Comment on the paper by Y.Komura and Y.Okabe [arXiv:1011.3321]

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Comment on the paper by Y. Komura and Y. Okabe, *J.Phys. A* **44**, 015002, 2011

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Abstract

We point out that the claim of strong universality of [1] is incorrect, as it contradicts known rigorous results.

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The generalised lattice ferromagnetic XY-model was introduced by two of us in [2]. Its simplest version involves 3–component unit spins on a D-dimensional lattice, parameterized by standard angles in the spherical coordinates and interacting according to the ferromagnetic nearest-neighbour Hamiltonian

$$H = -J \sum_{<i,j>} (sin\theta_i \, sin\theta_j)^q \; cos(\varphi_i - \varphi_j) \; , \;\; J > 0 \; .$$

When $D = 2$, the model yields orientational disorder at all finite temperatures, and produces a transition to a low–temperature Berezinskiĭ–Kosterlitz–Thouless (BKT) phase, possessing slow decay of magnetic correlations and infinite susceptibility [2]. In turn, the transition is expected, and was found by simulation [3], to exhibit the usual BKT scenario, at least for small values of q. On the other hand, this scenario does not hold for *all* values of q, i.e. the BKT transitions are *not universal*.

Indeed, we *proved* in [4] that the named transition turns first–order for sufficiently large q. Recall that a hint of this behaviour was already suggested by numerical analysis [3].

Notice that the above *rigorous* results entail that both existence and the type of transition *depend* on the parameter q, i.e. they exclude universality.

The above-mentioned generalized XY–model was recently addressed by simulation for various values of q in Ref. [1]. Although the authors found only a certain evidence of the BKT criticality, they claim that this behaviour holds for *all values* of q. This claim can not be correct since it contradicts to our rigorous results [4], which seem to have been unknown to the authors of the Ref. [1].

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