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# An ART Neural Network Model of Discrimination Shift Learning

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## Abstract

We present an ART-based neural network model (adapted from [2]) of the development of discrimination-shift learning that models the trial-by-trial learning process in great detail. In agreement with the results of human participants (4–20 years of age) in [1] the model revealed two distinct learning modes in the learning process: (1) a discontinuous rational learning process by means of hypothesis testing; and (2) a slow, yet discontinuous learning process. Categorical differences in behavior are the result of uniformly distributed dimensional preferences. In addition, it models the developmental differences between reversal and nonreversal-shift learning. The network implements attention-guided learning by selective sensory processing based on dimensional preferences mediated through reinforcement. The developmental differences consist of separate adjustment of the valuation of negative reinforcement, which is proposed in the empirical neuroscience literature [3].

**Keywords:** discrimination-shift learning, cognitive development, neural network.

## Introduction

Discrimination-shift learning is a long-standing paradigm in the investigation of human category learning and concept formation. A typical discrimination-shift learning task consists of two phases: discrimination learning and shift learning. Discrimination learning requires participants to choose the correct stimulus from a pair of stimuli. The stimuli presented differ on two or more dimensions (e.g. color and shape). Each of the dimensions has two possible values (e.g. white and black; circle and triangle), resulting in four possible stimuli. Reinforcement contingencies are such that only the choice of one value on a particular dimension, the so-called relevant dimension, is reinforced positively. Choosing the opposing value on the relevant dimension leads to negative reinforcement. The other dimension is irrelevant with respect to reinforcement and thus choices based on either value on this dimension lead to positive reinforcement in 50% of the choices made.

During the second, shift phase of the task, which starts once a specified learning criterion has been reached in the first phase, reinforcement contingencies are shifted. Numerous developmental differences have been found in comparing performance of children and adults on discrimination-shift tasks (see the literature on

(non)reversal-shift learning and reviews by [4-7]). In the current paper we focus on the developmental findings related to the difference between reversal shifts (RS) and nonreversal shifts (NRS). After a RS, all contingency relations are reversed leaving the same stimulus dimension relevant for discrimination (e.g., black is correct instead of white). A NRS makes a formerly irrelevant dimension relevant, shifting only two out of four contingency relations (e.g., circle is correct instead of white).

Developmental differences in discrimination learning, the first phase of the task, are mainly manifested as an increase in learning efficiency from childhood to young adulthood [6]. In the Levels-of-function theory two distinct learning modes are posited to account for the observed difference: (1) an hypothesis-testing, rational learning mode; and (2) a slow and incremental learning mode.

Subsequent analysis of these hypothesized learning modes has led to a refining of their proposed nature. A finite mixture model analysis on the error distribution of simple discrimination learning data gathered from subjects aged 6 to 10 was conducted in [8]. The error distribution was found to be bimodal, i.e. the distribution was best described as composed of two components. One component best describes a model of learning through hypothesis testing. The other component included slow learners who did not show any increase in performance during the first 16 trials (although criterion was reached in 48 trials).

These findings are extended by a more detailed investigation of the aforementioned learning modes [1]. Data from a simple discrimination-learning task (230 subjects, ages 4–20) was analyzed by fitting several mathematical learning models to the sequences of responses produced by the participants, i.e. the trial-by-trial learning process was modeled. Learning models were specified as latent Markov models (or mixtures thereof). The best fit on the data was obtained by a mixture model with a component of discontinuous rational learning through hypothesis testing and a component of slow, yet discontinuous learning (as opposed to incremental learning). The relative proportions of the components depend on age and relevant dimension (with respect to reinforcement contingencies) in the following two ways: (1) the proportion of rational learners

increases with age; and (2) in the younger age groups, the proportion of rational learners is higher when the relevant dimension is color than when the relevant dimension is shape. In the older age groups there was no such difference.

Several phenomena in childrens' and adults' discrimination-shift learning (DSL) are evident in the literature. One basic effect, with regard to discrimination learning in adults (and children older than 10), is that RS are executed more quickly than NRS.

In contrast, for children below 10, some studies show that the NRS require fewer trials to learn than the RS, whereas other studies fail to show any difference [5,6,9,12]. Another relevant finding is that learning RS on a child's 'preferred' dimension (e.g. color as opposed to shape) results in better performance than on a relatively less salient dimension relative to NRS; this was not found in experiments with adult subjects. Finally, the overtraining effect is also apparent, where training, continued after the learning criterion is reached, facilitates RS but not NRS in children.

The purpose of this paper is to model the two modes of discontinuous learning and the basic developmental results of discrimination-shift learning using a neural network architecture with incremental-learning rules. The network architecture will be implemented such that a cognitively plausible and developmentally relevant parameter accounts for systematic variation in learning behavior.

## Models of Discrimination-Shift Learning

Krushke [9] developed the AMBRY connectionist model of DSL that qualitatively fits adult data. Two important components are contextual bias and response-to-category mapping. The biased attention to input leads to preservation of dimensional attention, which would account for ease of RS over NRS. The separation of responses from the internal category structure is considered essential for shift learning.

Raijmakers, et al. [10] created a neural network utilizing the backpropagation error correction algorithm and one hidden layer to model DSL. The networks often learned NRS faster than RS, although a trial-by-trial analysis performed similarly to preschool children during shift learning. Overtraining however did not help the network to perform as adults, but also not as children. This lead to the conclusion that these particular feedforward networks that have been applied as models of higher cognitive tasks such as balance-scale learning, being bottom-up associative systems, are inadequate models of human learning as it is involved in discrimination learning, as they cannot incorporate the top-down categorization of adults and children on discrimination shift learning.

Sirois & Shultz [12] use a cascade-correlation algorithm to overcome the shortcomings of previous DSL neural network models. Given that these feedforward networks can adapt their topology, and therefore allow for structural plasticity, they are of great relevance in modeling developmental phenomena. Essentially, the architecture of the adult and child networks do not differ in DSL modeling as discrimination shifts are linearly separable problems. The

researchers argue that overtraining leads to differences in children and adults on these tasks. This coincides with findings in which overtraining preschoolers leads task performance similar to that of older subjects [5,6]. This is implemented in the network through a lowering of the allowed discrepancy between desired input and output. This lowered score-threshold fine-tunes pattern discrimination through extensive training, leading to fewer learning trials for child as opposed to adult networks; this is in contrast with humans. The results do reflect empirical data in that RS are learned more quickly than NRS in adult networks and equally quickly in child networks. The authors did not compare the trial-by-trial learning in the pre-shift phase of the network. However, it is typically expected that these incremental learning models show an incremental learning process in contrast to the all-or-none learning observed by humans, both children and adults.

## Model specification

To model sequential learning behavior on a simple discrimination task, an ART (Adaptive Resonance Theory, [23]) neural network architecture was chosen that was adapted to simulate perseveration behavior on the Wisconsin Card Sorting Task (WCST) [2]. The original architecture consists of an input-to-category node structure mediated by an attentional gating system of biases acting selectively on dimensional attributes in the input. Biases are mediated by so-called habit nodes which detect how often classifications have been made irrespective of feedback. Bias activation is controlled by a reinforcement signal. This signal is mediated by the reinforcement gain parameter. Lowering the value of this parameter will increase the likelihood of dimensional perseveration through the bias nodes. It thus models a decrease in the ability to adjust categorization behavior based on changing reinforcement contingencies.

The model was adapted to perform a simple discrimination task. Two major structural changes were made:

1. The reinforcement gain parameter was split in two to reflect the different influences of positive ( $\alpha+$ ) and negative ( $\alpha-$ ) reinforcement on responding. Frank et al. [15] made a similar distinction. They report empirical support for the dissociation between learning from positive reinforcement and learning from negative reinforcement in a study on how dopamine affects these distinct learning processes in Parkinson's patients. They present a computational model [16, 17] that accounts for this dissociation by proposing that two distinct dopamine-dependent pathways exist in the basal ganglia (BG), one excitatory pathway for positive reinforcement learning and one inhibitory pathway for negative reinforcement learning. These pathways are part of a larger model incorporating cortical structures as well as the basal ganglia and thalamus. The basal ganglia are influenced top-down by the orbitofrontal cortex (OFC) which represents positive and negative

reinforcement values separately in the medial OFC and lateral OFC, respectively.

2. A layer of two response nodes was added on top of the category layer. This allows the network to differentiate not only between categories, but also between responses to categories (as in AMBRY where internal category knowledge is separated from overt category responses in a similar manner). A stochastic process based on the negative reinforcement gain parameter ( $\alpha$ -) controls the update of the connections between the category nodes and the response nodes.

Learning in the ART network takes place in two ways: (1) updating the bias nodes and (2) updating the weights from category to response nodes. The final network topology is given in figure 1 (A full specification is supplied as an appendix). As can be seen a total number of four nodes code for input features. There are two nodes per dimension of the discrimination task (color and shape). Input signals propagate to category nodes, one for each of the four possible stimuli (white circle, white triangle, black circle, and black triangle). The category nodes mutually inhibit each other. Signals from the input to the category nodes are mediated by attentional gating through bias node activation. Response nodes determine the response of the network by comparing the activation values of the category nodes using weighted connections between category and response nodes. These weights are randomly initialized per dimensional pair as either zero or one. This thus expresses a response preference within each dimension.

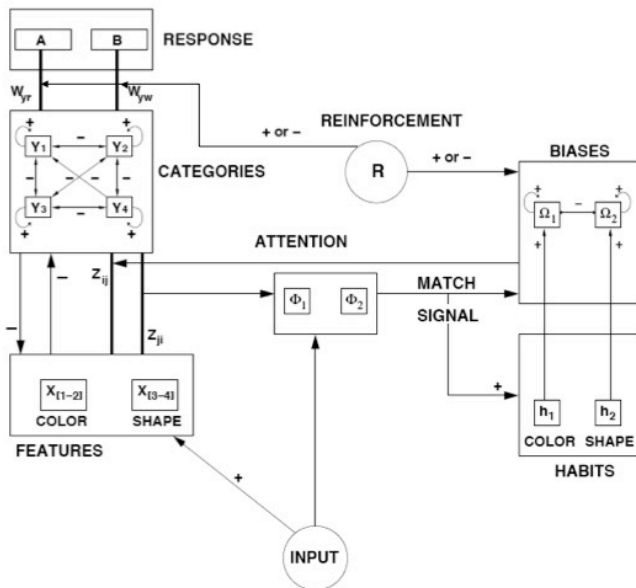


Figure 1: ART discrimination network.

Attentional gating of signals from input to category nodes is done through bias nodes acting on input dimensions. Bias nodes are initialized uniformly random as either high or low such that if one bias has a value greater than two, the other bias node is smaller than two by the same amount. The deviation from two is uniform. This expresses an initial

dimensional preference of the network. Bias activation is determined by six factors. A bias node is excited by (1) its own activation, (2) positive reinforcement mediated by the learning parameter, (3) by a match-signal computed between input and the category node with the highest activation value, and (4) by the corresponding habit node (provided its activation exceeds a specified threshold). It is inhibited by (5) the competing bias node and by (6) negative reinforcement. The separation of bias and habit subsystems in the current model is an implementation of the idea that memory for reinforcement value and (motor) response memory are functions of interacting, but distinct, subsystems in the brain [2]. A similar distinction has been shown to account for different types of errors subjects make on the WCST, namely failures to maintain set and perseverative errors [14]. Cortical-subcortical computational models have also been developed that make a similar distinction (e.g. [13,16,17]; see also [20] for a neurobiological discussion).

In [18] a neural model of perseveration is developed based on a distinction between active (in the prefrontal cortex) and latent memory representations (in the posterior cortex). This is comparable to, although not identical with, the distinction between active bias memory and latent habit memory in the current model. For any category-learning task (e.g. WCST, discrimination learning etc.) distinguishing between perseverative errors and errors of set-maintenance seems to require a distinction between a memory system that can implement active and flexible responses and a memory system for latent representations of previous responses. This is what we have attempted to implement using the bias and habit subsystems.

With respect to the current model, these two views are not radically different. The bias system is the driving force behind dimensional preference and thus responding in the network. Mediation through reinforcement controls response shifting (i.e. new response-behavior). The habit system keeps track of the responses gives and influences the bias-system to continue in its previous response-behavior (i.e. maintaining response-behavior).

**Modeling Development** Whereas originally reinforcement mediation was used to model differences in perseveration behavior on the WCST between healthy subjects and frontal patients [2], in the current study such mediation is used to model developmental differences in simple discrimination-shift learning. It has been observed that the perseverative errors seen in frontal patients behavior are comparable to the mistakes children make [21]. Our hypothesis is that these mistakes are the result of a reduced effect of corrective feedback, i.e. children have trouble shifting from one rule to another because responding to changing reinforcement contingencies is not as efficient in children as it is in adults.

We test this hypothesis by modeling a simple discrimination task (which is similar to the WCST) using the specified neural network model. Developmental differences

are modeled using different values for the negative feedback component of the reinforcement gain parameter.

As mentioned, individual networks differ with respect to their dimensional preference (bias) and response preference per dimension (response nodes). These differences are not, however, related in any systematic way to the value of the reinforcement gain parameter. Therefore, any difference in performance between adult and child networks will be the result of a difference in negative reinforcement gain.

## Simulations and Results

The networks were submitted to RS and NRS shift tasks. Stimuli follow one of four learning rules: (1) black maps to A, white maps to B, (2) white maps to A, black maps to B, (3) triangle maps to A, circle maps to B and (4) circle maps to A, triangle maps to B. Through a binary encoded vector of one of four stimuli, a black or white triangle or circle, was presented to the network. The network then chooses response category A or B and is given reinforcement dependent upon correct categorization. One fourth of each developmental group began with each rule. Initial dimensional preference was uniformly distributed over all networks via random initialization. The networks were given 48 trials to reach a learning criterion of 10 consecutive correct responses for this initial discrimination-learning rule. The rule was then shifted with either a reversal or non-reversal rule and 48 trials were presented. Networks that failed to reach the 10 sequentially correct criteria were removed from analyses (as is standard procedure in human studies). Further details and parameter values are presented in the Appendix.

**Discrimination Learning** An analysis of variance on the number of trials in the preshift phase reveals a main effect of age (i.e. adult versus child) on the number of trials. Adult networks learn significantly faster than child networks.  $F(1; 418) = 466.46$ ,  $p < 0.001$ . An interaction effect between initial bias and rule was also found. Networks with an initial bias-matching rule (i.e. the initial bias matches the rule to be learned) learn significantly faster than networks with an initial bias not matching rule (non-matching networks).  $F(3; 418) = 160.63$ ,  $p < 0.001$ . Finally, an interaction effect was found between age, initial bias and rule.  $F(3; 418) = 67.09$ ,  $p < 0.001$ . That is, the difference in number of trials to criterion between matching and non-matching networks is greater for child networks than for adult networks (a similar effect is reported in [1]). The reduced effect of corrective feedback for child networks thus magnifies, as expected, the differences in learning rate between matching and non-matching networks.

**Markov Model Analysis** Several hidden Markov models were fit to the trial-by-trial data generated by adult and child networks. These statistical models include single component models of incremental learning and of discontinuous learning. In addition also combination of all single component models, i.e. two-component models, are fitted to

the data. For the simulation data generated by the adult networks, a single component discontinuous-learning model provided the best fit. The learning parameter, i.e. the probability of moving from the presolution state to the learned state, was estimated at 0.19. For the data generated by the child networks, a two-component model consisting of two discontinuous learning modes provided the best fit. For the slow, discontinuous learning component (describing 70% of the data), the learning parameter was estimated at 0.064. For the fast learning component the learning parameter was estimated at 0.35. Estimated parameters were very similar to the parameter estimates in [1].

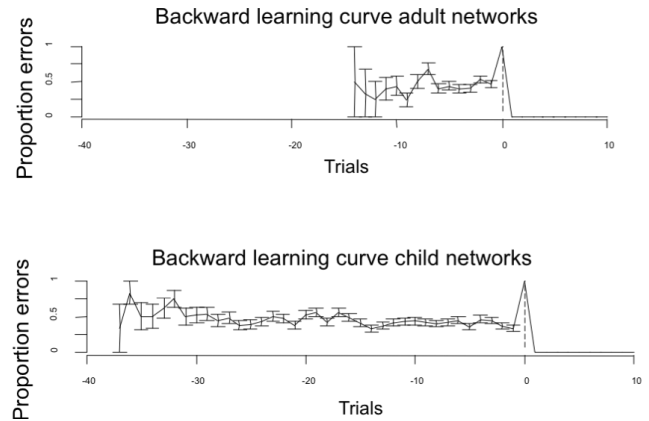


Figure 2: Backward learning curves of simulation data. Note that the peak at trial 0 is an artifact of the method because all sequences are aligned at the last error.

**Backward Learning Curves** The discontinuity of the learning process is apparent from the backward learning curves. As can be seen from figure 3 for both adult (a) and child (b) networks the curve is flat at the 0.5 level before learning (left of the dashed line). It then jumps to near zero once the criterion is mastered (right of the dashed line). This clearly indicates the existence of a discontinuous learning process.

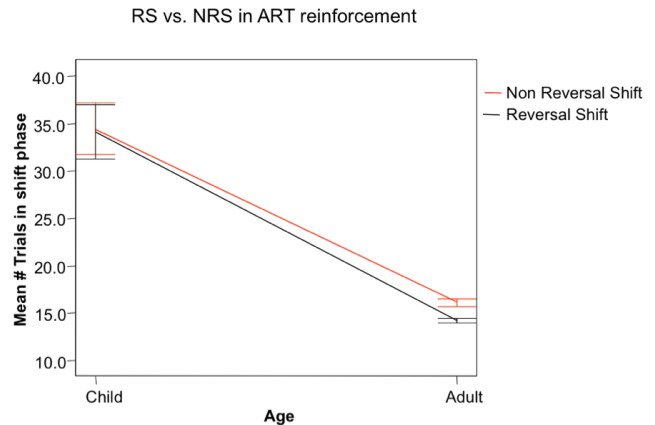


Figure 3: Shift learning in adult and child networks

**Discrimination-Shift Learning** The mean number of trials to learning criterion post-shift is less for adult than child networks. An ANOVA with number of trials to learning criterion as dependent variable and age (adult/child), rule matching bias (yes/no) and shift type (reversal/non-reversal) revealed a significant main effect for age:  $F(1,625)=586.22$ ,  $p<.001$ . Matching nor shift type revealed main effects,  $F(1,625)=1.83$ ,  $p=.176$  and  $F(1,625)=1.92$ ,  $p=.678$  respectively. A significant effect of shift type on trials-to-criterion is found for adult networks:  $F(1,398)=67.15$ ,  $p<.001$ . No significant result for child networks was found:  $F(1,398)=.869$ ,  $p=.176$ . See Figure 3.

## Conclusion

We have presented a neural network model of the development of discrimination-shift learning that shows discontinuous learning in distinct learning modes. The effectiveness of negative reinforcement was varied between adult networks (high) and child networks (low). Networks modeling adult performance learn fast and are modeled by a fast, discontinuous learning process. The learning process is discontinuous for all child networks, and is best modeled by a two-component model with components for fast and slow learners. That is, only a subset of the child networks learn slower than the adult networks. Differences in learning rate were caused by an interaction between initial bias and the rule to be learned. Child networks are less able to switch dimensional preference and category-to-response connections because of the decreased influence of negative reinforcement compared to the adult networks. For child networks this results in a mixture of learning modes as found in the empirical study [1]. Note that the two-component structure of the trial-by-trial learning data of the child networks is the result of a uniform distribution of initial dimensional preferences. That is, a continuous variation in networks' initial state results in categorical difference in performance.

The implementation of the stochastic process within the category-to-response connection based on the negative reinforcement parameter provided the required differentiation between stages of development after the shift has occurred. As expected, child networks executed shifts slower than adult networks. Also, reversal shifts were easier than non-reversals for adult networks, yet for child networks no significant difference was found. These results concur with empirical evidence (e.g. Espirito, 1975). From the model we can predict that children have a larger variation in the number of trials they need for shift learning than adults. This would also explain the inconsistent results presented in literature.

A next step towards a more complete model of discontinuous discrimination-shift learning is a network that is able to induce its own representational categories based purely on input. In addition, future research could examine whether the overtraining effect can also be reproduced,

whereby child networks with further training beyond criterion would perform like adult networks.

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## Appendix: Network Specification

The network specification agrees with [22] with a few modifications for this DSL model. Input  $I_i$  is set to 5 when feature  $i$  ( $i=1,2,3,4$ : 1=black, 2=white, 3=triangle, 4=circle) is present in the stimulus and 0 otherwise. The feature nodes  $x_{i=1,2,3,4}$  are activated depending on  $I_i$  and category node activation, represented by  $y_j$  ( $j=1,2,3,4$ : 1=black, 2=white, 3=triangle, 4=circle) and weighted by  $z_{ji}$ :

$$dx_i/dt = -Ax_i + (B - Cx_i)(I_i + \sum_{j=1}^4 f(y_j)z_{ji}) - Dx_i \sum_{j=1}^4 f(y_j), i = 1,2,3,4 \quad (1)$$

with  $A=10$ ,  $B=5$ ,  $C=1$ ,  $D=1$  and  $f$  defined by:

$$f(x) = \arctan(x - 1) + \pi/2 \quad (2)$$

Feature nodes  $x_i$  activate category nodes  $y_j$  weighted by  $z_{ji}$  and bias nodes  $\Omega_k$  ( $k=1,2$ : 1=color, 2=shape):

$$dy_j/dt = -Ay_j + (B - Cy_j)(f(y_j) + \sum_{i=1}^4 g(\Omega_{[(i+3)/4]}x_i)z_{ji}) \quad (3)$$

with  $I=100$  and  $g$  defined by:

$$g(x) = 0; x < .5, .5 \leq x \leq 2.5, x > 3 \quad (4)$$

Weights  $z_{ji}$  and  $z_{ji}$  between  $x$  and  $y$  are fixed. If  $i=j$ ,  $z_{ji}=5$  and otherwise  $z_{ji}=0$ ;  $z_{ji}=z_{ji}/5$  resulting in 0 and 1 values. The category node  $y_i$  with the highest activation value and the negative reinforcement gain  $\alpha$  determines the response  $a$  weighted by the weights between the two, initialized randomly at 1 or 0 and updated by:

$$a^{RESP1:2} = \sum_{j=1}^4 y_{choice_j} w_{yr,j} \quad (5)$$

with  $S=6.5$  and  $y_{choice}$  represents  $y$  node activity where  $y_{choice_j}=1$  if  $y_j=\max(y)$  and 0 otherwise. The stochastic process added to augment response choice samples from a uniform random distribution between 0 and 6.5; if the value is  $< \alpha$  then  $\sigma$  is 1 and otherwise 0. Weights  $w_{yr}$  are updated as follows:

$$\Delta w_{yr,j} = (t_{r,j} - \alpha^{RESP1:2}) y_{choice_j} \sigma \quad (6)$$

with  $t_r$  set to 1 if response is correct and 0 if incorrect. The bias node activates itself, is inhibited by the competing bias node and is activated by the corresponding habit node and the reinforcement value mediated by positive and negative reinforcement,  $\alpha^+$  and  $\alpha^-$  respectively.

$$d\Omega_k/dt = -E\Omega_k + \{(F - \Omega_k)((h_k - \theta_1)^+ + \gamma^+ R^+ + g(\Omega_k)) - \Omega_k(\gamma^- R^- + G \sum_{r \neq k} g(\Omega_r))\} f(\Phi_k) \quad (7)$$

with  $E=.01$ ,  $F=3$ ,  $G=1.1$  and  $\theta_1=1$ . The reinforcement value is 1 for correct and -1 for incorrect response. Initial bias values are sampled from a uniform random distribution between 1.7 and 2.3 for adult and child networks. Positive and negative reinforcement gain is set to 1.2 and 8 for adult networks and 1.2 and .8 for child networks. Variable is a match signal occurring between input  $I$  and category node  $y$ :

$$\Phi_k = \sum_{i=4k-3}^{4k} z_{ji} I_i \quad (8)$$

in which  $i$  is the index of feature node  $x$ .  $J$  is the index of  $\max(y)$ , and  $k$  the index of the corresponding bias node.  $I_i$  is the input signal (5 or 0). Activation of the habit nodes is given by with  $H=.1$ ,  $J=3$  and  $\theta_2=.5$ .

$$dh_k/dt = Hh_k\{(J - h_k) - (\Phi_k - \theta_2)^+ - (\Phi_k - \theta_2)^-\} \quad (9)$$