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OPTIMISING INVENTORY MANAGEMENT WITH PREPACKS

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Dissertation

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Biography and Acknowledgments

Ana Catarina Costa Azevedo was born in 27 of May of 1996 in Vila Nova de Famalicão, district of Braga.

In 2018, she finished her Bachelor degree in Management at Faculdade de Economia do Porto. In the same year, she had the opportunity to start her career as *Stock Analyst* at Sonae MC.

Furthermore, in 2019 she enrolled in the Master in Sales Management with the purpose of increasing and strengthen knowledge.

In 2021 she was promoted to *Category Planner* that is her current working responsibility.

The beautiful path I have travelled so far is the result of hard work, resilience and persistence, but I would not have been the same without those who were part of it and to whom I am sincerely grateful for being a crucial part of my life.

A special thanks to my family, Rui, Maria, Paulo and Florinda. All of you were the trampoline that helped me reach new highs. Thank you for the vital support and for believing in my work.

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Finally, to all of my friends, a thank you for all the patience and motivation given to make me keep on going.

Resumo

Atualmente, com o permanente desenvolvimento das empresas de retalho, a otimização das cadeias de abastecimento tem-se tornado numa preocupação constante. Devido à rápida rotação dos produtos, responder de forma rápida e lucrativa a estes problemas é uma exigência contínua.

É comum a mercadoria ser embalada em *prepacks* de forma a simplificar o seu manuseamento, especialmente na indústria têxtil, que será o foco do nosso estudo. Os *prepacks* são compostos por múltiplas unidades do mesmo *Stock Keeping Unit (SKU)* ou combinações *SKU's* diferentes.

Ao comprar *prepacks* em vez de *SKU's* individualmente, as empresas esperam melhorar o seu método de alocações, reduzindo assim os custos totais. Por outro lado, as cadeias de abastecimento necessitam de prestar especial atenção às ocorrências de discrepâncias de *stock* quando usam *prepacks*. Embora existam várias vantagens no seu uso, o facto de, tipicamente, ser o fornecedor a decidir a sua composição poderá ser um problema. Visto que, este pode não ter em conta os padrões de procura e/ou histórico de vendas do retalhista.

Deste modo, esta dissertação determina o plano ótimo de alocação de *prepacks*, garantindo a satisfação da procura de uma determinada empresa que foi objeto de estudo.

Assim sendo, a dissertação começa por usar um método estatístico para prever a procura que, posteriormente, é aplicada num modelo de otimização que foi formulado matematicamente e é apresentada uma nova solução de alocação da mercadoria.
Por conseguinte, é determinada a política de abastecimento que minimiza custos de manuseamento da mercadoria e custos de inventário e rutura, causados pela discrepância entre a procura e a oferta (custo stock, por excesso ou defeito).

Por fim, o método proposto é aplicado e validado comparando os resultados da política resultante da solução do referido modelo de otimização com a política atual da empresa. Analisando os resultados obtidos, conseguimos demonstrar a vantagem da nova política, dado que os custos totais são significativamente menores e a distribuição dos *stocks* é mais assertiva, i.e., está mais de acordo com a necessidade das lojas.

Palavras-chave: *Prepacks*, problema de alocação de *stock*, centro de distribuição, inventário, ruturas, previsão da procura.

Abstract

With retail companies becoming more sophisticated nowadays, optimising the supply chains is a current concern. Due to the fast rotation of goods, answering the demand quickly and profitably is essential.

Merchandise is commonly prepacked to simplify its handling, specifically in the fashion industry, which will be the focus of our study.
Prepacks are ordinarily composed of multiple units of the same Stock Keeping Unit (SKU) or combinations of different SKU's. By ordering prepacks rather than single SKU's, companies expect to improve their allocation method, thus reducing costs. On the other hand, supply chains need to pay special attention to stockout occurrences and inventory costs when using prepacks.

Although there are many advantages of using prepack allocation, the fact that, typically, the vendor decides on what constitutes a prepack may be a problem since such decisions may not consider the demand patterns and sales history of the specific retailer. Therefore, prepack allocation must be optimised in order to satisfy each store's forecast demand.

This dissertation starts by using statistical methods to forecast demand based on historical data. Then, the problem of determining the prepack allocation to each store, that is, how many prepacks of each type should be delivered to each store, is cast as a mathematical programming model.

The proposed optimisation model aims to minimise handling and inventory costs; the latter are due to mismatches between supply and demand (over and understocked items).

The proposed approach is applied and validated by comparing the policy resulting from solving the optimisation model with the current company policy.
The new policy allows for a significant reduction in the handling and inventory costs, as well as a drastic decrease in the number of stockout occurrences.

Keywords: Prepacks, allocation problem, distribution centre, inventory, stockout, demand forecast.

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1. Introduction

This dissertation introduces and analyses a new solution approach to optimise stock allocations for retailers. Moreover, the central objective is to develop a solution approach that finds an allocation plan that minimises total costs and helps to manage prepacks in our inventory.

Nowadays, retail companies are much more sophisticated, facing a variety of newness and challenges, which conducts to a significant concern with the optimisation of logistics costs and, consequently, financial costs (Mou, Robb, & DeHoratius, 2018).

Therefore, logistics impacts the companies, its products costs and the supply chain as a whole, becoming an efficiency focal point, particularly for retail companies (Wensing, Sternbeck, & Kuhn, 2018). Consequently, and due to the fast rotation of goods, retailers need to find a way to answer the continuous challenges of their companies, turning the entire supply chain into a process that is easier and cheaper.

Additionally, it is common to buy items in cases known as prepacks, which allows for easier handling. Prepacks are ordinarily prepared by the vendor and can consist of multiple units of the same Stock Keeping Unit (SKU) or combinations of different SKU's. SKU's are the lowest level of products in the supply chain, and they can differ in some attributes such as colour or size. By ordering prepacks rather than single SKU's, companies expect to improve their allocation method, thus reducing shipping and handling costs (Agrawal & Smith, 2019).

Although there are many advantages of using prepack allocation, the fact that, typically, the vendor decides on what constitutes a prepack may be a problem since such decisions may not consider the demand patterns and sales history of the specific retailer. For this reason, the downstream supply chain department must decide on the best way to allocate the prepacks, responding to the combined demand of several customers regarding the different items.

Hence, this is a very relevant topic as prepack ordering optimisation can improve items' availability and lead to inventory reductions as well as stockout reductions. From a sustainability point of view, these issues are even more focused and relevant because, in perishable products, a non-requested product (or at least not with the frequency assumed by the downstream) may end up going to be wasted.

Waste reduction has recently become the focus of attention since it not only implies the loss of the product and all costs assumed with having it ready for sale, but also, in many cases, implies additional costs to discard the products.

A sustainable supply chain addresses the incorporation of the triple bottom line, intending to balance the supply chain's different needs and goals, bearing in mind environmental, financial and social aspects, the three pillars of sustainability (Nabil, Afia, & Shihata, 2021). Inventory management of perishable products is a much higher challenge because of the distribution, lifetime (they lose their value in a short period) and customers' demand. When the lifetime ends, they must be discarded of waste or sold on sale, depending on the type of perishable product (food, pharmaceutical products, toxic items, or even fashion ones that become perishable at the end of their season). Our study will be focused on fashion items.

For this reason, the purpose of this study is to understand how inventory management can be optimised so that companies can better respond to demand, minimising inefficiencies, inventory levels, and stockouts occurrences, bearing in mind that companies cannot change existing prepacks. In other words, we will use a mixed-integer linear programming model to formulate the problem of allocating prepacks to a group of stores. Demand is an input that, based on historical data, is forecast by resorting to an econometric time series based method. Our model aims at minimising handling, inventory, and stockout costs.

Additionally, there is a personal motivation as the author working responsibilities are closely related to this topic. Therefore, for this study, we will use a real company problem. The validation of the new solution proposed will resort to a comparison with the retailer's current practice in terms of allocations coherence, inventory discrepancies, total costs and optimisation of the decisions.

This particular company is continuously suffering from poorly distributed stock, being some stores overstocked and others understocked at the same time. Also, the collaborators responsible for allocating the items are continuously suffering from a lack of information to make the decisions. The limitation of the company's allocation software requires prepacks to be manually managed, causing only single-size items to be automatically issued. So, although the focus of the study is optimising prepacks allocations (as it is currently a manual task), we will also evaluate the allocation of single-size items since they depend on each other.

The remaining of this document is divided into five chapters: the prepack optimisation problem, literature review, problem modelling and solving, case study, and conclusions and future work.

Chapter 2 presents the overview of all the supply chain moments that require optimisation due to uncertainties and various decisions, specifically with prepacks. This chapter describes the supply chain subsystems and how prepacks impact each one of them as well as the theoretical frameworks of inventory management, regarding different approaches and corresponding difficulties, based on the previous significant papers on the topic.

Chapter 3 - literature review focuses on the framework of the discussed matters, describes the most significant works and sources covering prepack allocation problems and concepts, and lists some literature gaps. Also, although store demand is not the focus of our study, it is impossible to optimise inventory management without considering the demand.

Chapter 4 provides a detailed description of the prepack allocation problem, followed by its mathematical formulation, then a general approach to how it will be solved.

Chapter 5 describes the case study and explains how data is collected and handled. Then, the solution approach is described: first, we use an econometric method to forecast the demand based on the historical data collected, and then we solve a mixed-integer linear programming model to find the minimum cost prepack allocation policy; this way improving some inventory management indicators. Finally, we demonstrate the effectiveness of the proposed approach by comparing it with the current company policy.

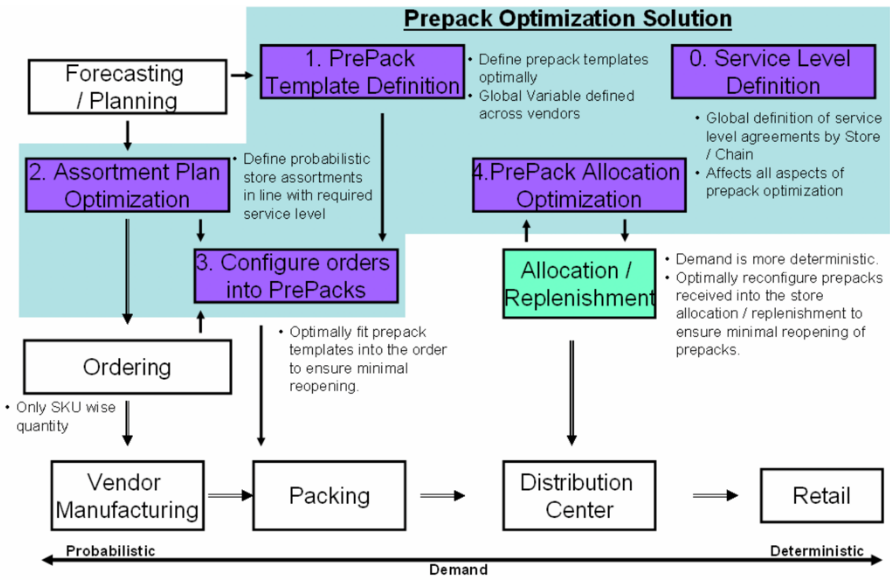
Finally, chapter 6 concludes the dissertation. After summarising the work proposed, its contributions and drawing some conclusions, we acknowledge some limitations and suggest future research directions.

2. Prepack Optimisation Problem

The prepack optimisation problem requires decisions and information at different moments of the supply chain. As figure 1 shows, one way to approach the problem is to define the prepack assortment, which, optimally, matches the store's demand and refines the prepack allocation, increasing efficiency and reducing the total cost.

There are many possibilities regarding the prepack optimisation problem involving a variety of decisions and uncertainty. So, it is essential to break the problem into smaller ones and follow step by step in order to achieve efficiency through the entire supply chain (Chettri & Sharma, 2012). Undoubtedly, the main goal of the supply chain is to be capable of answering the customer's (stores) demand on time at the lowest cost. Still, first, it is necessary to establish the desired service level for each customer. Thus, because the best way to satisfy store demand is to use prepacks, it is essential to consider prepack assortment, order configurations and allocation optimisation, which requires robust demand forecasting. These decisions are made in parallel with planning decisions between suppliers and retailers.

Figure 1 - Supply Chain Cycle



(Chettri & Sharma, 2012)

Therefore, our goal is to understand how inventory management can be optimised, using prepack allocation to each store, to respond better to demand, minimise inefficiencies, inventory costs and stockout occurrences, bearing in mind that companies cannot change existing prepacks.

2.1 Prepack Assortment

Prepacking consists of packing multiple units of the same SKU or combinations of related SKU's clubbed together (Agrawal & Smith, 2019) into bigger cases for easier handling and fewer expenses for the supply chain.

One major problem is deciding the optimal box configuration because retailers need to satisfy a particular demand, which includes responding to different necessities. So, understanding the best prepack composition can be a challenge since, if there are many different configurations of prepacks, the demand will be assured. Still, it will have a significant impact on operating costs. To reduce operation costs, bringing down the number of configurations is necessary, but this can cause overstocked or understocked items at the customers' premises (Hoskins et al., 2014). The former leads to increasing inventory costs and may also lead to waste and related costs. In contrast, the latter leads to lost sales and eventual consequences like customer willingness to accept a stockout.

Even though we assume that it is the vendor's responsibility to choose the prepack assortment, we consider that it is vital to understand the challenges of this topic because they will be reflected in the inventory management problem.

This subject has an important place in industrial applications, so it has been studied since the early 1960s. One of the problems mainly studied is the Bin Packing Problem, where items are aggregated and packed into a number of same size and shape cases (bins) to optimise transportation costs (Fischetti, Monaci, & Salvagnin, 2015). There are two versions of the problem: one-dimensional (Martello & Toth, 1990) and higher-dimensional (Lodi, Martello, & Monaci, 2002). The final goal is to achieve the minimum number of bins possible to pack all the items. These problems are frequently faced in engineering, logistics, and manufacturing processes.

An example of this is when an editor of a newspaper needs to fix articles dimensions into the pages regarding a layout (Dokeroglu & Cosar, 2014). When referring to shipping and transportation, the situation is identical. It is necessary to consider the packages heights and weights in order to load the minimum number of bins needed.

Furthermore, in the literature, the different inventory allocation applications are considered a prepacking problem because a distribution centre has to manage different customers, and therefore different ways of packing the bins. There so, it is essential to keep the number of varying box configurations under control to keep costs secured (Fischetti et al., 2015).

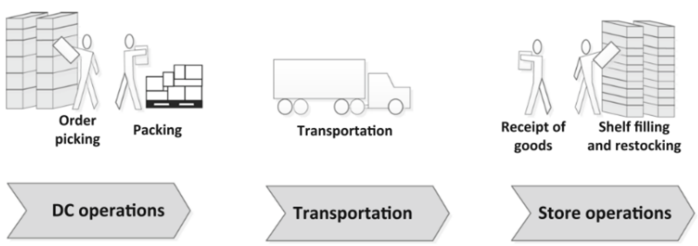
2.2 Prepack Allocation

The biggest challenge in a retail supply chain is to handle a vast number of orders, with different products to numerous customers. The difficulty is deciding how a vast number of SKU's will be distributed to a wide variety of stores.

Therefore, perishable items are usually shipped in prepacks due to their features, like a short life cycle; otherwise, the distribution centres could not have the required processing capacity as frequently as the requests; besides, the distribution costs would be prohibitive. Notwithstanding the previous comments, prepack allocation can lead to overstocked or understocked inventories. That is why optimising inventory management with prepacks is a dominant problem, which several retail companies are concerned about solving.

Hence, allocation implies product distribution that starts at a distribution centre (distribution centre) and ends at a store. According to (Sternbeck & Kuhn, 2014), this process includes three subsystems: distribution centre operations, transportation and store operations, as illustrated in figure 2. Consequently, it is vital to explore these subsystems in order to understand the impact of prepacks on each one of them.

Figure 2 - Subsystems of retail networks



Sternbeck & Kuhn (2014)

2.2.1 Distribution Centre Operations

Higginson & Bookbinder (2005) referred to distribution centres as facilities that "accumulate and consolidate products from various points of manufacture within a single firm, or several firms, for combined shipment to common customers". Here are all processes of goods reception after purchase orders are made, such as storage, picking, packing and shipping, which are also the cost drivers of this subsystem. It is known that prepacks influence picking and packing processes due to their dimensions and composition.

The picking process is usually considered the activity that costs the most (Koster, Le-Duc, & Roodbergen, 2007). Thus, it is essential to have prepacks since they influence the number of cases picked at distribution centres, which reflects directly on their picking costs. Overall, if distribution centres have most of the goods into prepacks, they have fewer cases to handle and lower picking costs.

2.2.2 Transportation

When referring to transportation, as figure 2 shows, it includes all the necessary processes to ship goods from distribution centres to stores. It encompasses processes like defining routes, delivery schedules, lead times, frequency of deliveries, and others.

Once more, it is expected that prepacks influence this subsystem because it impacts the vehicle's capacity in terms of space and frequency of deliveries. While working with prepacks, it is possible to accommodate the goods better and fulfil store demand using fewer deliveries. However, some authors proved that the modification of prepacks does not affect transportation costs (Wensing et al., 2018).

2.2.3 Store Operations

Store operations are also an essential subsystem of the entire logistics; nearly 50% of the logistics costs are due to all the manual work included in filling shelves (Sternbeck & Kuhn, 2014). The included activities are installing pallets and cases, unpacking the goods, identifying SKU's, filling and refilling shelves, and keeping the store organized and always stocked to respond to customer demand.

If companies use prepacks, they will have less frequent deliveries because they allow more SKU's to be packed in the same delivery, which will also impact store operators' time on unpacking. Then, it will affect handling costs regarding the decrease of operations complexity, which can transfer labour from backroom to instore operation, like giving attention to customers and keeping shelves full to attend better to demand (Wensing et al., 2018).

Our study will focus on allocation optimisation by knowing all the necessary decisions regarding the entire supply chain and breaking the problem into smaller ones. In fact, we will present a better approach to decide how many prepacks need to be allocated to each store, considering the demand and assuming costs minimisation.

3. Literature Review

Here, the theoretical framework of the matters being discussed are explained. The prepack allocation problem is issued, followed by its determinants and description of the recent relevant studies in the literature.

Additionally, we present a summary of some relevant literature about store demand because, although it is not the study's central question, it is pertinent to its solution.

3.1 Prepack Allocation Framework

There are two common ways to allocate merchandise to the stores: prepacks or single-size items.

A prepack contains a specific number of units of the same SKU or combinations of similar ones differentiated by size, colour, typology, etc. It is usually conceived by the vendors, with the concern that it needs to have an acceptable dimension to be sent to stores, most of the time, without distribution centre manipulation. A distribution centre manipulation does not exist when the boxes are received and shipped to the stores without being opened or changed.

A single-size item or a SKU, as the name implies, is the lowest level of products in the supply chain and, for instance, in the fashion industry, is differentiated by size. Additionally, when the purchase order is placed, the SKU's that were not bought as prepacks are received in master cases of single-size items. There is no concern about the number of items inside the master case because they are not sent to stores without manipulation.

To summarising, when a distribution centre picker[1] receives a store's order, he can send prepacks or single-size items. So, the challenge is to find the balance between the two.

Moreover, to decide on the allocation quantities, we need to consider the store's predicted demand. Sending a prepack could ensure the store demand of a size while over or underfulfil the demand of another. However, it may not be cost-effective, or even possible, to deliver every size demanded. Therefore, a lower limit of the supply of each size to each store is often imposed by authors (Hoskins et al., 2014; Wang, 2012). Note that we will not use

[1] Person who has the job to pull select items from storage and ready them for shipment, at the distribution centre.

this limitation in this study because the company that is the object of study is not just a textile retailer. Due to that, the distribution centre can mix other types of products in the same shipment, minimising the total costs involved.

As previously referred, most of the fashion apparel items become perishable. In other words, they have a specific seasonality, depending on the product (a puffer has a different selling season from a t-shirt). The decision process is based on this seasonality and, because of the long lead time (usually six months) associated with the textile industry, the retailers have a pre-season period of time, where all the processes before allocation are guaranteed, such as demand patterns, prepack assortment and purchased orders to the vendor. Additionally, in-season planning begins once the distribution centre receives the merchandise from the vendor. During this period, prepacks, or single-size items, need to be allocated to each store.

The two different methods have distinct implications, which is why it is so important to optimise the allocation problem. Focusing on minimising the total involved costs, such as handling fees at the distribution centre, at the stores and also the penalty costs of both over and understocking.

A further primary reason to have such concern about optimising allocation has to do with the fact that, usually, the forecast demand does not coincide with the actual demand, due to the difference between the time that is forecast (at the pre-season plan) and the actual sales of the item.

3.2 Allocation Problem Approaches

Since the prepack allocation problem is a decision-making process intrinsically complex, it is imperative to review the literature that focuses on and tries to solve some of the optimisation problems.

Although the academic relevance of this topic only started to appear in 2012, previous studies can be related to the prepack allocation problem, such as the knapsack problem. The issue concerns combinatorial optimisation, where the example of someone climbing a mountain with a knapsack is often given. The goal is to choose a set of items, each one with a weight and a value (in this case, nutritional value), in a way that the total weight is less than or equal to the limit, maximising the value (Martello & Toth, 1990).

Chettri & Sharma (2012) first introduced the overall decision-making process in prepack allocation to stores. They defended that we need to have an assortment plan where we take the critical decision whether to ship all the goods in prepacks or ship some individual units. Hence, when prepacks arrive at the distribution centre, the companies test the precision of the assortment plan. Then they start to decide the accurate allocation of the prepacks according to the store demand.

The best result is when prepacks are used as far as possible without under/overstocking stores. The authors mentioned above presented a second option, considering handling costs when the assortment plan is not perfect: opening prepacks and shipping individual units. However, this alternative is not often studied because it is not an option at many distribution centres due to high handling costs and working hours.

Here, the authors presented a problem formulation composed of an optimisation problem to feedbacking outputs of prepacks allocations to all the stores, coming from one distribution centre. Hence, they did not present any results to assume conclusions or limitations.

Wang (2012) determined a model for prepacks allocation to respond to each store's demand, which minimises handling costs and discrepancies between supply and demand, presuming that manufacturers provide the prepack configurations and an under and over-limit of allocation is set. An integer programming problem was formulated, but it was not adequate to solve the problem. Therefore, the author presented a dynamic programming to help understand the complexity of the matter.

Hence, two heuristic methods were proposed. The first one was a naive heuristic developed to treat each store individually. Secondly, a hierarchical decomposition heuristic divided the prepack allocation problem into smaller ones.

This study was based on the textile industry, with non-real-world data, so the information of each section was generated, such as the number of stores, sizes, prepack configurations, costs, demand, and others, which the author points out as a problem, affecting the quality of the solution.

Hoskins et al. (2014) considered the prepack assortment as fixed. They limited the overstocking value since they assumed that suppliers view the understocking as a decrease of customer value but, at the same time, overstocking causes incremental costs.

Consequently, they presented three approaches to the problem (with real-world data) and considered several prepack configurations, where the difference between the items was in colour. Also, the allocation capacities are restricted since the number of prepacks by each store is limited. First, constraint programming was tested. Nevertheless, it was only able to provide solutions when understocking was not requested. Then they formulated a mixed-integer program that performed with optimal results only in prepacks of one colour. Finally, the hybrid metaheuristics responded to all the scenarios and resulted in a better short-time solution. Nevertheless, the constraint and mixed-integer programs had better results because the hybrid metaheuristics used generated data.

Furthermore, Fischetti et al. (2015) introduced different prepacks combinations, mixing colours and sizes, also with a fixed limit of under and overstocking and with generated data. They presented metaheuristics approaches, where substructure problems were identified and fixed a subset of variables to solve the prepack allocation problem. For instance, they realized that the bigger the number of stores, the longer the processing time of the computational experiments. So, they first solved the problem for a subset of stores and then, stores were iteratively added.

Wensing et al. (2018) applied a Markov chain and a heuristic solution to approach the inventory model. They considered multiple periods of deliveries instead of static allocations to be closer in time to demand (most of the previous studies used stationary cycles), considering the process of filling shelves and assuming that shelf space is limited. They first tested with artificial data and then with real-world data. Also, it quantifies all the costs through the entire supply chain (considering the three subsystems previously mentioned). Finally, they developed an analytical formula and a non-linear optimisation procedure that shows that using only one type of prepacks for all stores is the best choice for cost reductions.

Sung & Jang (2018) studied the Assortment Packing and Distribution Problem (APDP), following previous studies of Hoskins et al. (2014) & Fischetti et al. (2015). Nevertheless, they based their study on an actual case study of a fashion brand. They proposed an NP-hard algorithm for optimising the model but, due to a large number of possibilities, they divided the problem into two phases: a computational algorithm to reduce the number of configurations; a mathematical model to find the real solution in the reduced number of options. Here, was set a limit of two prepacks allocated to each store. This decision was made to simplify the problem and, although the algorithm can be used, they still failed to prove its optimality condition.

Finally, Agrawal and Smith (2019) presented the prepack allocation problem using different combinations of prepacks that could be used at various retailing businesses, considering backorders. This could be an impediment at fashion distribution because most retailers place the order by planning season, which means that no backorder can happen, so it is a comprehensive limitation of the study. They solved, with non-real-world data, a Markov decision process using a stochastic dynamic program that was combined with a steady case to decipher the optimal allocation for finite time horizons. Hence, it was proved that it is more advantageous to ship prepacks rather than single units. Nevertheless, it is better to use different prepacks to respond to divergent demands regarding various stores, but this is, as presented before, a costly alternative.

Overall, many authors study the allocation problem because it has a crucial impact on the supply chain. Plus, it is a complicated question, depending on many other variables. That is why we found this topic so significant and current to address.

3.3 Literature Gap

Attending to all the reviewed literature, not many authors had the opportunity to study the problem using real-world data, except Hoskins et al., 2014; Sung & Jang, 2018; Wensing et al., 2018. In our study, we will use real-world data, which is a massive benefit for the quality of the solution.

Hence, as previously referred, it is common to impose a lower or even upper limit of how many prepacks and single-sizes can be shipped to each store (Fischetti et al., 2015; Hoskins et al., 2014; Sung & Jang, 2018; Wang, 2012). However, we will not impose this limit since the studied company is not exclusively a textile retailer; thus, the distribution centre can mix other company products in the same shipment.

Furthermore, some studies considered mixed-integer or integer programming models, but none achieved the expected results (Hoskins et al., 2014; Wang, 2012), and they had to resort to other methods.

Additionally, Chettri & Sharma (2012) considered handling costs of manipulating the goods when the assortment plan is not perfect: opening prepacks and shipping individual units. However, this alternative is not often studied because it is not an option at many distribution centres due to high handling costs and working hours.

According to the literature review discussed, we are basing our case study on the textile industry, as well as Agrawal & Smith (2019); Wang (2012). Nevertheless, we are using real-world data, not establishing a lower and upper limit of merchandise shipped to stores nor backorders (due to the long lead time of the fashion industry).

Also, the alternative of opening both prepacks and single-size items (Chettri & Sharma (2012)) is not considered. We only assume single-size items manipulation because they are received inside master cases and sent individually. Still, prepacks are never opened to be sent in individual units, in our study.

The allocations are divided and fixed into four quarters, but the decision for all quarters is made at the beginning of the year, and not over the course of time, not following a timeline as quarters go by, unlike (Wensing et al., 2018).

Overall, mixed integer programming is contemplated to achieve the intended results of allocating a mix of prepacks and single-size items to all the stores to minimise the total costs. Considering all the previously presented features, it is the first time this topic has been approached under these circumstances.

3.4 Store Demand Framework

Although the focus of our study is not to forecast demand, it is not possible to optimise inventory management without considering the demand since the best way to optimise allocation is to have supply following demand closely.

Thus, the fashion industry is distinguished by a short product life cycle and unpredictable demand (Choi, 2018; Soni & Kodali, 2010), where fashion products are characterised by some features (Bruce, Daly, & Towers, 2004):

- Short lifecycle: it is expected that the period in which the product will be saleable is brief, most likely weeks or at most a few months (Lee, 2003);
- High volatility: demand can be influenced by a diversity of facts, such as weather, public figures, influencers, and others;
- High impulse purchase: the consumer can be stimulated to buy the product when is confronted with it;
- Low predictability.

Due to the importance that demand forecast has in supply chain management and business strategy (Guo, Wong, & Li, 2013; Smith & Mentzer, 2010), there are many studies about the complexity of this problem.

Nenni, Giustiniano, & Pirolo (2013) added the product variety as a feature because the offer to the consumer is increasing, which turns the consumer more selective, and the demand even more challenging to predict.

Fashion retailers have different approaches. Most of them adopt traditional statistical methods, but there are new methods as Artificial Intelligence (AI) and even hybrid ones (which combine statistical and AI techniques (Ren, Chan, & Siqin, 2020). Statistical methods are used more often because they are easier to operate and implement, faster to process, and, finally, their form is specific (like a graphic or a table), making the combination with other business operations straightforward.

Mostard, Teunter, & De Koster (2011) presented a comparison study to predict single-period products. They analysed many SKU's of an apparel company and compared the validity of three quantitative forecasting methods based on the advance order data that allowed customers to pre-order (due to the lack of historical demand data).

Later, Yelland & Dong (2014) applied a Bayesian forecasting model and found that it offers better quantitative results than other methods of forecasting fashion demand.

Ren, Choi, & Liu (2015) employed a panel data forecasting model to predict fashion items sales. They took into consideration related items and the respective prices. They proved that this model has advantages over traditional time series models since it enables the correlation of three different demand impact factors in the same period (time-series trend of previous sales; forecasts of items prices; effects from correlations of items).

Kuo & Tang (2015) presented the Box-Jenkins methodology as the best framework regarding stationary data. It applies autoregressive moving average (ARMA), autoregressive integrated moving average (ARIMA), or seasonal autoregressive integrated moving average (SARIMA) models to find the best fit of a time-series model to a historical data time series. The ARMA model (AR – autoregressive; MA – moving average) adds, to the prediction equation, lags of the differenced series and the forecast errors.

ARIMA model has shown to have a better forecast than other time-series approaches and is the best at analysing the time series in the time domain instead of the frequency domain. One of its attractive features is that ARIMA processes have a large scale of models. It is usually possible to find the process that provides an adequate description of the data.

Although statistical methods are more straightforward and faster, they depend on historical sales data, which can be acceptable for raw data. Nevertheless, with the increase of information (the significant data era), it is necessary to evaluate new methods to forecast demand.

With the advance of data and technologies, artificial intelligence (AI) emerges as an adequate alternative method. A frequent AI method is an Artificial Neural Network (ANN), which can provide forecasting results with high accuracy. Notwithstanding, it consumes substantial processing time (Li & Lim, 2018). Even though AI models present advantages as forecasting models, they still do not solve the problem of big data. This limitation calls for new methods like hybrid models, which combine the advantages of statistical and AI methods (Ren et al., 2020).

Ren et al. (2015) used statistical methods to forecast linear components and AI methods to non-linear ones. Hybrid models, compared to statistical and AI ones, showed propitious forecasting performances.

Overall, it is essential to retain that there is no perfect way to predict demand due to all the previously referred features. Still, companies are always trying to find the best approaches for their products, continually seeking supply chain improvement.

Since we are not dealing with big data in this study, we will use a statistical method, more precisely, the Box Jenkins methodology, to forecast the demand applied to solve the optimisation model that will solve the problem being studied.

4. Problem Modeling and Solving

This section presents a detailed description of the problem followed by its mathematical formulation and a general approach to how it will be solved. Therefore, a straightforward method to expose the prepack allocation problem is a mixed-integer linear programming model (MILP) that consists of a specific case of a linear program in which some of the primary decision variables can only take integer values (Richards & How, 2005)

4.1 Detail Description and Formulation

In the specific problem being addressed, there is a set N of n stores, which we wish to allocate with a single item as a set S of s different sizes at a minimum cost, where a single distribution centre is used to fulfil all the stores.

There are two types of cases at the distribution centre, with fixed quantities, containing the items, namely P prepacks and S single-size items.

Furthermore, the prepack content is known. That is, each prepack has q_{ik} items of size k, $i \in P$ and $k \in S$. While each single-size item $k \in S$ has b_k units of the corresponding size inside a master case.

The merchandise allocation to the stores is made by shipping prepacks and/or a number of single-size items. The latter implies that the master case of the corresponding size is opened at the distribution centre, and the required number of single-size items is shipped. Since doing so, in addition to increasing costs, defeats the purpose of using prepacks, that is why only a prespecified percentage of the store delivery may be used to this strategy.

As previously studied, we wish to minimise the allocation costs, including handling prepacks and single-size items at the distribution centre and at stores, as well as costs associated with overstock and understock. The formers are typically related to perishable products at the end of their life cycle, or in the case of the textile industry, at the end of the season. At the same time, the latter is related to lost sales and customer satisfaction and willingness of coming back.

Before presenting the proposed model, let us summarise the notation used:

Sets and indices:

P – set of p prepacks, indexed by i;

S – set of s sizes, indexed by k;

N – set of n stores, indexed by j.

Parameters:

Q_i - quantity of prepacks of type i, available at the distribution centre, $i \in P$;

q_{ik} - quantity of items of size k in prepack of type i, $i \in P$ and $k \in S$ (note that not all prepacks need to have items of all sizes, therefore, q_{ik} may be zero for some k);

B_k - number of master cases of size k, $k \in S$;

b_k - number of items of size k in a master case, $k \in S$;

d_{kj} - demand for items of size k at store j, $k \in S$ and $j \in N$;

h_i^w - cost of handling the prepack of type i at the distribution centre, $i \in P$;

h_{ij}^s - cost of handling the prepack of type i at store j, $i \in P$ and $j \in N$;

h_k^{wv} - variable cost of handling a size k at the distribution centre, $k \in S$;

h_k^{wf} - fixed cost of handling a size k at the distribution centre, $k \in S$;

h_{kj}^{sv} - variable cost of handling a size k at store j, $k \in S$ and $j \in N$;

h_{kj}^{sf} - fixed cost of handling a size k at store j, $k \in S$ and $j \in N$;

ω_{kj} - cost of over ordering a size k at store j, the overstocked inventory cost, $k \in S$, $j \in N$;

φ_{kj} - cost of under ordering a size k at store j, the understocked inventory cost, $k \in S$, $j \in N$.

Variables:

1. **Main Decision:**

x_{ij} - number of prepacks of type i sent to store j, $i \in P$ and $j \in N$;

v_{kj} - number of items of size k sent individually (as non-prepack, we will call it, single-size items) to store j, $k \in S$ and $j \in N$;

2. **Auxiliary:**

μ_{kj} - number of understocked items of size k in store j, $k \in S$ and $j \in N$;

θ_{kj} - number of overstocked items of size *k* in store *j*, *k*∈*S* and *j*∈*N;*

γ_k - binary variable set to 1 if there are single items of size *k* shipped and 0, otherwise, *k*∈*S;*

τ_{kj} - binary variable set to 1 if store *j* receives the size *k* and 0, otherwise, *k*∈*S* and *j*∈*N*

At this point, with all the provided information, the prepack allocation problem can be defined as follow:

Minimise

$$\sum_{k\in S}\sum_{j\in N}\omega_{kj}\,\theta_{kj}+\sum_{k\in S}\sum_{j\in N}\varphi_{kj}\,\mu_{kj}+$$
$$\sum_{i\in P}(\,h_i^{w}\sum_{j\in N}x_{ij})+\sum_{k\in S}\sum_{j\in N}(h_k^{wf}\gamma_k+\,h_k^{wv}v_{kj})+$$
$$\sum_{i\in P}\sum_{\in N}h_{ij}^{s}\,x_{ij}+\sum_{k\in S}\sum_{j\in N}(h_{kj}^{sf}\tau_{kj}+\,h_{kj}^{sv}v_{kj})$$

Subject to

$$\sum_{j\in N}x_{ij}\leq Q_i,\ \forall_{i\in \mathrm{P}} \tag{4.1}$$

$$\sum_{j\in N}v_{kj}\leq B_k b_k,\ \forall_{k\in \mathrm{S}} \tag{4.2}$$

$$\mu_{kj}\geq d_{kj}-(\sum_{i\in P}q_{ik}\,x_{ij}+v_{kj}),\ \forall_{k\in \mathrm{S}},\forall_{j\in \mathrm{N}} \tag{4.3}$$

$$\theta_{kj}\geq(\sum_{i\in P}q_{ik}\,x_{ij}+v_{kj})-\ d_{kj},\ \forall_{k\in \mathrm{S}},\forall_{j\in \mathrm{N}} \tag{4.4}$$

$$\sum_{j\in N}v_{kj}\leq \mathrm{M}\gamma_k,\ \forall_{k\in \mathrm{S}} \tag{4.5}$$

$$\sum_{j\in N}v_{kj}\geq \mathrm{M}\,\tau_{kj},\ \forall_{k\in \mathrm{S}},\forall_{j\in \mathrm{N}} \tag{4.6}$$

$$x_{ij},v_{kj},\mu_{kj},\theta_{kj}\geq 0\ \forall_{\mathrm{i\in P}},\forall_{j\in \mathrm{N}}\forall_{k\in \mathrm{S}} \tag{4.7}$$

$$\mu_{kj},\theta_{kj}\geq 0,\forall_{k\in \mathrm{S}},\forall_{j\in \mathrm{N}} \tag{4.8}$$

$$x_j,v_{kj}\geq 0\ \text{and integers},\ \forall_{k\in \mathrm{S}},\forall_{j\in \mathrm{N}} \tag{4.9}$$

$$\gamma_k,\tau_{kj}:\text{binary},\ \forall_{k\in \mathrm{S}},\forall_{j\in \mathrm{N}} \tag{4.10}$$

The objective function specifies all the costs that the model will minimise. $\sum_{k\in S}\sum_{j\in N}\omega_{kj}\,\theta_{kj}$ and $\sum_{k\in S}\sum_{j\in N}\varphi_{kj}\,\mu_{kj}$ are the penalty costs of over or under ordering each size at each store, summed across all sizes stores, respectively.

At the distribution centre level, $\sum_{i\in P}(\,h_i^{w}\sum_{j\in N}x_{ij})$ specify the handling costs associated with all the prepacks sent to all stores. On the other hand, $\sum_{k\in S}(h_k^{wf}\gamma_k+\,h_k^{wv}v_{kj})$, represents the handling costs of sending all the single-size items to all stores, bearing in mind that can exist fixed (h_k^{wf}) and variable (h_k^{wv}) costs associated.

At the store level, the logic is similar. Therefore, $\sum_{i \in P} \sum_{\in N} h_{ij}^s \, x_{ij}$ are the handling costs of receiving all the prepacks at all stores and $\sum_{j \in N}(h_k^{sf} \, \tau_{kj} + h_k^{sv} v_{kj})$ specify the handling costs of all the single-size items allocated to all stores, taking into consideration fixed (h_k^{sf}) and variable (h_k^{sv}) costs.

Regarding the constraints, (4.1) explains that for each prepack type *i*, the number of prepacks sent to the stores cannot exceed more than the available at the distribution centre. Constraint (4.2) restricts that the number of items of size *k* sent to the stores cannot be more than the ones at the distribution centre.

In addition, (4.3) says that the number of understocked items of size *k* at the stores is greater than, or equal to, the differences between the demand of size *k* at the stores and the number of size *k* received. Otherwise, (4.4) explains that the number of overstocked items of size *k* at the stores is greater than or equal to the differences between the number of size *k* received and their demand.

Constraints (4.5) and (4.6) ensure that when the distribution centre ships a single item of size k (γ_k) and sent to the stores (τ_{kj}), respectively, the binary variables turn 1.

4.2 Problem Approach

To solve the specific problem being addressed, we will use real-world data from one particular retail company with a selected group of stores. We will gather stock information, the number of items allocated to each store, the assortment of cases, and cost parameters for 2020 concerning the current company policy.

Additionally, we will collect the sales information for the period of time of two years (2019 and 2020) so that we can use the first-year data to forecast the demand of 2020, applying an econometric time series based method since we do not have demand details.

Furthermore, the sales of 2020 are going to be used as the demand of the current policy. Then, the company's total costs of the decisions already registered will be calculated for each quarter of 2020.

The forecast results will be considered as the demand for the new policy following an optimisation model. For this reason, and using a specific software called CPLEX, we will determine a new optimised allocating approach for each quarter of the year of 2020.

Finally, we want to demonstrate the model's effectiveness by comparing its decisions with the current company policy. We will study stock analysis, where we will see the differences between the total quantity of understocked and overstocked items, the total costs and the proportion of the number prepacks versus the number of single-size items allocated in each quarter.

5. Case Study

In order to answer the central questions raised during this dissertation, we start by giving a framework of the company that is the object of study. Secondly, we collect and process all the necessary information to study and evaluate the problem. Additionally, we use an econometric time series based method to forecast the demand, and then an optimisation model, which is solved by an off the shelf software, to determine an optimal prepacks allocation policy for each quarter of the year. Finally, we demonstrate the model's effectiveness by comparing the approach prescribed by the model decisions with the company current policy for a sample period of one year.

5.1 Data Collection and Processing

Before proceeding with the application of the model, it is necessary to explain the information used and identify its data sources.

The company being study offers both food and non-food products and services both through physical and online stores. It is a retailer that operates in Portugal through multiple store types, such as large urban stores, proximity stores, e-commerce platforms, and proximity franchising stores. The proximity franchising stores are the most predominant, representing 49% of the whole stores. Large urban stores followed with a smaller percentage of 30%, and proximity stores accounted for the remaining 21%.

Currently, the company has 640 stores across the country. In this work, we are analysing a restricted group of 29 stores since these are the ones that have textile items in their product range. The item that will be the object of study is a man apparel item, more precisely a blue t-shirt.

The choice of textile products was made considering that the author's work responsibilities are closely related to this topic. Secondly, the selection of this specific item is associated with its typology due to its long-life cycle regarding textile products features.

We considered 2020 the information's time base, such as sales, stocks, allocation policy, and cost parameters. However, we also have collected the sales data of 2019 to forecast the model's demand; this is explained in section 5.1.1.

To collect the data information, we resorted to the company's Enterprise Resource Planning (ERP) and to queries and connections of a Structured Query Language (SQL) database that allows exporting data to Excel.

Furthermore, the data was worked in several Excel PivotTables, to organise better the information, which resulted in the best way possible to present a general overview of the needed information.

Since the company orders only one type of prepacks, the set of prepacks (*P*, indexed by *i*) is no longer used. Additionally, only the variable costs of handling are contemplated, both regarding stores and the distribution centre. The company's fixed costs are shared with other (non-food) products, so it is not possible to calculate them separately.

Finally, to clearly understand the process, it is essential to explain all the costs and demand parameters considered to design and test the studied model.

5.1.1 Store Demand

As previously referred, we collected the sales information for two years (2019 and 2020) to use the first-year data to forecast the demand of 2020, applying an econometric time series based method since we do not have demand details. Furthermore, the sales of 2020 are going to be used as the demand of the current policy. Then, the forecast results will be considered as the demand for the new policy following an optimisation model.

In section 3.4, we mentioned the Box-Jenkins approach as the one that will be used. The purpose is to use it to model the ARIMA process behind the sales series and, afterwards, use that information for forecasting. To proceed, we resort to an econometric program called EViews – Innovative Solution for Econometrics Analysis, Forecasting & Simulation, Version 12, that helped find the appropriate ARIMA process that fits the data.

When applying the software, the information passes by five stages of the process (Lewis-Beck, Bryman, & Futing Liao, 2012)

- Data preparation: the data is manipulated to stabilize the series variance, which is very common in economic data. Then, the differences between observations are differencing to correct patterns of trending or seasonality. That way, the data is easier to model.

- Model selection: the ARIMA processes that better fit the data are identified.
- Parameter estimation: computational algorithms find the better model coefficients to the data.
- Model-checking: the model is tested, and if it is inadequate, it will retreat to the model selection step.
- Forecasting: the final procedure of the process and its purpose. Here, the data is already fitted, the model is selected, estimated and checked.

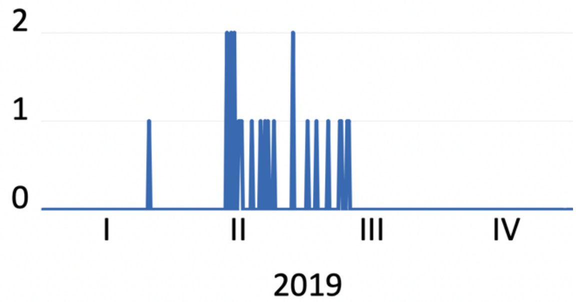
All these steps are computer made, resorting to EViews V12 software.

Finally, this approach finds the appropriate statistical model that better fits the data and convenient answers to the study needs to forecast demand.

Now, to illustrate the method of Box-Jenkins, we will present the whole process using one variable as an example, the size XL at store number 1 that ran from January 2nd of 2019 to December 31st, and it is available daily. The reason why it begins on January 2nd is that all the company stores are closed, every year at January 1st.

Figure 3 presents the sales behaviour of size XL at store number 1 during the year 2019.

Figure 3 – Size XL sales behaviour at store 1, the year 2019



The data presents some seasonal fluctuations, which is why it has been seasonally adjusted, using the moving average method implemented in the EViews program.

The first step in developing a Box-Jenkins model is to determine if the series is stationary. Box and Jenkins recommend differencing nonstationary series one or more times to achieve stationarity. It produces an ARIMA model, with the "I" stands for "Integrated". However, its first difference $\Delta_y = y_t - y_{t-1}$ is stationary, so y is integrated of order 1, or y ~ I (1).

For this, we have analysed the data series stationary using the Augmented Dickey-Fuller (ADF) test.

The results were statistically significant (p-value < 0.05), revealing that the null hypothesis is rejected. The series does not have a unit root and is stationary. So the initial series of the daily shirt XL is integrated of order zero.

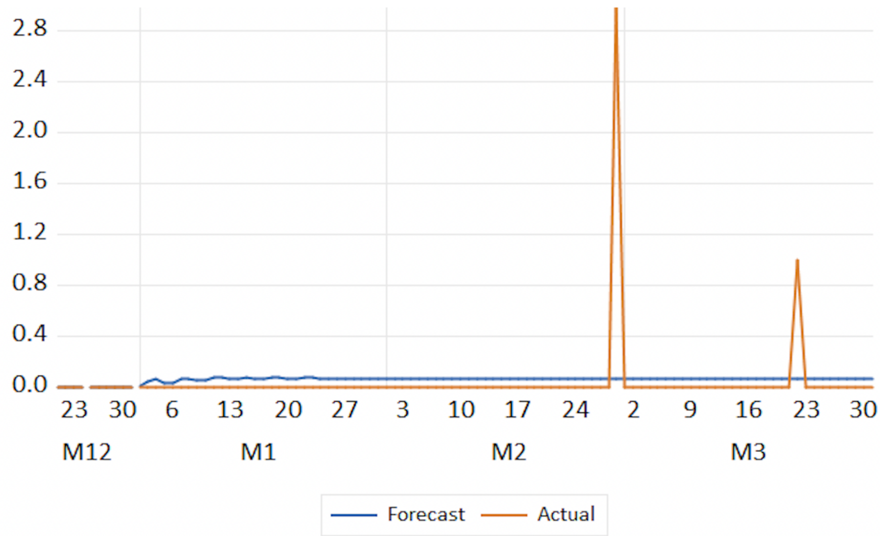
As a result, we have applied the Box- Jenkins procedure on the stationary data series, and we want to identify the corresponding ARIMA (p,q) process. The series correlogram has allowed us to choose appropriate p and q for the data series. We have estimated more models to determine the most suitable specification by choosing from the different models estimated on the informational criteria Akaike and generating predictions based on estimated models. The series correlogram suggests the necessity of introduction in the process estimation of both the analysed variable lags and the lags of the error term. We started with an AR (1) process and further analysed the residual correlogram to catch the correlations and autocorrelations from lags more significant than 1.

From Akaike Information Criteria's (AIC) point of view, the proper model to best adjust the data is ARIMA (4,0,4). It is the smallest obtained given by the $AIC = -2\frac{l}{T} + 2\frac{k}{T}$, where l is the value of the log of the likelihood function with the k parameter estimated, using T observations. The various information criteria are based on -2 times the average log-likelihood function, adjusted by a penalty function. The model with the lowest AIC is selected.

We estimate the model ARIMA (4,0,4), the parameters of the model are significantly different of 0 using a t-test. The other statistics portend a good fitting: the Durbin-Watson statistic value implies no autocorrelation for a level of significance of 5%. The F-statistic allows concluding that the model is statistically significant at 1%. The determination coefficient R-squared is 43%. The residual analysis is based on two criteria: the normality test points out that the average of residuals is approximately 0, and the residual is white noise, analysing the autocorrelation. Any term is not exterior to the confidence intervals, and the Q-statistic has a critical probability near 1. The residual may be assimilated to a white noise process. Therefore, the estimation of the ARIMA (4,0,4) model is validated, the time series can be described by an ARIMA(4,0,4) process.

As we can see below, we have the compared results between the forecast and actual sales of the first quarter of the size and store analysed.

Figure 4 - Comparison between actual and forecast sales, size XL, store 1, first quarter



The EViews outputs the results daily to each size and store. Still, since our study will always be analysed by quarter, we organised all the values by quarter, store and size.

The table below presents the sales forecast for the entire year of 2020 for size XL at store 1.

Table 1 - Forecast sales, size XL, store 1, by quarter

Store	Size	1Q	2Q	3Q	4Q	TOTAL
1	XL	6	6	36	5	53

These processes were repeated for all combinations of size, store and quarter. Still, we do not have the company's authorization to show complete information of sales results and, for that reason, we will not attach these results to our study.

5.1.2 Store Costs and Parameters

This section explains and quantifies all store costs and parameters.

As mentioned above, only the variable costs are contemplated since the company's fixed costs are shared with other (non-food) products.

Therefore, we consider marginal handling costs, marginal inventory overstock costs, and marginal understock (shortage) costs. These are the parameters that the textile team of the company understudy usually considers as directly involved with its activity.

Although a variety of costs could be considered, many of them are shared among several businesses units of the company, such as fixed handling costs, store maintenance costs etc. So, they are usually not directly considered in the costs control analysis of the textile business.

Additionally, in table 2 we can see a detailed description of each parameter and how it has been calculated. We have obtained the following data from the retailer:

- time spent on handling prepacks at stores;
- collaborators wage per minute at each store (obtained as the per minute average wage of the collaborators, of all stores in the same city);
- time spent on handling a single-size item at the stores (regardless of item size and store location, we assumed that the time spent on handling a single-size item is the same);
- margin loss (i.e., the margin loss associated with selling the items on sale, instead of the initial price, considering a VAT of 23% - according to the Portugal VAT rates to textile merchandise);
- time spent on re-tagging at the stores (items on sale need to be re-tag as they have a different price);
- overstock cost - cost of not selling a unit at the stores (i.e., the value that the store would have been earned with the sale of a unit (the profit margin), also calculated with a VAT of 23%);
- understock cost - considered as 20% of the lost sales and loss of customer goodwill;

It is essential to retain that all the time figures are in minutes and all the costs figures are in euros (€).

The table below uses the following notation: PHT – prepack handling time; IHT – single item handling time; CW – collaborators wage; LM – lost margin; RT – re-tagging time; LS – lost sales; LCG – loss of customer goodwill.

Table 2 - Store Parameters Description

Parameter	Description	Formula
$\boldsymbol{h_j^s}$	Cost of handling the prepack at store *j*. It includes opening the cases and placing security tags.	$PHT * CW$
$\boldsymbol{h_{kj}^{sv}}$	Cost of handling a single-size item of size *k* at store *j*. It includes opening the cases and placing security tags.	$IHT * CW$
$\boldsymbol{\omega_{kj}}$	Costs of over ordering items of size *k* at store *j*, that include costs of selling on sale and re-tagging the products.	$LM + RT * CW$
$\boldsymbol{\varphi_{kj}}$	Costs of under ordering items of size *k* at store *j*, due to lost sales and of customer goodwill.	$LS + LCG$

The complete information on parameters and costs of each store is provided in appendix I.

Note that overstock costs vary between 1.65€ and 1.76€ per unit and are lower than under-stock costs, where the amount is 1.83€ per unit.

Therefore, it is possible to conclude that the company prefers to have excessive stock that not being able to satisfy the customer

5.1.3 Distribution Centre Costs and Parameters

The distribution centre is responsible for processing goods reception after the purchasing, such as storage, picking, packing and shipping. Those are the main cost drivers. Once again, we are not considering all these drivers due to the company's capacity of sharing some costs amongst various businesses' units. For this reason, we are only contemplating variable costs of handling prepacks and single-size items.

Concerning apparel items, the goods are usually allocated to the stores in two different ways: prepacks containing several units of the same item in different sizes, respecting a size curve, or several units of just a single-size item. The purchase order is a mix of these two kinds of merchandise delivery. Thereby, the downstream supply chain department must decide on the best way of allocating the SKU's, responding to different demands in several stores.

In our study, the scope is to use the two different combinations of merchandise delivery previously defined by the vendor. The purchase plan is defined as:

- 80% prepacks of multiple sizes (Q) containing a prespecified number of units of each size of the same item; more specifically, each prepack contains one small item, two medium items, two large items, two extra-large items and one extra-extra-large item (q_k);
- 20% of single-size items clubbed into master cases of the same size (B_k), each one with 64 units $(b_k = 64)$.

Recall that we allow master cases manipulation only, which has not yet been done in the literature. Since other authors either allow complete manipulation (on prepacks and single-size items) or no manipulation at all. With manipulation, we assume not to send the merchandise as received at the distribution centre, for example, open the prepacks and send as single-size items.

Furthermore, table 3 shows a detailed description of each parameter and how it is calculated. We here obtained the following data from the retailer:

- time spent on handling prepacks at the distribution centre;
- collaborators wage per minute at the distribution centre (obtained as the per minute average wage of the collaborators who have the responsibility of picking the goods, assuming that they work 40 hours a day);
- time spent on handling single-size items at the distribution centre.

Table 3 – Distribution Centre Parameters Description

Parameter	Description	Formula
$\boldsymbol{h^w}$	Cost of handling the prepack at the distribution centre. It includes the process of picking the prepack from its distribution centre location to the store preparation set.[2]	$PHT * CW$
$\boldsymbol{h_k^{wv}}$	Cost of handling a single-size item *k* at the distribution	$IHT * CW$

[2] Each store has its setting place on the distribution centre, where the merchandise is placed before uploaded to transportation.

	centre. It includes the process of picking the prepack from its distribution centre location to the store preparation set.	

A prepack contains a total of eight units that are handled as one. However, when handling single-size items, just one unit is being handled. Based on the information in appendix II, and since the cost of handling a prepack is 0.28€ and the cost of handling a single-size item is also 0.28€, it is possible to conclude that it is more expensive to handle single-size items than a prepack.

Therefore, if we want to send the same quantity using prepacks versus single-size items, we turn everything more expensive with the latter. The same situation happens at store costs.

5.2 Model Application

After explaining the data, we can now rewrite the model, considering only the specific information we appraise in this study.

Minimise

$$\sum_{k \in S} \sum_{j \in N} \omega_{kj}\ \theta_{kj} + \sum_{k \in S} \sum_{j \in N} \varphi_{kj}\ \mu_{kj} +$$
$$h^{w} \sum_{j \in N} x_j + \sum_{k \in S} \sum_{j \in N} h_k^{wv}\ v_{kj} +$$
$$\sum_{j \in N} h_j^{s}\ x_j + \sum_{j \in N} \sum_{k \in S} h_{kj}^{sv}\ v_{kj}$$

Subject to

$$\sum_{j \in N} x_j \leq Q \tag{5.1}$$

$$\sum_{j \in N} v_{kj} \leq B_k b_k, \forall_{k \in S} \tag{5.2}$$

$$\mu_{kj} \geq d_{kj} - (q_k x_j + v_{kj}), \forall_{k \in S}, \forall_{j \in N} \tag{5.3}$$

$$\theta_{kj} \geq (q_k x_j + v_{kj}) - d_{kj}, \forall_{k \in S}, \forall_{j \in N} \tag{5.4}$$

$$\mu_{kj}, \theta_{kj} \geq 0, \forall_{k \in S}, \forall_{j \in N} \tag{5.5}$$

$$x_j, v_{kj} \geq 0 \text{ and integers}, \forall_{k \in S}, \forall_{j \in N} \tag{5.6}$$

As before, we seek to allocate prepacks (x_j) and single-size items (v_{kj}) to each store. Amongst all possible solutions, we are interested in one that minimises the total costs while complying with distribution centre availability.

The cost function has six components since it includes the penalty costs of over and under ordering each size at each store, summed across all sizes and stores; the costs of handling prepacks and single-size items at the distribution centre and stores, respectively.

Regarding the constraints, (5.1) and (5.2) ensure, respectively, that the total number of prepacks and single-size items sent to the stores do not exceed distribution centre availability. The number of understock units of each size in each store is determined by constraints (5.3), while the number of overstock units is determined by constraints (5.4).

Finally, constraints (5.5) and (5.6) defined the nature of the variables.

Using the costs and data explained in section 5.1, the allocation policy obtained by solving this model will be compared with the company's current policy.

Presently, in the company we are studying, Stock Analysts (people responsible for allocating items) allocate prepacks to stores manually. They simply ground their decisions on the business know-how and experience. Since the company has already identified the need for improvement and the computational experiments show an improved performance, we expect our approach to be implemented.

5.3 Computational Experiments and Results

This section reports the computational results obtained by solving the proposed optimisation model.

The MILP model was solved using the IBM ILOG CPLEX Optimization Studio OPL 20.1.0.0 model.

Although there are several other software packages, such as CPLEX, GUROBI and MATLAB, we chose CPLEX because of its known good efficiency and simplicity of implementation, not requiring extensive knowledge of programming languages (Atta & Sen, 2021; Bliek, Bonami, & Lodi, 2014; Wang, 2012). Additionally, it is free to access for students.

The model written in CPLEX language is provided in appendix III.

The experiments were conducted on a Lenovo laptop with an Intel (R) Core (TM) i5- 7200U CPU @ 2.50 GHz, with 8.0 GB of memory and a 64-bit operating system.

After inputting the parameters' values, the model was solved for each quarter of the year. The model was solved very quickly since, on average, it required only 12 seconds.

5.3.1 Comparison of solutions: allocations and stocks

Next, we summarise and analyse the results obtained. Detailed results of all variables (main and auxiliary) are provided in appendix IV and appendix V, regarding our solution and the current policy solution, respectively, by store and quarter.

5.3.1.1 Summary of the main results

Table 4 – Our solution: number of prepacks and single-size items sent to stores

	#prepacks sent to the stores (x_j)	#single size items sent to stores (v_{kj})					
		S	M	L	XL	XXL	TOTAL
1Q	67	24	40	73	11	28	176
2Q	88	24	63	114	18	12	231
3Q	363	19	116	226	13	75	449
4Q	80	32	87	148	25	65	357
TOTAL	598	99	306	561	67	180	1213

Table 4 sums up the number of prepacks and single-size items sent to all stores throughout the year using the proposed approach. There seems to be an increasing trend for the number of items shipped to the stores along the year, reaching its peak at the third quarter and then a decrease can be observed.

This behaviour is related to the typology of the product being study (a man basic blue t-shirt) and its seasonality. It is expected that a t-shirt will reach its sales peak in the summer season (third quarter-3Q), and then will reduce its performance in winter.

Nevertheless, besides its seasonality, there will always be a considerable demand over the year since this type of product has a long high life cycle.

Table 5 - Current policy solution: number of prepacks and single-size items sent to stores

	#prepacks sent to the stores (x_j)	#single size items sent to stores (v_{kj})					
		S	M	L	XL	XXL	TOTAL
1Q	180	0	0	0	0	0	0
2Q	418	0	0	4	0	16	20
3Q	0	99	306	509	67	164	1145
4Q	0	0	0	48	0	0	48
TOTAL	598	99	306	561	67	180	1213

Now, analysing the current policy solution, we can observe that one of the significant differences in this method is that the Stock Analyst only sends single-size items after running out of all the prepacks. This movement can be clearly observed in table 5.

Obviously, this difference in strategy is expected to have substantial implications on the number of understock and overstock units.

Tables 6 and 7 report the number of overstock and understock items for our approach and the current policy, respectively.

Table 6 – Our solution: number of overstocked and understocked items

	#overstocked items (θ_{ki})						#understocked items (μ_{ki})					
	S	M	L	XL	XXL	TOTAL	S	M	L	XL	XXL	TOTAL
1Q	9	20	17	14	4	64	3	2	0	0	0	5
2Q	6	2	13	11	9	41	1	0	0	0	2	3
3Q	115	37	109	58	67	386	55	33	39	45	19	191
4Q	22	33	29	10	23	117	0	19	2	0	0	21
TOTAL	152	92	168	93	103	608	59	54	41	45	21	220

Recall that the results reported are for all stores and all days of the quarter. Therefore, as it can be observed, we report overstocked and understocked units for the same quarter and size. Actually, this happens even for the same store, see appendix V.

Table 7 – Current policy solution: number of overstocked and understocked items

	#overstocked items (θ_{ki})						#understocked items (μ_{ki})					
	S	M	L	XL	XXL	TOTAL	S	M	L	XL	XXL	TOTAL
1Q	144	153	149	151	145	742	0	0	0	0	0	0
2Q	493	503	520	504	526	2546	0	0	0	0	0	0
3Q	24	1	29	28	18	100	435	472	432	437	450	2226
4Q	0	0	0	0	0	0	0	19	2	0	0	21
TOTAL	661	657	698	683	689	3388	435	491	434	437	450	2247

Comparing the two solutions, we can see that our approach has lower stock discrepancies values, both over and under (see table 6).

One of the main reasons the current policy solution has higher stock discrepancies is the initial exclusive use of prepacks until these run out. This decision will increase the number of overstocked items in the first quarters (1Q and 2Q) and the number of understocked items in the last quarters (see table 7).

Figure 5 - Comparison of overstock and understock

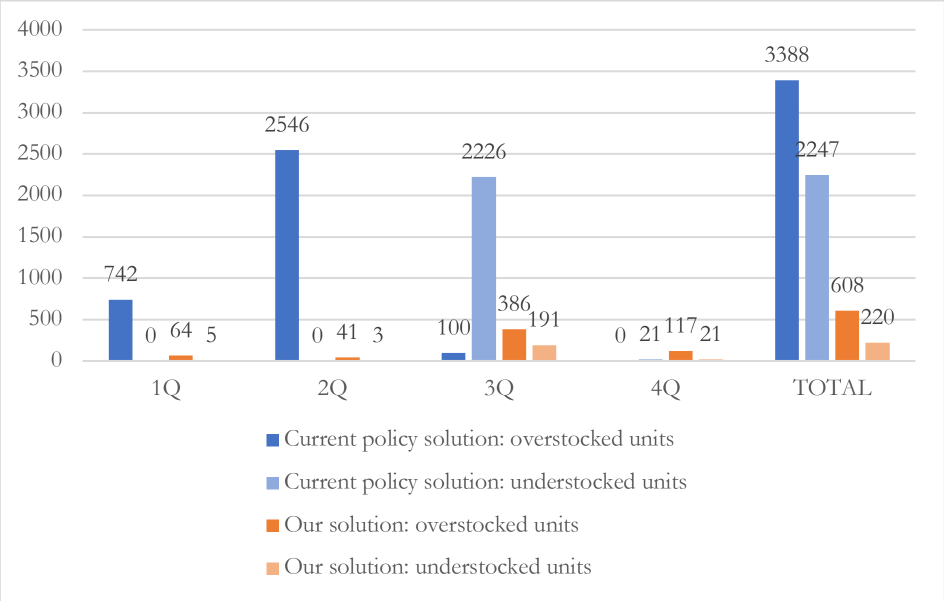


Figure 5 shows the vast discrepancy between the two solutions in terms of stock distribution. It's clear that the decision of not sending all the prepacks first is wisely made since a considerable reduction in both understock and overstock units is accomplished.

It can be concluded that our solution is more balanced and leads to much lower discrepancies both regarding stockout events (understock) and excessive inventory (overstock).

5.3.1.2 Detailed analysis

After summarising the main results, we now present another point of view of the analysis. For this discussion, we will define three store clusters:

- Cluster A, with nine stores, representing 50% of the Reported Net Sales (RNS)[3];
- Cluster B, also with nine stores and 36% of the RNS;
- Cluster C with ten stores and representing only 14% of the RNS.

Stores were allocated to one of the clusters based on the current company's cluster decision. That is, they commonly divide the clusters equally in terms of the number of stores, where they are ranked by sales performance.

[3] RNS represents the sum of a company's gross sales excluding its returns, allowances, and discounts.

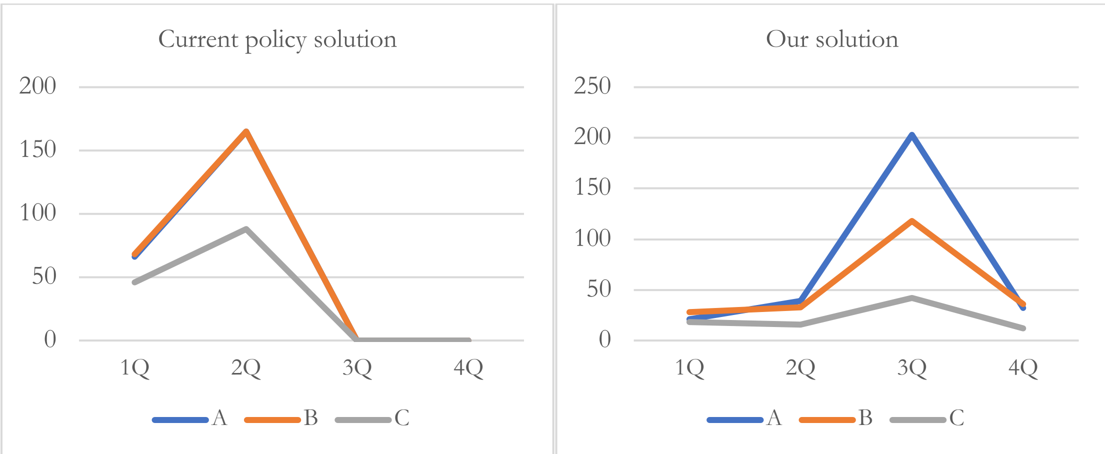
Table 8 - Number of prepacks sent to each store cluster

Cluster	Current policy solution	Our solution
A	231	295
B	233	215
C	134	88
TOTAL	598	598

Table 8 shows the difference between the two solutions regarding the number of prepacks sent to each cluster. Figure 6 also provides these numbers with a different point of view.

We observe that the difference in the number of prepacks sent to the stores in clusters A and B stores is negligible in the current policy solution. However, this is not the case in our solution. Also, our solution has a more adequate distribution of prepacks, bearing in mind the sales performance of each cluster.

Figure 6 - Number of prepacks sent to each store cluster over the year



Regarding single-size items, we summarise the number of single items of each size sent to each cluster in tables 8 and 9 for our solution and the current policy, respectively.

Table 9 - Number of single-size items sent to stores: our solution

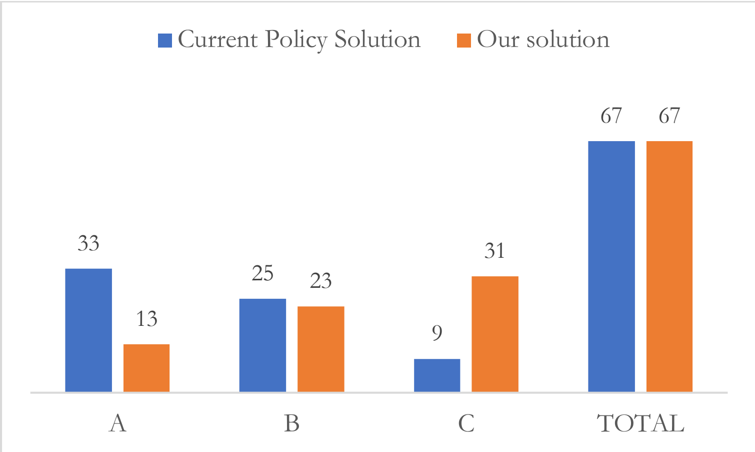
	Our solution					
Cluster	S	M	L	XL	XXL	TOTAL
A	38	68	190	13	33	342
B	34	131	191	23	81	460
C	27	107	180	31	66	411
TOTAL	99	306	561	67	180	1213

Table 10 - Number of single-size items sent to stores: current policy solution

	Current Policy Solution					
Cluster	S	M	L	XL	XXL	TOTAL
A	58	136	247	33	90	564
B	31	117	196	25	68	437
C	10	53	118	9	22	212
TOTAL	99	306	561	67	180	1213

Another view that we need to consider is the number of single-size items sent to each cluster. Figure 7 presents the behaviour of the single items of size XL sent to each cluster.

Figure 7 - Number of single items size XL sent to each store cluster



Then, this reveals that, for cluster A, the number of single items sent in size XL is higher in the current solution and starts to decrease. The same does not happen with our solution.

According to table 14 of appendix I, the time spent on prepack handling at the stores is 5 minutes and 3 minutes on single-size items. Since one prepack has eight units in its composition, it takes longer to fulfil store shelves with single-size items.

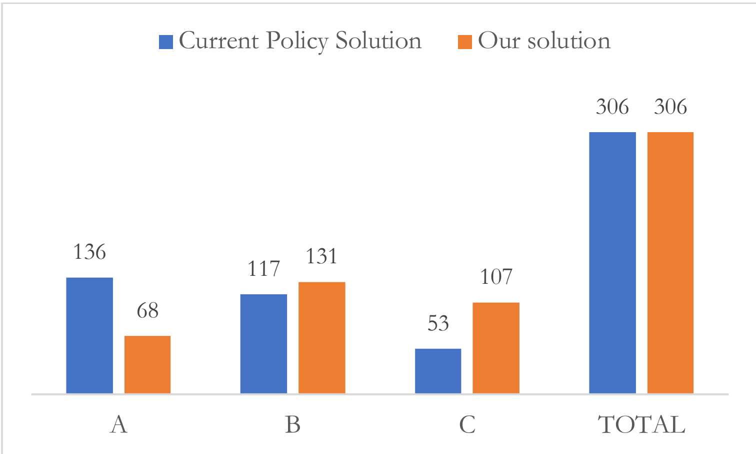
So, in stores with fast rotation of goods (cluster A), it is essential to have a higher proportion of prepacks than units of single-size items received.

Furthermore, having a higher proportion of prepacks and the smallest of single-size items on cluster A makes more sense. That is what our new solution presents to the detriment of the current one.

Additionally, after the above mentioned, it is essential to justify why cluster B has more single-size items than C in other cases where the total quantity of single size items is higher (see figure 8). That is related to their sales performances. Since cluster C only represents 14% of the RNS, it will automatically receive fewer units in some sizes, regardless of whether they are prepacks or single-size items. So, given the stipulated quantity of single-size items, and because cluster C does not have a great demand, the higher amount should go to cluster B rather than A.

It is essential to underline that we only show sizes XL and M because the others have similar behaviours, as shown in appendix VI.

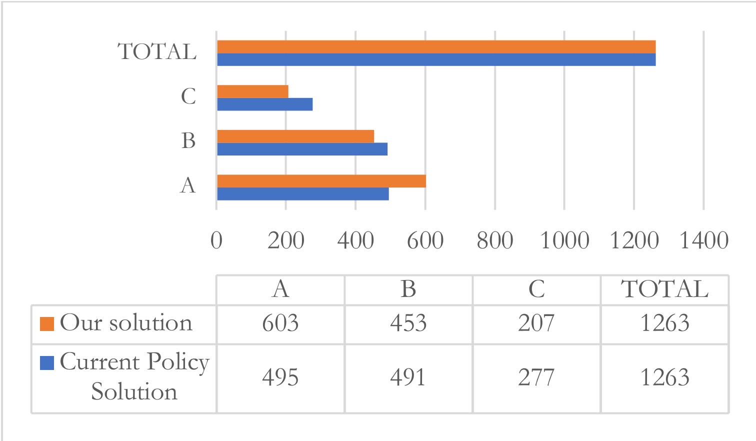
Figure 8 - Number of single items size M sent to each store cluster



To sum up, the stores in the best cluster need to receive a more significant number of items (because they have a higher demand) and should receive a higher proportion of prepacks to faster respond to the rotation of the goods.

Figure 9 shows the total quantity of sizes XL sent to each store cluster, whether sent on prepacks or single-size items (the other sizes are on appendix VI, since they have similar behaviours).

Figure 9 - Total quantity of size XL sent to each store cluster



It is possible to conclude that our new solution better follows the stores' demand and performance since each cluster of stores receives adequate quantities for their behaviours.

For example, suppose we calculate the proportion of the sent units presented in figure 9. We can observe the finest similarity between the %RNS of each cluster on our new solution comparing with the current one. Table 11 affirms the effectiveness of the new model on following the stores' sales performance (once more, the other sizes information can be found in appendix VI).

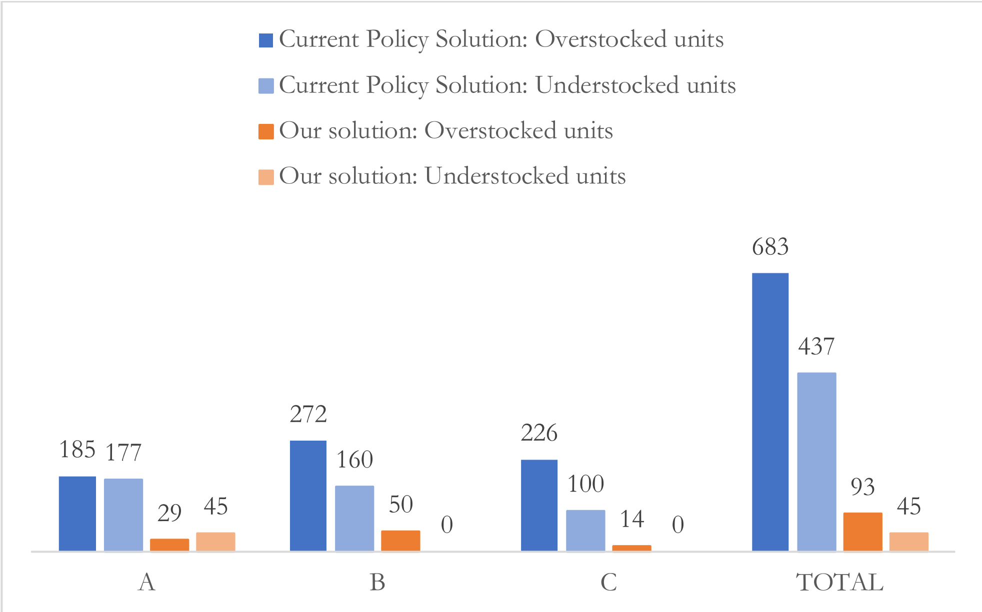
Table 11 – Proportion of the total quantity of size XL sent to each cluster

	% quantity of size XL items sent		% RNS
Cluster	Current Policy Solution	Our solution	
A	39%	48%	50%
B	39%	36%	36%
C	22%	16%	14%
TOTAL	100%	100%	100%

After confirming that it is essential that better stores (in terms of sales performance) need to have a higher proportion of prepacks received, it is crucial to certificate that they do not have higher amounts of overstocks due to that.

Figure 10 shows that besides cluster A receiving more prepacks (our solution), it can maintain a low number of overstocked items, comparing to the current solution. Still, although cluster C receives fewer prepacks, combined with single-size items, they are sufficient for its demand because cluster C does not have understocked units.

Figure 10 - Comparison of overstock and understock in each cluster: size XL



Overall, we can confirm that our solution is much more efficient in terms of inventory discrepancies. So, the allocations are more assertive with this methodology, which will directly impact costs reduction. In appendix VII, it is possible to check the other sizes similar behaviour.

5.3.2 Comparison of solutions: costs

This section summarises and discusses the impact on the costs incurred with the proposed solution, both regarding the stores and the distribution centre.

5.3.2.1 Summary of the main results

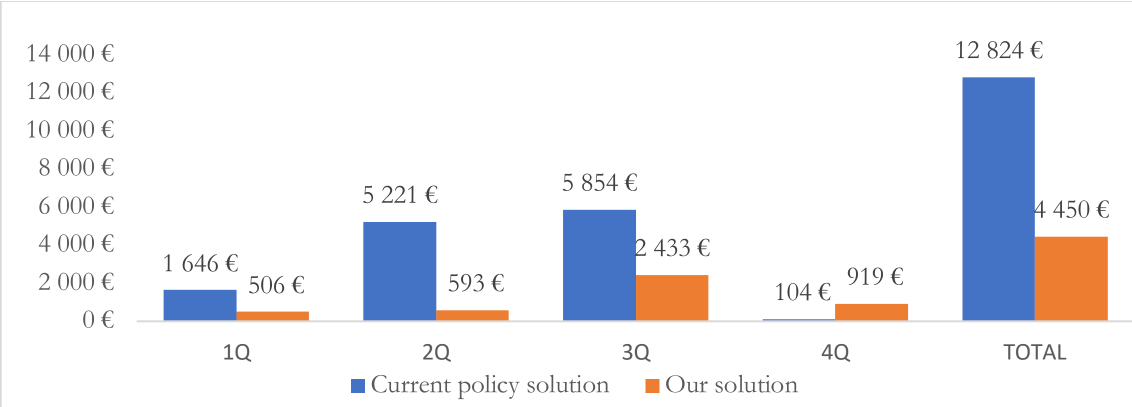
The costs associated with the prepacks and single-size items allocation policy derived by the solution proposed are differentiated from those associated with the company's current policy.

Recall that costs are incurred at the stores and also at the distribution centre:

- Store costs include handling costs and overstock and understock costs;
- Distribution centre costs include only handling costs. Therefore, the allocation policy does not affect distribution centre costs since all available prepacks and all available single-size items are delivered and thus handled.

Figure 11 depicts the total costs (per quarter and for the whole year) incurred by implementing the current policy as well as the ones incurred by implementing the proposed approach. Further detail on the costs is provided in appendix VIII.

Figure 11 - Comparison of total costs



As it can be observed, the yearly costs of the proposed approach are about one-third of those of the company's current policy. Regarding quarterly costs, they are much lower in the first three quarters (about 70%, 89%, and 58% lower, respectively) and quite larger (about 83%) in the last quarter. The main reason for the much higher costs, in the last quarter, of the proposed solution (in comparison to the current solution) is the fact that in the current solution, almost no units are delivered, see table 5.

Since the store costs are the only ones affected by the allocation policy, let us look at them in more detail. As seen in table 12, the handling costs are almost the same in both policies. This is expected since i) the total amounts of handled prepacks and single-size items are the same and ii) the handling costs incurred are proportional to the salary of the collaborators that vary with the store, although only slightly.

However, the costs associated with overstocked items and understocked items are significantly lower, about 82% and 90% lower, respectively. This massive decrease in both overstock and understock costs is achieved by having a number of units available at the stores much closer to the demanded quantity.

Table 12 - Comparison of store costs

	Current policy solution	Our solution
Handling prepacks	1 149 €	1 144 €
Handling single size items	1 366 €	1 381 €
Overstock	5 695 €	1 022 €
Understock	4 112 €	403 €
Total	12 323 €	3 949 €

5.3.2.2 Detailed analysis

Let us now perform a more detailed analysis of the cost reductions achieved by considering the store type, i.e., the three clusters presented before.

Table 13 and figure 12 shows the prepacks handling costs (total and over time, respectively) for each store cluster, as well as the total handling cost over all stores for the whole year.

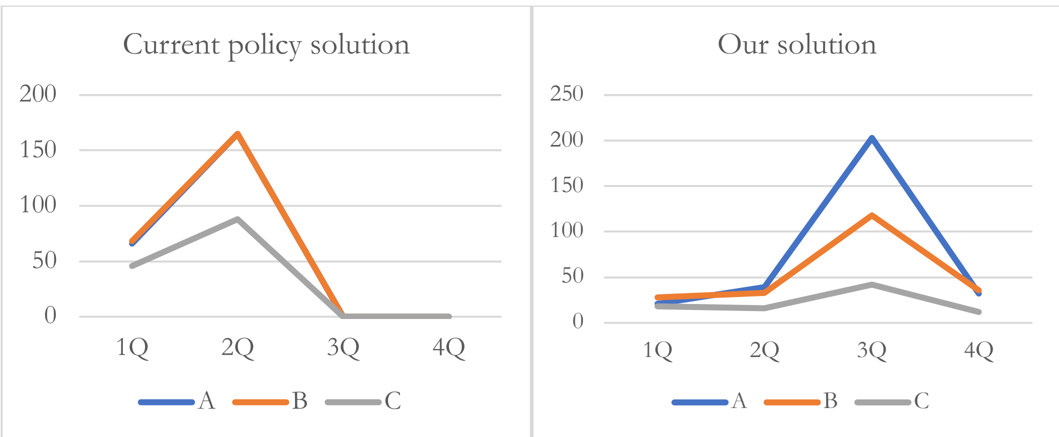
Table 13 - Total costs of handling prepacks in each cluster

Cluster	Current Policy Solution	Our solution
A	463 €	584 €
B	428 €	391 €
C	257 €	169 €
TOTAL	1 148 €	1 144 €

Although the yearly prepacks handling costs for all stores are almost the same in both policies, some differences can be observed for each cluster. Regarding the current policy, cluster A and B are responsible for nearly 80% of the costs, while cluster C accounts for just over 20% of the costs. In the policy we propose, cluster A is responsible for about 51% of the costs, cluster B for about 34%, and cluster C for about 15%.

The main reason for these differences is that our solution has a more adequate distribution of prepacks, bearing in mind the sales performance of each cluster. On the other hand, the current policy has a negligible difference between prepacks sent to clusters A and B (see table 8).

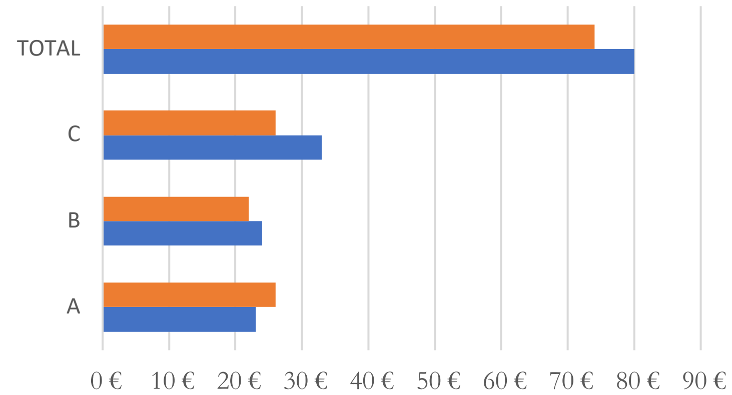
Figure 12 - Total costs of handling prepacks in each cluster over the year



It is also possible to observe that our solution costs meet better the peak sales observed previously in the third quarter, related to the product seasonality.

Regarding handling costs associated with single-size items, the behaviour is similar across all sizes. Here, we only discussed the results for size XL, see figure 13, but the results for the other sizes are provided in appendix VIII.

Figure 13 - Total costs of handling single-size items in each cluster: size XL



	A	B	C	TOTAL
Our solution	26 €	22 €	26 €	74 €
Current Policy Solution	23 €	24 €	33 €	80 €

The handling costs of the proposed policy are about 7.5% lower than those of the current policy. Additionally, the distribution of these costs among the clusters also presents some differences: in our approach, clusters A and C are responsible each for about 35% of the costs and cluster C is responsible for about 30% of the costs. For the current policy, these figures are 29%, 30%, and 41% for clusters A, B, and C, respectively.

These values are not only directly related to the number of single-size items sent to each store but also to the average wage of the collaborators. So, it is not possible to direct justify the differences between the two solutions.

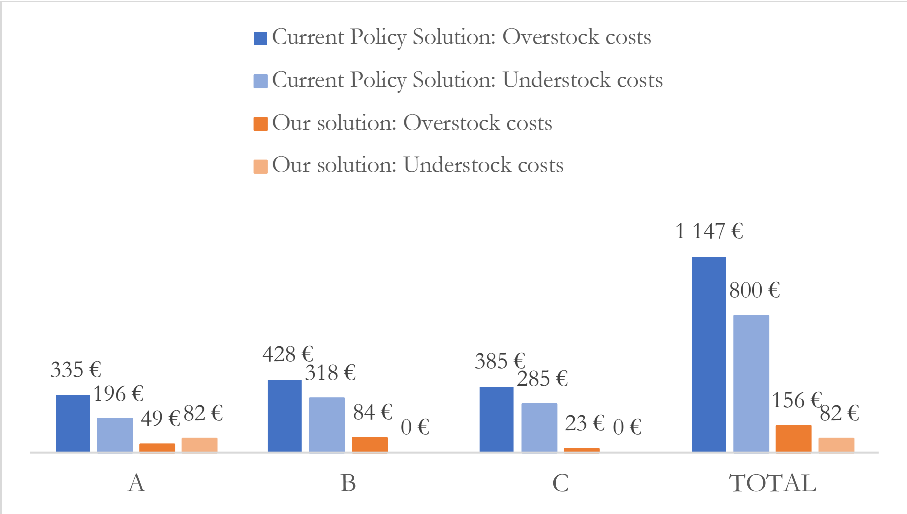
The essential point to retain is that our solution has lower handling costs than the current one.

Let us now analyse the overstock costs (inventory) and understock costs (stockout). These costs are much more relevant since they are significantly affected by the policy choice.

As before, we only consider size XL as the behaviour for other sizes is similar, see appendix VIII.

In figure 14 the yearly overstock and understock costs for each cluster and for all stores are depicted both for the current policy and for the proposed one.

Figure 14 - Inventory and stockout costs in each cluster: size XL



The yearly total overstock and understock costs are about, respectively, 86% and 90% smaller for the proposed policy. Looking at the clusters level and regarding overstock costs, we can see that the most considerable reduction occurs for cluster C (about 94% lower), then cluster A (about 85% lower), and finally cluster B (about 20% lower).

Regarding the understock costs, the reduction is about 58% for cluster A and 100% for clusters B and C, as they do not incur in any understock cost (as we can see in figure 10, our solution does not contain understock units for these two clusters).

6. Final Remarks

This chapter presents some concluding remarks of the outcomes in section 6.1 and discusses possible future research directions in section 6.2.

6.1 Conclusions

Nowadays, it is common to prepack merchandise for easier handling, especially in a textile retail company. Prepacks are ordinarily composed of multiple units of the same Stock Keeping Unit (SKU) or combinations of different ones. By ordering prepacks rather than single SKU's, companies expect to improve their allocation method, thus reducing costs. Therefore, since decisions are no longer made only on individual items but also on prepacks, supply chains need to be careful with stockout occurrences or inventory costs. Since using prepacks leads to higher mismatches between supply and demand.

Additionally, the use of prepacks can also lead to larger inventories.

This dissertation examines how the prepack allocation problem affects the cost efficiency of a specific retail company. In order to do so, we propose a MILP (mixed-integer programming) formulation for the prepack allocation problem. As referred to before, the first step regarding the optimisation of the prepack allocation problem is to forecast demand. We do so by resorting to the Box Jenkins methodology.

Furthermore, the solution approach incorporates demand forecasting and MILP model solving. In order to find a new allocation strategy that is expected to significantly improve the task's efficiency, in terms of stocks (understock and overstock) and costs, impacting the supply chain and its financial results.

One of the significant contributions of this dissertation is that, although many authors already tried to use MILP models to solve problems of prepack allocation, none, to the best of our knowledge, has achieved the expected results on using MILP and had to resort to other methods (Hoskins et al., 2014; Nabil et al., 2018; Wang, 2012)

Another one is that we do not consider the lower or upper limit on units allocated to stores. It was a prevalent practice in other studies to consider such limits. This is an important feature, at least for the company being studied, since it is not exclusively a textile retailer; thus, the distribution centre can mix other company products in the same shipment.

Additionally, we use real-world data, which is an advantage regarding results quality and applicability, a feature that most studies ignore.

The alternative of opening prepacks to further manipulation Chettri & Sharma (2012) is not considered since it is not an option to the distribution centre under study, as well as many others due to high handling costs and working hours. Also, there is no opportunity for backorders due to the fashion industry lead time.

The allocation policy devised by our approach was compared with the company's current policy regarding: stockout occurrences, inventory held, and costs. The costs considered include handling costs, both at the stores and the distribution centre, understock costs, and overstock costs.

We have shown that the handling and inventory costs, as well as stockout occurrences, were abruptly reduced.

Considering all the previously presented features, to our knowledge, it is the first time this topic has been approached under these circumstances. Thereby, in addition to its practical relevance, this work consolidates a significant contribution to the academic literature on this topic.

6.2 Future work

Although statistical methods are more straightforward and faster, they depend on historical sales data, which can be acceptable for raw data. Nevertheless, with the increase of information, it is necessary to evaluate new methods to forecast demand there so, the method used to forecast demand may not work in some more extensive databases.

The same is valid for the optimisation model since many studies refer that CPLEX has a big data issue (Atta & Sen, 2021; Bliek, Bonami, & Lodi, 2014; Wang, 2012). One of the solutions can be the use of heuristics.

The MILP model proposed in chapter 4 is general and has been adopted in chapter 5 to the specific retail business in hands. Therefore, it can be tailored to other business areas.

Another avenue of research within this type of problem is the quantification of the costs associated with lost sales. This topic is a considerable handicap in sales management and directly related to demand forecast and, consequently, stock allocations.

Additionally, future work can explore further the cost structure, for example, the cost of picking locations. Suppose we have different types of prepacks or single-size items. In that case, these will increase the distribution centre costs, since each type needs to have its specific picking location (i.e., each single-size item has its particular picking location and the same happens with each different prepack so the distribution centre will have as many different picking locations, as the number of different types of merchandise). Unfortunately, this was not possible to include in this study since the company has a fixed number of types of prepacks and single-size items.

Finally, as previously explained, this dissertation provides a solution for the prepacks allocation problem with a previously defined prepack. Some studies researched prepack assortment but without considering the prepack allocation. A very interesting problem in need of investigation can be defined by combining these two problems: the prepack assortment and allocation problem.

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Appendices

Appendix I – Store Costs

Table 14 - Handling costs at stores

#store	wage (€)	wage/min (€/min)	time spent on prepack handling (min)	h_j^s	time spent on single size item handling (min)	h_{kj}^{sv}
1	964.00 €	0.40 €	5	2.01 €	3	1.21 €
2	1 082.00 €	0.45 €	5	2.25 €	3	1.35 €
3	986.00 €	0.41 €	5	2.05 €	3	1.23 €
4	881.00 €	0.37 €	5	1.84 €	3	1.10 €
5	803.00 €	0.33 €	5	1.67 €	3	1.00 €
6	849.00 €	0.35 €	5	1.77 €	3	1.06 €
7	780.00 €	0.33 €	5	1.63 €	3	0.98 €
8	764.00 €	0.32 €	5	1.59 €	3	0.96 €
9	1 190.00 €	0.50 €	5	2.48 €	3	1.49 €
10	899.00 €	0.37 €	5	1.87 €	3	1.12 €
11	900.00 €	0.38 €	5	1.88 €	3	1.13 €
12	1 190.00 €	0.50 €	5	2.48 €	3	1.49 €
13	789.00 €	0.33 €	5	1.64 €	3	0.99 €
14	1 010.00 €	0.42 €	5	2.10 €	3	1.26 €
16	792.00 €	0.33 €	5	1.65 €	3	0.99 €
203	785.00 €	0.33 €	5	1.64 €	3	0.98 €
209	801.00 €	0.33 €	5	1.67 €	3	1.00 €
439	776.00 €	0.32 €	5	1.62 €	3	0.97 €
446	804.00 €	0.34 €	5	1.68 €	3	1.01 €
459	827.00 €	0.34 €	5	1.72 €	3	1.03 €
460	881.00 €	0.37 €	5	1.84 €	3	1.10 €
461	900.00 €	0.38 €	5	1.88 €	3	1.13 €
462	759.00 €	0.32 €	5	1.58 €	3	0.95 €
463	1 300.00 €	0.54 €	5	2.71 €	3	1.63 €
464	1 190.00 €	0.50 €	5	2.48 €	3	1.49 €
465	849.00 €	0.35 €	5	1.77 €	3	1.06 €
468	810.00 €	0.34 €	5	1.69 €	3	1.01 €
1050	827.00 €	0.34 €	5	1.72 €	3	1.03 €
801	832.00 €	0.35 €	5	1.73 €	3	1.04 €

Table 15 - Overstocked and understocked inventory costs

#store	wage (€)	wage/min (€/min)	time spent on re-tagging (min)	margin loss	ω_{kj}	profit margin	customer satisfaction loss	φ_{kj}
1	964.00 €	0.40 €	0.5	1.49 €	1.69 €	1.53 €	0.31 €	1.83 €
2	1 082.00 €	0.45 €	0.5	1.49 €	1.72 €	1.53 €	0.31 €	1.83 €
3	986.00 €	0.41 €	0.5	1.49 €	1.70 €	1.53 €	0.31 €	1.83 €
4	881.00 €	0.37 €	0.5	1.49 €	1.67 €	1.53 €	0.31 €	1.83 €
5	803.00 €	0.33 €	0.5	1.49 €	1.66 €	1.53 €	0.31 €	1.83 €
6	849.00 €	0.35 €	0.5	1.49 €	1.67 €	1.53 €	0.31 €	1.83 €
7	780.00 €	0.33 €	0.5	1.49 €	1.65 €	1.53 €	0.31 €	1.83 €
8	764.00 €	0.32 €	0.5	1.49 €	1.65 €	1.53 €	0.31 €	1.83 €
9	1 190.00 €	0.50 €	0.5	1.49 €	1.74 €	1.53 €	0.31 €	1.83 €
10	899.00 €	0.37 €	0.5	1.49 €	1.68 €	1.53 €	0.31 €	1.83 €
11	900.00 €	0.38 €	0.5	1.49 €	1.68 €	1.53 €	0.31 €	1.83 €
12	1 190.00 €	0.50 €	0.5	1.49 €	1.74 €	1.53 €	0.31 €	1.83 €
13	789.00 €	0.33 €	0.5	1.49 €	1.65 €	1.53 €	0.31 €	1.83 €
14	1 010.00 €	0.42 €	0.5	1.49 €	1.70 €	1.53 €	0.31 €	1.83 €
16	792.00 €	0.33 €	0.5	1.49 €	1.66 €	1.53 €	0.31 €	1.83 €
203	785.00 €	0.33 €	0.5	1.49 €	1.65 €	1.53 €	0.31 €	1.83 €
209	801.00 €	0.33 €	0.5	1.49 €	1.66 €	1.53 €	0.31 €	1.83 €
439	776.00 €	0.32 €	0.5	1.49 €	1.65 €	1.53 €	0.31 €	1.83 €
446	804.00 €	0.34 €	0.5	1.49 €	1.66 €	1.53 €	0.31 €	1.83 €
459	827.00 €	0.34 €	0.5	1.49 €	1.66 €	1.53 €	0.31 €	1.83 €
460	881.00 €	0.37 €	0.5	1.49 €	1.67 €	1.53 €	0.31 €	1.83 €
461	900.00 €	0.38 €	0.5	1.49 €	1.68 €	1.53 €	0.31 €	1.83 €
462	759.00 €	0.32 €	0.5	1.49 €	1.65 €	1.53 €	0.31 €	1.83 €
463	1 300.00 €	0.54 €	0.5	1.49 €	1.76 €	1.53 €	0.31 €	1.83 €
464	1 190.00 €	0.50 €	0.5	1.49 €	1.74 €	1.53 €	0.31 €	1.83 €
465	849.00 €	0.35 €	0.5	1.49 €	1.67 €	1.53 €	0.31 €	1.83 €
468	810.00 €	0.34 €	0.5	1.49 €	1.66 €	1.53 €	0.31 €	1.83 €
1050	827.00 €	0.34 €	0.5	1.49 €	1.66 €	1.53 €	0.31 €	1.83 €
801	832.00 €	0.35 €	0.5	1.49 €	1.66 €	1.53 €	0.31 €	1.83 €

Appendix II – Distribution Centre Costs

Table 16 - Costs of handling at the distribution centre

average wage (€)	average wage/min (€/min)	time spent on prepack handling (min)	h^{w}	time spent on single size item handling (min)	h_{k}^{wv}
665.00 €	0.28 €	1	0.28 €	1	0.28 €

Appendix III – CPLEX language

Figure 15 - Model on CPLEX language

```
/*********************************************
 * OPL 20.1.0.0 Model
 * Author: ACCAZEVEDO
 * Creation Date: 03/05/2021 at 18:10:08
 *********************************************/
{string} size=...;
{int} store=...;
int qk[size]=...;
int Bk[size]=...;
int bk[size]=...;
int dkj[size][store]=...;
float hw=...;
float hsj[store]=...;
float hwvk[size]=...;
float hsvkj[size][store]=...;
float wkj[size][store]=...;
float zkj[size][store]=...;

dvar int+ xj[store];
dvar int+ vkj[size][store];
dvar int+ ukj[size][store];
dvar int+ okj[size][store];



minimize
 sum(k in size, j in store) wkj[k][j]*okj[k][j] +
 sum(k in size, j in store) zkj[k][j]*ukj[k][j] +
 hw*sum(j in store) xj[j] +
sum(k in size, j in store) (hwvk[k]*vkj[k][j]) +
sum(j in store) hsj[j]*xj[j] +
sum(k in size, j in store) (hsvkj[k][j]*vkj[k][j]);




subject to {

sum(j in store) xj[j] <= 598;

forall (k in size)
sum(j in store) vkj[k][j]<=Bk[k]*bk[k];

forall (k in size,j in store)
ukj[k][j]>= dkj[k][j] - (qk[k]*xj[j] + vkj[k][j]);


forall (k in size,j in store)
okj[k][j]>= (qk[k]*xj[j] + vkj[k][j]) - dkj[k][j];

}
```

Appendix IV – Our solution: Variables results

Table 17 – Our solution: First Quarter Results

#store	θ_{kj}					μ_{kj}					v_{kj}					x_j
	S	M	L	XL	XXL	S	M	L	XL	XXL	S	M	L	XL	XXL	
1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	4
2	0	1	0	0	0	0	0	0	0	0	1	0	13	0	1	0
3	0	0	0	0	0	0	0	0	0	0	0	3	4	0	2	2
4	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	3
5	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	3
6	0	0	0	0	0	0	0	0	0	0	1	0	5	0	1	3
7	0	0	0	0	0	0	2	0	0	0	2	1	3	1	0	2
8	0	0	0	1	0	0	0	0	0	0	0	1	1	0	0	4
9	0	0	2	6	2	0	0	0	0	0	2	1	1	0	0	0
10	0	0	0	0	0	0	0	0	0	0	4	4	2	1	0	3
11	6	3	6	1	1	0	0	0	0	0	0	1	1	0	0	2
12	0	1	1	1	1	0	0	0	0	0	0	1	1	0	0	4
13	1	0	0	1	0	0	0	0	0	0	0	0	1	0	0	3
14	0	1	0	0	0	0	0	0	0	0	1	0	2	1	2	2
16	0	0	0	0	0	0	0	0	0	0	0	0	2	1	0	2
203	0	0	1	0	0	0	0	0	0	0	0	0	1	2	0	4
209	0	0	0	0	0	0	0	0	0	0	0	9	2	1	0	3
439	0	8	1	0	0	0	0	0	0	0	1	0	1	0	1	2
446	0	6	0	0	0	3	0	0	0	0	0	1	1	0	0	1
459	1	0	3	1	0	0	0	0	0	0	0	1	1	0	5	3
460	0	0	1	0	0	0	0	0	0	0	5	0	1	1	4	2
461	1	0	0	0	0	0	0	0	0	0	0	1	2	0	2	3
462	0	0	0	0	0	0	0	0	0	0	1	1	1	0	2	3
463	0	0	0	0	0	0	0	0	0	0	2	0	2	1	1	0
464	0	0	2	0	0	0	0	0	0	0	0	1	1	0	1	0
465	0	0	0	0	0	0	0	0	0	0	0	7	6	1	2	2
468	0	0	0	1	0	0	0	0	0	0	2	1	2	0	0	2
801	0	0	0	2	0	0	0	0	0	0	1	0	8	0	1	3
1050	0	0	0	0	0	0	0	0	0	0	1	5	5	0	1	2
Total	9	20	17	14	4	3	2	0	0	0	24	40	73	11	28	67

Table 18 – Our solution: Second Quarter Results

#store	θ_{kj}										v_{kj}					x_j
	S	M	L	XL	XXL	S	M	L	XL	XXL	S	M	L	XL	XXL	
1	0	1	0	3	0	0	0	0	0	0	1	4	3	0	1	4
2	0	0	0	0	0	0	0	0	0	0	0	3	6	0	2	4
3	0	0	0	0	0	0	0	0	0	0	3	4	7	1	0	2
4	0	0	0	0	2	0	0	0	0	0	2	4	4	0	0	4
5	0	0	0	0	0	0	0	0	0	0	0	3	3	1	0	2
6	0	0	1	1	0	0	0	0	0	0	1	3	4	0	0	7
7	0	0	0	0	0	0	0	0	0	0	0	3	3	2	0	2
8	1	0	0	2	0	0	0	0	0	0	0	2	5	0	0	3
9	0	0	2	0	1	0	0	0	0	0	2	2	5	2	0	5
10	0	0	0	0	1	0	0	0	0	0	0	2	4	0	0	1
11	0	0	1	1	2	0	0	0	0	0	3	0	3	0	0	3
12	0	0	0	1	1	0	0	0	0	0	1	1	2	0	0	5
13	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	5
14	2	0	1	0	0	0	0	0	0	0	0	1	3	2	0	2
16	0	0	1	0	0	0	0	0	0	0	0	0	4	1	0	2
203	0	0	0	0	0	0	0	0	0	0	0	3	4	1	1	5
209	0	0	0	0	0	0	0	0	0	0	0	3	2	0	0	3
439	1	0	1	0	0	0	0	0	0	0	0	5	3	0	0	0
446	0	0	0	0	0	1	0	0	0	0	0	3	6	0	0	2
459	0	1	4	1	2	0	0	0	0	0	0	2	6	0	0	3
460	1	0	1	0	0	0	0	0	0	0	0	0	2	2	0	2
461	1	0	0	0	0	0	0	0	0	0	0	0	3	1	0	4
462	0	0	0	0	0	0	0	0	0	0	2	2	2	0	1	3
463	0	0	0	0	0	0	0	0	0	0	2	3	3	1	2	6
464	0	0	0	0	0	0	0	0	0	0	1	2	5	2	1	1
465	0	0	1	0	0	0	0	0	0	0	2	1	2	0	1	2
468	0	0	0	2	0	0	0	0	0	2	2	1	5	0	0	2
801	0	0	0	0	0	0	0	0	0	0	2	2	7	1	1	3
1050	0	0	0	0	0	0	0	0	0	0	0	4	5	1	2	1
Total	6	2	13	11	9	1	0	0	0	2	24	63	114	18	12	88

Table 19 – Our solution: Third Quarter Results

#store	θ_{kj}					μ_{kj}					v_{kj}					x_j
	S	M	L	XL	XXL	S	M	L	XL	XXL	S	M	L	XL	XXL	
1	3	0	0	5	5	0	0	0	0	0	0	1	6	0	0	9
2	12	0	39	0	0	0	0	0	0	0	0	1	6	2	0	26
3	34	0	23	0	17	0	0	0	0	0	0	2	5	1	0	8
4	0	0	0	2	0	0	0	0	0	0	1	4	5	0	0	23
5	0	0	0	1	10	0	0	0	0	0	1	1	7	0	0	5
6	4	0	2	0	0	0	0	0	0	0	0	4	4	1	1	22
7	0	0	23	8	0	0	12	0	0	0	1	2	6	0	3	18
8	4	0	0	8	0	0	0	0	0	0	0	2	7	0	3	11
9	0	0	0	0	0	0	0	18	25	15	0	2	5	0	0	27
10	2	1	2	4	0	0	0	0	0	0	0	1	6	0	23	6
11	0	0	0	0	0	5	0	0	0	0	0	18	23	1	1	4
12	0	0	0	0	0	42	19	19	20	0	0	1	4	0	2	20
13	0	0	0	3	0	8	0	0	0	0	0	7	6	0	5	20
14	0	0	0	0	1	0	0	0	0	0	1	6	18	2	0	5
16	2	0	2	0	1	0	0	0	0	0	0	2	5	1	0	2
203	2	0	0	0	0	0	0	0	0	0	0	1	16	1	11	8
209	0	0	0	0	5	0	0	0	0	0	3	1	5	0	0	7
439	0	0	0	0	0	0	0	1	0	0	2	22	7	0	0	12
446	36	0	0	0	0	0	2	0	0	0	0	1	6	0	4	8
459	0	0	0	0	0	0	0	1	0	0	1	6	4	0	21	11
460	0	0	10	0	27	0	0	0	0	0	1	8	3	0	0	4
461	0	2	0	0	0	0	0	0	0	0	4	1	17	0	0	23
462	4	0	0	0	0	0	0	0	0	0	0	1	18	1	0	19
463	0	0	0	5	0	0	0	0	0	0	0	8	5	0	0	15
464	0	3	0	0	0	0	0	0	0	0	0	1	11	2	0	5
465	1	31	0	0	1	0	0	0	0	0	0	0	4	1	0	10
468	0	0	6	3	0	0	0	0	0	4	3	4	6	0	0	23
801	11	0	0	12	0	0	0	0	0	0	0	2	7	0	1	9
1050	0	0	2	7	0	0	0	0	0	0	1	6	4	0	0	3
Total	115	37	109	58	67	55	33	39	45	19	19	116	226	13	75	363

Table 20 – Our solution: Fourth Quarter Results

#store	θ_{kj}					μ_{kj}					v_{kj}					x_j
	S	M	L	XL	XXL	S	M	L	XL	XXL	S	M	L	XL	XXL	
1	1	2	1	0	0	0	0	0	0	0	0	1	3	0	5	5
2	0	0	0	0	0	0	0	0	0	0	3	2	14	2	7	4
3	0	0	0	0	0	0	0	0	0	0	4	2	3	1	0	2
4	0	0	0	0	0	0	0	0	0	0	0	5	3	0	0	3
5	0	0	0	0	0	0	0	0	0	0	0	2	8	2	0	3
6	0	0	0	0	0	0	0	0	0	0	0	1	16	2	6	3
7	0	0	0	0	0	0	19	0	0	0	4	0	16	1	2	3
8	0	0	0	0	0	0	0	0	0	0	4	6	5	0	1	7
9	0	0	9	8	0	0	0	0	0	0	4	6	3	0	0	4
10	0	0	0	0	0	0	0	0	0	0	0	2	5	0	4	2
11	2	10	1	0	0	0	0	0	0	0	0	2	3	1	0	2
12	3	1	7	0	7	0	0	0	0	0	0	1	3	0	0	7
13	1	11	1	0	8	0	0	0	0	0	0	0	5	1	0	3
14	0	0	0	0	0	0	0	0	0	0	0	15	3	2	3	0
16	1	0	0	0	0	0	0	0	0	0	0	2	4	0	13	0
203	0	0	0	0	1	0	0	0	0	0	0	2	4	2	0	4
209	4	0	0	0	0	0	0	0	0	0	0	1	3	1	1	2
439	1	5	10	2	0	0	0	0	0	0	0	3	3	0	3	2
446	2	4	0	0	2	0	0	0	0	0	0	3	2	0	0	2
459	0	0	0	0	3	0	0	2	0	0	1	4	0	0	0	3
460	0	0	0	0	0	0	0	0	0	0	0	2	5	2	7	1
461	7	0	0	0	0	0	0	0	0	0	0	1	5	0	2	5
462	0	0	0	0	0	0	0	0	0	0	3	1	3	1	0	3
463	0	0	0	0	0	0	0	0	0	0	1	4	3	2	1	3
464	0	0	0	0	0	0	0	0	0	0	0	1	5	1	0	2
465	0	0	0	0	0	0	0	0	0	0	3	5	3	1	7	2
468	0	0	0	0	2	0	0	0	0	0	3	5	6	0	0	0
801	0	0	0	0	0	0	0	0	0	0	1	5	5	2	1	3
1050	0	0	0	0	0	0	0	0	0	0	1	3	7	1	2	0
Total	22	33	29	10	23	0	19	2	0	0	32	87	148	25	65	80

Appendix V – Current policy solution: Variables results

Table 21 - Current policy solution: First Quarter values

#store	θ_{kj}					μ_{kj}					ν_{kj}					x_j
	S	M	L	XL	XXL	S	M	L	XL	XXL	S	M	L	XL	XXL	
1	4	3	2	4	2	0	0	0	0	0	0	0	0	0	0	8
2	3	5	6	5	3	0	0	0	0	0	0	0	0	0	0	9
3	18	3	5	9	2	0	0	0	0	0	0	0	0	0	0	5
4	2	3	2	3	3	0	0	0	0	0	0	0	0	0	0	6
5	2	2	2	2	9	0	0	0	0	0	0	0	0	0	0	5
6	2	2	5	3	3	0	0	0	0	0	0	0	0	0	0	8
7	6	4	2	5	14	0	0	0	0	0	0	0	0	0	0	4
8	5	2	4	4	2	0	0	0	0	0	0	0	0	0	0	7
9	2	5	10	12	6	0	0	0	0	0	0	0	0	0	0	9
10	3	3	1	2	8	0	0	0	0	0	0	0	0	0	0	5
11	16	9	10	7	5	0	0	0	0	0	0	0	0	0	0	6
12	6	9	5	7	5	0	0	0	0	0	0	0	0	0	0	9
13	5	18	6	5	18	0	0	0	0	0	0	0	0	0	0	9
14	5	5	9	5	2	0	0	0	0	0	0	0	0	0	0	5
16	6	4	5	7	4	0	0	0	0	0	0	0	0	0	0	4
203	6	4	5	10	6	0	0	0	0	0	0	0	0	0	0	7
209	4	9	5	7	10	0	0	0	0	0	0	0	0	0	0	7
439	5	16	11	4	7	0	0	0	0	0	0	0	0	0	0	4
446	3	10	20	14	4	0	0	0	0	0	0	0	0	0	0	5
459	5	7	9	5	12	0	0	0	0	0	0	0	0	0	0	5
460	6	4	7	3	0	0	0	0	0	0	0	0	0	0	0	5
461	7	3	2	2	3	0	0	0	0	0	0	0	0	0	0	6
462	2	1	3	2	1	0	0	0	0	0	0	0	0	0	0	5
463	2	2	2	1	1	0	0	0	0	0	0	0	0	0	0	11
464	9	2	4	9	2	0	0	0	0	0	0	0	0	0	0	7
465	4	3	1	3	4	0	0	0	0	0	0	0	0	0	0	4
468	2	4	1	5	3	0	0	0	0	0	0	0	0	0	0	5
801	1	10	0	4	2	0	0	0	0	0	0	0	0	0	0	6
1050	3	1	5	2	4	0	0	0	0	0	0	0	0	0	0	4
Total	144	153	149	151	145	0	0	0	0	0	0	0	0	0	0	180

Table 22 - Current policy solution: Second Quarter values

#store	θ_{kj}					μ_{kj}					v_{kj}					x_j
	S	M	L	XL	XXL	S	M	L	XL	XXL	S	M	L	XL	XXL	
1	13	18	11	15	16	0	0	0	0	0	0	0	0	0	0	18
2	22	11	27	20	20	0	0	0	0	0	0	0	0	0	0	19
3	27	12	30	16	22	0	0	0	0	0	0	0	0	0	0	13
4	7	9	3	14	13	0	0	0	0	0	0	0	0	0	1	22
5	13	10	11	5	15	0	0	0	0	0	0	0	0	0	0	13
6	10	11	19	8	17	0	0	0	0	0	0	0	0	0	1	19
7	16	28	21	23	8	0	0	0	0	0	0	0	0	0	2	13
8	11	9	8	16	11	0	0	0	0	0	0	0	0	0	0	20
9	20	18	30	32	23	0	0	0	0	0	0	0	0	0	2	23
10	13	2	10	9	29	0	0	0	0	0	0	0	0	0	0	10
11	18	34	19	21	10	0	0	0	0	0	0	0	0	0	0	10
12	27	22	26	21	23	0	0	0	0	0	0	0	0	0	2	22
13	12	30	22	22	36	0	0	0	0	0	0	0	0	0	2	15
14	36	21	25	32	18	0	0	0	0	0	0	0	0	0	1	12
16	20	8	29	21	26	0	0	0	0	0	0	0	0	0	0	6
203	20	21	11	29	21	0	0	0	0	0	0	0	0	0	0	19
209	22	33	14	34	20	0	0	0	0	0	0	0	0	0	2	14
439	15	30	21	22	16	0	0	0	0	0	0	0	0	0	0	11
446	27	22	22	8	22	0	0	0	0	0	0	0	0	0	0	10
459	22	17	32	23	28	0	0	0	0	0	0	0	0	0	1	16
460	9	22	19	24	34	0	0	0	0	0	0	0	4	0	0	12
461	19	14	18	10	13	0	0	0	0	0	0	0	0	0	1	18
462	15	7	11	4	13	0	0	0	0	0	0	0	0	0	0	14
463	11	14	9	10	4	0	0	0	0	0	0	0	0	0	0	19
464	14	10	10	14	6	0	0	0	0	0	0	0	0	0	1	5
465	20	24	35	15	19	0	0	0	0	0	0	0	0	0	0	13
468	17	11	8	10	13	0	0	0	0	0	0	0	0	0	0	13
801	9	11	0	6	8	0	0	0	0	0	0	0	0	0	0	12
1050	8	24	19	20	22	0	0	0	0	0	0	0	0	0	0	7
Total	493	503	520	504	526	0	0	0	0	0	0	0	4	0	16	418

Table 23 - Current policy solution: Third Quarter values

#store	θ_{kj}					μ_{kj}					v_{kj}					x_j
	S	M	L	XL	XXL	S	M	L	XL	XXL	S	M	L	XL	XXL	
1	0	0	0	0	0	5	20	5	16	4	2	17	21	0	7	0
2	5	0	6	0	0	0	6	0	13	24	7	19	25	4	5	0
3	13	0	0	0	0	0	1	4	8	3	0	4	11	0	3	0
4	0	0	0	0	0	5	3	3	1	5	6	14	28	1	11	0
5	0	0	0	0	0	16	4	8	2	12	0	17	14	0	0	0
6	0	0	0	0	0	4	15	8	19	15	4	6	6	3	2	0
7	0	0	7	4	2	19	15	0	0	0	6	3	12	4	1	0
8	0	0	0	0	0	10	10	9	16	19	2	16	25	0	7	0
9	0	0	0	0	0	28	4	36	76	25	20	24	39	7	21	0
10	0	0	0	0	0	13	2	18	2	28	2	10	19	0	6	0
11	0	0	0	0	0	53	40	31	20	5	0	4	8	0	2	0
12	0	0	0	0	0	56	31	39	28	18	8	23	17	0	18	0
13	0	0	0	0	0	14	61	17	29	58	7	16	35	5	10	0
14	0	1	0	0	0	39	0	58	34	1	6	9	22	7	1	0
16	1	0	1	0	0	0	3	0	22	1	0	2	2	0	0	0
203	0	0	0	0	0	6	11	16	40	25	5	15	19	2	2	0
209	0	0	0	0	0	39	36	4	21	21	2	7	11	2	3	0
439	0	0	0	0	0	19	66	41	24	16	3	9	4	0	13	0
446	0	0	0	0	0	10	39	22	8	40	0	3	5	0	2	0
459	0	0	0	0	0	19	20	40	31	57	3	20	30	2	6	0
460	0	0	5	3	3	4	36	0	0	0	0	0	2	0	2	0
461	0	0	0	0	0	33	0	28	5	23	0	9	45	2	1	0
462	0	0	0	0	0	0	0	18	3	6	3	7	22	1	6	0
463	0	0	0	0	0	4	13	2	0	3	0	5	26	10	12	0
464	0	0	0	0	0	24	2	24	12	2	2	7	15	2	6	0
465	5	0	3	6	13	0	13	0	0	0	5	9	13	4	5	0
468	0	0	7	0	0	9	15	0	3	27	3	18	18	10	7	0
801	0	0	0	0	0	2	3	0	4	6	3	13	15	1	5	0
1050	0	0	0	15	0	4	3	1	0	6	0	0	0	0	0	0
Total	24	1	29	28	18	435	472	432	437	450	99	306	509	67	164	0

Table 24 - Current policy solution: Fourth Quarter values

#store	θ_{kj}					μ_{kj}					v_{kj}					x_j
	S	M	L	XL	XXL	S	M	L	XL	XXL	S	M	L	XL	XXL	
1	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	10	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	19	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	5	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
203	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0
209	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
439	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
446	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0
459	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0
460	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
461	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
462	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
463	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
464	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0
465	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0
468	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0
801	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
1050	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Total	0	0	0	0	0	0	19	2	0	0	0	0	48	0	0	0

Appendix VI – Comparison of clusters: allocations

Figure 16 - Number of single items size S sent to each store cluster

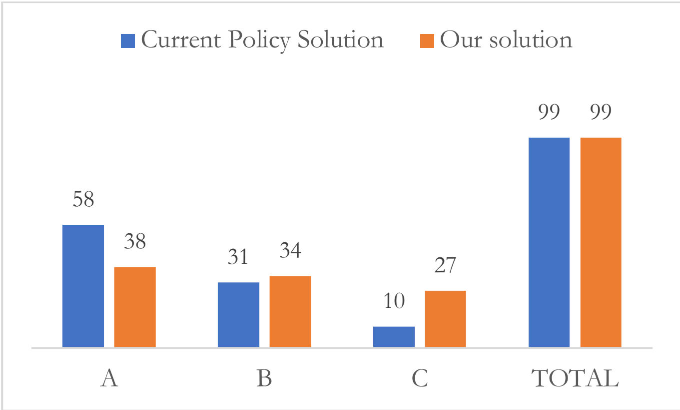


Figure 17 - Number of single items size L sent to each store cluster

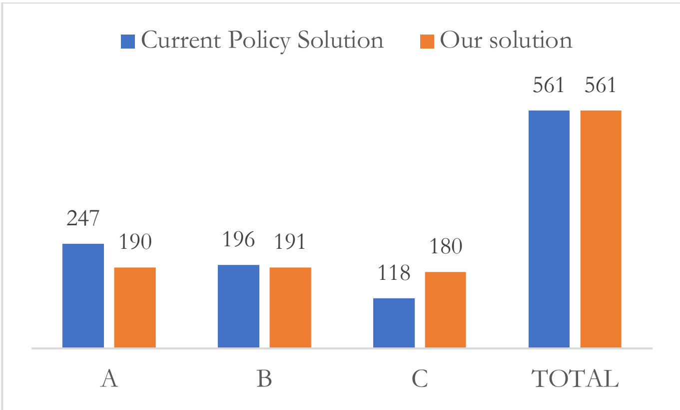


Figure 18 - Number of single items size XXL sent to each store cluster

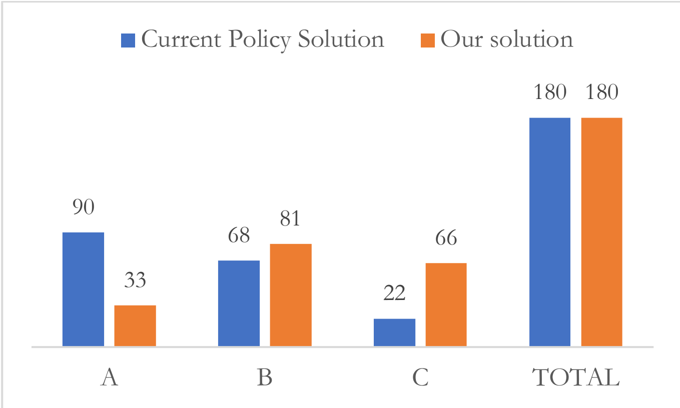
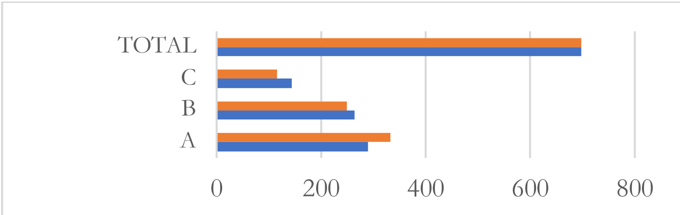
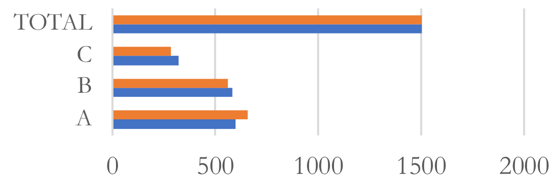


Figure 19 - Total quantity of size S sent to each store cluster



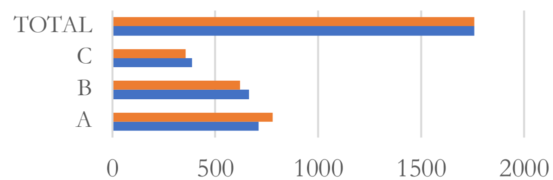
	A	B	C	TOTAL
Our solution	333	249	115	697
Current Policy Solution	289	264	144	697

Figure 20 - Total quantity of size M sent to each store cluster



	A	B	C	TOTAL
Our solution	658	561	283	1502
Current Policy Solution	598	583	321	1502

Figure 21 - Total quantity of size L sent to each store cluster



	A	B	C	TOTAL
Our solution	780	621	356	1757
Current Policy Solution	709	662	386	1757

Figure 22 - Total quantity of size XXL sent to each store cluster

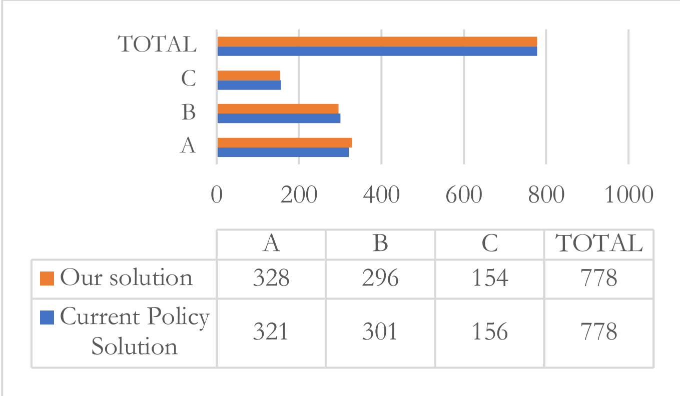


Table 25 - Proportion of the total quantity of size S sent to each cluster

	% quantity of size S items sent		% RNS
Cluster	Current Policy Solution	Our solution	
A	41%	48%	50%
B	38%	36%	36%
C	21%	16%	14%
TOTAL	100%	100%	100%

Table 26 - Proportion of the total quantity of size M sent to each cluster

	% quantity of size M items sent		% RNS
Cluster	Current Policy Solution	Our solution	
A	40%	44%	50%
B	39%	37%	36%
C	21%	19%	14%
TOTAL	100%	100%	100%

Table 27 - Proportion of the total quantity of size L sent to each cluster

	% quantity of size L items sent		% RNS
Cluster	Current Policy Solution	Our solution	
A	40%	44%	50%
B	38%	35%	36%
C	22%	20%	14%
TOTAL	100%	100%	100%

Table 28 - Proportion of the total quantity of size XXL sent to each cluster

	% quantity of size XXL items sent		% RNS
Cluster	Current Policy Solution	Our solution	
A	41%	42%	50%
B	39%	38%	36%
C	20%	20%	14%
TOTAL	100%	100%	100%

Appendix VII – Comparison of clusters: stocks

Figure 23 - Comparison of overstock and understock in each cluster: size S

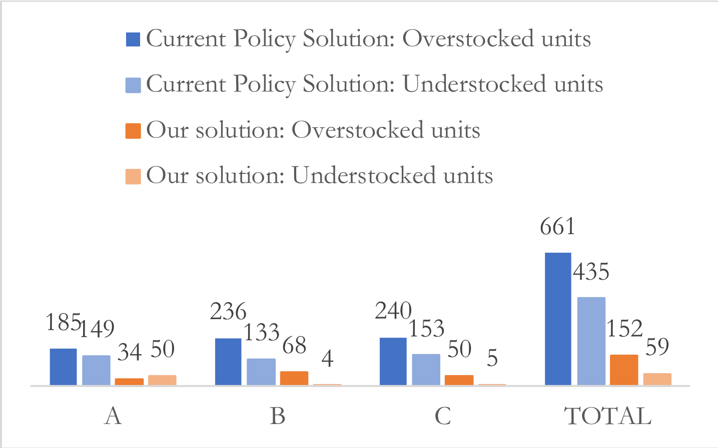


Figure 24 - Comparison of overstock and understock in each cluster: size M

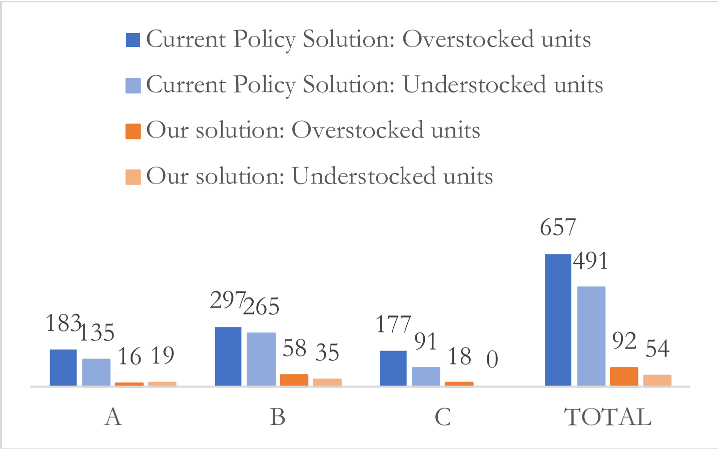


Figure 25 - Comparison of overstock and understock in each cluster: size L

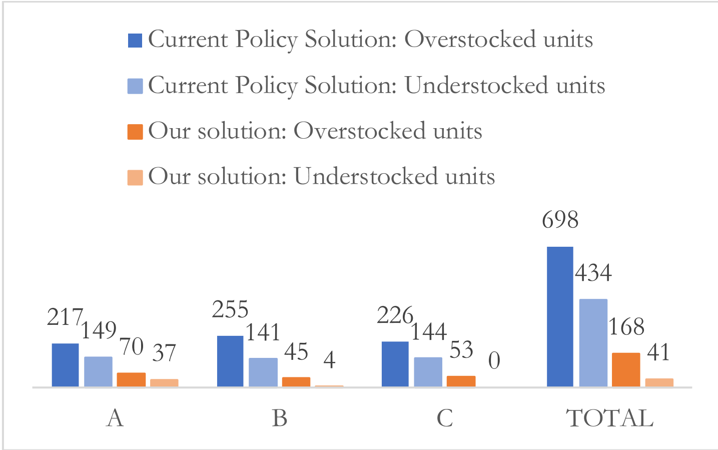
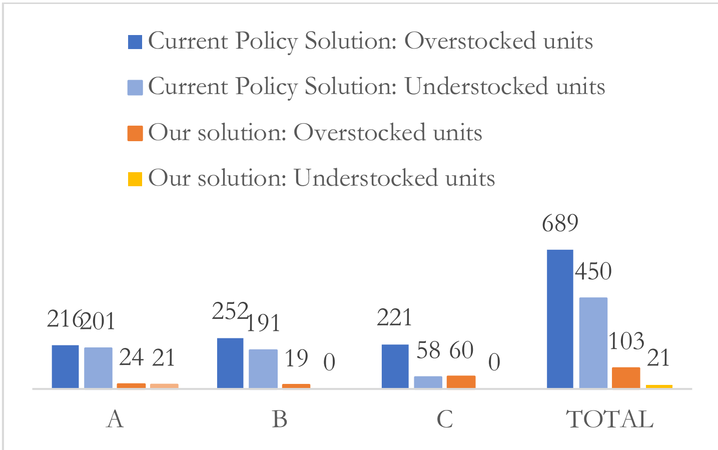


Figure 26 - Comparison of overstock and understock in each cluster: size XXL



Appendix VIII – Comparison of costs

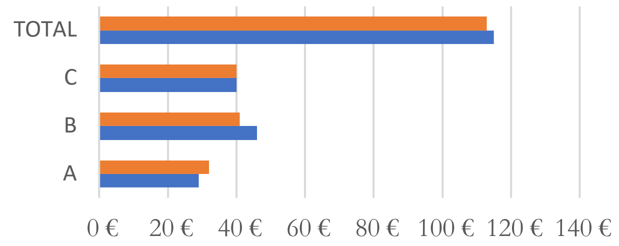
Table 29 – Our solution: total costs

	Our solution						
	Store Costs				DC Costs		Total Costs
	Handling prepacks	Handling single size items	Overstock	Understock	Handling prepacks	Handling single size items	
1T	120.54 €	201.30 €	107.81 €	9.15 €	18.56 €	48.77 €	506.13 €
2T	170.45 €	259.75 €	68.84 €	5.49 €	24.38 €	64.01 €	592.91 €
3T	698.64 €	511.57 €	647.86 €	349.53 €	100.58 €	124.41 €	2 432.58 €
4T	154.12 €	408.05 €	197.11 €	38.43 €	22.17 €	98.92 €	918.80 €
TOTAL	1 143.74 €	1 380.66 €	1 021.62 €	402.60 €	165.70 €	336.10 €	4 450.42 €

Table 30 - Current policy solution: total costs

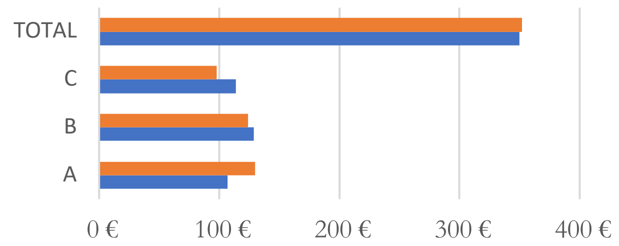
	Current policy solution						
	Store Costs				DC Costs		Total Costs
	Handling prepacks	Handling single size items	Overstock	Understock	Handling prepacks	Handling single size items	
1T	349.82 €	0.00 €	1 246.14 €	0.00 €	49.88 €	0.00 €	1 645.83 €
2T	799.32 €	21.33 €	4 279.07 €	0.00 €	115.82 €	5.54 €	5 221.08 €
3T	0.00 €	1 292.68 €	170.01 €	4 073.58 €	0.00 €	317.26 €	5 853.53 €
4T	0.00 €	52.26 €	0.00 €	38.43 €	0.00 €	13.30 €	103.99 €
TOTAL	1 149.13 €	1 366.27 €	5 695.21 €	4 112.01 €	165.70 €	336.10 €	12 824.42 €

Figure 27 - Total costs of handling single-size items in each cluster: size S



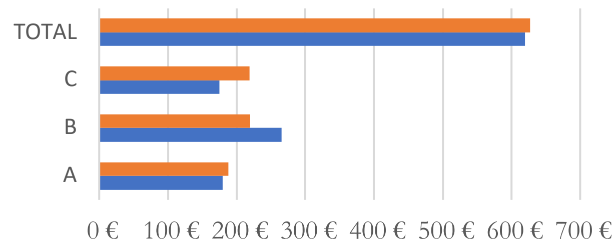
	A	B	C	TOTAL
Our solution	32 €	41 €	40 €	113 €
Current Policy Solution	29 €	46 €	40 €	115 €

Figure 28 - Total costs of handling single-size items in each cluster: size M



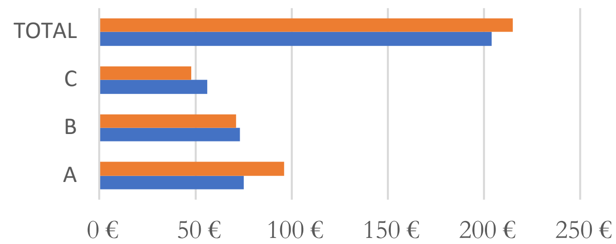
	A	B	C	TOTAL
Our solution	130 €	124 €	98 €	352 €
Current Policy Solution	107 €	129 €	114 €	350 €

Figure 29 - Total costs of handling single-size items in each cluster: size L



	A	B	C	TOTAL
Our solution	188 €	220 €	219 €	627 €
Current Policy Solution	180 €	265 €	175 €	620 €

Figure 30 - Total costs of handling single-size items in each cluster: size XXL



	A	B	C	TOTAL
Our solution	96 €	71 €	48 €	215 €
Current Policy Solution	75 €	73 €	56 €	204 €

Figure 31 - Inventory and stockout costs in each cluster: size S

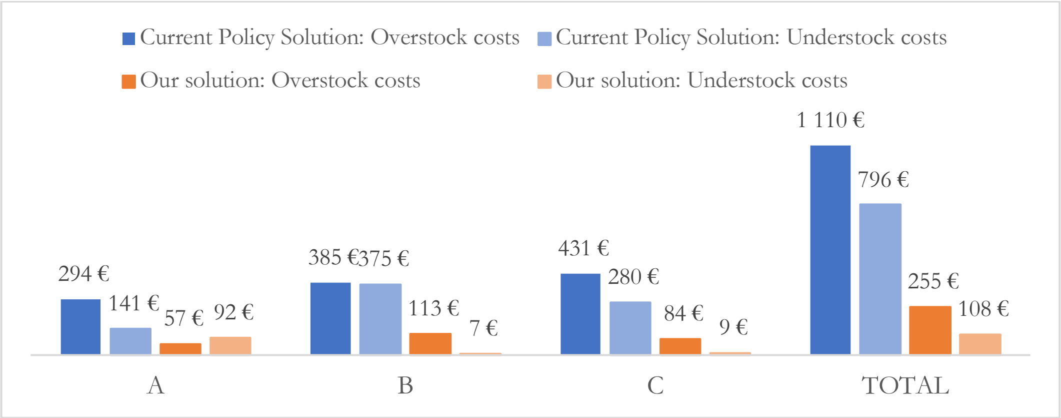


Figure 32 - Inventory and stockout costs in each cluster: size M

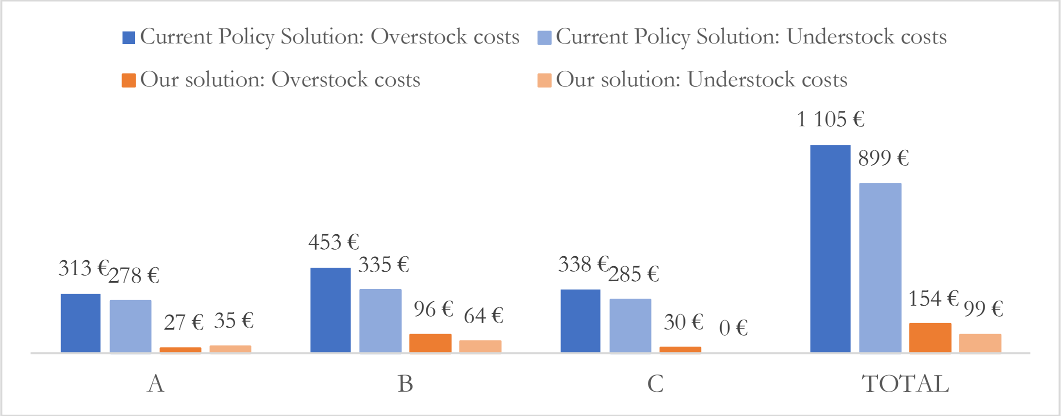


Figure 33 - Inventory and stockout costs in each cluster: size L

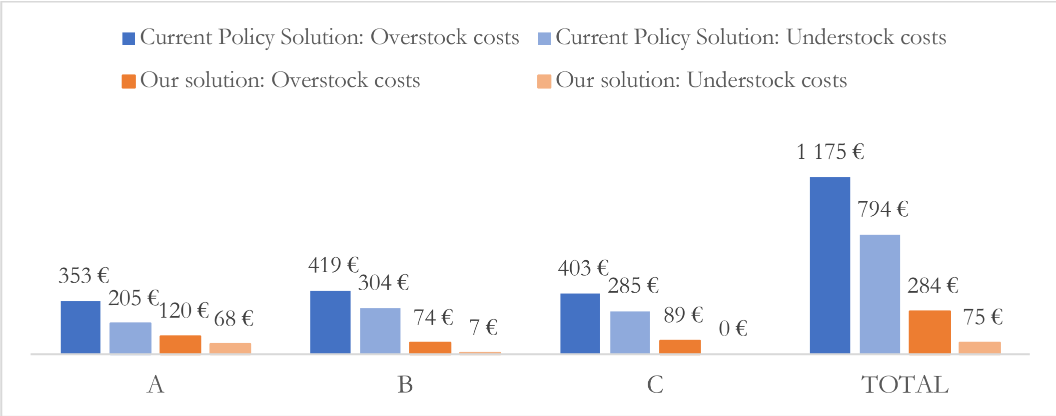
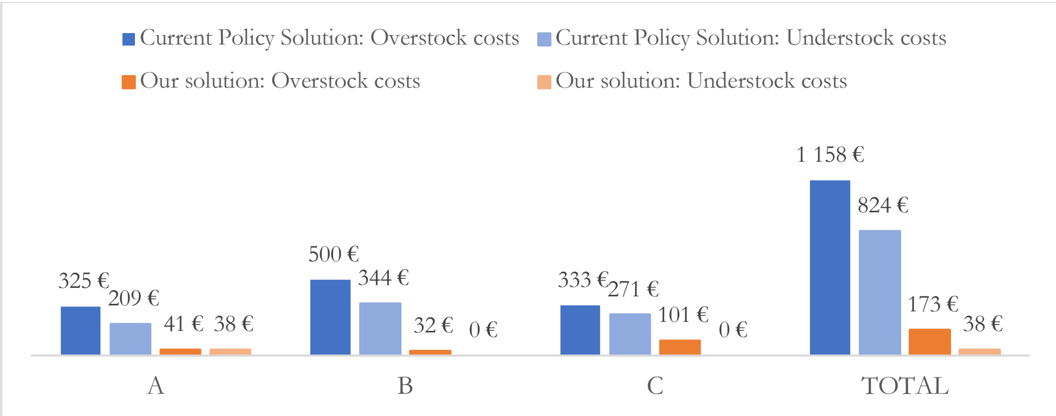


Figure 34 - Inventory and stockout costs in each cluster: size XXL



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