frac{A_t}{L_t}. \tag{11}$$

At the end of period t the pension fund's assets are the sum of the second-pillar contributions from workers in period t *minus* the second-pillar benefits paid to the retirees in period t *plus* the pension fund's assets at the end of period $t - 1$ grossed up by their return in the financial markets:

$$A_t = \left(\sum_{j=1}^{R} \frac{N_{j,t}}{I} \sum_{i=1}^{I} p_{i,j,t}^S - \sum_{j=R+1}^{D} \frac{N_{j,t}}{I} \sum_{i=1}^{I} b_{i,j,t}^S \right) + (1 + r_t^g) A_{t-1}, \tag{12}$$

where

$$1 + r_t^g = \left(1 - z^e - z^h\right)\left(1 + r_t^{lb}\right) + z^e \left(1 + r_t^e\right) + z^h \left(1 + r_t^h\right), \tag{13}$$

is the gross nominal rate of return on the pension fund's asset portfolio with a constant share z^e invested in equities, a constant share z^h invested in the housing market and the remainder in long-term bonds. In view of their long-term obligations, pension funds have a preference for investing in long-term debt. Here, we assume that the entire fixed-income part of the pension fund's portfolio consists of 10-year zero coupon bonds. Further, the net returns on the long-term bonds (r_t^{lb}), equity (r_t^e) and housing (r_t^h) are exogenous.

Our assumption that the pension fund always holds 10-year bonds, implies that at the end of each year bonds of 9-year maturity are sold to purchase new 10-year bonds. In more detail, the fund's annual portfolio rebalancing operation works as follows. In year $t - 1$, say, the pension fund buys 10-year zero-coupon bonds for an amount of B_{t-1}. Denoting the return on 10-year bonds by $r_{10,t-1}^b$, the value at maturity of the bonds is

$$P_{t+9} = B_{t-1} \left(1 + r_{10,t-1}^b\right)^{10}, \tag{14}$$

hence, the present value B_{t-1} of the bond holdings in year $t - 1$ is:

$$B_{t-1} = \frac{P_{t+9}}{\left(1 + r_{10,t-1}^b\right)^{10}}.$$

In year t, only 9 years of maturity are left, and the bond return is $r_{9,t}^b$. The present value B_t is then

$$B_t = \frac{P_{t+9}}{\left(1 + r_{9,t}^b\right)^9}.$$

Combining with (14) we obtain the following expression:

$$B_t = B_{t-1} \frac{\left(1 + r_{10,t-1}^b\right)^{10}}{\left(1 + r_{9,t}^b\right)^9} = B_{t-1} \left(1 + r_t^{lb}\right).$$

The fund's liabilities are the sum of the present values of current and future rights already accumulated by the cohorts currently alive:

$$L_t = \sum_{j=1}^{D} \frac{N_{j,t}}{I} \sum_{i=1}^{I} L_{i,j,t}. \tag{15}$$

The expected present value at time t of current and future benefits of a cohort j in skill group i is

$$L_{i,j,t} = \begin{array}{ll} E_t \left[\sum_{l=R+1-j}^{D-j} \frac{1}{\psi_{j,t-j+1}} \left(\prod_{k=0}^{l} \psi_{j+k,t-j+1} \right) \frac{1}{(1+d_{l,t})^l} M_{i,j,t} \right] & j \leq R, \\ E_t \left[\sum_{l=0}^{D-j} \frac{1}{\psi_{j,t-j+1}} \left(\prod_{k=0}^{l} \psi_{j+k,t-j+1} \right) \frac{1}{(1+d_{l,t})^l} M_{i,j,t} \right] & j > R, \end{array} \tag{16}$$

where future benefits are discounted at a rate $\{d_{k,t}\}_{k=1}^{D}$. Note that $\psi_{j,t-j+1}$ cancels out in the above equation. When $j \leq R$, furthermore, we discount all the future benefits to the current year t, but of course they will only be paid out once individuals have retired.

In this paper, we explore the consequences for welfare and pension buffers of different ways of discounting future pension payments. Our benchmark analysis considers four alternatives:

1. Discounting against the market swap curve, $\{d_{k,t}\}_{k=1}^{D} = \left\{r_{k,t}^s\right\}_{k=1}^{D}$, where $\left\{r_{k,t}^s\right\}_{k=1}^{D}$ describes the swap curve at time t, which will be generated through stochastic simulation of a model (described in Section 2.6 below) that we estimate on actual swap curve data.

2. Discounting against a weighted average of past swap curves, $\{d_{k,t}\}_{k=1}^{D} = \left\{r_{k,t}^{ws}\right\}_{k=1}^{D}$, where
$$r_{k,t}^{ws} = \sum_{l=0}^{L-1} w_l r_{k,t-l}^s \text{ and } \sum_{l=0}^{L-1} w_l = 1.$$

3. Discounting against the average swap curve $\{d_{k,t}\}_{k=1}^{D} = \{\overline{r}_k^s\}_{k=1}^{D}$, where $\{\overline{r}_k^s\}_{k=1}^{D}$ is the average swap curve. Hence, discounting of all future liabilities takes place using a set of constant (over time) discount rates.

4. Discounting at some constant and flat rate, $\{d_{k,t}\}_{k=1}^{D} = d$

In the second case, the mark-to-market approach is still followed and so potential structural changes in the market swap curve are still tracked. However, it tries to avoid as much as possible the effects of high-frequency fluctuations in the swap curve that reverse themselves later. Further, note that the final year in the term structures that we consider is D periods from now, so that all future pension payments associated with existing accumulated pension rights can be discounted.

2.3.3 Accidental bequests

Accidental bequests do not have any significant bearing on our results. Their only role is to ensure that resources do not "disappear" because people die. The financial assets left by those who die are all collected by the government. The aggregate of these accidental bequests in the economy amounts to:

$$H_t = \sum_{j=2}^{D} \left(1 - \psi_{j,t-j+1}\right) \frac{N_{j-1,t-1}}{I} \sum_{i=1}^{I} a_{i,j,t} = \sum_{j=2}^{D} \frac{(N_{j-1,t-1} - N_{j,t})}{I} \sum_{i=1}^{I} a_{i,j,t},$$

where $a_{i,j,t}$ are the assets accumulated by each individual in cohort j in skill class i at the end of period $t-1$ and which become available for collection by the government at the start of period t. The government redistributes H_t equally over all individuals alive at time t, resulting in a transfer to each individual of

$$h_t = \frac{H_t}{\sum_{j=1}^{D} N_{j,t}}. \tag{17}$$

2.4 The individual decision problem

In a given period t an individual of skill group i in cohort j chooses a sequence of nominal consumption levels for the rest of her life. Savings are then invested in a portfolio of bond, equity and housing assets. Hence, the individual solves:

$$V_{i,j,t} = \max_{\{c_{i,j+l,t+l}\}_{l=0}^{D-j}} E_t \left[\sum_{l=0}^{D-j} \frac{\beta^l}{\psi_{j,t-j+1}} \left(\prod_{k=0}^{l} \psi_{j+k,t-j+1} \right) u \left(\frac{c_{i,j+l,t+l}}{\prod_{k=0}^{l} (1+\pi_{t+k})} \right) \right],$$

where $u(.)$ is the period utility function, which we assume to be of the conventional CRRA format with coefficient of relative risk aversion $\gamma > 0$,

$$u(x) = \frac{x^{1-\gamma} - 1}{1 - \gamma},$$

subject to equations (1)-(17), and the intertemporal budget constraint

$$a_{i,j+l+1,t+l+1} = \left\{ \begin{array}{ll} \begin{array}{l} (1+r_{t+l+1})(a_{i,j+l,t+l} - c_{i,j+l,t+l}) \\ +y_{i,j+l+1,t+l+1} - p^F_{i,j+l+1,t+l+1} - p^S_{i,j+l+1,t+l+1} + h_{t+l+1} \end{array} & \text{if } j+l < R \\ \begin{array}{l} (1+r_{t+l+1})(a_{i,j+l,t+l} - c_{i,j+l,t+l}) \\ +b^F_{t+l+1} + b^S_{i,j+l+1,t+l+1} + h_{t+l+1} \end{array} & \text{if } j+l \geq R \end{array} \right\},$$

where $a_{i,j+l,t+l}$ are the financial assets in year $t+l$ of an individual in skill group i of cohort $j+l$ and

$$1 + r_{t+l+1} = \left(1 - x^e_{j+l} - x^h_{j+l}\right)\left(1 + r^{sb}_{t+l+1}\right) + x^e_{j+l}\left(1 + r^e_{t+l+1}\right) + x^h_{j+l}\left(1 + r^h_{t+l+1}\right),$$

is the overall return on her asset portfolio in period $t+l+1$, the composition of which is age-specific and characterised by the exogenous weights $\left\{x^e_{j+l}, x^h_{j+l}\right\}$ at the end of period $t+l$. The individual earns returns from the investment in short-maturity bonds r^{sb}_{t+l+1}, equities r^e_{t+l+1}, and housing market r^h_{t+l+1}. Note that in contrast to the pension fund's portfolio, the individual's portfolio does not include holdings in long-maturity bonds. The exclusion of long-term bonds from the individual's portfolio has no consequences for the results. Note that the individual's portfolio varies with his age, but for given age is assumed to be fixed across skill groups. The end of next period's assets equal the gross return on this period's assets minus consumption, plus "net income". For the workers, net income is labour income minus social security contributions plus the accidental bequest, while for the retired net income equals the sum of the social security benefits plus the accidental bequest. Note that only the second-pillar benefit is cohort- and skill-specific.

The Euler equation for the individual is

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$$u'\left(c_{i,j+l,t+l}\right) = \beta\psi_{j+l+1,t-j+1}E_{t+l}\left[\frac{1+r_{t+l+1}}{1+\pi_{t+l+1}}u'\left(\frac{c_{i,j+l+1,t+l+1}}{1+\pi_{t+l+1}}\right)\right]. \tag{18}$$

2.5   Shocks

We assume that there are only aggregate, hence no individual-specific shocks. In our model, eight types of aggregate exogenous shocks hit the economy. Specifically, we consider demographic shocks (to the growth rate of the newborns cohort and to the survival probabilities)**,** inflation rate shocks, nominal income shocks (which, together with the inflation shock, produce a shock to the productivity growth rate) and financial market shocks (to bond returns, equity returns, housing returns and the bond yield curve). All these shocks are collected in the vector $\omega_t = \left[\epsilon_t^n, \epsilon_t^\psi, \epsilon_t^g, \epsilon_t^\pi, \epsilon_t^e, \epsilon_t^h, \epsilon_{1,t}^b, ..., \epsilon_{D,t}^b, \epsilon_{1,t}^s, ..., \epsilon_{D,t}^s\right]$ with elements

- ϵ_t^n: shock to the newborn cohort growth rate, n_t

- ϵ_t^ψ: a vector of shocks to the set of survival probabilities $\left\{\psi_{j,t-j+1}\right\}_{j=1}^D$

- ϵ_t^g: shock to the nominal income growth rate, g_t

- ϵ_t^π: shock to the inflation rate, π_t

- ϵ_t^e: shock to the nominal equity return, r_t^e

- ϵ_t^h: shock to the housing return, r_t^h

- $\epsilon_{k,t}^s, k = 1, ..., D$: shock to the swap return at maturity k, $r_{k,t}^s$.

- $\epsilon_{k,t}^b, k = 1, ..., D$: shock to the bond return at maturity k, $r_{k,t}^b$.

All these shocks affect the size of the funding ratio (equation (11)), whereas only demographic shocks affect the first-pillar pension system (equation (7)). As a consequence, the key parameters of the pension system have to be adjusted to restore the balance in the first pillar and to maintain sustainability of the second pillar.

The demographic shocks are independent of each other and of all other shocks (at all leads and lags). The growth rate n_t of the newborns cohort depends on deterministic and random components:

$$n_t = n + \epsilon_t^n,$$

with n the mean and ϵ_t^n the innovation at time t, which follows an AR(1) process with parameter φ:

$$\epsilon_t^n = \varphi\epsilon_{t-1}^n + \eta_t^n, \quad \eta_t^n {\,}^\sim N\left(0, \sigma_n^2\right).$$

The survival probabilities evolve according to a Lee-Carter model (see Appendix 6.2.2 for details):

$$\ln\left(1 - \psi_{j,t-j+1}\right) = \ln\left(1 - \psi_{j,t-j}\right) + \tau_j\left(\chi + \epsilon_{t-j+1}^\psi\right), \quad \epsilon_{t-j+1}^\psi {\,}^\sim N\left(0, \sigma_\psi^2\right), \quad j = 1, ..., D.$$

with τ_j an age-dependent coefficient, χ a constant growth factor (to describe the historical trend increase in survival probabilities) and ϵ_{t-j+1}^{ψ} an innovation at time $t-j+1$ that follows an i.i.d. process with variance σ_{ψ}^2.

We allow the shocks to the inflation rate, the nominal income growth, the one-year bond return $r_{1,t}^b = r_t^{sb}$, the equity return and the housing return to be correlated with each other and over time. These variables feature the following multivariate annual process:

$$
\begin{pmatrix} \pi_t \\ g_t \\ r_{1,t}^b \\ r_t^e \\ r_t^h \end{pmatrix} = \begin{pmatrix} \pi \\ g \\ r_1^b \\ r^e \\ r^h \end{pmatrix} + \begin{pmatrix} \epsilon_t^\pi \\ \epsilon_t^g \\ \epsilon_{1,t}^b \\ \epsilon_t^e \\ \epsilon_t^h \end{pmatrix},
\tag{19}
$$

with means $\left(\pi, g, r_1^b, r^e, r^h\right)'$, and innovations $\left(\epsilon_t^\pi, \epsilon_t^g, \epsilon_{1,t}^b, \epsilon_t^e, \epsilon_t^h\right)'$ for year t following a VAR(1) process,

$$
\begin{pmatrix} \epsilon_t^\pi \\ \epsilon_t^g \\ \epsilon_{1,t}^b \\ \epsilon_t^e \\ \epsilon_t^h \end{pmatrix} = \mathbf{B} \begin{pmatrix} \epsilon_{t-1}^\pi \\ \epsilon_{t-1}^g \\ \epsilon_{1,t-1}^b \\ \epsilon_{t-1}^e \\ \epsilon_{t-1}^h \end{pmatrix} + \begin{pmatrix} \eta_t^\pi \\ \eta_t^g \\ \eta_{1,t}^b \\ \eta_t^e \\ \eta_t^h \end{pmatrix},
\tag{20}
$$

with

$$
\begin{pmatrix} \eta_t^\pi \\ \eta_t^g \\ \eta_{1,t}^b \\ \eta_t^e \\ \eta_t^h \end{pmatrix} \sim N\left(\mathbf{0}, \Sigma_f\right).
$$

We finally turn to the swap curve $\left\{r_{k,t}^s\right\}_{k=1}^D$ and the bond yield curve $\left\{r_{k,t}^b\right\}_{k=1}^D$. Consistently with the prevailing literature (see, e.g., Evans and Marshall, 1998; Dai and Singleton, 2000) we assume that the swap curve follows the process:

$$
\begin{pmatrix} r_{1,t'}^s \\ r_{2,t'}^s \\ \vdots \\ r_{D,t'}^s \end{pmatrix} = \begin{pmatrix} r_1^s \\ r_2^s \\ \vdots \\ r_D^s \end{pmatrix} + \begin{pmatrix} \epsilon_{1,t'}^s \\ \epsilon_{2,t'}^s \\ \vdots \\ \epsilon_{D,t'}^s \end{pmatrix},
\tag{21}
$$

where t' indicates the *month* and the vector of innovations follows a vector autoregressive distributed lag (VADL) process of order 1,[1]

$$
\begin{pmatrix} \epsilon_{1,t'}^s \\ \epsilon_{2,t'}^s \\ \vdots \\ \epsilon_{D,t'}^s \end{pmatrix} = \mathbf{\Gamma}_0^s + \mathbf{\Gamma}_1^s \begin{pmatrix} \epsilon_{1,t'-1}^s \\ \epsilon_{2,t'-1}^s \\ \vdots \\ \epsilon_{D,t'-1}^s \end{pmatrix} + \mathbf{\Gamma}_2^s \begin{pmatrix} \epsilon_{t'-1}^\pi \\ \epsilon_{t'-1}^g \\ \epsilon_{1,t'-1}^b \\ \epsilon_{t'-1}^e \\ \epsilon_{t'-1}^h \end{pmatrix} + \begin{pmatrix} \eta_{1,t'}^s \\ \eta_{2,t'}^s \\ \vdots \\ \eta_{D,t'}^s \end{pmatrix},
\tag{22}
$$

and

---

[1] The swap curve process is estimated (and simulated) at a monthly, rather than annual, frequency, in order to have enough observations.

$$\begin{pmatrix} \eta^s_{1,t'} \\ \eta^s_{2,t'} \\ \vdots \\ \eta^s_{D,t'} \end{pmatrix} \tilde{\ } N\left(\mathbf{0}, \Sigma_s\right).$$

Each time t', the swap return at maturity k, $r^s_{k,t'}$, is given by the sum of the average value r^s_k and the innovation $\epsilon^s_{k,t'}$ (see equation (21)). The innovation is a linear combination of deterministic and random components (see equation (22)). The deterministic part is a function of several variables at time $t'-1$: the innovations at all maturities k and the innovations in (20) converted to a monthly frequency (under the assumption that they are constant during the year – see Appendix 6.2.4). The random part is given by the shock $\eta^s_{k,t'}$ (allowed to be correlated across maturities).

Since $E\left[r^s_{k,t'}\right] = E\left[r^s_{k,t'-1}\right]$, because of stationarity, the average swap curve is given by the expression

$$\begin{pmatrix} \overline{r}^s_1 \\ \overline{r}^s_2 \\ \vdots \\ \overline{r}^s_D \end{pmatrix} = \begin{pmatrix} r^s_1 \\ r^s_2 \\ \vdots \\ r^s_D \end{pmatrix} + \left(I - \boldsymbol{\Gamma}^s_1\right)^{-1}\boldsymbol{\Gamma}^s_0. \tag{23}$$

The bond yield curve $\left\{r^b_{k,t}\right\}^D_{k=1}$ is constructed analogously. The one-year bond interest rate $r^b_{1,t}$ is already determined via (19). The remaining parts of the curve are modelled analogously to the swap curve:

$$\begin{pmatrix} r^b_{2,t'} \\ r^b_{3,t'} \\ \vdots \\ r^b_{D,t'} \end{pmatrix} = \begin{pmatrix} r^b_2 \\ r^b_3 \\ \vdots \\ r^b_D \end{pmatrix} + \begin{pmatrix} \epsilon^b_{2,t'} \\ \epsilon^b_{3,t'} \\ \vdots \\ \epsilon^b_{D,t'} \end{pmatrix},$$

with $\left(\begin{matrix} \epsilon^b_{2,t'} & \epsilon^b_{3,t'} & ... & \epsilon^b_{D,t'} \end{matrix} \right)'$ following a VADL(1) process similar to (22). Appendix 6.2.5 provides details on the computation of the parameters of the yield curve. Realisations to 9- and 10-year yield returns determine the return on long-term bonds $r^{lb}_t = \frac{\left(1+r^b_{10,t-1}\right)^{10}}{\left(1+r^b_{9,t}\right)^9}$.

The simulations conducted below take place at the annual frequency. Therefore, in those simulations we apply (22) and its analogon for the yield curve twelve times for a given year t, and use the last realisations as our annual swap curve $\left\{r^s_{k,t}\right\}^D_{k=1}$ and annual bond-yield curve $\left\{r^b_{k,t}\right\}^D_{k=1}$. For more details, see Appendices 6.2.4 and 6.2.5.

2.6 Policy intervention

We assume that the government automatically adjusts the contribution rate $\theta^F_t \in [0,1]$ to maintain (7) and thus a balanced first pillar of the social security system. On average, this contribution rate increases over the years along with the ageing of the population.

More policy options are available to affect the funding ratio of the second pillar. Indeed, there are three key parameters: the contribution rate $\theta^S_t \in \left[0, \theta^{S,\max}\right]$, the two indexation parameters $\{\kappa_t \geq 0, \iota_t \geq 0\}$ and, as a last resort, a reduction m_t in the nominal pension rights. Policymakers start with a benchmark parameter combination $\left\{\theta^S, \kappa, \iota\right\}$ and a funding ratio between the boundaries $1+\xi^m$ and $1+\xi^u$ ($\xi^m < \xi^u$). There is a third boundary, $1+\xi^l$ ($\xi^l < \xi^m$), which is considered

the level below which there is "underfunding". Policy adjustments take place as follows (the rule is formally described in Appendix 6.1). When the funding ratio (11) falls below $1+\xi^m$ a long-term restoration plan is started, while when it falls below $1+\xi^l$ a short-term restoration plan is started. When the ratio exceeds $1+\xi^u$, measures are taken to reduce the funding ratio. In effect, policy is aimed at moving the funding ratio back into the interval $[1+\xi^m, 1+\xi^u]$.

More specifically, policy parameter adjustments take place as follows. In the case of a short-term or long-term restoration plan, first productivity indexation $\iota_t \in [0,1]$ is reduced up to a minimum of zero. Then, if necessary, price indexation $\kappa_t \in [0,1]$ is reduced up to a minimum of zero, followed by an increase in the contribution rate $\theta_t^S \in \left[0, \theta^{S,\max}\right]$ up to a maximum $\theta^{S,\max}$. If this is still not enough in the case of underfunding, then nominal claims are scaled back by whatever amount is necessary to eliminate the underfunding within the allowed restoration period. In the case of a long-term restoration plan, no further action is undertaken and so nominal pension claims are unaltered. When the funding ratio is above $1+\xi^u$, first any reductions in nominal pension rights are given back. Then, if the funding ratio allows this, any missed price indexation is restored, followed by the restoration of any missed productivity indexation. If the funding ratio is still not expected to return to below $1+\xi^u$ within the allowed amount of time, the contribution rate is reduced up to a minimum of zero.

The exact policy parameter combination $\left\{\theta_{t+1}^S, \kappa_{t+1}, \iota_{t+1}\right\}$ for year $t+1$ is determined in year t on the basis of a projection $\widetilde{F}_{t+1}$ of the funding ratio at time $t+1$, computed from the size of the fund's assets A_t and liabilities $L_{j,t} = \frac{1}{I}\sum_{i=1}^{I} L_{i,j,t}$ of the various cohorts in year t (averaged over the skill groups), and under the assumption of no further shocks $\left(\omega_{t+1} = \underset{(2D+6)\times 1}{\mathbf{0}}\right)$.

2.7   Welfare measures

We consider three measures of welfare. One is cohort- and skill-specific and the other two are population-wide. The first is the intertemporal utility function $V_{i,j,t}$ for skill class $i \in \{1,..I\}$, cohort $j \in \{1,..D\}$ in year t. The second measure, S_t^A, is defined as the unweighted average of the intertemporal utilities of all individuals alive at $t=1$:

$$S_t^A = \sum_{j=1}^{D} \frac{N_{j,t}}{\sum_{j=1}^{D} N_{j,t}} \frac{1}{I}\sum_{i=1}^{I} V_{i,j,t}.$$

The third measure, S_t^T, is defined as the unweighted average of the intertemporal utilities of all individuals alive and yet unborn at $t=1$:

$$S_t^T = \sum_{j=1}^{D} \frac{N_{j,t}}{\sum_{j=1}^{D} N_{j,t}} \frac{1}{I}\sum_{i=1}^{I} V_{i,j,t} + \sum_{s=1}^{\infty} \frac{N_{1,t+s}}{\sum_{j=1}^{D} N_{j,t}} \frac{\frac{1}{I}\sum_{i=1}^{I} V_{i,1,t+s}}{(1+q)^s}. \tag{24}$$

with q the discount rate. In the simulations, we truncate the computation of welfare to 250 unborn generations, as the discounted welfare of any subsequent generations is negligible in equation (24). Note that in equation (24) the size of any unborn generation is normalised to the size of the population alive in year t.

To ease the interpretation of the three measures $V_{i,j,t}$, S_t^A and S_t^T, we report them in terms of constant consumption flows. As regards the cohort-specific measure $V_{i,j,t}$, we define "certainty

equivalent consumption" $CEC_{i,j,t}$ for skill group $i \in \{1,...,I\}$, cohort $j \in \{1,...,D\}$ in year t, as the certain, constant consumption level over the remainder of the cohort's lifetime that yields a level of the utility function identical to the level of the utility function obtained under the relevant scenario. Hence,

$$CEC_{i,j,t} = u^{-1} \left(\frac{V_{i,j,t}}{E_t \left[\sum_{l=j}^{D} \frac{\beta^{l-j}}{\psi_{j,t-j+1}} \left(\prod_{k=0}^{l-j} \psi_{j+k,t-j+1} \right) \right]} \right). \tag{25}$$

In a similar vein, for the economy-wide measures we define the constant consumption flow

$$C_t^\Upsilon = u^{-1} \left(\frac{S_t^\Upsilon}{\sum_{l=\overline{J}+1}^{D} \frac{\beta^{l-(\overline{J}+1)}}{\psi_{\overline{J}+1,t-\overline{J}}} \left(\prod_{k=0}^{l-(\overline{J}+1)} \psi_{(\overline{J}+1)+k,t-\overline{J}} \right)} \right), \quad \Upsilon = A, T, \tag{26}$$

of an agent with the average age $\overline{J}$ in the economy in year t,

$$\overline{J} = \text{integer} \left[\frac{\sum_{j=1}^{D} j N_{j,t}}{\sum_{j=1}^{D} N_{j,t}} \right],$$

where integer [.] is the function that generates the largest integer smaller than or equal to the number inside the square brackets. Note that this is the constant consumption stream of a person of age $\overline{J}$ that gives her a utility equal to social welfare S_t. It is *not* the constant consumption stream that gives a person of age $\overline{J}$ the utility level that he has under the relevant policy. Notice that there is a one-to-one relationship between the level of C_t and S_t as calculated in (26).

2.7.1 Comparison of policy scenarios

We evaluate welfare under a scenario A relative to the welfare under a scenario B. Scenario A is always the benchmark scenario of discounting against the actual swap curve. Scenario B is always completely identical to A apart from the choice of the discount rate in the second pillar ratio. Under both scenarios the parameters are initially identical and equal to those in the benchmark calibration. They remain unchanged in the ensuing years as long as the funding ratio remains between $1 + \xi^m$ and $1 + \xi^u$. Once the funding ratio falls below $1 + \xi^m$ or rises above $1 + \xi^u$, the policy parameters change according to the policy described above. Obviously, the year in which the policy parameter(s) need to be changed for the first time may vary depending on the chosen discount rate.

We consider four measures of welfare comparison between the two policies. A first measure is the constant percentage difference in certainty equivalent consumption between the two scenarios. For each skill class and cohort in a given period this measure is computed as:

$$\Delta CEC_{i,j,t}(A,B) \equiv \frac{CEC_{i,j,t}(B) - CEC_{i,j,t}(A)}{CEC_{i,j,t}(A)}, \tag{27}$$

where $CEC_{i,j,t}(s)$ denotes the value of $CEC_{i,j,t}$ under scenario $s \in \{A, B\}$. We consider three further, population-wide, measures of comparison. One is the "majority support" for policy B, that is the share of people that are better off under B rather than A:

$$D_t(A, B) \equiv \frac{1}{\sum\limits_{j=1}^{D} N_{j,t}} \sum_{j=1}^{D} \frac{N_{j,t}}{I} \sum_{i=1}^{I} \mathbf{1}\{CEC_{i,j,t}(B) > CEC_{i,j,t}(A)\}, \qquad (28)$$

where $\mathbf{1}\{.\}$ is an indicator function that assigns a value of one (zero) if the condition inside the curly brackets holds (does not hold). The final two measures are the "social welfare gain" from using policy B rather than policy A, excluding, respectively including, the welfare of the unborn generations:

$$\Delta C_t^{\Upsilon}(A, B) \equiv \frac{C_t^{\Upsilon}(B) - C_t^{\Upsilon}(A)}{C_t^{\Upsilon}(A)}, \quad \Upsilon = A, T, \qquad (29)$$

3   Calibration and details on the simulation

We follow the standard literature and calibrate the exogenous parameters of the model to reproduce the main features of the US economy. However, the pension arrangements are calibrated to the Dutch situation. Tables 1 and 2 summarise our benchmark calibration.

We assume that the economically active life of an individual starts at age 25. He works for $R = 40$ years until he reaches the age of 65. He lives for at most $D = 75$ years, until the age of 100. His coefficient of relative risk aversion is set to $\gamma = 2$, in accordance with a large part of the macroeconomic literature. The discount factor is set to $\beta = 0.98$, slightly above the usual choice of 0.96 because individuals also take into account their survival probabilities. To compute the welfare measure (24) we try several discount rates q for the utility of unborn generations. We find no qualitative differences and in what follows we report results based on $q = 4\%$. The age-dependent portfolio composition $\{x_j^e, x_j^h\}_{j=1}^{D}$ is taken from the mean values of the 2007 wave of the Survey of Consumer Finances (SCF, 2009).[2] Portfolio composition is reported by age groups, and we interpolate the data using the spline method. We keep the portfolio weights constant for ages as of 90. The efficiency index $\{e_i\}_{i=1}^{I}$ is given by the income deciles in the US for year 2000 taken by the World Income Inequality Database (WIID, 2008). We normalise the index to have an average of 1. The seniority index $\{s_j\}_{j=1}^{I}$ uses the average of Hansen's (1993) estimation of median wage rates by age group. We take the average between males and females and interpolate the data using the spline method.

The exogenous social security parameters are specifically calibrated to the Dutch situation. For the first social security pillar we set the benefit scale factor $\rho^F = 17\%$ to generate a realistic average replacement rate of 30.40%. The Dutch Tax Office ("Belastingdienst") reports for 2008 a maximum income assessable for first-pillar contributions of EUR $3,850.40$ per month. We therefore set our upper income threshold for contributions $\delta^u = 1.10$, roughly equal to $3,850.40 * 12/42,403$, where EUR $42,403$ is our imputation of the economy's average income as of 2008.[3] The lower income threshold is set to $\delta^l = 0.56$, in such a way as to generate a starting contribution rate $\theta_1^F = 16.42\%$,

---

[2] We aggregate assets into three categories: bonds (transaction accounts, certificates of deposit, savings bonds, and bonds), equities (stocks, investment funds, cash value of life insurance, other assets) and housing (residental properties).

[3] In Eurostat the most recent statistic on average income in the Netherlands refers to year 2005. The same source also provides the minimum income until year 2008. Exploiting the correlation between average and minimum income, we run an OLS regression of average income over time and minimum income. As a result, we predict the average income of year 2008 to be EUR $42,403$.

consistent with the reality. For the second social security pillar, historically the accrual rate has been between 1.5 and 2%, and most frequently at 1.75%. We therefore consider $\mu = 0.0175$ and set the franchise to $\lambda = 0.33$, to generate a realistic average replacement rate of 37.60%. In our simulations we consider a short-term restoration period of $K^s = 5$ years when the pension buffer falls below $1 + \xi^l$ and a long-term restoration period of $K^l = 15$ years when the pension buffer falls below $1 + \xi^m$, but remains at or above $1 + \xi^l$. Further, we set the thresholds $\left\{\xi^l, \xi^m, \xi^u\right\}$ for the buffer at $\left\{\xi^l, \xi^m, \xi^u\right\} = \{5\%, 25\%, 60\%\}$.

In general, the composition of the fund's investment portfolio depends on the size of the buffer. However, in our benchmark simulations we set $\left\{z^e, z^h\right\} = \{45\%, 5\%\}$ for any level of the funding ratio. This choice of $\left\{z^e, z^h\right\}$ corresponds to the balance sheet average for Dutch pension funds over the period 1996 - 2005 (source: DNB, 2009). Because the various assets in the pension fund's portfolio generally have different realised returns, at the end of each period t its portfolio is reshuffled such that the fund enters the next period $t + 1$ again with the original portfolio weights $\left\{z^e_{t+1}, z^h_{t+1}\right\} = \{45\%, 5\%\}$.

Finally, we set the starting levels of the indexation parameters at $\kappa_1 = \iota_1 = 100\%$ (hence the pension fund provides full indexation to nominal wages). The starting contribution rate is set such that aggregate contributions at $t = 1$ coincide with aggregate benefits in the absence of shocks. The rate that satisfies this condition is $\theta^S_1 = 17.56\%$, which is close to the average actual contribution rate in the Netherlands. We then choose initial assets A_0 that generate an initial funding ratio F_1 of 140% in the absence of shocks; initial assets amount to roughly 2.1 times the initial level of income in the economy. [4] The contribution rate is capped at $\theta^{S,\max} = 25\%$.

The deterministic growth rate of the newborn cohort, $\mu = 0.47362\%$, is the average growth from a regression using 20 observations on the annual variation in the number of births in the US between 1986 and 2005 (the source is the Human Mortality Database: HMD, 2009); details on the regression are in Appendix 6.2. This appendix also describes our calibration of the survival probabilities based on the Lee-Carter model (Lee and Carter, 1992). The combination of survival probabilities and birth rates determines the size of each cohort. The starting value of the old-age dependency ratio (i.e., the ratio of retirees over workers) is 25.23%, in line with OECD statistics for 2005.

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[4] This is on the high side compared to the actual Dutch situation. However, in our model every worker participates in the pension fund, while in the Netherlands this is only part (though a majority) of those who are employed. Moreover, a large fraction of the workers has his pension arranged through insurance companies, while the self-employed do not participate in pension funds either (they have the possibility to build up their pension through an insurance company, but the financial reserves of insurance companies are not considered part of the pension buffers).

Table 1. Calibration of the exogenous parameters

Symbol	Meaning	Calibration
General setting		
D	Number of cohorts (= maximum death age -25)	75
R	Number of working cohorts (= retirement age -25)	40
γ	relative risk aversion parameter	2
β	Discount factor	0.98
q	Unborn generation discount rate	4%
$\left\{x_j^e, x_j^h\right\}_{j=1}^D$	Household portfolio composition	SCF (2007)
$\{e_i\}_{i=1}^I$	Efficiency index	WIID (2008)
$\{s_j\}_{j=1}^R$	Seniority index	Hansen (1993)
First-pillar parameters		
ρ^F	Benefit scale factor	0.17
$\left\{\delta^l, \delta^u\right\}$	Income thresholds in the contribution formula	$\{0.56, 1.10\}$
Second-pillar parameters		
μ	Accrual rate	0.0175
λ	Franchise share	0.33
$\left\{K^s, K^l\right\}$	Length of restoration periods	$\{5, 15\}$
$\left\{\xi^l, \xi^m, \xi^u\right\}$	Buffer thresholds	$\{5\%, 25\%, 60\%\}$
d	Fixed discount rate	4%
$\left\{z^e, z^h\right\}$	Fund portfolio composition	$\{45\%, 5\%\}$
$\{\kappa_1, \iota_1\}$	Starting indexation	$\{100\%, 100\%\}$
θ_1^S	Starting contribution rate	17.56%
$\theta^{S,\max}$	Upperbound on ontribution rate	25%

Crucial is the calibration of the average annual values of price inflation, nominal income growth and the bond, equity and housing returns (see Table 2). We loosely follow the literature in this regard (see, e.g., Brennan and Xia, 2002; van Ewijk et al., 2006) and set the average inflation rate at $\pi = 2\%$, the average nominal income growth rate at $g = 3\%$ (which corresponds to an average real productivity growth of 1% per year), the average one-year bond interest rate at $r_1^b = 3\%$, and the average housing return at $r^h = 4\%$. Since our attention primarily concerns the volatility of the funding ratio, we assume that the expected return on the pension fund's long-term bond portfolio equals the expected return on short-term bonds, i.e. $r^{lb} = r_1^b = r^{sb}$. Since the average of $\frac{\left(1+r_{10,t-1}^b\right)^{10}}{\left(1+r_{9,t}^b\right)^9}$ in the simulations exceeds $1+r_1^b$, we correct the simulated long-term bond returns by subtracting in each simulation run from the long-term bond returns the average over the simulation run of $\frac{\left(1+r_{10,t-1}^b\right)^{10}}{\left(1+r_{9,t}^b\right)^9}$ and adding the constant r_1^b to that number. The average equity return is set at $r^e = 5.2\%$ to generate a funding ratio that is stable over time in the absence of shocks and policy parameter changes.[5] Innovations in these five variables follow the VAR(1) process described in Appendix 6.2.3. Appendixes 6.2.4 and 6.2.5 provide details on the calculation of the parameters of the process for the swap curve $\left\{r_{k,t}^s\right\}_{k=1}^D$ and the bond yield curve $\left\{r_{k,t}^b\right\}_{k=1}^D$.

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[5]In this situation, the ratio is approximately constant for the first 20 years, and still around 110% after 75 years. The first policy parameter adjustment is made only after 44 years.

Table 2. Calibration of averages of the random variables

Symbol	Description	Calibration
π	Inflation rate	2%
g	Nominal income growth rate	3%
r_1^b	Nominal one-year bond return	3%
r^e	Nominal equity return	5.2%
r^h	Nominal housing return	4%

Note: for the stochastic component, see the Appendix

To obtain the optimal consumption rules from equation (18) we solve the individual decision problem recursively by backward induction using the method of "endogenous gridpoints" (Carroll, 2006). Shocks to the inflation rate, the income growth rate and the bond, equity and residential housing returns introduce through equation (20) five state variables into the model. To avoid the curse of dimensionality caused by having too many state variables, we determine the optimal rule in year t under the assumption that the shocks in year $t-1$ are all equal to 0, $\epsilon_{t-1}^\pi = \epsilon_{t-1}^g = \epsilon_{t-1}^b = \epsilon_{t-1}^e = \epsilon_{t-1}^h = 0$. We approximate the random variable distributions by means of a Gauss-Legendre quadrature method (see Tauchen and Hussey, 1991), and discretise the state space using a grid of 100 points with triple exponential growth.[6] For points that lie outside the state space grid, we use linear extrapolation to derive the optimal rule.

We simulate $N = 1,000$ times a sequence of vectors of unexpected shocks over $2D-1+250 = 399$ years, drawn from the joint distribution of all the shocks. Our welfare calculation is based on the economy as of the Dth year in the simulation. Hence, we track only the welfare of the cohorts that are alive in that year, implying that those that die earlier are ignored, and we track the welfare of cohorts born later, the latest one dying in the final period of the simulation. The total number of years of one simulation run equals the time distance between the birth of the oldest cohort that we track and the death of the latest unborn cohort that we track. At each moment there are D overlapping generations. For the sake of simplicity, we relabel the Dth year in the simulation as $t = 1$. The purpose of simulating the first $D - 1$ years is to simply generate a distribution of the assets held by each cohort at the end of $t = 0$.

In each simulation run, we assume that the ageing process stops after $t = 40$. That is, mortality rates at any given age no longer fall. This assumption is in line with the fact that some important ageing studies, such as those by the Economic Policy Committee and European Commission (2006) and the United Nations (2009), only project ageing (and its associated costs) up to 2050, hence roughly 40 years from now. Moreover, it is hard to imagine that mortality rates continue falling for many more decades at the same rate as they did in the past. In particular, many of the common mortal diseases have already been eradicated, while it will become more and more difficult to treat remaining lethal diseases. Effective treatment of those diseases will also surely be held back by the fact that the share of national income that can be spent on health care is bounded.

To allow for the cleanest possible comparison among the various discounting policies, we use the same shock series under all policies, while, moreover, during the initialisation phase of each simulation run no policy responses occur (that is, there is constant and complete indexation and the contribution rate is kept constant). Hence, the situation at the end of $t = 0$ or at the beginning

---

[6]We create an equally-spaced grid of the function $log(1 + log(1 + log(1 + s)))$, where s is the state variable. The grid with "triple exponential growth" applies the transformation $exp(exp(exp(x) - 1) - 1) - 1$ to each point x of the equally-spaced grid. This transformation brings the grid back to the original scale of the state variable, but determines a higher concentration on the low end of possible values. A grid with triple exponential growth is more efficient than an equally-spaced grid as the consumption function is more sensitive to small values of the state variable.

of $t = 1$ (before choices are made) is identical in each run under the various policies. Because welfare depends on the size of the buffer after the initialisation period in the simulation run, we reset the stock of pension fund assets such that the buffer at the end of $t = 0$ equals 140%.[7] Finally, the process z_t is re-normalised to unity at the end of $t = 0$ and the nominal pension claims of the various cohorts are rescaled by the same amount. At the start of the preceding $D-1$ dummy years, liabilities are set at the steady state values implied by the income level at that moment. They are computed using (10) under the assumption of no shocks (i.e. expectations are treated as if they are realised).

4 Results

4.1 Benchmark

Our benchmark analysis considers the four variants mentioned earlier to discount the liabilities. In variant 1, we discount against the actual realisation of the swap curve. In variant 2, we discount against a weighted average of the present and past swap curves, setting $L = 5$ years and attaching an equal weight $w_l = \frac{1}{L}$ for each value of l. In variants 3 and 4 discounting is against a time-invariant maturity-yield profile: in variant 3, we discount against the average swap curve $\{\bar{r}_k^s\}_{k=1}^D$, which exhibits a quadratically-looking profile (see Figure 7, Appendix), while for variant 4 we choose a flat discount rate of 4% for all maturities of pension payments. This discount rate is exceeded by the average swap return at any maturity above $k = 1, 2$.

Figure 1 shows the median funding ratio under the alternative ways of discounting.[8] Not surprisingly, the adjustment policy we consider produces in all the variants a median funding ratio that lies between the thresholds $1 + \xi^m$ and $1 + \xi^u$, fluctuating in most cases between 135% and 145%.

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[7]For each of the four variants we consider, the initial 140% funding ratio is the one obtained by discounting the given set of nominal pension rights against the discount rate for that specific variant. This implies for each of the variants the initial amount of assets in the pension fund is different.

[8]We report the median rather than the average, because the former is not affected by the few extreme outcomes generated in our simulations.
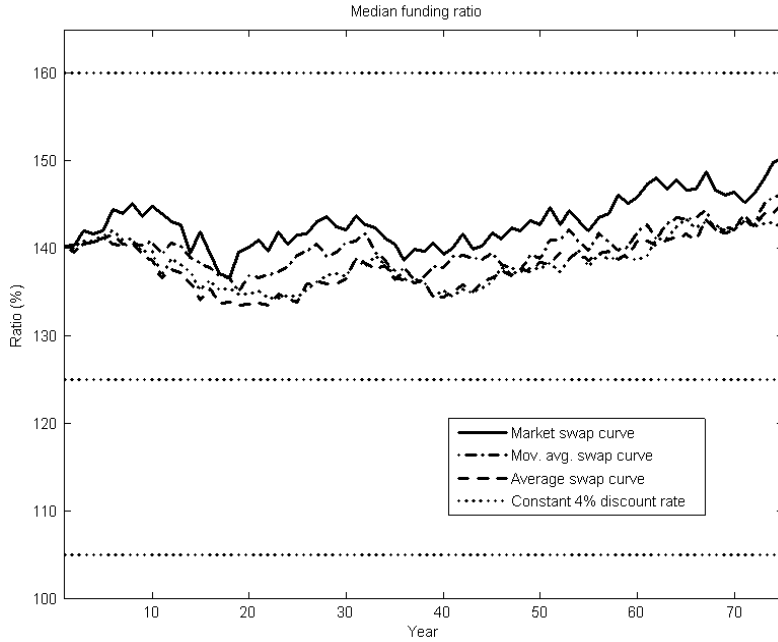
Figure 1. Median funding ratio under different discounting methods

The differences among the variants manifest themselves through the volatility of the funding ratio. Panel a. of Figure 2 shows the median coefficient of variation of the funding ratio, that is, the ratio between its median volatility (measured as half the interquartile range) and its median value. Compared to the case of discounting against the market swap curve, the volatility of the funding ratio is around 17% smaller if liabilities are discounted using a moving average swap curve and around 26% smaller if they are discounted using either a (constant) average swap curve or a constant and flat discount rate. This reduction of volatility is trivially driven by a more stable computation of liabilities (see Table 3). Differences in the volatility of the funding ratio also manifest themselves in the probability of underfunding, i.e. the likelihood that the funding ratio falls below $1 + \xi^l$ in the simulations (see panel b. of Figure 2 and Table 3).[9] The likelihood of underfunding under the variant based on the moving average swap curve is lower than in the case of discounting against the market swap curve (across periods it is on average around 12% against around 15%). In turn, the variants with constant discounting exhibit a substantially lower likelihood of underfunding than the other two variants (the likelihood under the former is below 10% in most periods).

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[9]Note that because the funding ratio is reset at 140% at the beginning of $t = 1$, the likelihood of underfunding is much lower during the first years of a simulation run than later on.
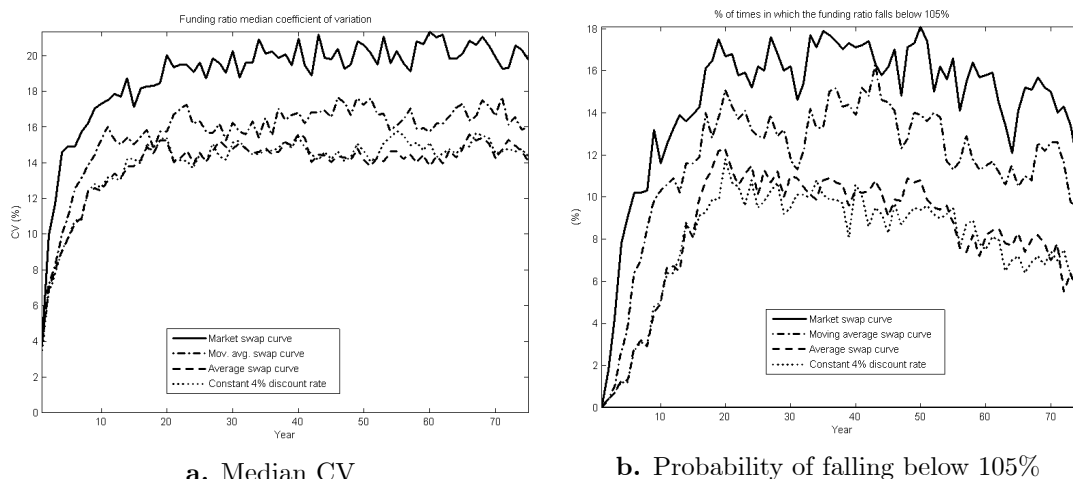
a. Median CV **b.** Probability of falling below 105%

Figure 2. Funding ratio volatility under different discounting methods

Table 3 reports further summary statistics from the simulation of the four variants. Policy parameters are adjusted more frequently when liabilities are discounted at the market swap curve: under this variant in 27% of the time, at least one policy parameters is changed as opposed to around 24% under the other variants. It is worthwhile to notice the high correlation between assets and liabilities. *Ceteris paribus*, a higher correlation dampens the volatility of the funding ratio. The high correlation is to a large extent driven by the shocks to the market swap curve. A fall in the swap curve raises liabilities, but it also raises the value of the fund's bond portfolio (given the correlation of the swap curve with the bond yield curve).[10] Further, changes in the policy parameters tend to stabilise the funding ratio, thus offsetting any exogenous shock to assets and liabilities. As a result, the two components of the funding ratio vary less and tend to go into the same direction. When we remove the policy responses to the funding ratio, the correlation between assets and liabilities drops to around 59%. These effects are present in all discounting policy variants, which explains why we observe similar correlations between the assets and liabilities under the four variants.

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[10] The correlation between the shocks to the swap returns, $\left\{\epsilon_{k,t}^{s}\right\}$, and to the bond returns, $\left\{\epsilon_{k,t}^{b}\right\}$, can be computed from (22) and the analogous model (indicated with superscript b) for the bond returns and is given by $\Gamma_{2}^{s}var\left(X_{t-1}\right)\Gamma_{2}^{b\prime}$, where $X_{t-1} = \left(\begin{array}{ccccc} \epsilon_{t'-1}^{\pi} & \epsilon_{t'-1}^{g} & \epsilon_{1,t'-1}^{y} & \epsilon_{t'-1}^{e} & \epsilon_{t'-1}^{h} \end{array}\right)'$. It ranges from 23.79% for $corr\left(y_{t}^{s}, y_{30,t}^{b}\right)$ to 39.51% for $corr\left(y_{t}^{s}, y_{3,t}^{b}\right)$.

Table 3. Benchmark comparison of the discounting variants

%	Market swap curve	Moving average swap curve	Average swap curve	Constant and flat rate $d = 4\%$
Prob. of intervention	27.3080	23.6617	24.7191	23.9636
Funding ratio				
Prob. funding ratio below $1 + \xi^l$	14.6160	11.6773	8.3480	7.8960
Prob. funding ratio above $1 + \xi^u$	33.5133	27.5067	24.0693	24.6040
Median coeff. var.	18.9212	15.6353	13.8674	13.9833
Components of the funding ratio				
Assets, median coeff. var.	21.9560	19.7686	21.4250	21.8924
Liabilities, median coeff. var.	21.3193	17.7707	16.5112	16.5407
Assets-liabilities correlation	86.7150	90.6523	92.5385	91.2677
Welfare				
C_1^A	83.2535	83.0916	83.2244	83.2856
C_1^T	38.4171	38.5755	38.6177	38.6195
ΔC_1^A	-	-0.1944	-0.0349	0.0386
ΔC_1^T	-	0.4124	0.5221	0.5269
D_1	-	75.7689	84.9668	91.3515

Notes: "Prob." is "Probability" and captures the fraction of time over all simulation runs. "Coeff. var." is "coefficient of variation". Finally, "intervention" means that at least one of $\left\{ \theta_t^S, \kappa_t, \iota_t \right\}$ is changed.

Overall, as measured by C_1^A the policy with constant and flat discount rate is the one preferred by the generations alive at $t = 1$. However, the welfare differences among the variants are very small. This policy remains the preferred one when we also include the welfare of the generations born after $t = 1$. The policy using the market swap curve is instead the worst in terms of welfare. The conclusion is the same when we consider the statistic D_1, which reports the fraction of those alive at $t = 1$ that are in favour of switching from discounting against market swap curve to another discounting method. Again the constant and flat discount rate is the most preferred policy ($D_1 = 91.35\%$).

To understand why the variants with a constant discount rate (the average swap curve and the 4% flat rate) are most preferred ones, it is instructive to see the average values of the policy parameters across the simulations under the four variants. These are reported in Table 4, together with their standard deviations. Both constant discount rate variants result into a lower contribution rate and higher indexation rates. Further, the parameters are also less volatile, implying less volatility in human wealth.

Table 4. Average policy parameters

%	Market swap curve	Moving average swap curve	Average swap curve	Constant and flat rate $d = 4\%$
θ_t^S	14.5108	14.8413	14.2271	13.7229
	(10.3002)	(10.3563)	(9.7637)	(9.9996)
κ_t	93.6530	96.2227	94.3892	93.8430
	(199.2245)	(176.4675)	(185.6677)	(187.9003)
ι_t	82.9622	87.5837	84.7772	84.3382
	(188.3907)	(186.9155)	(174.1260)	(173.5377)

Note: standard deviation is within parentheses.

While aggregate welfare differences across the variants are small, differences across cohorts are larger; see Figure 3, which reports a cubic interpolation over the cohort-specific measures in (27). Middle-aged and older generations prefer any discounting alternative to the one using the market swap curve, as this results in more stable benefits**.**

Only the young generations of workers aged 40 or less seem to be better off with a policy based on the market swap curve, which can provide them with up to 0.70% of additional certainty-equivalent consumption. The higher volatility of the funding ratio under mark-to-market discounting raises the likelihood of a marked drop in indexation early on in the simulation run. The indexation is on average restored later in the life of these cohorts when they hold more pension rights than at the moment indexation was reduced. Hence, on average, they profit from the "reduction-restoration" cycles in the indexation of the pension rights. In fact, average price (productivity) indexation in the years $t = [61, 75]$ is 101.26% (111.01%) using mark-to-market discounting, which is around 4% points higher than under the other variants (see Figure 4, which reports average indexation under the four variants).
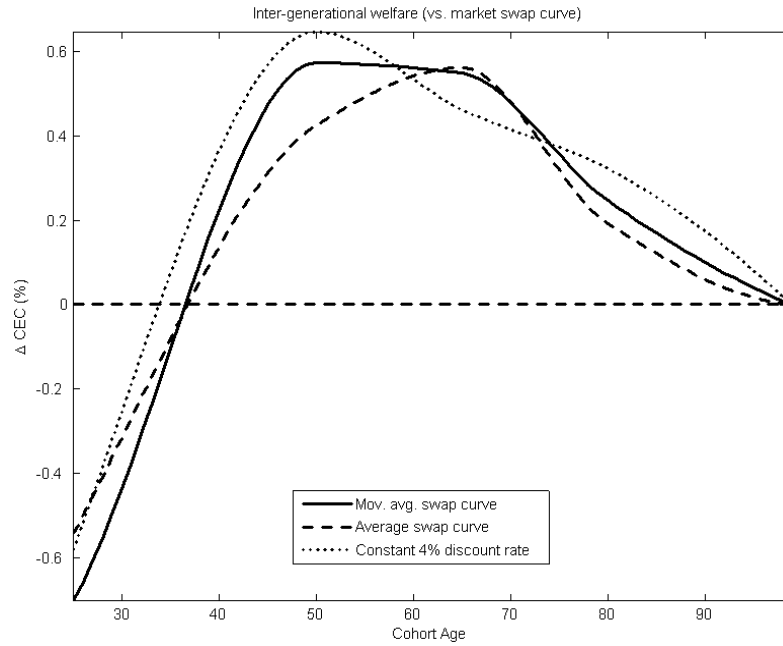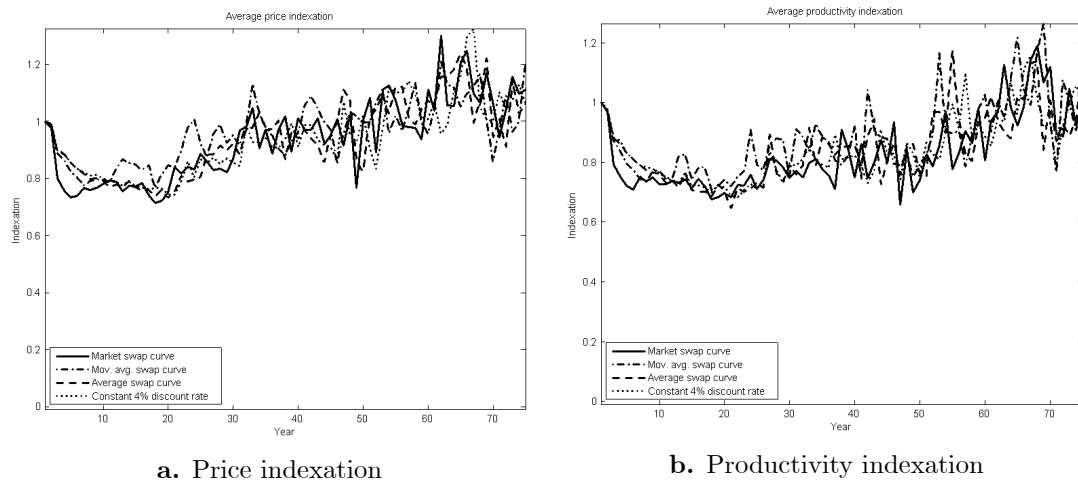
Figure 3. Welfare comparison existing generations

Note: $\Delta CEC > 0$ means better off under alternative to discounting against market swap rate.

a. Price indexation **b.** Productivity indexation

Figure 4. Average indexation

We now consider the following "counterfactual" experiment. We simulate the model assuming that policy interventions take place as before on the basis of the funding ratio computed under each of the four discounting variants. However, for each of these variants we also compute the counterfactual funding ratio based on discounting the future pension payments against the market swap curve. Only for variant 1 the funding ratio coincides with that shown in Figure 1, while for variants 2 and 3 the counterfactual funding ratios are rather close to those under variant 1 (see Figure 5, panel a). However, in the case of variant 4, discounting against a constant and flat discount rate, the counterfactual funding ratio is substantially higher and exceeds the threshold

$1 + \xi^u$ after 50 years. This is not surprising because the level of liabilities (for a given stream of future pension benefits) at which policy intervention takes place is "too low" when measured using the market swap curve. Hence, the amount of assets held by the pension fund is "excessively large" in this case (at the beginning of each simulation run it is 2.40 instead of 2.10 using the market swap curve). The volatility of the counterfactual ratio is smallest under constant and flat rate discounting and highest under mark-to-market discounting. In the other two cases the volatilities of the counterfactual funding ratio are very close and lie between those of the other two cases (see Figure 5, panel b).
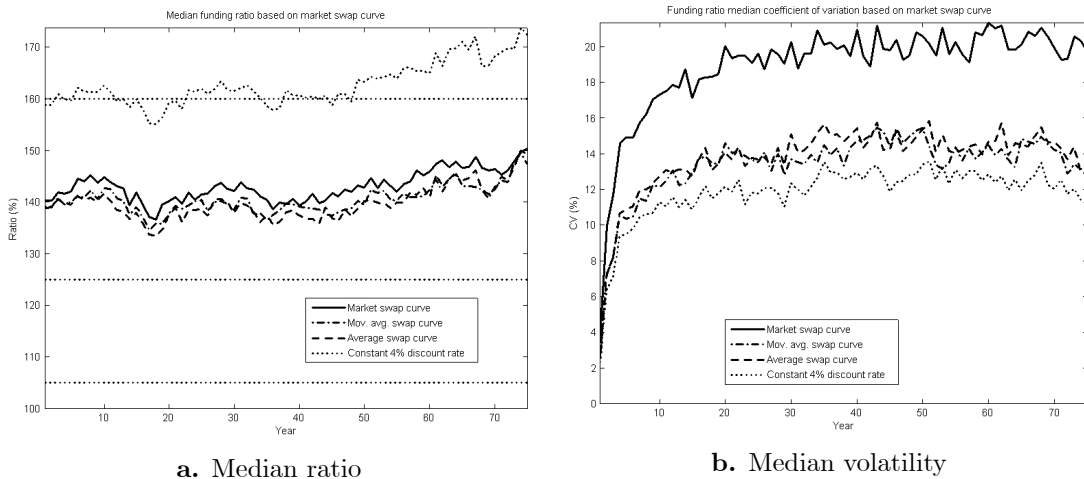

| **a.** Median ratio | **b.** Median volatility |

Figure 5. Counterfactual funding ratio, discounting against market swap curve

4.2 Alternative ways of discounting future pension payments

4.2.1 Alternative moving average swap curves

In this subsection we consider alternative moving averages for the swap curve. The first alternative takes an equally-weighted moving average over the past 20 curves ($L = 20$ and $w_l = \frac{1}{L}$ for $l = 0, .., L - 1$). Increasing the window L should generate a more stable moving average curve. The second alternative assumes that more recent swap curves receive a relatively higher weight in the computation of the moving average. In particular, under this alternative the weights are given by $w_l = (L - l) \frac{2}{L(L+1)}$ for $l = 0, .., L - 1$ with $L = 5$.

The summary statistics for these alternatives are found in columns 2 and 3 of Table 5. As expected, increasing the window L reduces the volatility of the funding ratio (to 14.36%, against 15.64% under the original 5-year window), which is close to the volatility obtained with the average swap curve and the constant and flat discount rate. The volatility with the weighted moving average lies between the volatility under mark-to-market discounting and the volatility with the simple moving average at window $L = 5$. The welfare measures suggests that a weighted moving average is inferior to a simple moving average, as it is less effective in stabilising the funding ratio. However, they disagree on the preferable window length ($L = 5$ according to ΔC_1^T and D_1, $L = 20$ according to ΔC_1^A; compare with Table 3).

4.2.2 Alternative constant and flat discount rate

Until recently, Dutch pension funds used a constant and flat rate $d = 4\%$ to discount pension payments for calculating the funding ratio. However, the resulting level of liabilities is substantially higher than under discounting against the average swap curve, implying that the pension fund needs to hold more assets initially to enter the simulation run with a funding ratio of 140%. In this subsection we replicate the benchmark analysis using a higher constant and flat discount rate. It is set to $d = 5.154\%$ to produce an initial level of liabilities equal to that under discounting against the average swap curve. This ensures a fairer comparison between the two discounting methods.

The last column of Table 5 reports summary statistics for this case. The findings are in line with those in the benchmark case, the main exception being that now fewer individuals then before prefer the constant and flat discount rate to discounting against the market swap curve (however, they are still a large majority of the population: $D_1 = 76.25\%$ instead of 91.35% as in the benchmark). Discounting at a constant and flat rate $d = 5.154\%$ generates a higher average contribution rate than discounting against $d = 4\%$ (14.20% instead of 13.72%). This increase hurts the workers. Those in their thirties and forties who in Figure 3 were only weakly in favour of the policy with $d = 4\%$ are now against the policy with $d = 5.154\%$. However, it should be kept in mind that comparison of the two scenarios is not directly possible as pension fund's assets are lower in the economy with $d = 5.154\%$.

Table 5. Comparison with alternative discounting methods

%	Market swap curve	Simple MA ($L = 20$)	Weighted MA ($L = 5$)	Constant and flat rate $d = 5.154\%$
Prob. of intervention	**27.3080**	24.4698	25.0467	24.6207
Funding ratio				
Prob. of a ratio below $1 + \xi^l$	**14.6160**	8.7840	13.1533	8.3920
Prob. of a ratio above $1 + \xi^u$	**33.5133**	25.3413	29.1160	24.1133
Median cv	**18.9212**	14.3585	16.9150	13.8817
Ratio components				
Assets median cv	**21.9560**	20.4808	20.6966	21.5020
Liabilities median cv	**21.3193**	15.6801	19.1301	16.6923
Assets-liabilities correlation	**86.7150**	90.4123	89.6654	93.1388
Welfare				
C_1^A	**83.2535**	83.1495	83.1125	83.2022
C_1^T	**38.4171**	38.5108	38.4564	38.5950
ΔC_1^A	**-**	-0.1248	-0.1694	-0.0616
ΔC_1^T	**-**	0.2439	0.1024	0.4630
D_1	**-**	73.2739	41.3243	76.2544

Note: see notes to Table 3.

4.2.3 Discounting against the bond yield curve

From a policy perspective, it may also be interesting to see how the results are affected if we use an alternative term structure of the interest rates to discount future pension payments. We now use the term structure based on yields on high-grade public debt (the "bond yield curve"). In the simulations all the shock series are identical to those before except that the shocks to the

bond yield curve replace the shocks to the swap curve. Because the bond yield curve generally lies below the swap curve, the average size of the liabilities is slightly higher. Under mark-to-market discounting, it becomes 1.7272 times GDP rather than 1.6166 times GDP. Also the correlation between the assets and liabilities of the fund becomes slightly higher – see Table 6, which reports the summary statistics. The main difference between the two mark-to-market discounting cases is that the funding ratio is much less volatile when the bond yield curve is used; its median coefficient of variation now drops from 19% to 14% while the fraction of time during which there is underfunding falls from 15% to 8%. It is worth pointing out, however, that the volatility of the funding ratio is the resultant of two separate forces: the uncertainty in the random variables and the movements in the policy parameters. If we were to keep the policy parameters fixed at their initial values, thus capturing only the volatility arising from the fundamental shocks, we would observe for all the policies using the yield curve a larger volatility of the assets component (around 40%) and a smaller correlation (around 65%) with liabilities (whose volatility is around 17%). As a result, the funding ratio would exhibit higher volatility (about 49%). In this setting, the correlation between assets and liabilities is, however, sustained by the fact that shocks to 9-year bond returns affect both components of the funding ratio. This effect is instead absent when discounting is based on the swap curve. In such case, the correlation reduces further to 59%, and liabilities are a little more volatile (around 18%) since in our calibrations the swap curve is subject to heavier fluctuations. As a result, the volatility of the funding ratio climbs to 56%.

Table 6. Comparison with discounting against the yield curve

%	Market swap curve	Market yield curve	Moving average yield curve	Average yield curve
Prob. of intervention	**27.3080**	27.9769	24.0938	23.2438
Funding ratio				
Prob. of a ratio below $1 + \xi^l$	**14.6160**	7.7467	6.6907	7.8960
Prob. of a ratio above $1 + \xi^u$	**33.5133**	26.0907	23.5893	22.7587
Median cv	**18.9212**	14.1021	13.1317	13.3595
Ratio components				
Assets median cv	**21.9560**	20.6534	20.2225	21.0817
Liabilities median cv	**21.3193**	18.1735	15.7523	16.3398
Assets-liabilities correlation	**86.7150**	88.3892	90.7078	91.4648
Welfare				
C_1^A	**83.2535**	82.4789	82.6300	82.6060
C_1^T	**38.4171**	38.2875	38.4156	38.3811
ΔC_1^A	**-**	-0.0093	-0.0075	-0.0078
ΔC_1^T	**-**	-0.3373	-0.0039	-0.0937
D_1	**-**	39.1249	48.5635	46.6974

Note: see note to Table 3.

Overall, the welfare differences using swap and bond yield curves are small. However, all the three measures indicate that welfare is higher under discounting against the market swap curve. An interesting observation arises from Figure 6, which makes a welfare comparison by cohort. Here, those aged roughly 40 and younger prefer a policy of discounting against the market swap curve, whereas all the remaining cohorts prefer a policy of discounting against the (market or average) bond yield curve. The preference of the young for discounting against the market swap curve is at

least partially explained by the fact that the contribution rate is smaller and more stable than in the other cases.[11] This is beneficial for young workers.
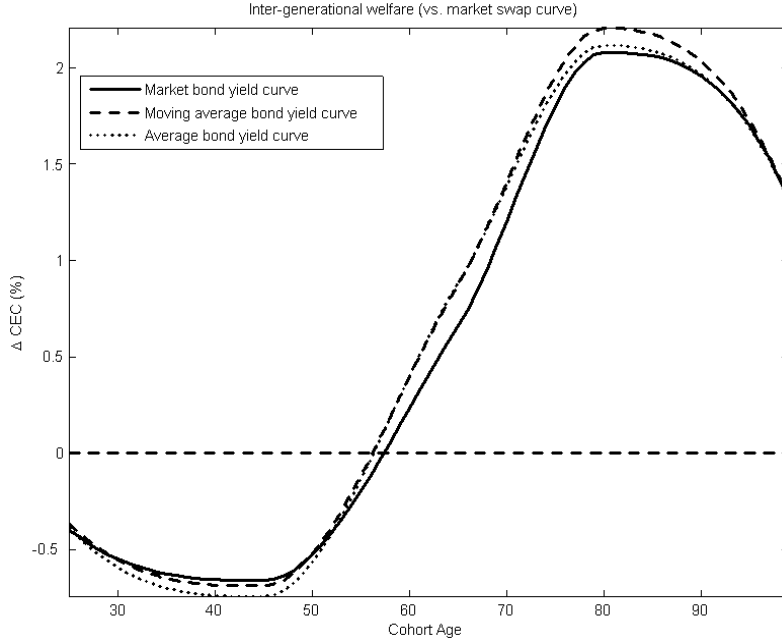

Figure 6. Welfare comparison - yield vs. swap curve

4.3 Only short-term bonds in pension fund portfolio

The analysis so far has assumed that the pension fund holds ten-year bonds. However, the maturity of the fund's bond holdings may not be an innocent choice. It is plausible that a reduction in the maturity of the fund's bond portfolio reduces the correlation between the fund's assets and liabilities and, hence, raises the volatility of the funding ratio. After all, a downward shift in the swap curve raises the level of the liabilities, but it also raises the value of the fund's bond holdings (because the swap curve and bond-yield curve tend to move rather closely together). The effect on the fund's bond portfolio will be smaller, though, the lower its duration, suggesting that effects of shocks other than those to the swap curve may now play a relatively more important role, thereby leading to a lower correlation between the value of assets and liabilities.

In this subsection we assume that the fund's bond portfolio consists entirely of one-year zero coupon bonds $r_t^{sb} = r_{1,t}^b$. Hence, the fund's total portfolio return now becomes:

$$1 + r_t^g = \left(1 - z^e - z^h\right)\left(1 + r_t^{sb}\right) + z^e\left(1 + r_t^e\right) + z^h\left(1 + r_t^h\right) \tag{30}$$

Table 7 reports the statistics for the simulation of this case. For mark-to-market discounting, we do indeed observe a reduction in the correlation between assets and liabilities and an increase in the volatility of the funding ratio (compare with Table 3). There is also an increase in the frequency with which the funding ratio falls below $1 + \xi^l$ (now 17.29% of the time against 14.62% in the benchmark case).

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[11]In the first 15 years, the average contribution rate is 13.46%, with standard deviation 6.80%, using the market swap curve. The average is around 0.50% higher, with standard deviation around 7%, using the other curves.

Table 7. Short-maturity bonds in the fund portfolio

%	Market swap curve	Moving average swap curve	Average swap curve	Constant rate $d = 4\%$
Prob. of intervention	**26.0506**	23.8673	21.7351	20.8164
Funding ratio				
Prob. of a ratio below $1 + \xi^l$	**17.2880**	13.9613	11.6253	11.5467
Prob. of a ratio above $1 + \xi^u$	**33.2880**	27.3240	20.7653	21.0693
Median cv	**20.6069**	16.5959	14.3937	14.4801
Ratio components				
Assets, median cv	**23.5234**	21.2811	23.6302	24.2011
Liabilities, median cv	**22.3498**	18.9565	18.8381	19.3349
Assets-liabilities correlation	**84.4161**	91.1018	90.3718	92.4121
Welfare				
C_1^A	**83.1545**	83.1818	83.1570	83.0921
C_1^T	**38.5096**	38.4303	38.4974	38.4675
ΔC_1^A	**-**	-0.0328	0.0030	-0.0751
ΔC_1^T	**-**	-0.2060	-0.0315	-0.1094
D_1	**-**	74.5268	58.6864	65.1446

Note: see note to Table 3.

4.4 Regime switches

An important rationale for applying mark-to-market discounting rather than discounting at some constant discount rate is that mark-to-market discounting allows the calculation of the liabilities to track structural shifts in the term structure. For example, a persistent fall in the real interest rate (holding constant inflation) that is not reflected in a reduction in the rate at which future pension payments are discounted, would over time lead to an increasing gap between the value of the fund's assets (which grow at a lower rate) and the "true" value of the liabilities as measured by applying the appropriate discount rate. At some point the payment of the pensions would be in danger.

In this subsection we allow for occasional shifts in the average one-year bond return $r_{1,t}^b$. In a given year t the realisation of the bond return follows from the process in equation (19), where its average r_1^b may take one of two values:

$$r_1^b = \left\{ \begin{array}{ll} \overline{r} + \widetilde{r} & \text{if } q_t = 1 \\ \overline{r} - \widetilde{r} & \text{if } q_t = 0 \end{array} \right\}, \quad \widetilde{r} > 0,$$

with q_t a random variable. The average return is assumed to remain the same for 10 years. After 10 years, the probability of having $q_{t+10} = 1$ or 0, that is, a relatively high or low average bond return, depends only on q_t:

$$\Pr\left(q_{t+10} = i | q_t = j\right) = p_{i,j} \text{ for } i = 0, 1 \text{ and } j = 0, 1.$$

A variation of $2\widetilde{r}$ in the average bond return has a direct effect on the level of returns in the swap curve (see equations (21), (22) and (33)), which shift by the same amount $2\widetilde{r}$. Notice that in the benchmark scenario we implicitly assume that $\widetilde{r} = 0$ (no change in average return). Here we set $\overline{r} = 0.03$ and $\widetilde{r} = 0.01$. For the purpose of comparability with the analysis so far, we impose the restriction that r_1^b coincides in expectation with the value in the benchmark case (3%),

$$E\left[r_1^b\right] = \left(\bar{r} + \tilde{r}\left(p_{1,1} - p_{0,1}\right)\right)\Pr\left(q_t = 1\right) + \left(\bar{r} + \tilde{r}\left(p_{1,0} - p_{0,0}\right)\right)\Pr\left(q_t = 0\right) = 0.03,$$

from which we have that $p_{1,1} = p_{0,1} = p_{1,0} = p_{0,0} = 0.5$.

Table 8 reports summary statistics from this case of regime switches. Most results are in line with the benchmark case reported in Table 3, though compared with this benchmark case we now observe a higher probability of intervention and a larger volatility of the funding ratio using the moving average swap curve. This finding is explained by the fact that, after a shift in the bond return has arisen, the moving average keeps memory of the past level of returns for 5 years. This creates a discrepancy in the way assets and liabilities are affected by shocks, and involves more frequent policy parameter adjustments to preserve the stability of the ratio.

Moving away from a policy that discounts against the market swap curve is still preferable according to two measures out of the three (ΔC_1^T and D_1), but the support for alternative policies is generally weaker. In particular, the best alternative policy seems the one using a constant and flat rate, as in the benchmark case. However, the majority support for this policy has shrunk ($D_1 = 80.62\%$ instead of 91.35%), and aggregate welfare shows a smaller gain ($\Delta C_1^T = 0.38\%$ rather than 0.53%) or even a tiny loss ($\Delta C_1^A = -0.06\%$ instead of 0.04%). A policy using the market swap curve is seen relatively more favourably because it neutralises the effect of regime switches in the average bond return, which appear in both the assets and liabilities components of the ratio.

Table 8. Comparison of the discounting variants under regime switches

%	Market swap curve	Moving average swap curve	Average swap curve	Constant and flat rate $d = 4\%$
Prob. of intervention	**27.1475**	25.2590	24.7093	23.6614
Funding ratio				
Prob. of a ratio below $1 + \xi^l$	**15.4440**	11.9720	8.2360	7.8933
Prob. of a ratio above $1 + \xi^u$	**33.0800**	28.5867	24.8000	25.4360
Median cv	**19.1596**	16.1744	13.9815	14.0944
Ratio components				
Assets, median cv	**21.9875**	19.8725	22.0207	22.2430
Liabilities, median cv	**22.2633**	18.8553	17.0756	16.5662
Assets-liabilities correlation	**87.8846**	90.3713	92.1139	92.6185
Welfare				
C_1^A	**83.4220**	83.2262	83.3103	83.3682
C_1^T	**38.6225**	38.6082	38.6742	38.7677
ΔC_1^A	**-**	-0.2347	-0.1339	-0.0645
ΔC_1^T	**-**	-0.0368	0.1341	0.3761
D_1	**-**	53.8323	57.3871	80.6222

Note: see note to Table 3.

5 Conclusion

In this paper we have investigated the inter- and intra- generational welfare implications of different methods of discounting future pension outlays. We have also explored the effect on the pension buffer itself. We described the economy with an OLG model of a small-open economy, featuring

a two-pillar pension system similar to the one in the Netherlands, a country that has traditionally featured a large second pension pillar. The economy was subject to demographic, economic and financial shocks that we calibrated from US data. We compared mark-to-market discounting against the swap curve (the method currently followed in the Netherlands), discounting against a moving average of past swap curves, discounting against the average swap curve and discounting against a constant and flat rate. Aggregate welfare differences among the methods are small and the same holds for differences in the behavior of the pension buffer. The benchmark simulations show, though, that mark-to-market discounting is dominated in terms of aggregate welfare by the alternative discounting methods. Intergenerational welfare differences are more pronounced. In particular, it is the younger workers who prefer mark-to-market discounting, while the opposite is true for the older generations. The higher volatility of the pension buffer under mark-to-market discounting tends to cause a reduction in indexation early on in the simulation run. For the younger generations the compensation of the missed indexation later is given over a, by then, larger accumulated stock of pension rights, enabling them to benefit from the earlier reduction in indexation. In contrast, older generations prefer discounting methods other than mark-to-market as the induced more frequent changes in indexation destabilise income over their remaining lifetime. Finally, we find that the presence of regime switches raises the relative attractiveness of mark-to-market discounting, although this effect is limited.

6 Appendix

6.1 Detailed rules for adjustment of policy parameters

The adjustment policy works as follows. In case *no restoration plan* from an earlier period is still active in t:

1. If $F_t < 1 + \xi^l$, a *short-term restoration plan* is started that after K^s years in the absence of shocks brings back along a linear growth path the funding ratio at $1+\xi^l$. Hence, the sequence of policy parameter combinations $\left(\theta_{t+1}^S, \kappa_{t+1}, \iota_{t+1}\right), ..., \left(\theta_{t+K^s}^S, \kappa_{t+K^s}, \iota_{t+K^s}\right)$ is set at period t such that the funding ratios $\widetilde{F}_{t+1}, \widetilde{F}_{t+2}, ..., \widetilde{F}_{t+K^s}$ projected from F_t in the absence of further shocks hit the target funding ratios $\bar{F}_{t+\tau} = F_t + \left[\left(1 + \xi^l\right) - F_t\right] \frac{\tau}{K^s}$ for years $\tau = 1, ..., K^s$. For every period $t + \tau$ along the restoration path, we first reduce productivity indexation $\iota_{t+\tau}$ up to a minimum level of zero. If this is not enough, we reduce parameter $\kappa_{t+\tau}$ up to a minimum level of zero. If this is still not enough, we raise the contribution rate $\theta_{t+\tau}^S$ up to a maximum of $\theta^{S,\max}$. If after applying all these measures the funding ratio still falls short of its target $\bar{F}_{t+\tau}$, we set $\theta_{t+\tau}^S = \theta^{S,\max}$, $\kappa_{t+\tau} = \iota_{t+\tau} = 0$ and apply a reduction in nominal rights $m_{t+\tau} > 0$ such that $\widetilde{F}_{t+\tau} = \bar{F}_{t+\tau}$.

2. If $1 + \xi^l \leq F_t < 1 + \xi^m$, a *long-term restoration plan* is started that after K^l years in the absence of shocks brings back along a linear growth path the funding ratio at $1+\xi^m$. Hence, the sequence of policy parameter combinations $\left(\theta_{t+1}^S, \kappa_{t+1}, \iota_{t+1}\right), ..., \left(\theta_{t+K^l}^S, \kappa_{t+K^l}, \iota_{t+K^l}\right)$ is set at period t such that the funding ratios $\widetilde{F}_{t+1}, \widetilde{F}_{t+2}, ..., \widetilde{F}_{t+K^l}$ projected from F_t in the absence of further shocks hit the target funding ratios $\bar{F}_{t+\tau} = F_t + [(1 + \xi^m) - F_t] \frac{\tau}{K^l}$ for years $\tau = 1, ..., K^l$. For every period $t + \tau$ along the restoration path, we first reduce productivity indexation $\iota_{t+\tau}$ up to a minimum level of zero. If this is not enough, we reduce price indexation $\kappa_{t+\tau}$ up to a minimum level of zero. If this is still not enough, we raise $\theta_{t+\tau}^S$ up to a maximum of $\theta^{S,\max}$. If after applying all these measures the funding ratio still falls

short of $\bar{F}_{t+\tau}$, we set $\theta^S_{t+\tau} = \theta^{S,\max}$, $\kappa_{t+\tau} = \iota_{t+\tau} = 0$, but we apply no reduction in nominal rights.

3. If $1 + \xi^m \leq F_t < 1 + \xi^u$, there are two cases:

   (a) In the absence of any missed nomimal rights (see below), the next-year policy parameters are set to $\theta^S_{t+1} = \theta^S_t$ and $\kappa_{t+1} = \iota_{t+1} = 1$.

   (b) In the presence of missed (unrestored) nominal rights, the next-year policy parameters are set to $\theta^S_{t+1} = \theta^S_t$ and $\kappa_{t+1} = \iota_{t+1} = 0$.

4. If $F_t \geq 1 + \xi^u$, m_{t+1} is set to restore any missed nominal rights (as described below) to the extent that the funding ratio does not fall below the target ratio $1 + \xi^u$.[12] If after restoring possible missed nominal rights still $\widetilde{F}_{t+1} > 1 + \xi^u$, then further adjustment to the policy parameters is made. First, we restore possible missed price indexation (see below). Then, we restore possible missed productivity indexation and, finally, we reduce the contribution rate θ^S_{t+1} up to a minimum of 0. If after applying all these measures the funding ratio in the absence of shocks still exceeds $1 + \xi^u$, we raise price indexation by an *extra* amount $\hat{\kappa}_{t+1} > 0$ such that over a period of three years along a linear path in the absence of shocks the funding ratio is back at $1 + \xi^u$.

In case a *long-term restoration plan* from an earlier period is still active in t:

1. If $F_t < 1 + \xi^l$, the long-term restoration plan is cancelled and the policymaker follows the above policy under "no restoration plan" from an earlier period still active in t. That is, it sets up a short-run restoration plan as determined above.

2. If $1 + \xi^l \leq F_t < 1 + \xi^m$, there are two cases:

   (a) If $F_t < \bar{F}_t$, we reduce productivity indexation up to a minimum of $\iota_{t+1} = 0$ to produce a projected ratio $\widetilde{F}_{t+1} = \bar{F}_{t+1}$ in the absence of shocks. If this is not enough, we reduce price indexation up to a minimum of $\kappa_{t+1} = 0$. If this is still not enough, we increase the contribution rate up to a maximum of $\theta^S_{t+1} = \theta^{S,\max}$. If after applying these measures next period's funding ratio still falls below $\bar{F}_{t+1}$, we set $\theta^S_{t+1} = \theta^{S,\max}$, $\kappa_{t+1} = \iota_{t+1} = 0$ and undertake no further action.

   (b) If $\bar{F}_t \leq F_t < 1 + \xi^m$, the policy parameters are those prescribed by the existing long-term restoration plan.

3. If $1 + \xi^m \leq F_t < 1 + \xi^u$, then the above policy under "no restoration plan" from an earlier period still active in t is followed.

4. If $F_t \geq 1 + \xi^u$, then the above policy under "no restoration plan" from an earlier period still active in t is followed.

In case a *short-term restoration plan* from an earlier period is still active in t:

1. If $F_t < 1 + \xi^l$, there are two cases:

---

[12] Dutch pension law says that a pension fund is not allowed to reduce contribution rates until any earlier reduction in nominal rights is undone.

(a) If $F_t < \bar{F}_t$, we reduce productivity indexation up to a minimum of $\iota_{t+1} = 0$ to produce a projected ratio $\widetilde{F}_{t+1} = \bar{F}_{t+1}$ in the absence of shocks. If this is not enough, we reduce price indexation up to a minimum of $\kappa_{t+1} = 0$. If this is still not enough, we increase the contribution rate up to a maximum of $\theta_{t+1}^S = \theta^{S,\max}$. If after applying these measures next period's funding ratio still falls below $\bar{F}_{t+1}$, we set $\theta_{t+1}^S = \theta^{S,\max}$, $\kappa_{t+1} = \iota_{t+1} = 0$ and $m_{t+1} > 0$ such that in the absence of shocks $\widetilde{F}_{t+1} = \bar{F}_{t+1}$.

(b) If $\overline{F}_t \le F_t < 1+\xi^l$, the policy parameters are those prescribed by the existing short-term restoration plan.

2. If $1 + \xi^l \le F_t < 1 + \xi^m$, then the above policy under no restoration plan from an earlier period still active in t is followed. That is, a long-term restoration plan is set up in the way described above.

3. If $1 + \xi^m \le F_t < 1 + \xi^u$, then the above policy under no restoration plan from an earlier period still active in t is followed.

4. If $F_t \ge 1 + \xi^u$, then the above policy under no restoration plan from an earlier period still active in t is followed.

We restore missed price and productivity indexation and missed nominal rights as follows. Let us take the case of price indexation. For this case, we define two processes, an "actual" process (tracking the actual indexation that has been given, where π is long-run average inflation),

$$p_t^{\kappa,a} = (1 + \kappa_t \pi)\, p_{t-1}^{\kappa,a}, \tag{31}$$

and a "shadow" process that corresponds to always having full indexation:

$$p_t^{\kappa,s} = (1 + \pi)\, p_{t-1}^{\kappa,s}. \tag{32}$$

We set the processes equal to unity at $t = 1$ (D periods into the simulation run): $p_1^{\kappa,a} = p_1^{\kappa,s} = 1$.

Suppose that in period t, the funding ratio exceeds $1+\xi^u$. Then, indexation for the next period will at least be equal to full indexation: $\kappa_{t+1} \ge 1$. In case $p_t^{\kappa,a} < p_t^{\kappa,s}$, the indexation in the next period will be set at most so high that the missed indexation is restored in expected terms. That is, κ_{t+1} will be set at most such that $p_{t+1}^{\kappa,a} = p_{t+1}^{\kappa,s}$, which is equivalent to $(1 + \kappa_{t+1}\pi)\, p_t^{\kappa,a} = (1 + \pi)\, p_t^{\kappa,s}$, which in turn is solved as:

$$\kappa_{t+1}^{restore} = \frac{1}{\pi}\left(\frac{p_t^{\kappa,s}}{p_t^{\kappa,a}} - 1\right) + \frac{p_t^{\kappa,s}}{p_t^{\kappa,a}}.$$

Finally, we define κ_{t+1}^u as the indexation rate that brings the funding ratio to $1 + \xi^u$ next year in the absence of further shocks. Actual indexation κ_{t+1} will be set at:

$$\kappa_{t+1} = \min\left\{\max\left\{1, \kappa_{t+1}^u\right\}, \kappa_{t+1}^{restore}\right\}.$$

The processes (31) and (32) continue further until the end of the simulation run.

For missed productivity indexation, we similarly define the "actual", respectively "shadow", processes:

$$
\begin{aligned}
p_t^{\iota,a} &= \left(1 + \iota_t\left(\frac{1+g}{1+\pi} - 1\right)\right) p_{t-1}^{\iota,a}, \\
p_t^{\iota,s} &= \left(1 + \left(\frac{1+g}{1+\pi} - 1\right)\right) p_{t-1}^{\iota,s},
\end{aligned}
$$

where $p_1^{\iota,a} = p_1^{\iota,s} = 1$. Restoration of indexation is completely similar to that in the case of price indexation.

Finally, for reductions in nominal rights (captured by $m_t > 0$), we define the "actual", respectively "shadow", processes

$$
\begin{aligned}
p_t^{m,a} &= (1 - m_t)\, p_{t-1}^{m,a}, \\
p_t^{m,s} &= p_{t-1}^{m,s},
\end{aligned}
$$

where $p_1^{m,a} = p_1^{m,s} = 1$. Again, if at some moment t, we have $p_t^{m,a} < p_t^{m,s}$ and the funding ratio exceeds $1 + \xi^u$, missed nominal rights can be given back up to a maximum level such that $p_{t+1}^{m,a} = p_{t+1}^{m,s}$. The exact formula for the restoration of missed nominal rights is

$$
m_{t+1} = \max\left\{ \min\left\{ 0, 1 - \frac{\widetilde{F}_{t+1}}{1 + \xi^u} \right\}, \min\left\{ 0, 1 - \frac{p_t^{m,s}}{p_t^{m,a}} \right\} \right\},
$$

where $\widetilde{F}_{t+1}$ is the projection at time $t + 1$ of the funding ratio in the absence of further shocks. To see the first argument of this expression, notice that if $m_{t+1} = 1 - \frac{\widetilde{F}_{t+1}}{1+\xi^u}$, all nominal rights are multiplied by the factor $\frac{\widetilde{F}_{t+1}}{1+\xi^u}$. Hence, all future pension benefits are multiplied by this same factor and, then, total liabilities are multiplied by this same factor, implying that the funding ratio becomes $1 + \xi^u$.

6.2 Details on the calibration

6.2.1 Growth rate of the newborn cohort

For the number of births in the US between 1985 and 2005 (source: HMD, 2009), we estimate the model:

$$
n_t = n + \epsilon_t^n,
$$
$$
\epsilon_t^n = \varphi \epsilon_{t-1}^n + \eta_t^n, \quad \eta_t^n \tilde{\ } N\left(0, \sigma_n^2\right).
$$

This yields $n = 0.0047362$, $\varphi = 0.4543931$ (standard error 0.2223041) and $\sigma_n = 0.0132662$ (standard error 0.0017105).

6.2.2 Survival probabilities

Our simulations require cohort life tables, which are incomplete for recent cohorts. Using easily available period life tables, however, leads to an over-estimate of mortality because of the well documented downward trend in mortality. To correctly estimate mortality, we follow the Lee-Carter model (Lee and Carter, 1992) and collect from HMD (2009) US period life tables from 1950 to 2005. These contain the total population on a year-by-year basis from ages 0 to 110. We call $\psi_{j,t}^p$ the probability of being alive in year t for individuals aged j, conditional on having been alive at age $j - 1$. To distinguish the trend from fluctuations, we estimate with singular value decomposition the parameters of the Lee-Carter model:

$$
\ln\left(1 - \psi_{j,t}^p\right) = \alpha_j + \tau_j \chi_t + \eta_t^\psi,
$$

where α_j and τ_j are age-varying parameters, χ_t is a time-varying vector and η_t^ψ is a random disturbance distributed as $N\left(0, \widetilde{\sigma}_\psi^2\right)$. Lee and Carter (1992) point out that the parameterisation is not unique. Therefore, we choose the one fulfilling their suggested restrictions:

$$\left\{ \begin{array}{l} \sum\limits_{t=1}^{T} \chi_t = 0 \\ \sum\limits_{j=1}^{D} \tau_j = 1 \end{array} \right\},$$

where $t = 1, .., T$ indicates the sample period. With these restrictions the estimated value for α_j will be the average probability over the sample that someone dies at age j , when having survived up to age $j - 1$.[13] Consistently with the existing literature we assume that the mortality index χ_t evolves as a random walk with drift χ:

$$\chi_t = \chi_{t-1} + \chi + \epsilon_t^{\psi},$$

with $\epsilon_t^{\psi} \tilde{} N \left(0, \sigma_{\psi}^2 \right)$. With our data we estimate $\hat{\chi} = -1.2595$ and $\hat{\sigma}_{\psi} = 0.0266$, thereby implying a trend fall in the probability of dying at any age j, conditional on having survived up to age $j - 1$. In the simulations we assume that $\hat{\chi} = 0$ after year $t = 40$, that is, there is no further population ageing after 40 years. We make this assumption to avoid dealing with very large contribution rates in the first- and second-pillar systems and on the assumption that the ageing process cannot continue forever.

From the period life table estimates and the trend of the mortality index we calculate the cohort life tables as follows:

$$\begin{aligned} \ln \left(1 - \psi_{j,t-j+1} \right) &= \hat{\alpha}_j + \hat{\tau}_j \left(\hat{\chi}_{t-j+1} + j\hat{\chi} \right) \\ &= \hat{\alpha}_j + \hat{\tau}_j \hat{\chi}_{t+1}, \end{aligned}$$

where $t-j+1$ is the year of birth of the cohort. Thus $\psi_{j,t-j+1}$ indicates the (estimated) probability of being alive at age j (end of period t) for the cohort of individuals born at the beginning of year $t - j + 1$, conditional on them being alive at age $j - 1$. In our model, the survival probabilities $\left\{ \psi_{j,D} \right\}_{j=1}^{D}$ of the cohort born in year $t = 0$ are set equal to those of the actual cohort of individuals born in 1950.

The survival probability for the cohort born in the following year $t - j + 2$ evolves according to:

$$\begin{aligned} \ln \left(1 - \psi_{j,t-j+2} \right) &= \hat{\alpha}_j + \hat{\tau}_j \left(\hat{\chi}_{t-j+2} + j\hat{\chi} \right) \\ &= \alpha_j + \hat{\tau}_j \left(\hat{\chi}_{t-j+1} + j\hat{\chi} + \hat{\chi} \right) \\ &= \alpha_j + \hat{\tau}_j \left(\hat{\chi}_{t+1} + \hat{\chi} \right) \\ &= \ln \left(1 - \psi_{j,t-j+1} \right) + \hat{\tau}_j \hat{\chi}. \end{aligned}$$

6.2.3   Economic shocks

We assume that the shocks to our five economic and financial variables (the inflation rate, the nominal wage growth rate, the one-year bond return, the equity return and the housing return) evolve according to a VAR(1) process. The underlying data are the following time series: for the inflation rate, the US Consumer Price Index; for the nominal income growth rate, the US hourly

---

[13]Notice that $\frac{1}{T} \sum\limits_{t=1}^{T} \ln \left(1 - \psi_{j,t}^p \right) = \frac{1}{T} \sum\limits_{t=1}^{T} \left(\alpha_j + \tau_j \chi_t + \eta_t^{\psi} \right) = \alpha_j + \tau_j \left(\frac{1}{T} \sum\limits_{t=1}^{T} \chi_t \right) + \left(\frac{1}{T} \sum\limits_{t=1}^{T} \eta_t^{\psi} \right) = \alpha_j + \left(\frac{1}{T} \sum\limits_{t=1}^{T} \eta_t^{\psi} \right)$

$= \hat{\alpha}_j + \left(\frac{1}{T} \sum\limits_{t=1}^{T} \hat{\eta}_t^{\psi} \right) = \hat{\alpha}_j$, where $\hat{\alpha}_j$ is the estimate of α_j and $\hat{\eta}_t^{\psi}$ is the regression residual. The last equality is obtained by using that the sum of the residuals is zero.

wage (source for both series: OECD, 2009); for the one-year bond return, the US end-of-year public debt yield at maturity one year (source: Federal Reserve, 2009); for the equity return, the MSCI US equity index (source: Datastream, 2009); for the housing return, the OFHEO house price index (now FHFA index, source: FHFA, 2009). All the series are annual over the period 1976-2005 (30 observations). For each series we take the deviations from the historical average.

Our shocks consist of a deterministic component, which is a linear combination of previous-year shocks, and a purely random component, given by realisations from i.i.d. innovations. The estimation of the deterministic component is shown in panel a of Table 6. It is worth pointing out that no variable in the specification of the equity return is significantly different from zero; indeed, a Wald chi-squared test does not reject the hypothesis that equity returns follow a purely random (white noise) process.

Table 6. VAR(1) regression

a. Deterministic coefficient estimates (matrix **B** in (20))

Variable	Inflation	Wage	Bond	Equity	Housing
Inflation (-1)	0.7864***	0.3060**	0.3694**	-1.5158	-0.8204***
	(0.1747)	(0.1192)	(0.1840)	(2.1683)	(0.2660)
Wage (-1)	0.0185	0.6609***	-0.0786	0.3825	1.0658***
	(0.1930)	(0.1317)	(0.2033)	(2.3953)	(0.2938)
Bond (-1)	-0.0555	-0.1661**	0.6857***	1.3535	-0.2609
	(0.1104)	(0.0753)	(0.1163)	(1.3700)	(0.1681)
Equity (-1)	0.0094	0.0125	0.0252	-0.0247	0.0119
	(0.0148)	(0.0101)	(0.01554)	(0.1831)	(0.0225)
Housing (-1)	0.2903***	0.0957*	0.1533**	-1.0446	0.6839***
	(0.0779)	(0.0531)	(0.0821)	(0.9669)	(0.1186)
Wald chi-squared	149.1552	233.2539	171.2329	3.9514	93.5409
p-value	0.0000	0.0000	0.0000	0.5564	0.0000

Note: standard deviations in parentheses.

***: significant at 1%; **: significant at 5%; *: significant at 10%

Wald chi-squared: test on the joint significance of the coefficients in each column.

The test follows a chi-squared distribution with 5 degrees of freedom.

b. Residual covariances and correlations (%)

Variable	Inflation	Wage	Bond	Equity	Housing
Inflation	0.0136	*50.2306*	*54.9103*	*20.8439*	*-15.2365*
Wage	0.0047	0.0063	*48.3280*	*-25.8828*	*-0.6701*
Bond	0.0079	0.0047	0.0151	*7.0268*	*4.7483*
Equity	0.0353	-0.0299	0.0125	2.1005	*0.2007*
Housing	-0.0032	-0.0001	0.0010	0.0005	0.0316

Note: correlations in italic; (co-)variances are in non-italic.

6.2.4 The swap curve

Deviations from the average swap returns follow the VADL(1) process of equation (22), in which each deviation is a function of all the deviations and other exogenous variables observed one month earlier. The exogenous variables are the innovations to the inflation rate, wage growth and the bond, equity and housing returns. Our dataset is a time series of US swap interest rates at any annual maturity from 1 to 10, plus maturities 12, 15, 20, 25 and 30 (source: Datastream, 2009). Many of these time series are not available before 1997. To obtain a reasonable number of observations, we therefore collect annual returns at monthly frequency to cover the period from 1997 to 2006 (120 observations).[14]

The VADL specification explains 15 variables observed in a given month (the swap return deviations) with an intercept and 20 variables observed one month earlier (the 15 swap return deviations, and the innovations to the 5 economic variables). The regression output is available upon request. For each dependent variable we reject the hypothesis that it follows a white noise process, and the R-squared statistic lies between 0.9480 and 0.9967. The shocks are assumed to follow a multivariate normal distribution, with mean 0 and covariance matrix given by the covariance among the residuals of the regression. The volatility of the shock at maturity one (standard deviation 0.0111) is close to that for the shock to one-year bond returns (standard deviation 0.0123). Shocks at near maturities have very high correlations around 98%; the lowest correlation we observe – between shocks at maturities 1 and 30 – is however still pretty high (42%).

We use the regression output to generate random swap returns at the observed maturities. The time period in the model is one year, but the regression is conducted on monthly data. Therefore, for each period in the simulation we generate a sequence of 12 subsequent swap curves using the estimated VADL(1) process. Each draw requires as input the monthly innovations to the inflation rate, the nominal wage rate and the returns to the one-year bond, equity and housing. However, only annual innovations are known through the process (20). Thefore, we construct monthly shocks from annual shocks after noticing that equation (20) coincides with

$$
\begin{pmatrix} \epsilon^{\pi}_{t'+12} \\ \epsilon^{g}_{t'+12} \\ \epsilon^{sb}_{t'+12} \\ \epsilon^{e}_{t'+12} \\ \epsilon^{h}_{t'+12} \end{pmatrix} = \mathbf{A}^{12} \begin{pmatrix} \epsilon^{\pi}_{t'} \\ \epsilon^{g}_{t'} \\ \epsilon^{sb}_{t'} \\ \epsilon^{e}_{t'} \\ \epsilon^{h}_{t'} \end{pmatrix} + \sum_{j=1}^{12} A^{12-j} \begin{pmatrix} \eta^{\pi}_{t'+j} \\ \eta^{g}_{t'+j} \\ \eta^{sb}_{t'+j} \\ \eta^{e}_{t'+j} \\ \eta^{h}_{t'+j} \end{pmatrix},
$$

where t' indicates the month, shocks at $t' = 0$ are set to 0, $A = B^{\frac{1}{12}}$ is obtained with single value decomposition and the monthly i.i.d. shocks arise from the (observed) annual i.i.d. shocks,

$$
\begin{pmatrix} \eta^{\pi}_{t'+j} \\ \eta^{g}_{t'+j} \\ \eta^{sb}_{t'+j} \\ \eta^{e}_{t'+j} \\ \eta^{h}_{t'+j} \end{pmatrix} = \left(\sum_{j=1}^{12} A^{12-j} \right)^{-1} \begin{pmatrix} \eta^{\pi}_{t-1} \\ \eta^{g}_{t-1} \\ \eta^{sb}_{t-1} \\ \eta^{e}_{t-1} \\ \eta^{h}_{t-1} \end{pmatrix},
$$

under the assumption that the shocks in months $t' + j, \quad j = 1, ..., 12$ are identical. We use these shocks to compute for each month the shocks to the yields at maturity 1-10, 12, 15, 20, 25 and 30 of the swap curve, according to equation (22).

From this sequence of 12 swap curve yields we consider the last one (say, the December one) in the simulations. We then adopt a linear interpolation over the available swap rates to obtain swap

---

[14]We ignore observations in later periods to satisfy the assumption of stationarity. After 2006 one enters the highly unusual situation of the current crisis.

rates at any discrete maturity between 1 and 30. Rates at maturity longer than 30 are set equal to the rate at maturity 30. Swap returns are then built as the sum of the VADL(1) realisations and a vector of constants, derived from

$$
\begin{pmatrix} r_1^s \\ r_2^s \\ \vdots \\ r_D^s \end{pmatrix} = \begin{pmatrix} r_1^b \\ r_1^b \\ \vdots \\ r_1^b \end{pmatrix} + \begin{pmatrix} \overline{r_{1,t}^s - r_{1,t}^b} \\ \overline{r_{2,t}^s - r_{1,t}^b} \\ \vdots \\ \overline{r_{D,t}^s - r_{1,t}^b} \end{pmatrix},
\tag{33}
$$

where r_1^b is the calibrated average one-year bond return (see Table 2), and the difference $\overline{r_{k,t}^s - r_{1,t}^b}$ is the sample average of the swap return at maturity k in excess of the sample average of the one year bond return. We use this formula to make swap returns in magnitude comparable to the calibrated average one-year bond return.

The average swap curve $\{\overline{r}_k^s\}_{k=1}^D$, defined in equation (23), follows the quadratically-looking profile shown in Figure 7. Only the average returns at the two shortest maturities $k \leq 2$ are below the constant discount rate of 4%.

6.2.5 The bond yield curve

We assume that the one-year interest rate coincides with the one-year bond return, while the interest rates for other maturities follow a similar process as for the swap curve. Our dataset is a time series of US yield returns at maturities 2, 3, 5, 7, 10, 20 and 30 (the only observed maturities – source is Federal Reserve). To make the comparison with the swap curve consistent, we take the same sample period (from 1997 to 2006) and frequency (monthly), even though we could use a longer series in this case. In the sample there are occasionally missing values for the yields at maturities 20 and 30. We impute the missing values using a linear interpolation method.

The regression output is available upon request. As for the swap curve, we obtain large R-squared statistics (between 0.9104 and 0.9958) and always reject the hypothesis that the interest rates follow a white noise process. Shocks at near maturities are highly correlated (usually above 95%, and never below 46%); they exhibit lower volatility than the corresponding shocks to the swap curve, especially at longer maturities. For instance, the standard deviation of the yield return at maturity 30 is only 53% of the standard deviation of the swap return at the same maturity. Both volatilities are, however, small compared to those of one-year bond returns.

We use the regression output to generate random yield returns at the observed maturities. As for the swap curve, for each year of the simulation we generate a sequence of 12 random yield returns, and make use of only the last realisation. We then adopt a linear interpolation over these yields to obtain the interest rates at any discrete maturity between 1 and 30. Interest rates at maturities longer than 30 are set equal to the interest rate at maturity 30. Yield returns at maturity $k \geq 2$ are then built as the sum of the VADL(1) realisations and a vector of constants, derived from

$$
\begin{pmatrix} r_2^b \\ r_3^b \\ \vdots \\ r_D^b \end{pmatrix} = \begin{pmatrix} r_1^b \\ r_1^b \\ \vdots \\ r_1^b \end{pmatrix} + \begin{pmatrix} \overline{r_{2,t}^b - r_{1,t}^b} \\ \overline{r_{3,t}^b - r_{1,t}^b} \\ \vdots \\ \overline{r_{D,t}^b - r_{1,t}^b} \end{pmatrix},
$$

where r_1^b is the average one-year bond return, and the difference $\overline{r_{k,t}^b - r_{1,t}^b}$ is the sample average of the yield return at maturity k in excess from the sample average of the one-year bond return. We use this formula to make bond yield returns comparable to the (assumed) one-year bond return.

Figure 7 shows the average bond yield curve, in comparison with the average swap curve and a constant and flat discount rate of 4%. The average bond yield curve shows a quadratic-looking profile similar to that of the average swap curve, although at each maturity the return is around 0.5% points lower than the corresponding return of the average swap curve. All the returns at maturity $k \leq 6$ are below the constant discount rate of 4%.
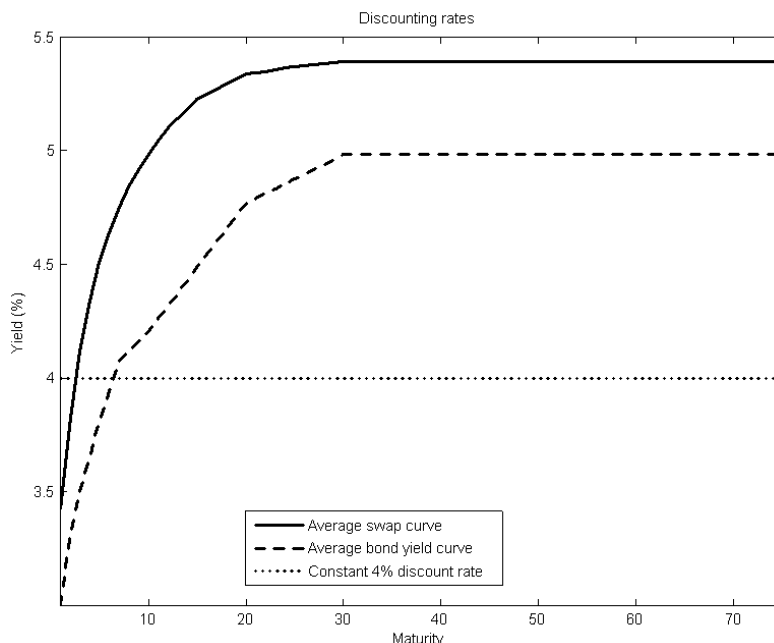

Figure 7. Average swap and bond yield curves

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