Hankel matrix-based Mahalanobis distance for fault detection robust towards changes in process noise covariance

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Abstract: Statistical subspace-based change detection residuals have been developed to infer a change in the eigenstructure of linear systems. Their statistical properties have been properly evaluated in the case of a known reference and constant noise properties. Previous residuals have favored the family of null space-based approaches, whereas the possibility of using other metrics such as the Mahalanobis distance has been omitted. This paper investigates the development and study of such a norm under the premise of a varying noise covariance. Its statistical properties have been studied and tested on a numerical example of a mechanical system.

Keywords: Change detection, Local approach, Mahalanobis distance, Linear systems

1. INTRODUCTION

Damage detection in mechanical or civil structures corresponds to detecting changes in the eigenstructure of a linear system, usually under unknown inputs. Among the many model-based or data-driven methods for damage detection (Carden and Fanning, 2004; Fan and Qiao, 2011; Dong et al., 2012), methods based on direct model-data matching are particularly appealing for an automated damage diagnosis, where current measurement data are directly confronted to a reference. For instance, such methods include non-parametric change detection based on novelty detection (Worden et al., 2000), whiteness tests on Kalman filter innovations (Bernal, 2013) or null space tests (Dong et al., 2012). Another method within this category, the local asymptotic approach to change detection (Benveniste et al., 1987) focuses on the detection of small changes in some chosen system parameters. Note that the considered changes in the system parameters affect the observed linear system in a non-additive way, unlike for example (Dong and Verhaegen, 2009).

Based on Hankel matrices of output covariances, subspace methods are a large family of system identification methods that have been developed and extensively studied for linear systems to identify their eigenstructure. Based on subspace methods but avoiding the cumbersome and fine-tuned exercise of identification, a family of residuals in the local approach framework were proposed for fault detection (Balmes et al., 2006; Döhler et al., 2016; Döhler and Mevel, 2013; Döhler et al., 2014; Viefhues et al., 2020). A particular difficulty for their design is the robustness of the resulting Generalized Likelihood Ratio (GLR) statistical test towards changing process noise covariance Q. A change in Q exerts a change in both its sensitivity with respect to the parameter and its covariance, which then affects the distribution properties of the GLR, with or without any change considered in the system, as shown in Döhler and Mevel (2013). A fault detection residual robust towards changes of Q was proposed in Döhler and Mevel (2013), under the assumption that the reference model is deterministic, while in practice it is evaluated from measurements and subjected to estimation uncertainty. For these null-space based residuals, the uncertainty of the reference model was accounted for in the residual proposed in Viefhues et al. (2018), and extended to the problem of robustness to temperature and noise characteristics changes in Viefhues et al. (2020) by a variance scheme involving recomputation of the residual covariance for every new tested data set and no proper formulation of the associated sensitivity. The previously considered residuals that have been extensively studied in the context of the local approach framework need to define a reference null space for the comparison of column spaces of Hankel matrices. Alternatively, the direct and simple difference between the two Hankel matrices is also a possible metric of change that can be favored since it avoids this null space definition. Including the classic and well adopted Mahalanobis distance, the notion of features comparison through a distance metric is a widely spread choice in change detection. The current paper builds upon the previous works and extends them such that the Mahalanobis distance benefits of the good theoretical properties of the local approach framework.

This paper will explain how the Mahalanobis distance fits in the local approach framework and how uncertainty in the reference and robustness to change in the process noise covariance can be theoretically approached. In Section 2, the modeling and classical Mahalanobis distance will be recalled. Section 3 will present the new robust residual, whose asymptotic distribution will be established, as well as the exact formulation of the necessary sensitivity and covariance matrices. This will allow to formulate the asymptotic law of the hypothesis test deciding about a change in the system under the assumption of a simultaneous change in the variance of the process noise. Then, it will be finally shown that the mean of the test under the assumption of no system change is independent of the process noise. In Section 4, this new hypothesis test will be validated on a simulated numerical system.

2. HANKEL MATRIX DIFFERENCE-BASED FAULT DETECTION

Hereafter a fault detection residual based on a difference of Hankel matrices is introduced and its statistical properties are derived.

2.1 Preliminaries

Consider the discrete time state-space model

$$\begin{cases} x_{k+1} = Ax_k + v_k \\ y_k = Cx_k + w_k \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n$ are the states, $y_k \in \mathbb{R}^r$ are the outputs, $A \in \mathbb{R}^{n\times n}$, $C \in \mathbb{R}^{r\times n}$ are respectively the state and the observation matrices, where n is the known system order and r is the number of the observed outputs. The process noise v_k is assumed to be a stationary process with zero mean and covariance matrix $Q = \mathrm{E}(v_k v_k^T)$, w_k denotes the zero-mean output noise with covariance matrix $R = \mathrm{E}(w_k w_k^T)$, and the covariance between v_k and w_k is $S = \mathrm{E}(v_k w_k^T)$, where $\mathrm{E}(\cdot)$ denotes the expectation operator.

Let $\mathcal{R}_i = \mathrm{E}(y_k y_{k-i}^T) = CA^{i-1}G$ be the theoretical output covariances of the measurements, where $G = \mathrm{E}(x_{k+1} y_k^T) = A\Sigma^s C^T + S$ and $\Sigma^s = \mathrm{E}(x_k x_k^T)$. There have been many methods to analyse the output covariance matrices, in particular the system identification techniques where modes of the system are extracted by subspace methods. They rely on the construction of Hankel matrices of these output covariances, i.e.

$$\mathcal{H} = \begin{bmatrix} \mathcal{R}_1 & \mathcal{R}_2 & \dots & \mathcal{R}_q \\ \mathcal{R}_2 & \mathcal{R}_3 & \dots & \mathcal{R}_{q+1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{R}_{p+1} & \mathcal{R}_{p+2} & \dots & \mathcal{R}_{p+q} \end{bmatrix} \in \mathbb{R}^{(p+1)r\times qr}, \tag{2}$$

where p and q are chosen such that $\min(pr, qr) \geq n$ with often $p + 1 = q$. Consistent estimates $\hat{\mathcal{R}}_i$ and $\hat{\mathcal{H}}$ can be computed from the output covariances of the measurements $Y_N = \{y_1 \dots y_N\}$ of length N. The matrix $\mathcal{H}$ enjoys the factorization property

$$\mathcal{H} = \mathcal{O}(C, A)\ \mathcal{C}(A, G) \tag{3}$$

into observability times the controllability matrix. The matrices $\mathcal{O}(C, A)$ and $\mathcal{C}(A, G)$ are classically obtained from a singular value decomposition (SVD) of $\mathcal{H}$ thanks to the factorization property (3). Then, eigenvalues and eigenvectors of the system are retrieved from $\mathcal{O}(C, A)$ using numerical algebra operations. Finally, a comparison of the resultant eigenstructure for two different data sets can indicate a possible model change therein. These approaches are usually cumbersome, hard to automatize and need engineering expertise. Alternatively, the output covariance matrices capture the whole dynamics of the structure. Monitoring changes in the output covariances will be equivalent to monitoring changes in the eigenstructure of the linear system. Monitoring changes of the column space of the Hankel matrix has been an investigated approach that yields change detection tests taking into account uncertainty information and sensitivity to the monitored change by means of a χ^2 hypothesis test. In the next section a simple change detection metric based on the difference between two Hankel matrices is developed.

2.2 Hankel matrix difference for change detection

Analyzing the differences between two different features is a classical approach in data analysis. Squaring the difference and taking into account the variance of the reference variable yields the Mahalanobis distance in many occasions. The proposed approach here is at the crossroad between the family of variants of Mahalanobis distances and the local approach framework developed previously by Benveniste et al. (1987) and recalled later.

Let θ denote a parameterization of the properties of system (1), and let θ_* be its value in the reference state. A single set of measurements Y_N is generated from the system under (unknown) θ, and covariance matrices Q, R and S. The considered fault detection problem relates to monitoring the changes of the system from its nominal behavior characterized by θ_*, based on Y_N. In this section, no change in the value of Q between different measurement sessions is assumed. Hereafter, the statistical context for detecting changes is defined.

Let $\mathcal{H}_{\mathrm{ref}}^{\theta_*}$ and $\mathcal{H}_{\mathrm{test}}^{\theta}$ be exact Hankel matrices of rank n and let $\hat{\mathcal{H}}_{\mathrm{ref}}^{\theta_*}$ and $\hat{\mathcal{H}}_{\mathrm{test}}^{\theta}$ be their estimates obtained respectively from data sets of lengths M and N. Then,

$$\mathcal{H}_{\mathrm{test}}^{\theta} - \mathcal{H}_{\mathrm{ref}}^{\theta_*} = 0 \quad \text{iff} \quad \theta = \theta_*, \tag{4}$$

and

$$\mathcal{H}_{\mathrm{test}}^{\theta} - \mathcal{H}_{\mathrm{ref}}^{\theta_*} \neq 0 \quad \text{iff} \quad \theta \neq \theta_*. \tag{5}$$

In this context, a decision about a parameter change needs to take into account the uncertainty information of the Hankel matrices and their sensitivity towards the considered parameterization. A validated approach for evaluating this change is to define a residual whose properties can be evaluated properly as the number of samples goes to infinity. For this, the above hypotheses are rewritten as the "close hypotheses"

$$\begin{aligned} &\mathrm{H}_0 : \theta = \theta_* \ \ \text{(reference state)}, \\ &\mathrm{H}_1 : \theta = \theta_* + \delta/\sqrt{N} \ \ \text{(faulty state)}, \end{aligned} \tag{6}$$

where $\delta = \sqrt{N}(\theta - \theta_*)$ is unknown but a fixed change vector. This is known as the local approach (Benveniste et al., 1987). Based on both (4) and (5), a change detection residual can be defined as

$$\hat{\xi}^{\theta} \overset{\mathrm{def}}{=} \sqrt{N}\mathrm{vec}(\hat{\mathcal{H}}_{\mathrm{test}}^{\theta} - \hat{\mathcal{H}}_{\mathrm{ref}}^{\theta_*}), \tag{7}$$

The use of the normalization factor $\sqrt{N}$ in the residual is related to the local approach and will become apparent when analyzing its distribution in the next section.

2.3 Change detection framework

The asymptotic local approach for change detection developed in Benveniste et al. (1987) is used to characterize the distribution of the residual (7). Thanks to the close hypotheses assumption and the Central Limit Theorem (CLT), the local approach ensures that $\hat{\mathcal{H}}_{\mathrm{test}}^{\theta}$ is asymptotically Gaussian, and it holds

$$\text{H}_0 : \sqrt{N}\,\text{vec}(\hat{\mathcal{H}}^{\theta}_{\text{test}} - \mathcal{H}^{\theta_*}_{\text{test}}) \xrightarrow{\mathcal{L}} \mathcal{N}(0, \Sigma_{\text{test}}), \tag{8}$$

$$\text{H}_1 : \sqrt{N}\,\text{vec}(\hat{\mathcal{H}}^{\theta}_{\text{test}} - \mathcal{H}^{\theta_*}_{\text{test}}) \xrightarrow{\mathcal{L}} \mathcal{N}(\mathcal{J}^{\mathcal{H}_{\text{test}}}_{\theta_*}\delta, \Sigma_{\text{test}}), \tag{9}$$

where Σ_{test} is the asymptotic covariance of $\text{vec}(\hat{\mathcal{H}}^{\theta}_{\text{test}})$, and $\mathcal{J}^{\mathcal{H}_{\text{test}}}_{\theta_*} = \partial \text{vec}(\mathcal{H}^{\theta}_{\text{test}})/\partial\theta\ (\theta_*)$. Similarly, $\hat{\mathcal{H}}^{\theta_*}_{\text{ref}}$ is asymptotically Gaussian distributed

$$\sqrt{M}\,\text{vec}(\hat{\mathcal{H}}^{\theta_*}_{\text{ref}} - \mathcal{H}^{\theta_*}_{\text{ref}}) \xrightarrow{\mathcal{L}} \mathcal{N}(0, \Sigma_{\text{ref}}), \tag{10}$$

where Σ_{ref} is the asymptotic reference Hankel matrix covariance. The formulation of the Jacobian $\mathcal{J}^{\mathcal{H}_{\text{test}}}_{\theta_*}$ can easily be derived following the ideas of Döhler et al. (2016).

The residual $\hat{\xi}^{\theta}$ depends on both $\hat{\mathcal{H}}^{\theta_*}_{\text{ref}}$ and $\hat{\mathcal{H}}^{\theta}_{\text{test}}$, therefore before inferring its statistical properties the distribution of joint $\hat{\mathcal{H}}^{\theta_*}_{\text{ref}}$ and $\hat{\mathcal{H}}^{\theta}_{\text{test}}$ is derived. Define the joint vectors

$$\hat{h} = \begin{bmatrix} \text{vec}(\hat{\mathcal{H}}^{\theta_*}_{\text{ref}}) \\ \text{vec}(\hat{\mathcal{H}}^{\theta}_{\text{test}}) \end{bmatrix}, \quad h = \begin{bmatrix} \text{vec}(\mathcal{H}^{\theta_*}_{\text{ref}}) \\ \text{vec}(\mathcal{H}^{\theta_*}_{\text{test}}) \end{bmatrix}.$$

Since both Hankel matrices are computed on different data sets, they are statistically independent, and it follows for the joint distribution

$$\text{under H}_0 : \sqrt{N}\,(\hat{h} - h) \xrightarrow{\mathcal{L}} \mathcal{N}(0, \Sigma_h),$$
$$\text{under H}_1 : \sqrt{N}\,(\hat{h} - h) \xrightarrow{\mathcal{L}} \mathcal{N}(\mathcal{J}^{h}_{\theta_*}\delta, \Sigma_h),$$

where

$$\mathcal{J}^{h}_{\theta_*} = \begin{bmatrix} 0 \\ \mathcal{J}^{\mathcal{H}_{\text{test}}}_{\theta_*} \end{bmatrix}, \quad \Sigma_h = \begin{bmatrix} c\Sigma_{\text{ref}} & 0 \\ 0 & \Sigma_{\text{test}} \end{bmatrix}. \tag{11}$$

and $c = \lim \frac{N}{M}$ such that $\text{cov}(\sqrt{\frac{N}{M}}\sqrt{M}\text{vec}(\hat{\mathcal{H}}^{\theta_*}_{\text{ref}})) \approx c\Sigma_{\text{ref}}$.

The estimate of the residual $\hat{\xi}^{\theta}$ is asymptotically Gaussian with the distribution properties

$$\text{H}_0 : \hat{\xi}^{\theta_*} \xrightarrow{\mathcal{L}} \mathcal{N}(0, \Sigma_{\xi}), \tag{12}$$

$$\text{H}_1 : \hat{\xi}^{\theta} \xrightarrow{\mathcal{L}} \mathcal{N}(\mathcal{J}^{\xi}_{\theta_*}\delta, \Sigma_{\xi}), \tag{13}$$

where $\mathcal{J}^{\xi}_{\theta_*} = \partial \text{vec}\left(\mathcal{H}^{\theta_*}_{\text{ref}} - \mathcal{H}^{\theta}_{\text{test}}\right)/\partial\theta\ (\theta_*)$ is the residual sensitivity and $\Sigma_{\xi} = c\Sigma_{\text{ref}} + \Sigma_{\text{test}}$ its covariance.

Let $\widehat{\mathcal{J}}$ and $\widehat{\Sigma}$ respectively be consistent estimates of $\mathcal{J}^{\xi}_{\theta_*}$ and Σ_{ξ}. Then a Generalized Likelihood Ratio (GLR) test to decide between H_0 and H_1 writes as

$$t = (\hat{\xi}^{\theta})^T\,\widehat{\Sigma}^{-1}\,\widehat{\mathcal{J}}\left(\widehat{\mathcal{J}}^T\,\widehat{\Sigma}^{-1}\,\widehat{\mathcal{J}}\right)^{-1}\widehat{\mathcal{J}}^T\,\widehat{\Sigma}^{-1}\hat{\xi}^{\theta}\ . \tag{14}$$

Assuming Σ_{ξ} to be invertible, under H_0, t follows a χ^2 distribution with $\text{rank}(\mathcal{J}^{\xi}_{\theta_*})$ degrees of freedom. Under H_1, it follows a non-central χ^2 distribution with $\text{rank}(\mathcal{J}^{\xi}_{\theta_*})$ degrees of freedom and noncentrality parameter

$$\lambda = \delta^T(\mathcal{J}^{\xi}_{\theta_*})^T\,\Sigma^{-1}_{\xi}\,\mathcal{J}^{\xi}_{\theta_*}\delta\ .$$

Due to the independence between the two data sets, the distribution of (14) is derived along the same lines as in Basseville et al. (2000) without any hurdle, under the assumption that the unknown matrix Q is equal for the two different data sets. This assumption is now relaxed.

3. NORMALIZED HANKEL MATRIX DIFFERENCE FOR FAULT DETECTION

In this section, the development of a residual robust towards changes in Q is motivated. Hereafter a normalization scheme for the metric in (7) is developed and the statistical properties of the resulting robust residual are derived.

3.1 Motivation

As recalled above, the formulation of the analytical form of the output covariance matrices depends on G. Then, those matrices vary with the amplitude or the characteristics of the noise. In that sense, a simple difference between the same output covariance features computed for different data sets would be meaningless if this variability is not considered in the design of the metric. Any change in the variance of the noise will lead to a change in mean and the variance of the designed residual, and an inability to establish the GLR test as a random variable with a known χ^2 distribution. To avoid the analysis of a possibly complicated and non-tractable distribution of the GLR test under changes in the noise properties, a different residual can be formulated that takes into account changes in the noise statistics in its design. In the following, such a residual is derived and its distribution properties are studied under the local approach framework.

3.2 Normalization scheme

Let $\mathcal{H}^{\theta_*}_{\text{ref}}$ and $\mathcal{H}^{\theta_*}_{\text{test}}$ be two exact Hankel matrices of rank n for a system in the reference state θ_*, subjected to process noise with unknown, possibly different covariances Q_{ref} and Q_{test}. An SVD of the juxtaposed matrices $\mathcal{H}_{\text{ref}}$ and $\mathcal{H}_{\text{test}}$ writes

$$\left[\mathcal{H}^{\theta_*}_{\text{ref}}\ \mathcal{H}^{\theta_*}_{\text{test}}\right] = \left[U_{\text{s}}\ U_{\text{ker}}\right] \begin{bmatrix} D_{\text{s}} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V^T_{\text{s}} \\ V^T_{\text{ker}} \end{bmatrix}, \tag{15}$$

where $\text{rank}\left(\left[\mathcal{H}^{\theta_*}_{\text{ref}}\ \mathcal{H}^{\theta_*}_{\text{test}}\right]\right) = n$, $U_{\text{s}} \in \mathbb{R}^{(p+1)r\times n}$ contains the left singular vectors, $D_{\text{s}} \in \mathbb{R}^{n\times n}$ contains the non-zero singular values and $V_{\text{s}} \in \mathbb{R}^{2qr\times n}$ contains the right singular vectors, which are split into $V^T_{\text{s}} = \left[V^T_{\text{s,ref}}\ V^T_{\text{s,test}}\right]$ corresponding to $\mathcal{H}^{\theta_*}_{\text{ref}}$ and $\mathcal{H}^{\theta_*}_{\text{test}}$ respectively. Now define

$$\mathcal{Z}_{\text{ref}} = D_{\text{s}}V^T_{\text{s,ref}}, \quad \mathcal{Z}_{\text{test}} = D_{\text{s}}V^T_{\text{s,test}}, \tag{16}$$

where both $\mathcal{Z}_{\text{ref}}$ and $\mathcal{Z}_{\text{test}}$ are full row rank. The exact Hankel matrices share the same image in the reference state

$$\left[\mathcal{H}^{\theta_*}_{\text{ref}}\ \mathcal{H}^{\theta_*}_{\text{test}}\right] = U_{\text{s}}\left[\mathcal{Z}_{\text{ref}}\ \mathcal{Z}_{\text{test}}\right]. \tag{17}$$

To compare $\mathcal{H}^{\theta_*}_{\text{ref}}$ with $\mathcal{H}^{\theta_*}_{\text{test}}$ an appropriate normalization is given by

$$\overline{\mathcal{H}}_{\text{test}} = \mathcal{H}^{\theta_*}_{\text{test}}\mathcal{Z}^{\dagger}_{\text{test}}\mathcal{Z}_{\text{ref}}, \tag{18}$$

where $\overline{\mathcal{H}}_{\text{test}}$ now shares the same $\mathcal{C}(A, G)$ as $\mathcal{H}^{\theta_*}_{\text{ref}}$.

3.3 Normalized residual

Let $\hat{\mathcal{H}}^{\theta_*}_{\text{ref}} \to \mathcal{H}^{\theta_*}_{\text{ref}}$ be obtained from a data set of length M generated under a process noise covariance Q_{ref}. Similarly, assume $\hat{\mathcal{H}}^{\theta}_{\text{test}} \to \mathcal{H}^{\theta}_{\text{test}}$ obtained from a data set of length N generated under the process noise covariance Q_{test}, with Q_{ref} possibly different from Q_{test}. Then, (17) yields

$$\mathcal{H}^{\theta}_{\text{test}}\mathcal{Z}^{\dagger}_{\text{test}}\mathcal{Z}_{\text{ref}} - \mathcal{H}^{\theta_*}_{\text{ref}} = 0 \quad \text{iff} \quad \theta = \theta_*, \tag{19}$$

and

$$\mathcal{H}^{\theta}_{\text{test}}\mathcal{Z}^{\dagger}_{\text{test}}\mathcal{Z}_{\text{ref}} - \mathcal{H}^{\theta_*}_{\text{ref}} \neq 0 \quad \text{iff} \quad \theta \neq \theta_*, \tag{20}$$

where $\mathcal{Z}_{\text{ref}}$ and $\mathcal{Z}_{\text{test}}$ are obtained from an SVD of $\left[\mathcal{H}^{\theta_*}_{\text{ref}}\ \mathcal{H}^{\theta}_{\text{test}}\right]$ analogous to (15) that is truncated at order n. Based on both (19) and (20), the change detection residual is defined as

$$\hat{\zeta}^{\theta} \stackrel{\text{def}}{=} \sqrt{N}\text{vec}(\hat{\mathcal{H}}^{\theta}_{\text{test}}\hat{\mathcal{Z}}^{\dagger}_{\text{test}}\hat{\mathcal{Z}}_{\text{ref}} - \hat{\mathcal{H}}^{\theta_*}_{\text{ref}}), \tag{21}$$

where $\hat{\mathcal{Z}}_{\text{ref}}$ and $\hat{\mathcal{Z}}_{\text{test}}$ are defined from the SVD of the juxtaposed Hankel matrix estimates partitioned at system order n

$$\left[\hat{\mathcal{H}}_{\text{ref}}^{\theta_*}\ \hat{\mathcal{H}}_{\text{test}}^{\theta}\right] = \left[\hat{U}_{\text{s}}\ \hat{U}_{\text{ker}}\right] \begin{bmatrix} \hat{D}_{\text{s}} & 0 \\ 0 & \hat{D}_{\text{ker}} \end{bmatrix} \begin{bmatrix} \hat{V}_{\text{s,ref}}^T & \hat{V}_{\text{s,test}}^T \\ \hat{V}_{\text{ker,ref}}^T & \hat{V}_{\text{ker,test}}^T \end{bmatrix},$$

as $\hat{\mathcal{Z}}_{\text{ref}} = \hat{D}_{\text{s}}\hat{V}_{\text{s,ref}}^T$, $\hat{\mathcal{Z}}_{\text{test}} = \hat{D}_{\text{s}}\hat{V}_{\text{s,test}}^T$.

3.4 Residual distribution

Hereafter, the asymptotic distribution of the new residual in (21) is derived.

Theorem 1. The residual in (21) is asymptotically Gaussian with the following properties

$$\text{H}_0 : \hat{\zeta}^{\theta_*} \xrightarrow{\mathcal{L}} \mathcal{N}(0, \Sigma_\zeta), \tag{22}$$

$$\text{H}_1 : \hat{\zeta}^{\theta} \xrightarrow{\mathcal{L}} \mathcal{N}(\mathcal{J}_{\theta_*}^{\zeta}\delta, \Sigma_\zeta)\ , \tag{23}$$

where $\Sigma_\zeta = \mathcal{J}_h^\zeta \Sigma_h \mathcal{J}_h^{\zeta T}$ is the residual covariance, whose exact formulation is derived in Appendix A, which boils down to

$$\Sigma_\zeta = \mathcal{J}_{\mathcal{H}_{\text{ref}}}^{\zeta} c\Sigma_{\text{ref}}(\mathcal{J}_{\mathcal{H}_{\text{ref}}}^{\zeta})^T + \mathcal{J}_{\mathcal{H}_{\text{test}}}^{\zeta}\Sigma_{\text{test}}(\mathcal{J}_{\mathcal{H}_{\text{test}}}^{\zeta})^T, \tag{24}$$

where $\mathcal{J}_{\mathcal{H}_{\text{ref}}}^{\zeta} = I_{(p+1)r} \otimes U_{\text{ker}}U_{\text{ker}}^T$ and $\mathcal{J}_{\mathcal{H}_{\text{test}}}^{\zeta} = (\mathcal{Z}_{\text{test}}^\dagger \mathcal{Z}_{\text{ref}})^T \otimes U_{\text{ker}}U_{\text{ker}}^T$, and $\mathcal{J}_{\theta_*}^{\zeta}$ is the residual sensitivity with respect to the chosen parameterization defined as

$$\mathcal{J}_{\theta_*}^{\zeta} = \left(\left(\mathcal{Z}_{\text{ref}}^\dagger \mathcal{Z}_{\text{ref}}\right)^T \otimes U_{\text{ker}}U_{\text{ker}}^T \right) \mathcal{J}_{\theta_*}^{\mathcal{H}_{\text{ref}}}, \tag{25}$$

where $\mathcal{J}_{\theta_*}^{\mathcal{H}_{\text{ref}}} = \partial\text{vec}\,(\mathcal{H}_{\text{ref}})/\partial\theta\ (\theta_*)$.

Proof: Due to the involvement of two different model states, the Jacobian is a function of two variables corresponding to both the reference and the possibly changed parameter values. Let θ' and θ'' be the corresponding values in the parameter space. Let the related Hankel matrices be $\mathcal{H}_{\text{ref}}^{\theta'}$ and $\mathcal{H}_{\text{test}}^{\theta''}$, then along the lines of (15) define

$$\mathcal{Z}_{\text{ref}}^{\theta',\theta''} = D_{\text{s}}^{\theta',\theta''} V_{\text{s,ref}}^{\theta',\theta''\,T}$$

$$\mathcal{Z}_{\text{test}}^{\theta',\theta''} = D_{\text{s}}^{\theta',\theta''} V_{\text{s,test}}^{\theta',\theta''\,T}$$

Those two variables are depending on both parameters. This is a technical difficulty not present in the study of the previous null space residuals, where all involved matrices were statistically decoupled. Notice that the asymptotic residual can be defined as a function of θ' and θ'' as

$$\zeta^{\theta',\theta''} = \text{vec}\left(\tilde{\mathcal{H}}_{\text{test}}^{\theta',\theta''} - \mathcal{H}_{\text{ref}}^{\theta'}\right)$$

where $\tilde{\mathcal{H}}_{\text{test}}^{\theta',\theta''} = \mathcal{H}_{\text{test}}^{\theta''}(\mathcal{Z}_{\text{test}}^{\theta',\theta''})^\dagger \mathcal{Z}_{\text{ref}}^{\theta',\theta''}$

The derivative of $\zeta^{\theta',\theta''}$ with respect to θ'' evaluated at the point (θ_*, θ_*) coincides with the Jacobian matrix $\mathcal{J}_{\theta_*}^{\zeta}$ and writes $\frac{\partial\zeta}{\partial\theta''}(\theta_*, \theta_*) = \breve{\mathcal{J}}_{\mathcal{H}_{\text{test}}}^{\zeta} \breve{\mathcal{J}}_{\theta_*}^{\mathcal{H}_{\text{test}}}$, where

$$\breve{\mathcal{J}}_{\mathcal{H}_{\text{test}}}^{\zeta} = \frac{\partial\text{vec}\left(\mathcal{H}_{\text{test}}^{\theta''}(\mathcal{Z}_{\text{test}}^{\theta',\theta''})^\dagger \mathcal{Z}_{\text{ref}}^{\theta',\theta''}\right)}{\partial\text{vec}\left(\mathcal{H}_{\text{test}}^{\theta''}\right)}\ (\theta_*, \theta_*),$$

$$\breve{\mathcal{J}}_{\theta_*}^{\mathcal{H}_{\text{test}}} = \frac{\partial\text{vec}\left(\mathcal{H}_{\text{test}}^{\theta''}\right)}{\partial\theta''}\ (\theta_*).$$

Derivatives $\breve{\mathcal{J}}_{\mathcal{H}_{\text{test}}}^{\zeta}$ and $\breve{\mathcal{J}}_{\theta_*}^{\mathcal{H}_{\text{test}}}$ coincide respectively with $\mathcal{J}_{\mathcal{H}_{\text{test}}}^{\zeta}$ and $\mathcal{J}_{\theta_*}^{\mathcal{H}_{\text{test}}}$ since the derivative of the singular vectors are continuous in θ'', see Appendix B. After dropping θ_*, the expression for $\mathcal{J}_{\theta_*}$ writes as

$$\mathcal{J}_{\theta_*}^{\zeta} = \left((\mathcal{Z}_{\text{test}}^\dagger \mathcal{Z}_{\text{ref}})^T \otimes U_{\text{ker}}U_{\text{ker}}^T\right) \mathcal{J}_{\theta_*}^{\mathcal{H}_{\text{test}}} \tag{26}$$

Notice that $\mathcal{H}_{\text{ref}}^{\theta_*} = \mathcal{H}_{\text{test}}^{\theta_*}\mathcal{Z}_{\text{test}}^\dagger \mathcal{Z}_{\text{ref}}$, $\mathcal{H}_{\text{test}}^{\theta_*} = \mathcal{H}_{\text{ref}}^{\theta_*}\mathcal{Z}_{\text{ref}}^\dagger \mathcal{Z}_{\text{test}}$. Subsequently, since $U_{\text{ker}}^T \mathcal{H}_{\text{ref}}^{\theta_*} = 0$, $\mathcal{J}_{\theta_*}^{\zeta}$ in (26) writes as

$$\begin{aligned}
\mathcal{J}_{\theta_*}^{\zeta} &= \left(\left(\mathcal{Z}_{\text{test}}^\dagger \mathcal{Z}_{\text{ref}}\right)^T \otimes U_{\text{ker}}U_{\text{ker}}^T\right) \text{vec}\left(\frac{\partial\mathcal{H}_{\text{ref}}}{\partial\theta}\ (\theta_*)\mathcal{Z}_{\text{ref}}^\dagger \mathcal{Z}_{\text{test}}\right) \\
&\quad + \underbrace{\left(\left(\mathcal{Z}_{\text{test}}^\dagger \mathcal{Z}_{\text{ref}}\right)^T \otimes U_{\text{ker}}U_{\text{ker}}^T\right) \text{vec}\left(\mathcal{H}_{\text{ref}}^{\theta_*}\frac{\partial\mathcal{Z}_{\text{ref}}^\dagger}{\partial\theta}\ (\theta_*)\mathcal{Z}_{\text{test}}\right)}_{=0} \\
&\quad + \underbrace{\left(\left(\mathcal{Z}_{\text{test}}^\dagger \mathcal{Z}_{\text{ref}}\right)^T \otimes U_{\text{ker}}U_{\text{ker}}^T\right) \text{vec}\left(\mathcal{H}_{\text{ref}}^{\theta_*}\mathcal{Z}_{\text{ref}}^\dagger \frac{\partial\mathcal{Z}_{\text{test}}}{\partial\theta}\ (\theta_*)\right)}_{=0} \\
&= \left((\mathcal{Z}_{\text{ref}}^\dagger \mathcal{Z}_{\text{ref}})^T \otimes U_{\text{ker}}U_{\text{ker}}^T\right) \mathcal{J}_{\theta_*}^{\mathcal{H}_{\text{ref}}} \qquad \square
\end{aligned}$$

By Slutsky's theorem, the asymptotic true values can be replaced by their estimates in the CLT.

Theorem 2. $\mathcal{J}_{\theta_*}^{\zeta}$ in (25) is invariant towards change in Q.

Proof: Let $\mathcal{H}_{\text{ref}}^{\theta_*}$ and $\mathcal{H}_{\text{test}}^{\theta_*}$ be the Hankel matrices corresponding to the same model under different process noise covariance. Their joint SVD yields the matrices U_s, $\mathcal{Z}_{\text{ref}}$ and $\mathcal{Z}_{\text{test}}$ (17), which thus depend on the process noise properties of both matrices. It is now shown that the factors $U_{\text{ker}}U_{\text{ker}}^T$ and $\mathcal{Z}_{\text{ref}}^\dagger \mathcal{Z}_{\text{ref}}$ in (25) can both be linked to the properties of $\mathcal{H}_{\text{ref}}^{\theta_*}$ only, independently from potentially different noise properties of the tested matrix $\mathcal{H}_{\text{test}}^{\theta_*}$.

Let the SVD of $\mathcal{H}_{\text{ref}}^{\theta_*}$ be $\mathcal{H}_{\text{ref}}^{\theta_*} = \tilde{U}_{\text{s}}\tilde{D}_{\text{s}}\tilde{V}_{\text{s}}^T$. From (17), it follows $\mathcal{H}_{\text{ref}}^{\theta_*} = U_s\mathcal{Z}_{\text{ref}}$. Since both U_{s} and $\tilde{U}_{\text{s}}$ define an orthogonal basis of the column space of $\mathcal{H}_{\text{ref}}^{\theta_*}$, there exists a matrix T with $U_{\text{s}} = \tilde{U}_{\text{s}}T$. It holds

$$I_n = U_{\text{s}}^T U_{\text{s}} = (\tilde{U}_{\text{s}}T)^T \tilde{U}_{\text{s}}T = T^T T,$$

hence T is orthogonal. Then, $U_{\text{ker}}U_{\text{ker}}^T = I - U_{\text{s}}U_{\text{s}}^T = I - \tilde{U}_{\text{s}}\tilde{U}_{\text{s}}^T$ and thus only related to $\mathcal{H}_{\text{ref}}^{\theta_*}$. Furthermore, from the SVD of $\mathcal{H}_{\text{ref}}^{\theta_*}$ and (15) follows

$$\mathcal{H}_{\text{ref}}^{\theta_*} = \tilde{U}_{\text{s}}TT^T\tilde{D}_{\text{s}}\tilde{V}_s^T, \quad \mathcal{Z}_{\text{ref}} = T^T\tilde{D}_{\text{s}}\tilde{V}_s^T.$$

Then, $\mathcal{Z}_{\text{ref}}^\dagger \mathcal{Z}_{\text{ref}}$ simplifies to

$$\mathcal{Z}_{\text{ref}}^\dagger \mathcal{Z}_{\text{ref}} = \tilde{V}_{\text{s}}\tilde{D}_{\text{s}}^{-1}TT^T\tilde{D}_{\text{s}}\tilde{V}_{\text{s}}^T = \tilde{V}_{\text{s}}\tilde{V}_{\text{s}}^T$$

which only depends on $\mathcal{H}_{\text{ref}}^{\theta_*}$ (the reference matrix). $\square$

As a consequence of Theorem 2 Jacobian $\mathcal{J}_{\theta_*}^{\zeta}$ is invariant for different values of the process noise variance Q. Hence, rank$(\mathcal{J}_{\theta_*}^{\zeta})$ is constant, and the resultant mean test value is not affected by different values of Q under H_0. It implies the robustness of the test (14) to false alarms under H_0 when the residual (21) is used.

4. NUMERICAL APPLICATION

In this section, the proposed change detection residual is applied in a numerical experiment on a 6 DOF mechanical chain-like system that, for any consistent set of units, is modeled with spring stiffness $k_1 = k_3 = k_5 = 100$ and

$k_2 = k_4 = k_6 = 200$, mass $m_i = 1/20$ and a proportional damping matrix such that all modes have a damping ratio of 3%. It is illustrated in Figure 1. The system is excited by a white noise signal acting at all DOFs. Two cases for Q are considered, namely first a base-case, where Q is the identity matrix I_6, and second where Q is a positive definite matrix that is randomly computed as $Q = bb^T$ where $b \in \mathbb{R}^{6\times 6}$ is a matrix whose entries are drawn from a standard normal distribution. The structural accelerations are simulated at DOFs 1, 3 and 5 at a sampling frequency of 50 Hz, and white measurement noise with 5% of the standard deviation of the output is added to each response measurement.

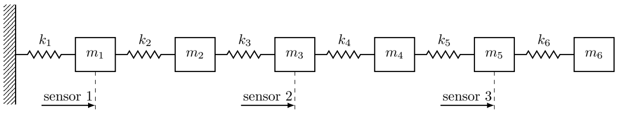

Fig. 1. 6 DOF chain system sketch

The simulation campaign consists of a reference built with data length $N_0 = 2{,}000{,}000$, and tested data sets each simulated with data length $N = 100{,}000$. In total 1000 tested states for each case of Q are realized. The damage is modeled as a gradual stiffness reduction of the second spring by 5% and 10%. First, the simple residual (7) and the corresponding GLR test (14) are evaluated for the case where Q is changing randomly between each simulated data set. Figure 2 illustrates that the distributions of the test evaluated for different damage levels are mixed and inseparable due to the changes in the value of Q.

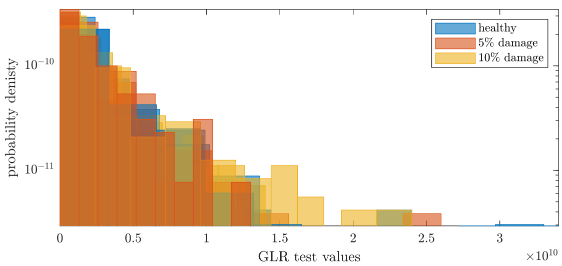

Fig. 2. Histograms of the GLR test (14) with the classic residual (7), for multiple Q at the reference state and two different damage extents.

Next, the normalized residual (21) and the corresponding GLR test are computed. In Figure 3 the distributions of the test for Monte Carlo simulations are shown for three different values of Q. In the reference case, all three distributions are superposed, confirming the robustness of the test. The test values due to model change are well separated from the ones corresponding to the reference state. They show some fluctuations due to Q in the changed states, which does not impair the overall damage detection. In Figure 4, Q is changed randomly for each simulated data set. There, one can observe that the different test conditions are well separated, even when Q changes for each data realization. Note that for the results in both Figure 3 and Figure 4 the mean of the test in the reference state is stable and very close to the theoretical value.

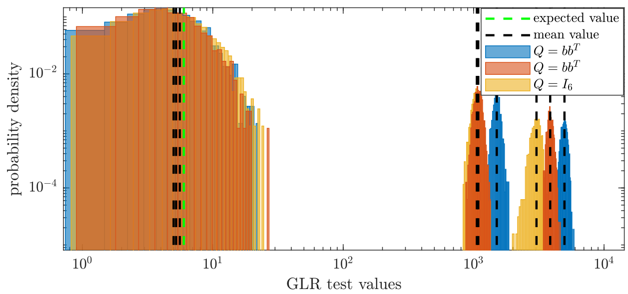

Fig. 3. Histograms of the GLR test (14) with the robust residual (21) in healthy and two damaged states. Three different values of Q in the tested data sets.

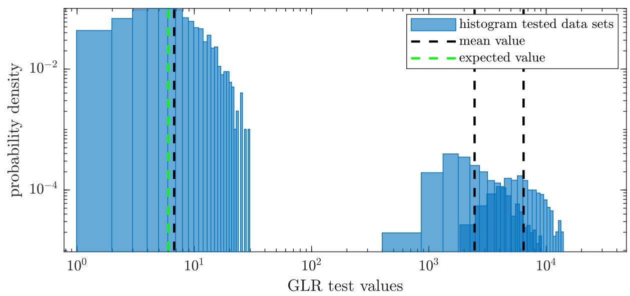

Fig. 4. Histograms of the GLR test (14) with the robust residual (21). Q changing for each tested data set.

5. CONCLUSIONS

This paper proposed a new hypothesis test coupling the Mahalanobis distance and the local approach to detect a change in the LTI system eigenstructure. The residual and thus the test has been shown to be independent of a change in the process noise in the reference state. The proposed change detection method has been validated numerically on a simple mechanical model.

Appendix A. RESIDUAL COVARIANCE

The covariance of the residual (24) is developed from the first order perturbation of (21), assuming $\theta = \theta_* + \delta/\sqrt{N}$,

$$\begin{aligned}\Delta\hat{\zeta}^\theta/\sqrt{N} &= \Delta\left(\mathrm{vec}\left(\hat{\mathcal{H}}^\theta_{\text{test}}\hat{\mathcal{Z}}_{\text{test}}{}^\dagger\hat{\mathcal{Z}}_{\text{ref}} - \hat{\mathcal{H}}^{\theta^*}_{\text{ref}}\right)\right) \qquad \text{(A.1)}\\ &= \Delta\left(\mathrm{vec}\left(\hat{\mathcal{H}}^\theta_{\text{test}}\hat{\mathcal{Z}}_{\text{test}}{}^\dagger\hat{\mathcal{Z}}_{\text{ref}}\right)\right) - \mathrm{vec}\left(\Delta\hat{\mathcal{H}}^{\theta^*}_{\text{ref}}\right).\end{aligned}$$

Since $(\hat{\mathcal{Z}}^\dagger_{\text{test}}\hat{\mathcal{Z}}_{\text{ref}})^T\hat{\mathcal{Z}}^\dagger_{\text{test}}$ is bounded and $\sqrt{N}\Delta\hat{\mathcal{Z}}_{\text{test}}$ is approx. Gaussian, with bounded moments too, then the sensitivity w.r.t.$\Delta\hat{\mathcal{Z}}_{\text{test}}$ and its cross-terms with $\Delta\hat{\mathcal{H}}^{\theta^*}_{\text{ref}}$ and $\Delta\hat{\mathcal{H}}^\theta_{\text{test}}$ can be neglected and $\Delta\hat{\zeta}^\theta/\sqrt{N}$ resolves to

$$\begin{aligned}&\approx \left(V_{\text{ker,ref}}V^T_{\text{ker,ref}} + V_{\text{s,ref}}V^\dagger_{\text{s,test}}V_{\text{ker,test}}V^T_{\text{ker,test}}\right)\\ &\otimes \mathcal{H}^\theta_{\text{test}}\mathcal{Z}^\dagger_{\text{test}}U^T_{\text{s}}\,\mathrm{vec}(\Delta\hat{\mathcal{H}}^{\theta^*}_{\text{ref}})\\ &+ \left(V_{\text{ker,ref}}V^T_{\text{ker,test}} - V_{\text{s,ref}}V^\dagger_{\text{s,test}}V_{\text{ker,test}}V^T_{\text{ker,test}}\right)\\ &\otimes \mathcal{H}^\theta_{\text{test}}\mathcal{Z}^\dagger_{\text{test}}U^T_{\text{s}}\,\mathrm{vec}(\Delta\hat{\mathcal{H}}^\theta_{\text{test}}) - \mathrm{vec}\left(\Delta\hat{\mathcal{H}}^{\theta^*}_{\text{ref}}\right)\\ &+ \left((\mathcal{Z}^\dagger_{\text{test}}\mathcal{Z}_{\text{ref}})^T \otimes I_{(p+1)r}\right)\mathrm{vec}\left(\Delta\hat{\mathcal{H}}^\theta_{\text{test}}\right)\end{aligned}$$

Since $\mathrm{rank}([\mathcal{H}^{\theta_*}_{\text{ref}}\ \mathcal{H}^{\theta_*}_{\text{test}}]) = n$

$$\begin{bmatrix} V_{\mathrm{s,ref}} \\ V_{\mathrm{s,test}} \end{bmatrix} \begin{bmatrix} V_{\mathrm{s,ref}}^T & V_{\mathrm{s,test}}^T \end{bmatrix} + \begin{bmatrix} V_{\mathrm{ker,ref}} \\ V_{\mathrm{ker,test}} \end{bmatrix} \begin{bmatrix} V_{\mathrm{ker,ref}}^T & V_{\mathrm{ker,test}}^T \end{bmatrix} = I_{2n}$$

Consequently,

$$\Big(V_{\mathrm{ker,ref}} V_{\mathrm{ker,ref}}^T + \hat{V}_{\mathrm{s,ref}} \hat{V}_{\mathrm{s,test}}^\dagger \hat{V}_{\mathrm{ker,test}} \hat{V}_{\mathrm{ker,test}}^T\Big) \otimes \mathcal{H}_{\mathrm{test}}^{\theta_*} \mathcal{Z}_{\mathrm{test}}^\dagger U_s^T = \Big(I_n \otimes \mathcal{H}_{\mathrm{test}}^{\theta_*} \mathcal{Z}_{\mathrm{test}}^\dagger U_s^T\Big)$$

and

$$\Big(V_{\mathrm{ker,ref}} V_{\mathrm{ker,test}}^T - V_{\mathrm{s,ref}} V_{\mathrm{s,test}}^\dagger V_{\mathrm{ker,test}} V_{\mathrm{ker,test}}^T\Big) \otimes \mathcal{H}_{\mathrm{test}}^{\theta_*} \mathcal{Z}_{\mathrm{test}}^\dagger U_s^T = \Big(-V_{\mathrm{s,ref}} V_{\mathrm{s,test}}^\dagger\Big) \otimes \mathcal{H}_{\mathrm{test}}^{\theta_*} \mathcal{Z}_{\mathrm{test}}^\dagger U_s^T.$$

Approximation of (A.1) writes

$$\Delta\hat{\zeta}^\theta/\sqrt{N} \approx \big(I_{(p+1)r} \otimes U_{\mathrm{ker}} U_{\mathrm{ker}}^T\big) \operatorname{vec}(\Delta\hat{\mathcal{H}}_{\mathrm{ref}}^{\theta_*}) + \Big((\mathcal{Z}_{\mathrm{test}}^\dagger \mathcal{Z}_{\mathrm{ref}})^T \otimes U_{\mathrm{ker}} U_{\mathrm{ker}}^T\Big) \operatorname{vec}(\Delta\hat{\mathcal{H}}_{\mathrm{test}}^\theta).$$

After squaring and going to the limit with N, it yields

$$\Sigma_\zeta = \lim_{N\to\infty} \Delta\hat{\zeta}^\theta \Delta\hat{\zeta}^{\theta^T} = \begin{bmatrix} \mathcal{J}_{\mathcal{H}_{\mathrm{ref}}}^\zeta & \mathcal{J}_{\mathcal{H}_{\mathrm{test}}}^\zeta \end{bmatrix} \Sigma_h \begin{bmatrix} (\mathcal{J}_{\mathcal{H}_{\mathrm{ref}}}^\zeta)^T \\ (\mathcal{J}_{\mathcal{H}_{\mathrm{test}}}^\zeta)^T \end{bmatrix}$$

Appendix B. CONTINUITY OF THE SINGULAR VECTOR DERIVATIVE

For a given set of continuous matrices X^θ, where $\theta \to \theta_*$, $\operatorname{rank}(X^\theta) = 2n$ for $\theta \neq \theta_*$, and $\operatorname{rank}(X^{\theta_*}) = n$. The sensitivity of the first n left singular vectors of an estimate of X^θ is examined during convergence, i.e. for $\tilde{D}_1 \to 0$ in the SVD of X^θ

$$X^\theta = \begin{bmatrix} U_1 & \tilde{U}_1 & \check{U}_2 \end{bmatrix} \begin{bmatrix} D_1 & 0 & 0 \\ 0 & \tilde{D}_1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ \tilde{V}_1^T \\ \check{V}_2^T \end{bmatrix},$$

where U_1 and V_1 denote the first n left and right singular vectors. $D_1 = \operatorname{diag}(d_1, \ldots, d_n)$ contains the first n non-zero singular values. Matrices $\tilde{U}_1$ and $\tilde{V}_1$ denote the left and right singular vectors corresponding to the last n non-zero singular values in $\tilde{D}_1 = \operatorname{diag}(d_{n+1}, \ldots, d_{2n})$. Matrices $\check{U}_2$ and $\check{V}_2$ define the left and right null spaces of X^θ respectively. Let u_f be the f-th column of U_1, where $f = 1 \ldots n$. A first order perturbation of u_f, developed in Liu et al. (2008), writes

$$\begin{aligned} \Delta u_f &= \begin{bmatrix} U_1 & \tilde{U}_1 \end{bmatrix} \begin{bmatrix} D_f & 0 \\ 0 & \tilde{D}_f \end{bmatrix} \begin{bmatrix} U_1^T \\ \tilde{U}_1^T \end{bmatrix} \Delta X v_f d_f \\ &+ \begin{bmatrix} U_1 & \tilde{U}_1 \end{bmatrix} \begin{bmatrix} D_f & 0 \\ 0 & \tilde{D}_f \end{bmatrix} \begin{bmatrix} D_1 & 0 \\ 0 & \tilde{D}_1 \end{bmatrix} \begin{bmatrix} V_1^T \\ \tilde{V}_1^T \end{bmatrix} \Delta X^T u_f \\ &+ \check{U}_2 \check{U}_2^T \Delta X v_f d_f^{-1}, \end{aligned} \tag{B.1}$$

where $D_f \in \mathbb{R}^{n\times n}$ is a diagonal matrix with entries $D_f(g,g) = 1/(d_f^2 - d_g^2)$ for $g \neq f$, and $D_f(f,f) = 0$. Matrix $\tilde{D}_f \in \mathbb{R}^{n\times n}$ is diagonal with entries $\tilde{D}_f(g,g) = 1/(d_f^2 - d_{g+n}^2)$, analogously. Suppose now $X^\theta \to X^{\theta_*}$, so the non-zero singular values $d_{n+1} \ldots d_{2n}$ converge to zero, $\tilde{D}_1 \to 0$. Then $\tilde{D}_f \to I_n d_f^2$, and $\tilde{U}_1$ converge to vectors in the left null space of X^{θ_*}, which becomes $U_2 = \begin{bmatrix} \tilde{U}_1 & \check{U}_2 \end{bmatrix}$. Thus, the first order perturbation of u_f follows from (B.1) as

$$\Delta u_f = U_1 D_f U_1^T \Delta X v_f d_f + U_1 D_f D_1 V_1^T \Delta X^T u_f + U_2 U_2^T \Delta X v_f d_f^{-1}$$

which coincides with the sensitivity of the f-th left singular vector of the matrix X^{θ_*} with $\operatorname{rank}(X^{\theta_*}) = n$. Hence, the derivatives of the first n singular vectors of matrix X^θ are continuous in θ_*.

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