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Transformational Geometry Unit

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TRANSFORMATIONAL GEOMETRY UNIT

A Project Report

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by

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The study included the development and writing of a unit on transformational geometry which involved a holistic approach including the cognitive, psychomotor, and affective domains. This unit was taught to the eighth grade class in the Oakville School District in Oakville, Washington. The results showed support that the teaching of this unit was effective.

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Chapter 1

BACKGROUND OF THE STUDY

Introduction to the Problem

The study of mathematics is a building process. Students use what they have learned as a foundation to learn more. The first element a student uses is intuition. Every student has intuitive ideas about mathematics. These ideas help a student learn more about what he believes to be true. It is necessary to bring these intuitive ideas to the surface and to use them.

Intuition can be used in the process of problem solving. One proposed plan of problem solving has four main steps: 1.) understand the problem, 2.) devise a plan, 3.) carry out the plan, and 4.) examine the solution.[1] A student's intuition can be used to understand the problem to be solved and to devise a plan to use. Intuition can also assist the student in making an estimation of the solution, which is also part of problem solving.

"One of the most important tasks of education is to develop the decision-making process of each individual. If mathematics is to aid in this process, its presentation

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[1]George Polya, How To Solve It (Princeton, New Jersey: Princeton University Press, 1945), p. vii.

must elicit from the student creative, problem-solving abilities. A strictly axiomatic approach to the teaching of mathematics often stifles this creative thinking. Although axiomatics and structure are important, they are secondary considerations in problem solving."[2]

Mathematics has several branches. Geometry is the branch which deals with space and logical argument. It can be defined as the study of the three undefined terms: point, line, and plane; length and angle measurement; and equality, parallelism and perpendicularity relationships.[3] Another way to define geometry is to say, that it is a science that studies and selects the properties of geometric figures that remain unchanged by motions of the figures.[4] Traditionally, the geometry course in high school is Euclidean geometry and is taught in the tenth grade. This course includes the study of two and three dimensional space and uses a deductive approach.[5]

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[2]Arthur A. Hiatt, "Problem Solving in Geometry," Mathematics Teacher, November 1972, p. 595.

[3]E. A. Maxwell, Geometry by Transformations (Cambridge: Cambridge University Press, 1975), p. xii.

[4]Allen Shields, trans., Geometric Transformations, by I. M. Yaglom (New York: Random House, 1962), p. 10.

[5]Howard F. Fehr, "The Present Year-Long Course in Euclidean Geometry Must Go," Mathematics Teacher, February, 1972, p. 102.

Several sources agree that the study of geometry should be more than one year in length and begin earlier than the tenth grade. Many recommendations have been made by numerous committees to include informal geometry in the seventh and eighth grades. "As educators come to realize how people learn mathematics, many are deciding that the study of geometry as a deductive system can be enhanced by the use of inductive reasoning. Informal geometry provides an excellent vehicle with which to use inductive reasoning."[6]

Statement of the Problem

Previous to the traditional Euclidean geometry course most students probably have not had much exposure to working with a plane and other geometrical figures. Students may benefit from an informal introduction to geometric concepts earlier in their education that would prepare them for the study of Euclidean geometry. This introduction may motivate and develop the student's concept of space, and ways of thinking which would aid the student in his further study of not only geometry but also of mathematics. It is hoped that this project will yield information about a particular part of this introduction,

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[6]John C. Peterson, "Informal Geometry in Grades 7-14," Geometry in the Mathematics Curriculum, Thirty-sixth Yearbook of the National Council of Teachers of Mathematics (United States of America: N.C.T.M., 1973), pp. 52 and 60.

which is, transformational geometry.

Purpose of the Study

The purpose of this project is to develop a unit on transformational geometry for use in junior high school. This approach will be informal and inductive. It will use the student's intuition and problem solving skills.

Procedure of the Study

Current literature and textbooks pertaining to this project will be reviewed. A unit will be written which will involve a holistic approach including the cognitive, psychomotor, and affective domains. For each domain a list of concepts, skills, or qualities will be made. In turn, each concept, skill, or quality will become an objective in a lesson. Each lesson will contain: the teacher's instructional input, the students' behavior, and the materials and resources when necessary. Incorporated into the project will be several manipulative activities using primarily a compass, straightedge, and mira. These activities will encourage the use of intuition, inductive reasoning, and problem solving by the students. This unit will be taught to an eighth grade class in the Oakville School District in Oakville, Washington. A pretest and post-test will be based on the same concepts. Results from these two tests will be compared to see if there has been a gain of knowledge through this unit.

Definition of Terms

The references used to define these terms were: The American Heritage Dictionary of the English Language, Geometry in the Mathematics Curriculum, and Webster's New Collegiate Dictionary.

axiomatic - pertaining to an undemonstrated proposition concerning an undefined set of elements, properties, functions, and relationships

geometry - the mathematics of the properties, measurement, and relationships of points, lines, angles, surfaces, and solids

Euclidean geometry - a system of geometry based on the undefined concepts of point, line, and plane

informal geometry - an approach to geometry which uses exploration and intuition to present geometric facts and relationships

deductive - based on reasoning from the general to the specific

inductive - based on reasoning from the specific to the general

plane - surface containing all the straight lines connecting any two points on it

geometric figures - diagrams and symbols representing terms used in geometry such as point, line, triangle, and quadrilateral

transformational geometry - an approach to geometry which utilizes the transformations of the plane including reflection, slide, and rotation, and which utilizes a student's intuition

holistic - emphasizing the functional relation between parts and wholes

cognitive - pertaining to knowledge

psychomotor - pertaining to muscular activity associated with mental process

affective - pertaining to emotions or feelings

manipulative - pertaining to a skilled use of the hands

mira - a device through which points are reflected

Chapter 2

REVIEW OF RELATED LITERATURE

Research for this chapter included a review of articles found in current mathematics journals, Educational Resources Information Center documents, and classroom textbooks. Many articles focused on the need for a change in the geometry curriculum, and several textbooks reflected this change through additions in content, variations in methods, and expansion to additional grade levels. Most changes included the use of intuition, problem solving, and informal geometries.

Intuition

It is difficult to find agreement on the definition of intuition. There are many characteristics of intuition. According to Jerome Bruner intuition does not follow a clearly defined process, it can not be explained, it can be used at any age, it produces good ideas, it depends on confidence in process not product, and it comes before a proof. In relation to mathematics he feels that it brings a quick answer after working for a time on a problem, helps to make quick first guesses about solutions, and involves making a mathematical idea concrete.[1] William F. Irmscher

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[1]Susan E. Goethe, A Study of Intuitive Thinking, U. S., Educational Resources Information Center, ERIC Document ED 131 438, 1976; Jerome S. Bruner, On Knowing, (Cambridge, Massachusetts: Belknap Press of Harvard University Press, 1964), pp. 102-105.

in a paper on intuition pointed out that intuition is a sudden uncontrollable process of which we are unaware. Intuition is a kind of built-in teacher, a form of intelligent behavior, and a practical resource which should be developed and made stronger.[2]

Intuition can be used in the study of mathematics specifically in geometry and problem solving. Edwin E. Moise stated that geometric definitions can first be illustrated with pictures, which are intuitively understood by students, and with this understanding comes a belief in the formal definitions.[3] Howard F. Fehr feels that students can gain in intuition about lines, rays, segments, and angles in Euclidean Geometry by using an informal and manipulative approach.[4] As a practical resource that everyone possesses and uses, intuition can aid in problem solving by helping to make estimations of the solution, creating plans to follow, and by bringing the solution to the surface. Indirectly, intuition aids

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[2]William F. Irmscher, Acknowledging Intuition, U. S., Educational Resources Information Center, ERIC Document ED 150 640, 1977.

[3]Edwin E. Moise, "The Meaning of Euclidean Geometry In School Mathematics," Mathematics Teacher, October, 1975, p. 475.

[4]Howard F. Fehr, "The Present Year-Long Course in Euclidean Geometry Must Go," Mathematics Teacher, February, 1972, p. 153.

in problem solving by increasing knowledge about a subject. For example, as the knowledge of geometric definitions increases so can the ability to solve problems with this knowledge.

Problem Solving

As with intuition, problem solving has many different characteristics. George Polya states that problem solving is a specific achievement of intelligence, a way to attain a goal which had been blocked, and a practical skill which is acquired by imitation and practice.[5] Donovan A. Johnson and Gerald R. Rising believe that problem solving helps students learn new concepts, practice computations, allows for the transfer of knowledge, motivates students to learn more, requires background knowledge, requires a followed procedure, and is a complicated mental process.[6]

Many suggestions have been made on how teachers can help students improve their skills in problem solving, use and demonstrate it in class so students can help and learn by imitation, and assist students with their problem solving.[7]

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[5]George Polya, How to Solve It, (New Jersey: Princeton University Press, 1945), pp. 5-6.

[6]Donovan A. Johnson and Gerald R. Rising, Guidelines for Teaching Mathematics, (Belmont, California: Wadsworth Publishing Company, Inc., 1972), p. 234-238.

[7]George Polya, Mathematical Discovery, (United States: John Wiley & Sons, Inc., 1962 and 1965), p. 100 and 105.

Two articles in a Yearbook of the National Council of Teachers of Mathematics suggest that teachers should provide the necessary background knowledge, help students understand the problem, guide them through the process, allow for variation of the process, recognize students who are becoming good problem solvers,[8] relate the unknown to something known in the problem, and limit the number of problems to solve, but not the ways to solve each one.[9]

The teaching of problem solving according to Polya will teach students how to think and to use information they have learned.[10] The abilities to think and to use information are necessary at the present time and also in the future. No one is sure of the problems to be faced in the future, but the skills of problem solving will be

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[8]Kenneth B. Henderson and Robert E. Pingry, "Problem Solving in Mathematics," The Learning of Mathematics: Its Theory and Practice, Twenty-first Yearbook of the National Council of Teachers of Mathematics (Washington, D.C.: N.C.T.M., 1953), pp. 248-255.

[9]John R. Clark and Howard F. Fehr, "Learning Theory and the Improvement of Instruction - A Balanced Program," The Learning of Mathematics: Its Theory and Practice, Twenty-first Yearbook of the National Council of Teachers of Mathematics (Washington, D. C.: N.C.T.M, 1953), pp. 342-344.

[10]Polya, Mathematical Discovery, p. 100.

necessary in order to effectively deal with these problems.

Geometry Curriculum

Traditionally, the senior high geometry course is based on a deductive development of Euclidean Geometry. Several sources recommended that the content of this course be expanded. One possibility, suggested by Carl B. Allendoerfer, included the introduction of informal geometry early in the curriculum, an informal approach to solid geometry, and the inclusion of geometric transformations.[11]

With the increase of content, several sources agreed that geometry should not be confined to a one year course. Instruction of this subject could be expanded to include all levels just as arithmetic and algebraic concepts are now. Bruce E. Meserve sees the solution of a shortage of time spent on geometry to be "The extensive previous contacts with informal geometry inside school and outside school."[12] Allendoerfer also suggested that informal plane and solid geometry along with geometric transformations should be part of the elementary curriculum and that, on

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[11]Carl B. Allendoerfer, "The Dilemma in Geometry," Mathematics Teacher, March, 1969, p. 168.

[12]Bruce E. Meserve, "An Improved Year of Geometry," Mathematics Teacher, February, 1972, p. 104.

the junior high level there should be more informal geometry.[13] Meserve felt that the study of geometry should be conducted in the seventh through twelfth grades. He proposed that the instruction should start "with a physical, informal study, using drawings, paper folding, measuring, and physical objects to gain an intuitive feeling for figures in Euclidean 2- and 3-space, especially for lines, rays, segments, and angles."[14]

Another recommended possibility is to include a study of geometric transformations in the junior high school curriculum. According to Zalman P. Usiskin and Arthur F. Coxford, transformational geometry can be used to help students with problem solving. It is closely related to a student's intuition and it can organize the study of geometry.[15] They also believe that transformations provide a link between a student's intuitive ideas and the formal definitions of congruence, similarity, and symmetry.[16]

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[13]Allendoerfer, p. 169.

[14]Meserve, p. 153.

[15]Arthur F. Coxford, "A Transformation Approach To Geometry," Geometry in the Mathematics Curriculum, Thirty-Sixth Yearbook of the National Council of Teachers of Mathematics (United States: N.C.T.M., 1973), p. 152.

[16]Zalman P. Usiskin and Arthur F. Coxford, "A Transformation Approach to Tenth Grade Geometry," Mathematics Teacher, January, 1972, p. 21.

These formal definitions are a large part of a tenth grade geometry course. Transformations provide many opportunities for students to have visual and tactile experiences. These experiences help the student have an active role instead of a passive role in learning about geometry.[17] Other countries have used transformational geometry since they have changed from the Euclidean approach. Some textbooks have been written on the subject, and Allendoerfer believes that "they deserve widespread attention of the United States."[18]

Twenty-two eighth grade, and twelve geometry textbooks were reviewed. Seven of the eighth grade books contained no information about transformations, eleven had a few pages or sections, and four had a complete chapter or chapters about transformations. Of the geometry books, four had no information, three had a small amount of information, eight had at least one chapter, and one was centered around the transformations. The approaches in these books varied in the degree of formality and complexity. Three of these books were published in 1967 or 1968 and the remaining twenty-one were published between 1970 and 1978.

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[17]Stanley B. Jackson, "Applications of Transformations To Topics in Elementary Geometry: Part I," Mathematics Teacher, November, 1975, p. 554.

[18]Allendoerfer, p. 167.

Chapter 3

PROCEDURES OF THE STUDY

Previous to a tenth grade geometry course most students have probably had little exposure to geometric figures. An informal introduction to geometric concepts presented earlier in the curriculum may prepare these students for this course. In particular, this introduction could include transformational geometry.

This study developed a unit on transformational geometry to be used as an introduction to geometric concepts before a tenth grade course. A holistic approach including the cognitive, psychomotor, and affective domains was used. A pretest, homework assignments, and a post-test were also written.

Before teaching the unit the pretest, which was based on general knowledge of geometric figures and knowledge of geometric transformations, was given to the eighth grade students in the Oakville School District in Oakville, Washington. They were to match geometric figures with their names, select reflection, slide, and rotation images, and perform constructions. The students were told to answer the questions that they could, and that they would not be graded on the results of this test.

The teaching of the unit began with some of the

psychomotor skills that involved the use of a mira, compass, and straightedge. After the students were proficient with these skills, the teaching of concepts began. Soon daily lessons consisted of a combination of concepts, skills, and affective qualities. As the students progressed through this unit, several geometric figures were presented and used in homework assignments.

Each lesson proceeded in an inductive manner from the specific to the general. As the students moved through the different levels of thinking they also moved through the steps of problem solving, and this movement was aided by their intuition. Students gained an understanding of the problem on the knowledge level, devised a plan on the comprehension level, carried out the plan on the application level, and they examined their results on the analysis level. The approach was also informal in that there were several manipulative activities, and discovery activities.

In the cognitive domain, the students progressed through each level of thinking from knowledge to evaluation. The teacher began a lesson on the knowledge level of one concept by asking the students questions about the measurement and reflection image of a segment. They recalled the reflection image of a segment and told how to measure a

segment. At the next level, comprehension, the students predicted the measure of the reflection image of given segments. For the application level they illustrated the reflection images of given segments and utilized a ruler to measure them. On the analysis level, the students examined and compared their results. They formulated a general statement about the length of the reflection image of a given segment on the synthesis level. Finally, at the evaluation level, the students selected the correct measures for the reflection images of given segments.

In the psychomotor domain the students gained mastery of a skill through four steps: 1.) by attempting the skill, 2.) by practicing isolated parts, 3.) by varying the skill, and 4.) by attempting increased difficulty. For example, the teacher demonstrated the construction of an angle, using a protractor, for the class. Then each student attempted this construction. Each part of the construction was then practiced separately. The students constructed a ray, correctly positioned the protractor on the ray, located and marked the angle measurement, and constructed the second ray to complete the construction of the angle. For variation of the skill, they constructed angles with measures less than ninety degrees and greater than ninety degrees for increased difficulty.

In the affective domain the students increased their appreciation, respect, willingness, and positive attitude toward different affective qualities. To begin one lesson the teacher demonstrated a construction with a collapsible compass while the students observed. Then they stated some difficulties encountered with the collapsible compass, and some advantages of a modern non-collapsible compass. Finally, they thought of possibilities for a new compass.

The main concept of the unit was that a geometric figure may be moved in three ways. This generalization was taken through each level of thinking. The students formed small groups and recalled the types of images for a geometric figure, explained the procedures for finding the images, demonstrated these procedures, compared their results, formulated a general statement about the images of a geometric figure, and considered other possible ways that a set of points may be moved.

The students transferred their knowledge of transformations from the classroom to their out-of-school environment, and composed a list of examples for the three types of transformations that are seen outside the classroom.

The teacher reviewed the unit with the students in preparation for the post-test, which was given on the

sixteenth day. This test covered the same knowledge that was contained in the pretest. It was composed of questions from the categories of memory, translation, interpretation, application, analysis, synthesis, and evaluation.

Chapter 4

RESULTS OF THE STUDY

Many students may not be exposed to geometrical concepts until the tenth grade. Then, the traditional approach has been Euclidean geometry which is formal and deductive. An earlier introduction to geometric concepts might prepare the students for this approach. The introduction proposed was: informal and inductive, utilized the students' intuition and problem solving skills, and included transformational geometry.

Current related literature and textbooks were reviewed. A unit was developed and written using a holistic approach and including the cognitive, psychomotor, and affective domains. The unit was taught to an eighth grade class, and these students participated by using their intuition and inductive reasoning. They also participated in problem solving and manipulative activities. A pretest and post-test, covering knowledge of geometric figures and transformations, were given to the students.

A matched-pairs test was performed on the scores of the pretest and post-test. The null hypothesis was, that there has been no improvement from the performance on the pretest to that on the post-test. The difference for each pair of post-test scores and pretest scores was

calculated. Each difference was then squared. The sum of the differences and the sum of the differences squared were also calculated. The mean of the matched-pairs differences was calculated to be 14.95 and the standard deviation of the matched-pairs differences was calculated to be 8.308. Next, the test statistic value was computed to equal 8.247. The tabled student t-value with twenty degrees of freedom, and twenty-one pairs of scores was found to equal 3.850 on the .0005 level. The null hypothesis was rejected because the test statistic value was greater than the critical value at the .0005 level. This meant that the chances of the scores improving by accident was .05%. Therefore, conclusion is that the teaching of the unit was effective.

CALCULATION OF MATCHED-PAIR DIFFERENCES

FOR PRETEST AND POST-TEST SCORES

i	Post-test Scores	Pretest Scores	Difference d_i	d_i^2
1	41	17	24	576
2	3	2	1	1
3	18	12	6	36
4	32	15	17	289
5	34	34	0	0
6	39	14	25	625
7	20	16	4	16
8	31	7	24	576
9	34	17	17	289
10	32	3	29	841
11	28	6	22	484
12	16	5	11	121
13	24	4	20	400
14	26	19	7	49
15	41	25	16	256
16	33	10	23	529
17	16	2	14	196
18	27	14	13	169
19	21	11	10	100
20	19	8	11	121
21	26	6	20	400
Totals:	561	247	314	6074

Mean of the matched-pairs differences:

$$\bar{d} = \frac{\sum d_i}{n} = \frac{314}{21} = 14.95$$

Standard deviation of the matched-pairs differences:

$$s_{d\text{-paired}} = \sqrt{\frac{\sum d_i^2 - n\bar{d}^2}{n-1}} = \sqrt{\frac{6074 - 21(14.95)^2}{20}} = \sqrt{69.0224} = 8.308$$

Test statistic:

$$T = \frac{\bar{d}\sqrt{n}}{s_{d\text{-paired}}} = \frac{14.95\sqrt{21}}{8.308} = \frac{14.95(4.583)}{8.308} = 8.247$$

Critical value:

$$t_{.0005}^{(n-1)} = 3.850$$

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APPENDIX A

TRANSFORMATIONAL GEOMETRY UNIT

CONCEPTS

1. Given a point and one line, the given point and its reflection image are the same distance from the line and the segment connecting them is perpendicular to the line.

2. Through a line of reflection, each point has one and only one reflection image.

3. Given a line of reflection and a point, the reflection image of that point has the given point as its reflection image.

4. A set of points is reflected one point at a time.

5. The reflection image of a line through a given line is also a line.

6. Given line l and reflection line m, if the reflection image of line l is line 2 then the reflection image of line 2 is line 1.

7. If three points of a line are reflected through a line, then their images are in the same order as the given points.

8. The reflection of a ray through a line is also a ray.

9. The reflection of a segment through a line is a segment.

10. Given a segment and its measure then the reflection of the segment through a line has the same measure.

11. The reflection of an angle with a given measure through a line is an angle with the same measure.

12. If points in a figure are labeled clockwise, then the reflection image of this figure will be labeled counter-clockwise.

13. The line of symmetry of a plane figure divides the figure in two parts of the same size and shape.

14. To slide a point it is reflected through parallel lines.

15. To rotate a point, it is reflected through two or more nonparallel lines.

16. The center of rotation is the point of intersection of the reflection lines.

17. The reflection of a point through a point is a half-turn.

18. Slides and rotations preserve segments, rays, lines, angles, triangles, and other sets of points.

19. Slides and rotations preserve segment and angle measure.

20. Slides and rotations preserve betweenness.

21. If the points of a figure are labeled clockwise, then the slide image of this figure will also be labeled clockwise.

22. If the points of a figure are labeled clockwise, then the rotation image of this figure will also be labeled clockwise.

23. The bisector of an angle is the line of symmetry of the angle.

Generalization: There are three ways that a geometric figure may be moved.

Transfer: List examples of reflections, slides, and rotations that are seen outside the classroom.

Concept: Given a point and a line, the given point and its reflection image are the same distance from the line and the segment connecting them is perpendicular to the line.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will ask questions about measuring the distance from a point to a line and about perpendiculars.	KNOWLEDGE: Students will answer questions about measuring distance and perpendiculars.	
	COMPREHENSION: Students will state in their own words the meaning of two points the same distance from a line and a segment perpendicular to a line.	
Teacher will give some specific examples to do and will demonstrate two in class.	APPLICATION: Students will demonstrate how to locate the reflection image of a point through a line.	Paper, rulers, miras, and protractors
Teacher will provide additional examples.	ANALYSIS: Students will examine and compare their results. They will also experiment with other examples.	
Teacher will evaluate the statement	SYNTHESIS: Students will formulate a statement about the location of the reflection image of a point through a line.	
Teacher will explain directions and answer questions	EVALUATION: Students will select the correct reflection images and measure to locate reflection images.	Worksheet

Concept: Through a line of reflection, each point has one and only one reflection image.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will evaluate the meanings given and provide necessary background knowledge.	KNOWLEDGE: Students will recall the meanings of a line and a point.	Miras and paper
	COMPREHENSION: Students will predict the number of reflection images any given point will have.	
Teacher will provide help when necessary.	APPLICATION: Students will demonstrate how to reflect several points through a line and locate the image of each point.	
	ANALYSIS: Students will compare results.	
Teacher will evaluate the statement.	SYNTHESIS: Students will propose a general rule for the number of reflection images that a point has.	
Teacher will explain directions and answer questions.	EVALUATION: Students will select the reflection image of each given point.	Worksheet

Concept: Given a line of reflection and a point. The reflection image of that point has the given point as its reflection image.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will ask students questions about finding the reflection image of a given point.	KNOWLEDGE: Students will recall the general rule for the number of images for a given point and the procedure used to find the image.	Overhead projector or Chalkboard
Teacher will initiate discussion by asking questions about reflecting an image point.	COMPREHENSION: Students will discuss possible results of reflecting an image point through the given line of reflection.	
Teacher will provide help when necessary.	APPLICATION: Students will illustrate the results of reflecting an image point through the given line of reflection.	Paper, miras
Teacher will suggest other cases with which to experiment.	ANALYSIS: Students will compare their results and experiment with other given points, lines of reflection, and their images.	
Teacher will evaluate the statement.	SYNTHESIS: Students will formulate a rule for the results of reflecting an image point through the given line of reflection.	
Teacher will explain directions and answer questions.	EVALUATION: Given a point and a line of reflection, students will select the reflection image of the given point and then	Worksheet

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
	EVALUATION: select the reflection image of this point. They will also reflect the given point and its reflection image.	

Concept: A set of points is reflected on point at a time.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will list the steps on the chalkboard.	KNOWLEDGE: Students will list the steps used to reflect a point through a line and recall the number of reflection images that a point has.	
Teacher will lead the discussion.	COMPREHENSION: Students will discuss procedures which could be used to reflect a set of points through a given line.	
Teacher will provide help when necessary.	APPLICATION: Students will demonstrate a procedure to reflect a set of points through a given line.	Paper, miras
	ANALYSIS: Students will compare their results and procedures.	
Teacher will evaluate the statement.	SYNTHESIS: Students will propose a rule stating the procedure of reflecting a set of points through a line.	
Teacher will explain directions and answer questions.	EVALUATION: Students will select the reflection of a given set of points through a given line and draw reflection images of sets of points.	Worksheet

Concept: The reflection image of a line through a given line is also a line.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will write the procedure on the chalkboard.	KNOWLEDGE: Students will recall the procedure used to reflect a set of points through a given line.	
Teacher will ask questions about the predictions.	COMPREHENSION: Students will predict what the reflection of a line through a given line will be.	
Teacher will provide help when necessary.	APPLICATION: Students will illustrate the reflection of a line through a given line.	Paper and miras
	ANALYSIS: Students will examine and compare their results.	
Teacher will evaluate the statement.	SYNTHESIS: Students will propose a rule for the result of reflecting a line through a given line.	
Teacher will explain directions and answer questions.	EVALUATION: Students will select the correct reflection in each problem.	Worksheet

Concept: Given line 1 and reflection line m. If the reflection image of line 1 is line 2 then the reflection image of line 2 is line 1.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will ask students to recall the reflection of a line.	KNOWLEDGE: Students will recall that the reflection of a line is a line.	
	COMPREHENSION: Students will predict what will happen if a reflection image of a given line is reflected.	
Teacher will provide help when necessary.	APPLICATION: Students will demonstrate how to locate the reflection image of the reflection image of a given line.	Paper and miras
Teacher will suggest possible locations for the first line.	ANALYSIS: Students will experiment by placing the first line in several locations.	
Teacher will evaluate the statement.	SYNTHESIS: Students will propose a general rule for the reflection of the reflection of a given line.	
Teacher will explain directions and answer questions.	EVALUATION: Students will consider the outcomes when the reflection image of other geometric figures are reflected.	Worksheet

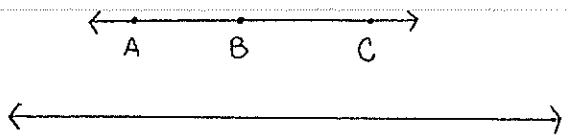
Concept: If three points of a line are reflected through a line, then their images are in the same order as the given points.

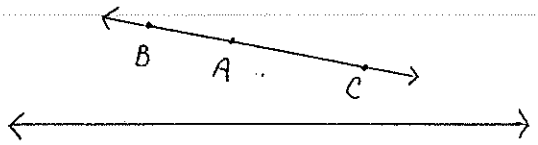
TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will ask students to recall how to reflect a set of points.	KNOWLEDGE: Students will recall how to reflect a set of points.	
Teacher will start discussion.	COMPREHENSION: Students will discuss how the reflection images of the points could be arranged.	
Teacher will provide students with several situations to consider.	APPLICATION: Students will demonstrate location of the images of three points on a line.	Paper and miras
	ANALYSIS: Students will compare their results.	
Teacher will evaluate the statement.	SYNTHESIS: Students will formulate a general result for the arrangement of the images of three points on a line.	
Teacher will explain directions and answer questions.	EVALUATION: Students will select the correct images of three given points on a line.	Worksheet

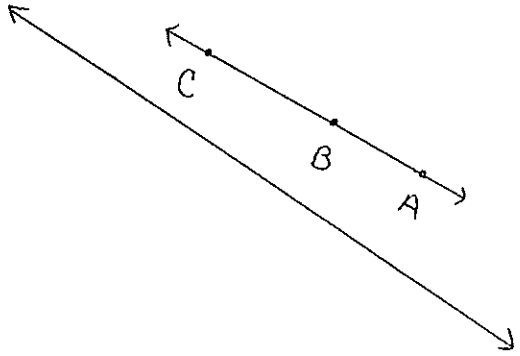
TRANSFORMATIONAL GEOMETRY NAME _____

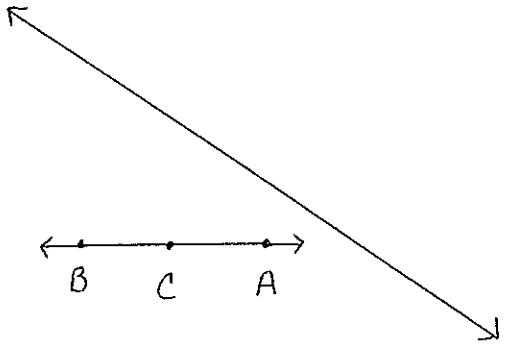
WORKSHEET DATE _____

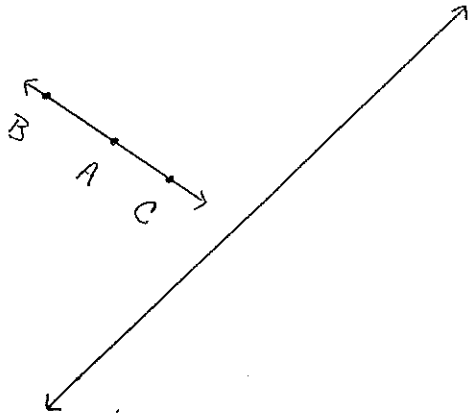
Directions: Draw the reflection of the line and locate the images of A, B, and C.

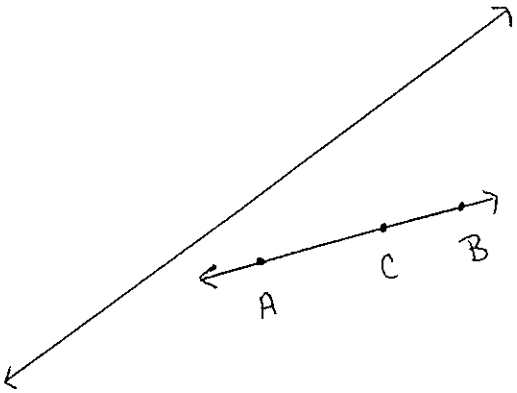












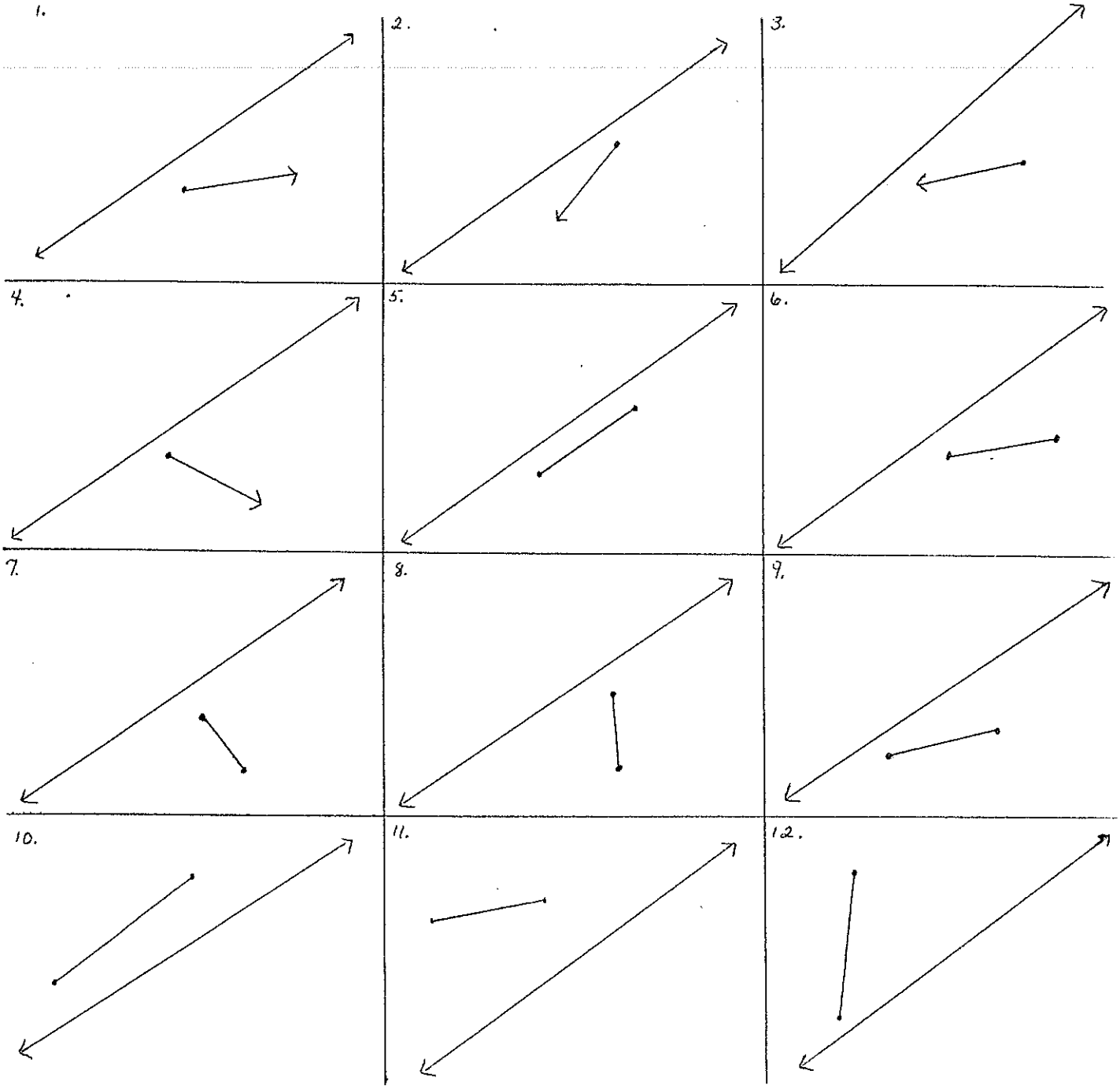
Concept: The reflection image of a ray through a line is also a ray.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will ask students to recall the reflection of a point and a line.	KNOWLEDGE: Students will recall that the reflection image of a point is a point and the reflection of a line is a line.	
Teacher will ask students to predict what the reflection of a ray will be.	COMPREHENSION: Students will predict the reflection image of a ray.	
Teacher will provide help when necessary.	APPLICATION: Students will illustrate reflection of a ray.	Paper and miras.
	ANALYSIS: Students will examine and compare their results.	
Teacher will evaluate the statement.	SYNTHESIS: Students will propose a general statement about the reflection image of any ray.	
Teacher will explain directions and answer questions.	EVALUATION: Students will select the correct reflection images for the given rays.	Worksheet

TRANSFORMATIONAL GEOMETRY NAME_____

WORKSHEET DATE_____

Directions: #1-8 draw the reflection images. #9-12 write the measure for the reflection images.



Concept: The reflection of a segment through a line is a segment.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will confirm their responses.	KNOWLEDGE: Students will recall the reflection images of a point, a line and a ray through a line.	
	COMPREHENSION: Students will predict the reflection image of a segment.	
Teacher will provide help when necessary.	APPLICATION: Students will illustrate the reflection of a segment.	Paper and miras
	ANALYSIS: Students will compare and examine their results.	
Teacher will evaluate the statement.	SYNTHESIS: Students will formulate a general statement about the reflection image of a segment.	
Teacher will explain directions and answer questions.	EVALUATION: Students will select the correct reflection images for the given segments and judge if other reflections are correct.	Worksheet

Concept: Given a segment and its measure then the reflection of the segment through a line has the same measure.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will ask students questions about measures of reflection images of segments.	KNOWLEDGE: Students will recall the reflection image of a segment and then tell how to measure a segment	
	COMPREHENSION: Students will predict the measures of the reflection image of given segments.	
Teacher will provide help when necessary	APPLICATION: Students will illustrate the reflection images of given segments and utilize a ruler to measure them.	Paper, rulers, miras
	ANALYSIS: Students will examine and compare their results.	
Teacher will evaluate the statement.	SYNTHESIS: Students will formulate a general statement about the length of the reflection image of a given segment.	
Teacher will explain directions and answer questions.	EVALUATION: Students will select the correct measures for the reflections of given segments.	Worksheet

Concept: The reflection of an angle with a given measure through a line is an angle with the same measure.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will ask students to recall reflections of a point, line, segment, ray, set of points, and the order of images.	KNOWLEDGE: Students will recall the reflection images of a point, line, segment, ray, set of points, and the order of images.	
	COMPREHENSION: Students will predict the reflection image of an angle and measure of the angle	
Teacher will provide help when necessary.	APPLICATION: Students will demonstrate how to locate the reflection image of an angle.	Worksheet Miras, protractors, rulers.
	ANALYSIS: Students will examine and compare their results.	
Teacher will evaluate the statement.	SYNTHESIS: Students will formulate a general statement about the reflection image and its measure.	
Teacher will explain directions and answer questions.	EVALUATION: Students will select the correct reflections of given angles and measures of reflections of given angles.	Worksheet

Concept: If points in a figure are labeled clockwise then the reflection image of this figure will be labeled counter-clockwise.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will write the procedure on the board.	KNOWLEDGE: Students will recall the procedure used to reflect a set of points.	
Teacher will present 4-6 examples of sets of points to be reflected.	COMPREHENSION: Students will predict the location of the image of each point in a set of points to be reflected through a line.	
Teacher will provide help when necessary.	APPLICATION: Students will demonstrate the reflections of these sets of points. The image of each point will be labeled as such.	Paper, miras
Teacher will suggest other sets of points with which to experiment.	ANALYSIS: Students will examine and compare results. They will also experiment with their own sets of points.	
Teacher will evaluate the statement.	SYNTHESIS: Students will formulate a general statement about the arrangement of the images in comparison to the arrangement of the given points.	
Teacher will explain directions and answer questions.	EVALUATION: Students will select the correct locations for points on reflections of given sets of points through a line.	Worksheet

Concept: The line of symmetry of a plane figure divides the figure in two parts of the same size and shape.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will show students some examples of figures which are symmetrical and non-symmetrical.	KNOWLEDGE: Students will record on a chart the figures which are symmetrical and non-symmetrical.	Rulers
Teacher will ask students to tell why a figure is symmetrical.	COMPREHENSION: Students will tell in their own words why a specific figure is symmetrical.	
Teacher will provide help when necessary.	APPLICATION: Students will sketch figures which are symmetrical.	
	ANALYSIS: Students will examine and compare their sketches.	
Teacher will check the arrangements.	SYNTHESIS: Students will arrange their figures along a line so that the line cuts the figure in two symmetric halves.	
Teacher will explain directions and answer questions.	EVALUATION: Students will select figures which are symmetrical and select the correct line of symmetry.	Worksheet

Concept: To slide a point, it is reflected through parallel lines.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will ask students questions about parallel lines.	KNOWLEDGE: Students will answer questions about parallel lines. They will also state in their own words when two lines are parallel.	
Teacher will give students several pairs of lines on chalkboard.	COMPREHENSION: Students will determine which pairs of lines are parallel.	Chalkboard
Teacher will provide help when necessary.	APPLICATION: Students will utilize the parallel lines and knowledge of reflections to reflect a point through both parallel lines.	Mira, rulers, paper
Teacher will suggest possible locations of points to reflect.	ANALYSIS: Students will examine and compare their results. They will also experiment with various locations of the point to be reflected.	
Teacher will evaluate the statement.	SYNTHESIS: Students will formulate a statement about their results and propose a term for the procedure.	
Teacher will explain directions and answer questions.	EVALUATION: Students will select the correct images of given points which were reflected through two or more parallel lines	Worksheet

Concept: To rotate a point, it is reflected through two or more non-parallel lines.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will ask students to recall the meaning of parallel lines.	KNOWLEDGE: Students will state the meaning of parallel.	
Teacher will evaluate the students' interpretations.	COMPREHENSION: Students will interpret the meaning of non-parallel lines.	
Teacher will demonstrate this procedure and assist students.	APPLICATION: Students will adapt the procedure for sliding a point through parallel lines to non-parallel lines.	
	ANALYSIS: Students will compare the results between the two procedures.	
Teacher will evaluate the statement.	SYNTHESIS: Students will formulate a statement about the results and propose a term for the procedure.	
Teacher will explain directions and answer questions.	EVALUATION: Students will select the correct rotation for a given point through given non-parallel lines.	Worksheet

Concept: The center of rotation is the point of intersection of the reflection lines.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will ask students to recall the procedure for rotation.	KNOWLEDGE: Students will recall the procedure for reflecting a point through two or more non-parallel lines.	
Teacher will provide examples of locating intersection points.	COMPREHENSION: Students will locate the point of intersection of the reflection lines and the image of a given point reflected through these lines.	Rulers
Teacher will provide help when necessary.	APPLICATION: Students will demonstrate that the distance between the given point and the intersection point is the same as the distance between the image of the given point and the intersection point.	Rulers, miras
Teacher will suggest other examples to examine.	ANALYSIS: Students will examine and compare their results. They will also experiment with other examples.	
Teacher will evaluate the statement.	SYNTHESIS: Students will formulate a general statement about their results.	
Teacher will explain directions and answer questions.	EVALUATION: Students will select the correct centers of rotation and then measure to check their selection.	Worksheet

Concept: The reflection of a point through a point is a half-turn.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher asks students questions about the procedure to reflect a point through a line.	KNOWLEDGE: Students will recall the procedure used to reflect a point through a line.	
	COMPREHENSION: Students will predict the location of a point reflected through a point.	
Teacher will provide help when necessary.	APPLICATION: Students will demonstrate locating the reflection image of a point through a second point.	Rulers
Teacher will suggest other cases to consider.	ANALYSIS: Students will test their procedure using several other points.	
Teacher will evaluate the statement.	SYNTHESIS: Students will formulate a general statement about the location of the reflection image of a point through a point and propose a term to name it.	
Teacher will explain directions and answer questions.	EVALUATION: Students will select the correct locations for the reflection images of points through a point.	

Concept: Slides and rotations preserve segments, rays, lines, angles, triangles, and other geometric figures.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will ask questions about the reflection image of several sets of points.	KNOWLEDGE: Students will recall the reflection image of a segment, ray, line, angle, triangle.	
Teacher will record the predictions.	COMPREHENSION: Students will predict the slide and rotation images of a segment, ray, line, angle, and triangle.	
Teacher will provide help when necessary.	APPLICATION: Students will employ their knowledge of the slide image and rotation image of a point to determine the slide and rotation images for a segment, ray, line, angle, and triangle.	
	ANALYSIS: Students will examine and compare results.	
Teacher will evaluate the statement.	SYNTHESIS: Students will propose a general statement about slides and rotations of segments, rays, lines, angles, triangles and other geometric figures.	
Teacher will explain directions and answer questions.	EVALUATION: Students will select the correct slide and rotation images for each set of points.	Chalkboard or worksheet

Concept: Slides and rotations preserve segment and angle measure.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will ask students questions about reflection images of segments and angles, their measures, and procedures used to find slide & rotation images.	KNOWLEDGE: Students will recall reflection images of a segment and angle. They will also review the procedure used to find a slide image and a rotation image.	
Teacher will provide segment and angle measures.	COMPREHENSION: Students will predict the measure of the slide and rotation images for given segments and angles.	
Teacher will provide the students with examples.	APPLICATION: Students will demonstrate finding the slide and rotation images of the segments and angles.	Rulers, protractors, miras
	ANALYSIS: Students will examine and compare their results.	
Teacher will evaluate the statement.	SYNTHESIS: Students will formulate a general statement about measures of segments and angles and their slide and rotation images.	
Teacher will explain directions and answer questions.	EVALUATION: Students will select the correct slide and rotation images and measure to select the correct slide and rotation images.	

Concept: Slides and rotations preserve betweenness.

TEACHER'S INSTRUCTIONAL INPUT	STUDENT'S COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will ask students questions about the possible order of three points on a line and about betweenness.	KNOWLEDGE: Students will answer questions.	
Teacher will provide the students with problems.	COMPREHENSION: Students will predict the order of the slide and rotation images of three points on a line.	
Teacher will provide help when necessary.	APPLICATION: Students will demonstrate how to locate the slide and rotation images for the three points on a line.	
	ANALYSIS: Students will examine and compare their results.	
Teacher will evaluate the statement.	SYNTHESIS: Students will formulate a conclusion about slides and rotations in relation to the order of three given points.	
Teacher will explain directions and answer questions.	EVALUATION: Students will select the correct slide and rotation images for each problem of the worksheet.	Worksheet or chalkboard

Concept: If the points of a figure are labeled clockwise then the slide image of this figure will also be labeled clockwise.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will ask students to recall the reflection image of a triangle and how the triangle and its image are labeled.	KNOWLEDGE: Students will recall the reflection image of a triangle and identify the images of each vertex of the triangle.	Overhead - Chalkboard
Teacher will present 4-6 figures to the students for their predictions.	COMPREHENSION: Students will predict the location of the slide image of each point in a set of points (triangle, rectangle, etc.).	
Teacher will provide help when necessary.	APPLICATION: Students will demonstrate the slides of these sets of points. The slide image of each point will be labeled as such.	Rulers
Teacher will suggest sets of points to use.	ANALYSIS: Students will examine and compare results. They will also experiment with own sets of points.	
Teacher will evaluate the statement.	SYNTHESIS: Students will formulate a general statement about the arrangement of the slide images in comparison to the arrangement of the given points.	
Teacher will explain directions and answer questions.	EVALUATION: Students will select the correct slide images of given sets of points.	Worksheet - Chalkboard

Concept: If the points of a figure are labeled clockwise, then the rotation image of this figure will also be labeled clockwise.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will ask students to recall the rotation image of a triangle.	KNOWLEDGE: Students will identify the rotation image of a triangle.	Overhead - Chalkboard
Teacher will present several figures to the students.	COMPREHENSION: Students will predict the location of the rotation images for Vertices of several figures.	
Teacher will provide help when necessary.	APPLICATION: Students will demonstrate the location of the rotation images for the vertices of several figures.	Compass, Ruler Protractor
	ANALYSIS: Students will examine and compare their results.	
Teacher will evaluate the statement.	SYNTHESIS: Students will formulate a general statement about the arrangement of the rotation images in comparison to the arrangement of the given points.	
Teacher will explain directions and answer questions.	EVALUATION: Students will select the correct rotation images of given sets of points.	Worksheet or Chalkboard

Concept: The bisector of an angle is the line of symmetry of the angle.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will ask students to recall the location of lines of symmetry for given geometrical figures and also meaning of bisect.	KNOWLEDGE: Students will identify the correct line of symmetry for several geometrical shapes. They will state the meaning of bisect.	Overhead or chalkboard figures
Teacher will ask students to locate the bisectors for several angles.	COMPREHENSION: Students will predict the location of the bisectors of several angles.	
Teacher will provide help when necessary.	APPLICATION: Students will illustrate the correct locations for the bisector of each angle.	Protractor or miras
	ANALYSIS: Students will examine and compare their results.	
Teacher will evaluate the statement.	SYNTHESIS: Students will propose a general statement about the bisector of an angle and the line of symmetry of that angle.	
Teacher will explain directions and answer questions.	EVALUATION: Students will select the angle bisector for each angle by checking for symmetry.	Worksheet

Generalization: There are three ways that a geometric figure may be moved (3 images).

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will ask students to state the three types of images for a geometric shape.	KNOWLEDGE: Students will form small groups and recall the three types of images for a geometric shape.	
	COMPREHENSION: Students will explain the procedures used to find the types of images.	
Teacher will provide help when necessary.	APPLICATION: Students will demonstrate these procedures.	
	ANALYSIS: Students will examine and compare the results of their demonstrations.	
Teacher will evaluate the statement.	SYNTHESIS: Students will formulate a general statement about the images of a geometric figure.	
Teacher will explain directions and answer questions.	EVALUATION: Students will consider other ways in which a set of points may be moved.	

Transfer: List examples of reflections, slides, and rotations that are seen outside of the classroom.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' COGNITIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will ask students to recall the meanings of reflections, slides and rotations.	KNOWLEDGE: Students will recall the meanings of reflections, slides and rotations.	Butcher paper markers
	COMPREHENSION: Students will translate these meanings into everyday usage.	
Teacher will provide help when necessary.	APPLICATION: Students will classify objects into three categories: reflections, slides, rotations. Class will be divided into groups.	
	ANALYSIS: Groups will compare their lists.	
Teacher will evaluate the statement.	SYNTHESIS: Students will assemble one list using ideas from the groups.	
Teacher will explain directions and answer questions.	EVALUATION: Students will evaluate items that they see in a day and revise the master list.	

SKILLS

1. Locate reflections through a line by folding paper and tracing.

2. Using a mira to locate and construct the images of given geometric figures.

3. Using a compass and straightedge to draw reflections through a given line.

4. Using a protractor to construct an angle.

5. Locate the line of symmetry for symmetrical plane figures.

6. Locating slide images.

7. Locating rotation images through two intersecting lines.

Skill: Locate reflections through a line by folding paper and tracing.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' PSYCHOMOTOR BEHAVIOR	MATERIALS AND RESOURCES
Teacher will demonstrate how to reflect a point through a line by folding the paper and tracing.	ATTEMPT AS A WHOLE: Reflecting a point through a given line by folding the paper and tracing.	Tracing paper
	PRACTICE ISOLATED PARTS: Folding the paper on the line. Locating the reflection of the point.	
	PROVIDE FOR VARIATION: Reflecting a segment by folding and tracing.	
	ATTEMPT INCREASED DIFFICULTY: Reflecting triangles, quadrilaterals by folding and tracing.	

Skill: Using a mira to locate and construct the images of given geometric figures.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' PSYCHOMOTOR BEHAVIOR	MATERIALS AND RESOURCES
Teacher will demonstrate how to use a mira.	ATTEMPT AS A WHOLE: Constructing the image of a geometric figure from the worksheet. Begin with point-segment-line.	Overhead projector miras worksheet with geometric figures
	PRACTICE ISOLATED PARTS: Practice placing the mira on a straight line and finding the reflected image.	
	PROVIDE FOR VARIATION: Constructing the image of the remainder of the geometric figures on the worksheet.	
	ATTEMPT INCREASED DIFFICULTY: Constructing the image of a geometric figure which the student has drawn.	Worksheet

Skill: Using a compass and straightedge to draw reflections through a given line.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' PSYCHOMOTOR BEHAVIOR	MATERIALS AND RESOURCES
Teacher will demonstrate how to reflect a point through a given line using a compass and straightedge.	ATTEMPT AS A WHOLE: Reflecting a point through a given line using a compass and straightedge.	Compass and straightedge
	PRACTICE ISOLATED PARTS: 1.) Placing point of compass on point and marking two areas on the given line, 2.) Placing point of compass on intersection points found in #1 and making two arcs on opposite side of line, 3.) Marking last intersection point.	
	PROVIDE FOR VARIATION: Reflecting a segment through a given line using a compass and straightedge.	
	ATTEMPT INCREASED DIFFICULTY: Reflecting quadrilaterals and polygons through a given line using a compass and protractor. Drawing own figures and then reflecting.	Worksheet

Skill: Using a protractor to construct an angle.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' PSYCHOMOTOR BEHAVIOR	MATERIALS AND RESOURCES
Teacher will demonstrate how to construct an angle using a protractor.	ATTEMPT AS A WHOLE: Constructing an angle using a protractor.	Protractors Worksheet
Teacher will give an angle measurement.	PRACTICE ISOLATED PARTS: 1.) Constructing a ray, 2.) positioning the protractor, 3.) locating and marking the correct measuring, and 4.) constructing the second ray to complete the angle.	
	PROVIDE FOR VARIATION: Constructing angles with different measurements, less than 90 degrees.	
	ATTEMPT INCREASED DIFFICULTY: Constructing angles with measures greater than 90 degrees.	

Skill: Locate the line of symmetry for symmetrical plane figures.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' PSYCHOMOTOR BEHAVIOR	MATERIALS AND RESOURCES
Teacher will demonstrate how to locate the line of symmetry for segments.	ATTEMPT AS A WHOLE: Drawing the line of symmetry for segments.	Ruler Overhead or chalkboard
Teacher will help students individually.	PRACTICE ISOLATED PARTS: 1.) Placing a ruler on the figure to locate pairs of points directly opposite each other. 2.) Locating the point halfway between each pair of points. 3.) Drawing the line of symmetry.	
	PROVIDE FOR VARIATION: Drawing the line of symmetry for a triangle, rectangle and square and angles.	
	ATTEMPT INCREASED DIFFICULTY: Drawing the line of symmetry for a pentagon, hexagon and octagon.	Worksheet

Skill: Locating slide images.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' PSYCHOMOTOR BEHAVIOR	MATERIALS AND RESOURCES
Teacher will locate the slide image of a point.	ATTEMPT AS A WHOLE: Locate the slide image of a point through two parallel lines.	Compass Ruler
	PRACTICE ISOLATED PARTS: 1.) Reflect the point through the first line. 2.) Reflect again through the second line.	
	PROVIDE FOR VARIATION: Locate the slide image of a segment, ray, line.	Worksheet
	ATTEMPT INCREASED DIFFICULTY: Locate the slide image of a triangle, rectangle, pentagon, hexagon.	

Skill: Locating rotation images through 2 intersecting lines.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' PSYCHOMOTOR BEHAVIOR	MATERIALS AND RESOURCES
Teacher will demonstrate how to locate the rotation image of a point.	ATTEMPT AS A WHOLE: Locate the rotation image of a point through two intersecting lines.	Ruler Mira, compass
	PRACTICE ISOLATED PARTS: 1.) Reflect the given point through the first line, 2.) reflect this point through the second line.	
	PROVIDE FOR VARIATION: Locate the rotation image of a line, ray, segment, triangle.	Worksheet
	ATTEMPT INCREASED DIFFICULTY: Use a compass to locate the rotation image of a given point or set of points.	

QUALITIES

1. Appreciation of modern methods in mathematics.

2. Appreciation of neatness in work.

3. Willingness to share equipment with others and help others.

4. Willingness to accept correctness.

5. Positive attitude toward courtesy.

6. Willingness to try new things.

7. Appreciation of the contribution each makes to the whole.

8. Respect for other people's property.

Affective Quality: Appreciation of modern methods in mathematics.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' AFFECTIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will demonstrate a construction with a collapsible compass.	RECEIVING: Students will observe the demonstration of the collapsible compass.	Collapsible compass, Non-collapsible compass and Straightedge
Teacher will ask the students what some of the problems are when using a collapsible compass.	RESPONDING: Students will state some problems encountered with collapsible compass.	
	VALUING: Students will tell some of the advantages of a modern non-collapsible compass in a brainstorm session.	
	ORGANIZING: Students will think of a new type of compass.	

Affective Quality: Appreciation of neatness in work.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' AFFECTIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will demonstrate a procedure to the class in a messy manner.	RECEIVING: Students will take notes on the procedures.	
Teacher will answer questions.	RESPONDING: Students will ask questions about the problem.	
	VALUING: Students will discuss the difficulty they had taking notes and difficulties they would encounter in attempting to duplicate the procedure.	
Teacher will suggest some areas if discussion lags.	ORGANIZING: Students will discuss areas where neatness is especially important.	

Affective Quality: Willingness to share equipment with others and help others.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' AFFECTIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will assign three short problems to do at the beginning of class which require the use of a mira and set a time limit. Students are to share equipment.	RECEIVING: Students will work on the problems.	Enough miras for only half of the class.
At the end of time, the teacher will collect the papers and then ask how the students thought they performed.	RESPONDING: Students will tell how they performed on the assignment.	
Begin a discussion on sharing equipment.	VALUING: Students will discuss why sharing equipment and helping others is important.	
	ORGANIZING: Students will compose a list of some occupations or areas where sharing is necessary and where helping improves situations.	

Affective Quality: Willingness to accept correctness.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' AFFECTIVE BEHAVIOR	MATERIALS AND RESOURCES
While doing a problem at the chalkboard, the teacher will make a deliberate and obvious mistake.	RECEIVING: Students will observe and take notes	
Teacher will not admit that a mistake has been made.	RESPONDING: Students will observe the mistake, point it out to the teacher, and attempt to correct the teacher.	
Teacher will admit the mistake after the students attempt to correct it several times.	VALUING: Students will discuss their reactions to the teacher's unwillingness to be corrected. Also, they will discuss the importance of being correct.	
	ORGANIZING: Students will share personal personal experiences where they have been sure that they were correct, but actually were wrong.	

Affective Quality: Positive attitude toward courtesy.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' AFFECTIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will begin class by being rude to students.	RECEIVING: Students will observe the teacher's actions and attitude.	
	RESPONDING: Students will state how they felt when the teacher was rude.	
Teacher will aid in the discussion.	VALUING: Students will discuss how other people might feel when they are rude to them.	
	ORGANIZING: Students will discuss why they should be courteous.	

Affective Quality: Willingness to try new things.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' AFFECTIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will tell students about a situation where a new thing took a long time to be tried and accepted.	RECEIVING: Students will listen.	
	RESPONDING: Students will ask questions and comment on the situation.	
	VALUING: Students will discuss why some people don't want to try something new.	
	ORGANIZING: Students will discuss what might happen if people stopped trying new things.	

Affective Quality: Appreciation of the contribution each makes to the whole.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' AFFECTIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will present an activity where each person in the group has something to contribute. (Puzzle and each one has one piece)	RECEIVING: Students will perform the activity in groups.	
	RESPONDING: Students will state their reactions to the activity.	
	VALUING: Students will discuss how they feel when they are able to contribute in the completion of a project.	
	ORGANIZING: Students will compose a list of situations where each person's contribution is necessary.	

Affective Quality: Respect for other people's property.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' AFFECTIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will pre-arrange with one student to break his pencil.	RECEIVING: Students will watch teacher break a student's pencil.	
	RESPONDING: Students will react verbally to this action.	
	VALUING: Students will discuss how they would feel if someone broke one of their belongings.	
	ORGANIZING: Students will compose a list of actions that show disrespect for property of others.	

Affective Quality: Willingness to come to class prepared.

TEACHER'S INSTRUCTIONAL INPUT	STUDENTS' AFFECTIVE BEHAVIOR	MATERIALS AND RESOURCES
Teacher will be unprepared for an activity (not enough worksheets, no equipment, unsure of directions).	RECEIVING: Students will wait for teacher to get ready.	
	RESPONDING: Students will voice their reactions.	
	VALUING: Students will discuss why people should be prepared.	
	ORGANIZING: Students will compose a list of their responsibilities as students.	

PRETEST

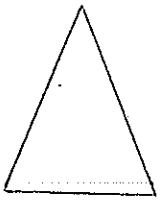
NAME _____
TRANSFORMATIONAL GEOMETRY
DATE _____

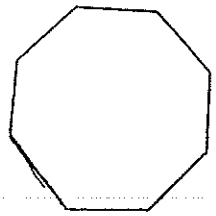
EOMETRIC TERMS AND FIGURES

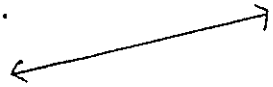
Match each term in Column A with its corresponding picture in Column B. Use each picture once.

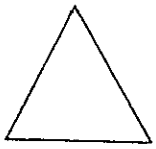
	COLUMN A
_____1.	point
_____2.	segment
_____3.	ray
_____4.	line
_____5.	acute angle
_____6.	obtuse angle
_____7.	right angle
_____8.	parallel lines
_____9.	perpendicular lines
____10.	intersecting lines
____11.	polygon
____12.	scalene triangle
____13.	isosceles triangle
____14.	equilateral triangle
____15.	right triangle
____16.	quadrilateral
____17.	parallelogram
____18.	rectangle
____19.	square
____20.	trapezoid
____21.	pentagon
____22.	hexagon
____23.	octagon
____24.	circle

COLUMN B

a. 

b. 

c. 

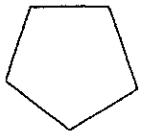
d. 

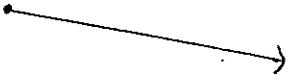
e. 

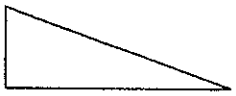
f. 

g. 

h. 

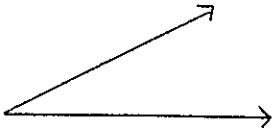
i. 

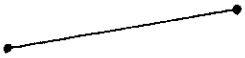
j. 

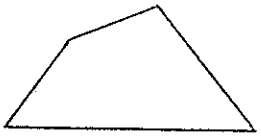
k. 

l. 

m. 

n. 

o. 

p. 

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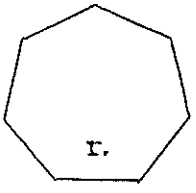
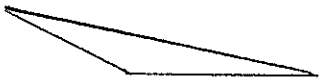
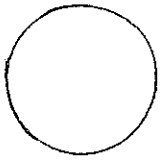
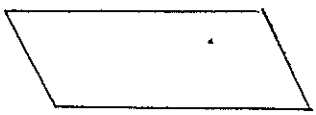
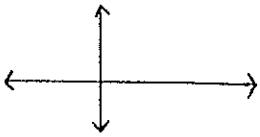
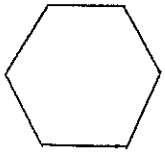
r. 

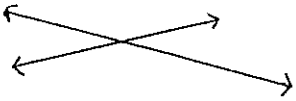
s. 

t. 

u. 

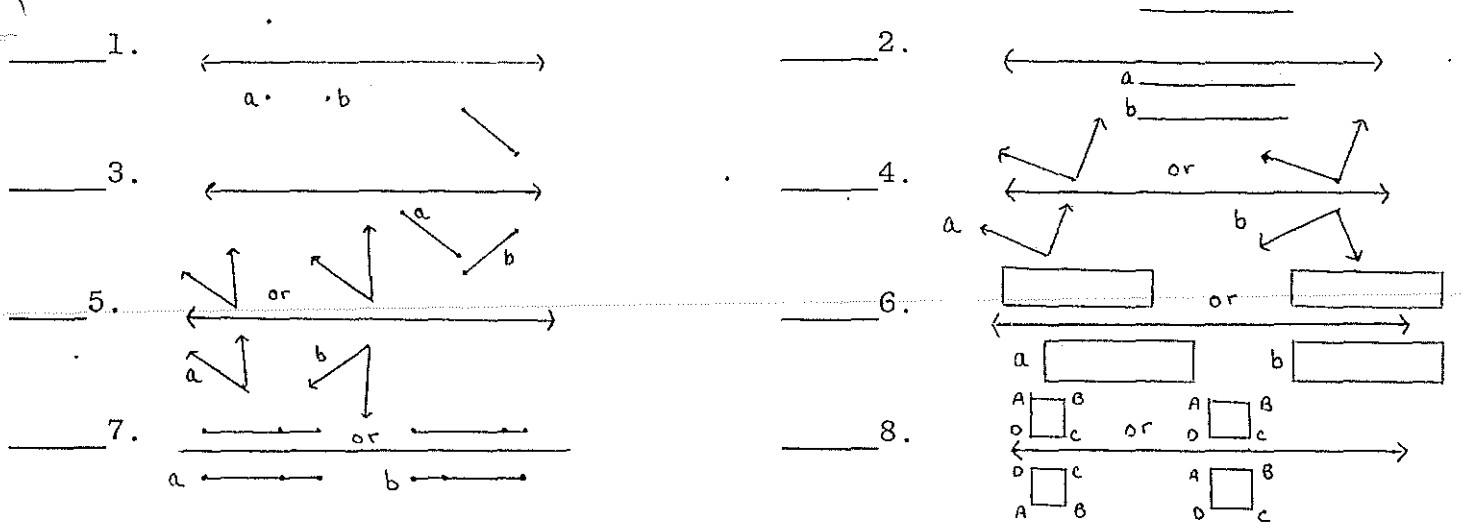
v. 

w. 

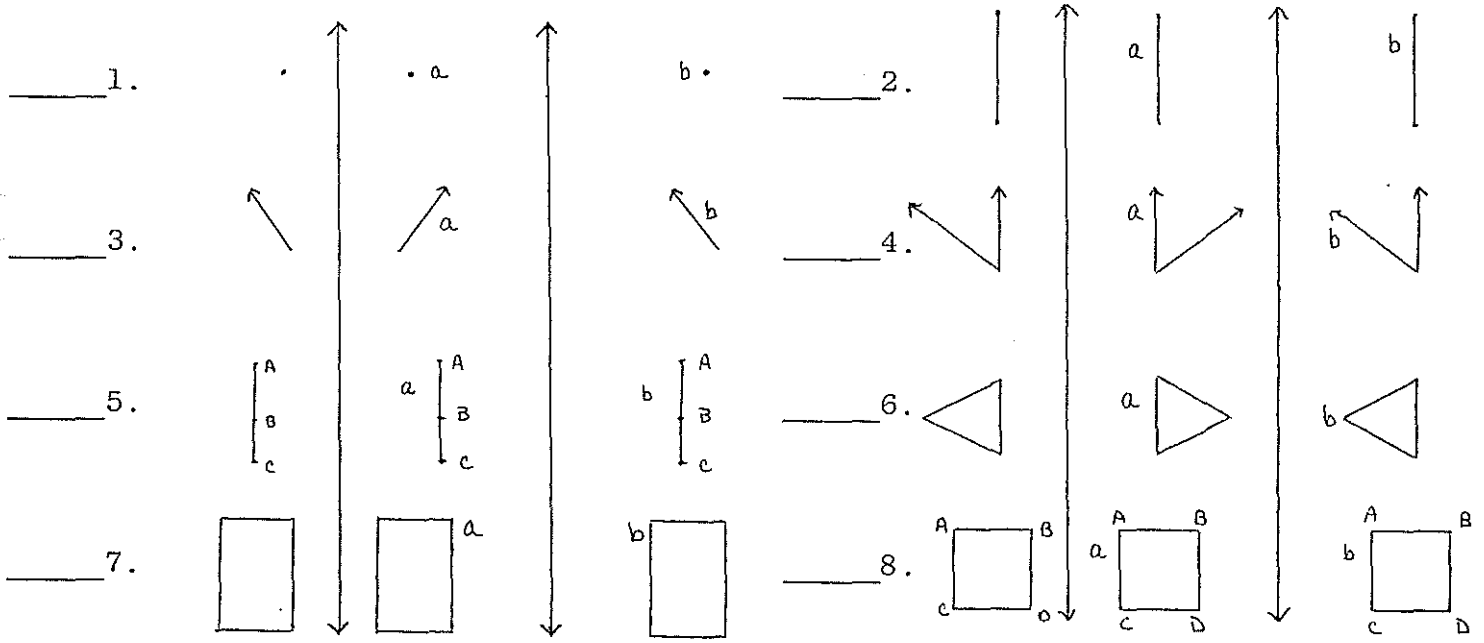
x. 

IMAGES

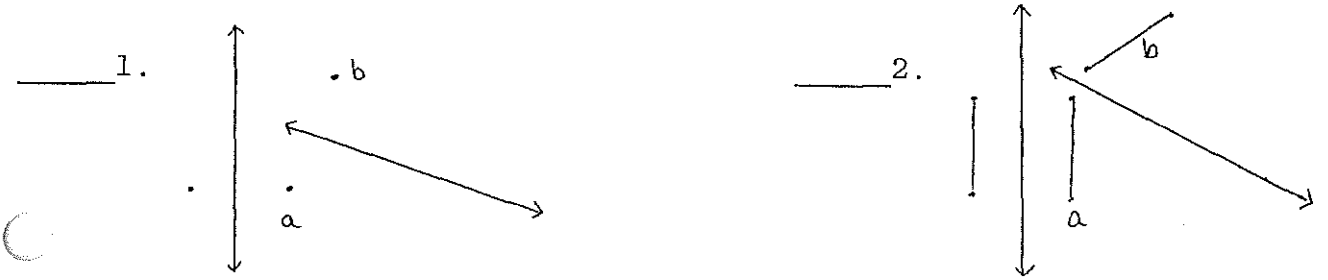
A. Select the correct reflection image in each diagram and write its corresponding letter in the blank.

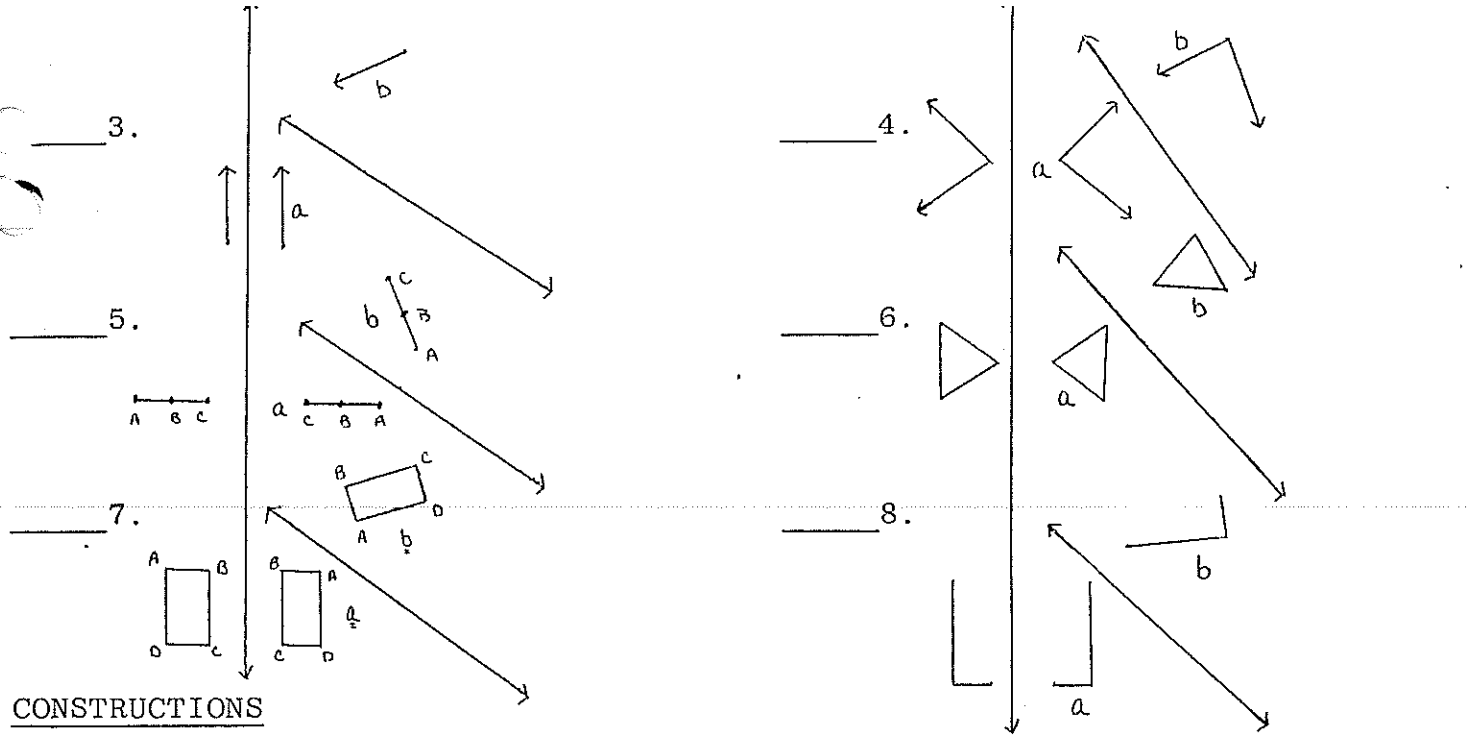


B. Select the correct slide image in each diagram and write its corresponding letter in the blank.



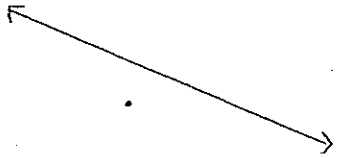
C. Select the correct rotation image in each diagram and write its corresponding letter in the blank.



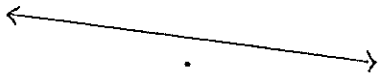


CONSTRUCTIONS

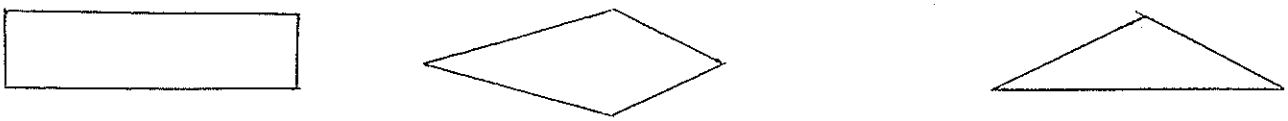
Use a compass and ruler to locate the reflection image of the given point.



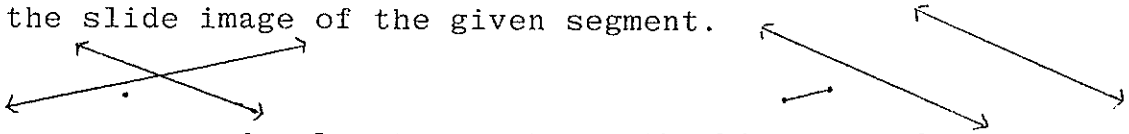
Use a protractor to locate the reflection image of the given point.



Use a ruler to locate and draw the line of symmetry for the figures.



Use a compass and ruler to locate the rotation image of the given point and the slide image of the given segment.



Use a compass and ruler to construct the bisector of the angle.

Use a compass and ruler to construct the perpendicular bisector of the segment.

TRANSFORMATIONAL GEOMETRY NAME_____

POST-TEST DATE_____

MEMORY

1. State how many reflection, slide, and rotation images that a point has.

2. List the reflection images of the following:

   a) point b) ray

3. List the slide images of the following:

   a) line b) segment

4. List the rotation images of the following:

   a) angle b) polygon

5. Name the center of rotation when you find the rotation image of a set of points.

TRANSLATION

6. In your own words describe the distance relation between:

   a) a point and its reflection image

   b) a point and its slide image

   c) a point and its rotation image

7. Describe the line of symmetry for a geometric figure.

INTERPRETATION

8. In finding the slide image of a segment which of the following conditions are necessary:

   a) the segment measures 5 centimeters.
   b) the segment is reflected through two or more parallel lines.
   c) the segment is reflected through two or more intersecting lines.

9. Arrange the following steps in correct order: for constructing an angle

_____place center of protractor on end point of ray

_____mark the location for the correct measure

_____construct a ray

_____locate the measure for the angle

_____construct a second ray with the same endpoint as the first

10. Arrange the following steps in correct order for finding the reflection image of a point through a line

_____cross the line twice using compass on same setting

_____place tip on one place of intersection and mark an arc

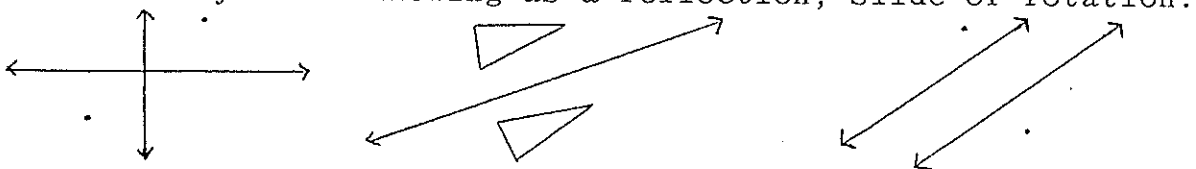
_____locate reflection images where arcs intersect

_____place tip of compass on the point

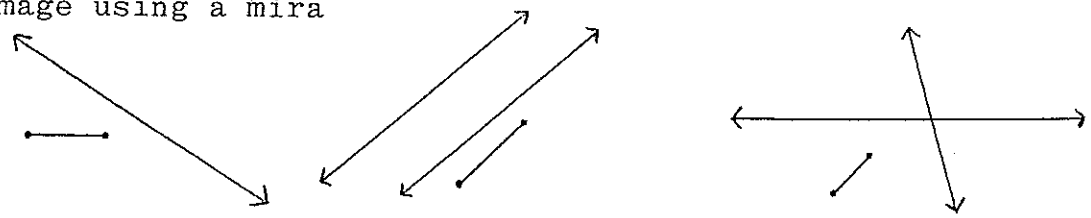
_____place tip on second place of intersection and make arc

APPLICATION

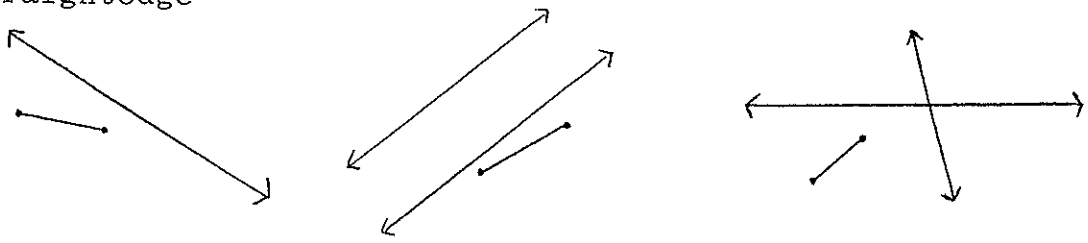
11. Classify the following as a reflection, slide or rotation.



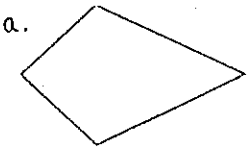
12. Construct the reflection image, slide image, and rotation image using a mira

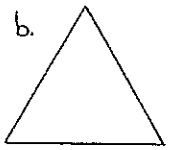


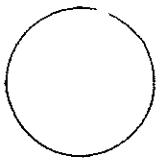
13. Follow the directions for #12 except use a compass and straightedge

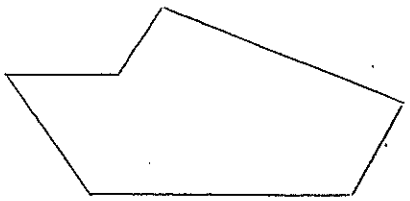


14. Select the symmetrical shapes.

a. 

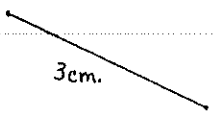
b. 

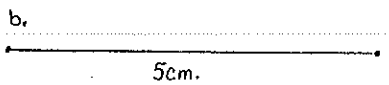
c. 

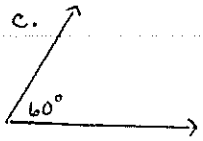
d. 

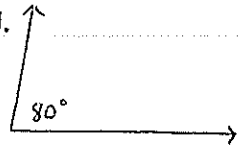
ANALYSIS

15. Calculate the measure of the reflection image, slide image, and rotation image for the following segments and angles.

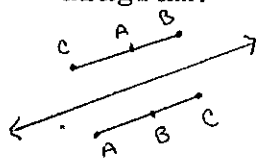
a. 

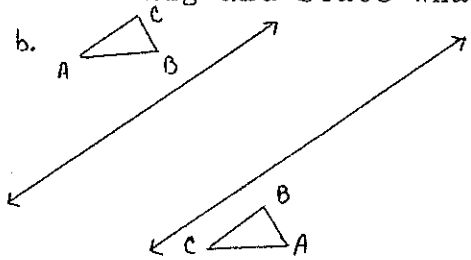
b. 

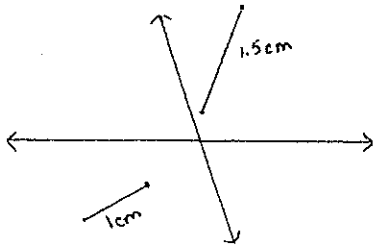
c. 

d. 

16. Examine the following and state what is wrong with each diagram.

a. 

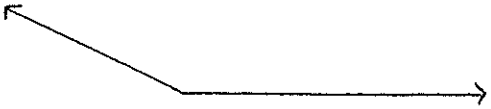
b. 

c. 

17. Construct an angle which measures 60 degrees.

SYNTHESIS

18. Construct the angle bisector for the given angle



19. Create a design using images of a triangle

EVALUATION

20. List the advantages and disadvantages of using a mira to locate a reflection image.

21. List the advantages of a non-collapsible compass over a collapsible compass.

NAME _____
TRANSFORMATIONAL GEOMETRY
DATE _____

MEMORY

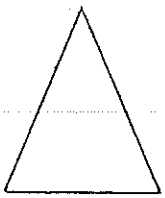
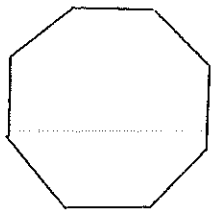
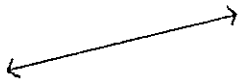
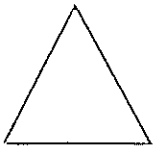
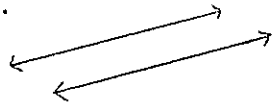
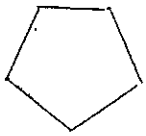
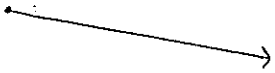
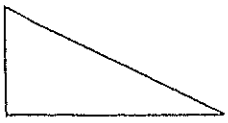
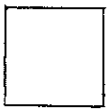
GEOMETRIC TERMS AND FIGURES

Match each term in Column A with its corresponding picture in Column B. Use each picture once.

COLUMN A

_____1. point
_____2. segment
_____3. ray
_____4. line
_____5. acute angle
_____6. obtuse angle
_____7. right angle
_____8. parallel lines
_____9. perpendicular lines
_____10. intersecting lines
_____11. polygon
_____12. scalene triangle
_____13. isosceles triangle
_____14. equilateral triangle
_____15. right triangle
_____16. quadrilateral
_____17. parallelogram
_____18. rectangle
_____19. square
_____20. trapezoid
_____21. pentagon
_____22. hexagon
_____23. octagon
_____24. circle

COLUMN B

a. 
b. 
c. 
d. 
e. 
f. 
g. 
h. 
i. 
j. 
k. 
l. 
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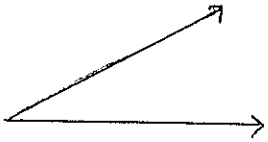
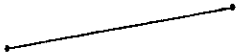
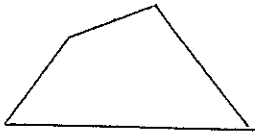
n. 
o. 
p. 
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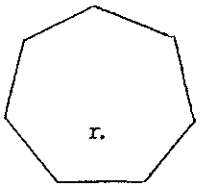
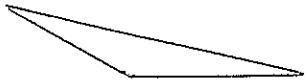
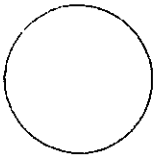
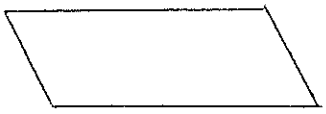
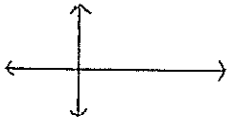
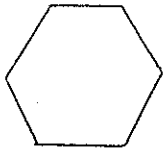
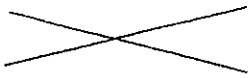
r. 
s. 
t. 
u. 
v. 
w. 
x. 

BEHAVIORAL OBJECTIVES

1. Students will be able to state the meaning of the reflection image of a geometric figure.

2. Students will be able to state the number of reflection images for a geometric figure.

3. Students will be able to state that the reflection of a reflection image is a given set of points.

4. Students will be able to state the procedure to reflect a set of points.

5. Students will be able to state the reflection image for a given geometric figure.

6. Students will be able to state the correct order of points for the reflection image of a geometric figure.

7. Students will be able to state the measure of the reflection image of an angle or segment.

8. Students will be able to correctly label the points for a reflection image.

9. Students will be able to define the line of symmetry of a plane figure.

10. Students will be able to state the procedure used to locate the slide image of a geometric figure.

11. Students will be able to state the procedure used to locate the rotation image of a geometric figure.

12. Students will be able to state the center of rotation for a given construction.

13. Students will be able to state the meaning of a half-turn.

14. Students will be able to state the results of slides and rotations of geometric figures.

15. Students will be able to state the measurements of slide and rotation images of given segments and angles.

16. Students will be able to state the order of points for slide and rotation images.

17. Students will be able to correctly label the points for slide and rotation images.

18. Students will be able to locate the bisector of an angle.

19. Students will be able to state three ways that a geometric object may move.

20. Students will be able to list examples of reflections, slides, and rotations that are seen outside the classroom.

21. Students will be able to locate reflection, slide, and rotation images of geometric figures by folding paper and tracing.

22. Students will be able to locate reflection, slide, and rotation images of geometric figures using a compass and straightedge.

23. Students will be able to construct an angle with a given measurement using a protractor.

24. Students will be able to locate the line of symmetry for symmetrical figures.