

## Micro and nanostrips in spintronics: how to keep them cool.

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### ABSTRACT.

This tutorial explores the problem of Joule heating on metallic micro or nanostrips, still one of the most popular geometries in modern spintronics. Many of the effects that result from the interaction of a spin polarized current and the local magnetization, require of a sizeable current density. This implies, quite often, an unneglectable Joule heating. Despite the few articles devoted to some aspects of Joule heating, there is still disinformation and many misconceptions in this topic, which is key for the correct interpretation of the scientific results. In this tutorial, we highlight the material parameters that are important to keep the temperature of the strip under control and those that give only a marginal advantage. In the vast majority of papers, at least one of these parameters is missing. We also focus on some misconceptions, such as the belief that performing the measurement on a cryostat, rules Joule heating out. In fact, for a fixed current density, measuring in a cryostat decreases the temperature but not enough to justify the use of such a costly measuring set-up. At the practical level, we put forward a 1D model to calculate, in few seconds, if Joule heating is present and if it should be taken into account when interpreting the results. Finally, and importantly, we describe a simple fabrication route to enhance the dissipation of heat in the strip considerably. This fabrication strategy is more effective at keeping the temperature under control than performing the experiment at cryogenic temperatures.

## I. INTRODUCTION.

The introduction of the concept of spin transfer torque<sup>1,2</sup> and the race track memory<sup>3</sup>, opened the eyes of the scientific community to the development of spintronic devices based on the control of magnetic domain walls (DWs) in ferromagnetic nano or microstrips. In the last 15 years, we have seen a very intense research devoted to current induced movement<sup>4</sup>, pinning or depinning<sup>5,6</sup> or transformations of DWs<sup>7,8</sup>. Most of the early work was done using Permalloy nanostrips<sup>9</sup>. In these experiments the current density required to see any effect, was often in the range of  $10^{12}$  A/m<sup>2</sup>. More recently, researchers moved towards materials with Perpendicular Magnetic Anisotropy (PMA)<sup>10</sup>, such as Pt/Co, or antiferromagnetic materials<sup>11,12</sup>, where a spin current, generated at the interfaces<sup>13</sup>, exert torques on the local magnetization and, for instance, assists the magnetic switching. Nevertheless, even when using materials with PMA or antiferromagnets, it is not uncommon to use a sizable current density<sup>11,12,14,15,16</sup>, more often than not, larger than  $10^{11}$  A/m<sup>2</sup>.

The use of such a large current density unavoidably raises questions about the role of Joule heating. Some authors deal with the problem by including some simulations in their work<sup>17,18</sup>, or by estimating the heating in their system with the help of a theoretical expression<sup>19,20,21</sup> or by doing some experimental estimations<sup>22,23,24</sup>. In other cases, Joule heating is simply not addressed<sup>11,12</sup>, especially when using pulsed current, a cryostat or a current density close to  $10^{11}$  A/m<sup>2</sup>. Not addressing Joule heating may or may not compromise the main claims of the work, but it could lead an incomplete interpretation of the results such as, for instance, to an underestimation of the value of a critical current density.

An accurate answer to the question ‘how hot does the micro or nanostrip get for a given current density?’, would probably require the combination of at least two of the above approaches, namely a simulation plus an experimental estimation. For instance, in Ref. 25, the authors managed to predict, with precision of few degrees, that the temperature in a Nickel nanowire was reaching almost exactly its Curie temperature. The accuracy in this work required the combination of a meticulous COMSOL simulation where all the material parameters were either measured or carefully chosen and a micromagnetic simulation that included the thermal behaviour of the structure.

The problem of heating in nanostructures is a research field in itself<sup>26</sup> and perhaps it is not practical to expect a meticulous thermal analysis in every work related to magnetic micro and nanostrips. At the same time, it is still quite common to find in the bibliography mistakes or misconceptions related to the thermal problem in magnetic strips. For instance, some works make use of microsecond long pulses as a strategy to reduce the Joule heating in their metallic strips. This is ineffective as, in most systems involving a metallic strip deposited on a substrate, the temperature reaches steady state in less than 1  $\mu$ s, as shown in previous works<sup>18</sup> or in theoretical models<sup>19,20</sup>. Another example of a potentially inefficient approach is to perform the measurement in a cryostat, even at very low temperatures. For a large current density, the strip may still get too hot.

There are of course many other issues that may lead to a wrong estimation of the temperature of the metallic strip. For instance, using a substrate that is a good thermal conductor, does not necessarily imply an effective dissipation from the strip. The interface between the strip and the substrate hampers the phonon transmission leading to an interface thermal resistance. This interface thermal resistance can be present even if the metallic strip is deposited by epitaxy on the substrate. In previous works, we have shown how important this parameter can be<sup>27,28</sup>, to the point that it can ruin the advantages of choosing a very good thermal conductor as a substrate. Normal values are in the range of  $1-5 \times 10^{-8}$  m<sup>2</sup>K/W. Admittedly, measuring the interface contact resistance is not straightforward<sup>27</sup> and, unless the evaluation of the local temperature in the strip has to be very precise, it is not practical to evaluate this parameter for every experiment. Anyhow, it is good to keep the existence of the interface thermal resistance in mind and assume that the temperature in the strip may likely be larger than the one predicted by a quick estimation.

With the existing studies dealing with Joule heating in ferromagnetic nano and microstrips, most experimentalists working in the field of spintronics know that it is not easy to evaluate if Joule heating has to be included in the interpretation of the results. On the other hand, considering the volume of research related to current induced effects in magnetic nano and microstrips, we clearly need easy tools or strategies to estimate if our device may be heating during a particular experiment. Also, it is important to know the details that need to be included in an article, so the reader can critically assess if the results can be affected by Joule heating. For instance, to say that the film is deposited on a Si/SiO<sub>2</sub> substrate but not to give the thickness of the SiO<sub>2</sub> layer<sup>29</sup>, makes the estimation of the Joule heating impossible.

The results shown in this work have been obtained with COMSOL<sup>30</sup> simulations. In general, we have used the material properties given by COMSOL. In our own experience, the properties of the materials can be quite different when deposited in different chambers and with different conditions or techniques, leading often to a large difference in the temperature reached for a given current density. On the other hand, our main goal here is to make this work comprehensive and as general and useful as possible. For instance, we have used the resistivity of the Permalloy that we deposit in our laboratory, which is larger than the bulk value, but it is useful to highlight the difference that the resistivity makes in the final temperature. In order to make the simulations faster, we have used a 0.5  $\mu\text{m}$  thick substrate and we have fixed the temperature at the bottom surface of the substrate to the ambient temperature. This is enough to work on the main points of this article and the difference with the temperature that we would obtain using a thicker substrate is usually few tens of degrees. Only for the simulations involving thermally insulating substrates, we needed to simulate thicker substrates (2.5  $\mu\text{m}$ ). In any case, it is important to remember that a meticulous evaluation of Joule heating would require to simulate a thicker substrate, usually 6 to 10  $\mu\text{m}$  thick at least.

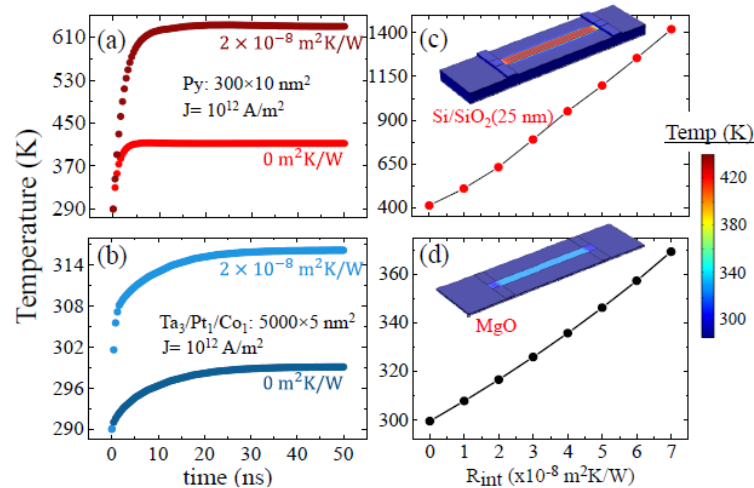
In this work we first summarize the main issues related to the problem of Joule heating in ferromagnetic nanostrips, we show when some strategies are ineffective keeping the temperature under control, we propose a quick and very simple method to estimate the temperature and finally, but importantly, we show a simple strategy to fabricate nanostrips that keep them as cool as possible during the measurement.

## II. SUBSTRATE, RISE TIME AND INTERFACE THERMAL RESISTANCE.

One of the challenges of trying to find a simple and comprehensive approach to the problem of Joule heating in magnetic strips, is the variety of materials, substrates and dimensions that the different authors use. In Figure 1, we summarize the results of a COMSOL simulation in two of the most common experimental setups. Fig. 1(a) shows the temperature of a Permalloy nanostrip with cross section  $300 \times 10 \text{ nm}^2$ , deposited on a Si/SiO<sub>2</sub>(25nm) substrate, when a DC current density of  $10^{12} \text{ A/m}^2$  is delivered to the strip. The calculation is done for two values of interface thermal resistance, 0 and  $2 \times 10^{-8} \text{ m}^2\text{K/W}$ . For the sake of simplicity, at this point, we ignore factors that may affect the accuracy of the simulation, such as the thermal gradients that arise from the contacts<sup>27,31</sup> or the importance of selecting the correct material parameters<sup>27</sup>. We simply have a nanometric conductive strip and a large current density ( $10^{12} \text{ A/m}^2$ ) is kept constant during the whole heating process, starting at  $t=0 \text{ ns}$ . The problem is more complicated when using nanosecond long pulses<sup>27,28</sup> as we briefly discuss later, but it is more comprehensible to speak about a constant current density. In the end, one needs to reach a particular current density to see a given effect related to spin transfer: we speak about critical current density, etc. It is the current density the figure we keep in mind in this type of experiments.

As we can see in Fig. 1(a), the temperature of the nanostrip reaches the steady state temperature in less than 10 ns. The heating process is very quick. In Fig. 1(b), we show the typical Hall bar used in experiments dealing with PMA materials. In this case, we simulate a Ta(3 nm)/Pt(1 nm)/Co(1 nm) multilayer, 5  $\mu\text{m}$  wide, deposited on a crystalline MgO substrate, when a DC current density of  $10^{12} \text{ A/m}^2$  is flowing through the strip. We also use two values of the interface thermal resistance, 0 and  $2 \times 10^{-8} \text{ m}^2\text{K/W}$ . Although in this case the final temperature is much lower than in the case of Fig. 1(a), the rise time is still in the range of few nanoseconds. For both strips, Permalloy and Pt/Co, we can see the dramatic effect of the interface thermal resistance in Figs. 1(c) and 1(d) respectively. Although the increase of temperature for the Pt/Co sample is generally small, we can see that the interface thermal resistance can lead to an increase of temperature of tens of degrees. Few tens of degrees of heating may seem like irrelevant but, in materials with PMA, a small increase in temperature can lead to big changes in the magnetization process or even to an increase of the domain wall velocity of one order of magnitude<sup>32</sup>.

Figure 1 is a good starting point to discuss several important points. Firstly, we can see that the temperature reaches steady state very quickly, in a matter of few nanoseconds. In fact, it is quite unlikely that a metallic ferromagnetic strip would take more than few hundreds of nanoseconds to reach steady state. Therefore, as it has been stressed before<sup>18,33</sup>, using microsecond long current pulses does not help preventing Joule heating in ferromagnetic metallic strips. Note that the situation may be different when using semiconductor strips<sup>34</sup>.



**FIG 1.** (a) Temperature in a Permalloy nanostrip of resistivity  $\rho_{Py}(300K) = 62 \mu\Omega \text{ cm}$  and cross section  $300 \times 10 \text{ nm}^2$ , deposited on a Si/SiO<sub>2</sub>(25 nm) substrate (dimensions of the Si  $10 \times 2.4 \times 0.5 \mu\text{m}$ ), when exposed to a  $10^{12} \text{ A/m}^2$  current density that starts at  $t=0 \text{ ns}$ , with zero rise time. The temperature is calculated for 0 and  $2 \times 10^{-8} \text{ m}^2\text{K/W}$  of interface thermal resistance. (b) Temperature of a wide microstrip of Ta/Pt/Co with resistivity  $\rho(300K) = 21 \mu\Omega\text{cm}$  and cross section  $5000 \times 5 \text{ nm}^2$ , deposited on a MgO substrate (dimensions  $100 \times 24 \times 0.5 \mu\text{m}$ ) when exposed to a  $10^{12} \text{ A/m}^2$  current density that starts at  $t=0 \text{ ns}$ , with zero rise time. Also here, the temperature is calculated for 0 and  $2 \times 10^{-8} \text{ m}^2\text{K/W}$  of interface thermal resistance. (c) and (d) Temperature of both nanostructures, Permalloy and Ta/Pt/Co respectively, versus different values of the interface thermal resistance  $R_{int}$ .

In terms of the maximum temperature that the strip may reach, there are two main reasons why the device in Fig. 1(a) heats up to a moderate temperature while the one in Fig.1(b) remains almost at room temperature: the strip of Fig.1(b) is thinner, it has lower resistivity and it is deposited on top of a substrate that is a good thermal conductor. Note that the width of the strip is none of the main reasons. The heat generated per unit of volume and unit of time is  $J^2\rho$ , where  $\rho$  is the resistivity of the nanostrip and  $J$  the current density. Therefore, a difference in the strip resistivity has a direct impact in the maximum final temperature. The resistivity for Permalloy was taken from Ref. 27, while we used the COMSOL library for the resistivity of Ta, Co and Pt for the strip in Fig. 1(b). Although the resistivity is temperature dependent, as a guide, the resistivity of Permalloy at RT in this simulation is  $\rho_{Py}(300K) = 62 \mu\Omega \text{ cm}$  and the average resistivity of the Ta/Co/Pt stack is  $\rho(300K) = 21 \mu\Omega \text{ cm}$ . The lower resistivity will lead to smaller heating. As the experimental resistivity of a material depends on many factors (thickness, depositions conditions, impurities, etc.), it is very important to use the experimental resistivity for every particular case, otherwise, the error in the final temperature can be quite large.

Additionally, as  $J^2\rho$  is the power per unit of volume, a larger cross section in the strip leads to larger heat generated, because the dissipation is only done through the substrate, i.e. proportional to the width. If the thickness is fixed (which it is usually the case as one needs a particular set of multilayers to perform a given experiment), for a constant current density, having a wider strip does not help preventing Joule heating, because the dissipation surface with the substrate increases as much as the cross section. Having a wide strip is in fact slightly worse in terms of heating than having a narrow strip, as we will see later.

In Figs. 1(c) and 1(d) we also see the effect of having a different value of the interface thermal resistance: the larger it is, the hotter the device gets. Previous works<sup>27,28</sup> have stressed the importance of this parameter that accounts for an imperfect thermal conduction between the strip and the substrate. It is always present because the strip and the substrate are made out of different materials and, often, with different crystalline structure. Normal values are in the range of  $1\text{-}5 \times 10^{-8} \text{ m}^2\text{K/W}$ . Despite its potential importance, the interface thermal resistance is always ignored because it is very difficult to measure its value and it will likely depend not only on the materials used, but also on the particular deposition system and conditions. Nevertheless, as we have mentioned in the introduction, it is important to keep it in mind and to be aware that the temperature may be higher that one would expect bearing in mind the geometry and the materials involved. In the conclusions, we explain how to take this parameter into account when performing an estimation of the Joule heating in a ferromagnetic strip.

Finally, we shall also quickly discuss the role of the substrate. When performing transport experiments on magnetic strips, where the current density can be large, the substrate must be a good thermal conductor. Of course, it has to be also an electric insulator, so all the electric current flows through the strip. The difference between using a thermally conductive substrate such as Si and using a more insulating one such as SiO<sub>2</sub>, could easily lead to an increase of the temperature of the strip of several hundreds of degrees. Crystalline substrates such as MgO or GGG, are an option if the strips are micrometers wide and they can be fabricated by optical lithography. But, if the strips are in the nanometric range, the processing may be quite complicated because the substrate builds up charge during the e-beam lithography. A common choice for nanometric devices is to use doped Silicon with a thin layer of thermal SiO<sub>2</sub> on top. This is also a cheap option. An important question to bear in mind in this case is that, when using pulsed current in timescales shorter than a microsecond, the oxide layer has to be thick enough to avoid capacitive current leakage through the substrate. For instance, if the device is shaped like an 'S' with the contacts at the top and at the bottom of the 'S' and it is deposited on doped Si with only few nanometers of native SiO<sub>2</sub>, a nanosecond pulse would flow largely through the substrate. This would lead to a milder heating but also to an apparent milder effect of the current on the local magnetization, simply because there is less current flowing through the strip. Using nanosecond pulses is not straightforward and it requires impedance matching of the device and the transmission line. In any case, even if one has done the homework on how to deliver nanosecond long pulses, at least in your first experiments, it is a good idea to see the pulse that comes out of the device on a high frequency oscilloscope. Otherwise you can never be sure of how much of the pulse is going through the device and how much is reflected. Going back to the discussion on what substrate to choose, while any electrically insulating crystalline substrate may work well for micron size strips (processed with optical lithography), for nanometric strips, doped Si with thermal SiO<sub>2</sub>(25 nm) is a very good option that would allow good thermal dissipation and the use of nanosecond long pulses.

### III. A QUICK ESTIMATION OF THE TEMPERATURE IN THE STRIP

In this field, a particularly useful tool would be a simple formula that could allow us to perform a quick calculation and find out, in a matter of seconds, if Joule heating could be a matter of concern or not. A COMSOL simulation can be fairly quick but it requires experience and a good knowledge of the properties of the materials deposited. Otherwise, it can lead to a completely wrong value. In any case, experimentalists often need, at the very least, a rough idea of the build-up of temperature in the strip, so they can make a decision of whether or not it is worth spending resources in a thorough thermal characterization.

One option is to use a theoretical model. You et al. (2006) deduced a simple expression for the temperature of the metallic strip with time, which has been used by several authors to estimate the Joule heating in their experiments:

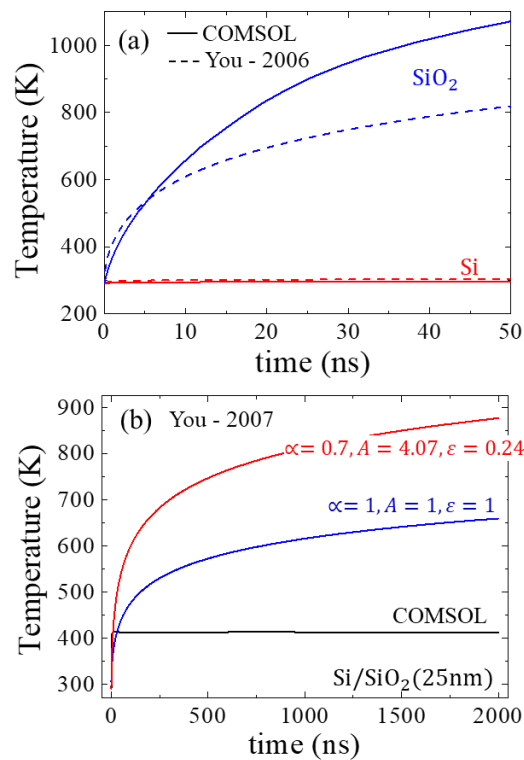
$$T(t) = T_{final} \left( \ln \left( \frac{4\sqrt{\mu_s t}}{w_G} \right) - \theta(t - t_p) \ln \left( \frac{4\sqrt{\mu_s(t-t_p)}}{w_G} \right) \right) \text{ where } T_{final} = \frac{wzJ^2\rho}{\pi k_{sub}}$$

with  $w$ ,  $z$  and  $\rho$  the width, thickness and resistivity of the nanowire.  $J$  the current density with a duration of  $t_p$ ,  $k_{sub}$  the thermal conductivity of the substrate,  $\mu_s$  the diffusivity of the substrate and  $w_G = \alpha w$  the gaussian profile width. This expression (Ref. 19) assumes material parameters non-dependent on the temperature and it is valid only for a substrate made out of a single material (for instance, it is valid for a Si substrate, but not for a Si/SiO<sub>2</sub> substrate). In Fig. 2(a), we compare the result of a COMSOL simulation with the You model, using the same material properties (although COMSOL includes the temperature dependency of each property). As it can be seen in Fig. 2(a), the You model works well for substrates that are good thermal conductors and where the build-up of temperature is very small, although it deviates from the simulation when using a poor thermal conductor as a substrate, such as SiO<sub>2</sub> (although this is an unlikely choice of substrate in an experiment). The same authors improved their model<sup>20</sup> to include an insulating layer in the substrate (thickness  $s$ ) and therefore account for very popular substrates such as Si/SiO<sub>2</sub>:

$$T_{final} = \frac{AwzJ^2\rho}{\pi k_{sub}} \left[ \frac{k_{sub}}{k_s} + \left( \frac{\varepsilon\pi S}{w_G} \right) \left( 1 - 2 \left( \frac{k_{sub}}{k_s} \right)^2 \right) + 4 \left( \frac{\varepsilon\pi S}{w_G} \right)^2 \left( \frac{k_{sub}}{k_s} \right)^3 \right]$$

This formula provided by You et al. (2007), has three adjustable parameters or 'correction factors', all of the 'order of the unity':  $A$ ,  $\varepsilon$  and  $\alpha$ . Increasing the value of  $A$  and  $\varepsilon$ , or decreasing the value of  $\alpha$ , would lead to a hotter temperature. Only  $\alpha$  affects mildly to the value of the rise time. The value of these parameters has to be chosen by simulating the experiment and adjusting the model to the real temperature

behaviour. Therefore, this model cannot be used as a quick tool to check if the strip in your experiment is getting hot or not. For instance, in Fig. 2(b) we compare our case study, Permalloy nanostrip in the same conditions as in Fig. 1(a), with the 2007 You model, using the same values for the correction factors that were used in the original work (red line) and setting all the values to unity (blue line). As we can see, getting the right values for the correction factors is very important. By setting the values of the correction factors to  $\alpha = 0.7, A = 0.75, \varepsilon = 0.24$ , we can match the steady state temperature with one obtained with COMSOL (black line in Fig.2b), although the rise time of the temperature is always considerably longer in the You model.



**FIG 2.** (a) Temperature on a Permalloy nanostrip of cross section  $300 \times 10 \text{ nm}^2$ , deposited on a Si substrate (red) and on a SiO<sub>2</sub> substrate (blue), when a current density of  $J=10^{12} \text{ A/m}^2$  is flowing through the nanostrip, starting at  $t=0$  ns. Solid lines represent the temperature predicted by COMSOL simulation and dash lines represent the temperature obtained with the 2006 You model [Ref.19]. (b) Same COMSOL simulations as in Fig. 1(a) (black line), using a Si/SiO<sub>2</sub> (25nm) substrate and the prediction obtained with the 2007 You model [Ref. 20], with the correction factors used in the original paper (red line) and with all the correction factors equal to 1 (blue line). The values of the correction factors required to fit our case study (black line) are  $\alpha = 0.7, A = 0.75, \varepsilon = 0.24$ .

We have seen how the You model can provide ambiguous results because it is largely dependent on three unknown correction parameters. Therefore, here we propose a very simple formula based on a one dimensional (1D) model. Let us first deduce the expression, then discuss its validity and finally compare it with some previous experimental estimations of the Joule heating. We are going to assume that the temperature increases exponentially, which is not far from the reality,

$$T(t) = T_{final} - (T_{final} - T_{RT})e^{-t/\tau} \quad (1)$$

where there are two parameters to calculate:  $T_{final}$  and the characteristic heating (or cooling) time  $\tau$ . In a similar fashion to the ‘Lumped-Heat’ approximation for unsteady state<sup>35</sup>, we are assuming the heating time to be  $\tau = R_{th}C_{th}$ , where  $C_{th}$  accounts for the capacity of the nanostrip to accumulate thermal energy and  $R_{th}$  is the thermal resistance of the substrate to dissipate the heat. The thermal capacity of the strip is given by  $C_{th} = C_V AZ$ , where  $C_V$  is the heat capacity per unit of volume of the strip,  $A$  the area of contact between the strip and the substrate, and  $z$  the thickness of the nanostrip. In a 1D model, where the equithermal lines run parallel to the length of the strip, the dissipation thermal resistance can be expressed as  $R_{th} = s/(k_{sub}A)$ ,  $A$  the contact area with the strip<sup>35</sup> and  $s$  and  $k_{sub}$  the thickness and the thermal conductivity of the ‘active’ part of the substrate. The term ‘active’ will become clearer later but basically refers to the part

of the substrate that sustains most of the temperature drop which, in a Si/SiO<sub>2</sub> substrate, is going to be the SiO<sub>2</sub> layer. Therefore, the heating (or cooling) time is given by,

$$\tau = \frac{C_V z s}{k_{sub}} \quad (2)$$

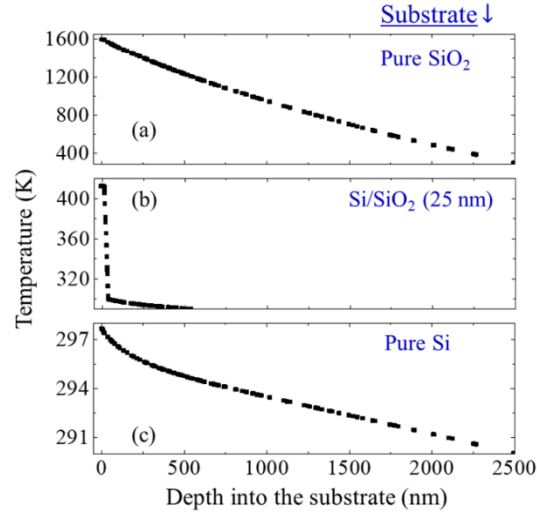
In order to estimate  $T_{final}$ , we assume that all the heat generated per unit of time in the strip  $J^2 \rho A z$ , is dissipated exclusively by the 1D thermal resistance of the substrate. Therefore, in steady state, we can approximate

$$J^2 \rho A z \approx \frac{T_{final} - T_{RT}}{R_{th}} \quad (3)$$

Therefore, substituting (3) in (1), we obtain,

$$T(t) = \frac{J^2 \rho s z}{k_{sub}} (1 - e^{-t/\tau}) + T_{RT} \quad \text{where} \quad \tau = \frac{C_V z s}{k_{sub}} \quad (4)$$

This approximation assumes linear drop of temperature in the substrate and only 1D dissipation of the heat vertically down the substrate. At the same time, it is such a simple expression that it is a good exercise to see when it may be applicable. Notice that the expression (4) is a similar simplified version of the one deduced by Kim *et al.*<sup>21</sup> in their model.



**FIG. 3.** Steady state temperature (after 2  $\mu$ s of the beginning of the current flow) underneath a Permalloy nanostrip of cross section  $300 \times 10$  nm<sup>2</sup>, deposited on different substrates (top SiO<sub>2</sub>, middle Si/SiO<sub>2</sub>(25 nm) and bottom Si), when a DC current density of  $J=10^{12}$  A/m<sup>2</sup> flows through the nanostrip. The temperature at the base of the substrate is fixed to Room Temperature. The temperature of the strip is the one at depth 0 nm.

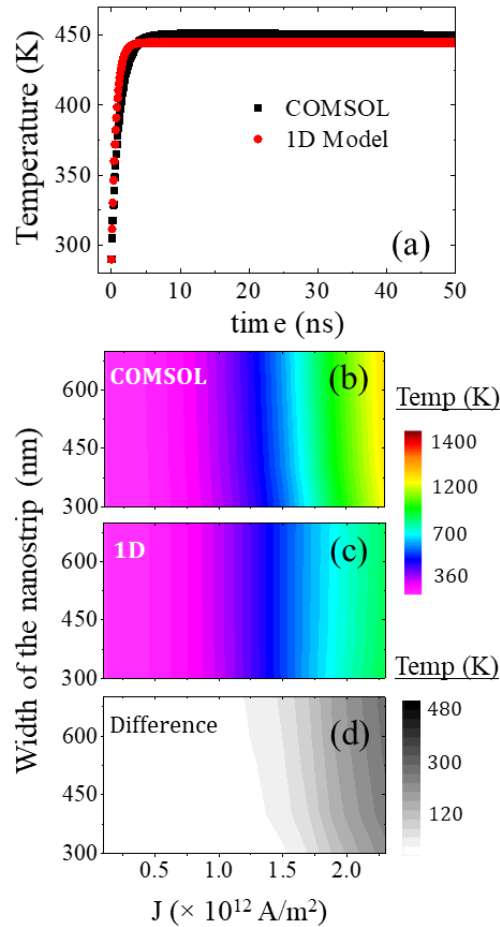
Figure 3 shows the temperature drop in the substrate, underneath the ferromagnetic strip. For a substrate that is made out of a single material, whether it is a bad thermal conductor such as SiO<sub>2</sub> in Fig. 3(a) or a good thermal conductor such as Si in Fig. 3(c), the temperature underneath the strip drops exponentially and therefore the above 1D model is, in principle, not applicable. On the other hand, for the most commonly used substrate, Si with a SiO<sub>2</sub> layer on top, most of the temperature drop is linear and it happens within the thermally resistive SiO<sub>2</sub> layer. In this case, which again, is the most popular choice of substrate, the simple 1D model given by expression (3) is applicable if the parameters  $s$  and  $k_{sub}$  refer to the layer of SiO<sub>2</sub>, where practically all the temperature drop is happening.

In Fig. 4 we compare the results of the 1D model with a COMSOL simulation for our case study, a Permalloy nanostrip 10 nm thick,  $\rho_{Py}(300K) = 62 \mu\Omega$  cm, deposited on Si/SiO<sub>2</sub>(25 nm). Figure 4(a) shows the behaviour of the temperature with time for a particular cross section of the nanostrip ( $400 \times 10$  nm<sup>2</sup>) and a current density of  $1.1 \times 10^{12}$  A/m<sup>2</sup>, both for a COMSOL simulation (black) and the 1D model (red), with  $s = 25$  nm and  $k_{sub} = 1.4$  Wm<sup>-1</sup>K<sup>-1</sup> which refer only to the SiO<sub>2</sub> layer, what we called earlier the ‘active’ part of the substrate, where most of the temperature drop happens. In this particular case, the 1D model gives a very accurate result in both the rise time and the steady state temperature.

Indeed, the rise time obtained with the 1D model is surprisingly accurate even with different thicknesses of SiO<sub>2</sub>. Using  $k_{SiO_2} = 1.4$  Wm<sup>-1</sup>K<sup>-1</sup>,  $C_V = 4.4 \times 10^6$  JK<sup>-1</sup>m<sup>-3</sup> for Permalloy and  $z=10$  nm, we obtain a characteristic heating time of 0.8 ns for a thickness of the SiO<sub>2</sub> layer of  $s=25$  nm, i.e. the nanostrip would reach 95% of its final temperature in  $3\tau = 2.4$  ns. If we use 100 nm thickness for the SiO<sub>2</sub> layer, the Permalloy nanostrip would reach 95% of its final temperature in about 10 ns. These values obtained with



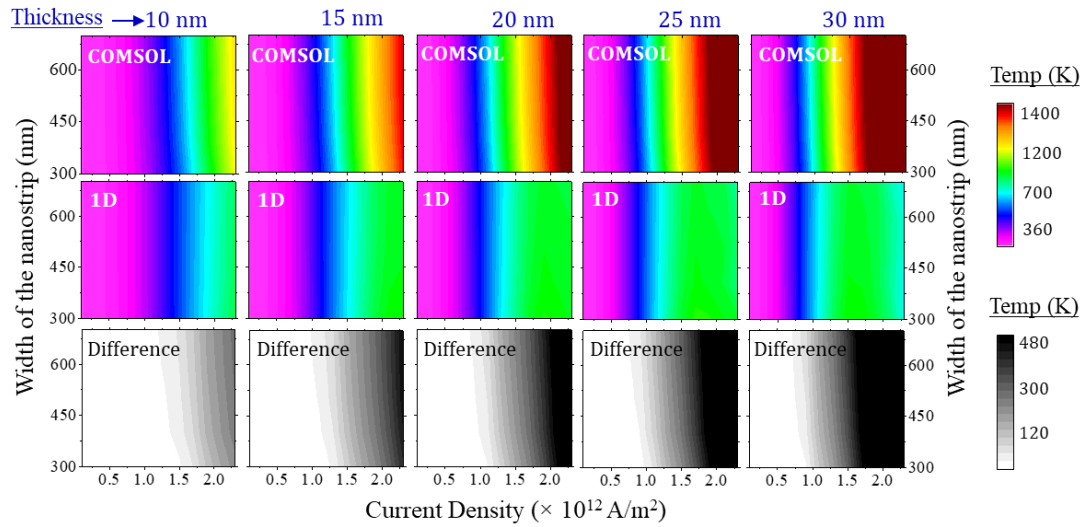
such a simple calculation are quite close to the results obtained with a simulation (see for instance Figure S3 in the Supplementary Information of Torrejon et al.<sup>18</sup>).



**FIG. 4.** (a) Temperature versus time in a Permalloy nanostrip of cross section  $400 \times 10 \text{ nm}^2$ , deposited on a Si/SiO<sub>2</sub>(25 nm) substrate, when exposed to a  $1.1 \times 10^{12} \text{ A/m}^2$  current density that starts at  $t=0 \text{ ns}$ , with zero rise time. The black curve is the COMSOL simulations and the red curve is the result of the 1D model. (b) and (c) compare the temperature obtained with COMSOL (b) and the 1D model (c) for different widths and current densities. (d) Represents the deviation of the 1D model from the result obtained with COMSOL.

In terms of the steady state maximum temperature obtained with the 1D model, Fig. 4(b) and Fig. 4(c) compare the maximum temperature for the Permalloy nanostrip of thickness 10 nm and different widths. Figure 4(d) plots the difference between the values in Fig. 4(b) and in Fig. 4(c). As it can be seen, the 1D model gives a good prediction even for a very large current density. As shown in Figure 5, as the thickness of the nanostrip increases, the 1D model is increasingly inaccurate, especially when the strip gets very hot. Importantly, the 1D model predicts very well the onset of heating (boundary between purple and blue in the maps of Fig. 5) and it underestimates the maximum temperature. Therefore, the 1D model can be considered as a warning sign: if the 1D model predicts some heating, the reality is likely going to be worse and the temperature should be taken into account in the analysis of the results. We should note that this 1D model can be used to evaluate the effect of the interface thermal resistance when using any substrate, as we explain in the conclusions. We should also note that Fig. 5 shows that wider strips heat up more for a given current density. The model of You et al. (2007) already predicted that narrow strips would heat up less than wider strips although, for most practical purposes, the width of the strip should not play an important role.





**FIG. 5.** Comparison of the maximum temperature in a Permalloy strip with resistivity  $\rho_{Py}(300K) = 62 \mu\Omega\text{cm}$ , deposited on a Si/SiO<sub>2</sub> (25 nm) substrate and different dimensions. The width of the strip is in the y-axis of each panel. The result of the COMSOL simulation is in the top panels, the result of the 1D model in the middle panels and the difference between the top and middle panels is represented in the bottom panels. The thickness of the strip is indicated on top of the panels in blue.

In Table 1 we have selected some relevant articles where the authors evaluated experimentally the temperature of the strip. In the works where the substrate used was a single material (Si, MgO, etc.), in order to apply the 1D model, we have to input a value of the interface thermal resistance as explained in the conclusions. We always selected a small value for the interface thermal resistance, except for Ref. 27 where the value was found experimentally. For the calculation of the increment of temperature following the 1D model,  $\Delta T_{1D}$ , we have used  $k_{SiO_2} = 1.4 \text{ Wm}^{-1}\text{K}^{-1}$ . By looking to the results in Table 1 we can conclude that, despite its simplicity, the 1D model is a good quick tool to give an approximated idea of the presence of any relevant Joule heating, using only values that are known or that can be found experimentally without much trouble, such as the resistivity.

Material	Cross Section (nm <sup>2</sup> )	Substrate	$J(\text{A/m}^2)$	$\rho$ ( $\mu\Omega\times\text{cm}$ )	ITR ( $\text{m}^2\text{K/W}$ )	$\Delta T_{\text{experiment}}$	$\Delta T_{1D}$	Ref.
Permalloy	110×34	Si/SiO <sub>2</sub> (Native)	$2.1\times 10^{12}$	7*	$1\times 10^{-8}$ *	60-100 K	70 K	[36]
Permalloy	300×10	Si/SiO <sub>2</sub> (25 nm)	$2.8\times 10^{12}$ †	34	0	450 K	475 K	[24]
Permalloy	300×40	Si/SiO <sub>2</sub> (25 nm)	$1.6\times 10^{12}$ †	40	0	300 K	730 K	[24]
Permalloy	300×12	Si/SiO <sub>2</sub> (500 nm)	$4\times 10^{11}$	62	$5.5\times 10^{-8}$	250 K	472 K	[27]
Permalloy	120×5	Si/SiO <sub>2</sub> (600 nm)	$2.1\times 10^{11}$	7*	0	100 K	7 K	[22]
Permalloy	240×30	Si/SiO <sub>2</sub> (100 nm)	$5.5\times 10^{11}$	80†	0	400 K	515 K	[23]
Permalloy	275×25	MgO(111)	$8\times 10^{11}$	68†	$1\times 10^{-8}$ *	200 K	80 K	[23]
Cu	340×20	Si/SiO <sub>2</sub> (100 nm)	$2\times 10^{12}$	2*	0	200 K	114 K	[23]
Ta/Pt/Co	340×8	Si/SiO <sub>2</sub> (50 nm)	$6.5\times 10^{11}$	11*	0	33 K	15 K	[32]
PMA stack	5500×18	Sapphire	$4\times 10^{11}$	12	$1\times 10^{-8}$ *	90 K ♦	3 K	[37]

**TABLE 1.** Comparison of the experimental temperature measured by different authors with the result obtained with the 1D model described in the text. (\*) These values are unknown, so we chose the tabulated value. For the interface thermal resistance (ITR), a value is needed in order to apply the 1D model when the substrate is a single material such as Si or MgO. (†) Current density taken at the end of a nanosecond long pulse. The strips in this study were only 4  $\mu\text{m}$  long, so dissipation through the contacts may account for a lower experimental temperature for a given current density. (‡) Values taken at the corresponding current density (i.e it is not the value of resistivity at RT). (♦) This value was calculated using a model<sup>34</sup>, it is not an experimental value.

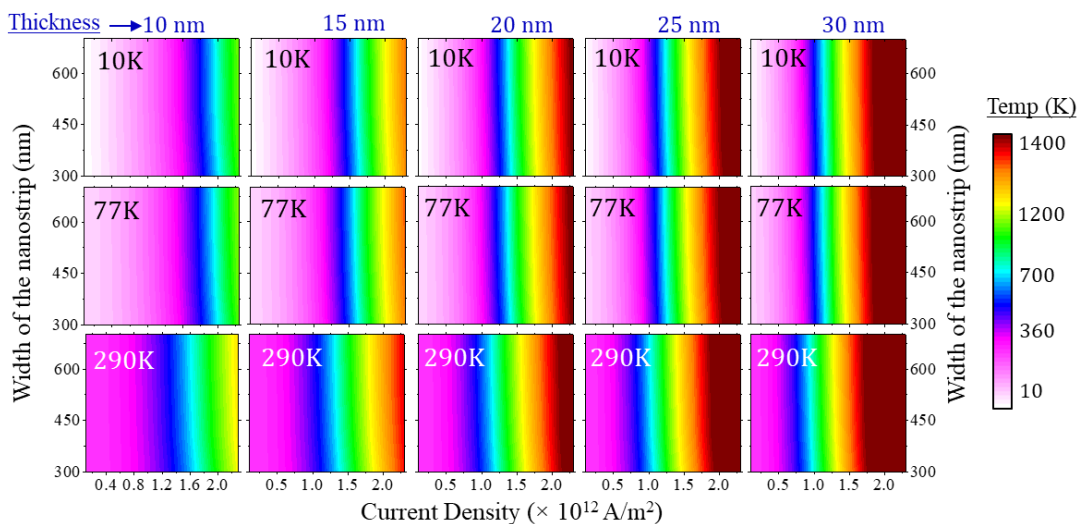
#### IV. KEEPING THE STRIP COOL.

In the previous sections we have summarized the main issues that contribute to the Joule heating when a given current density flows through a ferromagnetic strip. In most experiments the physics involved may leave you with no choice on the selection of the thickness for the strip or the value of the maximum current density or the substrate. Therefore, we still need to find out the best strategy to reduce the Joule heating as much as possible.

The two main strategies that researchers have used so far are: the use of short current pulses and the measurement in a cryostat. We comment now on these two approaches and then we suggest a third option that, to the best of our knowledge, it has not been used before and it is the simplest and most effective option to keep the Joule heating under control.

Delivering short pulses is, by far, the most common strategy to minimize the impact of the Joule heating. As we have seen, the rise time of the temperature is very fast. Also, by looking to formula (1) one can immediately see that the rise time is even shorter when the substrate is a good thermal conductor (large  $k_{sub}$ ). In order to make this approach useful<sup>18,28</sup>, very likely, the pulse will have to be shorter than 5 ns and possibly, shorter than 2 ns. As we have mentioned already, delivering such a short pulse is not only a question of purchasing an expensive pulse generator, it also requires proper impedance matching. In reality, it is not an approach as easy as it may seem and not necessarily as effective as one may hope. Of course, using nanosecond long pulses may be good for the durability of the device, as it sustains a large temperature only in very short periods of time. On the other hand, one can never be sure if the magnetic properties of the strip remain the same once the temperature has gone well above the ordering temperature.

A second option is to use a cryostat<sup>38</sup>. Considering the minute size of the strip in comparison to the substrate, one may intuitively think that, by keeping the substrate very cold, the strip will remain cold. This is true to some extent but, sadly, not as effective as one may think. As the heat generated per unit of volume and unit of time is  $J^2\rho$ , if the dissipation through the contact area with the substrate is not sufficiently effective, the temperature will still increase. Perhaps the best way to understand this intuitively, is by using formula (3), but substituting  $T_{RT}$  by the temperature of the cryostat (for instance 10 K). For a given dissipation thermal resistance  $R_{th}$ , if the term  $J^2\rho$  is large,  $T_{final}$  can still be very large, no matter how cold the substrate is. In Figure 6 we illustrate this for some particular values. The simulation is done for current densities in the range of  $10^{12}$  A/m<sup>2</sup> and for our case study, a Permalloy nanostrip with resistivity  $\rho_{Py}(300K) = 62 \mu\Omega\text{cm}$ , deposited on a Si/SiO<sub>2</sub> (25 nm). We have selected three cryostat temperatures, 10 K (top panels), 77 K (middle panels) and Room Temperature (bottom panels). In the simulation we fix the temperature of the bottom plane of the substrate to the cryostat temperature and the temperature in all the system at  $t=0$  ns is also the temperature of the cryostat.



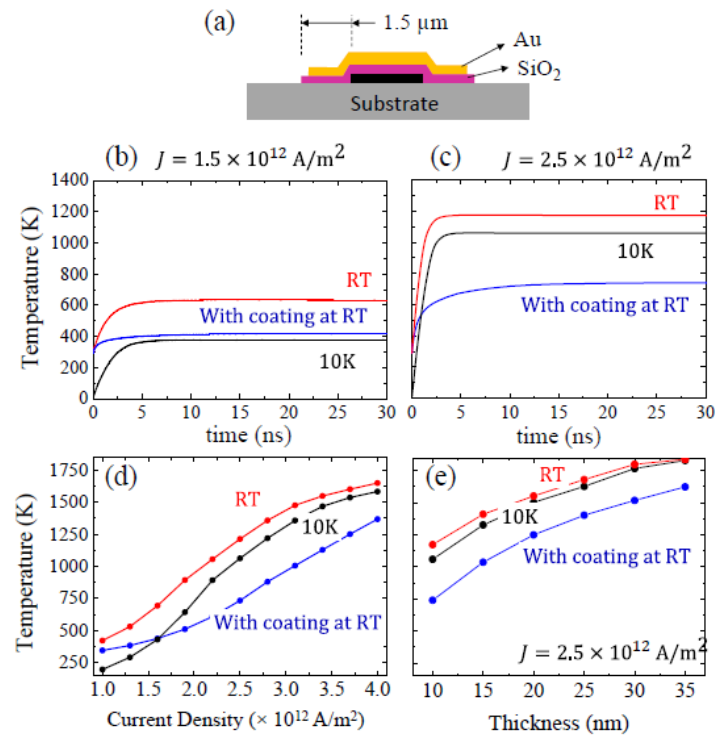
**FIG. 6.** Comparison of the steady state temperature of a Permalloy nanostrip with resistivity  $\rho_{Py}(300K) = 62 \mu\Omega\text{cm}$ , deposited on a Si/SiO<sub>2</sub> (25 nm) and different dimensions (width represented in the y-axis of each panel and thickness marked at the top of the panels). Each row of panels corresponds to one temperature of the cryostat.

We begin our discussion with the three panels on the left, thickness 10 nm. If we take as a reference the dark blue colour, the onset of a relevant heating ( $\sim 550$  K), at Room Temperature (bottom panel), this regime

is obtained for  $\sim 1.3 \times 10^{12}$  A/m<sup>2</sup>. When the measurement is done in a cryostat, the current density that the strip can hold before reaching this temperature is  $\sim 1.7 \times 10^{12}$  A/m<sup>2</sup>. It is an improvement, but possibly not enough to justify the cost of the measuring setup. As we have mentioned in the previous paragraph, once the Joule power is large, the strip cannot dissipate the heat fast enough to the substrate, even if the measurement is done in a cryostat, and the temperature will rise. The cryostat approach is even less effective for a larger cross-section of the strip, as shown in the rest of the panels (different thicknesses labelled at the top of the panels). In terms of the maximum current density that can be delivered to the strip without producing an aggressive heating, using a cryostat in the experiment gives such a small advantage, that any other experimental parameter, such as the resistivity of the strip or the interface thermal resistance, can ruin the gain of using the cryostat.

Finally, we propose a third strategy to reduce the maximum temperature reached by the strip due to Joule heating. For a given current density, the strip generates heat proportional to the cross section but dissipates only by the bottom surface, in contact with the substrate. The top surface of the strip, in contact with the air, does not contribute to the dissipation. Therefore, it would be very useful to habilitate a dissipation channel through the top surface.

In Fig. 7(a) we show the cross section of the strip (black) with a coating of SiO<sub>2</sub> (12 nm)/Au (50 nm) deposited on a window with the same length of the strip and extending only 1.5  $\mu$ m on each side of the strip, as indicated in Fig 7(a). The SiO<sub>2</sub> layer between the strip and the Au top layer is required to avoid shorting the strip. This coating of SiO<sub>2</sub>/Au allows the flow of heat through the top surface. The heat goes through the SiO<sub>2</sub> layer onto the Au layer on top and it is distributed effectively by the Au over a larger area, to finally sink back to the substrate. Of course, the Au layer can be substituted by a cheaper material that is good thermal conductor, such as Al or Cu, as we will see in the next figure. The cost of this strategy is having to include two extra steps in the fabrication route, lithography and deposition for the SiO<sub>2</sub> and Au layers, although the Au layer can be done together with the contact pads, so the cost would be only one extra lithography step in the fabrication process.

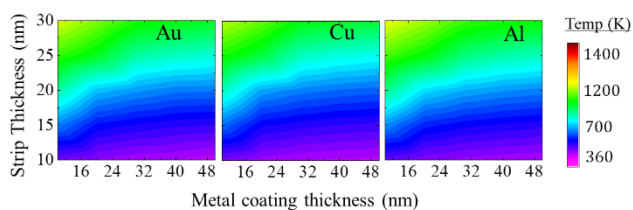


**FIG. 7.** (a) Schematic of the cross section of the strip (black) with a coating bilayer of SiO<sub>2</sub>/Au on top to habilitate a dissipation channel through the top surface. (b) Temperature versus time in a Permalloy nanostrip of cross section  $300 \times 10$  nm, deposited on a Si/SiO<sub>2</sub>(25 nm) substrate, when exposed to a current density of  $1.5 \times 10^{12}$  A/m<sup>2</sup> that starts at  $t=0$  ns. The figure shows the heating in three cases: the substrate is at RT (red), at 10 K (black) and at RT with the SiO<sub>2</sub>/Au coating on top (blue). (c) Same simulation as in (b) but using a current density of  $2.5 \times 10^{12}$  A/m<sup>2</sup>. (d) Same simulation as in (b) and (c) for a range of current densities. In this figure we only plot the final steady state temperature for the three cases: the substrate is at RT (red), at 10 K (black) and at RT with the SiO<sub>2</sub>/Au coating on top (blue). (e) Same simulation as in (c) with a current density of  $2.5 \times 10^{12}$  A/m<sup>2</sup>, but changing the thickness of the 300 nm wide strip.

In Fig. 7(b) we can see how effective the SiO<sub>2</sub>/Au coating is at reducing the maximum temperature. In a Permalloy nanostrip of cross section 300×10 nm<sup>2</sup>, deposited on a Si/SiO<sub>2</sub>(25 nm) substrate, we plot the behaviour of the temperature when a DC current density of 1.5×10<sup>12</sup> A/m<sup>2</sup> flows through the strip, starting at  $t=0$  ns. Despite the large current density, the overcoat strategy reduces the maximum temperature to a very manageable value and it is almost as effective as performing the measurement in a cryostat at 10 K. For larger current densities, the coating strategy is considerably more effective than using a cryostat at 10 K, as shown in Figs. 7(c) and 7(d). Fig. 7(d) also shows that, once the current density goes over  $\sim 3 \times 10^{12}$  A/m<sup>2</sup>, there is little one can do to avoid very intense heating and possibly destruction of the strip. Any experiment intending to use current densities larger than 3×10<sup>12</sup> A/m<sup>2</sup>, may require to combine several strategies to minimize Joule heating, all at once.

Finally, in Fig. 7(e), we can see the effect of increasing the thickness of the strip. The heating becomes more aggressive but the overcoat strategy still decreases the temperature by almost the same amount ( $\sim 300$  K). In general, for the vast majority of strip geometries used in the bibliography, the coating strategy would allow a substantial decrease of the maximum temperature, for a given current density.

As we have mentioned, we do not need to use Au or even such a thick layer of Au to reduce temperature with the top coating strategy. In Fig. 8, we plot the final steady state temperature that a 300 nm wide Permalloy strip reaches for different top coating layers, following the same schematic as in Fig. 7(a). The current density used in this figure is 1.5×10<sup>12</sup> A/m<sup>2</sup>. As we can see in Fig. 8, any coating should work fairly well (refer to Fig. 5 to find the temperature without the coating in these conditions). On the other hand, if the thickness of the strip is large, the temperature would be large whatever the thickness of the top coating layer is.



**FIG. 8.** Contour plot of the steady state temperature reached on a Permalloy strip 300 nm wide, of different thicknesses (y-axis), with a current density of 1.5×10<sup>12</sup> A/m<sup>2</sup> and for different thicknesses of the metal coating (x-axis), following the same configuration displayed in Fig. 7(a). The SiO<sub>2</sub> layer (see Fig. 7a) has the thickness of the strip plus 2 nm in all cases. The three panels study the effect of using different metals on the top coating, as marked in each of the panels. The temperature without coating can be extracted from Fig. 5.

## V. CONCLUSIONS

In this tutorial we have summarized the main issues that affect the build-up of temperature in experiments using magnetic (metallic) strips when a large current density flows through the strip. We have also seen how to estimate the temperature of the strip under a given current density, using a very simple 1D model. We have shown that this model gives good results for a Si/SiO<sub>2</sub> substrate, where most of the dissipation thermal resistance is coming from the SiO<sub>2</sub> layer. This model can also be used with other substrates, such as MgO, simply by assuming the (likely) case that there exists an interface thermal resistance. The trick is to compare the interface contact resistance to the thermal resistance of a layer of SiO<sub>2</sub> of a given thickness. The heat flow in steady state in one dimension is described by the Fourier law,  $q = -kA \times \partial T / \partial x$ , where  $q$  is the heat flow (in Watts),  $k$  the thermal conductivity and  $A$  the area. In an infinite plane this equation can be re-written as  $q = \Delta T / R_{th}$ , where  $R_{th} = s/kA$  is the thermal resistance of the plane, as we have seen already in the 1D model. Therefore, a 10 nm thick layer of SiO<sub>2</sub> with thermal conductivity of  $k \sim 1 \text{ Wm}^{-1}\text{K}^{-1}$ , would set a thermal resistance per unit of area of  $R_{th} = 10^{-8} \text{ K/W}$ , equivalent to an interface thermal resistance of  $1 \times 10^{-8} \text{ m}^2\text{K/W}$ . With this in mind, we can use the 1D model to predict the temperature of the strip in Fig. 1(d), using  $J=10^{12} \text{ A/m}^2$ ,  $\rho = 21 \mu\Omega \text{ cm}$ ,  $z=5 \text{ nm}$  assuming a  $2 \times 10^{-8} \text{ m}^2\text{K/W}$  interface thermal resistance, which is roughly equivalent to a 20 nm SiO<sub>2</sub> layer between the strip and the substrate ( $s=20 \text{ nm}$  and  $k_{sub}=1.4 \text{ W/m}^{-1}\text{K}^{-1}$ ). Therefore, for  $T_{RT} = 290 \text{ K}$ , the maximum temperature is 305 K, within range of

the COMSOL estimation. Also, we have applied the 1D model to some previous experimental works using MgO, Si or Sapphire as a substrate, as shown in Table 1, obtaining a result within the range of the experimental temperature.

As the 1D model provides a good indication of the presence of a relevant Joule heating, in general, articles dealing with current induced effects on ferromagnetic strips, should provide the full details of the substrate, the experimental resistivity of the ferromagnetic material and, of course, all the dimensions. Only if all these data are provided, the reader can evaluate the role (if any) of the Joule heating in the experiment. For instance, in the work of Sato et al.<sup>29</sup>, the authors observed that the magnetic domain wall disappeared from the corner of a zig-zag strip for a current of 4.5 mA ( $8.1 \times 10^{11}$  A/m<sup>2</sup>). As the thickness of the SiO<sub>2</sub> layer in the substrate is unknown, the increase in temperature  $\Delta T$  for that current density could be anywhere from 40 K to 550 K (using the 1D model with  $\rho = 15.5 \times 10^{-8}$  A/m<sup>2</sup>,  $z = 15$  nm and the thickness of the SiO<sub>2</sub> layer from 25 nm to 500 nm).

Finally, we are aware that in some cases, the interface thermal resistance may be the parameter that determines if heating is present or not, especially when using a substrate of a single material (Si, MgO, GGG, etc.). The only way of having a quick idea of how large this parameter might be, is sacrificing a twin strip. In our own experience<sup>25,27,28</sup> and with the information provided by other authors<sup>23</sup>, although a ferromagnetic strip or nanowire can reach its Curie temperature repeatedly and remain unaffected, it usually gets destroyed around 1000 K, using a DC current density. Therefore, if the strip is prepared on a chip with sufficient twin devices, it is a good exercise to test the maximum current density that one of the strips can withstand before destruction. If we assume that the strip blows up at 1000 K, knowing the rest of the parameters in the 1D model, one should have a very rough (but possibly useful) idea of how large is the interface thermal resistance and what is the range of current densities that can be used in the experiment without excessive heating.

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#### DATA AVAILABILITY STATEMENTS

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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