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# Frequency Coupling Admittance Modeling of Quasi-PR Controlled Inverter and Its Stability Comparative Analysis Under the Weak Grid

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**ABSTRACT** This paper intends to comparatively study the stabilities of grid-connected inverters with three closely related controllers: quasi-proportional resonance (quasi-PR), proportional integral (PI), and proportional resonance (PR) under the weak grid. Firstly, considering the influence of frequency coupling characteristic, a frequency coupling admittance model of quasi-PR controlled inverter is established. Then, the admittance characteristics of the quasi-PR, PI and PR controlled inverters are compared. Admittance characteristics of the PI and PR controlled inverters are similar while the quasi-PR controlled inverter is quite different: the amplitude of the quasi-PR controlled inverter is larger than that of the PI controlled inverter and the phase difference between the two inverters is obvious in the mid-high frequency areas, which are mainly caused by the resonance bandwidth of the quasi-PR controller. Furthermore, the stabilities of the quasi-PR, PI and PR controlled inverters are analyzed. The stabilities of the PI and PR controlled inverters are similar but the quasi-PR controlled inverter is more sensitive to weak grid and high inverter output power. To achieve the same system stability, the voltage outer-loop bandwidth of the quasi-PR controlled inverter should be designed narrower than that of the PI and PR controlled inverters. Finally, experiments verify the correctness of the analyses.

**INDEX TERMS** Frequency coupling, admittance modeling, quasi-PR control, stability analysis.

## I. INTRODUCTION

With the development of renewable energy generation, more and more power electronic converters are connected to the grid [1]–[3]. Inverters are used as the interface for renewable energy to connect to the grid, thus its control method has received a lot of attention [4], [5].

The PI control method in dq-domain (the synchronous reference frame) [6], [7], the PR and quasi-PR control methods in phase-domain (the stationary reference frame) [8], [9] are often used in the inner-loop control of grid-connected inverters. As a typical grid-connected inverter

control method, the stability problem of the PI controlled inverter has been widely studied in [10], [11]. A small-signal transfer function model for the PI controlled inverter to analyze system stability has been established in [10] and the stability of the PI controlled inverter through the impedance-based method has been analyzed [11]. However, since the zero steady-state error performance of the PI control method should be achieved in dq-domain, several times of coordinate transformation are required during the control process [12]. Therefore, the conversion relationship between the control in dq-domain and the control in phase-domain is established, and the PR control method in phase-domain which is equivalent to the PI control method in dq-domain is proposed [13]. To reduce the sensitivity of

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where  $k_{p_v}$ ,  $k_{p_i}$  and  $k_{pr}$  are proportional coefficients;  $k_{i_v}$ ,  $k_{i_i}$ , and  $k_r$  are integral coefficients;  $\omega_r$  is resonance bandwidth of quasi-PR controller; the center frequency of the quasi-PR controller  $\omega_0 = 2\pi f_1$ ;  $f_1$  is the fundamental frequency.

$$G_{PR}(s) = G_{PI} \left( \frac{s^2 + \omega_0^2}{2s} \right) = k_{pr} + \frac{2k_r s}{s^2 + \omega_0^2} \quad (3)$$

Existing research shows that, after the conversion in (3), the equivalent controller, PR controller  $G_{PR}(s)$  in AC system, can be obtained from the PI controller  $G_{PI}(s)$  in DC system [13].

The bode plot for the transfer function of the PR controller is shown as the solid blue curve in Fig. 2. The PR control method can realize the zero steady-state error for tracking the fundamental signal.

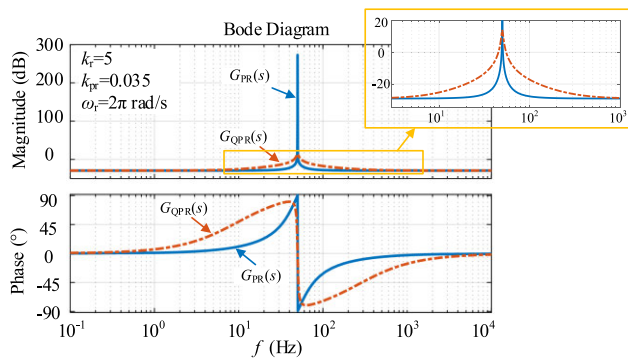


FIGURE 2. Bode diagram of transfer function in different control methods.

Considering the frequency fluctuations in the grid, a resonance bandwidth is set to make the controller can track the signal almost without steady-state error [14], which is called quasi-PR control method. According to EN 50438, the frequency fluctuation in the grid is less than  $\pm 1$  Hz while the grid frequency is 50 Hz [15]. During the operation of the grid in China, the grid frequency is generally fluctuating in 49.5 Hz to 50.5 Hz. Thus,  $\omega_r$  is set as  $2\pi$  rad/s in this paper to meet the requirements of the control performance. The bode plot for the transfer function of the quasi-PR controller is shown with the dotted orange curve in Fig. 2. This controller can nearly realize zero steady-state error performance in the resonance bandwidth  $\omega_r$  with the center frequency  $\omega_0$ .

Fig. 2 shows the difference between  $G_{QPR}(s)$  and  $G_{PR}(s)$  exists in the relatively wide frequency range. The difference between the magnitudes of  $G_{QPR}(s)$  and  $G_{PR}(s)$  exists in 10 Hz to 300 Hz. What's more, the obvious difference between the phases of  $G_{QPR}(s)$  and  $G_{PR}(s)$  exists in 2 Hz to 1000 Hz.

The PR control method in phase-domain can be derived from the PI control method in dq-domain, thus they can make the grid-connected inverters achieve a similar performance in different coordinate systems. However, the quasi-PR control method is obtained from the PR control method through introducing resonance bandwidth, which seems make it different with the PR control method. Therefore, it is worth

to be studied whether the stability of the quasi-PR controlled inverter is consistent with that of the PR and PI controlled inverters. From the view of frequency coupling admittance, this paper intends to study the stabilities of the quasi-PR, PI and PR controlled inverters.

### III. FREQUENCY COUPLING ADMITTANCE MODELING OF QUASI-PR CONTROLLED INVERTER

Admittance characteristics of grid-connected inverters can be affected by control loops, control delay and frequency coupling characteristic. Thus, the admittance model of quasi-PR controlled inverter will be built considering these factors.

After a positive-sequence disturbance voltage at the disturbance frequency  $f_p$  being injected into the quasi-PR controlled inverter system. A positive-sequence disturbance response current at  $f_p$  and a negative-sequence response current at coupling frequency  $f_{p1} = f_p - 2f_1$  will be generated in the output current of the grid-connected inverter due to frequency coupling characteristic. The negative-sequence response current flowing through the grid impedance will generate the negative-sequence voltage at  $f_{p1}$ . Thus,  $i_a$  and  $v_a$  can be expressed in frequency-domain as below.

$$\mathbf{I}_a[f] = \begin{cases} \mathbf{I}_1, & f = \pm f_1 \\ \mathbf{I}_p, & f = \pm f_p \\ \mathbf{I}_{p1}, & f = \pm f_{p1}, \end{cases} \quad \mathbf{V}_a[f] = \begin{cases} \mathbf{V}_1, & f = \pm f_1 \\ \mathbf{V}_p, & f = \pm f_p \\ \mathbf{V}_{p1}, & f = \pm f_{p1} \end{cases} \quad (4)$$

where  $\mathbf{I}_1 = (I_1/2)e^{\pm j\varphi_{i1}}$ ,  $\mathbf{I}_p = (I_p/2)e^{\pm j\varphi_{ip}}$ ,  $\mathbf{I}_{p1} = (I_{p1}/2)e^{\pm j\varphi_{ip1}}$ ,  $\mathbf{V}_1 = V_1/2$ ,  $\mathbf{V}_p = (V_p/2)e^{\pm j\varphi_{vp}}$ ,  $\mathbf{V}_{p1} = (V_{p1}/2)e^{\pm j\varphi_{vp1}}$ ;  $I_1, I_p, I_{p1}$  are the amplitudes of the currents, respectively;  $V_1, V_p$  and  $V_{p1}$  are the amplitudes of the voltages, respectively;  $\varphi_{i1}, \varphi_{ip}, \varphi_{ip1}, \varphi_{vp}$  and  $\varphi_{vp1}$  are the initial phase angles of the corresponding current and voltage components, respectively.

#### A. MODELING OF THE VOLTAGE OUTER-LOOP

The voltage outer-loop control is the important part of the control strategy of the grid-connected inverter in Fig. 1. To accurately describe the admittance characteristics of the grid-connected inverter, the voltage outer-loop control should be considered.

According to the law of energy conservation, the power on the DC-side is equal to that on the AC-side.

$$(i_{pv} - sC_{dc}v_{dc})v_{dc} = (v_a + sL_f i_a) i_a + (v_b + sL_f i_b) i_b + (v_c + sL_f i_c) i_c \quad (5)$$

The expression of  $v_{dc}$  at  $f = \pm(f_p - f_1)$  can be obtained from (5) after the convolution calculation in frequency-domain.

$$v_{dc} = \frac{3 [\mathbf{I}_1 \mathbf{V}_{p1} + (sL_f \mathbf{I}_1 + \mathbf{V}_1) \mathbf{I}_{p1} + (\mathbf{V}_1^* + sL_f \mathbf{I}_1^*) \mathbf{I}_p + \mathbf{I}_1^* \mathbf{V}_p]}{i_{pv} - sC_{dc}V_{dc}} \quad (6)$$

where the superscript “\*” represents the conjugated variable;  $V_{dc}$  is the steady-state component of the DC-side voltage.

In order to simplify the expression, we can define

$$v_{dc} [f] = F_i \mathbf{I}_p + F_{i1} \mathbf{I}_{p1} + F_v \mathbf{V}_p + F_{v1} \mathbf{V}_{p1} \quad (7)$$

where

$$\begin{bmatrix} F_i & F_{i1} \\ F_v & F_{v1} \end{bmatrix} = \begin{bmatrix} \frac{3(\mathbf{V}_1^* + s_2 L_f \mathbf{I}_1^*)}{i_{pv} - s_2 C_{dc} V_{dc}} & \frac{3(\mathbf{V}_1 + s_2 L_f \mathbf{I}_1)}{i_{pv} - s_2 C_{dc} V_{dc}} \\ \frac{3\mathbf{I}_1^*}{i_{pv} - s_2 C_{dc} V_{dc}} & \frac{3\mathbf{I}_1}{i_{pv} - s_2 C_{dc} V_{dc}} \end{bmatrix}.$$

It is defined that  $s = \pm j2\pi f_p$ ,  $s_1 = \pm j2\pi f_1$ ,  $s_2 = \pm(j2\pi f_p - j2\pi f_1)$ ,  $s_{p1} = \pm(j2\pi f_p - j4\pi f_1)$ .

The reference value of the output current in dq-domain  $i_{dr}$  can be expressed as shown in below.

$$i_{dr} = (v_{dc} - V_{dc}) G_v (s_2) \quad (8)$$

Submitting (6) into (8), it can be obtained

$$i_{dr} [f] = \begin{cases} I_{dr}, & \text{dc} \\ G_v (s_2) G_{vf} (s_2) v_{dc}, & f = \pm (f_p - f_1) \end{cases} \quad (9)$$

## B. MODELING OF THE PLL

Considering the disturbance, the PLL output can be expressed as  $\theta_{PLL}(t) = \Delta\theta_{PLL}(t) + \theta_1(t)$ . According to the derivation method in [5], the relationship between  $\Delta\theta_{PLL}$  and the disturbance voltage in PCC at  $f = \pm(f_p - f_1)$  can be obtained. The PLL transfer function  $G_{PLL}(s_2) = (k_{p\_PLL} + k_{i\_PLL}/s_2)/s_2$ .

$$\Delta\theta_{PLL}[f] = T_p (s_2) G_{vf} (s) \mathbf{V}_p + T_{p1} (s_2) G_{vf} (s_{p1}) \mathbf{V}_{p1} \quad (10)$$

$$T_p (s_2) = \frac{\mp j G_{PLL} (s_2)}{1 + V_1 G_{PLL} (s_2)}, \quad T_{p1} (s_2) = \frac{\pm j G_{PLL} (s_2)}{1 + V_1 G_{PLL} (s_2)} \quad (11)$$

The output current reference value in phase A  $i_{ar}$  can be obtained by the following formula.

$$i_{ar} [f] = \cos(\theta_1 [f]) \otimes (i_{dr} [f] - \Delta\theta_{PLL} [f]) \otimes i_{qr} [f] - \sin(\theta_1 [f]) \otimes (\Delta\theta_{PLL} [f]) \otimes i_{dr} [f] + i_{qr} [f] \quad (12)$$

where the steady-state component of  $i_{qr}$  is zero, the symbol “ $\otimes$ ” represents a convolution calculation.

Combining (9), (10) and (12), the specific expression of  $i_{ar}$  can be derived as below.

$$i_{ar} [f] = \begin{cases} 0.5 (i_{dr} [s_2] \pm j I_{dr} \Delta\theta_{PLL} [s_2]), & f = \pm f_p \\ 0.5 (i_{dr} [s_2] \mp j I_{dr} \Delta\theta_{PLL} [s_2]), & f = \pm (f_p - 2f_1) \end{cases} \quad (13)$$

## C. MODELING OF THE QUASI-PR CONTROLLED INVERTER

According to the control block diagram shown in Fig. 1(c), the expression of the modulation signal  $m_1$  can be obtained.

$$m_1 = \frac{\mathbf{V}_1 + j2\pi f_1 L_f \mathbf{I}_1}{K_m V_{dc} G_{del} (s_1)} \quad (14)$$

where the control delay is expressed as  $G_{del}(s) = e^{-1.5T_s s}$ , and  $T_s$  is the sampling period.

From Fig. 1(c), the relationship between the output voltage and inductor current of the grid-connected inverter can be obtained as follows.

$$sL_f i_a = K_{pwm} G_{del} v_{dc} \otimes [(i_{ar} - i_a) G_{QPR} (s) + K_f v_a] - v_a \quad (15)$$

Combining (5), (13) and (15), the admittance matrix of the grid-connected inverter can be calculated as

$$-\begin{bmatrix} \mathbf{I}_p \\ \mathbf{I}_{p1} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_p \\ \mathbf{V}_{p1} \end{bmatrix} \quad (16)$$

The expressions of  $Y_{11}$ ,  $Y_{12}$ ,  $Y_{21}$  and  $Y_{22}$  in the admittance matrix are shown in (17) to (20). Those variables in (17) to (20) as shown at the bottom of the page.

$$\begin{bmatrix} J_i \\ J_{i1} \end{bmatrix} = \begin{bmatrix} -V_{dc} G_{QPR} (s) G_{if} (s) \\ -V_{dc} G_{QPR} (s_{p1}) G_{if} (s_{p1}) \end{bmatrix} \quad (17)$$

$$\begin{cases} H_p = 0.5 V_{dc} G_{vf} (s_2) G_v (s_2) G_{QPR} (s) + m_1 \\ H_{p1} = 0.5 V_{dc} G_{vf} (s_2) G_v (s_2) G_{QPR} (s_{p1}) + m_1^* \end{cases} \quad (22)$$

$$Y_{11} = \frac{[s_{p1} L_f - (F_{i1} H_{p1} + J_{i1}) K_{pwm} G_{del} (s_{p1})] [(F_v H_p + D_{v1}) K_{pwm} G_{del} (s) - 1] + F_{i1} H_p (F_v H_{p1} + D_{v3}) K_{pwm}^2 G_{del} (s) G_{del} (s_{p1})}{- [s L_f - (F_i H_p + J_i) K_{pwm} G_{del} (s)] [s_{p1} L_f - (F_{i1} H_{p1} + J_{i1}) K_{pwm} G_{del} (s_{p1})] + F_{i1} H_p F_i H_{p1} K_{pwm}^2 G_{del} (s) G_{del} (s_{p1})} \quad (17)$$

$$Y_{12} = \frac{[s_{p1} L_f - (F_{i1} H_{p1} + J_{i1}) K_{pwm} G_{del} (s_{p1})] (F_{v1} H_p + D_{v2}) K_{pwm} G_{del} (s) + [(F_{v1} H_{p1} + D_{v4}) K_{pwm} G_{del} (s_{p1}) - 1] F_{i1} H_p K_{pwm} G_{del} (s)}{- [s L_f - (F_i H_p + J_i) K_{pwm} G_{del} (s)] [s_{p1} L_f - (F_{i1} H_{p1} + J_{i1}) K_{pwm} G_{del} (s_{p1})] + F_{i1} H_p F_i H_{p1} K_{pwm}^2 G_{del} (s) G_{del} (s_{p1})} \quad (18)$$

$$Y_{21} = \frac{F_i H_{p1} [(F_v H_p + D_{v1}) K_{pwm} G_{del} (s) - 1] K_{pwm} G_{del} (s_{p1}) + [s L_f - (F_i H_p + J_i) K_{pwm} G_{del} (s)] (F_v H_{p1} + D_{v3}) K_{pwm} G_{del} (s_{p1})}{- [s L_f - (F_i H_p + J_i) K_{pwm} G_{del} (s)] [s_{p1} L_f - (F_{i1} H_{p1} + J_{i1}) K_{pwm} G_{del} (s_{p1})] + F_{i1} H_p F_i H_{p1} K_{pwm}^2 G_{del} (s) G_{del} (s_{p1})} \quad (19)$$

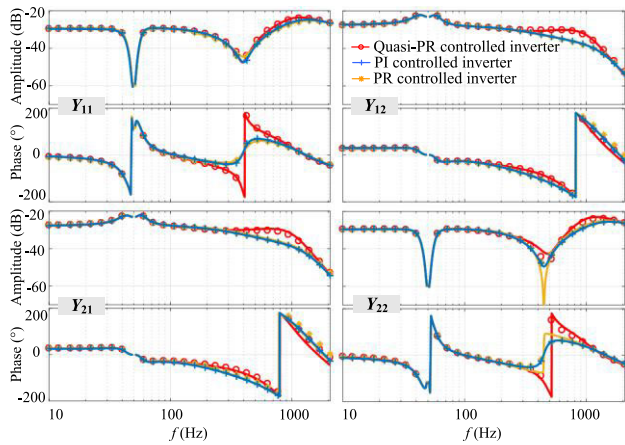
$$Y_{22} = \frac{F_i H_{p1} (F_{v1} H_p + D_{v2}) K_{pwm}^2 G_{del} (s) G_{del} (s_{p1}) + [s L_f - (F_i H_p + J_i) K_{pwm} G_{del} (s)] [(F_{v1} H_{p1} + D_{v4}) K_{pwm} G_{del} (s_{p1}) - 1]}{- [s L_f - (F_i H_p + J_i) K_{pwm} G_{del} (s)] [s_{p1} L_f - (F_{i1} H_{p1} + J_{i1}) K_{pwm} G_{del} (s_{p1})] + F_{i1} H_p F_i H_{p1} K_{pwm}^2 G_{del} (s) G_{del} (s_{p1})} \quad (20)$$



$$\begin{cases} D_{v1} = (\pm 0.5jI_{dr}G_p(s_2)G_{QPR}(s) + K_f)V_{dc}G_{vf}(s) \\ D_{v2} = \pm 0.5jI_{dr}V_{dc}G_{vf}(s_{p1})G_{p1}(s_2)G_{QPR}(s) \\ D_{v3} = \mp 0.5jI_{dr}V_{dc}G_{vf}(s)G_p(s_2)G_{QPR}(s_{p1}) \\ D_{v4} = (\mp 0.5jI_{dr}G_{p1}(s_2)G_{QPR}(s_{p1}) \\ \quad + K_f)V_{dc}G_{vf}(s_{p1}) \end{cases} \quad (23)$$

**IV. COMPARISON OF ADMITTANCE CHARACTERISTICS OF THE QUASI-PR, PI AND PR CONTROLLED INVERTERS**

The parameters of the quasi-PR controlled inverter studied in this paper are shown in Table 1. In order to verify the correctness of the established admittance model, an admittance measurement platform was established in Matlab/Simulink. The admittance measurement results of the quasi-PR controlled inverter are shown in Fig. 3. The solid red line indicates the established frequency coupling admittance model of the quasi-PR controlled inverter, and the red circle represents the admittance measurement result. It can be seen from Fig. 3 that the admittance measurement results essentially agree with the established frequency coupling admittance model, which proves the correctness of the established frequency coupling admittance model of the quasi-PR controlled inverter.



**FIGURE 3. Admittance characteristic curves of the grid-connected inverter under these three control methods.**

The frequency coupling admittance models of the PI and PR controlled inverters are shown as (24)-(25).

$$-\begin{bmatrix} \mathbf{I}_p \\ \mathbf{I}_{p1} \end{bmatrix} = \begin{bmatrix} Y_{11\_PI} & Y_{12\_PI} \\ Y_{21\_PI} & Y_{22\_PI} \end{bmatrix} \begin{bmatrix} \mathbf{V}_p \\ \mathbf{V}_{p1} \end{bmatrix} \quad (24)$$

$$-\begin{bmatrix} \mathbf{I}_p \\ \mathbf{I}_{p1} \end{bmatrix} = \begin{bmatrix} Y_{11\_PR} & Y_{12\_PR} \\ Y_{21\_PR} & Y_{22\_PR} \end{bmatrix} \begin{bmatrix} \mathbf{V}_p \\ \mathbf{V}_{p1} \end{bmatrix} \quad (25)$$

In order to compare and analyze the admittance characteristics of the quasi-PR, PI and PR controlled inverters more conveniently, the admittance model and measurement results of the PI and PR controlled inverters also are drawn in Fig. 3. The admittance modeling of the PI controlled inverter has been studied a lot [5], in addition, the frequency coupling admittance model of the PR controlled inverter only need replace the  $G_{QPR}(s)$  in the established admittance model of the PR controlled inverter with  $G_{PR}(s)$ . Thus, the frequency

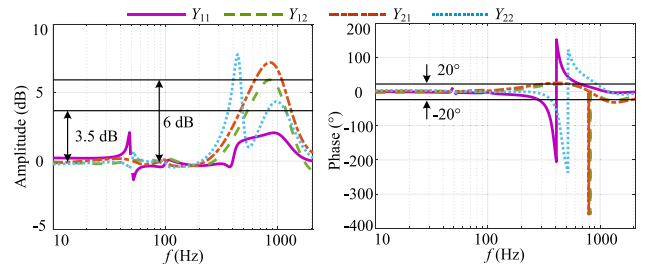
**TABLE 1. Parameters of the Quasi-PR controlled inverter.**

Symbol	Quantity	Values
$L_f$	filter inductance	3 mH
$C_f$	filter capacitor	20 $\mu$ F
$R_d$	damping resistance of filter capacitor	1.5 $\Omega$
$V_{ref}$	DC-side voltage	700 V
$V_i$	grid voltage	311 V
$f_i$	fundamental frequency	50 Hz
$K_{pwm}$	modulation coefficient	0.5
$i_{dc}$	the DC-side current	14.29 A
$T_s$	the sampling period	$5 \times 10^{-5}$ s
$k_{p\_PLL}$	proportional coefficient in $G_{PLL}(s)$	0.2659
$k_{i\_PLL}$	integral coefficient in $G_{PLL}(s)$	10.9988
$\omega_r$	resonance bandwidth of QPR controller	$2\pi$ rad/s
$k_{pr}$	proportional coefficient of QPR controller	0.035
$k_r$	integral coefficient of QPR controller	5
$k_{p\_v}$	proportional coefficient in $G_v(s)$	8.3398
$k_{i\_v}$	integral coefficient in $G_v(s)$	698.5
$\omega_{vf}$	the cut-off frequency of the sampling filters for the voltage	$2\pi \cdot 4000$ rad/s
$\omega_{if}$	the cut-off frequency of the sampling filters for the current	$2\pi \cdot 4000$ rad/s
$K_f$	voltage feed forward coefficient	1/350

coupling admittance models of the PI and PR controlled inverters will not be deduced in this paper.

In Fig. 3, the frequency coupling admittance model of the PI controlled inverter is represented by a solid blue line, and the admittance measurement result is represented by the blue cross; the frequency coupling admittance model of the PR controlled inverter is represented by a solid yellow line, and the admittance measurement result is represented by the yellow star. Their simulation parameters are consistent with that of the quasi-PR controlled inverter.

According to the comparison of the three admittance models, the admittance characteristics of the PR controlled inverter is very similar to that of the PI controlled inverter. However, the quasi-PR controlled inverter is quite different from that of the PI and PR controlled inverters.



**FIGURE 4. Admittance differences between quasi-PR and PI controlled inverters.**

To describe this difference clearly, the admittance differences between the quasi-PR and PI controlled inverters are shown in Fig. 4. The different colors and curves are used to present the admittance differences of  $Y_{11}$ ,  $Y_{12}$ ,  $Y_{21}$ , and  $Y_{22}$ . In the frequency areas 350 Hz to 1300 Hz, the amplitude difference is larger than 3.5 dB. In the frequency areas 400 Hz to 500 Hz and 600 Hz to 1100 Hz, the amplitude difference is

larger than 6 dB which indicates the amplitude of the quasi-PR controlled inverter is more than twice as large as that of the PI controlled inverter. In 250 Hz to 2000 Hz, the phase difference is larger than 20 degrees.

Therefore, the admittance differences between the quasi-PR and PI controlled inverters are quite large.

In order to further explore the factor caused the above admittance differences, we try to focus on the only different parameter  $\omega_r$  between the PI and quasi-PR controllers. The admittance characteristics of the quasi-PR controlled inverter when the  $\omega_r$  is changed are observed and their admittance characteristic differences are compared as shown in Fig. 5.

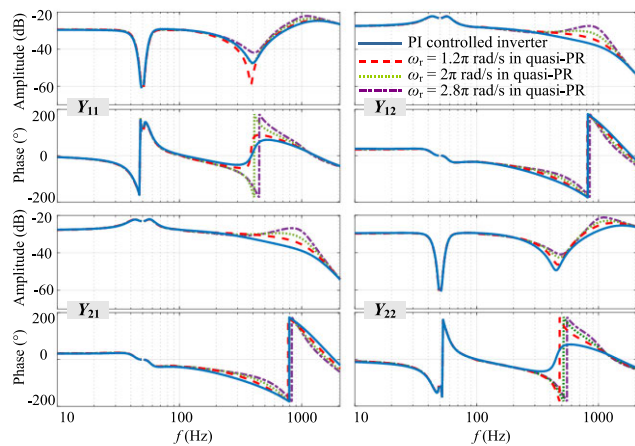


FIGURE 5. Admittance characteristic curves when  $\omega_r$  changes.

In Fig. 5, the change of  $\omega_r$  has a great impact on admittance characteristics in the mid-high frequency areas. As  $\omega_r$  increases, the admittance peaks become sharper. The smaller the  $\omega_r$ , the closer the admittance characteristics of the quasi-PR and PI controlled inverters.

Thus, the resonance bandwidth of the quasi-PR controller is an important factor that causes differences in admittance characteristics of the grid-connected inverters under the quasi-PR and PI control methods.

### V. COMPARATIVE ANALYSES OF SYSTEM STABILITY

Based on the established frequency coupling admittance model and generalized Nyquist criterion, the influence of the grid impedance, inverter output power  $P_s$  and the outer-loop bandwidth  $BW_v$ , on system stability are studied and the system stabilities for the quasi-PR, PI and PR controlled inverters are compared. The equivalent return matrix of the grid-connected inverter system in Fig. 1(a) can be defined as below.

$$L = Z_g \cdot Y_{inv} = \begin{bmatrix} Z_{g11} \cdot Y_{11} & Z_{g11} \cdot Y_{12} \\ Z_{g22} \cdot Y_{21} & Z_{g22} \cdot Y_{22} \end{bmatrix} \quad (26)$$

where  $Z_{g11} = Z_g(s)$ ,  $Z_{g22} = Z_g(s - j4\pi f_1)$ .

The filter capacitor branch is not included in the control of the grid-connected inverter. Thus, the filter capacitor branch can be regarded as a part of the grid impedance:

$$Z_g(s) = \frac{(sL_g + R_g) [R_f + 1/(sC_f)]}{sL_g + R_g + R_f + 1/(sC_f)} \quad (27)$$

According to (26),  $\lambda_1$  and  $\lambda_2$  are defined as the eigenvalues of the return matrix. The system stability is analyzed by the Nyquist diagrams of  $\lambda_1$  and  $\lambda_2$  which are indicated by the solid and dotted line, respectively.  $a = 1$ ,  $b = -(Z_{g11}Y_{11} + Z_{g22}Y_{22})$ ,  $c = Z_{g11}Z_{g22}(Y_{11}Y_{22} - Y_{12}Y_{21})$ .

$$\lambda_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad \lambda_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad (28)$$

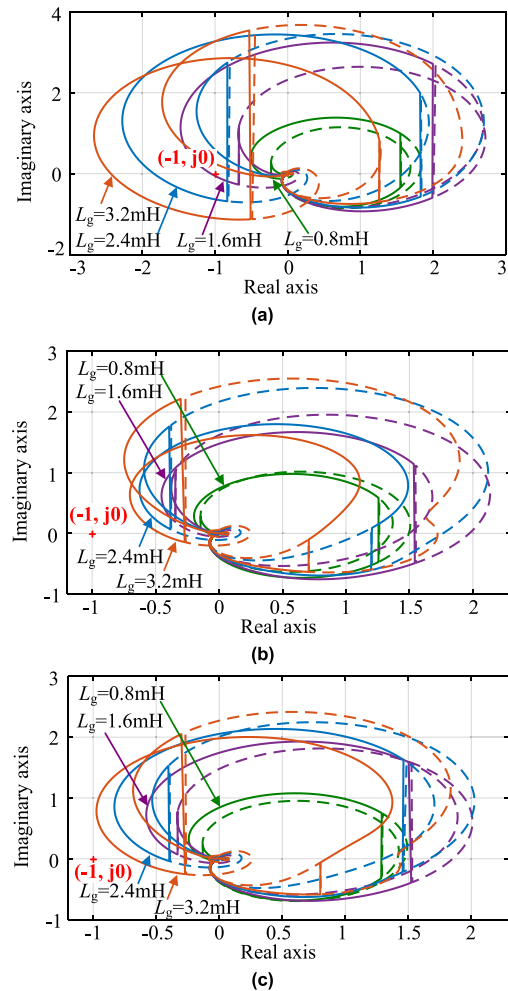


FIGURE 6. The Nyquist diagram when  $L_g$  is changed. (a) The quasi-PR controlled inverter. (b) The PI controlled inverter. (c) The PR controlled inverter.

### A. THE EFFECTS OF THE GRID IMPEDANCE ON SYSTEM STABILITY

To comparatively study system stability when  $L_g$  is changed, the Nyquist diagrams in Fig. 6 are analyzed. In Fig. 6(a), when the  $L_g$  is 1.6 mH, 2.4 mH, or 3.2 mH, the Nyquist curves of the quasi-PR controlled inverter have surrounded the point  $(-1, j0)$ , which indicates the system is unstable. As shown in Fig. 6(b) and Fig. 6(c), the Nyquist curves of the PI and PR controlled inverters do not surround the point  $(-1, j0)$  even the  $L_g$  is 3.2 mH, which indicates the systems can remain stable.

Therefore, under the same conditions, the PI and PR controlled inverters have a similar system stability while the

quasi-PR controlled inverter is more sensitive to the weak grid.

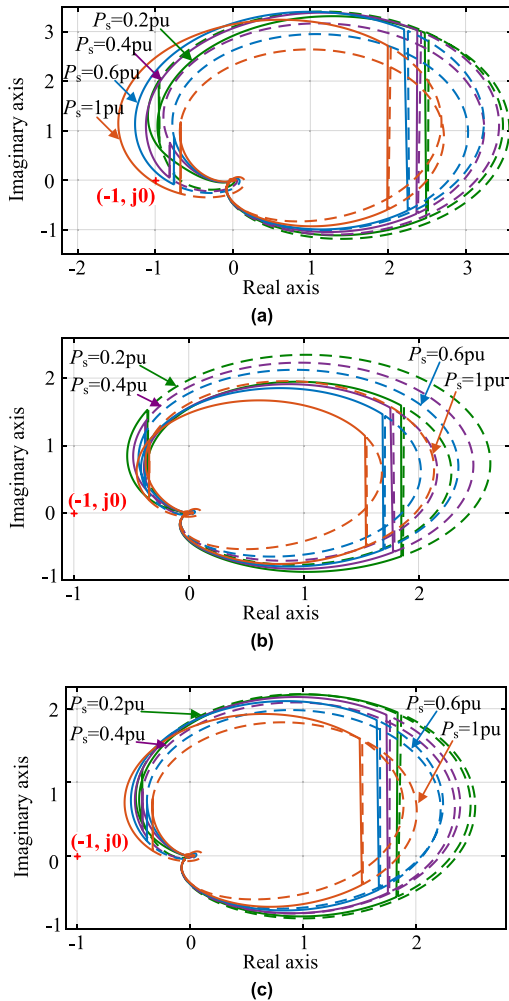


FIGURE 7. The Nyquist diagram when  $P_s$  is changed. (a) The quasi-PR controlled inverter. (b)The PI controlled inverter. (c) The PR controlled inverter.

**B. THE EFFECTS OF THE INVERTER OUTPUT POWER ON SYSTEM STABILITY**

The Nyquist diagrams when the  $P_s$  is 1 pu, 0.6 pu, 0.4 pu, or 0.2 pu in Fig. 7 are analyzed while  $L_g$  is 1.6 mH. The Nyquist curves in Fig. 7(b) and (c) are not surrounds the point  $(-1, j0)$  and the systems are stable. However, the Nyquist curve in Fig. 7(a) surrounds the point  $(-1, j0)$  when the  $P_s$  is 1 pu and the quasi-PR controlled inverter system is unstable.

Thus, under the same operating conditions, compared with the PI and PR controlled inverters, the quasi-PR controlled inverter is more sensitive to high  $P_s$ .

**C. THE EFFECTS OF THE OUTER-LOOP BANDWIDTH ON SYSTEM STABILITY**

In order to study the effects of the outer-loop bandwidth on system stability, while  $L_g$  is 0.8 mH, the Nyquist diagrams when the  $BW_v$  is changed in Fig. 8 are analyzed.

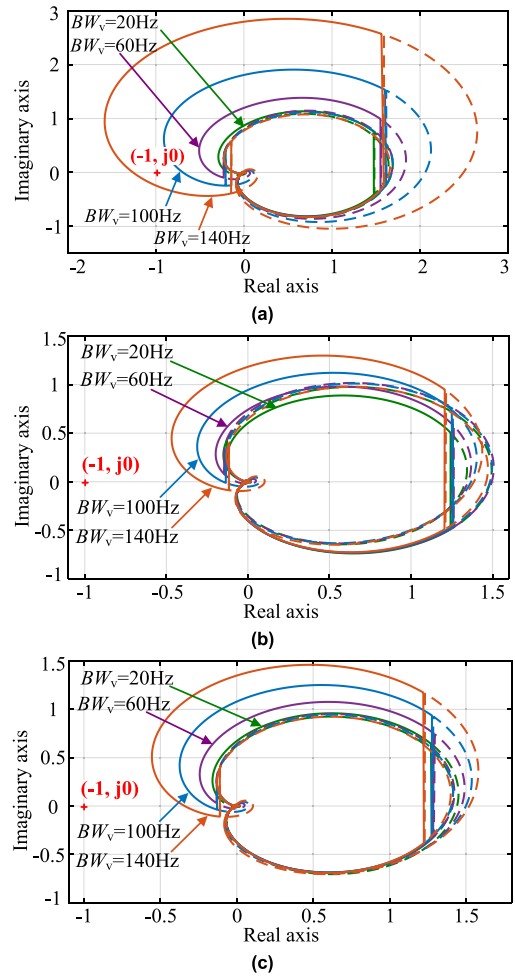


FIGURE 8. Nyquist diagram when  $BW_v$  is changed. (a) The quasi-PR controlled inverter. (b)The PI controlled inverter. (c) The PR controlled inverter.

As shown in Fig. 8(a), when  $BW_v$  is 140 Hz, the Nyquist curve of the quasi-PR controlled inverter has surrounded the point  $(-1, j0)$ , which indicates the system is unstable. In Fig. 8(b) and Fig. 8(c), the Nyquist curves of the PI and PR controlled inverters do not surround the point  $(-1, j0)$ , which indicates the systems are stable.

Thus, under the same operate condition, to achieve the same system stability, the outer-loop bandwidth of the quasi-PR controlled inverter should be designed narrower than that of the PI and PR controlled inverters.

**D. THE EFFECTS OF THE RESONANCE BANDWIDTH ON SYSTEM STABILITY**

To analyze the system stability when  $\omega_r$  is changed, the Nyquist diagram in Fig. 9 is studied while  $L_g$  is 1.6 mH and  $P_s$  is 0.6 pu.

In Fig. 9, when  $\omega_r$  is increased to  $2.8\pi$  rad/s, the Nyquist curve of the quasi-PR controlled inverter has surrounded the point  $(-1, j0)$ , which indicates that the system is unstable. Therefore, with the increasing of  $\omega_r$ , the quasi-PR controlled inverter system tends to be unstable.



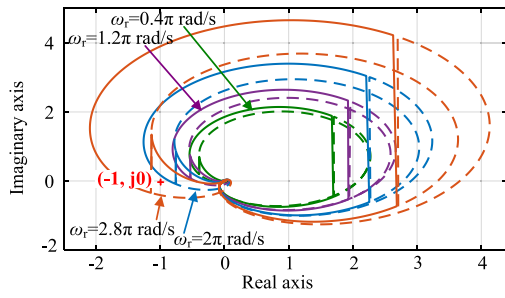


FIGURE 9. Nyquist diagram of quasi-PR controlled inverter when  $\omega_r$  is changed.

From the above analysis, it can be seen that the stabilities of the PI and PR controlled inverters are similar while the stability of the quasi-PR controlled inverter is worse than that of the PI and PR controlled inverters.

In the quasi-PR controlled inverter system, the  $\omega_r$  is an important factor that affects the system stability. To reduce the impact on system stability, the  $\omega_r$  should be set as small as possible in the area which can meet the control requirement.

### VI. EXPERIMENTAL RESULTS

To verify the correctness of the above analysis, experiments were carried out on the controller hardware-in-the-loop experimental platform, as shown in Fig. 10. Experimental parameters are consistent with Section V.

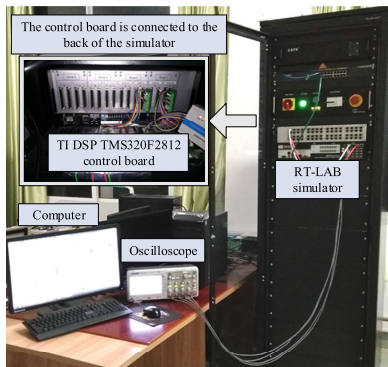


FIGURE 10. The HIL experimental platform.

The design process of the PR controller is introduced below. Firstly, it can be defined that

$$\frac{y(s)}{u(s)} = G_{PR}(s), \quad \frac{y_1(s)}{u(s)} = \frac{2k_r s}{s^2 + \omega_0^2} \quad (29)$$

$y_1(s)$  can be expressed as shown below.

$$y_1(s) = (2k_r u(s) - v(s)) / s \quad (30)$$

where  $v(s) = \omega_0^2 y_1(s) / s$ .

Combining (3), (29) and (30),  $y(s)$  can be expressed as (31) and the block diagram shown is shown in Fig. 11.

$$y(s) = k_{pr} u(s) + (2k_r u(s) - v(s)) \frac{1}{s} \quad (31)$$

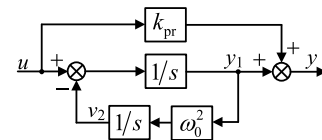


FIGURE 11. The decomposition block diagram of the  $G_{PR}(s)$ .

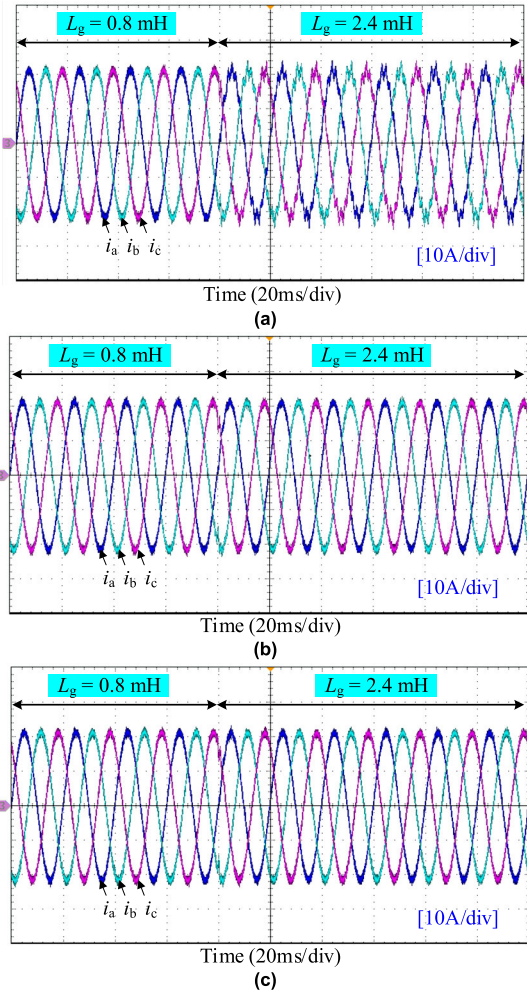


FIGURE 12. The experimental results when  $L_g$  is increased from 0.8 mH to 2.4 mH. (a) The quasi-PR controlled inverter. (b) The PI controlled inverter. (c) The PR controlled inverter.

After discretizing, (31) is shown as below.

$$\begin{cases} y(k) = y(k-1) + T_P [k_{pr} u(k-1) + (2k_r u(k-1) - v(k-1))] \\ v(k) = v(k-1) + \omega_0^2 T_P y(k) \end{cases} \quad (32)$$

where  $T_P$  is PWM carrier period.

According to the discrete expressions of  $y$ ,  $G_{PR}(s)$  can be realized in the TI DSP TMS320F2812 control board.

#### A. THE EFFECTS OF GRID IMPEDANCE ON SYSTEM STABILITY

Fig. 12 shows the experimental results when  $L_g$  is increased from 0.8 mH to 2.4 mH.

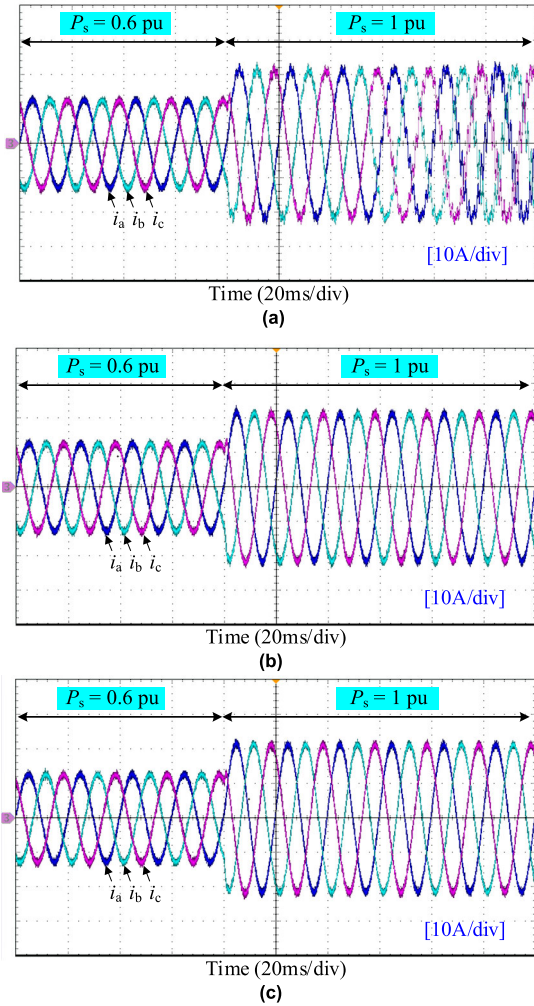


FIGURE 13. The experimental results when  $P_s$  is increased from 0.6 pu to 1 pu. (a) The quasi-PR controlled inverter. (b) The PI controlled inverter. (c) The PR controlled inverter.

When  $L_g$  is increased to 2.4 mH, the quasi-PR controlled inverter is unstable while the PI and PR controlled inverters can still maintain stable. Therefore, under the same operating conditions, the PI and PR controlled inverters have a similar system stability, but the quasi-PR controlled inverter is more sensitive to the weak grid.

**B. THE EFFECTS OF THE INVERTER OUTPUT POWER ON SYSTEM STABILITY**

The experimental results when  $P_s$  is changed are shown in Fig. 13. When  $P_s$  is increased from 0.6 pu to 1 pu, the quasi-PR controlled inverter is unstable while the PI and PR controlled inverters can keep stable.

Thus, under the same operating conditions, compared with PI and PR controlled inverters, the quasi-PR controlled inverter is more sensitive to high inverter output power.

**C. THE EFFECTS OF THE OUTER-LOOP BANDWIDTH ON SYSTEM STABILITY**

The experimental results reflecting the influence of  $BW_v$  are shown in Fig. 14. When  $BW_v$  is increased from 100 Hz

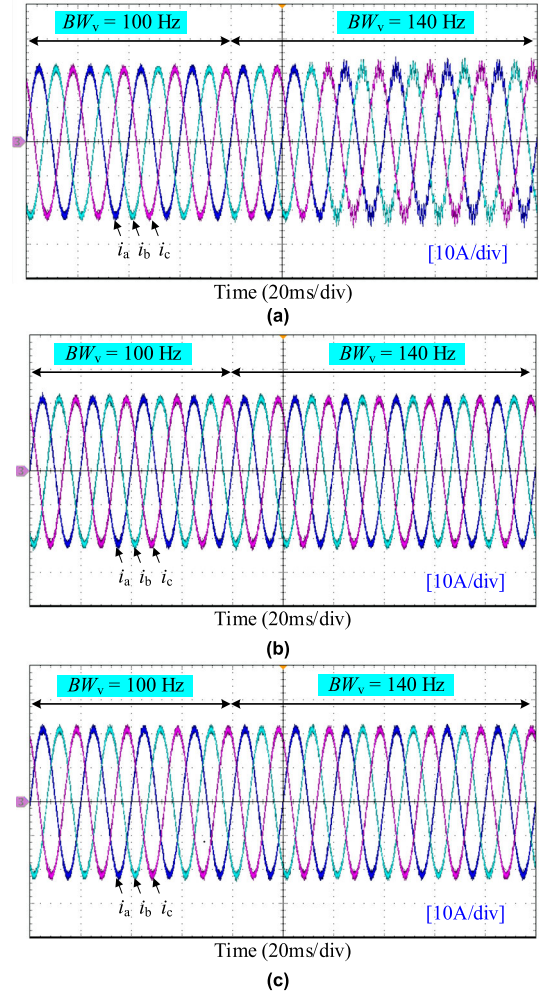


FIGURE 14. The experimental results when  $BW_v$  is increased from 100 Hz to 140 Hz. (a) The quasi-PR controlled inverter. (b) The PI controlled inverter. (c) The PR controlled inverter.

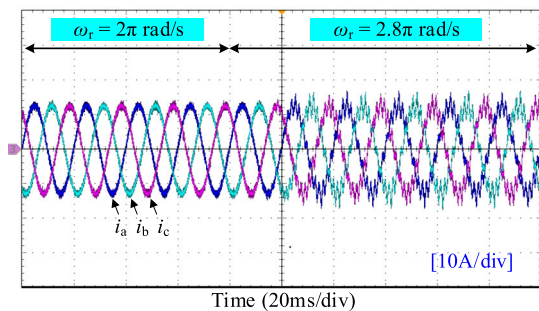


FIGURE 15. The experimental results of the quasi-PR controlled inverter when  $\omega_r$  is increased from  $2\pi$  rad/s to  $2.8\pi$  rad/s.

to 140 Hz, the quasi-PR controlled inverter is unstable while the PI and PR controlled inverters can still maintain stable.

Therefore, under the same conditions,  $BW_v$  of the quasi-PR controlled inverter should be designed narrower than that of the PI and PR controlled inverters to keep the system stable.

## D. THE EFFECTS OF THE RESONANCE BANDWIDTH ON SYSTEM STABILITY

The experimental result when  $\omega_r$  is changed is shown in Fig. 15. When  $\omega_r$  is increased from  $2\pi$  rad/s to  $2.8\pi$  rad/s, the quasi-PR controlled inverter system oscillates. The experimental result is consistent with the theoretical analysis in Fig. 9.

Therefore, the stability of the quasi-PR controlled inverter is also affected by the resonance bandwidth of the quasi-PR controller. The theoretical analysis is verified by the experimental results.

## VII. CONCLUSION

In this paper, considering the influence of frequency coupling characteristic, a frequency coupling admittance model of the quasi-PR controlled inverter is established, and its stabilities are compared with the PI and PR controlled inverters. Following conclusions are drawn:

- 1) Admittance characteristics of the PI and PR controlled inverters are similar while the quasi-PR controlled inverter is quite different: the amplitude of the quasi-PR controlled inverter is larger than that of the PI controlled inverter and the phase difference between the two inverters is obvious in the mid-high frequency areas.
- 2) The resonance bandwidth of the quasi-PR controller is an important factor that causes the mentioned admittance difference. With the increasing of the resonance bandwidth, the quasi-PR controlled inverter system tends to be unstable. To reduce the impact on system stability, the resonance bandwidth should be set as small as possible in the area which can meet the control requirement.
- 3) The admittance difference has a great impact on system stability. Compared with the PI and PR controlled inverters, the quasi-PR controlled inverter is more sensitive to weak grid and high inverter output power. To achieve the same system stability, the outer-loop bandwidth of the quasi-PR controlled inverter should be designed narrower than that of the PI and PR controlled inverters.

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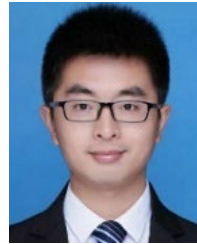
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