Sensitivity analysis for two wave breaking models used by the Green-Naghdi equation

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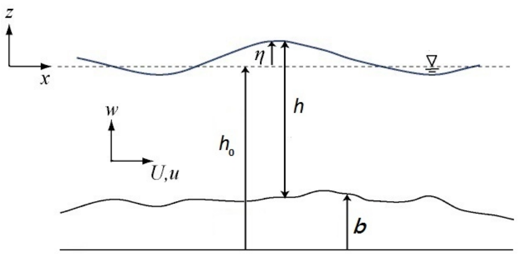
Context and motivation

Near shore: wave transformation due to interaction with complex bathymetry

- Strong non-linearity, dispersion
- Refraction diffraction
- Shoaling
- Breaking
- Induced currents
- Run-up, inundation

Modeling and discretisation



$$h_t + (hu)_x = 0$$

$$(hu)_t + (hu^2)_x + gh\eta_x = \phi$$

$$(I + \alpha\mathcal{T})\,\phi = \mathcal{W} - \mathcal{R} + (r_{wb})_x$$

$$\mathcal{W} = g\mathcal{T}(h_0\eta_x) \qquad (r_{wb})_x = (\nu_t H u_x)_x$$

$$\mathcal{R} = h\mathcal{Q}(u)$$

$$\mathcal{T}(\cdot) = -\frac{1}{3}h^2(\cdot)_{xx} - \frac{1}{3}hh_x(\cdot)_x + \frac{1}{3}\left[h_x^2 + hh_{xx}\right](\cdot) + \left[b_x h_x + \frac{1}{2}hb_{xx} + b_x^2\right](\cdot)$$

$$\mathcal{Q}(\cdot) = 2hh_x(\cdot)_x^2 + \frac{4}{3}h^2(\cdot)_x(\cdot)_{xx} + b_x h(\cdot)_x^2 + b_{xx}h(\cdot)(\cdot)_x + \left[b_{xx}h_x + \frac{1}{2}hb_{xxx} + b_x b_{xx}\right](\cdot)^2$$

- Elliptic part: Continuous Galerkin FE method
- Hyperbolic part: Third order MUSCL FV metod

Breaking closure: Shallow water dissipation -Hybrid

1. Detect Breaking Regions

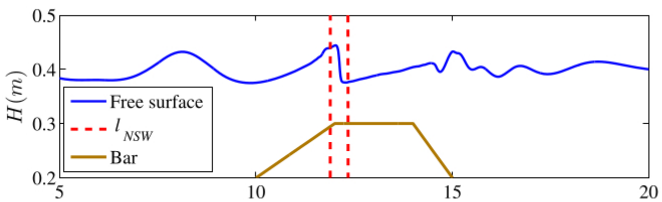
2. Remove dispersive terms

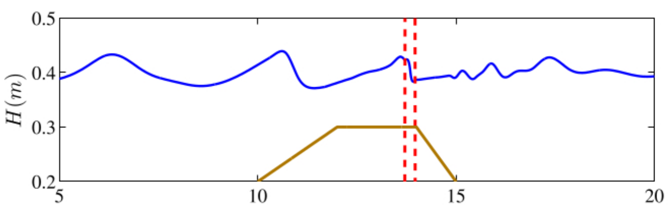
3. Shallow water shock

4. Dissipation

Breaking closure: Shallow water dissipation -Hybrid

1. Detect and flag breaking regions

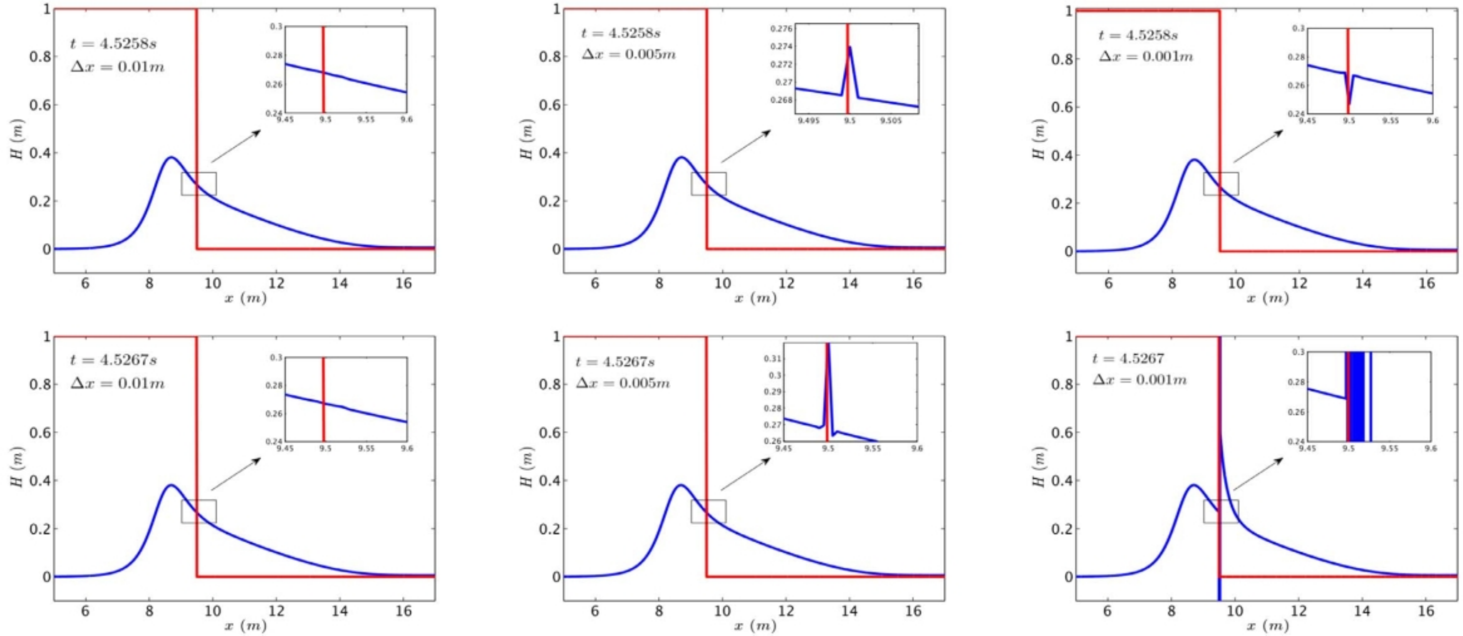




- Surface variation criterion:
$$|\eta_t| \geq \gamma\sqrt{gH}$$
- Local slope angle criterion:
$$||\nabla\eta|| \geq \tan\varphi_c$$
- The Froude number of each wave is computed and breaking is deactivated if Fr<1.3

Breaking closure

- Codes using this closure: Funwave, BOSZ, Coulwave, Mike 21 Tucwave, Slows etc…
- Drawback: can not be used on variable meshes*



Solitary wave breaking on a slope: hybrid treatment with order reduction at the coupling interface. Wave height at times $t = 4.5258$ s, 4.5267 s (top and bottom rows), on mesh sizes (from left to right) $\Delta x = 0.01$ m, 0.005 m, 0.001 m.

*

- P. Bacigaluppi,M. Ricchiuto, P.Bonneton, *Implementation and Evaluation of Breaking Detection Criteria for a Hybrid Boussinesq model*, *Water Waves* ,2, 2020
- Kazolea, M., Ricchiuto, M.: *On wave breaking for Boussinesq-type models. Ocean Modelling* 123, 16–39 (2018)
- Shi, F., Kirby, J.T., Harris, J.C., Geiman, J.D., Grilli, S.T.: *A high-order adaptive time-stepping TVD solver for Boussinesq modeling of breaking waves and coastal inundation*. Ocean Modelling 43, 36–51 (2012)

Breaking closure: Eddy viscosity model via a PDE based TKE model

1. Detect breaking Regions (same as before) γ, φ_c
2. Dissipation required through the definition of artificial viscosity:

$$v_t = C_\nu \sqrt{\kappa_b k_b + \kappa_j k_j} A$$

$$k_t + u k_x = \mathcal{D} + \mathcal{P} - \mathcal{E}$$

$$\mathcal{D} = \sigma \nu_t k_{xx} \qquad \mathcal{E} = -C_\nu^3 \frac{k^{3/2}}{\ell_t} \qquad \mathcal{P} = B(t,x) \frac{\ell_t}{\sqrt{C_\nu^3}} (u^s)^3$$

$$\ell_t = \kappa H$$

$\kappa,\ \sigma,\ C_\nu = \kappa_b,\ \sigma_b,\ C_{\nu_b}$ Bore

$\kappa,\ \sigma,\ C_\nu = \kappa_j\ \sigma_j\ C_{\nu_j}$ Hydraulic jump

Sensitivity analysis model

- Surrogate model – Kriging model
- ANOVA
  - Sobol decomposition

$$Y = F(\bar{\mathbf{x}}) = F_0 + \sum_{i=1}^{M} F_i(x_i) + \sum F_{ij}(x_i x_j) + \sum_{1 \le i < j \le M} F_{ij}(x_i, x_j) + \cdots + F_{1,2,\cdots M}(\bar{\mathbf{x}}) \quad \bar{\mathbf{x}} = [x_1, x_2, \cdots, x_M]$$

$$F_0 = E(Y)$$

$$F_i(x_i) = E(Y|x_i) - F_0$$

$$F_{ij}(x_i, x_j) = E(Y|x_i, x_j) - F_0 - F_i - F_j$$

UQLab
The Framework for Uncertainty Quantification

UQLab: A Framework for Uncertainty Quantification in MATLAB,
Stefano Marelli and Bruno Sudret,
In *The 2nd International Conference on Vulnerability and Risk Analysis and Management (ICVRAM 2014)*,
University of Liverpool, United Kingdom, July 13-16, 2014, pp. 2554–2563.
DOI: 10.1061/9780784413609.257 ➜

Sensitivity analysis model

– First order indices $$S_i = \frac{Var_{x_i}(P(Y|x_i))}{Var(Y)}$$

– Second order indices $$S_{i,j} = \frac{Var_{x_i,x_j}(P(Y|x_i, x_j))}{Var(Y)}$$

– Total Sobol index of an input variable == sum of all Sobol indices involving this variable.

UQLab
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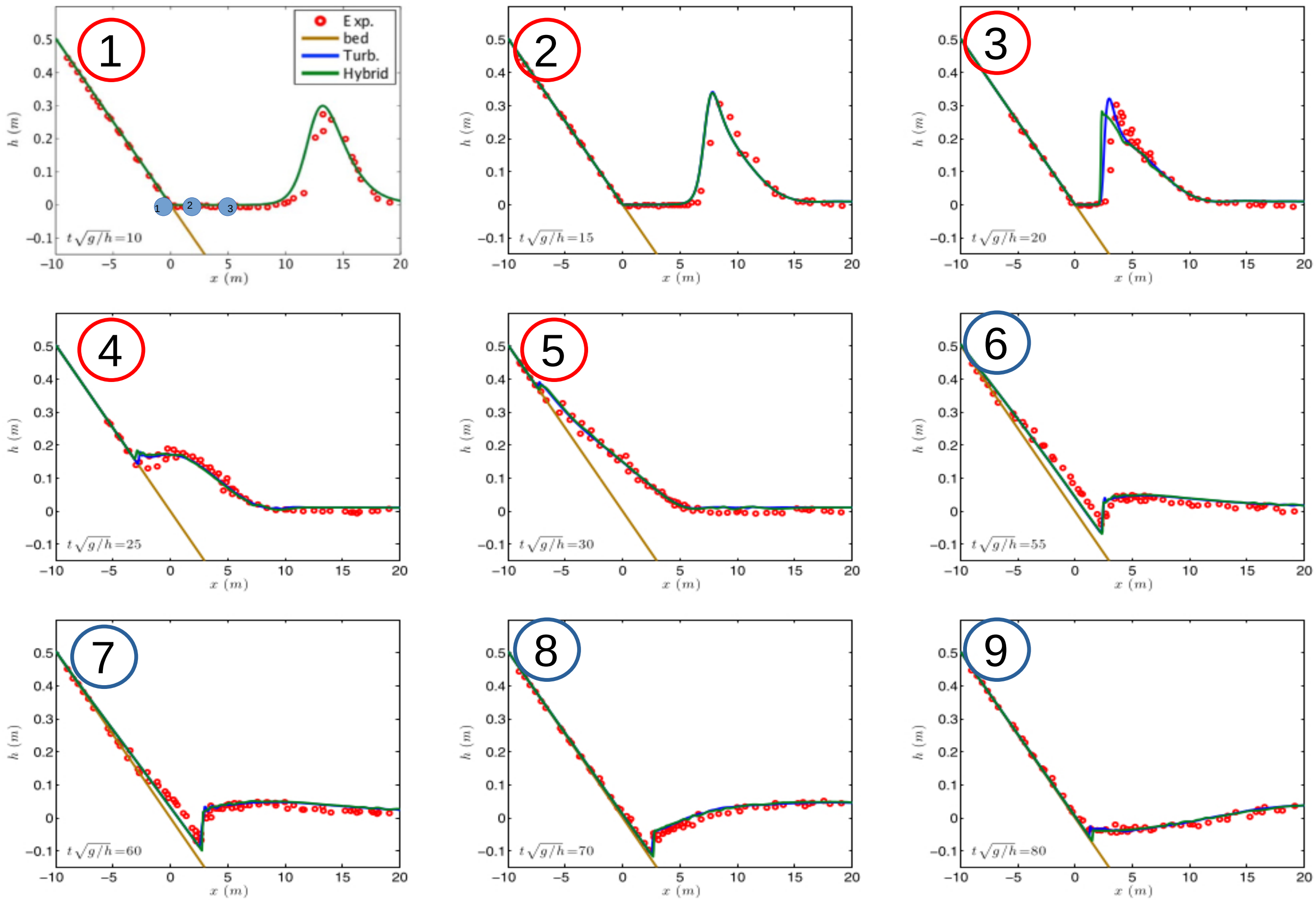
DOI: 10.1061/9780784413609.257 ➜

Solitary wave on a plane beach

- Set up: *A/d* = 0.28m, *d*=1m, *x*=[-20, 80]m, *Nm*=0.01, *WGs*: 0,2,5m
- 1-5 : First phase of the flow (shoaling breaking and runup)
- 6-9 : Second phase of the flow (run-down and breaking)



Free surface elevation of solitary wave run-up on a plane beach for the GN model.

Solitary wave on a plane beach:
Input parameters

- Surface variation criterion:

$$|\eta_t| \geq \gamma\sqrt{gH} \qquad \gamma \in [0.2,\ 0.6]$$

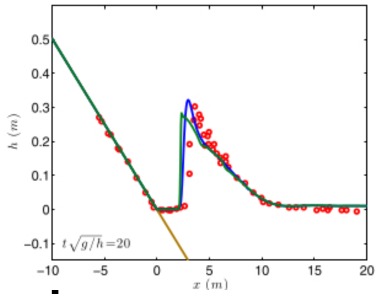
- Local slope angle criterion:

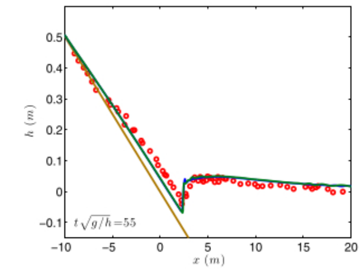
$$||\nabla\eta|| \geq \tan\varphi_c \qquad \varphi_c \in [0.3,\ 0.53]$$

- Constant parameters: $\Delta x = 0.05,\ Cfl = 0.01,\ Fr < 1.3$

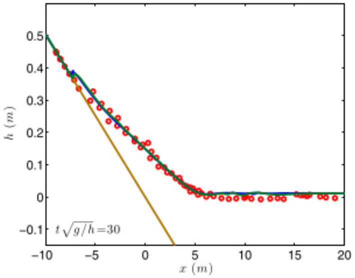
Solitary wave on a plane beach:
Output parameters

- Wave's breaking point location.

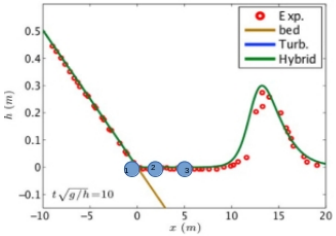




- Wave's height at the breaking point.



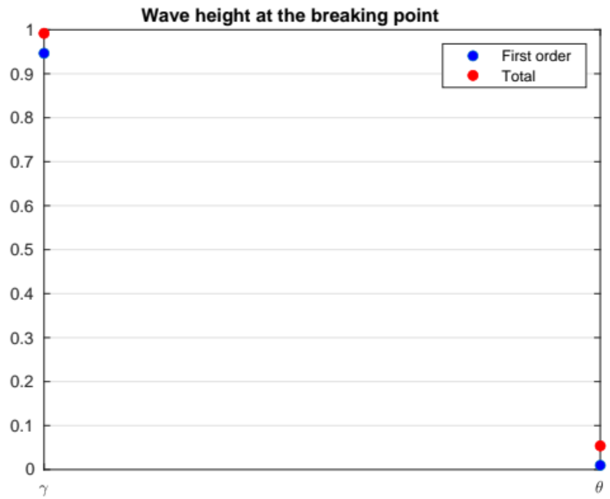
- Maximum run up.
- Maximum wave height at wave gauges.

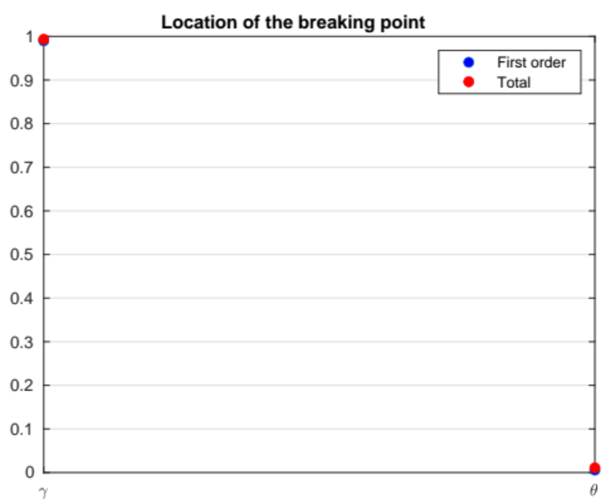


Solitary wave on a plane beach (Hybrid)

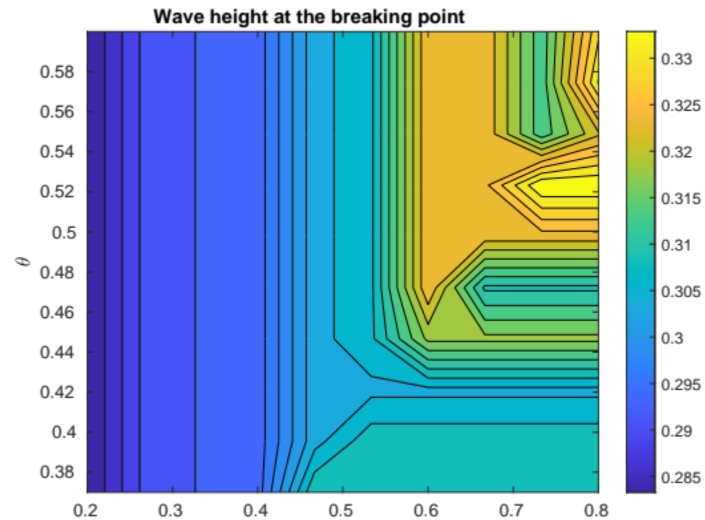
First phase of the flow 1-5

Sobol's indices

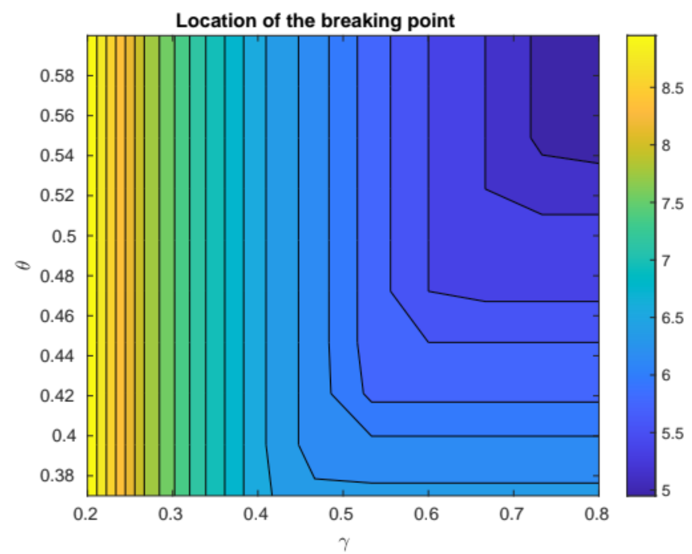




Contours



Range: 0.0521
Std(standard deviation) :0.0139

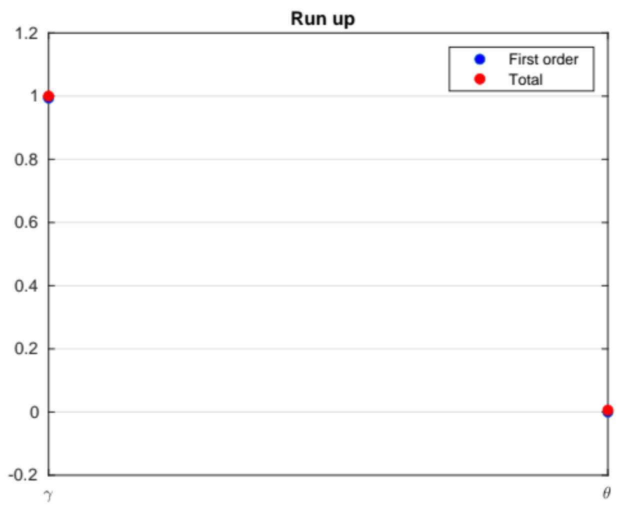


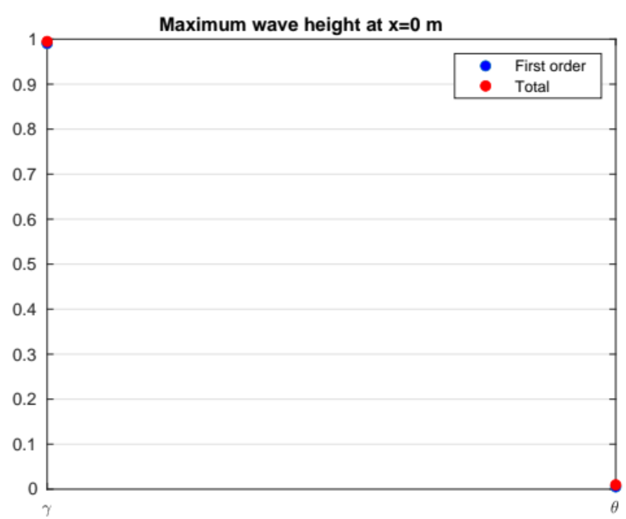
Range: 4.2
Std: 1.175

Solitary wave on a plane beach (Hybrid)

First phase of the flow 1-5

Sobol's indices





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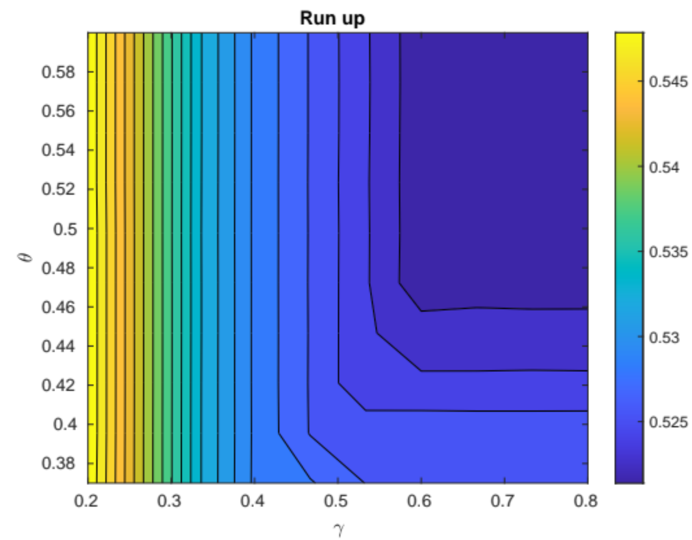
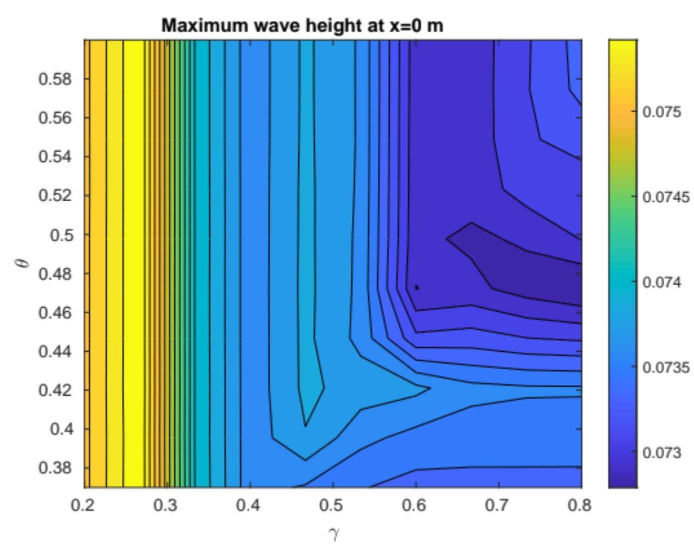


Range: 0.0278
Std: 0.0087

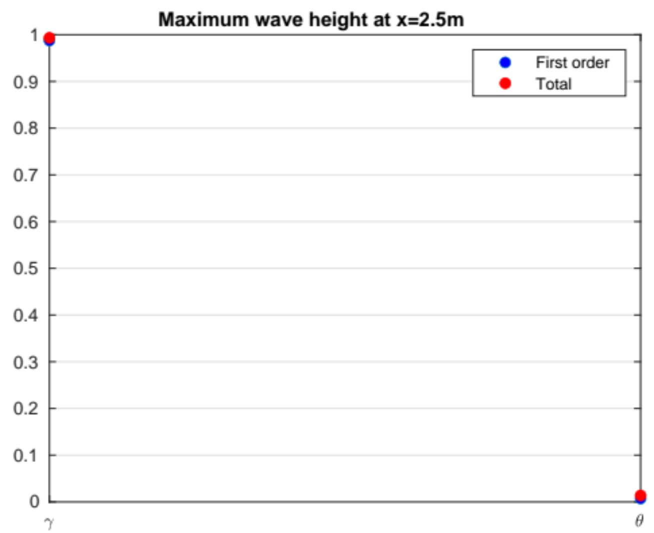


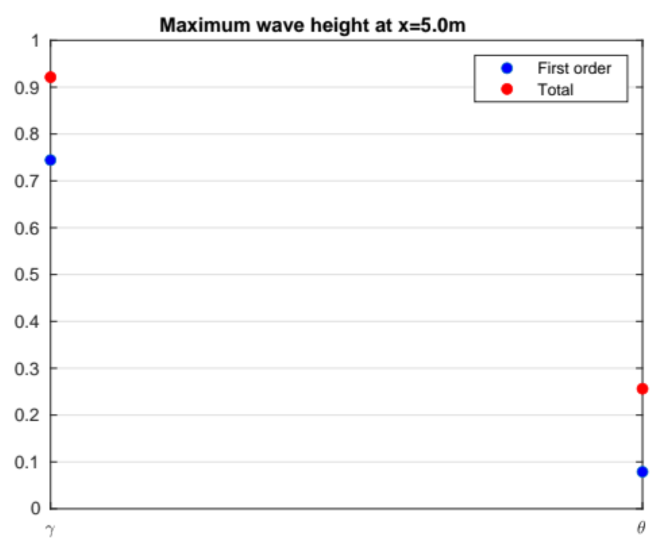
Range: 0.0028
Std: 8.1753*10^-4

Solitary wave on a plane beach (Hybrid)

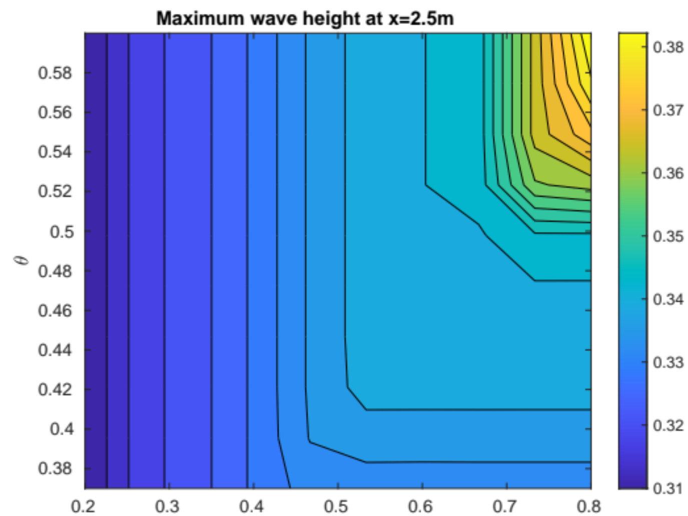
First phase of the flow 1-5

Sobol's indices

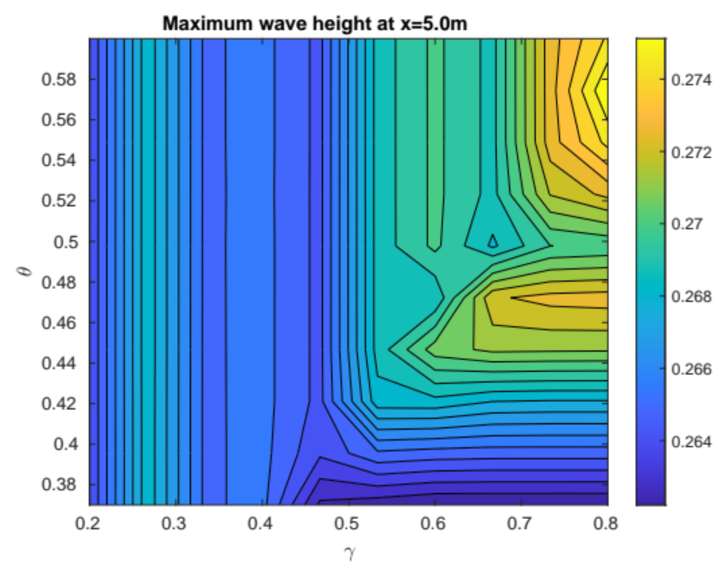




Contours



Range: 0.0758
Std: 0.0151

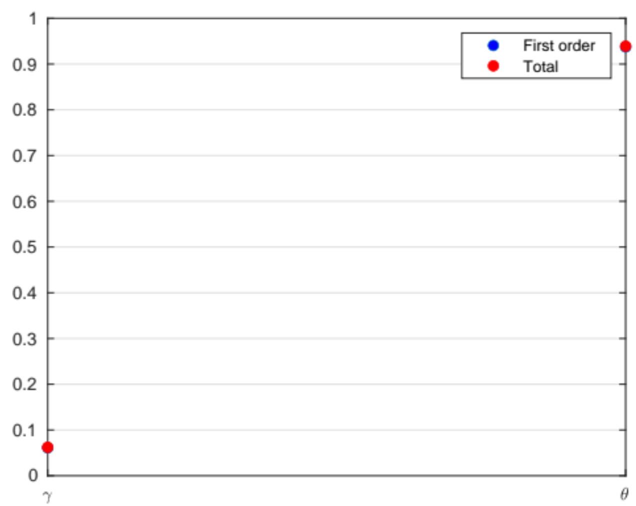


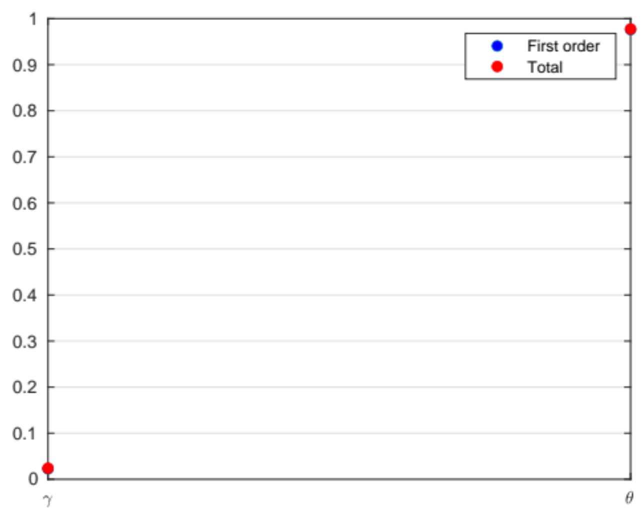
Range: 0.0135
Std: 0.0032

Solitary wave on a plane beach (Hybrid)

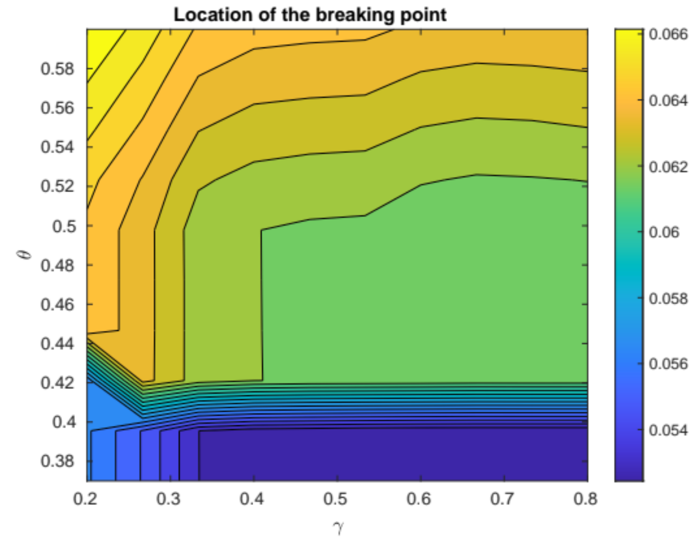
Second phase of the flow 6-9

Sobol’s indices

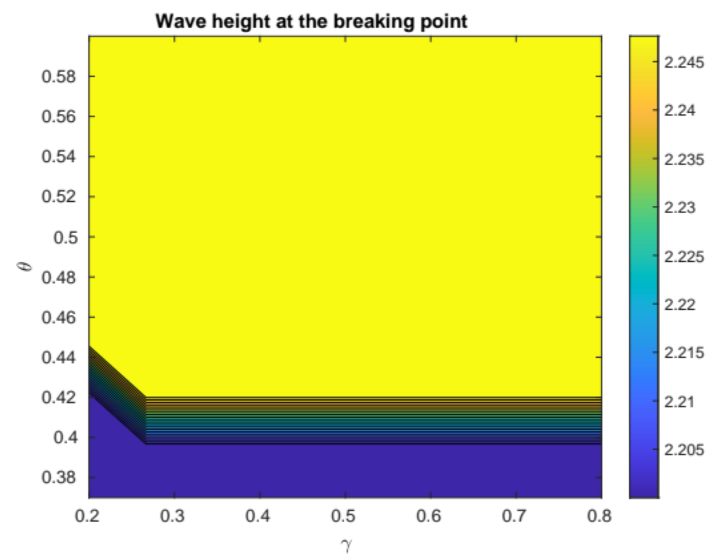




Contours



Range: 0.0144
Std: 0.0041



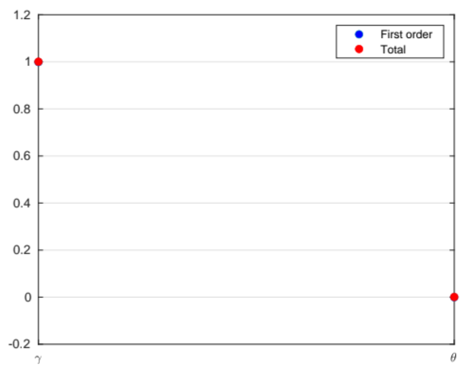
Range: 0.05
Std: 0.0205

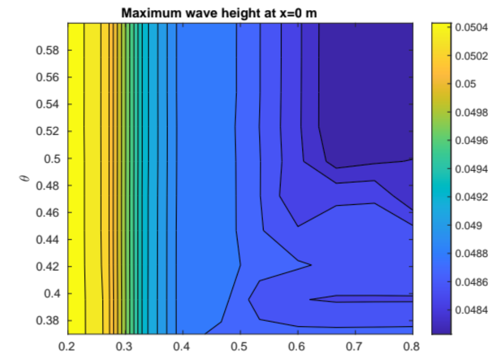
Solitary wave on a plane beach (Hybrid)

Sobol's indices

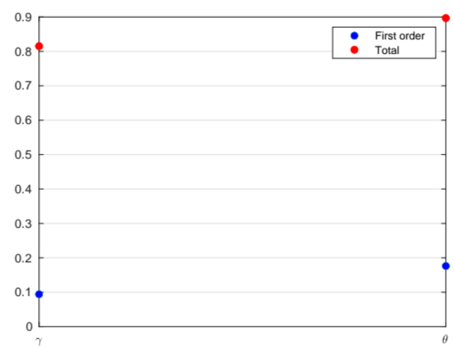
Contours

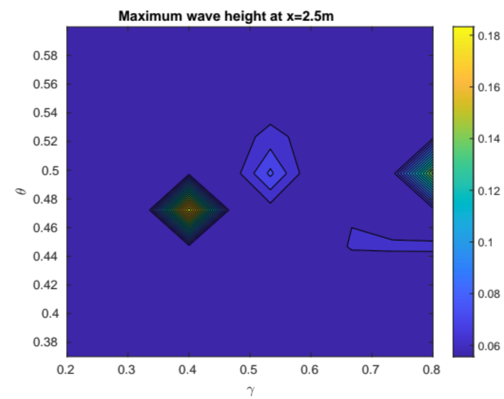
Second phase of the flow 6-9



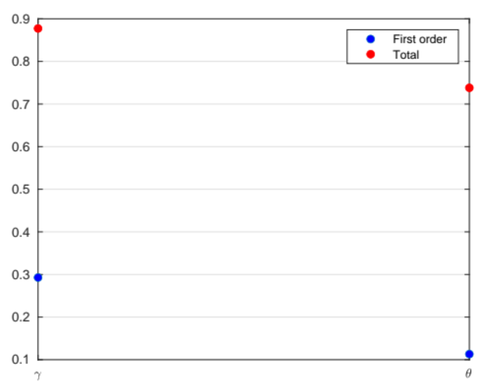


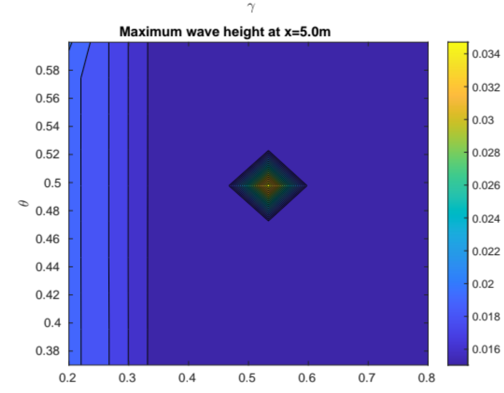
Range: 0.023
Std: 7.48*10^-4





Range: 0.01340
Std: 0.0161





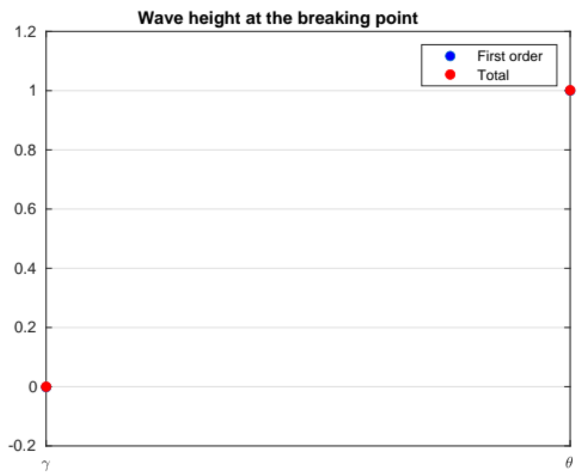
Range: 0.0207
Std: 0.0024

Solitary wave on a plane beach (Hybrid)

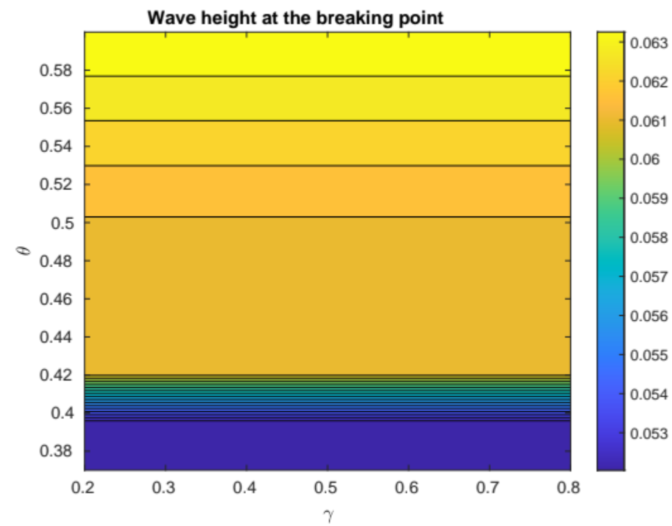
Second phase of the flow 6-9

Same initial condition

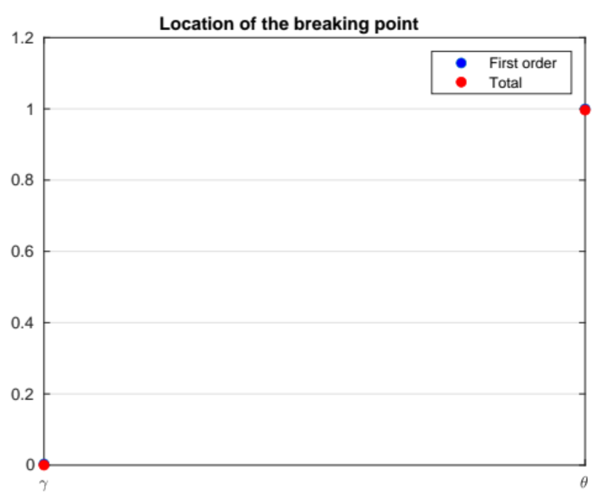
Sobol's indices

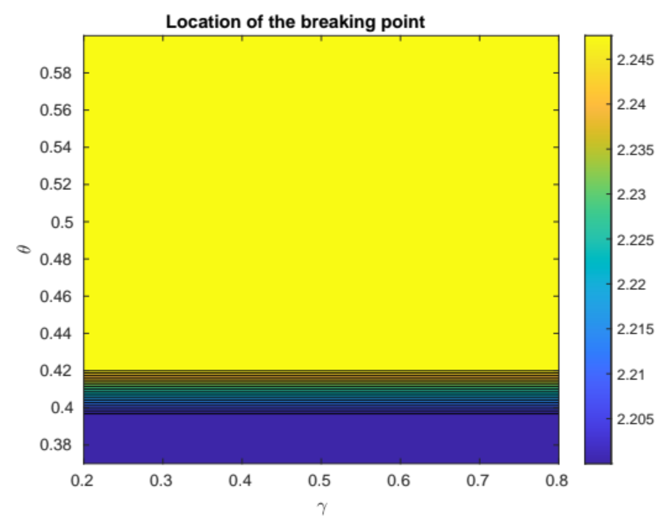


Contours



Range: 0.0118
Std: 0.0041





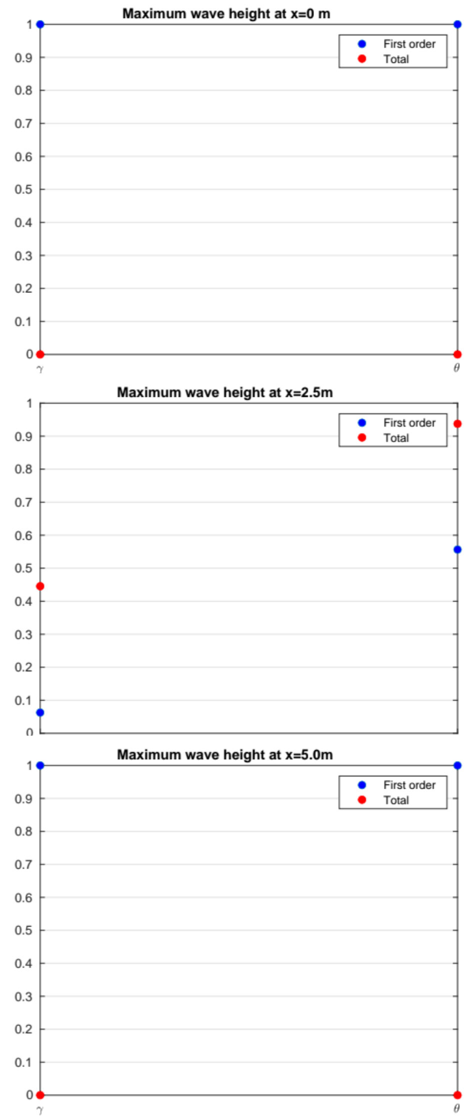
Range: 0.05
Std: 0.0201

Solitary wave on a plane beach (Hybrid)

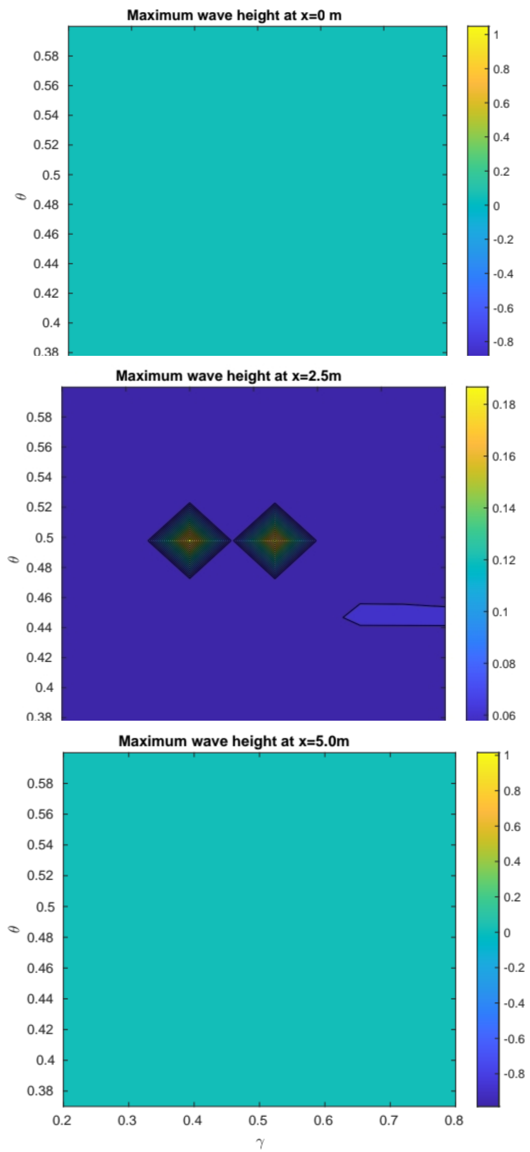
Second phase of the flow 6-9

Same initial condition

Sobol's indices



Contours

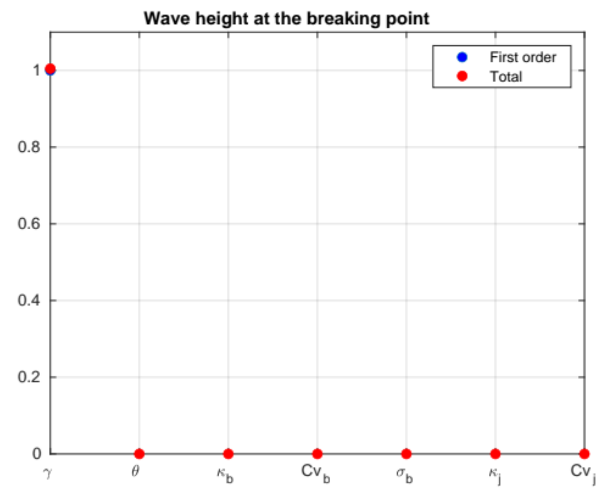


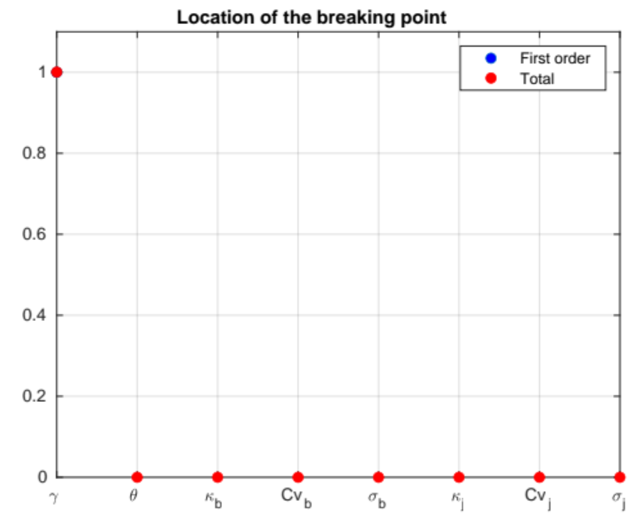
Range: 0.1402
Std: 0.019

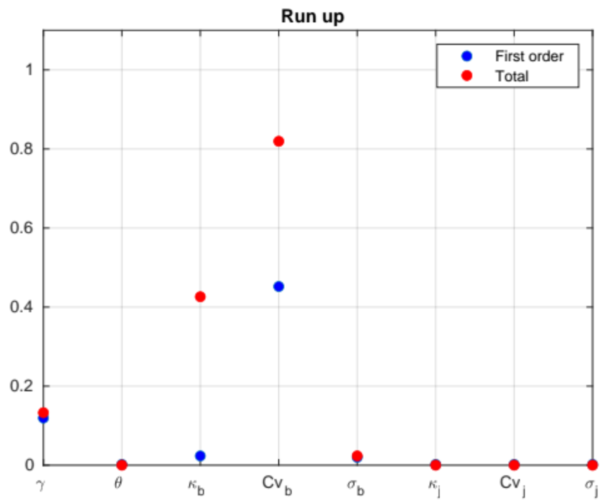
Solitary wave on a plane beach (TKE)

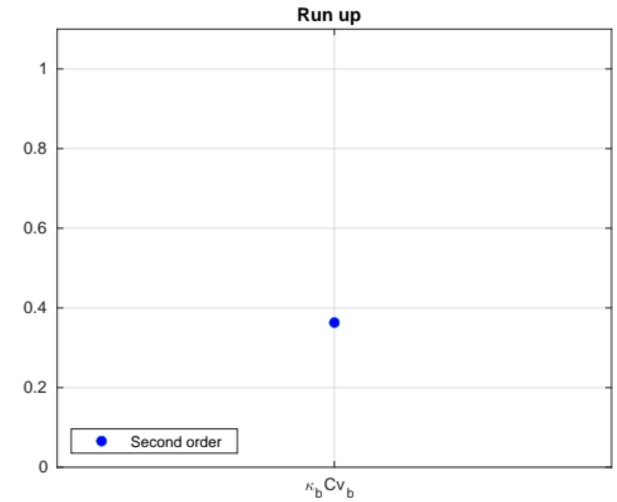
First phase of the flow 1-5

Sobol's indices







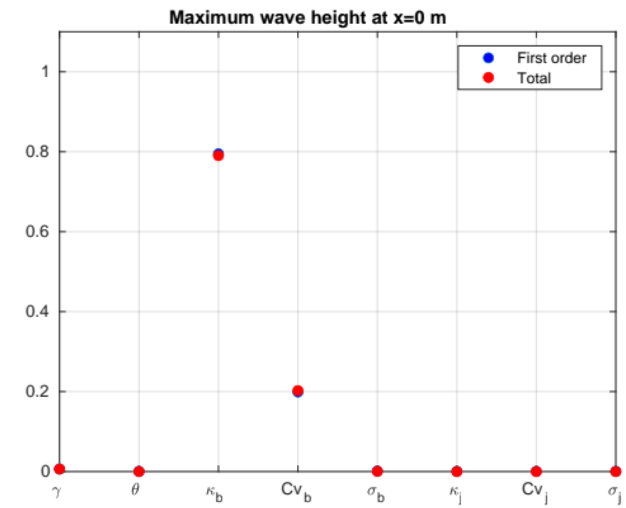


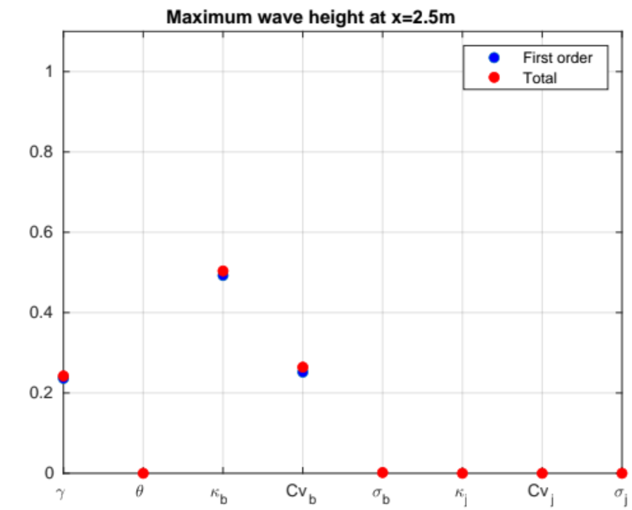
Range: 0.027
Std: 0.0076

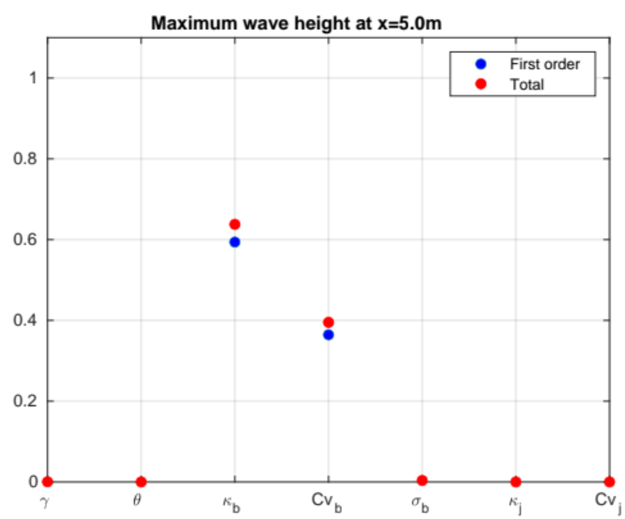
Solitary wave on a plane beach (TKE)

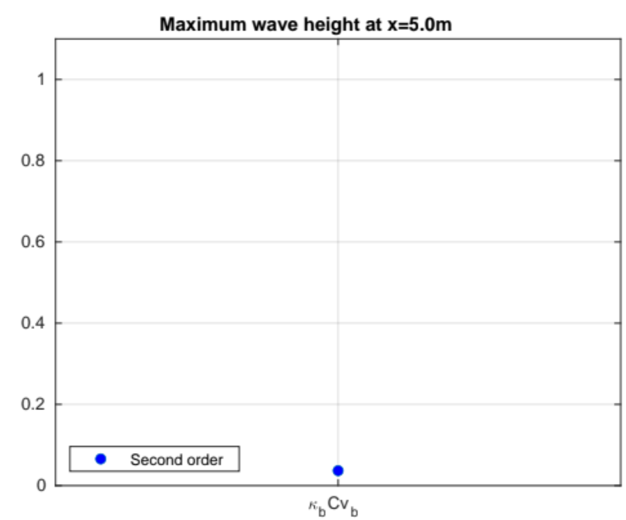
First phase of the flow 1-5

Sobol's indices-wave gauges







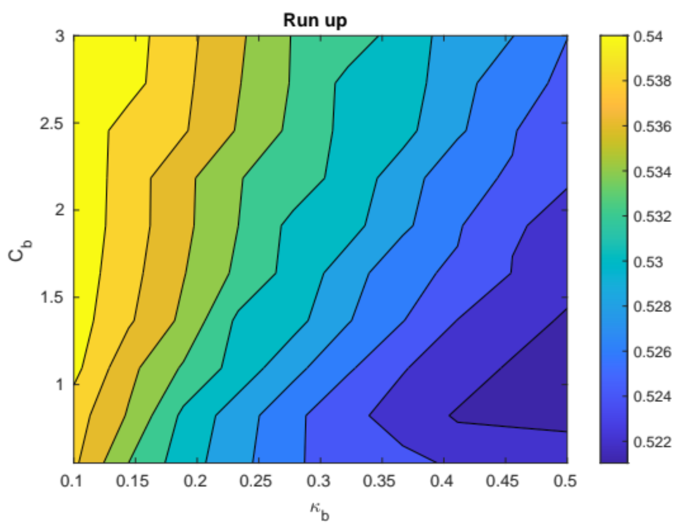


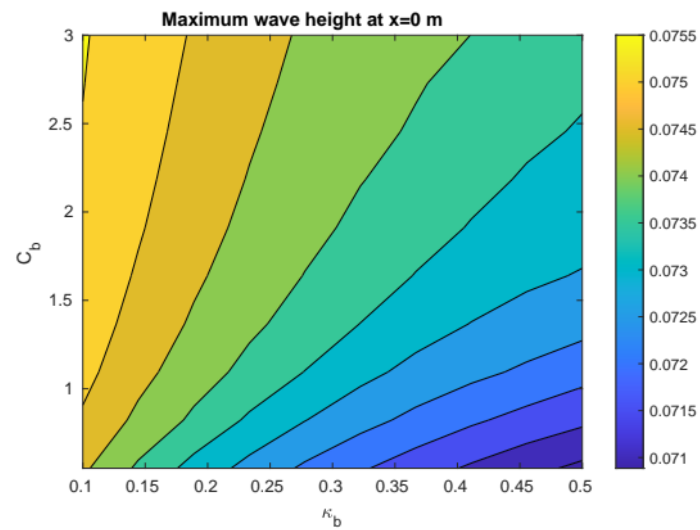
Range: 0.1501
Std: 0.0515

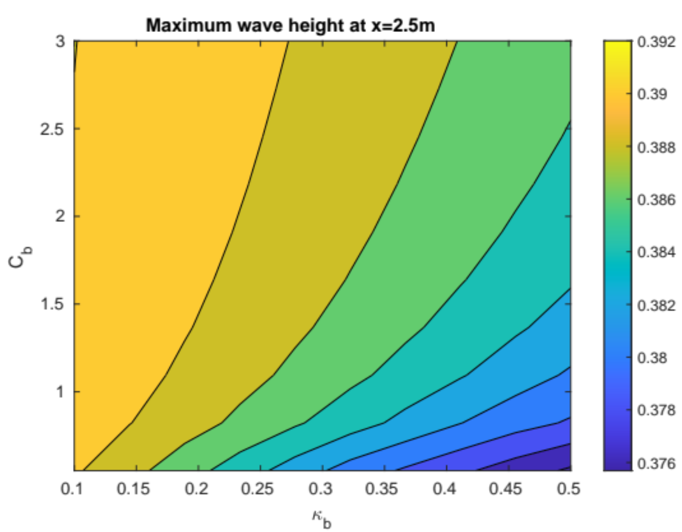
Solitary wave on a plane beach (TKE)

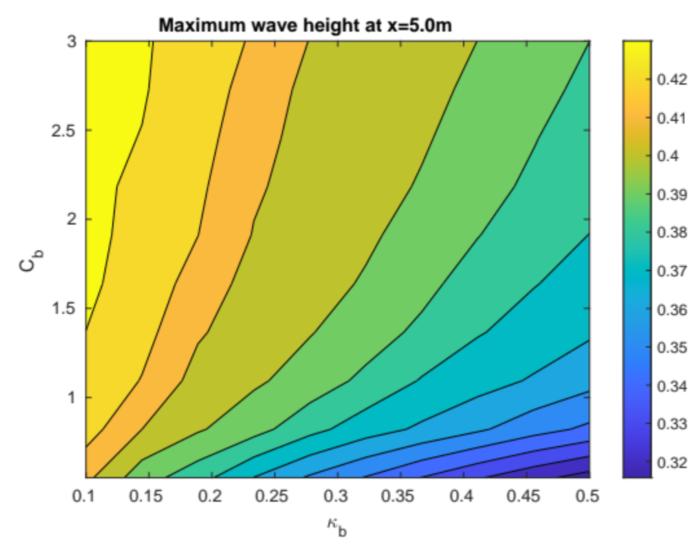
First phase of the flow 1-5

If we freeze all the input parameters except κ_b and c_b





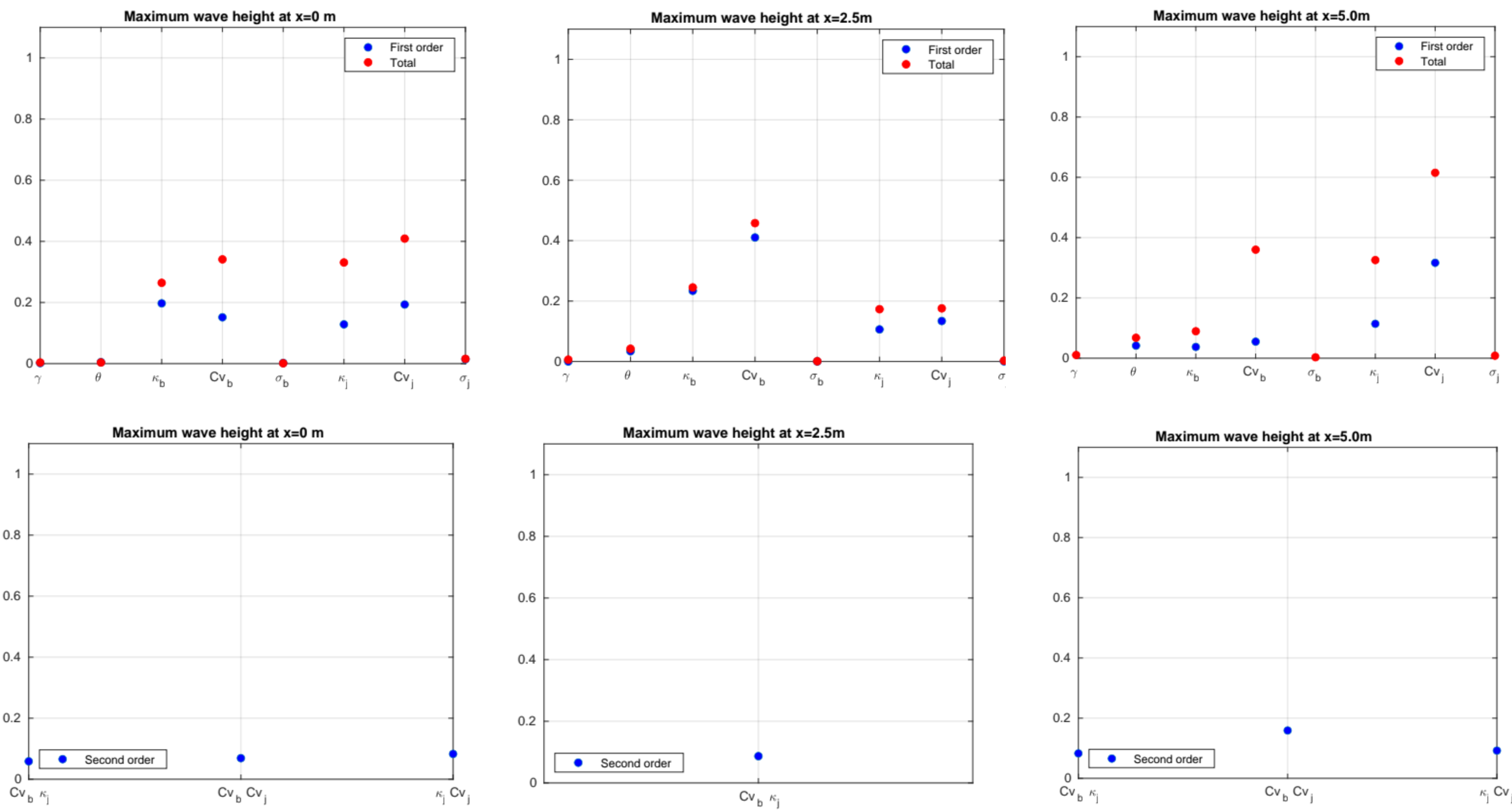




Solitary wave on a plane beach (TKE)

Second phase of the flow 6-9

Same initial condition

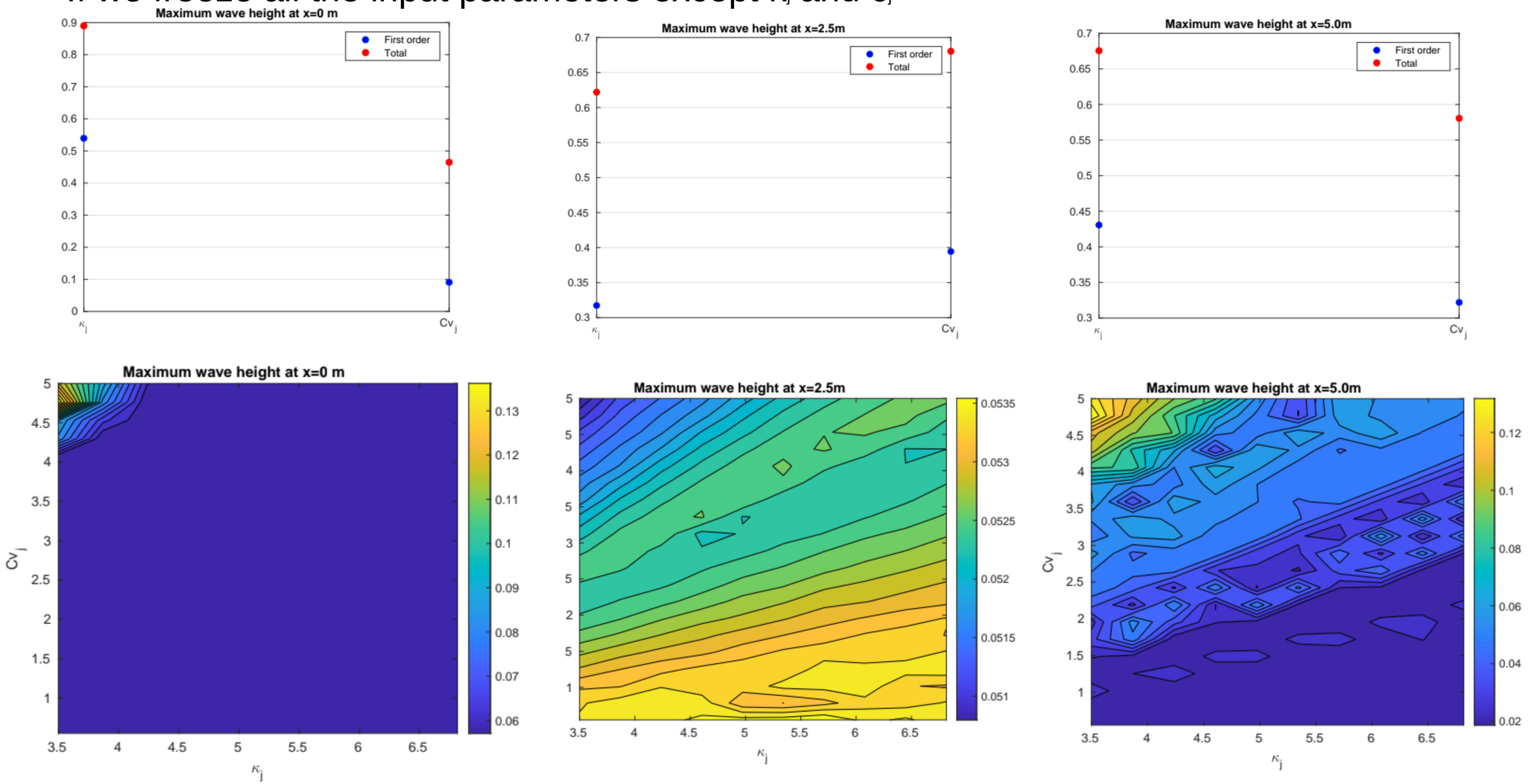


Solitary wave on a plane beach (TKE)

Second phase of the flow 6-9

Same initial condition

If we freeze all the input parameters except κ_j and c_j



Conclusions

- Hybrid closure: when works , more robust
- Tke always works but many more parameters to tune
- The variation with TKE quite smooth /no discontinuity in the output parameters

Thank you!!