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ABSTRACT: Structural health monitoring (SHM) of civil structures often is limited due to changing environmental conditions, as those changes affect the structural dynamical properties in a similar way like damages can do. In this article, an approach for damage detection under changing temperatures is presented and applied to a beam structure. The used stochastic subspace-based algorithm relies on a reference null space estimate, which is confronted to data from the testing state in a residual function. For damage detection the residual is evaluated by means of statistical hypothesis tests. Changes of the system due to temperature effects are handled with a model interpolation approach from linear parameter varying system theory. From vibration data measured in the undamaged state at some few reference temperatures, a model of the dynamic system valid for the current testing temperature is interpolated. The reference null space and the covariance matrix for the hypothesis test is computed from this interpolated model. This approach has been developed recently and was validated in an academic test case on simulations of a mass-spring-damper. In this paper, the approach is validated experimentally on a beam structure under varying temperature conditions in a climate chamber. Compared to other approaches, the interpolation approach leads to significantly less false positive alarms in the reference state when the structure is exposed to different temperatures, while faults can still be detected reliably.

KEY WORDS: Damage detection, Subspace methods, Temperature effects, Model interpolation, Laboratory beam structure, Climate chamber

1 INTRODUCTION

Damage detection in the context of automated vibration-based structural health monitoring (SHM) often is limited due to environmental effects [1], such as temperature. Those methods aim at detecting changes in the dynamical behavior of the structure, as it is affected by damages. However, the dynamical properties usually are also affected by changing temperatures, which may lead to false alarms in the undamaged state. Consequently, a damage detection method is required, which can differentiate between changes due to damages and those caused by variation in temperature.

In the literature some approaches for temperature rejection in vibration-based damage detection methods can be found. Features robust to temperature effects are used, such as in [2], where a regression analysis of the natural frequency is done after a system identification step. Others, e.g. [3], use novelty detection approaches, defining normal reference conditions firstly and then the corresponding system properties.

This paper uses an interpolation approach to account for temperature changes in stochastic subspace-based damage detection methods. Those output-only algorithms are promising for the application to automated vibration-based SHM, as they do not require information about the excitation and are able to handle the data uncertainties. A damage detection method introduced in [4] relies on a reference null space estimate of the output covariance Hankel matrix, which is confronted to data from the testing state in a residual function. The residual is evaluated statistically, to decide if it is significantly different from 0. The test value of this evaluation can be used as damage indicator.

First temperature rejection approaches for the stochastic subspace-based damage detection are proposed. They use averaged system matrices, computed from data at several reference temperature states [5]. In this algorithm no information about the current testing temperature is needed. In another approach [6] several reference matrices are considered, which are valid for a specific temperature range in the damage detection step, using the information of the testing temperature.

In [7], the authors have presented an interpolation approach to account for temperature changes with the stochastic subspace-based damage detection, based on a model interpolation method from [8]. For a system whose behavior depends on an external parameter p, but shows linear behavior at fixed working points of this parameter, an interpolation approach for linear parameter varying (LPV) systems can be used to find a model that is valid at the new parameter value. From this interpolated model a reference null space is computed, corresponding to the parameter in the testing state. In the present context the parameter is set to be the temperature.

Up to now, the interpolation-based temperature rejection approach was validated theoretically on numerical data of a mass-spring-damper with a very simple temperature model in [7] and [9]. This motivates for a deeper look into the applicability of the algorithms to real structures in this study, and an application is presented to data from a reinforced concrete beam recorded in a climate chamber at defined temperatures and at three different system states. Furthermore, some conclusions regarding the choice of the weighting functions are made.

This paper is organized as follows: In section 2 the theoretical background of the stochastic subspace-based damage detection method and the model interpolation approach to account for temperature changes is recalled. Section 3 introduces the laboratory tests in the climate chamber and the application of the algorithms to the laboratory data. The section concludes with some suggestions for the choice of the weighting function.

2 THEORETICAL BACKGROUND

2.1 Stochastic subspace-based damage detection

The dynamical behavior of a civil structure at some temperature $T = T_j$ can be described by a stochastic discrete time state-space model

$$x_{k+1,j} = A_j x_{k,j} + w_{k,j}$$
$$y_{k,j} = C_j x_{k,j} + v_{k,j}\ , \tag{1}$$

where $x \in \mathbb{R}^n$ is the state of a system with model order n, $y \in \mathbb{R}^r$ is the measured output at r sensor positions, and k denotes the time index within the sample. The system matrices $A_j = A(T_j)$ and $C_j = C(T_j)$ are the state transition matrix with dimension $n \times n$ and the observation matrix with dimension $r \times n$, respectively. For the state noise w and the output noise v white noise is assumed.

From a data sample of length N a block Hankel matrix is built from the output covariances $R_{i,j} = \boldsymbol{E}\big(y_{k+i,j} y_{k,j}^T\big) = C_j A_j^{i-1} G_j$, where i denote the time shift and $G_j = \boldsymbol{E}\big(x_{k+1,j} y_{k,j}^T\big)$. The Hankel matrix writes as

$$\mathcal{H}_{\mathrm{j}} = \begin{bmatrix} R_{1,j} & \cdots & R_{q,j} \\ \vdots & \ddots & \vdots \\ R_{p+1,j} & \cdots & R_{q+p,j} \end{bmatrix}. \tag{2}$$

Due to the relation between the system matrices and the Hankel matrix, changes in the dynamical behavior of the structure will lead to changes in the Hankel matrix.

For damage diagnosis the left null space S_j of the Hankel matrix is used, computed in the undamaged reference state. It holds $S_j{}^T \cdot \mathcal{H}_j = 0$, and with the estimate $\widehat{\mathcal{H}}$ of the Hankel matrix in the testing state a data-based residual ζ can be formulated as

$$\zeta = \sqrt{N}\ \mathrm{vec}(S_j{}^T \widehat{\mathcal{H}}), \tag{3}$$

where $'\mathrm{vec}'$ denotes the stacking vectorization operator. If the system has changed, S_j is no longer the left null space of the tested Hankel matrix and the residual mean value will differ from zero.

The residual is evaluated statistically in a hypothesis test (e.g. from [10]), which writes as

$$t = \zeta^T\ \Sigma_\zeta^{-1}\ \zeta. \tag{4}$$

The asymptotic residual covariance Σ_ζ can be computed from the asymptotic covariance $\Sigma_{\mathcal{H}}$ of the vectorized Hankel matrix with

$$\Sigma_\zeta = J_{\zeta,\mathcal{H}}\ \Sigma_{\mathcal{H}}\ J_{\zeta,\mathcal{H}}^T\ , \tag{5}$$

where $J_{\zeta,\mathcal{H}} = I \otimes S^T$. An estimate of the covariance from a data sample $y_k, k = 1, \dots, N$ separated into n_b blocks of length N_b, with $n_b \cdot N_b = N$, writes as [10]

$$\hat{\Sigma}_{\mathcal{H}} = \frac{N_b}{n_b - 1} \sum_{h=1}^{n_b} \mathrm{vec}(\widehat{\mathcal{H}}^{(h)} - \widehat{\mathcal{H}})\ \mathrm{vec}(\widehat{\mathcal{H}}^{(h)} - \widehat{\mathcal{H}})^T. \tag{6}$$

In a training phase a threshold for the asymptotically χ^2-distributed test statistic t is computed with data from the healthy structure, allowing a given type I error (false positives). The test value in the monitoring state is compared to this threshold.

2.2 Model interpolation in the stochastic subspace-based damage detection for temperature rejection

When a mechanical structure is exposed to changing temperature, this will affect the dynamical behavior of the structure. Consequently, the Hankel matrix $\widehat{\mathcal{H}}$ in the monitoring state under temperature T does not correspond to the null space S_j in the reference state if the reference temperature T_j varies from the current temperature T. Besides changes due to damage, also changes due to temperature variation will lead to a residual mean deviating from 0, which may result in an increasing number of false alarms.

For a robust and reliable damage detection with the stochastic subspace-based method the choice of an adequate reference null space and a proper covariance is of crucial importance. In this context the model interpolation approach for linear parameter varying systems from [8] is used in [7] for a damage detection algorithm robust to temperature variation.

For the model interpolation, local models of the state space models in (1) are formulated for $j = 1, \dots, m$ reference temperatures T_j. Due to different state bases used for the estimation of the local state space models, the state space matrices cannot be interpolated directly. Instead the m local models are combined to a global state-space model, without requiring formulation of coherent local models [8]:

$$\underline{x}_{k+1} = \underline{A}\ \underline{x}_k + \underline{w}_k$$
$$y_k = \underline{C}(T)\ \underline{x}_k + \underline{v}_k\ , \tag{7}$$

where

$$\underline{x}_k = \begin{bmatrix} x_{k,1}^T & \dots & x_{k,m}^T \end{bmatrix}^T,$$

$$\underline{A} = \begin{bmatrix} A_1 & & \\ & \ddots & \\ & & A_m \end{bmatrix},$$

$$\underline{w}_k = \begin{bmatrix} w_{k,1}^T & \dots & w_{k,m}^T \end{bmatrix}^T,$$

$$\underline{C}(T) = [\rho_1(T) C_1 \quad \dots \quad \rho_m(T) C_m]\ ,$$

$$\underline{v}_k = \textstyle\sum_{j=1}^m \rho_j(T) v_{k,j}(t)\ .$$

The local model for a testing temperature is interpolated with a weighting function $\rho_j(T)$. The weighting function can be chosen by taking the effect of the temperature to the dynamical behavior qualitatively into account. For section-wise approximated linear effects e.g. bell-shaped functions may work, centered at the testing temperature T:

$$\textstyle\sum_{j=1}^m \rho_j(T)\ =\ 1. \tag{8}$$

Damage detection with the data-driven subspace-based residual (3) does not require the state space matrices of the reference state explicitly. It operates on the null space of the

Hankel matrix of the output covariances (2). As the null spaces relating on an arbitrary state basis cannot be interpolated easily, the Hankel matrices in the reference state are interpolated instead. Consider the parameter dependent formulation of the output covariances as

$$\underline{R}_i(T) = \underline{C}(T)\underline{A}^{i-1}\underline{G} = \sum_{j=1}^{m} \rho_j^2(T) C_j A_j^{i-1} G_j$$
$$= \sum_{j=1}^{m} \rho_j^2(T) R_{i,j} \;, \tag{9}$$

where the global $\underline{G}$ writes as

$$G = \begin{bmatrix} \rho_1(T)G_1 \\ \vdots \\ \rho_m(T)G_m \end{bmatrix}. \tag{10}$$

The interpolated Hankel matrix can be computed to

$$\mathcal{H}(T) = \sum_{j=1}^{m} \rho_j^2(T)\mathcal{H}_j \;. \tag{11}$$

The interpolated null space matrix $\underline{S}(T)$ is the left null space of (11) and replaced in the residual

$$\underline{\zeta} = \sqrt{N}\,\mathrm{vec}(\underline{S}(T)^T\hat{\mathcal{H}}). \tag{12}$$

The covariance of the interpolated Hankel matrix follows as

$$\underline{\Sigma}_{\mathcal{H}}(T) = \sum_{j=1}^{m} \rho_j^4(T)\Sigma_{\mathcal{H}_j} \;, \tag{13}$$

and the residual covariance as

$$\underline{\Sigma}_{\zeta} = \underline{J}_{\zeta,\mathcal{H}}\,\underline{\Sigma}_{\mathcal{H}}\,\underline{J}_{\zeta,\mathcal{H}}^T \tag{14}$$

where $\underline{J}_{\zeta,\mathcal{H}} = I \otimes \underline{S}(T)^T$. Finally, the corresponding test writes

$$\underline{t} = \underline{\zeta}^T\underline{\Sigma}_{\zeta}^{-1}\underline{\zeta}. \tag{15}$$

3 APPLICATION TO A BEAM STRUCTURE UNDER CHANGING TEMPERATURES

In former studies in [7] and [9], the model interpolation approach for a robust subspace-based damage detection was evaluated on numerical data from a mass-spring-damper with very simple temperature models. The effect of temperature on the dynamical behavior of the structure was simulated by decreasing stiffness of the springs with increasing temperature. In this work, the method is evaluated under more realistic conditions on experimental data of a laboratory beam structure.

3.1 Laboratory tests of a beam structure in a climate chamber

A reinforced concrete beam is tested in a climate chamber at defined temperatures and at different system states. The test setup is shown in Figure 1. The beam is bore at two points and the span is 2.72 m. Three geophones measure vertical vibration velocities on the top of the beam.

The structure is excited by an external shaker with a white noise signal in the range of 30 Hz to 2000 Hz, to simulate ambient excitation as it will occur similarly in e.g. bridges under operational conditions. The shaker is installed upside down (Figure 2) and isolated against the changing temperature conditions. The data is recorded with 5 kHz.

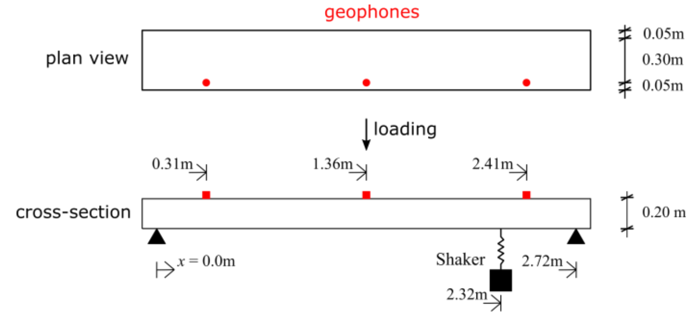

Figure 1. Test setup of the concrete beam with locations of geophones, shaker, and loading for the damage introduction.

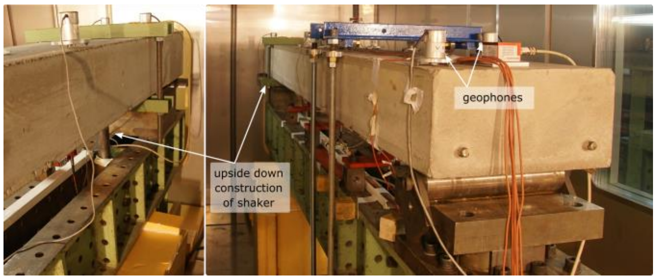

Figure 2. Concrete beam in the climate chamber.

Three different system states were considered: the undamaged reference state, damage level 1 and damage level 2. For damage level 1 the beam was loaded in the midspan with 20 kN. In the second damage state the structure was loaded up to 28 kN, that was when obvious cracks occur in the middle of the beam. The load was removed after introducing the damage. The experimental temperatures were chosen to be -25°C, -5°C, 5°C, 25°C, and 40°C. The temperatures were measured at several points inside the chamber and at two locations inside the beam. In each system state and at all 5 testing temperatures the velocities were recorded, after reaching a constant temperature level in the beam. Further details on this experiment are described in [11].

With an SSI-algorithm the eigenstructure of the system is identified. Due to high noise impact above 500 Hz, the data is sampled down to 400 Hz. Figure 3 shows the first three eigenfrequencies in dependence of the temperature, which can be clearly identified in every system state and at every temperature. The first and the second eigenfrequency are quite close. This is due to the construction of the shaker installation, which works as a tuned mass damper to the beam structure. Simulation results with a model of the corresponding structure with a mass damper have shown a good accordance to these first three eigenfrequencies.

Each frequency is plotted within its 5%-range to give an idea of the quantity of the temperature and the damage effect. The black full line marks the frequencies in the undamaged state. Effects due to temperature become more obvious for the second and the third frequency. While the damage level 2 leads to significant decrease of the eigenfrequencies of about 5%, the effect of damage level 1 does not become obvious in the first and the second eigenfrequency. The third frequency increases by about 3%. This was not expected and cannot be explained from the information gained from the experiments. However, this fact does not affect the usability of the data for the application study.

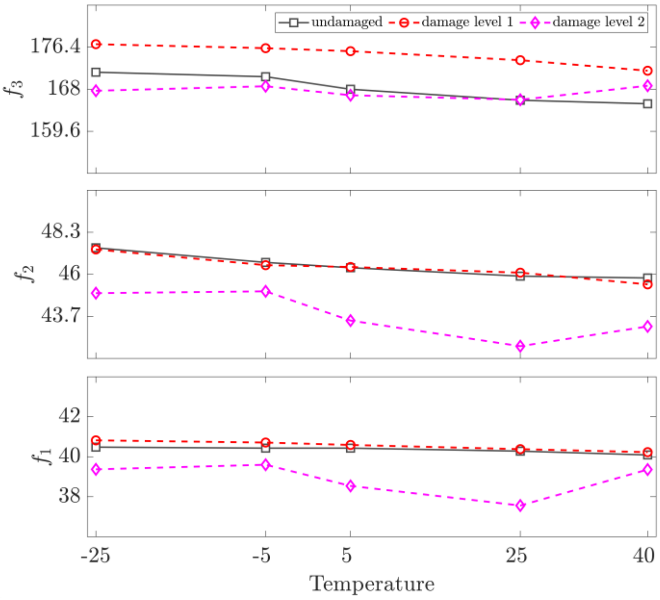

Figure 3. 1st to 3rd eigenfrequencies at different system states and at different temperatures. The y-axis is scaled to about 5% of each frequency.

3.2 Damage detection of the concrete beam under changing temperatures

To illustrate the effect of the changing temperature to damage detection of the concrete beam, the proposed method based on interpolation will be compared to 1/ taking a fixed reference at one of the temperatures (without consideration of the temperature effect), and 2/ the averaging approach from [5] that defines a fixed reference by mixing data from different reference temperatures.

Fixed reference

The tests are done firstly for the case where only data from one fixed reference temperature is considered in the reference state. The null space in (3) and the residual covariance in (5) are computed from a 5 minutes data sample at 5°C. Figure 4 shows the test values of the hypothesis test (4) at different testing temperatures in the undamaged state and at the two damage levels. For the setup of the testing Hankel matrices $\widehat{\mathcal{H}}$ data samples of 100 seconds are used. From the test values in the reference state for the undamaged structure at 5°C (grey bars) a threshold is defined, which allows for false positive alarms of 1% maximum.

It becomes obvious that changes in the temperature in the undamaged state (green bars) effect the test values in a similar way as the different damages do. This results in a high number of false alarms in the undamaged state, namely for all cases where the test temperature is different from the reference temperature. While all test values in the damaged cases also exceed the threshold, they cannot be distinguished from the test values in the reference state due to the temperature effect.

Averaging approach from [5]

In [5] a temperature rejection approach has been proposed, which uses data from different temperatures in the reference state together. From this merged data set the averaged reference Hankel matrix and its left null space is computed, as well as the covariance of the resulting residual. In Figure 5 the test values from this method are presented, where 5 minute data samples from three reference temperatures, -25°C, 5°C and 40°C, are merged in one reference data set. The testing Hankel matrices $\widehat{\mathcal{H}}$ are computed from a 100 s data sample. A threshold is defined from data in the reference state, thus of the undamaged structure at the three reference temperatures (grey bars), allowing for 1% false alarms.

The structure is correctly identified to be undamaged if the testing temperature corresponds to one of the used reference temperatures. Information about all reference states is contained in the merged reference data set and thus considered in the reference null space and the covariance matrix.

If the temperature differs from the reference temperatures, the changes in the structure due to temperature lead to higher test values even in the healthy states, exceeding the threshold. The structure is detected to be damaged. These changes due to new testing temperatures have quantitatively the same effect to the test values as damages do. While damages can be detected at all temperature levels, they cannot be distinguished from the healthy state at testing temperatures that are not contained in the reference data.

Proposed interpolation approach

In the interpolation approach, the information about the temperature depending behavior of the structure at testing temperatures different from the reference temperatures can qualitatively be carried by an adequate weighting function.

The test values in Figure 6 are computed with the interpolation approach using the parameter depending Hankel (11) and covariance matrices (14) of the undamaged system at the three reference temperatures -25°C, 5°C, and 40°C. For the weighting of these matrices a Gaussian function with a variance of 5.5, centered at the testing temperature, is applied. Data samples of 5 minutes are used for the reference set up, and the testing Hankel matrices are computed from 100 s data samples. A threshold is computed in the reference state of the undamaged structure at the three reference temperatures (grey bars), allowing for 1% false alarms.

With this approach the undamaged state can be distinguished clearly from the damaged state for any testing temperature. There is no difference in the test values, whether data from the testing temperature was available for the reference set up or not. An adequate null space can be found in any case, and false alarms in the undamaged state are avoided. The damages can be detected reliably.

In general, the test values of the interpolation method are higher than in the other computation methods. This can be explained from the choice of the weighting of the local reference models. For any testing temperature, the Gaussian function with variance of 5.5 leads to a weighted mixture between *all* of the used reference models. This means that even at the reference temperatures, where data for a more precise reference null space exists, an interpolated reference null space is used, which is affected by the other reference null spaces. This leads to some bias and as a result to higher test values of the hypothesis test. However, in the damaged case this effect persists, and damages still can be detected reliably.

In the following, some conclusions are made for the choice of weighting function. It could be confirmed that the usage of bell-shaped functions centered at the testing temperature is adequate, if the temperature effects can be assumed to be approximately linear for each section. However, the choice of the variance of these weighting functions is quite important. If a function with a high variance is chosen, this leads to results very similar to the averaging approach in Figure 5. If the variance is very low, the test values are similar to those in Figure 4, when only one reference temperature is used. In the present case the interpolation worked well with a function with $\sigma^2 = 5.5$ variance, but the effect of the width should be studied in further applications. In any case it is recommended not to use weighting functions that neglect nearby local models, even if a good local model is known from existing reference data. In this case the test values for testing temperatures different from the reference ones would be higher and could lead to false alarms.

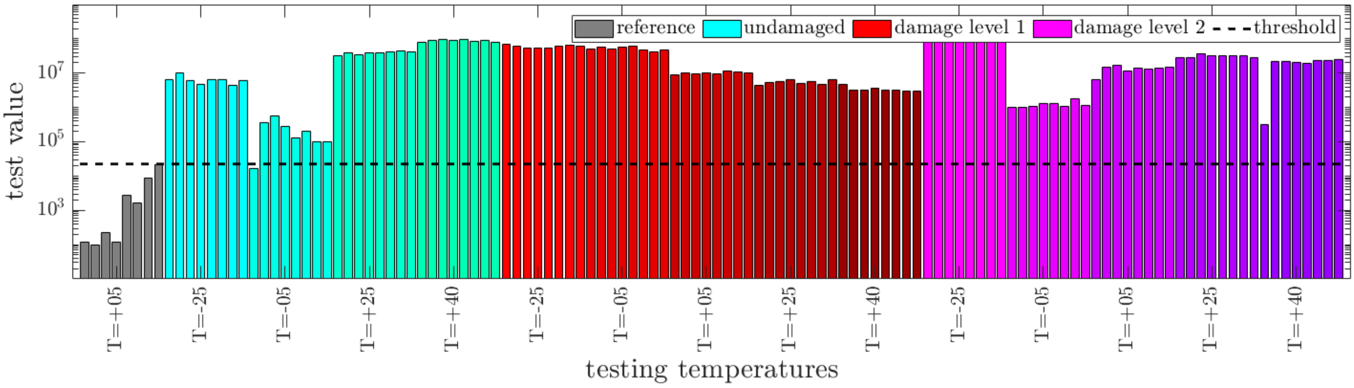

Figure 4. Test values at different testing temperatures in the undamaged state and at two damage levels. **Fixed** reference null space and covariance matrix are computed with data from one data set recorded at 5°C.

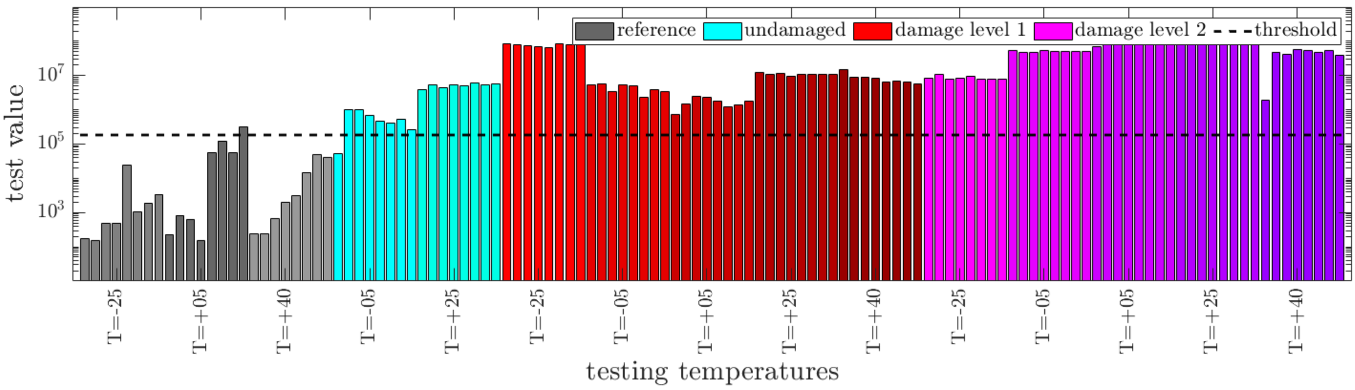

Figure 5. Test values at different testing temperatures in the undamaged state and at two damage levels. **Averaged** reference null space and covariance matrix are computed from a merged data set recorded at -25°C, 5°C, and 40°C.

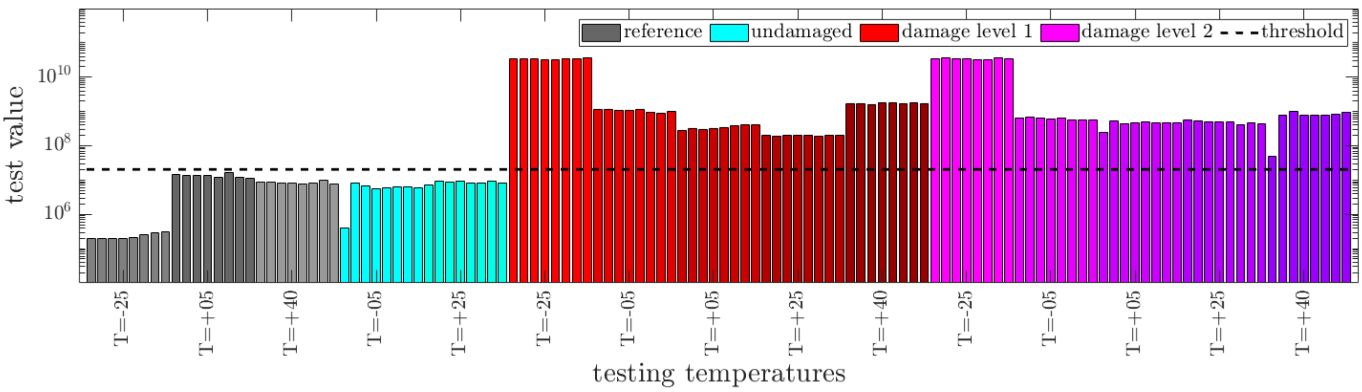

Figure 6. Test values at different testing temperatures in the undamaged state and at two damage levels. **Interpolated** reference null space and covariance matrix are computed from local matrices at -25°C, 5°C, and 40°C.

4 CONCLUSIONS

In this paper the application of an interpolation approach is presented for the consideration of the temperature effect in stochastic subspace-based damage detection to experimental data. It could be shown that the interpolation approach is able to distinguish between damaged and undamaged states reliably, and false alarms can be avoided. Some suggestions are made regarding the choice of an adequate weighting function.

Further research should address the method's performance and the choice of weighting functions for more complex temperature effects, especially discontinuous behavior as shown in [2]. Moreover, the effect of a more detailed consideration of the uncertainties of the reference null space as proposed in [12] should be investigated on the test performance of the interpolation method. In this context the question also arises about the minimum damage size that can be detected with an interpolated reference null space, and if the bias in the interpolated reference null space resulting from combining several local models affects the test sensitivity.

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