The conceptualisation and measurement of pedagogical content knowledge and content knowledge in the COACTIV study and their impact on student learning

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Abstract

An ongoing question is the extent to which teachers' professional knowledge has an impact on their teaching and, in particular, on their students' achievement. The COACTIV[1] study surveyed and tested the mathematics teachers of the classes sampled for PISA 2003/04 in Germany. The study's key components were newly developed tests of teachers' pedagogical content knowledge and content knowledge. This article gives a report of the conceptualisation and operationalisation of both domains of knowledge and describes the construction of the COACTIV tests. Findings from the tests show that there are differences with respect to both knowledge domains regarding teachers' school types, but that pedagogical content knowledge and content knowledge astoundingly both do not depend on teaching experience. Furthermore we show that the two domains of knowledge correlate positively with constructivist teachers' subjective beliefs, on the one hand, and with some crucial aspects of their instruction, on the other hand. Finally, we show that pedagogical content knowledge – but not pure content knowledge *per se* – significantly contributes to students' learning gains.

The COACTIV Study 2003/2004

Although the essential influence of teachers on students' learning is obvious, empirical studies which assess aspects of the teachers' professional knowledge systematically, and link them with the students' achievement, are

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1 COACTIV was a collaborative project, running 2002–2008, based at the MPI Berlin (Max-Planck Institute for Human Development; project director: Jürgen Baumert, project staff: Stefan Krauss, Mareike Kunter *et al.*), with the Universities of Kassel (director: Werner Blum) and Oldenburg (director: Michael Neubrand) as partner institutions. The COACTIV study was funded by the German Research Foundation (DFG) as a component of its BIQUA priority program on the quality of schools; see Kunter, Baumert, Blum, Klusmann, Krauss and Neubrand (forthcoming) for more details.

very rare. The main goal of the German COACTIV study (**Co**gnitive **Activ**ation in the Classroom: Professional Competence of Teachers, Cognitively Activating Instruction, and Development of Students′ Mathematical Literacy) was the investigation and testing of mathematics teachers of German PISA classes. The international PISA[2] study 2003, whose main focus lay in the subject of mathematics, has been extended in Germany both to a study based on whole classes (220 altogether) and to a longitudinal study, which means that the students of the grade 9 classes which were tested in PISA 2003 were examined again in grade 10 in the following year. Following this pattern, the COACTIV study investigated the mathematics teachers who taught these PISA classes in grade 9 and grade 10 at both PISA study dates (April 2003 and April 2004; therefore "COACTIV 03/04").

The COACTIV study 03/04, together with PISA, offered a unique opportunity to collect a broad range of data about both the students and their teachers, and to analyse them mutually. Due to the data of the COACTIV study it is not only possible to get an idea of the competencies and experiences of German secondary mathematics teachers, but it is possible to identify characteristics of a teacher empirically as well, which are relevant for the learning progress of students (or for different target criteria of mathematics lessons). In the context of the COACTIV study, numerous instruments for the investigation of mathematics teachers were newly developed or adapted (they include the measurement of professional knowledge, of motivational orientations, beliefs and values, aspects of work-life experiences etc.; a more detailed overview on that study is available in the book: "*Teachers' professional competence: Findings of the COACTIV research program*" (Kunter, Baumert, Blum, Klusmann, Krauss and Neubrand, forthcoming).

Figure 1 illustrates various aspects in which COACTIV collected data together with PISA. Together with the instruments which were used in PISA to examine the students, the teachers were presented with both questionnaires (regarding biography, interests, beliefs and more) and tests (e.g., regarding professional knowledge) in COACTIV. But what should a 'test' for teachers look like? With which knowledge should mathematics teachers be equipped?

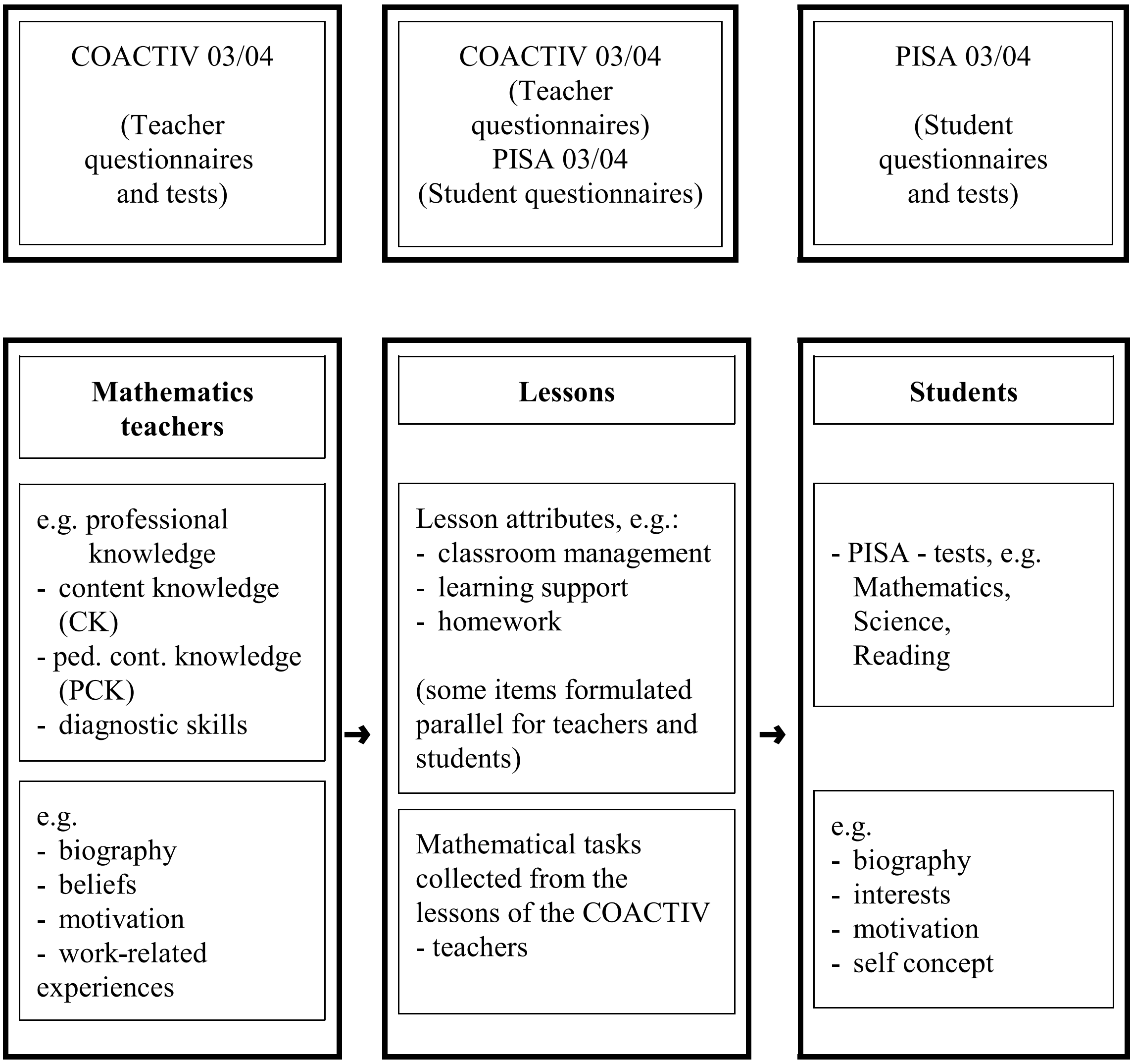
From the point of view of mathematics, the pedagogical content knowledge (PCK) and the content knowledge (CK) are of special interest as central parts

2 The OECD Programme for International Student Assessment, see http://www.oecd.org/pisa/

of the professional knowledge base (see Figure 1, left column). In the context of the COACTIV study, tests for mathematics teachers were developed for both knowledge categories which form the core of this study and which will be presented in the present paper in more detail.

In the above-mentioned book (Kunter *et al*., forthcoming) the interested reader can learn more about results of other aspects which have been examined in the COACTIV study, for example, about the teachers' experience of stress and 'burn out', about enthusiasm or about beliefs (see also left column in Figure 1), about aspects of mathematics lessons in PISA classes from the point of view of teachers and students, and about the mathematics tasks used by teachers (middle column in Figure 1). Interesting results about students (right column in Figure 1) can be taken from the respective PISA book (OECD, 2004).

Figure 1: Conceptual connection of the COACTIV 03/04 study and the PISA 03/04 study and sample aspects of three columns examined: mathematics teachers, mathematics lessons and students



Professional knowledge of mathematics teachers

At the outset of studies about teachers in the first half of the last century, the notion of *personality* was in the foreground. From the 1950s onwards, it was the teachers' *behaviour* in particular which was the object of research. Today it is the general opinion that above all the *professional knowledge* of teachers plays a crucial role in the regulation of behaviour and therefore in the control of the teaching and learning processes (a famous quote from Elbaz, 1983, says: "The single factor which seems to have the greatest power to carry forward our understanding of the teachers' role is the phenomenon teachers' knowledge."). Note that 'knowledge' here cannot be identified with declarative knowledge, in fact it must, in large parts, be regarded as procedural knowledge as well (routines, skills, abilities, competence) (cf. Weinert, Schrader and Helmke, 1990).

However, with which knowledge should teachers be equipped? The theoretical structuring of the teachers' knowledge into distinguishable categories is traced via so-called taxonomical approaches. One of the most influential knowledge taxonomies for teachers is the one of Lee Shulman (1986). Shulman introduced, among other categories, the domains of *pedagogical knowledge, content knowledge and pedagogical content knowledge.* These three categories form, seen from a contemporary point of view, the generally accepted core categories of the teachers' professional knowledge (e.g., Kunter *et al.*, forthcoming).

Considering the teachers' professional knowledge, lots of questions have remained unanswered over a long time: When is this knowledge acquired? How can it be measured? How does this knowledge influence lesson planning and the learning progress of students? An empirically valid answer to these questions requires that the relevant knowledge categories are made measurable. COACTIV sought to fill the gap in research concerning the two special knowledge categories, the pedagogical content knowledge and the content knowledge of mathematics teachers. Pedagogical knowledge (including the knowledge for the optimisation of the teaching-learning situation in general, e.g., classroom management, lesson structure, time management, discipline and the like), which should be essentially the same for teachers of different subjects, is not addressed in the present paper. A corresponding test construction for pedagogical knowledge has been developed (see Kunter *et al.*, forthcoming) in the context of the follow-up study COACTIV-R (in which trainee teachers have been assessed).

How can pedagogical content knowledge and content knowledge of mathematics teachers be conceptualised? In the following, we introduce Shulman's (1986) characterisation, which forms the base of the test construction in the COACTIV-study.

Pedagogical content knowledge (PCK) of mathematics teachers

In simple terms, Shulman (1986) defines pedagogical content knowledge as knowledge about "making content accessible". The core meaning of pedagogical content knowledge can best be taken from Shulman's original quote:

> Within the category of pedagogical content knowledge I include, for the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others. Since there are no single most powerful forms of representation, the teacher must have at hand a veritable armamentarium of alternative forms of representation, some of which derive from research whereas others originate in the wisdom of practice. Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. If those preconceptions are misconceptions, which they so often are, teachers need knowledge of the strategies most likely to be fruitful in reorganising the understanding of learners, because those learners are unlikely to appear before them as blank slates (p. 9–10).

Simply, Shulman describes two aspects of pedagogical content knowledge: on the one hand he emphasises the knowledge about explaining and representing ('the ways of representing and formulating the subject'), and on the other hand he underlines the importance of knowledge on subject-related student cognitions ('conceptions', 'preconceptions', 'misconceptions').

Attention should be paid to the fact that Shulman's description is true for every subject: teachers of all subjects should be able to represent content of their subject appropriately and should be conscious of typical misconceptions of students. It is well known that in mathematics lessons mathematical tasks play a decisive role (e.g. Christiansen and Walther, 1986; Neubrand, Jordan, Krauss, Blum and Löwen, forthcoming). Mathematical tasks offer efficient learning opportunities, and the majority of time in mathematics lessons is spent solving mathematical tasks. A substantial knowledge base on the characteristics of tasks is therefore of particular importance in mathematics lessons. It has to be taken into account that by 'knowledge about tasks' we do not mean the ability to solve mathematical tasks, but we mean the pedagogical knowledge about the potential of tasks for the learning of students (i.e. the knowledge about what a task can contribute to the students' successful knowledge construction).

Pedagogical content knowledge of the subject of mathematics was therefore conceptualised in COACTIV with three key components of knowledge:

- *knowledge about explaining and representing mathematical contents ('E&R')*
- *knowledge about mathematics related student cognitions ('StCog')*
- *knowledge about the potential of mathematical tasks ('Task')*

In Figure 2, test items can be found illustrating these three sub-facets of pedagogical content knowledge. In addition, a sample item of the test on content knowledge (see under **Content knowledge (CK) of mathematics teachers**) can be found there. The conceptualisation of the three sub-facets of pedagogical content knowledge was defined more precisely for the purpose of the operationalisation of test items in the following way:

Explaining and representing ('E&R'): Operationalisation based on lesson scenarios

The student's knowledge construction can quite often only be successful because of instructional guidance (e.g. Mayer, 2004). Mathematics teachers should be able to explain and represent mathematical issues in an appropriate way. When operationalising this aspect of pedagogical content knowledge, 11 situations in mathematical lessons were constructed in which direct support for local processes of understanding was necessary (see sample item 'minus 1 times minus 1' in Figure 2). As profound knowledge on mathematical representations means the availability of a broad range of explanations for mathematical problems, the knowledge on representations was thereby brought into focus.

Mathematics related student cognitions ('StCog'): Operationalisation as knowledge about typical errors and difficulties of students

In order to be able to teach adaptively, a teacher has to be equipped with knowledge about typical content-related student cognitions. Difficulties and errors, especially, reveal the implicit knowledge of the problem solver and therefore make cognitive processes noticeable (e.g. Matz, 1982). In order to utilise the students' errors and typical difficulties as a pedagogical opportunity, mathematics teachers must be able to identify, classify conceptually and analyse the students' errors. For operationalising pedagogical content knowledge about the students' cognitions, seven situations in mathematics lessons were constructed in which the students' errors and difficulties had to be identified and/or analysed (see sample task 'parallelogram' in Figure 2).

The potential of mathematical tasks ('Task'): Operationalisation as knowledge about the multiple potential to solve mathematical tasks

It has been pointed out repeatedly that mathematical understanding can be developed by comparing different solutions of mathematical tasks (e.g., Rittle-Johnson and Star, 2007). In order to make this issue accessible in lessons, mathematics teachers have to be able to recognise the potential of tasks for multiple solutions and they have to know what kind of structural differences are featured by these different solutions of a mathematical task. For operationalising pedagogical content knowledge about the potential of mathematical tasks, four mathematical tasks were chosen, each including an instruction for the teacher to explicate as many substantially different solutions as possible (see sample item 'square' in Figure 2).

The pedagogical content knowledge test in COACTIV therefore consists of three subtests, namely on knowledge on explaining and representing (11 items), knowledge on the students' errors and difficulties (seven items), and knowledge on multiple solutions of tasks (four items). Altogether, pedagogical content knowledge was assessed by 22 items.

Note that this conceptualisation (including the respective operationalisations) can easily be embedded into a simple model of mathematics lesson: mathematics lessons can – using the briefest phrasing – be taken as making mathematical contents accessible to students. Because of the sub-facets of the

COACTIV test for pedagogical content knowledge it is assured that each of the three pillars of mathematics lessons (contents, students, making accessible) is covered by one component of knowledge: the 'content' aspect is covered by knowledge of the potential of tasks (multiple solutions of tasks), the 'students' aspect is covered by knowledge of subject-related student cognitions (errors and difficulties of students), and the 'making accessible' aspect is covered by knowledge of 'making contents comprehensible' (explaining and representing). Of course, the present conceptualisation cannot cover the pedagogical content knowledge completely; it can rather be seen as an attempt to assess relevant facets of pedagogical content knowledge related to crucial aspects of teaching mathematics.

Other conceptualisations of PCK have been developed, in particular, by Grossman (1990), Ball, Hill and Bass (2005), Hill, Rowan and Ball (2005) or Tatto, Schwille, Senk, Ingvarson, Peck and Rowley (2008). While Tatto *et al.* (2008) investigated, in the Teacher Education and Development Study – Mathematics (TEDS-M), teacher trainees and pre-service teachers, Ball *et al.* (2005) examined the Mathematical Knowledge Needed for Teaching (MKT) of practicing elementary teachers. It is interesting to note that all projects regard knowledge on explanations and knowledge on student errors as crucial aspects of teachers' pedagogical content knowledge. All projects use the format of lesson scenarios, but whereas most of the TEDS-M items and of the items of the Michigan group (e.g., Ball *et al.*, 2005; Hill *et al.*, 2005) have multiple choice format, all PCK and CK items in the COACTIV study have an open-ended format, thus avoiding the problems typically associated with multiple choice items. The different approaches are compared in detail in Krauss, Baumert and Blum (2008) (also see the article by Adler and Patahuddin in the present volume, where the authors explicate the approach of Ball and colleagues).

Content knowledge (CK) of mathematics teachers

Content knowledge is generally seen as a necessary but not sufficient requirement for pedagogical content knowledge (e.g. see Kunter *et al.*, forthcoming). Although nobody queries that profound content knowledge is an unalienable basic requirement for successful lessons, this category of knowledge is treated a lot less extensively in literature – in comparison to pedagogical content knowledge. Shulman's conceptualisation of content knowledge says:

> To think properly about content knowledge requires going beyond knowledge of the facts or concepts of a domain. It requires understanding the structures of the subject matter [. . .]. For Schwab (1978) the structures of a subject include both the substantive and syntactic structure. The substantive structures are the variety of ways in which the basic concepts and principles of the discipline are organized to incorporate its facts. The syntactic structure of a discipline is the set of ways in which truth or falsehood, validity or invalidity, are established. [. . .]. The teacher need not only to understand that something is so, the teacher must further understand why it is so, on what grounds its warrant can be asserted, and under what circumstances our belief in its justification can be weakened and even denied (p. 9).

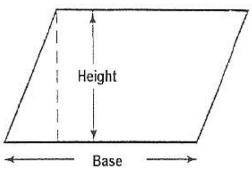
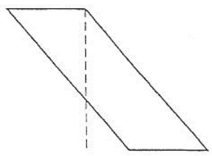
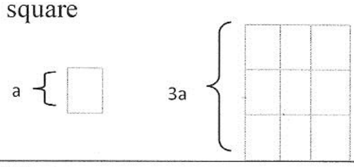
According to Shulman (1986), a teacher should be equipped – besides knowledge of mathematical facts – with the competence of argumentation and justification, e.g. for proofs or connections, within the discipline. However, Shulman's description leaves the question open, with which level of content knowledge a teacher should be equipped in particular. Does he only mean the subject matter of the school curriculum, or is it crucial to have a broad basis of university-related knowledge available? The term 'mathematical content knowledge', in principle, can refer to the following different levels:

1. mathematical everyday knowledge
2. knowledge of the subject matter of the mathematical curriculum (contents which have to be learned by students)
3. advanced background knowledge of the subject matter of the mathematical curriculum
4. mathematical knowledge which is exclusively taught at university

'Mathematical content knowledge' was conceptualised in COACTIV on the third level, i.e. as advanced background knowledge about the subject matter of the mathematical school curriculum. In order to be able to cope with mathematically challenging situations in a lesson in a competent way, teachers are expected to conceive the subject matter that they teach on an appropriate level which is obviously above the common work-level in the lessons.

Altogether, 13 items with mathematics on an advanced school knowledge level were presented to teachers in the COACTIV test of content knowledge. For content knowledge, no sub-facets were postulated theoretically (but see, e.g. Blömeke, Lehmann, Seeber, Schwarz, Kaiser, Felbrich and Müller, 2008, for such sub-facets). A sample item of this COACTIV test can be found in Figure 2.

Figure 2: Sample items (and respective sample solutions) from the COACTIV tests of mathematics teachers' PCK and CK

Category	Sample items	Sample solutions
Pedagogical content knowledge (PCK) "E&R"	**"Minus 1 times minus 1"** A student says: I don't understand why $(-1)\times(-1) = +1$ Please outline as many different ways as possible of explaining this mathematical fact to your student.	Although the principle of permanence does not prove that $(-1)x(-1) = +1$, it could be used here to promote students' conceptual understanding and to establish mental connections between concepts: -1 $2 \cdot (-1) = -2$ +1 $1 \cdot (-1) = -1$ $0 \cdot (-1) = 0$ $(-1) \cdot (-1) = 1$
Pedagogical content knowledge (PCK) "StCog"	**"Parallelogram"** The area of a parallelogram can be calculated by multiplying the length of its base by its height.  Please sketch an example of a parallelogram where students might not be able to apply this formula.	Students may have difficulties if the foot of the height is outside the parallelogram: 
Pedagogical content knowledge (PCK) "Tasks"	**"Square"** How does the area of a square change when the side length is tripled? Show your reasoning. Please write down as many different ways as possible of solving this problem (with explanations).	<u>Algebraic:</u> Area of the original square: a^2 Area of the "new" square: $(3a)^2 = 9a^2$, that is 9 times larger. <u>Geometric:</u> Nine times the size of the original square 
Content knowledge (CK) (This item was taken from the pilot phase of the study.)	**"Prime number"** Is $2^{1024} - 1$ a prime number?	No, because: $a^2 - b^2 = (a - b)(a + b)$. Therefore, $2^{1024} - 1$ can be broken down into $(2^{512} - 1)(2^{512} + 1)$

Administration of the tests

Altogether, 198 mathematics teachers were examined with both the pedagogical content knowledge test and the content knowledge test. As the tests were administered at the second measurement date of COACTIV in 2004, the participating teachers were recruited from the mathematics teachers of the grade 10 classes which were examined for the German longitudinal PISA 03/04 component. Thus, this sample can be regarded as representative. In Germany, all candidates entering a teacher training program must have graduated from the highest track in the school system, the so-called ‘Gymnasium’ (Gy), and received the so-called ‘Abitur’ qualification (corresponding to the Grade Point Average in the USA). At university, those aspiring to teach at the secondary level must choose between separate degree programs qualifying them to teach either at Gy or in the other secondary tracks (e.g., ‘Realschule’ or ‘Sekundarschule’). Gy and non-Gymnasium (NGy) teacher education students are usually strictly separated during their university training. One of the main differences in their degree programs is the subject matter covered: Students trained to teach at Gy cover an in-depth curriculum almost comparable to that of a master’s degree in mathematics. Relative to their colleagues who receive less subject-matter training (and usually spend less time at university), Gy teachers may therefore be considered mathematical experts. NGy teachers, in contrast, study less subject matter but they are trained with more pedagogical content in university. 85 of the 198 teachers who were working on the tests were teaching at Gy (55% of them were male), and 113 of those were teaching at NGy (43% of these were male). The average age of the participating teachers was 47.2 years (with a standard deviation of 8.4); the teachers received an expense allowance of 60 Euro for their participation. The tests were administered in a single session with attendance of a trained test guide, normally in the afternoons of the PISA test day in a separate room of the school. For completing the tests, there was no time limit. On average teachers spent two hours to complete both tests (about 65 min for dealing with the 22 items of the pedagogical content knowledge test and about 55 min for dealing with the 13 items of the content knowledge test). The use of hand-held calculators was not allowed for the completion of the tests.

All of the 35 items were open questions. An instruction for coding was developed, and eight of the best mathematics teacher students of the University of Kassel were instructed carefully in coding teachers’ answers. Each answer of the teachers was then coded by two of those trained students independently (whereby sufficient results of agreement have been achieved). The procedure of test construction (including the scoring scheme of the item 'square' as an example) and resulting psychometric test properties can be found in detail in Krauss, Blum, Brunner, Neubrand, Baumert, Kunter, Besser and Elsner (forthcoming).

Results

Figure 3 gives an overview of the test results, divided into the two different German school types. Note that all results refer to our specific conceptualisations and operationalisations of mathematics teachers' PCK and CK

Figure 3: CK and PCK: means M (standard deviations SD) and empirical maxima by teacher group

	M (SD) Gy (N=85)	M (SD) NGy (N=113)	Effect size d (Gy vs NGy)	Emp. max. Gy	Emp. max. NGy
CK (13 items)	8.5 (2.3)	4.0 (2.8)	1.73	1.3e+09	1.2e+08
PCK (22 items)	22.6 (5.9)	18.0 (5.6)	0.80		
E&R (11 items)	9.3 (3.4)	7.1 (3.2)	0.67		
StCog (7 items)	5.8 (2.3)	4.3 (1.9)	0.71		
Tasks (4 items)	7.5 (1.8)	6.6 (2.0)	0.47		

Gy academic track teachers, NGy non-academic track teachers. According to Cohen (1992), d=0.20 is a small effect, d=0.50 a medium effect, and d=0.80 a large effect. All differences are significant at p<0.01

One specific feature of the pedagogical content knowledge test has to be mentioned: while in every item of the test for content knowledge one point could be scored with each correct answer (the maximum score which could be achieved was therefore 13), in the pedagogical content knowledge test in 9 of the 22 items multiple answers were allowed (and even asked for, see Figure 2). Considering the sub-facet 'E&R', this was the case in 3 out of 11 items, considering 'StCog', in 2 out of 7 items and referring to 'Task', in all the 4 items. Therefore a theoretical maximum for pedagogical content knowledge does not exist, but an empirical one: 37 points, which were achieved by one teacher; that means she was able to solve all items correctly and therefore got one point for each, and that she provided, on average, 2–3 correct alternatives in the multiple tasks.

As expected, due to the quite intensive training in subject matter of Gy teachers, a major difference could be recognised between Gy teachers and NGy teachers in their content knowledge. Considering the pedagogical content knowledge, Gy teachers also achieved more points on average, especially due to their higher competence level considering students' errors and explaining and representing (see Figure 3). However, it should be noted that Brunner, Kunter, Krauss, Baumert, Blum and Neubrand *et al.* (2006) showed that, when CK is statistically controlled for (i.e., when only teachers with the same CK level are compared), the NGy teachers slightly outperform the GY teachers with respect to PCK. The following results are worth mentioning as well (see Krauss *et al*., forthcoming, for more detailed results):

1. Basically, the development of professional knowledge in our conceptualisation seems to be completed at the end of teacher training: surprisingly no positive correlations between both knowledge categories, on the one hand, and teaching experience or age, on the other hand, could be found. Of course, this does not mean that there are no other aspects of teachers' competence that increase with age and experience, for instance certain routines of classroom management. It only means that this kind of knowledge assessed by the COACTIV tests is obviously acquired during the time of the teacher training already. Indeed, from an additional construct validation study an intense increase of both knowledge categories could be found from the beginning of university studies to the end of teacher education. However, in this construct validation study only cross-sectional data were gathered (the examined samples were, among others, students at the end of Gymnasium and teacher training students at the end of university; for details of this construct validation study see Krauss, Baumert and Blum, 2008).

2. A strong relation between pedagogical content knowledge and content knowledge exists. Such a correlation is in line with the theoretical assumption of Shulman (1987) and other authors that pedagogical content knowledge is a certain 'amalgam' of content knowledge and

pedagogical knowledge. The relationship between the two knowledge categories can be examined directly by calculating the correlation between PCK and CK, which in the COACTIV data was $r = 0.60$ indicating that PCK seems actually to be built upon a reliable base of CK. Note that this connection was much stronger in the Gy group; indeed, modelling PCK and CK as latent constructs led to a latent correlation in the Gy group that was no longer statistically distinguishable from 1 (see Krauss, Brunner, Kunter, Baumert, Blum, Neubrand and Jordan, 2008). Why was this correlation less strong in the NGy group? Closer inspection of the teacher data revealed that about 15% of NGy teachers who performed very poorly on CK (e.g., scoring only 1–2 points) nevertheless showed above-average performance on PCK (note that all teachers worked on all items and that the content areas of the PCK items differs from the content areas of the CK items). In other words, although our data support the claim that PCK profits substantially from a solid base of CK, CK is only one possible route to PCK. The greater emphasis on didactics in the initial training provided for NGy teacher candidates in Germany seems to be another route.

3. Voss, Kunter and Baumert (forthcoming) theoretically and empirically analysed the structure of subjective beliefs of the COACTIV teachers by using a 2x2 table with the first dimension *nature of mathematical knowledge* opposed to *teaching and learning of mathematics* and the second dimension *transmissive* vs. *constructivist orientation toward learning*. They found that only constructivist orientation but not a transmissive orientation contributes positively to the quality of lessons. Krauss *et al*. (forthcoming) found, for example, that teachers with high PCK and CK scores tended to disagree with the view that mathematics is ‘just’ a toolbox of facts and rules that ‘simply’ have to be recalled and applied. Rather, these teachers tended to think of mathematics as a process permanently leading to new discoveries. At the same time, the knowledgeable teachers rejected a receptive view of learning (“mathematics can best be learned by careful listening”), but tended to think that mathematics should be learned by self-determined, independent activities that foster real insight (for the development of beliefs see Voss *et al*., submitted; or Schmeisser *et al*., forthcoming).

4. Because COACTIV was ‘docked’ onto the PISA study, it was possible to relate teachers’ PCK to their students’ mathematics achievement

gains over the year under investigation. Possibly the most important result of the COACTIV study is that pedagogical content knowledge – but not content knowledge itself – contributes to the quality of lessons and to the students' learning decisively. Very briefly, when their mathematics achievement in grade 9 was kept constant, students taught by teachers with higher PCK scores performed significantly better in mathematics in grade 10. By means of structural equation modelling, Baumert, Kunter, Blum, Brunner, Voss, Jordan, Klusmann, Krauss, Neubrand and Tsai (2010) or Baumert and Kunter (forthcoming) could show that PCK can explain students' achievement gains in a substantial way. To be more precise, if the pedagogical content knowledge of the teachers differed by one standard deviation (which means about 6 points in the PCK test according to Figure 3), the mathematical achievement of their students differed by nearly two thirds of a standard deviation after one year of schooling (which is really a lot taking into account that the average learning progress in grade 10 in PISA was a third of a standard deviation). Because student learning can be considered the ultimate aim of teaching, the discriminant predictive validity of both knowledge constructs can be considered a main result of the COACTIV study, especially in the light of the high correlation between both knowledge constructs (for more details see Baumert *et al*., 2010; or Baumert and Kunter, forthcoming).

Summary and discussion

The construction of knowledge tests for teachers has been demanded emphatically (e.g., Lanahan, Scotchmer, McLaughlin, 2004). COACTIV focused on specifying pedagogical content knowledge and content knowledge for the subject of mathematics, constructing appropriate tests, and utilising them for a representative sample of German mathematics teachers of secondary schools.

Referring to the understanding of mathematics lessons as making mathematical contents accessible to the students, pedagogical content knowledge was conceptualised and operationalised in COACTIV as knowledge on explaining and representing ('making accessible' aspect), on errors and difficulties of students ('students' aspect) and on multiple solutions of mathematical tasks ('contents' aspect). Considering the content knowledge test, it has to be emphasised that due to the chosen curriculum-focused conceptualisation (advanced background knowledge of school mathematics) no empirically verified statements about the importance of high-level content knowledge that in Germany is gained at university could be deduced. In order to be able to investigate the relevance of this high-level content knowledge for student learning, a new test construction would be necessary.

Essential results, considering pedagogical content knowledge and content knowledge of COACTIV teachers, are the following: teaching experience does not seem to make a relevant contribution to the development of the two knowledge domains, which suggests that pedagogical content knowledge and content knowledge (as conceptualised in COACTIV) obviously are primarily acquired during teacher training. In order to examine the exact time and process of the acquisition in both knowledge domains, more studies with the COACTIV tests are necessary (e.g. with teacher trainees or student teachers; see COACTIV-R). Gy teachers show higher scores on content knowledge. The fact that Gy teachers are equipped with significantly more PCK (even if the difference is less noticeable in this case than in the case of CK) can be taken as an indication of the importance of content knowledge for the development of pedagogical content knowledge. However, a small group of NGy teachers (about 15%) shows that it is possible to possess outstanding pedagogical content knowledge with less content knowledge.

A result of great significance is the fact that pedagogical content knowledge of a teacher – but not content knowledge *per se* – contributes substantially to the learning development of the students. Therefore it is worth investing in teacher training of mathematics teachers, especially with respect to pedagogical content knowledge, with a sound basis of content knowledge.

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