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# Common wrench capability evaluation of a human-robot collaborative system

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## 1. Introduction

Wrench capacity metrics are well known tools for the evaluation and the analysis of the performance of robotic manipulators. Arguably, the most widely used approximative capacity metrics are manipulability ellipsoids (Yoshikawa 1985) but the true capabilities have the form of polytopes (Chiacchio et al. 2002). Ellipsoids and polytopes both express the robot's ability to move and apply forces and torques in arbitrary directions.

Even though these metrics have been developed for robotics systems, they have been used for human upper limb (arm) analysis as well, in particular for rehabilitation (Rezzoug et al. 2021; Sasaki et al. 2010) and sports (Kathib et al. 2009).

Recently, with the development of collaborative robotics these metrics have been extended to evaluation of the common wrench capacity of the collaborative systems consisting of multiple robotic manipulators (Skuric et al. 2020).

Therefore this paper proposes to combine these techniques to calculate the common wrench capacity of the human-robot collaboration system in the form of the wrench polytopes.

## 2. Methods

### *2.1 Human arm wrench capacity polytope*

The musculoskeletal model of the human arm can be expressed by the equation

$$N(\boldsymbol{q}_h)\boldsymbol{t} = J_h(\boldsymbol{q}_h)^T\boldsymbol{f}_h$$

where $N$ is the moment arm matrix, $J_h$ is the arm's jacobian matrix, $\boldsymbol{t}$ is the vector of muscle's isometric forces, $\boldsymbol{q}_h$ is the vector of joint angles and $\boldsymbol{f}_h$ is the cartesian wrench vector. Furthermore the muscular isometric forces are constrained in between:

$$\boldsymbol{t} \in [\boldsymbol{t}_{min}, \boldsymbol{t}_{max}]$$

In order to find the polytope of achievable cartesian wrenches $\boldsymbol{f}_h$ that can be generated by the muscles $\boldsymbol{t}$ in a given configuration $\boldsymbol{q}_h$, the problem can be separated into two problems. First the achievable set of joint torques $\boldsymbol{\tau}_h$ can be found using:

$$N(\boldsymbol{q}_h)\boldsymbol{t} = \boldsymbol{\tau}_h$$

In order to find the achievable $\boldsymbol{\tau}_h$, the *hyper plane shifting method* (Gouttefarde and Krut 2010) can be used. This method transforms the equation above into a half-space representation form

$$H\boldsymbol{\tau}_h \leq \boldsymbol{d}$$

Finally, the half-space representation of the cartesian wrench $\boldsymbol{f}_h$ polytope can be found by substituting $\boldsymbol{\tau}_h$ with the $J_h^T(\boldsymbol{q}_h)\boldsymbol{f}_h$:

$$HJ_h(\boldsymbol{q}_h)^T\boldsymbol{f}_h \leq \boldsymbol{d}$$

This equation represents a set of inequalities which fully define the polytope of achievable wrenches $\boldsymbol{f}_h$.

### *2.2 Robot wrench capacity polytope*

The dual relationship between generalised joint torque vector $\boldsymbol{\tau}_r$ and the generated cartesian wrench $\boldsymbol{f}_r$ for a serial robotic manipulators can be written as

$$J_r(\boldsymbol{q}_r)^T\boldsymbol{f}_r = \boldsymbol{\tau}_r$$

where $J_r$ is robot's jacobian matrix, $\boldsymbol{q}_r$ is the joint configuration, $\boldsymbol{f}_r$ is the cartesian wrench vector. Furthermore $\boldsymbol{\tau}_r$ is defined by the mechanical characteristics of the robot and is limited to:

$$\boldsymbol{\tau}_r \in [\boldsymbol{\tau}_{r,min}, \boldsymbol{\tau}_{r,max}]$$

The robot wrench polytope half-space representation is straight forward by stacking the jacobian matrices and the torque limits:

$$\begin{bmatrix} J_r^T(\boldsymbol{q}_r) \\ -J_r^T(\boldsymbol{q}_r) \end{bmatrix} \boldsymbol{f}_r \leq \begin{bmatrix} \boldsymbol{\tau}_{r,max} \\ -\boldsymbol{\tau}_{r,min} \end{bmatrix}$$

### *2.3 Common wrench capacity polytope*

Once the cartesian wrench polytopes of the human arm $\boldsymbol{f}_h$ and the robot $\boldsymbol{f}_r$ have been calculated, two cases can be distinguished: human and robot apply forces/wrenches in the same direction, human and robot apply forces/wrenches in opposite directions, cancelling each other.

When the robot and the human apply wrenches in the same direction then, for example if they carry an object where each of them carries a certain part of the weight, their common capacity of achievable wrenches is the sum of the two capacities. And in that case, the polytope can be represented as the Minkowski sum:

$$\boldsymbol{f}_{\oplus} = \boldsymbol{f}_h \oplus \boldsymbol{f}_r$$

On the other hand, when the human and the robot are applying wrenches in opposite directions, for example

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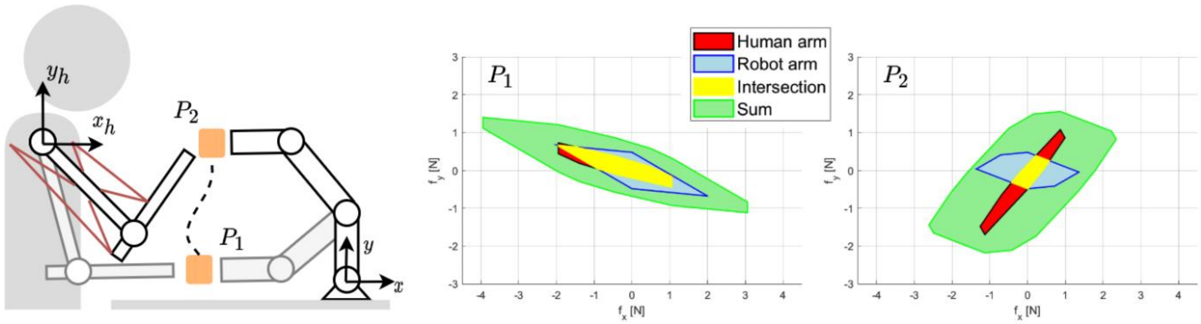

*Figure 1. Planar human-robot interaction example. Human arm, 6 muscles (in red) and 2 joints, on the left and 3 DOF robot on the right. Two graphs on the right show the cartesian force polytope comparison for two distinct poses $P_1$ and $P_2$. Graphs show human arm $\boldsymbol{f}_h$ , robot $\boldsymbol{f}_r$, intersection $\boldsymbol{f}_\cap$ and Minkowski sum $\boldsymbol{f}_\oplus$ polytopes.*

if the robot's task to hold an object still while the human manipulates it. Then the robot needs to be capable of counteracting all the wrenches applied by the human. Therefore in that case the common capacity can be represented as the intersection of the capacities:

$$\boldsymbol{f}_\cap = \boldsymbol{f}_h \cap \boldsymbol{f}_r$$

The intersection of wrench capacity polytopes can be found by stacking already calculated inequalities.

$$\begin{bmatrix} HJ_h^T(\boldsymbol{q}_h) \\ J_r^T(\boldsymbol{q}_r) \\ -\ J_r^T(\boldsymbol{q}_r) \end{bmatrix} \boldsymbol{f}_\cap \leq \begin{bmatrix} \boldsymbol{d} \\ \boldsymbol{\tau}_{r,max} \\ -\boldsymbol{\tau}_{r,min} \end{bmatrix}$$

## 3. Results and discussion

To demonstrate the proposed methods, a simple planar collaborative scenario is proposed, where a 3 degrees of freedom (DOF) robot and a human arm, modelled with 6 muscles and 2 joints, interact to carry an object through the common workspace. Points $P_1$ and $P_2$, on the trajectory are taken to visualise the evolution of their common capabilities. The isometric force of each muscle is limited to $\boldsymbol{t} \in [0, 1]$, robot joint torques are limited to $\boldsymbol{\tau}_r \in [-1, 1]$. Human arm's shoulder and elbow joint angles for $P_1$ are $\boldsymbol{q}_h = [\frac{\pi}{3}, \frac{2\pi}{3}]$ and $P_2$ are $\boldsymbol{q}_h = [\frac{2\pi}{3}, \frac{\pi}{2}]$. Robot's joint angles for $P_1$ are set to $\boldsymbol{q}_r = \left[\frac{\pi}{2}, \frac{2\pi}{3}, -\frac{\pi}{3}\right]$ and for $P_2$ as $\boldsymbol{q}_r = \left[\frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{3}\right]$.

The results of the proposed methods are shown on the figure 1. It can be seen that the human-robot collaborative capacity is considerably augmented by considering both robot's and human's capabilities. Additionally, the results show that the force polytope's shape and size depend greatly on the pose of the robot and the human.

Therefore, both sum and intersection of force polytopes, as true capacity measures, have a great potential to be used for the real-time robot control and improve the degree of collaboration and safety in between the human and the robot. However, due to the complexity of the human arm musculoskeletal models, this approach is still destined to the offline use and present an interesting research topic for the future.

## 4. Conclusions

In this paper a set of methods for calculating the common wrench capacity polytope of a collaborative system, consisting of serial robotic manipulator and a human arm, based on the musculoskeletal model is proposed.

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