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From the President's Desk…

Dear Mathematics Educators:

I am excited that the Winter issue of the Transformation Journal is ready for your use. This Journal is made available online through NSUWorks. I encourage you to submit your research articles so that we can share with the mathematics educators around the country. I also invite you to nominate a colleague or self-nominate to serve on our Board so that we can help make a difference in the K-22 mathematics education community in the State of Florida and throughout the country.

As an affiliate of the Florida Council of Teachers of Mathematics (FCTM), I am looking forward to achieving the following goals over the next two years:

1. Annual FAMTE Conference to promote the improvement of Florida's mathematics instructional programs and to promote cooperation and communication among the teachers of mathematics and mathematics teacher educators in Florida.
2. FAMTE Board represented by at least one K-12 Mathematics Teacher educators.
3. Promote scholarly publications.

With Warm Regards,

Hui Fang Huang "Angie" Su,
FAMTE President and Editor of
Transformation

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NOVA SOUTHEASTERN UNIVERSITY

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Article 1

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Don’t Count – Count on Visual Perception!

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Don't Count – Count on Visual Perception!

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Abstract

Verbal counting is the first step in the child's number building and this is why it seems natural to start arithmetic in school based on this competence. Under a cultural historical view, the development of number does not start with verbal counting but rather with ‚concrete counting'. Number words and the number word sequence developed after experience with concrete numbers.

This article describes the roots, basics, and first practical steps of a didactics based on perception rather than on verbal counting.

This proposed change allows inclusive lessons that prevent all students from misunderstanding calculation as a quick or clever form of counting and from becoming stuck in solidified counting.

Keywords

math didactics, primary school, computational problems, reasonable computation, inclusion, concrete number, number sequence, solidified counting, cultural history, first grade math, place value

Solidified Counting as a Key Problem

Don't count is an unusual imperative in an article about computation and the development of number sense. Can we not say that counting is the first property that children learn about numbers? Further, is it not generally agreed that counting is the first step towards cardinal understanding? „Counting provides children with the bridge between concrete but limited perception and abstract but general mathematical ideas. It is counting that puts abstract number and simple arithmetic within the reach of the young child“ (Barroody 1987, p.33, quoted after Moser-Opitz 2008, p. 63). For Freudenthal, the *Zählzahl* (counting-number) plays the first and most important role in the genesis of the concept of number. In addition, he denies that quantity might be a concept that grows out of perception and the understanding of invariance (Freudenthal 1977, p. 177-178). There seems to be no need of further debating this crucial aspect.

As a result of this universal conviction, the learning of number and computation starts with verbal counting. „From counting to a structured understanding of numbers“ is Gaidoschik's title of a chapter in his dissertation where he describes the methods that help children on their path to reasonable computation (Gaidoschik 2010, p. 208). Lessons might strengthen the acquisition of the cardinal aspect by using patterns or manipulatives with five-based structures. Lessons might use certain types of tasks to help children to construct number as ‚verbal or abstract unit items‘ (Steffe quoted in Fuson 1988, p. 54). All these concepts are based on the idea of starting with counting and developing the student's number concepts from the number sequence as a starting point. This view has the consequence that computation starts with addition followed later by subtraction. This seems natural as both operations can be easily solved by counting.[1]

On the other hand, we should aim to keep the standard high. Our goal should be *for all students* to learn to calculate with a cardinal understanding. Further, *all students* should understand the decimal system and should be able to use this cardinal structure in smart calculation methods. We should not except that „a significant proportion of second graders, perhaps the majority“ constructs ten only as a ‚numercial composite‘ (Cobb&Wheatley 1988, p. 5), and 19% of the 4th-grader group performed below the NAEP basic level (NCES 2021). Even the 3.6-6.6 % of children with mathematical learning disablities or a diagnosis of dyscalculia should be still in our focus (Dorheim 2007, p. 13 ff.), as should the even larger group of children with learning and other disabillities. While schooling is not successful for *all* students, it makes sense to question the basis of teaching methodology in this field. This is especially so when solidified counting is one typical symptom of children with difficulties in understanding calculations.

The symptoms of weak performance are broadly described in works such as Moser-Opitz (2008), Gerster&Schultz (2004), and Gaidoschik (2010). In addition to the phenomenon of solidified counting, children may lack the concept of a part-whole-scheme, and they may also lack the concept of ten as a reversible unit together with an absence of understanding and reasonable use of the place-value-system. However, the causes of this failure are not sought in the lessons and even less in the basic didactic assumptions of the curricula. Instead, they are

[1] Cobb&Wheatley's (1988) research on „Children's Initial Understanding of Ten“ also shows that verbal counting seems to be the only basis for developing number concept. The described development shows up in the ability to use tens and ones in counting. Other solutions based on the concept of operation and contra-operation and/or on step-by-step approximation are not mentioned. The performance of the children indicates their use of mechanical and systematic counting. However, they have not yet learned to think in reversible operative arithmetic structures, and they have not managed to overcome particular hurdles as given in 5 and 6 (see down, p. 7-8).

sought in the students, in their cognitive abilities, in perceptual problems, and in their social background. Weak performance in calculation had become something as a medical diagnosis.

Meyerhöfer (2011) criticises this approach. He describes how didactics such as Raddatz or Gerster have moved the focus away from the analysis of personal disabilities towards the analysis of neccessary processes of understanding. The term ‚Dyscalculia' was replaced by ‚Rechenschwäche' (‚weak calculation') which both names and describes the symptom. But – following Meyerhöfer – the math-didactic community has still remained focussed on the child's difficulties.

Meyerhöfer demands a change to this view. The math-didactic community should ask the question as to which obstacles lie in the subject itself, and ask what exactly must be understood when the goal is reasonable calculation. A further question is which of these obstacles are basic, given that they block the further learning process?
The factual hurdles that must be overcome Meyerhöfer terms „besondere stoffliche Hürden" (particular factual hurdles), and *he claims that it is the school's task to enpower every child to overcome them.* However, he does not give more precise information about what he identifies as a ‚besondere stoffliche Hürde'. In light of this, a crucial goal of this article is to propose an answer to this important question.

My goal in this paper is to propose a sequence of hurdles or obstacles that are crucial for understanding numbers written in a place-value-system. Here, I describe and justify the relevance and the order of those hurdles. One argument I give here is that we can analyze these hurdles by searching the steps of development in the cultural history of number and computation. It is an important background to my thinking and of my didactic concept *‚Math Inclusive – Calculation through Acting'* that cultural history can teach us what has to be understood and also illuminate which sort of tasks enable us to acheive understanding (Rödler 1998, 2006a, 2010, 2011, 2015, 2016a, 2018, 2020).
We can ask which tasks can initiate processes of accommodation, and that is the central question. This is so because it is accomodation that is necessary to change and develop a concept (von Glaserfeld 1997, pp. 72 ff., 168 f., 191). And it was the development of accommodations of number concepts that led mankind over many thousands of years to the modern level of understanding of number in a place-value-system.

In contrast to our familiar point of view, the starting point of mankind was not the number word sequence - not even the single number word. Words followed the experience of concrete counting. Cardinality, number-words and the part-whole-scheme were all rooted in experiences of perception in a context of a numercial problem (Menninger 1979, Ifrah 1987, Damerov et al. 1994a/b, Zaslavski 1999). Before I come to the main points of my article *(‚What has to be understood'* and ‚*How it can be understood')*, I want to give a short overview about the first steps of human number building from cultural historical perspective.
I am convinced that it is this misinterpretation of the starting point that narrows the analysis of computational problems and therefore the spectrum of didactic methods. The idea that number necessarily starts with verbal counting and therefore depends on competences that are based on the number word sequence, is, for me, the key mistake and has a constricting effect. It is important to detect that this assumption is a constraint (von Glaserfeld 1997, pp. 62-77) in recognizing the relevance of concrete numbers and in that context of perception. I argue that this view allows new and valuable discussion on the topic.

1. ***The Beginning of Number Sense under a Cultural Historical Perspective***

If we change our view and look to the origin of number, we must go more than 20,000 years into the past. By doing this, we can find people that did not have numbers, just as very young children do not (Menninger 1979, Ifrah 1988). So, we must ask the question: Why did humans develop numbers? There must have been a reason.

We do not have a time-machine, but we know that concepts are constructions and grow out of acting and reflected experience (von Glaserfeld 1979). New concepts are found when there are relevant problems that demand new solutions. Maybe those early humans wanted to ensure that no sheep of the flock was lost. Maybe they wanted to know how often the sun rises from full moon to full moon. Maybe they wanted to count dead enemies to find out who of a tribe is the most powerful warrior. For whatever reason, there was a cardinal reality that was important. But, there were no words for precise quantities and thus they they counted with objects. Humans took stones or shells or they carved wood or bones. Counting was a material process! The number was a *‚concrete number‘* (Rödler 2006a, 2011, 2016, 2020). *„So many"* was the only expression they needed.[2] Maybe, they had words with the meaning of ‚some', ‚many' or ‚a lot'; there was no need for more precision.

From this beginning, we can learn two things about the root of our numbers: First, the evolution did not start with the counting-process – *it started with the need to get hold of a cardinal reality.* Without a need, there is no action. Second, counting did not start as a verbal concept. It started with a concrete one-to-one or an *‚analog mapping‘* (Rödler 2011, 2020), based on a material process. This mapping fixed the cardinal reality into a material ‚re-presentation' (von Glaserfeld 1979).

There was no need for verbal counting in order to create cardinal meaning. The cardinal was already a concept. *Fingers, carving, stones or shells* were the items that reduced a heterogeneous reality into a collection of homogeneous unit items. At the same moment, these collections of unit items re-presented the counted reality and thus stood for something. In this sense, it was a „single whole" (Fuson 1988, p. 8); it was an ‚abstract composite unit' (Gerster&Schultz 2004, p. 58).

Small quantities up to three or four could be identified and distinguished by subitizing. So, it is not surprising that the first number words had the meaning of our one, two, three (four); larger quantities were termed as ‚many' (Menninger 1979, I, p. 33).

In the beginning, those first number words were used as adjectives (Menninger 1979, I, p. 33 ff.). They were closely connected to what they designate. But because of the experience of material counting and *the perceptive identity of for example III and III (even if the both III counted different objects)*, the concept of the number as an abstract whole was possible to be reflected and in this context arose the option of creating abstract number words that were not bound to a specific counted reality (Menninger 1979, I, p. 48).

This development of *number words* did not automatically create a *number sequence*. New words were built out of existing words or visible structures: A pair of pairs, a double four, two more than a hand. Step by step, number words for larger quantities were found and ‚many' gained new meanings. Menninger writes about „Zählgrenzen" (borders of counting) und „Rangschwellen" (thresholds) that had to be overcome. Concrete patterns and bundles create visual gradiations and so the verbal number sequence could be built by designating these grades and designating the gaps between those grades. Menninger describes this principle of building larger number words as ‚Reihung und Bündelung' (sequencing and bundling). He stresses that this process was possible because the counting and finding of words no longer occured at the

[2] There has been a second form of counting using parts of the body. Those ‚body-numbers' allowed to name quantities higher than four and were strictly ordinal (Ifrah 1987, p. 30-31). But when we research the number-systems of higher cultures and especially those our decimal place value system is rooted in, we find no links to these counting techniques. Obviously they have been something like a dead-end.

counted quantities in reality but on the concrete number, which was built by concrete counting in the form of a material one-to-one-mapping (Menninger 1979, pp. 49 ff.).[3]

The familiar view on learning numbers and computation describes the necessity of developing a cardinal understanding of the number word originally found by verbal counting. This development of the child is described in steps of knowledge. The counting scheme develops (Gelman&Gallistel 1978, Fuson 1988) and the understanding of counting items has to develop from perceptual unit items to the construction of number as a 'composite unit' (Steffe&Cobb 1988 after Gerster&Schultz 2004, p. 56-58)The fact that there is so much research in this field underlines the difficulties that some children have. There is so much research as even if most of the children succeed in learning reasonable computation, there are many who do not. For such children, the concept of number stays basic and hinders them from reasonable computational development. And thus they are stuck with solidified counting.

The cultural historical view shows us that the development from ‚perceptual unit item' to an ‚abstract composite unit' originally happened *before (!)* the number word sequence was developed. When number words arose, the cardinal aspect of number was already constructed. Words were named cardinality from the very beginning.
This fact helps us to understand why the existing number word sequence is not only a chance for most children but also a burden for some: A constraint! It hinders those students from the development of the basic ideas, and that is the reason why it makes sense to search for alternative curricula that are not based on the number word sequence but are based rather in concrete counting and perceptual reflection, just as things began more than 20,000 years ago.

This introduction aims to make my rethinking of teaching calculation understandable. I assume that children are just as much at the beginning of their conceptual number development as early humans. Analogous to Norbert Elias (Elias 1976, p. LXXIV), I assume that during their mental and conceptual development, children need to overcome the similar conceptual hurdles that we see throughout cultural history. These basic assumptions make it possible to formulate the epistemological hurdles more precisely. I think this is exactly what Meyerhöfer's concept of „nicht bearbeitete stoffliche Hürden" calls for. It helps us to realize what a student must understand in order to prevent the development of arithmetic impairment.

2. ***What has to be Understood? – 10 Particular Hurdles***

In this section, I describe *what* must be understood; however, I do not describe here *how* this is done. I discuss that aspect in the next section.
The ten hurdles I describe here are those that I feel to be essential for an understanding of number and place-value. They also build a sequence. In particular, the first six hurdles build on each other and form the basis for discussion of the final four. And within the first six the first four hurdles again are crucial.
This does not mean that lessons should thematisize one after the other. Learning processes and especially understanding does not go step by step and is therefore not a stairway of perfect stages. Knowledge grows by processes of approximation. This requires a complex process of dealing with experiences on different levels at the same time. The teacher must acquire an overview in order to understand the connection of basics and progress to more challenging aspects. It is this complexity which makes this sort of curriculum an inclusive one. It allows learning and understanding on different levels at the same time.

[3] Similar developments are described in Zaslavski (1999). Here it is striking that in Bantu there are „wide variations in the words for 6, 7, 8 and 9" but „we find similarity in the names for 2, 3, 4 and 5." (Zaslavski 1999, p. 39). The word for 5 is included here, because numbers are closely related to finger-gestures and the ‚full hand' gives a perceptive five.

- *First Particular Hurdle: The Need for Building Numbers*

It is amazing: The first and most important hurdle is ignored in school, namely the *need* for building numbers. Courses and tutorials start with verbal counting, learning the number signs, and giving these number-representations cardinal meaning and structure. Nobody seems to ask what relevance numbers have apart from teaching courses. Besides counting rhymes, number seems to be nothing more than a cultural technique that must be learned as useful for future life.

In textbooks, authors emphasize this learning with child orientated content: for instance, lighting and blowing out birthday candles, sitting and flying birds, children at the playground and cars in the street, and so on. But all these situations in a child's life have no relevance besides counting. All other aspects are irrelevant and do not really matter. This fact strengthens the child's impression that numbers have nothing to do with their own actual life. They are artificial and something that becomes relevant in the future. *This false impression needs to be changed!*
Counting must be made relevant to the child's life *now!* Counting contexts must be provided that are rooted in the enviroment of the child and are therefore relevant for his or her life. For example, when the next excursion is planned, it becomes important to ascertain whether the majority prefers the zoo, the playground, or a museum, for instance. Such experiences build the motivation to count and bring number and patterns of the surrounding world into the child's focus of interest. This is the first and most important basic to built up a competent concept of number.

- *Second Particular Hurdle: Cardinality (Number as a Collection of Single Unit Items)*

The child that becomes interested in a certain quantity makes these elements into perceptual unit items, which then makes it possible to count them. This is usually done by saying the number word sequence while touching one object after the other with their fingers or by eye-contact. However, this verbal counting does not mean that the child already understands the final word as the number for the whole quantity. There are different steps of development. For verbal or concrete counting, further development requires understanding the cardinality of the counted number in terms of a collection of countable single unit items (Gerster&Schultz 2004, p. 56-62).

- *Third Particular Hurdle: Number as a Cardinal Whole (Single Whole/Abstract Composite Unit)*

Numbers such as five or six should be understood not only as collections of single unit items. That is not sufficient, because it means that computation is dependent on counting processes. Numbers should also be understood as wholes. In German, we can formulate the difference easily. IIIII should not only ‚fünf', but also a ‚Fünfer'; not only five, but a ‚fiver'. A child needs to understand that numbers are wholes, which can be used as modules or building blocks.
Such a whole is invariant, and it can stand for many different quantities in reality. A ‚Fünfer' can stand for five fingers as well as for five toes or five candles on the birthday cake. This understanding of the number as a whole makes it possible to build bigger numbers out of smaller ones and to divide a number into parts, not only into ones.

- *Fourth Particular Hurdle: The Part-Whole-Scheme*

For Gerster&Schultz (2004), the part-whole-scheme is the crucial point in the development of number understanding as it allows computation without counting. They cite Resnick:
„The Part-Whole schema specifies relationships among triples of numbers. In the triple 2-5-7, for example, 7 is allways the whole; 5 and 2 are allways the parts. Together, 5 and 2 satisfy the equivalence constraint for the whole: 7. The relationship among 2, 5 and 7 holds whether the

problem is given as 5+2=?, 7-5=?, 7-2=?, 2+_=7, or _+5=7" (Resnick, 1983, 115) (Gerster&Schultz 2004, p. 79).

What should be understood is that the decomposition provides the parts that build the whole. That is why the knowledge of partitions gives the answer to all equations with addition and subtraction.
Numbers are building blocks that build bigger numbers, and bigger numbers can be broken into such blocks. A subtraction takes away one of the blocks and leaves the other, and addition rebuilds the whole. An equation such as 2+_=7 or 7=5+_ asks for the other block, and an equation such as 7-_=2 or 5=7-_ does the same in that it asks which part has been removed when a certain part is left.

This fundamental understanding of number in the part-whole-scheme is the goal that should be reached as soon as possible; not only because it allows fast and secure calculation, but also because two-digit numbers are built out of two such building blocks: the tens and the ones.

- *Fifth Particular Hurdle: Tens-Ones Breakdown of the Two-Digit Number*

Our place value system[4] builds large numbers from building blocks that are encoded in decimal place values. In the two-digit number range, these are the tens and ones.
To understand that a 35 is built out of a thirty and a five does not need a change in concept. For a child with a solid part-whole-scheme there is nothing new, besides the number words for the tens (ten, twenty, thirty, …) and their cardinal understanding. The tens-ones-breakdown can be learned by assimilation!

- *Sixth Particular Hurdle: Reversible Tens*

More difficulties relate to the sixth hurdle, and weak performers often fail at this point (Gerster&Schultz 2004, p. 80-99). For operations with two-digit numbers as well as for the understanding of place values in bigger number ranges, it is equally important to understand the reversible relation between ones and tens: Tens are compositions of ten units and therefore can be decomposed if necessary. Tens are not just a new unit; they are not just another ‚one'. Tens and ones do not exist independently of each other side by side – they must be understood in their reversible relation.
This competence makes it possible to calculate tasks such as 15-8= , 47+9= , 43-18=, 25+47= or 25+_=42 and 71-_=39 in reasonable steps.

- *Seventh Particular Hurdle: Number Signs are Coded Quantity, which Get Value by a Convention*

A number sign such as 125 seems to carry value. Our calculation techniques, especially written calculation methods, strengthen that impression. Experienced calculators can decode the number sign into the decimal building blocks by mental calculation. In lessons, children get materialisations such as Dienes-material which translates these building blocks into a visual cardinal reality by using rods as compositions of ten unit items, plates as compositions of hundred (ten rods), and big cubes as composition of thousand (ten plates). The number sign seems to be the starting point and carries value that can be visualized.

However, the number sign is just a sign just as the number word is just a word. It is a medium of communication and does not carry value. Rather, it carries information and allows the discussion or determinationof value, but this happens only for those who have already

[4] Not only place value system! The Sumerian and Egyptian number signs, too, were based on tens and ones. The concept of reversible decimal bundling is more basic and much older than our modern number system.

constructed this value in their mind. The concept of decimal building needs to be already constructed to decode the sign.

Under a cultural historical view, number signs and number words grew as re-presentations of concrete numbers. They allowed communication about material experiences. ‚M‘ does not carry a thousand, but it could be used for a thousand just as the Egyptian lotus-flower or a tree (if I define a tree as a symbol for thousand in my own number system). It is a question of convention to represent cardinal meaning into signs. That must be understood.

- *Eighth Particular Hurdle: Idea of Bundling /Concept of Decimal Value Levels*

Hurdles 7 and 8 are closely related. The two aspects of knowledge must interact to understand the problem and overcome these two hurdles.
In order to understand a convention and to re-build the coded value, it is a prerequisite that the user of the number sign has already built up the fundamental ideas that are decoded. Concerning our place-value system, this means that the concept of reversible decimal building blocks needs to be established. If not, and if a child‘s concept is still based in the number sequence, she or he will decode the three digits in 125 as one, two and five instead of a hundred, twenty and five. On this conceptual basis, such a child will solve the task 43+24= correctly with ‚67‘ by calculating with the digits as numbers: 4+2=6 and 3+4=7, but they will fail with 43-24= , because typically he or she computes the result 21 by 4-2=2 and 3-4=1.

In order to understand the concept of a place-value, it is not enough to think in tens, hundreds and thousands as long as these value levels are material concepts like ‚tens are rods‘ and ‚hundreds are plates‘, the concept of a bundle is something like a unit item of higher value. This concept fits to the cultural stage of Sumerian and Egyptian numbers. In addition, we still find it it in the ‚Roman‘ numbers of the middle ages.
Place value develops, when bundling becomes a process: ‚Ten of the smaller unit create a new value level.‘ This idea made it possible to break out of material presentation of a definite quantity and opened the space to the structural idea of computation. This breakthrough happened first in computation techniques by using tools such as the calculation board, the abacus, or the soroban. Counters became valuable depending on the position. Based on the experience of thousands of years with material place value computation, it was possible to understand bundling as a process that goes on and on and on. This experience made it meaningful for humans to develop a number system that maps iteration: the place value system.

It is important to understand that overcoming this hurdle is a necessary condition but is not sufficient. For some thousands of years, cultures had invented place value tools for computation but did not take the step to a corresponding writing of numbers, and this shows that the understanding of place value writing is a further hurdle to take.

- *Nineth Particular Hurdle: Place Value Numbers ‚upwards‘*

The first understanding of value by bundling is based on the unit item, the one. Higher value levels like tens, hundreds, thousands and so on can be created by bundling. It now must be understood that every additional level is coded in a new position to the left. Instead of using different signs for the different levels, we can use equal signs (digits) that get their value in connection with the position.

Why was it so difficult to make this step in historical development and what makes it difficult still for children to decode a multidigit number different from a collection of one-digit numbers? It is the problem of the ‚zero‘!

Number signs are a coded quantity, and it is natural to build the whole number out of cardinal modules. MMXXI means that we need two thousands, two tens and a one to have the number of our actual year. Place value numbers make it necessary to write down what is missing: no hundreds! Otherwise, the number 2021 would be missunderstood as 221.
And there is another problem: The correct number sign shows two ‚2'. But they have a completely different value. It is obvious that it is much easier to think of two thousands and two tens when they are shown with MM and XX. Clearly that cannot mean the same. To decode the one ‚2' as two different values demands a high level of developed construction of decimal value levels. One needs to search for and expect them in the number sign.

- *Tenth Particular Hurdle: Complete Place Value System (including downwards)*

Decimal bundling is the process that creates ascending new value levels, shown in number signs with new positions on the left. Because these processes can be inverted, the bigger and the smaller value level is in a reversible relation: thousands can be debundled into ten hundreds, hundreds into ten tens and tens into ten ones.
Therefore, the system becomes complete when we demand that the indefinite process of bundling should be equally indefinite in the direction of debundling.
What happens when the idea of ‚every value level can be debundled into ten smaller units' is used on the level of ones? We get tenths! And if we continue descending, we create hundredths, thousandths and so on. And every new smaller value level gets a position to the right.
This last step completes the full understanding of a decimal place value system.

3. How to Overcome these Particular Hurdles

A cultural historical view into the past has shown us that the process of number building and computation started with concrete numbers. It has also helped us to understand the changes in conceptual understanding and especially the obstacles that had to be overcome. The fact that all these steps in development lasted thousands of years proves that there were hurdles to this development. It was no easy and smooth continous process.
The thesis of the following section is that the epistomological question as to what a child must build up in order to understand numbers and computation is closely related to the cultural historical process. Here we can identify the problems and we can identify the hurdles and the prerequisites that support certain changes in concept.
Concepts develop based on experiences. A change of a concept in terms of accommodation depends on new experiences that disturb former interpretations. Therefore, didactics should ask how the environment of the child can be changed in order to provide situations in which number becomes relevant and the particular hurdles become a real problem *in the child's view*. I will describe here the start of this curriculum to give an impression as to how lessons change under this view.[5]

Taking the first three particular hurdles '(need for building numbers, cardinal number and number as a whole) is the basis for all further understanding. When students fail to compute with understanding, the reason often is that they did not overcome these first three hurdles. They are stuck in solidified counting computation because they did not overwind the number concept of the number word sequence. They compute with the digits as numbers and with tricks that allow them to solve problems by keeping on counting. New types of tasks are edited by assimilation.
That leads us to the question of which setting helps children:

[5] The further curricular in arithmetic, up to grade 4, is described in the following publications: Rödler 2012, 2013, 2016a, 2016b, 2018, 2020. Also in my youtube-channel, especially in playlist ‚Rechenprobleme'.

1. to become interested in building numbers.
2. to understand the aspect of cardinality (including invariance and classification)?
3. to understand numbers as cardinal wholes that can become building blocks for other numbers.

This includes the question: Which setting helps children to stop assimilation to the number word sequence and demands accomodation? A change in concept will not occur while our curriculum begins with verbal counting and supports solutions on this basis. I am convinced that we need to help children to experience the relevance of perception by solving counting and computation tasks on this basis.

The methods I describe have developed through 25 years of practical work as a teacher in first to fifth grade classes (Rödler 1997, 1998, 2006a, 2006b, 2012, 2013). All groups were very heterogenious, and some were inclusive in sense of that they included children with special needs. Inspired by Norbert Elias (Elias 1976, p. LXXIV) and it was based on the idea that early cultural ideas of number might fit better to childrens' natural concepts that I changed my concept of teaching. Step by step and by trial and error, I tested the effect of using early cultural historical number and calculation concepts. Out of this experience I developed the concept of ‚Calculation on Different Levels of Abstraction' (Rödler 2011). The following section describes some fundamental didactical changes that have proven the worth of this basic idea. Under this concept, perception becomes a key aspect. It marks the starting point, and it gives impulses for conceptual development of understanding number.

A: Concrete Counting Instead of Verbal Counting (The Construction of Number)

Children seem to begin with numbers by learning the number word sequence, but as the term ‚number word' says: these are only words, and more is required to understand the concept of number. The long way from the knowledge of words and sequence to the concept of the word as a cardinal number has been broadly described (Gelman&Gallistel 1978, Fuson 1988, Gerster&Schultz 2004, Dornheim 2008). This is the familiar view of the child's construction of number based on verbal counting.[6]

Under a cultural historical view, the number arises earlier than the word. This is possible because counting is a material process. It is concrete counting that creates the number as a perceptible collection and as a whole of unit items. („So many!") This concrete number allows understanding of relevant consequences of the construction out of the counting process directly by perception.

- A number is an abstraction that fixes the cardinal aspect of an unhomogeneous reality and is thus a creation by purpose (Frege 1987, p. 93 ff.).
- A number is a fixed quantity of unit items.
- Because of the counted backround, changing position of the singles in space does not change the value of the whole (invariance).
- Concrete numbers may differ in size and thus it is possible to order them as less, more, and equal (ordinal aspect and idea of number sequence).
- Equal concrete numbers show that every number can stand for different quantities in reality. So, every concrete number re-presents a class of counting results (classification).

[6] That does not mean the relevance of perception is ignored. The role of ‚perceptual and conceptual subitizing' (Clements (1999) the importance of figurative gesture and patterns and of manipulatives that visualize numbers and allow structured operations are common methods and suggestions, when it comes to the question how to prevent weak calculation (see, Gerster&Schultz 2004, Moser-Opitz 2008, Gaidoschik 2010). However, this all happens based on and still linked to the starting point of counting.

From the very beginning, the number is understood in its cardinal function. And this is not a sophisticated process. Rather, it is the direct consequence of the need for counting, and therefore this starting point is an inclusive access to number and computation.

To prevent verbal counting, it is important that our questions and problems lead to larger quantities. Otherwise, children will assimilate the counting problem and have no reason for accomodation (von Glaserfeld 1997, p. 84ff. /Piaget 1974, p.154 ff.). If we are no more bound to start with a small number range, it becomes possible to follow every question that arises in the class: Are there more boys or girls among the first graders? How many doors do we find in school? What is the favourite subject of the school's students? Which car brand is the most popular? It is obvious that projects like these demand something different rather than knowing just the number sequence. It leads to a cooperative process of collecting information by concrete counting in groups. It is a social process as every child is involved! Nobody stands aside because this concrete counting does not require special prerequisites, and, importantly, nobody knows the solution in advance, not even the teacher! *What nobody knows, creates an inclusive setting! That is a very important didactical rule.*

To return to our subject, concrete counting creates concrete numbers, but differently to the construction of the number out of the word sequence, this starting point creates and uses the concept of cardinality from the very beginning. Cardinality as the basis of number is understood by every learner.

Figure 1: Abstract and Concrete Number

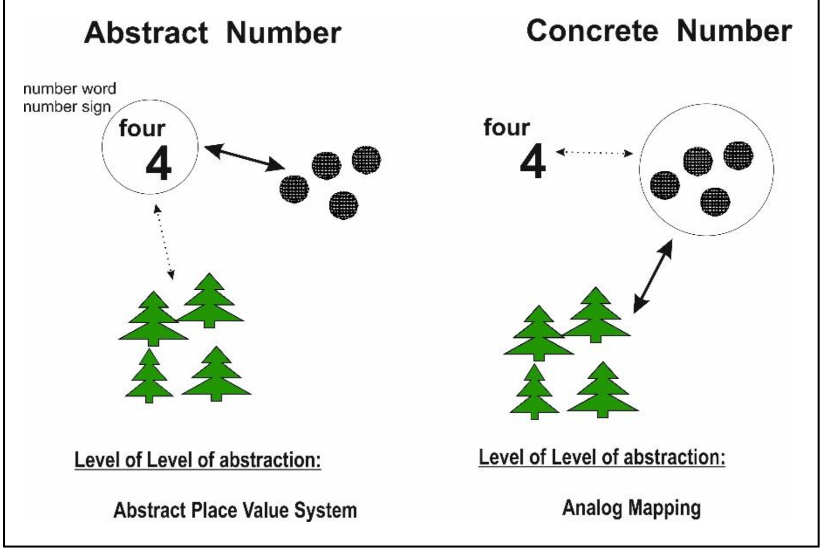
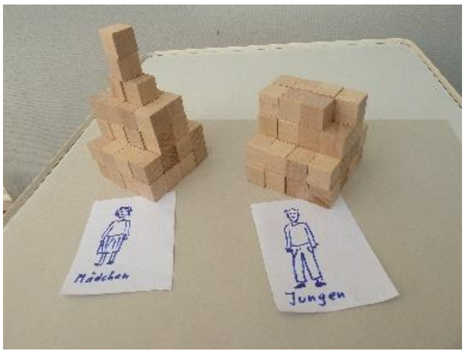


Figure 1 shows the difference. When we locate the number in the number word or number sign on the left, we must underlie the cardinality in a second step. Only after this understanding of cardinality is it possible to calculate with understanding. Without this understanding, calculations stay mechanical and lead to solidified counting instead of computation.
When we locate the number on the right, because this collection of unit items was created by mapping a certain concept of reality, we have a cardinal number that shows its value simply by perception. Number word and number sign are only media of communication that are added afterwards. They allow us to talk about our created numbers. Importantly, number words and number signs are not the numbers! At least not in the beginning.

B: Giving Structure (Making Perception Possible)
Finding the answer to a real question by concrete counting automatically leads to the limits of perception. If there is not a large difference, we can only distinguish small quantities up to four (Gelman&Gallistel 1979, Fuson 1989, Clements 1999, Gerster& Schultz 2004). So, when it comes to evaluating whether there are more boys or girls in school, we need to give the counting result, the two concrete numbers, a perceptiple figure.

By giving the students the experience of spontaneous perception of quantities, they find out that this is easy up to three and four, bur that it is not possible with larger quantities. There is a limit of competence in subitizing and thus it is plausible to use this knowledge by structuring the cubes that represent the boys and the girls (Graphic 2). We get ‚buildings' that make these concrete numbers perceptible. With these two buildings, the question can be answered by every child and there is no need for verbal counting.
Every child can find the answer by perception even in this quite huge number range.

Figure 2: More girls than boys



This has a strong effect on children in that they do not have to be convinced and nothing needs to be trained, thus there is no need for teaching. Arithemtic stays a practical experience close to real problems in the child's reality, and therefor close to the child's individual cognitive abilities.
‚So many' girls. , 'so many' boys. Counting is not necessarily a verbal process. The number does not emerge out of the number word sequence as the number already exists.[7] This is therefore the new experience.

C: Words, Symbols and Signs (Language and Scripture of Arithmetic)
Numbers up to four can be distinguished without verbal counting, but rather by subitizing. If we put one, two, three or four cubes under our hand and lift the hand for just a part of a second, children are able recognize the number. We just need words for what we see. One, Three, Four, Two – it is reasonable to give words to these visible differences.
The experience of naming the number without counting fosters the understanding that a number is not only a collection of singletons but also a single whole. Thus, III is not only three in sense of three singles but also a trinity that is different from a fourth or a pair.

Number words are a medium of communication as they describe value. Verbal counting becomes necessary when perception does not allow naming a quantity spontaneously. Importantly, verbal counting is a help in finding the word; it is not a process of creating number.

Figure 3: Which parts do you see?

Observe the building in figure 3. Maybe you first view the two trinities.

Then you can describe the bulding with $2 \cdot 3 + 2 \cdot 3 + 1$.
Or you see the single trinities. Then the term would be $4 \cdot 3 + 1$.
If your first view perceives the layers $(2 \cdot 2)$, your building is built out of $2 \cdot 2 + 2 \cdot 2 + 2 \cdot 2 + 1$.

[7] Of course, it is possible to ask how many cubes there are exactly just as it is possible to name visible substructres like two, four, eight, sixteen or even to count or distinguish the the numbers 52 and 47. It is even possible to talk about the difference ‚five more girls'. It is a typical complex situation that allows natural differentiation not only for the weak performers but also for the very gifted. The main point is this: *the starting question, the basis of the project, has been already completely answered by perception. The number exists, before it is counted verbally* and that is why it is an inclusive project.

Let us do it the other way round. Take cubes and lay a 3·3-structure. Now put a 2·2 as a second floor and on top a last single cube. This building-manual we can write with $3 \cdot 3 + 2 \cdot 2 + 1$.

Every building can be described in terms and each term can be understood as a construction manual for a building! Every building provides an opportunity to talk about visible numbers and about multplicative structures, and it is an occasion to also write down the words in the scripture of arithmetic. Every term is an oppotunity to translate the signs into material reality.
Working in this field helps children to overcome the particular hurdles three and four as set out above. Children become familiar with numbers such as 1, 2, 3 and 4 as cardinal wholes, and they get used to recognizing structures of 4, 6, 8 and 9 built out of smaller parts. This is a first step towards the understanding of number in the part-whole-scheme.

D: Concrete Calculations (Supporting Structures, Not Verbal Counting)

The entry into calculation, too, must support this important change in number concept. In the beginning, computation should be less a question of finding the correct result than of learning about operations and structured numbers. This starts at the first week in school!

If learning to compute has the goal of developing number concept, we must provide tasks that focus numbers as wholes and as structures of wholes. If we want to prevent solidified counting, we should not give tasks that are easily solved by verbal counting. Therefore, the familiar approach of starting with addition is counter productive.
A child whose number-concept is based on (a certain level) of number word sequence will solve 3+4= easily by her or his way of counting. There is no natural disturbance that demands a change in concept. The following subtraction will be solved in a fitting interpretation. Namely, either by counting backwards, or, more often, by using fingers or a manipulative and count forward three times. ($7 - 4 = 3$, because first the counting what is there: One, two, …,six, seven and then putting singles away: One, two, three, four, and at the end counting what stays: One, two, three.)
Manipulatives such as fingers, chains of pearls, or the abacus in a 20 number range are used as a counting aids. Teachers show and explain how those manipulatives have a structure with five and ten that can be used. However, weak performers do not integrate this information. When they are on their own, they return to the verbal counting procedure that is deeply rooted in their understanding of number. The correct result confirms that they have done well and thus they are held in wrong thinking and they stuck in a dead-end.

Constructivism emphasizes that accomodation arises only when assimilation fails (von Glaserfeld 1997). Therefore, the question is which entry in calculation provides such a disturbance? Which operation focuses structure instead of counting? Which sequence of operations helps the child to finally solve additions and subtractions within the framework of the part-whole-scheme?
This is easy to solve by not starting with $3 + 4 =$ but with $3 \cdot 4 =$!

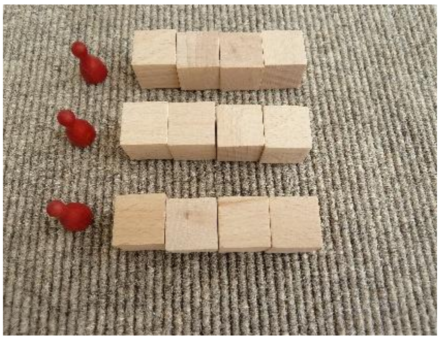
- *Starting Calculation by Multiplication and Division*

Starting with multiplication instead of addition creates an inclusive situation. Unlike with $3 + 4 =$, no child in first grade will know the answer for $3 \cdot 4 =$, and therefore this task does not divide the class. Every single child needs to approach the meaning of this operation and the possibilities of the solution. However, every child is able to understand the problem when it is translated into a real situation such as: There are 3 children, and every child has 4 cubes. How many do they have altogether?

In this introduction, the calculation happens at the level of reality. Every child understands what is going on. Even the child who might not be able to count up to 12 and whose answer might be ‚so many‘ understands the concept of multiplication.
The next step is therefore to transform the calculation from reality to the first level of number building, to analog mapping (Figure 4). It is the stoneage-level, when concrete numbers were used in concrete counting, and it is thus the lowest level of using a number.

Counting processes in class, as already described, create numbers by using cubes as single unit items. Patterns and buildings make it possible to talk about the perceptible numbers up to four. Number words and number signs are connected with these first numbers. Operation signs become relevant when descriptions of patterns and buildings are written down.
Parallel to this process, the same numbers and signs emerge in computing tasks when we take the same cubes. Thus, a ‚4‘ out of a counting process or in a building shows the same cardinality as in the task $3 \cdot 4 =$.

Figure 4: Entry by Multiplication
3·4=12



The cubes that are used in a concrete calculation are not manipulatives nor are they illustrative or visual aids. Thus, they do not visualize abstract symbols as this transition is already done by translating the abstract task $3 \cdot 4 =$ into reality. The cubes and the cones map this reality onto the ‚calculation carpet‘[8]: Three children (cones) with four cubes each. The cubes are concrete numbers!
Children who do not know the number signs, get a ‚number sign table‘ (Figure 5).

Figure 5: Number sign table **(4 / four)**

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23

This makes it possible for *all* children to start to work on the same tasks. And *all* children can find the results by acting with concrete numbers.

To start with multiplication has the advantage that all children are willing to calculate with concrete numbers. There is nothing discriminating in calculating by acting. In addition, this acting shows significant advantages with respect to developing the number concept in a direction of numbers as a whole and the concept of structured numbers as shown here:

- By laying $3 \cdot 4 =$ on the carpet, a counting child will count the first four and the second, and maybe the third also. With every new task the child will learn that the second and the third factor looks same as the first. Thus, when they have identified the first factor, they can just rebuild the others by perception.
- Only giving tasks with the factors zero to four will consolidate the visual concept of these concrete numbers. This allows children to name them by subitizing and to grab them at once. The items don’t need to be counted by verbal counting.
- The solution of $2 \cdot 3 = / 3 \cdot 2 =$ or $3 \cdot 4 / 4 \cdot 3 =$ shows the same result. This can easily be understood: Children only need to put the corns on the other side of the rectangle. This eqality can be seen and understood even better when it is connected with the experience of patterns in buildings: Every rectangle (of cubes) can be named in two ways, depending on the side of view.

[8] All calculation happens on a carpet. This helps children to focus on the act of calculation and the partcipating numbers.

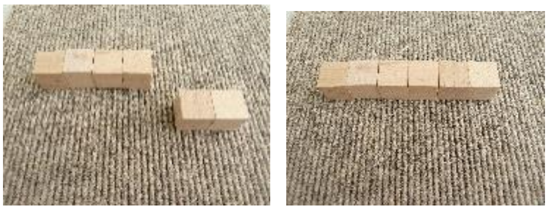
- Many children start to memorize 6 as a double-three, 8 as a double-four or 9 as three threes.[9] They build up the concept of numbers constructed out of smaller numbers.
- The following operation division allows the insight that operations might be reversibly related. The solution of $12: 3 =$ (There are 12 cubes. Three children share them. How many does each get?) shows the same picture as the multiplication $3 \cdot 4 =$. (Every child gets four, because three times four equals twelve.)

To start with multiplication followed by division helps children to build up a cardinal view on operations and supports the aspect of operation in correlations. This sequence supports the change in number concept in direction of numbers as structured wholes that is the fundamental prerequisite for calculations based on the part-whole-scheme. This start makes it possible to do this work in an inclusive setting where all children work on the same tasks with the same methods.

- *Subtraction Before Addition*

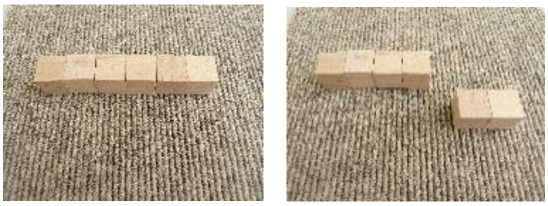
If we use the cubes as concrete numbers and accept calculation as the solving of a task concerning quantity or size, addition is an operation that merges two parts.
This has the effect that the result often overruns the limits of perception even in small additions like $4 + 2 =$ (Figure 6) and this means that the result must be determined by counting.

Figure 6: Addition $\mathbf{4 + 2 = 6}$



If we instead solve the counter-operation, the task $6 - 2 =$ on the level of analog mapping, things are different (Figure 7). Though the minuend must be counted, the operation itself and the finding of the result is possible on the basis of subitizing. Two cubes can be grabbed at once and the remaining four can be perceived at once. This supports the concept of numbers as a whole.

Figure 7: Subtraction $\mathbf{6 - 2 = 4}$



If the subtrahend ‚Two‘, does not disappear in the operation, because subtraction means *‚Take away and let lie!‘*, the student realizes that subtraction is a form of segmentation. It splits the minuend into one part that is taken away and one part that remains. Here we get a second argument as to why subtraction should be first and addition should be second: The evidence of the partition makes it possible to take notice of the operational context. It is obvious that when $6 - 2 = 4$ the task $6 - 4 =$ can be solved with 2. Both subtractions split 6 into a 2 and a 4.
Further, it is natural to realize that uniting the two parts 2 and 4 will rebuild the 6. Addition from the very beginning is something that is recognized as a task in the part-whole-scheme. Addition and subtraction are counter-operations – one merges what the other has divided. From $6 - 2 = 4$, we find $4 + 2 = 6$ and $2+4 = 6$. Subtraction as an action on the level of analog mapping leads directly into the operational connection of partition, addition, and subtraction. In other words, it leads into calculations in the part-whole-scheme.

[9] This is not for early learning of multiplication tables, but it is rather an early opportunity to get acquainted with numbers that are built out of other numbers: It is a first evidence of the part-whole-scheme!

There is a third aspect to the curricula I recommend that is used to root the thinking in the childs concept of number and operation. The children are not only given simple subtractions and addition but also the whole bandwith of equations.

This starts in about the sixth week in the very small number range up to 2, 3 and 4 and is carefully extended to 5. Always in connection with the ‚Zerlegungshaus' (Figure 8), where the different partners have to be filled in.

Figure 8: Zerlegungshaus of 2, 3 and 4

2	
2	
0	
1	
	0
	1
	2

3	
2	
0	
1	
3	
	1
	0
	2

4	
2	
0	
1	
3	
	2
	1
	3
	4

After filling in the possible partitions of three, the child works on a sheet with tasks like
2 + _ = 3 , _ + 1 = 3 , 3 = _ + 0 , 3 = _ + 3 ,
3 − 1 =_ , 3 − _ = 1 , 2 = 3 − _ .
If this works for ‚three' without counting, the next step consists of tasks with one, two and three mixed. And if this works, the number range is extended to four and then to five. This deceleration guarantees that children gain trust in calculation without counting. They come to understand that verbal counting is an aid for when the partitions are not known. Verbal counting is not the calculation that we are aiming at. Rather, it is an auxilliary because of a lack of knowledge.

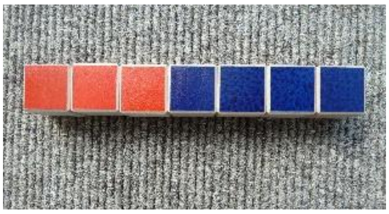
<u>*E: How to Encourage Concept Development*</u>
The use of concrete numbers on the level of analog mapping purely without any further didactical additions helps children understand the cardinal basis of number and operation. Every intervention, for example by putting the cubes on a 20th-field, prevents the intuitive natural acting of the child and therefore limits the options of tasks. Multiplication and division do not make sense on such a field and are nearly impossible by using a 20-abacus or a pearl-chain. The realisation of a subtraction in form of a partition is equally impossible or, at least, it does not give a clear picture.

All the familiar manipulatives have been invented in order to visualize an idea that does not lie in the focus of the learner. The power of five and ten, for example, are worked into those fields, chains, and abacuses, in order to introduce the idea into the childs thinking. However, experience and research show that this does not necessarily happen (Cobb&Wheatley 1988, Lorenz 1998).

However, starting with concrete numbers allows the child to calculate intuitively and correctly. Practical calculation is just a use of daily experiences thereby taking more or giving away or of sharing and multiplying. Thus, it is nothing new! It is only new in the sense that it happens in analog mapping on a calculation carpet.
This intuitive acting leads to problems when numbers become bigger and when numbers need to be named. It is the experience of a problem that legitimizes the intervention by a teacher. *There needs to be a disturbance for a child to be open to accomodation.* I have described such stitations when the task of comparing the cubes of boys and girls was not easily solved and, here, it makes sense to intervene in order to explain the reason for the problem and to demonstrate the limits of perception as well as the useful effect of structure and patterns. *A good intervention does not aim towards an unknown future. It aims to solve a present problem!*

If the intervention is a good one, there is no need for a long explanation or for training. The child understands immediately, and he or she is capable of using this hint from then on. If, on the other hand, the child needs a lot of words and teaching, there is something wrong. Mostly this happens when there is too much future in the intervention, which means that there is too much of what the child is not able to understand from his or her actual experience.

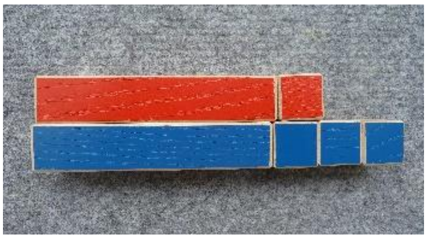
Figure **9**: Holding the summands visible



When children have begun to understand that addition joins parts that have been parted by subtraction, it makes sense to keep those parts visible (Figure 9). Now it is the point to introduce two-coloured cubes for use in addition.

When children work on solving tasks such as 6+8=, they find that neither summand can be controlled visually if they are built out of singles and that the result therefore must be counted. It is evident that both problems disappear when a concrete fiver makes six perceptive as a 5/1 and eight as a 5/3.
And if we lay the two concrete numbers under each other, we can see the result ‚14' immediately (Figure 10) because the two fiver build ten and the rests one and three build four.

Figure 10: 6+8=14



It is a fiver that is introduced and not a tenner! The tens, at this point in development, are not apparent to many children. Tens become important when the number range develops in the direction of a hundred. In the range up to 20, there is no need for tens. The five is legitimate because it it possible to overcome the limit of perception, which is four. This is why in cultural history there is the five before the ten. And that is why children spontanously understand the relevance and the worth of the ‚power of five' (Easley 1983, Flexer 1986, Gerster&Schultz 2004), at least, when we gave them the chance to experience the problem.

Tens become important and their relevance in our number system is not experienced until the recurring sequence of ones up to ten becomes apparent to children. This happens when they start counting to one hundred. Tens become equally important when decimal structures need to be named. This is possible before (!) the number sequence is secure. This, too, is something we can learn from cultural history (Menninger 1979) and that we can translate into primary didactics. I discuss this important aspect in the next section.

F: Lookout and Repeat as a Permanent Principle
Knowledge grows step by step, but not in the sense of one perfect step after the other. It is a process of gradual approximation in parallel fields, and it needs groping movements of thought. Its development needs reassurances in order to arrive at an assured knowledge. Some important aspects such as the recursive decimal structure can only be understood in a larger number range. On the other hand, calculation without verbal counting needs a view on structures and starts in a smaller range.
Time is needed to build up calculation in a part-whole-scheme up to ten. We always have to play in both fields: securing calculation in the small number range and building up the knowledge of the large two and three digit numbers.

The introduction of the two digit number range and its translation into signs and words starts early. In fact, it starts in the 5th week of school when our ‚school-day-counter' overcomes 23

(Rödler 2014). Now it is possible to experience and discuss the connection between the two 10-rods and the three cubes and the written number.

This concept of numbers built out of decimal structures becomes central when the hundredth school-day occurs. On the school-day-counter, the ten tens are changed into a hundred. Keeping on with concrete counting and writing the number signs daily helps children to understand that a sign like 111 does not mean that there are three ones. The school-day-counter shows that the ‚1‘ in a place value system can stand for very different value (Figure 11).

Figure 11: The school-day-counter (111 days in school)

In parallel, the children sort big numbers by tens and ones and lay tens and ones in patterns (Figure 12). This, too, allows children to connect the results with the written number sign. In this process, again, every child can take part and this is possible because we have lowered the level of abstraction.[10] All a child needs to be capable of is to count up to ten, or at least lay pairs of peas to each finger of one hand to create a heap.

We want to see the quantity as clearly as a written number, and there should be no more counting when the number is named. This becomes possible when we lay the tens and ones in patterns. This recurs to the very beginning, and very weak performers can use this entry into the large number range at least for securing the small range up to nine and the one digit number signs. But for the vast majority, this connection of concrete number, laid in perceptive decimal patterns and corresponding number sign repeats not only structures numbers up to nine but also clarifies the cardinal basis of our decimal place value system.

- Which picture shows 29 peas?
- What is the connection between the number sign and the cardinality we see on the first carpet?
- The ‚2‘ does not mean ‚two‘. It means two heaps. The ‚2‘ is a ‚twenty‘ not a ‚two‘! The ‚2‘ is not a number. It is a digit. And it contains the whole twenty.

Figure 12: Concrete number and number sign

29 **35** **20**

[10] No more concrete fiver, no more 5-cent-coins, no more two coloured cubes: the concrete number shows up as countable in homogeneous ones. After having calculated on the level of concrete bundling and symbolic bundling, we go back to the level of analog mapping. *This lowering of abstraction is always used when numbers are large or problems become complex!*

- That also applies for the peas on the third carpet. The whole ‚twenty‘ is included in the ‚2‘.
- So why do we need the ‚0‘? – Just let it away. What stays? ‚2‘. We read two. Two ones. But there are no ones. The ‚0‘ tells us: There are no ones!

The sign 20 says: Twenty and nothing.

In this way, it is easy to introduce all children in first grade to the cardinal meaning of two (and three) digit numbers and to let them understand how to write those numbers with our place value system.

Figure 13: The sum of 29, 35 and 20, calculated on the level of first-graders

On this basis, we also can start with computation! If, for example, we ask how many peas there are together on those three carpets, the result can be found by acting. We only need to sort heaps and ones.
It is evident then that the nine peas need only one more to build another heap and so we move one out of the five and find the result: eight heaps and four; eighty and four: 84

Tasks like $34 + 17 =$ or $45 - 28 =$ are an exercise in the understanding of two digit numbers. They are a practise in understanding the place value system, and they provide a first insight into operational proceedings.[11] The goal at this point is not to practise mental calculation with large numbers. Rather, the goals are more basic and children should understand the following:

- tens are built out of ten ones, which is a prerequsite for the concept of ‚reversible tens‘.
- multi digit numbers are built out of tens and ones.
- sometimes tens must be dissolved into ones. (For example, in $45 - 28 =$). This means that they are a reversible structure.
- patterns allow spontaneous naming of numbers, and this makes it possible to use this concrete numbers for calculation without counting.
- patterns allow spontanious answering, whether an addition creates a new ten (or a subtraction effords to dissolve a ten) or not.

This example from the middle of the first grade proves just how complexity allows lookout and repetition in the same situation. Complexity creates inclusive learning by natural differentiation. Weak and poor performers are equally addressed and challenged. Differentiation therefore does not split the class and it happens in a common learning process while doing the same things on the same tasks.
It is also an example of interventions that are orientated towards the children’s focus and concept of number. By lowering the level of abstraction, it is possible to get into large number ranges and into complex questions. Raising the level of abstraction by using concrete or symbolic bundles helps to rationalize concepts of computation.

Summary:

[11] It is evident that this way of starting calculations leads to completely different number concepts and, in particular, to different concepts of operation other than a path that is based on verbal counting and the word sequence of ones and tens. Just compare this concrete calculation with the solutions of the students in Cobb&Wheatley (1988). We don’t have to count verbally when we are capable of using structures.

The perspective of cultural history provides a history of concrete numbers and concrete computation. It begins with numbers that are created by analog mapping. They then develop by integrating patterns and bundles to maintain the detectability, even in larger number ranges.
Number words develop out of the need to talk about what is perceptive. New number words are created when patterns, structures, and concepts of bundling combine to overcome the limitations of perception.
Historically, the extension of number range happened in the interplay of concrete number, number word, and number sign. Larger and larger quantities were identified, named, and designated. On the bases of this knowledge and experience, the verbal number sequence arose.

The verbal number sequence was not the starting point. Rather, it is a late conceptualization. Maybe this is the reason that our cultural approach to reversing this natural order fails for some children who know the verbal sequence but do not understand the cardinal basis.
The fact that it needed thousands of years to get to a number word sequence, to get into decimal bundling and into concepts of reversible levels of decimal value, and, finally, to develop the concept of writing number signs in a place value system shows that there are hurdles to overcome. This should make us sensitive to the fact that it is precisely the same set of hurdles that students must overcome currently when they start with calculation. Whereas humans needed thousands of years, we should focus on these particular hurdles and give children the time and experience that is necessary to develop the concept of number.
This development happens when children are drawn into concrete counting and concrete calculation. It is based on the childs own concept of counting because this development happens on the basis of the child's thinking, the child is capable of reflecting on what is happening and can therefore improve the procedure.
To enable perception is a key driver in this process. Without perception there is no control. In order to sustain perception, the number concept must be developed. A developed number concept is the basis of reasonable computation as it allows the use of structures rather than simple counting.

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An Alternative Sudoku Puzzle with Letters While Addressing Math Anxiety

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Abstract

Math anxiety remains a critical issue affecting student performance and confidence across grade levels throughout the world. This paper looks at the impact of math anxiety on students and also how using letters instead of numbers with Sudoku puzzles can perhaps alleviate math anxiety and number anxiety as an alternative to doing Sudoku puzzles and turning students on to the logic of magic squares and Sudoku puzzles. This paper shares data on math anxiety levels by grade level from a study, provides some examples of some Sudoku puzzles with Greek letters and our English alphabet along with much research, and recommendations of best practices for teaching math and addressing such concerns in light of the reality of math anxiety existing in a world where we are preparing young people for a STEM world. The data in this study shows an upward trend in higher math anxiety levels as students increase in grade level. It is evident teachers need to do more starting in the early grades and each grade to use best practices for teaching math and also use math anxiety reduction strategies to work on reducing math anxiety as students advance each grade level. Research, best practices for teaching mathematics, strategies, and a survey are included.

Keywords: Math Anxiety, Best Practices, Sudoku Alternative, Attitudes, STEM

Introduction

		c				b		
	f		i	h			d	c
d	i			c	a			f
i		g				h	f	
	d			i	h			
		e	d		g	a		i
f					c	i		e
e		h	a				g	b
b		i		e	f		c	h

Directions:

Letter Sudoku is played on a grid of 9 x 9 spaces. Within the rows and columns are 9 “squares” (made up of 3 x 3 spaces). Each row, column and square (9 spaces each) needs to be filled out with the letters a to i, without repeating any letter within the row, column or square.

Figure 1. Sudoku Puzzle Using English Alphabet

Can you solve the above Sudoku puzzle with letters as opposed to the traditional numbers? [See Figure 1.] Would it be easier for young people or people afraid of numbers or math to solve these puzzles if they used letters instead of numbers? When many people see numbers they panic, many people today have math anxiety and numbers and math cause them panic and make them freeze or shut down. Maybe an alternative to Sudoku can help. According to Danesi (2019) "“Sudoku has a simple structure, a simple set of rules for solving it, but it still presents a challenge. Unlike the crossword, however, it requires no "external knowledge" (names of people, events, linguistic knowledge, etc.). It just requires us to place symbols (usually the first nine digits) in cells in a logical way. Despite its Japanese name, the concept behind Sudoku crystallized in the United

States in the form of "Numbers in Place," which appeared for the first time in the May 1979 issue of Dell Pencil Puzzles and Crossword Games magazine. It went virtually unnoticed, except by readers of the magazine. By the way, the late architect Howard Garns is pegged as being its inventor. In 1984, an editor for Nikoli magazines in Japan came across one of the puzzles, changed its name to Sudoku (meaning "only single numbers allowed") and included it in his magazines. Sudoku yet, since they could be solved in more than one way. But they had a similar layout. And, of course, there are magic squares, which go back to ancient times in China. Magic squares are number placement puzzles, but are solved by considering the actual value of a number since, in a magic square, the rows, columns, and diagonals must all add up to the same total (known as the magic constant). After being told by the AP reporter what Sudoku was all about, I pointed out to her that the idea can probably be traced back to magic squares or to "Latin Squares," invented by Swiss mathematician Leonhard Euler (1707-1783). A Latin square is a square arrangement of digits placed in such a way that no digit appears twice in the same row or column. Sudoku, I mentioned to the reporter, seems to simply expand upon Euler's invention. Sudoku is a simple puzzle with no tricks or twists built into it. In its usual form, it is made up of a nine-by-nine grid, with heavy lines dividing it into nine three-by-three boxes. The challenge is to fill the layout with the digits from 1 through 9, so that every row, every column, and every three-by-three box contains these digits, without repeating-that is, once and only once. The puzzle-maker provides some of the numbers in the layout, and these are the initial clues to be used in solving the puzzle. How is the level of difficulty determined? I am not sure, really, although the implicit principle seems to be that that the fewer the initial clues given, the harder it is to solve the puzzle" (Danesi, 2019). One of the oldest known Latin Squares known today is the Sator Square. This Square was supposedly found amongst the ruins of Pompeii in some volcanic ash as a result of the Mount Vesuvius eruptions in 79 AD, they were pressed in clay or perhaps carved in stone that were found after, more about it shows up on Glenn Westmore's blog (glennwestmore.com.au). Nonetheless, magic squares have been around for maybe more than 4000 years, used and found in Europe, China, and Africa and the Sudoku puzzle is based on magic squares.

When I tell someone that I am a math teacher, almost always they say, "I hated math" or "Math was my worst subject." Unfortunately, many people do not like math or have had bad experiences with taking the subject. The purpose of this review math anxiety research and what can be done to address such concerns while looking at Sudoku puzzles and connecting them to letters to more easily relate them to young learners. In addition to determine some other demographic information about math anxiety and preferred teaching styles for learning math. Many statistics/percentages are shared from a larger study conducted using K-12 students taking math courses. Some ideas for using letters/symbols instead of numbers for Sudoku puzzles is also explored. Negative attitudes toward mathematics and math anxiety are serious obstacles for students in all levels of schooling today (Süren & Kandemir,2020; Widjajanti, Listyani, & Retnowati,2020; Furner, 2021: Furner, 2019: Furner, 2017; Geist, 2010). Yet only limited attention has been devoted to the antecedents of math anxiety, which may include social factors like exposure to teachers who themselves suffered with math anxiety (Brewster & Miller, 2020; Maloney & Beilock, 2012). This study looks at the math anxiety levels among students K-12.

An elementary school principal from the school that this data in this paper is from told the author once that she always interviews all new students coming into the school and always asks students, "What is your favorite subject?" She said that most of the younger children always say to her, "math." The same school had decided to as part of their Southern Association of Colleges and Schools (SACS) accreditation as a K-12 international school (USA based curriculum) in Latin/South America to survey 25% of their students at each grade level (they have approximately 100 students per grade), Grades 1-12, and administer the Abbreviated Version of the Mathematics Anxiety Rating Scale(MARS) (Alexander and Martray, 1989) to see how their students feel about their math attitudes (See Figures 1 and 2). The results are somewhat inconclusive, but the graph shows primarily that as students increase in grade, their level of math anxiety increases for the most part (Furner, 2019) (See Figures 2 and 3). This may not be a complete surprise and seems consistent with the Third International Mathematics and Science Study (TIMSS) math results in the USA, whereas students increase in grade, their level of math achievement drops significantly from elementary, to middle, then to high school (Schmidt, 1998). Today in an age of preparing our young people for fields in the areas of Science, Technology, Engineering, and Mathematics (STEM), it is critical young people have positive dispositions and attitudes toward mathematics.

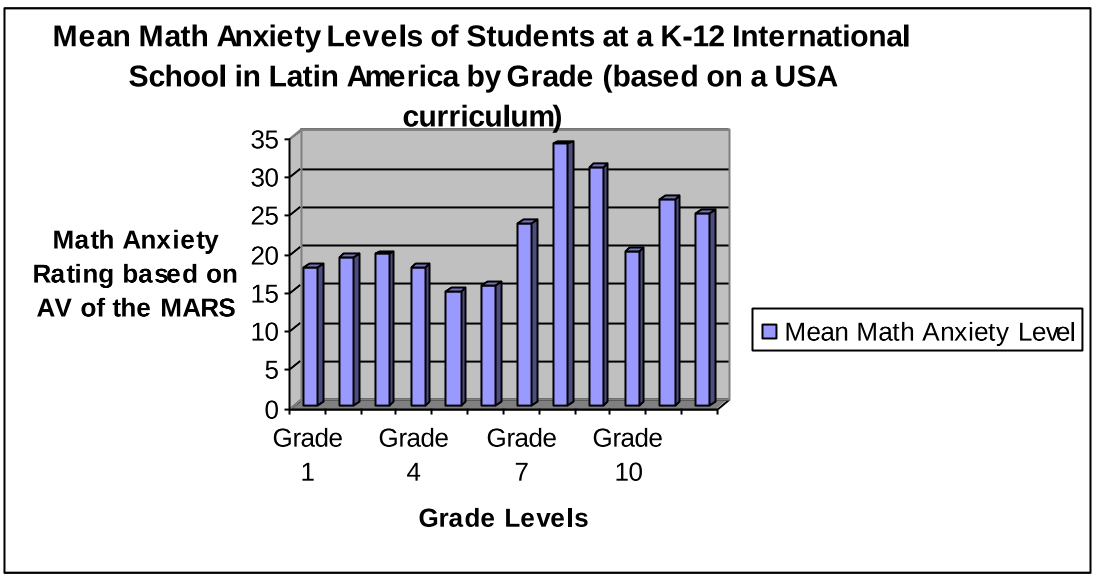

Figure 2. Mean Math Anxiety Levels by Grade

Mean Math Anxiety Levels by Grade

Grade	Average of the MARS
Grade 1	18

Grade 2	19.3
Grade 3	19.7
Grade 4	18
Grade 5	15
Grade 6	15.7
Grade 7	23.8
Grade 8	34
Grade 9	31
Grade 10	20
Grade 11	26.8
Grade 12	25

Figure 3. Raw Data of Math Anxiety Levels by Grade

According to Reuters (2007) and the American Association for the Advancement of Science in San Francisco, math anxiety saps working memory to do mathematics. Often times, worrying about doing math takes up a large part of a student's working memory which then spells disaster for the anxious student who is taking high-stakes tests. Today math teachers from around the world almost have to take on the role of counselors in their classrooms to address the many students who dislike or are fearful of mathematics. Mathematics teachers are encouraged to work with school counselors and Exceptional Student Education (ESE) teachers in helping to address the many math anxious students in today's schools. It really has become a pandemic in our society where so many young people and adults have negative feelings and poor past experiences with mathematics instruction. Metje, Frank, & Croft, (2007) believe that math anxiety is a worldwide phenomenon and that many people are not going into math fields like engineering and that more and more math instructors at the university level are not prepared to deal with the increased number of students who fear math to be able to teach and reach them during instruction, addressing math anxiety has become one of the largest challenges for a lecturer is supporting the students overcoming this fear of mathematics.

Anyone today can easily take an informal poll on the street and find that most respondents will not report positive experiences, feelings, and attitudes toward mathematics. However, we are now living in an age that depends so heavily on one being good at mathematics and problem solving. We are living in a world in which our students will soon be competing with young people from all parts of the globe for jobs. It is imperative that our students develop positive dispositions toward mathematics and the sciences in an information age of which has become so technologically oriented. Young people today need to be well prepared in the areas of math, science, and technology for all career choices. Nurses, engineers, architects, lawyers, teachers, along with many other fields will continue to use more advanced forms of technology that require one to know more mathematics and problem solving to perform their jobs more effectively. Sequencing, ordering, patterning, logic, spatial sense, and problem solving are some of the truly basic skills that all careers require (NCTM, 2000). By the time our young people reach middle school they have developed certain dispositions toward mathematics. Students' confidence and ability to do mathematics and apply these skills in many diverse settings is essential for success; therefore, our young people need to be well prepared to do the mathematics of the 21st century.

Steen (1999) found that "national and international studies show that most U.S. students leave high school with far below even minimum expectations for mathematical and quantitative literacy." Neunzert (2000) contends that we have to understand ourselves as MINT-professionals, where MINT is M=mathematics, I=informatics, N=natural sciences, T=technology. Neunzert (2000) believes that mathematics is critical for people living in the 21st Century for them to be successful. Neunzert feels as educators we need to encourage our students in all countries to study more mathematics and to see it as a tool for success in life. Most schools and states in the USA today are adhering to the Common Core Math Standards (National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO), 2010) which provide math teachers core math standards at each grade level to reach all students with much higher and rigorous levels of mathematics for US students. Today most states use these standards or a variation of them for teaching and learning mathematics to better prepare young people for many of the STEM fields. A broad overview of common core standards in mathematics include number sense concepts and operations, measurement, geometry and spatial sense, algebraic thinking, and statistics and probability. Teacher education programs have included such math standards in their curriculum so that teachers are better prepared to teach to these newer math standards covered in our K-12 schools in the USA.

Math Anxiety

What is math anxiety? Well, to put it simply, it is anxiety when confronted with math, especially about one's own performance in solving math problems. It can range from slight nervousness to all-out panic. This anxiety makes it more difficult for students to focus in class, learn math, and solve math problems. Frequently students would rather give up than have to face their fears. This means that they never get better at math and can therefore never overcome their anxiety. If this anxiety is not overcome, the student may suffer from this anxiety for their entire life, even beyond their time in school. Math anxiety is a well-documented phenomenon that has affected our society for over sixty years, and not enough is being done to address it in our classrooms or in the way we teach math (Szczygiel, 2020; Beilock & Willingham, 2014; Boaler, 2008; Dowker, Sarkar, & Looi, 2016; Geist, 2010; Metje, Frank, & Croft, 2007). Negative attitudes toward mathematics and math anxiety are serious obstacles for students in all levels of schooling today (Geist, 2010). Beilock and Willingham (2014) state that "Because math anxiety is widespread and tied to poor math skills, we must understand what we can do to alleviate it" (p. 29). Even Sudoku puzzles with numbers, because of the numbers can be anxiety provoking for math anxious learners.

What Causes Math Anxiety? Math anxiety is caused by a combination of external and internal factors; however, we cannot change internal factors within the student, so as teachers it makes more sense to focus on what we can control (Chernoff & Stone, 2014). Studies show that math anxiety is caused primarily by the way the student learns math: the type of authority the teacher uses, an emphasis on right answers and fear of getting wrong answers, requirements that the student respond with an answer sooner than he or she might be ready, and exposure to the rest of the class and their potential condemnation of a student who responds poorly, in short the traditional way of teaching math (Chernoff & Stone, 2014, Finlayson, 2014). Traditional teaching emphasizes:

- "Basic skills
- Strict adherence to fixed curriculum
- Textbooks and workbooks
- Instructor gives/students receive
- Instructor assumes directive, authoritative role
- Assessment via testing/correct answers
- Knowledge is inert
- Students work individually." (Finlayson, 2014)

Unfortunately, these methods can cause and increase math anxiety in the classroom (Finlayson, 2014).

According to Demirtaş & Uygun-Eryurt (2020) math anxiety can also be transmitted and learned from others, usually from parent to child or teacher to student, but occasionally student-to-student. If someone teaching math, whether to their own child or to a class, experiences math anxiety, they are more likely to rush through things in order to "get it over with". They would not be sure of their methods, so they would focus more on the correct answer. Like the student with math anxiety, they are also likely to become exasperated and give up rather than continue helping the student. This teaches the student that math is something to be afraid of and that, if they are not good at it, their parent or teacher will become angry with them and potentially leave. They also learn in class that, if their peers see that they are bad at math, they will be ridiculed publicly. Embarrassment is a very big deal for children, especially in middle and high school.

Another problem for those who suffer from math anxiety is the nature of anxiety itself. According to Rubinstein et al (2015), anxious individuals tend to focus on negative stimuli more than positive stimuli, essentially making themselves more anxious. The same thing is true of individuals with math anxiety; the only difference is that for people with math anxiety, math is negative stimuli (Rubinstein et al, 2015). This suggests that math anxiety could be handled through therapies designed to lessen anxiety, such as cognitive behavioral therapy and exposure therapy (exposing a person little by little to the thing that they are afraid of) (Rubinstein et al, 2015). While this is not something that a teacher could do with a full class to manage, it is something that tutors could be trained to help with; naturally, a licensed therapist would be the best option, but not all therapists are trained to help students with math. A combination of the two fields would be optimal.

Math anxiety remains a perplexing, persistent, and only partially understood problem from which many people suffer, NCTM (1991, p. 6) says, "Classrooms should be mathematics communities that thrive on conjecturing, inventing, and problem solving, and that build mathematical confidence. Unfortunately, currently, many kids and adults do not feel confident in their ability to do math. Mathematics anxiety in students has become a concern for our high-tech world. Is it possible that only about seven percent of Americans have positive experiences with math classes from kindergarten through college study (Jackson, C. D. & Leffingwell, 1999)? Burns (1998) in her book *Math: Facing an American Phobia* tackles an interesting subject and has found that two-thirds of American adults' fear and loathe math. Whether it is 93% or two-thirds of Americans experiencing negative math experiences it is clear that there is a problem and we need to do something about it as educators. If math anxiety is such a problem, one has to wonder why isn't as much being done about it in our schools today?

Evidence of students' poor attitudes and high levels of anxiety toward math is abundant today. In the midst of a technological era, declining mathematics (math) scores on the Scholastic Aptitude Test (SAT) have been widely publicized. Some reports have shown that American students rank last when compared with students from all other industrialized countries on 19 different assessments. The TIMSS study has shown a trend in U. S. students' math scores as they decline as students increase in age group from grade four to grade twelve (Schmidt, 1998). What is happening to our students that so many of them lose interest in math and lack the confidence to do and take more math classes?

How Do We Repair Math Anxiety Concerns in our Schools?

To put it simply: better teaching. Finlayson suggests the constructivist style of teaching which emphasizes these ideas:

- Begin with the whole – expanding to parts in learning process
- Pursuit of student questions/interests
- Use primary sources/manipulative materials
- Learning is interaction – building on what students already know, constructivism
- Instructor interacts/negotiates with students

- Assessment via student work, observations, points of view, and tests. Process is as important as product
- Knowledge is dynamic/change with experiences
- Students should work in groups (2014)

This style of teaching is very different from the traditional style which can cause and increase math anxiety. The constructivist style is much less intimidating and does not emphasize timed assessments or correct answers; instead, it focuses on the process of doing mathematics. Students are also likely to feel more engaged in class due to the more participatory style of teaching, making them want to work harder, instead of "getting it over with" oblivious of how this affects their performance.

However, frequently the problems in the classroom that cause math anxiety are due to a teacher with math anxiety (Chernoff & Stone, 2014). These teachers choose the easiest ways of teaching (rote memorization of formulas, practice using one method to get one right answer, timed tests, etc.) in order to minimize their own math anxiety, not realizing that they are passing their own anxiety onto their students (Chernoff & Stone, 2014). Therefore, we must first remove math anxiety from teachers, so they may teach their students not to experience math anxiety. Math is not inherently frightening, but that is the message that is modeled and expressed to many children, even from their parents and teachers.

As mentioned previously, math anxiety is a form of anxiety and therefore treatable through the same types of therapy we use to treat general anxiety and phobias (Rubinstein et al, 2015). This may prove especially helpful for adults with math anxiety, especially teachers; by working to handle their own math anxiety, adults would be able to prevent transmission of their anxiety to their children or students (Chernoff & Stone, 2014).

Discussing the Data from the K-12 School's Math Anxiety Levels Presented in this Study

The major trend from this data shows a notable upward trend in math anxiety in students as students increase in grade level and age (See Figures 2 and 3). As students take more math classes and are exposed to more math teaching, unfortunately their level of math anxiety increased in this data set of a K-12 International School in South America with a US-based curriculum. In discussions with the administrators and teachers, little is often done year to year with students as they pass from grade to grade in respect to addressing a students' math anxiety. This math anxiety can fester and continue to pass on and increase as students continue through their studies. The author of this paper worked with this school for two years during this data collection in the school as part of the SACS accreditation. He also worked as the 9th Grade Geometry teacher for the first year prior to the data collection year and has extensive expertise in math anxiety research and implemented extensive math anxiety reduction and prevention techniques. The author employed these techniques with the 9th Grade mathematics students the year prior to the data collection. It is visible to see that the 10^{th} Grade Students had reduced levels of math anxiety, likely due to the preventative and reductive math anxiety techniques used. Preventative strategies: like using "Best Practice" in mathematics include using: manipulatives, cooperative groups, discussion of math, questioning and making conjectures, justification of thinking, writing about math in math journals, using a problem-solving approach to instruction, content integration, using technology Geometer's Sketchpad, assessment as an integral part of instruction, such as homework quizzes and math portfolios. Along with math anxiety reductive strategies which include using: psychological techniques such as anxiety management, desensitization, counseling, support groups, bibliotherapy, and classroom discussions of how students feel about math and what they are learning. These insights can better help to understand why the 10^{th} Grade class had significantly lower math anxiety than the other middle school and high school grades. Students in elementary school often start out with little math anxiety, but this anxiety can increase as students go from grade to grade in their learning process. It is critical in an age of STEM (Science, Technology, Engineering, and Mathematics) that schools and teachers work to correct this trend of increase in math anxiety age students go from Grades K-12. More schools need to include affective aspects into their improvement plans, like checking for math anxiety, and then compare such data to their students' achievement levels. Unfortunately, like TIMSS showed for US schools, the trend of math achievement went down as students increased in grade like this study shows with math anxiety and it is likely correlations exist with how students feel about mathematics and how they perform. School leaders need to start looking at both affective and cognitive aspects of learning to see the relationships and to better address achievement and performance of their students in mathematics and likely all subjects. Higgins, Furner, and Gerencser (2020) in their work with 9^{th} Grade math students found bibliotherapy and some systematic desensitizing and counselling and group work effective in addressing students math anxiety.

Teachers, Counselors, and ESE Teachers Working Together to Improve Math Scores

To address the issue of math anxiety, classroom teachers need to team up with school counselors, ESE teachers, and professional development experts in teaching mathematics and make this all a part of their improvement plan, to assess attitudes toward math to then work toward improving math achievement. Teachers need to be sensitive to students' needs, feelings, and experiences with mathematics. Brigman & Campbell (2003) and Parker (1997) have found based on their research that when school counselors' team up with classroom teachers they can have a profound effect on student achievement scores. A counselors' psychological expertise can serve as a real asset to classroom teachers and students who struggle with a fear of mathematics or poor past math instruction experiences. As educators, we need to remember that not all students are alike, yet all students deserve equal opportunities in the mathematics classroom (NCTM, 2000). A math teachers' job is not only to teach the subject area. One of NCTM's goals for all learners was that as math teachers, we should help students become confident in their ability to do mathematics (NCTM 1989). NCTM (1989, 2000) contends that students should be exposed to numerous and varied interrelated experiences that encourage them to value math, to develop mathematical habits of the mind, they should understand the role of math in human affairs: they should be encouraged to guess, read, write, make conjectures and make errors so that they can gain confidence to solve complex problems. With this in mind, it is clear then that math teachers are not only instructional leaders, they are also counselors and confidence builders for their clients, their students.

Math anxiety may be defined as an inconceivable dread of mathematics that can interfere with working with numbers and solving word problems within a variety of everyday world and academic situations. NCTM (1989 & 1995) recognizes math anxiety as a problem and has specifically included in its assessment practices as a teacher's job to assess for their students' mathematical dispositions as NCTM Standard #10 (NCTM, 1989) (See Appendix A)

Today there are many things teachers and schools can do to help prevent math anxiety from occurring in our students. It really is a complicated matter and may involve what happens to students in and outside of the classroom. Both parents and teachers can play vital roles in helping to develop positive dispositions toward math in students. It is important that teachers check for these positive attitudes and dispositions toward mathematics at an early age. Often students can develop such anxieties toward math very early on in their math classrooms due to poor teaching, drill and practice, strained testing situations, parental and teacher insecurities about their own math abilities, etc. The elementary and middle school years are critical to developing positive perceptions toward mathematics in children. The NCTM (2000, 1995, & 1989) has made recommendations for preventing and reducing math anxiety (See Appendix A).

Reducing math anxiety is much different from preventing math anxiety. Teachers need to work with school counselors and to act as psychologist or counselors themselves to help lower or overcome such anxiety toward math in their students. It is critical that math teachers team up with school counselors to address reducing math anxiety in their students. Researchers in math anxiety propose systematic desensitization (Higgins, Furner, & Gerencser, 2021; Arem, 2003; Furner, 1996; Schneider & Nevid, 1993; Hembree, 1990; Trent, 1985; Tobias, 1993; Olson & Gillingham, 1980) as one of the most effective approaches for helping people reduce their math anxiety. Systematic desensitization in the context of math anxiety may be defined as a gradual exposure to the mathematical concepts that are causing students to become distressed and teaching them how to cope with that fear they are dealing with. Each time students are exposed to the math they fear, they should improve in their techniques in coping with their anxious feelings. Being able to talk about their history with math and releasing their anger, hatred and fear of the subject may be therapeutic in nature and then eventually students can work toward, come to terms with this anxiety, and overcome it (Higgins, Furner, & Gerencser, 2020). Through these types of counseling approaches, students will be able to come to understand that their anxiety was a learned behavior, which they were not born with these feelings toward math, and they can be taught to overcome them by consistently implementing their self-monitoring strategies to become less anxious and assessment and evaluation play a critical role in all of this (Ghaderi, Gask, & Jamali, 2020; Szczygiel, 2020).

How is math anxiety reduced? Teachers must help students understand how their math anxiety was created (See Appendix A) and work toward overcoming this fear while developing confidence. As Reuters (2007) and the American Association for the Advancement of Science in San Francisco reported, a relationship does exist between math anxiety levels and math achievement levels. Teachers can work with school counselors and be counselors themselves to ease such anxiety and work toward helping the students gain more confidence in doing math so that math achievement levels improve. In the case of the school mentioned here that is also assess math attitudes using the abbreviated Version of the MARS, they are using this information to work more closely with students to then help them overcome their math anxiety so that the school will hope to see high math achievement levels in the years to come.

Alternative Sudoku Puzzles with Letters

Students may find Sudoku puzzles more interesting and less threatening when they are presented with using letters as opposed to numbers since many students with math anxiety freeze when they see numbers [See Figures 4 and 5]. Evans, Lindner, & Shi (2011) advocate that students generate and create their own Sudoku puzzles and that by doing so it has many applications in having a better understanding of mathematics. While Sudoku really does not involve computational skills in math, it does involve logic and deduction, also Sudokus can be done completely without numbers as symbols or letters may be used instead in the creating of them. Pantaleon, Payong, Nendi, Jehadus, & Kurnila (2020) suggest when addressing math anxiety consider creative thinking approaches as part of teaching mathematics and allow students to explain and proving their work. Students may create Sudoku puzzles using symbols, letters, or perhaps the Greek alphabet, if students fear numbers, then while addressing math anxiety, they may also use letters as opposed to numbers in solving such puzzle and even creating them. According to Güven, Gültekin, & Dedeoğlu (2020) even your children who have gone through a math program with hands-on and a Montessori approach can benefit from doing Sudoku puzzles, and starting with simple one's like the 4 x 4 below [See Figure 4], you can show the students how to do it with numbers and even letters or other symbols or signs and maybe even manipulatives to have them fill in the grids with no repeats horizontally, vertically, and diagonally when doing problem solving and focusing on strategies. Posamentier & Poole (2020) contend that a lot of math can be learned and better understood when it is presented through problem solving. Sudoku is a great problem-solving game using logic and deduction. Students can also use letters as opposed to numbers to substitute in doing the puzzles like the 4 x 4 example below [See Figure 4]. Henle (2020) talks about how in grades 1-5 math should be taught more with art, as an educator, the author has always thought this too, one may use the Dürer's Magic Square and in his artwork show such a magic square to create and explore other magic squares like the one below which may be simple with Greek letters of the alphabet.

Jiang, Liu, Star, Zhen, Wang, Hong, & Fu (2020) researched how mathematics anxiety affects students' inflexible perseverance in mathematics problem-solving and they examined how the mediating role of cognitive reflection is critical as students learn math so to help prevent such anxiety. There is much research on the neurological aspects of learning math, self-regulating, stress, and math anxiety and executive function like neural influences of task switching on arithmetic processing as people learn math and how anxiety is created (Choi,Taber, Thompson, & Sidney,2020; Gabriel, Buckley, & Barthakur, 2020; John, Nelson, Klenczar, & Robnett, 2020; Pizzie, Raman, & Kraemer, 2020). As students often may see numbers as more threatening than letters or other symbols due to some of this research, perhaps using letters instead allow students to solve and explore with Sudoku puzzles with such letters like the examples in this paper as a less threatening way to solve, deduce, and use logic without allowing the numbers to pose a threat to their cognitive function or bring stress, or a shutdown of the processing [See Figures 4 and 5]. Eventually as students get comfortable doing the Sudoku puzzles with letters, they may be substituted with numbers as students gain more confidence while doing them systematically desensitizing them to the numbers working toward developing more confidence to use numbers. Math anxiety and number anxiety are real and if you can teach young people how to do Sudoku puzzles with letters first if it is less anxiety provoking, then lead to using numbers this can help the students gain more confidence and success with math, as many people believe the Sudoku puzzles are only for "math" or "smart" people when really, they are a diversion and an exercise for anyone. Consider the example Sudoku puzzles below as lead-ins for such exercises with young learners.

α	β		
		β	α
β	ε		
Δ		ε	β

Complete the 4 x 4 Sudoku/Magic Square with the correct Greek letters.

Figure 4. 4 X 4 Sudoku/Magic Square Using Greek Letters

		Δ				β		
	λ		Ω	π			ε	Δ
ε	Ω			Δ	α			λ
Ω		μ				π	λ	
	ε			Ω	π			
		Κ	ε		μ	α		Ω
λ					Δ	Ω		Κ
Κ		π	α				μ	β
β		Ω		Κ	λ		Δ	π

Complete the 9 x 9 Magic Square/Sudoku with the correct Greek letters. Each row, column and square (9 spaces each) needs to be filled out with the correct Greek letters without repeating any letter within the row, column or square.

Figure 5. 9 x 9 Magic Square/Sudoku with Greek Letters

Summary

Math anxiety is a major problem in today's world. As the old adage says, "attitude is everything," when students have bad attitudes toward mathematics, it can affect their lives forever. Maybe even make them as adults to be afraid of or not like math or doing Sudoku puzzles too. As adults, we need to be aware of our own anxiety in order to prevent it from being transmitted to our children and students; for those who are unduly impacted by math anxiety or for those who are more likely to transmit this anxiety to children, it may be helpful to receive assistance from a therapist. The data presented in this paper show little math anxiety with the early grades and increasing as students go up in grade. As math teachers, we need to make our classrooms a safe haven for students with math anxiety by altering our teaching styles; this will help all students, not just those with math anxiety. In order to fix this problem, we need to go straight to the source, even if that source is in our own anxieties. Only then can we prevent future generations from becoming part of the pandemic of math anxiety. Teachers of mathematics need to take on the role of counselors to address the math anxious students they have in their classrooms, addressing both preventative and reduction techniques to address such math anxiety.

Math teachers should be teaming up with school counselors and ESE teachers to employ the many suggestions and recommendations mentioned in this article in their classrooms/schools to help prevent and reduce math anxiety. Today, teachers need to put on their educational psychologists hats on in their classrooms to help address the issue of math anxiety. Teachers may also want to work with school counselors as well as encourage their schools to have family math nights where parents come with children and together they can "do math" and see its importance and value in life. As a society, we must work together to extinguish the discomfort that our youngsters are having toward mathematics, especially as students increase in age. It is important that all students feel confident in their ability to do mathematics in an age that relies so heavily on problem solving, technology, science, and mathematics. Today's educators must make the difference in our children's attitudes toward math. Math teachers working with school counselors and ESE teachers can strive toward creating mathematically literate and confident young people for the new millennium. The data in this study shows an upward trend in higher math anxiety levels as students increase in grade level. It is evident teachers need to do more starting in the early grades and each grade to use best practices for teaching math and also use math anxiety reduction strategies to work on reducing math anxiety as students advance from one grade to another. It would be nice to hear more young people and adults when asked how they feel about math say, "Math was my favorite subject" or "I am great at math!" not "I hate it" which seems to be more often heard by adults in the 21st Century. We need to flat line this trend, not allowing it to create an escalating a bar graph of increases as students increase in grade level. As math educators, we need to correct these poor attitudes toward mathematics so not to hold young people back in their lives from pursuing STEM fields and/or making important decisions, maybe even allow them to like doing Sudoku puzzles too!

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Appendix A:

Standards and Strategies to Address

Math Anxiety including the *Mathitudes Survey*

Standards and Strategies to Address Math Anxiety

Mathematics teachers need to be counselors too…

What NCTM says about Mathematics Anxiety and Dispositions Toward Mathematics

Standard 10: Mathematical Disposition (NCTM 1989)

As mathematics teachers it is our job to assess students' mathematical disposition regarding:

-confidence in using math to solve problems, communicate ideas, and reason;
-flexibility in exploring mathematical idea and trying a variety of methods when solving;
-willingness to persevere in mathematical tasks;
-interests, curiosity, and inventiveness in doing math;
-ability to reflect and monitor their own thinking and performance while doing math;
-value and appreciate math for its real-life application, connections to other disciplines and cultures and as a tool and language.

A Synthesis on How to Reduce Math Anxiety

1. Psychological Techniques like anxiety management, desensitization, counseling, support groups, bibliotherapy, and classroom discussions.

2. Once a student feels less fearful about math he/she may build their confidence by taking more mathematics classes.

3. Most research shows that until a person with math anxiety has confronted this anxiety by some form of discussion/counseling no "best practices" in math will help to overcome this fear.

A Synthesis on How to Prevent Math Anxiety

1. Using "Best Practice" in mathematics such as: manipulatives, cooperative groups, discussion of math, questioning and making conjectures, justification of thinking, writing about math, problem-solving approach to instruction, content integration, technology, assessment as an integral part of instruction, etc.

2. Incorporating the NCTM *Standards* and your State Standards into curriculum and

instruction.

3. Discussing feelings, attitudes, and appreciation for mathematics with students regularly

Name____________________________________

Grade____________________________________

Math Class________________________________

Age______________________________________

Career or Career Interest____________________

Mathitudes Survey

1. When I hear the word math I.......

2. My favorite thing in math is......

3. My least favorite thing in math is.......

4. If I could ask for one thing in math it would be...........

5. My favorite teacher for math is______________because______________

6. If math were a color it would be.....

7. If math were an animal it would be.....

8. My favorite subject is________________ because___________________

9. Math stresses me out: True or False Explain if you can.

10. I am a good math problem-solver: True or False Explain if you can

Appendix B:

Blank Grid for Creating Sudoku Puzzles with Symbols

Blank grid for creating Sudoku puzzles using any types of symbols

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Transformations

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Word Problems in the Mathematics Textbook: An Instructional Resource Guide to support writing instruction.

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Word Problems in the Mathematics Textbook: An Instructional Resource Guide to support writing instruction

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Abstract
Mathematics textbooks typically include word problems or story problems that require students to develop extended written responses. Yet, the answers to these prompts can vary so widely that preservice and inservice teachers must be prepared for multiple levels of interpretation of the language used to capture mathematical thinking. Based on an analysis of word problems within two teacher's editions of elementary mathematics textbooks, we describe a series of strategies and tasks to scaffold teachers' understanding of planning for word problems during mathematics instruction. We detail the following components; (1) the use of the Instructional Resource Guide, which assists in the decision-making process to support preservice and inservice teachers as they plan and analyze word problem language aiding in the selection of tasks based on specific objectives or instructional goals; (2) the creation of a consistent instructional sequence for integrated literacy instruction during mathematics instruction.

Keywords
Writing, Mathematics Education, Professional Development, Word problems, Problem Solvers, Instructional Planning

Math word problems - this simple phrase often strikes fear in the hearts of elementary students, especially for those who are not confident in math or for those who do not use written words to think and process mathematical information. Yet, scattered across mathematics textbooks are word problems that require students to construct written responses that potentially help students solidify concepts beyond the computation of digits (Colonneselyn, Armspaugh, LeMay, Evans & Field, 2018) and possibly provide teachers with a *window* into student thinking (Sowder, 2007). However, a window can become a Pandora's box when student answers to a single math prompt can be so varied and unwieldy that the teacher must engage in multiple levels of interpretation and draw upon a confluence of skills (Verschaffen, Schukajlow, Star & Van Dooren, 2020).

These skills include mathematics reasoning, problem solving, along with language and visual analysis (of drawings)—all skills that require transdisciplinary thinking across mathematics and literacy. To mediate these challenges, we provide a breakdown of the typical word problems presented in elementary mathematics teacher editions and suggest a corresponding framework that provides content support and guidance for preservice and inservice teachers as they use word problems to make instructional decisions.

Background Literature

The Mathematics Textbook as Key Instructional Resource

Textbooks have a major influence on content and instruction in the mathematics classroom (Banilower, Smith, Weiss, Malzahn, Campbell & Weiss, 2013). Major publishing companies typically follow guidelines of the National Council of Teachers of Mathematics (NCTM, 2000) to provide lessons and instructional activities that follow the scope and sequence of the math curriculum while connecting to state standards. Joseph (2012) noted, "As a result, commercially-published materials are used in 85% of classrooms in grades K-5 and 81% of classrooms grades 6-8 (Banilower, Smith, Weiss, Malzahn, Campbell & Weiss, 2013, p. 91)." Additionally, in other reports such as the Center for Education Policy Research (CEPR) from Harvard University (2019), noted that teachers reported covering 82% of mathematics textbook chapters over the course of a school year (p. 15). These findings suggest that the influence of the textbook could potentially impact students' opportunities to learn and achievement levels.

Mathematical Word Problems

Writing to communicate mathematically has many advantages for conceptual understanding (Casa, et. al., 2016; Pugalee, 2005). For quite some time, the NCTM Principles and Standards for School Mathematics (PSSM) have explicitly called for multiple forms of communication (including writing) and researchers have suggested that writing in math increases students' understanding (PSSM 2000; Fortescue, 1994). For example, in a math intervention study, Cohen, Miller, Casa & Firmender (2015) found that when students engaged in explicit conversations and wrote about their reasoning on an ongoing basis, they demonstrated an increased ability to provide reasoning and use math vocabulary in their oral language and written products in comparison to control groups.

To encourage extended forms of communication, writing prompts are used for different communicative purposes—to explore, inform, argue, and create (Colonneselyn, Armspaugh, LeMay, Evans & Field, 2018). According to Sowder (2007), using writing as a formative assessment provides a window into student reasoning and justifications. Moreover, this can assist in planning for next steps of instruction by identifying student levels of understanding from their written processes. To this end, the range of mathematical writing can span from students by listing steps in a solution, to students writing elaborate justifications for *why* an answer is correct. These writing prompts are commonly known as word problems, story problems, problem solvers, higher order thinking problems, or extensions in math textbooks. However, the reading of these prompts (or what we refer to as "word problems" throughout this paper), requires students to pay attention to every symbol and word in the problem with consideration to the genre of the task encountered (Sherman & Gabriel, 2017).

Academic Vocabulary/Mathematical Symbols

The amount of academic vocabulary within a mathematics word problem may increase the complexity of comprehending the problem, impacting the student solution process (Joseph, 2012; Kozdras, Joseph, & Schneider, 2015). For example, in order to write mathematically, the understanding of academic vocabulary is fundamental towards conceptual understanding. Academic vocabulary such as domain specific words, or what Beck, McKeown & Kucan, (2013) refer to as Tier 3 words, are more challenging concepts and require explicit instruction (e.g., hypotenuse, rhombus, addend, sum, etc.). Furthermore, students also need explicit instruction in understanding how to interpret signs and symbols (e.g., +, -, x, etc.) to words, and these words to their corresponding processes in order to fully comprehend the problem (Thompson, Kersaint, Richards, Hunsader, & Rubenstein, 2008; Baumann & Graves, 2010; Beck, McKeown, Kucan, 2013). In thinking about developing students' mathematical literacy, this academic vocabulary needs to be addressed with appropriate scaffolds in place to support conceptual understanding.

Genres of Writing Prompts

In addition, special attention must also be given to the forms of writing elicited by the word problem. In mathematics, a word problem can be classified into four different types of *prompts.* These writing prompts in mathematics can be classified as 1) process 2) content 3) narrative, and/or 4) affective in description (Baxter et al., 2001; Dougherty, 1996; Shield and Galbraith, 1998; Urquhart, 2009). A *process* prompt is a word problem that would require students to explain the process they encounter when solving the problem such as a strategy for a solution, or to reflect as to why they used the steps or the specific strategy communicated to solve the word problem (Dougherty, 1996; Urquhart, 2009). Dougherty (1996) notes the following as a process prompt, "*The most important part of solving this problem is...*" (p. 2). Following, if the word problem has the affordance of mathematics relationships and/or content then it can be classified as a *content* prompt (Urquhart, 2009). Urquhart (2009) notes a content problem example as the following, "*Define parallel in your own words"* (p.7). These content prompts provide student with the opportunity of explaining, relationships, comparing and contrasting, or defining a specific concept. Next, a *narrative* prompt is a word problem that requires a student to demonstrate an understanding of mathematics concepts aligned to imaginary or real-world application. These types of mathematical *narratives* are often complemented with mathematics children's literature (Joseph, 2018; Russo & Russo, 2017, Schneider, 2016; TESS-India, nd). The Teacher Education through School-based Support ("TESS-India," n.d.) note a *narrative* prompt as the following, "*Use your imagination to create a story around the given problem of 4 + 7. (Sample response: A girl was playing 'Snakes and Ladders' with her brother ...)"* (p.4). The final genre of mathematics writing prompts would be classified as *affective.* This type of prompt would require the student to write a response utilizing some type of affect or feeling/opinion about a specific mathematics concept or topic. (Baxter et al., 2001; Williams & Brian, 2000; Shield & Galbraith, 1998). Williams and Brian, (2000), note the following as an affective prompt, "*Explain how you organize your math notebook. How does your notebook help you?"* (p.133).

Challenges of Constructed Responses

Given the complexity of responses required from the four types of mathematical writing prompts, and the specialized word knowledge and language needed to respond to a mathematical prompt, it is clear that all constructed responses are not created equally and successful student responses to these written prompts require a deep understanding of concepts, a sophistication with language, and the expansion of thought (Vygotsky, 1978). Similarly, the complexity of responses and ranges of writing ability require teachers to have an understanding of several instructional components: 1) deep knowledge of mathematics, 2) intuitive understanding of students' mathematics concept development, and 3) knowledge of writing development for teaching and learning (Burns, 2004; Martin, Polly, McGee, Wang, Lambert & Pugalee, 2015; 2019).

Furthermore, teachers must also understand how to facilitate close reading (Fisher & Frey, 2012) whereby complex text can be read multiple times with annotating, questions, and prompting for further understanding. Additionally, teachers should be prepared to develop their content knowledge in order to interpret children's responses (Sipe, 2008). In other words, students may answer problems in a variety of ways, using alternative language and novel phrasing in order to describe their thinking.

Methods

Textbook Prompt Analysis: Minimal Support and Missed Opportunities

To determine the type of instructional support preservice and inservice teachers may need, we built on the first author's (Christine) analysis of the teacher editions of two fourth-grade level math series (*enVision MATH* and *Everyday Mathematics)* (See Joseph, 2012 for details). By analyzing 100% of the lettered or number exercises in the two student editions and corresponding teachers' editions and resources, Christine documented the type of teacher edition support teachers received regarding mathematics word problem instruction:

1. **No Student Sample or Teacher Support**: The teacher edition provided *no* student sample of a response or directions of support for the word problem.
2. **Written Directions:** The word problem included *some* form of directions of support for the teacher. However, there was *no* student sample response.
3. **Student Sample Problem with Correct Response:** These word problems had only *one* student sample provided. There were no other directions of support for the word problem.
4. **Student Sample with Correct Response and Teacher Support:** The prompts included a form of support for writing *along with* a student sample of the response. These written directions included a brief description in the form of instructional notes.

The majority of prompts (90%) in the two teacher editions required students to construct responses to questions that could be interpreted in multiple ways. Although the students could answer in numerous ways, the teacher editions provided limited support for the teacher to provide instruction for various responses. Specifically, the teacher editions were lacking in the area of direction of support in how to teach, select or assign word problems to match learning goals and objectives. Additionally, the teacher editions did not provide instructional suggestions based on the word problem even thought a sample response may have been provided. As a result, the limited instructional scaffolding for mathematical writing in the teacher edition indicated a key opportunity for professional development and support.

Given that we understood the range and types of writing prompts used across two major mathematics textbooks, we also recognized the need for additional professional development regarding mathematics writing prompts. Specifically, 1) selecting mathematics writing prompts for instruction; and 2) supports needed regarding the use of mathematics writing prompts for instruction.

Determining Interest and Usage

To determine how teachers used mathematics writing prompts and what barriers existed regarding the use of mathematics writing prompts for instruction, we focused on inservice teachers (n=35) in a Title 1 school in which 83% of the student population (n=689) were economically disadvantaged and 31% of the students were dual language learners. Christine, a district math coach at the time, met with the teachers during collaborative planning sessions in Professional Learning Communities (PLC's). These teachers represented Grades 1-5 and the PLC's were held once a week for 16 weeks.

Initial discussions focused on the school's selected math series and teacher edition (*Go Math* by Houghton Mifflin). The teachers worked together to locate, identify, and categorize mathematical writing prompts in order to gain a sense of the information these prompts could yield. Throughout the PLC meetings, Christine recorded anecdotal notes to summarize the following findings. The teachers identified four categories regarding their use of writing prompts: (1) as a formative assessment measure, (2) as a vehicle for teaching and uncovering skills/strategies, (3) as a discourse method for communicating mathematically, and (4) as a tool for the facilitation of real-world mathematics.

Across the grade level teams, the teachers stated that they valued mathematical writing prompts as an important component during mathematics instruction. Moreover, intermediate grade level teachers emphasized the extensive amount of writing prompts on high-stakes assessments in mathematics and the impact these assessments have on teaching and learning. Approximately 75% of the teachers stated they consistently used mathematical word problems in a formative matter to confirm strategies and assess their students' learning of the mathematics. The teachers also expressed a need for support in planning for word problem instruction. Specifically, they wanted to know *when* and *how* to use mathematical word problems during their instructional time with students.

Implementing a Prompt Selection Tool and an Instructional Sequence

Because the teachers identified a need to know *when* and *how* to use word problems, and the lack of scaffolded support in the teacher editions for writing in mathematics, this cause necessitated the development of the Instructional Resource Guide (See Figure 1). The IRG provided the planning support needed as a guide for implementing problems solvers within an instructional sequence.

Instructional Resource Guide

The Instructional Resource Guide (IRG, See Figure 1) breaks down the decision-making process to help teachers select prompted tasks based on specific objectives or instructional goals. To use the guide, teachers begin by analyzing the objective of their instruction (to introduce, to review, to instruct, to practice, to intervene, to assess). Placing the *objective* as the focal decision was essential for the teachers to determine the method of instruction to follow. With the objective in place, the teachers could also identify the most relevant prompt to administer and determine the delivery of instruction. While making these decisions, the teacher would also consider student affordances elicited from the prompt. In other words, how might the student answer the task? Did the problem solver require a description, narration, elaboration, or synthesis of mathematics content that would help the teacher provide the proper instructional supports? In analyzing the level of support teachers required, the *Instructional Resource Guide* developed into a tool that teachers used on a daily basis to plan instruction and address these topics.

Instructional Sequence

The IRG supported the teacher's selection of writing tasks within the various components of the mathematics instructional block. In addition, the IRG also led teachers to develop a consistent instructional sequence that corresponded to specific prompt selection. In other words, in selecting a purpose and corresponding writing prompt, the teachers also considered their gradual release of instructional support:

> ***Formative Assessment:*** select a prompt to "gather information about the learning in mathematics to directly improve that learning" (Popham, 2008).
>
> ***Warm up/Review:*** select a prompt relevant to strategies for content previously taught. ***Introduction of content:*** select a prompt for tapping prior knowledge, identifying strategies, and understanding student thinking regarding new content.
>
> ***Practice of content:*** select a prompt to practice skills, concepts, and strategies.
>
> ***Summative assessment:*** Select a prompt to serve as a final judgment on student success and the quality of instruction regarding the mathematics content (Popham, 2008).

By using the guide to select the appropriate *type of prompts* to meet the instructional goal, teachers were able to select the *method of instruction* within the mathematics block to administer the writing prompt.

Initial Results

Across the professional development series offered during PLC meetings in which the teachers implemented an instructional sequence and used the *Instructional Resource Guide*, teachers stated that they increased in the type of word problems used during the mathematics block. Specifically, two fourth grade teachers and one fifth grade teacher reported an increase in their use of writing tasks by selecting warm up/review, introduction of content, practice of content, and during the intervention block as enrichment or remediation. Prior to the PLC meetings, these three teachers only assigned word problems as outlined in the textbook.

Implementation of the Instructional Resource Guide

In the process of tracing the development and introduction of the *Instructional Resource Guide* (IRG) and the corresponding instructional sequence, we engaged in design-based research (Reinking & Bradley, 2008) to examine the instructional modifications necessary to support teacher's implementation of the guide into their classroom instruction. Over the course of 16 weeks, Christine met with each of six inservice teachers during their planning periods, once a week for approximately 40 minutes. During the first meetings, the teachers consulted the mathematics' teacher edition to identify the Chapter or Unit aligned to the standard to be taught. Next, the teachers identified the tasks regarding the learning goal of the instruction. For example, if a teacher wanted to use the task in order to practice working with content or vocabulary then a *warmup/review* task would be selected.

During this selection process, each teacher used the curriculum materials available to select tasks that were aligned to the standards and objective of the lesson. Their conversations centered on the language of the task, and the student affordance (how students may or may not answer).

Data Collection

Christine conducted the professional development training for writing in mathematics to K-5 grade level teachers in the following format:

Day 1: Gauge Interest to Determine Differentiated PD. Christine met with each grade level team during their PLC's to discuss the teachers use of word problems. At the beginning of the meeting, presented each team member with a copy of the *Instructional Resource Guide* (Figure 1) to determine if they had any interest in using the tool. The teachers made the following comments regarding their first impressions of the *Instructional Resource Guide*:

> "*I never thought of using word problems in all these different ways and formats. I am excited to begin the unit with a writing task and end with a writing task.*"
>
> *"I might end up skipping a "step" – that way it gives me a goal to incorporate more word problems into planning. This is a huge importance for the literacy integration in mathematics."*

"*This chart provided me with a way to understand where my students are and where to go next with my instruction*"

Given the teachers positive response, Christine asked the teachers to collaboratively select the *type* of instruction they wanted to model. Teachers in grades K, 1, 3 and 4 chose *Practice of Content* (see Figure 1) because these grade level teams were in the middle of an instructional unit. Grade 5 selected an end of unit task to assess student learning. This task was selected as a *Summative Assessment*. The Grade 2 team chose *Formative Assessment* to determine what students knew about the content that was going to be encountered in the upcoming unit.

Day 2: Select Word Problems and Textbook Selection. On Day 2, the grade-level groups reviewed the teacher editions to identify word problems in the textbook that would facilitate a constructed response. Based on the content within the standard, and discussions of misconceptions, the teachers decided to focus on a specific word problem lifted from the textbook per grade level team.

Day 3: Modeling and Student Collaboration. Christine modeled the instructional delivery of the word problem with students. At the end of the lesson, Christine showcased purposeful selections of student work while facilitating collaborative discussions with the students. Christine selected exemplars and highlighted common errors to support conceptual development. During the student collaborative, Christine addressed misconceptions and pointed out efficient strategies in real time. This real time intervention allowed for students to develop a deeper understanding of the content by the type of discourse that began to unfold from the task response. The teachers observed the process.

Day 4: Analyzing Student Responses to Determine Next Steps. Teachers communicated their analysis of student responses. For example, the Grade 2 team discovered, through conversations with students and analysis of student data, that several students had misconceptions regarding academic vocabulary and pictorial representations. The Grade 2 teachers then decided to create tasks that encouraged pictorial representations that were similarly aligned to the textbook word problem. In Grade 5, the teachers decided to build conceptual understanding through additional writing extensions. These writing extensions facilitate building on word problems in the textbook to promote real world application. In addition, these teachers determined that the tasks selected for further practice should include a student response with a visual representation. Furthermore, if the word problem from the textbook aligned to the standard and objective of the instruction but did not provide the opportunity for a written response, the teachers made certain modifications.

- **(Original)** Does the following array model represent the multiplication sentence of 3x2?

- **(Modification)** Does the following array model represent the multiplication sentence of 3x2? Explain your reasoning.

Adding the modification of "Explain your reasoning" extended the prompt by requiring the student to write a solution or provide justification.

Summary. The teachers specifically discussed the value of the IRG and the coaching sequence. They also expressed the need for additional PD focused on mathematics writing instruction and methods for supporting students when modifying textbook word problems to meet student's needs. These recommendations form the next phases of our work as outlined below.

Writing Instruction is Needed in the Mathematics Classroom

The lack of support surrounding word problems in mathematics teacher editions is a clear indication that professional development is necessary and urgent. In support of this matter, the following has been reported by the Partnership for Assessment and Readiness for College and Careers (PARCC), notes:

> "The PARCC (2018) Item Development correspondence:
> Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. Separating the practices from the content is not helpful and is not what the standards require. The practices to do not exist in isolation; the vehicle for engaging in the practices is mathematical content (p. 45)."

As a result, instructional supports for writing in mathematics should be considered. More specifically supports aligned to mathematics strategies, literacy structures, and mathematics processes. These supports should provide teacher with the awareness of how to reflexively move from each element as the process of writing is complex. In addition, writing in the disciplines requires instruction in the specific genres used within the field. In support of these suggestions, Joseph (2012) notes the paradigm shift for support in literacy as stated by Moje, Overby, Tysvaer, & Morris (2008):

> "We need to consider the larger contexts in which strategies are drawn up and the practices that various strategies support. It may be most productive to build Disciplinary literacy instructional programs rather than merely encourage content teachers to employ literacy teaching practices and strategies (p. 96)."

Additional research is necessary in order to fully implement how teachers can instruction mathematical writing successfully.

A survey published on writing in mathematics suggests that instructional support of writing in mathematics has not changed at all or is growing too slowly to have any observational measurement and that mathematics writing may often be considered less sophisticated in terms of composition (Kosko, 2016). Given the requirements of the NCTM *Principles* *and Standards for School Mathematics* (PSSM) (2000) note that the content standards in mathematics are developed through reasoning and proof, problem solving, communication, representation and connections. In thinking about the processes, writing certainly plays a central role. However, current methods of writing instruction, such as the Writer's Workshop or the 6 Traits of Writing instruction (Culham, 2003), may not have a clear alignment to these processes.

Mathematics Instruction is Needed in the Language Arts Classroom

Teachers and researchers in writing have identified common characteristics now widely recognized in traits models: ideas, organization, voice, word choice, sentence fluency, conventions, and presentation (e.g., Culham, 2003). These characteristics, based on the work of Diederich (1974) who sorted stacks of student writing into *good*, *fair*, and *poor* categories, have become essential components in the process of writing, providing students with a common language for writing assessment. Similarly, other researchers have developed scoring assessments and features guides to analyze students' spelling development (e.g., Bear, Invernizzi, Templeton, & Johnston, 2020).

Borrowing concepts and procedures from these models, we are calling for a new look at writing instruction in connection to informal strategies such as when writing is used as a formative tool for assessing understanding and instructional decision making. Elbow and Sorcinelli (2006) noted the difference in low stakes writing as an instructional strategy compared to more formal or high stakes writing (i.e., essays, term papers). With low stakes writing, students are removed from the boundaries of high stakes writing and are able to write freely through many forms such as exploratory or focus questions, free writing in response to a question, summary writing or reflective journals (White, Reichelt, & Woods 2011).

Using the IRG, preservice and inservice teachers can begin to address the appropriate time for writing instruction to occur during mathematics. This planning guide does not address all the areas of writing support that are needed in the mathematics classroom. However, it is the first step in planning for the utilization of how low stakes writing such as mathematics word problems can facilitate high stakes learning such as measurements of ability and conceptual understanding. Teachers and students can begin to build on mathematical concepts through the appropriate objective, method, type and delivery of word problems. This planning process is the beginning of understanding how one field can successfully inform the other.

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Figure 1: Instructional Resource Guide (IRG)

Objective of Instruction.	Method of Instruction	Type of Prompt	Delivery of Instruction (Teacher Led or Supported)	Assessment
To assist in the development of instruction for the **upcoming objective** through the use of student interviews and analysis of student data.	Formative Assessment	Prompt will encompass the upcoming benchmark or standard.	Whole Group Small Group Independent	Formative Rubric Learning Scales (Not graded)
To continue practice in working with content, vocabulary, and strategies of **previous objective**s.	Warm Up/Review	Prompt will be a review of standard previously taught	Whole Group Small Group Independent	Formative or Summative (Grading Optional)
To instruct in the area of content, vocabulary, and strategy development of the **current objective**.	Introduction of Content	Prompt will encompass the **upcoming** benchmark or standard.	Whole Group Small Group	Formative (Rubric Optional/Not graded)
To practice content, vocabulary and application of strategies of **current objective**.	Practice of Content	Prompt will encompass the **current** benchmark	Whole Group Small Group Independent	Formative (Rubric Optional/Not graded)
To assess the mastery of the skills/concepts taught within the **current objective.**	Summative Assessment	Prompt will encompass the benchmark or standard **previously** taught.	Whole Group Small Group Independent	Formative or Summative (Rubric/Graded)

Transformations

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Context-responsive equitable strategies for developing gender-responsive curriculums in Nepal

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Context-responsive equitable strategies for developing gender-responsive curriculums in Nepal

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Abstract

We argue that context-responsive equitable strategies support the development of a gender-responsive curriculum in the context of higher education in Nepal. This paper is our reflective journey of curriculum content analysis of the two Master's programs (Mathematics and English) from an inclusive cultural perspective of gender which engaged us to explore the answer to the question- How can we develop a gender-responsive curriculum? Adapting inclusive cultural perspective and participatory design we engaged with students and faculties and management representatives in the process of gender mainstreaming through action-reflection cycles. Further, we braided discussion with poetry, that is, a poetic inquiry to tell our praxis in a realistic and/or literary way. Finally, we discuss the three context-responsive equitable strategies such as (1) adapting the collaborative approach, (2) promoting 'the 3 pillars', and (3) enhancing inclusiveness that supported us for ensuring gender equality.

Keywords

inclusive cultural perspective, curriculum content analysis, equitable strategies, PAR

Introduction

Considering gender as a social issue and mainstreaming gender as an equitable strategy seems an empowering process when we give more value to power than to knowledge. If we culturally perceive gender (i.e. masculine and feminine) as two inherent human qualities that each of us possesses as a cultural understanding, gender and gender mainstreaming might not remain an empowering process rather become the process of cultural liberation. Seemingly, some Nepali people are de-cultured by non-inclusive modern worldviews which did not provide sufficient space to respect our deeply rooted cultural perspective, particularly of gender. Here, we are not against the modern worldviews in the educational context rather seeking "localness" while meeting globalization (Parajuli, 2015). Instead, we preferred to discontinue the illusionary perspective of 'gender equity through empowerment' as the only way of liberating from gendered situations. As illusion over clouds, our perception and obstructs for liberation and consciousness support us to shed light on illusion (Osborne, 2014), we need to go beyond the existing culture of blaming others and thereby recommending the framework. For it, we preferred to appreciate and critically reflect our cultural perspective and thereby continue the discussion of gender mainstreaming processes in the higher education curriculum development and improvement context.

According to Lamptey et al. (2015, p.11) gender mainstreaming, a strategy, addresses gender equality concerns in policies, programs and activities "to ensure that all development initiatives integrate the concerns of both men and women, and their needs are considered equal and equitably with the aim of attaining gender equality." The study shows that the development and/or improvement process of policy (i.e. curriculums or courses) of the universities plays a vital role not only in the learning of the students but also for gender mainstreaming to ensure gender equity. Here arises a question- what is gender? According to UNESCO (2015 (p. 9-10))

> *"Gender refers to the socially constructed relations between men and women… Gender equality ensures that men and women enjoy the same status and have equal opportunities to exercise their*

human rights and realize their full potentiality. Gender equality in education ensures that female and male learners treated equally, have equal access to learning opportunities and benefit from education equally. They become empowered and can fulfill their potential so that they can contribute to and benefit from social, cultural, political, and economic development equally. Special treatment/action can be taken to reverse the historical and social disadvantages that prevent female and male learners from occurring and benefitting from education on equal grounds."

Seemingly, the notion of gender equality of UNESCO that envisioned treating females and males treating equally (2015) is similar to the concept of gender balance that we have deeply rooted in our (Nepali) culture.

In our culture, Ardhanarishwar is considered an all-inclusive and balanced metaphor (Dhungana, 2020; Mishra, 2017) (see Figure 1). Ardhanarishwar, a Sanskrit word, refers to the union of the Hindu god Shiv and the goddess Parvati. Moreover, Ardhanarishwar is the metaphor of the "receptive, all-inclusive, holistic, integrated, self-sustained and balanced form of dialogic inquiry" (Dhungana, 2020, p. 52). The image itself shows our inherent male and female qualities metaphorically. With this reference, Ardhanarishwar seems a post-gender metaphor.

In other words, Ardhanarishwar can be used as a cultural harmonious lens of gender balance or gender equality. According to Hooks's (2002) we need to harmonize and generate better perspectives to enhance gender equity. Further, Hooks (2002, p. 117) claimed that "visionary feminism offers us hope for the future. By emphasizing ethics of mutuality and interdependency feminist thinking offers us a way to end domination while simultaneously changing the impact of inequality." Seemingly, Ardhanarishwar can be a context-responsive metaphor of post-gender perspective metaphor as it is supported to ensure gender equity in the graduate curriculum content analysis process.

However, for many years, overlooking the strength of mutuality and disregarding our cultural framework of gender balance, school and university curriculums have used the western framework for gender mainstreaming that valued one gender (mainly female) over the other (male). We might be limited to analyzing the school level curriculum and proposing guidelines to look for gender-neutral language, traditional gender roles, gender stereotypes, pictures and images, and gender parity. However, exploring gendered situations examining the only physicality would not be sufficient. We see the possibility of influencing ourselves, others and social formation by living the value of equality promoting a deeply rooted cultural inclusive perspective of gender. It is because gender is not only a hidden curriculum (unintended learning outcome) but masculine (i.e. logical) and feminine (i.e. intuitive) perspective and quality in the form of an intended learning outcome. It is because seemingly intended learning outcomes highly control learners' gender knowledge. Here, gender knowledge refers to the knowledge about gender and/or knowledge about multiple perspectives of gender.

Being Master's level curriculum designers and implementers, in line with Lamptey et al. (2015), our engagement in content analysis can engage us in a continuous process of mainstreaming. We can explore the nature of the curriculum that we need to address the needs of 21st-century learners. However, we might not envision holistic development without questioning our own perspective. Moreover, without respecting our deeply rooted cultural values, we might not explore what matters to us and the people with whom we live. In addition to that, curriculum needs to address the everyday needs of the learners; curriculum needs to be changed regularly to meet the needs of the 21st Century Educational and to bring improvement in teaching as well as students' learning and also ensure the SDG Goal 5, gender equality. However, the Western framework might not be sufficient in our local contexts.

At first glance, it may seem like a given framework has been ensuring gender equity however they failed to address the diverse gendered situations. For instance, Acar-Erdol and Gozutok (2018) recommended that social awareness of gender equity is a prerequisite to implementing a gender equality curriculum. However, paradoxically, their prescribed “The Taba Model” could not help us in our context as we were yet to explore the gender gap. To explore the status of students who were possibly lagging behind to enjoy provided learning opportunities because of gender, it was our responsibility to dig out the gender gap and thereby address that gap. Then we found Manuel’s (2018) study that motivated us to involve students in our project which could enhance students’ academic achievement. Although the participation of the students via exchanging experiences, meeting new people and helping and having fun (Manuel, 2020), the COVID-19 context discouraged having face-to-face interactions and discussions. Meanwhile, we saw the opportunity of online methods of participating in them. Thus, we realized that we need a context responsive framework for mainstreaming gender in this period of COVID-19 context.

Therefore intending to enhance gender equity in the Master's course, we looked for a suitable framework but we could not find any. Perhaps, the change in the context, COVID-19 context, provided us with an opportunity to seek a new framework. Meanwhile, Hermans & Thissen (2009) also inspired us to develop our own framework with the help of stakeholders as they introduced actor-analysis methods for public policy analysis as a context responsive method. Therefore, adopting Kincheloe's (2005) active perspective of a researcher, we did not follow any prescribed framework rather looked for the possibility of context-responsive approaches. Like Tolhurst et al. (2012) post-gender perspective addressing women issues like gender parity, inclusion and gender mainstreaming, we could engage ourselves and also other stakeholders like students and teachers in the gender mainstreaming process. However, we believed that our cultural post-gender perspective could support the meaningful engagement of adult multiple stakeholders in a respectful environment.

If we continue to believe that the western framework is the only one, the standard framework, we’ll never explore context-responsive, indigenous, relevant, practical frameworks. We’ll never explore what works well in our context unless we acknowledge our framework. We’ll continue to have trouble exploring other’s frameworks to understand our problems and solutions. By rethinking, re-using the inclusive cultural perspective we can fix our local problems; we can address gendered situations in the school (Dhungana, in Press), university and that brings ripple effects to the schools and other educational institutions in the country where the top-down approach is high; we can influence students, teachers by walking the talk.

Thus, as gender is in flux it needs to be dealt with in a context responsive way. Exploring the dynamic nature of gender and mainstreaming strategies, we took this as an opportunity to explore context responsive approaches for mainstreaming gender in our courses. As we were planning to design the Master's curriculum for Spring-2020, we began to ask- *How can we develop a gender-responsive curriculum?*

Therefore, the purpose of this content analysis is to explore context-responsive approaches adapting a cultural (inclusive) perspective of gender equality/equity. Before discussing the four context-responsive approaches, we discuss the research background and methods in the following sections.

Research background

The paper is based on our (the first, and the third authors, Ph. D. fellows and teacher educators) and (the second author, Post. Doc. and a teacher educator) collective reflective story on the actual gender-related experiences in the process of participatory content analysis of the two Master of Education Programs while developing a teacher's manual for mainstreaming gender in XXX University. We were engaged in content analysis and thereby gender manual development process from June 2020 to November 2020. In the process, we encountered manifold gender issues, which were more context-responsive than we find in literature which we discuss in the following paragraphs. In addressing the issues, we critically reflected upon those gender-related experiences and adapted our inclusive perspective of gender in mainstreaming our courses. Being global citizens and educators, we took this project as a social responsibility for enhancing gender equity in our work and an opportunity to connect other individuals in the COVID-19 context. We observed the ten courses of the following two subjects.

A course, YYY Master of Education, was launched in 2006 and another was launched in 2004. The course prepares the graduates to follow the latest principles and methodology of teaching, undertake small scale research to improve their own pedagogical skills, deliver short term teacher professional development packages, develop appropriate curricula, textbooks, modules and projected and non-projected materials, and educate pre-service and in-service teachers for effective teaching. Furthermore, another curriculum of ZZZ prepares students to possess conference skills, work in a team, master ICT skills, be independent practitioners, and be able to facilitate teacher development workshops.

Both the courses were revised in 2013 and 2018 to prepare competent teachers, teacher educators, materials developers and researchers; have similar key features and they are need-based, pedagogical content knowledge, applied mathematics, modelling of pedagogy, project-based and skills oriented that helps students master the skills of collaboration, investigation, presentation; aim to develop teacher educators, school leaders, teacher education experts, teacher development organizers and material developers.

Research Methodology

We chose a critical participatory action research design (Kemmis, 2008) to examine the existing curriculum and thereby seek the possibility of improving gender mainstreaming practices by engaging in an action-reflection cycle with the students and colleagues through dialogues. Inspired by living-theory-methodology that integrated (i.e. both critical and appreciative) approaches which made methodological inventiveness possible, we adapted an appreciative approach (that is to appreciate deeply rooted cultural practices) intending to complement the critical approach of participatory action research (Dhungana, 2020). Aiming to engage in action reflection we adapted dialogue as a research method (Delong, 2020). Further, the study of Wolstenholme, Rosscobb, and Bowen (2016, p. 1218), supported us to work with adult learners with participatory design as their participatory design valued adults and thereby allowed them to engage meaningfully and develop shared understandings and goals. Roughly we divided our research process into four phases.

First phase: In the first phase, we conducted a context analysis. In June 2020 we began to review the two Masters' courses from gender perspectives as a part of a gender manual development program that intended to guide teachers to make their curriculum gender-responsive. At this stage, borrowing the gender perspective from UNESCO (2015) we explored gender issues. Then we felt the need to participate in the content analysis process. In this phase, we felt to empower students or to make them able to critically self reflect the taken for granted assumptions of gender and contribute their views to improve curriculum. Then we planned for the second phase.

Second phase: In the second phase, we sent emails to the students of 2017-2019 batches to participate in the group discussions (on the issue of gender) voluntarily. We took their consent and conducted the FGDs with guideline questions via Google meet. We discussed (2 male and 2 female students). In this phase, we explored the reason for gender biases and context responsive ways out. However, we felt the need of becoming more inclusive and thereby exploring context-responsive ways out by involving teachers or faculty members. Meanwhile, we felt the need to explore more ways of being with colleagues.

Third phase: In the third phase, intending to explore context-responsive ways out we invited the representatives of the school management representatives (e.g. Head of Departments). Doing so, we had hope of receiving a safe environment and thereby making a positive influence. It is because we wanted to explore more ways to improve curriculum, appreciate teachers' best practice of mainstreaming gender to some extent, and enhance mutual relationships by engaging faculty and school management in gender mainstreaming processes. Our intention of gender mainstreaming was not to challenge existing practices rather improve best practices harmoniously and collaboratively. Then we sent an email requesting their voluntary participation in the Focus group discussion. We discussed it with six (5 males and 1 female) colleagues including the two Head of Department.

Fourth phase: In this phase, we revisited and reflected on our research and developed this paper. In the first phase while analyzing contents a question emerged - w*hat were the gendered issues in the curriculum?*- which guided our research. In the second phase another two questions emerged while discussing with students- *Why was gender a problem? How can we address the gender gap to ensure gender equity?*-that guided us further. We deepened our discussion with faculty and school management using all three questions. In short, the three questions emerged in the process and thereby guided the research, but not necessarily in a linear way.

Then, intending to make sense of our (authors, students, faculties and school management) lived and living experiences by engaging both mind and heart, we used poetic inquiry, 'the methodology of heart", (Owton, 2017, p. 103). Our intention of using poetic inquiry, which includes different forms of poetry, was to seek our essence of key experiences in the precise form that other modes of presentations (e.g. prose) might not bring forth (Owton, 2017). Therefore, we 'crystallized data' or framed poetry blurring field data and interpretation as we could not separate distinctly data and interpretation while writing (Jackson & Mazzei, 2018).

Results and Discussions

Our context analysis, discussions with students and teachers, and our reflective notes hold the evidence of (1) adapting collaborative approach, (2) promoting 'the 3 pillars', and (3) enhancing inclusiveness.

Adapting collaborative approach

We began inquiry with this initial question (*What were the gendered issues in the curriculum?)* which explored the need for a gender-responsive curriculum in graduate classes. In the process, we adapted collaborative inquiry (Belenky & Stanton, 2000). Here, collaborative inquiry refers to the inquiry of the two authors by engaging in action-reflection cycles. Such teacher-teacher collaboration in the context analysis process was new in our context, which we believe, made it possible for our gender awareness and or enhancement in our gender sensitivity. One of us could lead the project and another could assist, however, we felt we could work collaboratively. Perhaps we were going beyond the male/female binary construct through collaborative inquiry (Belenky & Stanton, 2000).

In collaboration, we chose one curriculum for each of us on the basis of our background. We shared our analysis, discussed and thereby tried to made sense of them through the following poem:

I'm happy for getting an opportunity
I am inviting you by reciting poetry.

I am the program designer and implementer
a female teacher educator, sensitive in gender

My course promotes enough reflective and critical thinking,
include issues of gender equity with frequent revisiting.

I am very conscious of using gender-neutral language,
It would be challenging if I did not use the English language.

Books and reference materials; articles and literary pieces,
With my careful selection of not gender stereotyped texts.

I am aware of gender, gender roles, and authorship of women,
Dugas & Allard's article, Plath's poetry to name a few of them.

Teaching and learning by the individual, pair works, and group works
Through reading and writing, presentation, discussion and field works.

I instruct, I facilitate, and I teach what to teach, how to teach and why to teach,
Through the module, auto tutorials, CD, face-to-face, online, games, activity and research.

In-semester 50% and end-semester 50% , my assessment system,
I evaluate all the assignments and follow the letter grade system.

My teaching and assessment are not of learning but for learning.
I claim a gender-sensitive environment for conducive learning.

Finally, I would like to thank you all for listening,
Drop your queries as/for gender mainstreaming.

This poem reflects our lack of gender responsiveness which was the main issue in the existing curriculum. However, in the beginning, we could not explore it as we might make shallow observations in a single attempt. For instance, we explored that curriculum designers and/or developers had gender awareness and sensitiveness. It was because the language used in the curriculum was gender-neutral. We believed that gender neutrality would be enough for gender justice. Then we felt that our presence in our department was addressing gender disparity. We were kind of happy being representative of the females. We believed that our presence, gender awareness and sensitiveness would be enough for gender mainstreaming.

But when we discussed and made a second observation we explored that gender was not an intended learning area rather a hidden curriculum in Masters of Education. Hidden curriculum or “informal curriculum”, other than intentional curriculum or “formal curriculum”, refers to those aspects of schooling that influence learners values, perceptions and behaviours UNESCO (2015). UNESCO defines gender analysis as the examining and exploration of the reasons for gender inequality, the disparity in given circumstances and situations. For examination and exploration of gender inequality, in the context of university settings, we first assumed that the curriculum had enough space. However, we continuously value gender as a ‘hidden curriculum’ that can influence the learning of the students (Schubert …) and teachers.

Then we explored the need of enhancing gender sensitivity. For instance, we thought that many times, teachers are aware of gender issues but they lacked gender sensitivity. To be gender inclusive, teachers needed to be aware of using proper learning material to ensure gender equity. There could be an inclusion of values, ethics, norms and beliefs. Besides that, a teacher could provide a safe learning environment where students can exercise human rights and challenge one’s own deep-rooted cultural issues. By adopting peer learning, cooperative learning, collaborative learning, peer evaluation, group evaluation, and group work, one could provide a space for learning.

However, we felt enhancement of gender sensitivity alone falls short when we practice teaching, learning, and assessing promote individual learning rather than collaborative learning and/or evaluation. Meanwhile, we were inspired by Lebler (2008) who provided three functions of assessing students: (1) Assessment of learning (to examine the students’ achievement to ensure learning outcomes); (2) Assessment for learning (to provide feedback and direction for future activities); (3) Assessment as learning (to produce learning in itself by involving students actively in the assessment process). That taught us that our practice of assessment should not be limited to ‘assessment of learning and assessment for learning’ rather move toward ‘assessment as learning'. Here, we felt that we might need to embrace cultural and/or indigenous knowledge to develop a gender-responsive curriculum.

Reaching this stage, being aware of the gendered curriculum, we saw the possibility of improving learning resources as gender equity, which might support our students to enjoy a gender-equitable learning environment.   We could use not only multiple resources like books, journal articles and literary pieces but also artefacts, natural phenomena, media, family, and mythologies as learning resources.

We had some reflective queries. For instance, were the existing learning materials enough for gender equity? We could connect values, ethics, norms and beliefs. For it, we thought a safe learning environment is a must but had we given space in our curriculum for exercising our human rights? Was my pedagogy enough to explore deep-rooted cultural issues? Perhaps not! But we were given space for peer learning and group work. Are they enough for peer learning, cooperative learning, collaborative learning and peer and group evaluation? No! We had highly promoted individual learning, not collaborative learning and evaluation.

The answer to the question of uneasiness was because of building consensus for gender mainstreaming without the involvement of the students and/or having students' consensus on a decision. Participatory assessment could be for the betterment of the curriculum and while talking about students' assessment, their involvement is equally important. Without their involvement, the empowerment of the students could not be done. Furthermore, it is not about ending patriarchy and Western Modern Worldview in higher education, it is about empowering those who are influenced by the patriarchy (Shackelford, 1992) for "cultural emancipation" (Taylor, 2013) through nurturing inclusive perspective.

Then we realized that realizing gender as a hidden curriculum might not be sufficient in our context. It is because students from diverse contexts come to the university and there might be genuine gender issues in the higher education context, not limited to male and female issues (e.g. Paudyal, 2015). Social inclusion and exclusion might function beyond male/female issues. Therefore, in line with Lamptey et al (2015), we thought that gender-neutral content scope would not support us to disrupt existing gender relations. Here emerged a question-*Why was gender a problem?* We explored the answer in the following section.

Promoting 'the 3 pillars'

Our query-*Why was gender a problem?*- explored the hegemony of binary perspectives of gender as male and female (Belenky & Stanton, 2000) but not as inherent (i.e. naturally gifted) qualities of masculine and feminine of each individual (Mishra, 2017). We made sense of it through the poems, for instance,

When my two voices argued,

A separate topic/unit!
No! Be inclusive.

A separate pedagogy!
No! Be gender-responsive!

A separate quota!
No! Make me feel equal!

Continue 'research on'!
No! 'Research with'!

Gender parity!
No! Equity!

My third voice said,

"Curriculum, community, and university,
the 3 pillars!"

The third voice of the poem refers to the inclusive voice which broke the boundary of first and second voice or binary voices. The third voice suggested the connection of curriculum with the issues of community and thereby collaboratively work on it being like the 3 pillars. Similarly, the discussion with teachers and school management explored collaboration in a context-responsive way. So, the third voice came not to empower any other voice rather connect and collaborate. The sense of oneness provided us with the ways out to move beyond binary perspective. To move and dismantle binary perspective, the all-inclusive metaphor of Ardhanarishwar supported us.

We think our inclusive perspective that involved students in the content analysis process was our belief in students as 'critical students' (Johnston, Mitchell, Myles & Ford, 2011) who explore the hegemony of the binary perspective and move beyond. Like Johnston, Mitchell, Myles and Ford (2011), we believed that critical students having the following personal qualities and values:

(1) a well-developed, robust, confident and aware self, able where necessary to challenge and reconstruct existing understanding and modes of operation; (2) an awareness of the values, priorities and power structures implicit in a context and a capacity to be constructively critical to them; (3) appropriate values such as respect for reasons, an inquiring attitude, open-mindness, independent-mindedness (p. 80)

A student of the 21st century is critically aware of self and others who challenge the hegemonic policy and practices. Moreover, this exploration was possible when we valued students' participation; their specific needs and multiple intelligences. From the discussion with the students, we saw the possibility of introducing varieties of contents of multiple contexts, including gender issues, in participation with the students to develop gender-friendly content. It was because, although the objective of my program was to foster students' critical thinking, however, the program itself lacked a critical look.

The notion of the 3 pillars (curriculum, community and university) seems a foundation for nurturing gender equity. For it, we needed to embrace the issues (e.g. gender) of community or society in curriculum contents. We need to invite community members to our class to discuss gender issues. We can bring artefacts in the class to discuss gender. Yes, curriculum, community and university are the three pillars of gender justice! The three pillars have equal value and also equal responsibility to ensure gender equity.

However, we should be aware of Schubert's (1986) notion of "curriculum as a cultural reproduction". In Schubert's words, the metaphor of "curriculum as cultural reproduction" refers to the curriculum that uncritically adopts and implements cultural and social practices mandatorily particularly in the school curriculum. We think, by promoting enough evaluation skills, creative thinking and affective domains of learning among teachers and students we can critically examine cultural practices to ensure gender justice.

Moreover, like us, teachers need to ask ourselves questions like- Is community-university participation necessary for the university curriculum designing and implementation? Do I need to give equal value to community knowledge, indigenous knowledge, in the university class? Is my curriculum contextualized, and connected to the community fully?

Thus, embracing the issues of community, inviting community members in the class to discuss issues and bringing artefacts from the community are some ways to improve the curriculum. Curriculum, community and the university are the three pillars of gender justice that have equal value and responsibility to ensure gender equity. By not giving value to the community, by not incorporating indigenous knowledge in the classroom and by not participating with the community members, the curriculum is decontextualized. It is hard for one individual to contextualize the university curriculum. Feminist pedagogy might emphasize dialogue and collaborative culture (Shackel, 1992), however, in the Nepalese context to initiate the contextualization of the curriculum classroom interaction, collaboration and dialogue are prerequisites (Luitel, 2019).

In line with Parajuli (2015) our attempt was to explore a cultural gap in education intending to make education responsive to the local needs. In the school context, exploring context-responsive approaches for contextualizing curriculum was possible through collaborative approaches Dhungana, et.al, 2020) and through living collaboration as a professional value (Dhungana, 2020). Moreover, exploration of a cultural perspective (i.e. satvic framework) was possible within university classrooms through self-study (Dhungana, 2021).

Decontextualized and decultured curriculum of the university seems one of the major existing challenges of higher education which fuelled for ensuring gender injustice. Disregarding the collaboration and connection of curriculum with family issues, culture, society and community might not address gender issues in our context. For contextualizing curriculum and ‘cultural emancipation’ university-community collaboration might be helpful. Although contextualizing the university curriculum might not be possible by my individual effort, we can continuously attempt to do so. Here emerged a question- *who is responsible for gender equity?*

Enhancing inclusiveness

Who is responsible for gender equity? The inquiry led to explore a rarely discussed (at least in our context) issue of teachers' self-inquiry and "Self-enquiry" (Osborne, 2014). According to LaBoskey (2004, p. 826) "each self is different, all offer an important, yet necessarily constrained perspective. Therefore, the knowledge of teaching can only be developed in a diverse and inclusive, particularly of previously marginalized voices, teacher-learning community." However, self-inquiry might not be sufficient in the Nepali context which has a deeply rooted cultural knowledge of 'Self-enquiry.' In line with Osborne (2014), who was inspired by the teachings of Ramana Maharshi, we believe that 'self' might dwell in the egoistic self whereas 'Self-enquiry' might take towards pure consciousness or inclusive experience. Therefore, self-inquiry is the inquiry of 'self' based on our practices whereas 'Self'-enquiry is the inquiry of the 'Self' or our Pure consciousness (means a sense of inclusiveness or weness). In our context, besides 'self- inquiry' we feel the need for inquiry of 'Self' which might play a vital role, particularly in the educational setting.

For instance, we explored teachers' collaboration as a context-responsive way to ensure gender equity. Here collaboration was not only the approach (Dhungana et. al, 2020) but also a living consciousness (Dhungana, 2020), and inclusive context responsive cultural perspective. We made sense through the following poem.

My loud voice claimed,
"I am pedagogy and I am fine."

My mild voice said,
"We, students, teachers, university
family, culture, society,
Content, learning materials, assignment,
research topics, university policy, relationships,
need improvement!"

My low voice whispered,
"collaboration with colleagues!"

Discussion with the school management and colleagues explored a context-responsive way of 'collaboration'. However, we found it paradoxical because through curriculum teachers intended to enhance collaborative learning (a few courses), but in practice, teachers themselves were overlooking the strength of collaborative culture which is deeply rooted in Nepali societies. In White Head's (2008) words, we were "living contradictions" by not living the value of collaboration fully. For instance, a teacher said, "I think teacher-teacher collaboration might work in our context." Although we had incorporated a few group activities for students in our curriculum, we needed to walk the talk!

Here, we realized that we all are responsible for ensuring gender equity. For instance, not only the teachers and school management, students also need to be inclusive and be able to accept change in the classroom; to bring change in our dualistic perspective.

Similarly, the university should encourage research on gender; should let the individual course facilitators make personal decisions about the course; should change their existing policy and be gender inclusive. Next, the issue of gender is an important content that needs to be integrated in the curriculum. Change in the curriculum is essential. There should be gender inclusion in content, learning material, assignment. Both males and females' voices (including texts) should be incorporated in the curriculum. Moreover, besides university, family and society need to acknowledge and contribute cultural or indigenous knowledge.

For all these, collaboration as an inclusive context responsive perspective is a prerequisite. Promotion of openness among teachers, the connection of curriculum with community or society and collaboration among students, colleagues and school management would create a gender-friendly learning environment and enhance gender equity.

Seemingly, we have been ignoring the third or the collaborative voice and promoting the egoistic (i.e. first voice) and the victim attitude (i.e. second) voice. We never heard our low voice, the problem-solving voice. Being adult professionals dealing with adults, role modelling could be a suitable strategy for transformative learning (Mezirow, 2000) that could enhance gender equity in the curriculum. We can be role models to our students and colleagues by collaboration. Like "curriculum as currere" (1986), we can be living curricula. Collaboration with colleagues seems possible in teaching, learning and assessing in our context but-How could we collaborate with our colleagues for enhancing gender equity in the curriculum?-emerged as an unanswered question that can be a question for further research.

Final reflections

The query-How can we develop a gender-responsive curriculum? -gave rise to the idea of all-inclusive context-responsive equitable strategies such as (1) adapting collaborative approach, (2) promoting 'the 3 pillars', and (3) enhancing inclusiveness.

At first, we explored collaboration as a gender gap that led us towards seeking the possibilities of respecting and nurturing the cultural perspective of gender equity. In other words, rather than focusing on problems and seeking ways for problem-solving, we could see what had been working well in our context and thereby continue being like a critical student (Johnston, Mitchell, Myles & Ford, 2011).

For instance, we can promote collaborative and cooperative learning communities of practices among students and faculty. Next, we can promote collaborative methodologies like action research, participatory action research, self-study methodologies which encourage collaboration, participation and improvement of professional practices. Similarly, cross-cultural projects like NORHED Rupantanran and NORAD QUANTICT can enhance collaboration among colleagues and the community. Collaboration between Nepali universities might be helpful in addressing gender issues. For global collaboration, mutual relationships within university members can sustain and thereby satisfy the stakeholders for the long run (Gaskins-Scott, 2020). University education can be a role model in the Nepali context if it has a foundation of collaborative culture and mutual relationships that might enhance gender equity in a sustainable way and support the community.

Moreover, the promotion of an integral worldview that moves beyond binary conflicts might be supportive for gender balance and equity. For it, the respect for both worldviews, Western Modern Worldviews and Eastern Wisdom Tradition seem the urgent need to realize their potentiality of complementing each other with their distinct potentialities. According to Timmers, Willemsen, and Tijdens (2010), a multi-perspective framework of policy awareness could help evaluate their gender equity policy measures. Therefore, being like Kincheloe's (2005) 'active researcher' and using van

Manen's (1991) pedagogical tactfulness, we can integrate both world views respecting so-called indigenous knowledge and non-indigenous knowledge.

Similarly, the promotion of both critical inquiry and appreciative inquiry seems urgent to realize our cultural practices, explore indigenous knowledge and practices to address contextual issues rather than waiting for the best theories and methods from non-indigenous contexts.

An inclusive approach prepared us to 'walk the talk' and thereby prepared enough space for classroom reformation and policy development in the university setting (LaBoskey, 2004). The critical self-examination might create tension in the university setting (Savage & Pollard, 2018), however, university culture could be a role model to influence students, teachers and faculties, and the society and beyond (LaBoskey, 2004). Further, continuous mainstreaming of gender with pedagogical tactfulness in the classroom with the hope of students getting informed and empowered with the strength of cultural practices would support deconstruction and then reconstruct hegemonic policies and practices.

Conclusion

Finally, we explored that openness, the culture of inquiry, the culture of respect, mutual trust, and shared values like cooperation and inclusiveness are prerequisites for developing, improving and nurturing an all-inclusive context-responsive perspective. All-inclusive perspectives can evolve a new (i.e. context-responsive) framework for gender equity. Moreover, we envision university curriculum and policy developers in collaboration with students, teachers, school management and the community representatives to explore context-responsive equitable strategies in diverse contexts to develop curriculum and thereby to execute university policies adapting participatory approach.

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