The London School of Economics and Political Science

Collective Bargaining, Wage Setting and Downward Adjustments in the Continental European Labour Market: Evidence from Portugal

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for the degree of Doctor of Philosophy

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Declarations

Declaration of Authorship

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

I warrant that this authorisation does not, to the best of my belief, infringe the rights of any third party.

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Statement of Conjoint Work

I confirm that Chapter 1 was jointly co-authored with John T. Addison and Pedro Portugal, and I contributed 50% of this work. A version of this chapter has been published in the British Journal of Industrial Relations, Volume 55 Issue 3, September 2017, pages 551-576.

I confirm that Chapter 2 was jointly co-authored with John T. Addison and Pedro Portugal, and I contributed 50% of this work.

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Without great solitude, no serious work is possible.
Picasso

Statement of inclusion of previous work

I confirm that work on Chapter 1 began prior to my enrollment in the PhD. The research paper has not been submitted for any prior degree.

Statement of Financial, Logistical and Technical Support

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Statement of Titles

The title of the third chapter is inspired in the song *Bridge over Troubled Water* of Simon and Garfunkel, which was part of their 1970 album with the same name. The title of the fourth chapter is inspired in the *Last Dance* documentary, produced by Netflix in 2020.

As for the future, your task is not to foresee it, but to enable it.
Antoine de Saint Exupery

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If I have seen further, it is by standing on the shoulders of giants.
Isaac Newton

This is the dawn I waited for
The new day clean and whole
When we emerge from night and silence
To freely inhabit the substance of time

Sophia de Mello Breyner Andersen

For the dream we will,
moved and speechless.
Have we arrived? Haven't we arrived?
Whether or not there is fruit,
for the dream we go.

Just have faith in what we have,
Just have hope on what
maybe we won't have.
It's enough that the soul we gave,
with the same joy,
to what we don't know
and to what is day-to-day.

Have we arrived? Haven't we arrived?
– We departed. We go. We Are.

Sebastião da Gama

You speak of civilization, and of not having to be,
Or that it shouldn't be like that.
You say that everyone suffers, or most of everyone,
With human things put this way,
You say that if were different, they would suffer less.
You say if were as you want, would be better.
I listen without hearing you.
What would I want to listen to you for?
Hearing you, I would know nothing.
If things were different, they would be different: that's all.
If things were how you want them, they would be just how you want them.
Woe to you and to everyone who lives
Wanting to invent the happiness-making machine!

Alberto Caeiro, Fernando Pessoa Heteronym

Abstract

With the Great Recession as enduring scenery, this journey dwells in widening the acumen on Continental European Labour Markets, particularly during sizable downturns. The yacht, the Portuguese case, is one of its representative designs. The course departs with the specific mannerisms of wage setting institutions, where the first leg reasons the dynamics of collective bargaining in distressed periods, and the second focuses on the influence of trade unions in this wage setting protocol. Then, we have a regatta for an empirical structural analysis of the dynamics of the wage setting and its resilience under significant distress. The last straw deals with the interaction between the labour market institutions and firm liquidity; and its influence on the dynamics of wages and unemployment throughout recessionary periods. At the final harbour, we have drawn a consistent sketch of our yacht's blueprints, and hopefully of those alike. As the zenith of the journey, we have broaden the knowledge about its conduct on rough waters.

JEL Codes: A31, C14, C21, C23, C25, C51, C55, C61, C62, C63, C68, C78, E17, E21, E22, E23, E24, E61, E64, J23, J24, J31, J33, J41, J51, J52, J53, J63, J64

Science proceeds more by what it has learned to ignore than what it takes into account.
Galileo Galilei

Preface

Labour market institutions provide the framework by which labour relationships are governed, postulates the scope of contract adjustments, particularly relevant during downturns, and they set the rules and conduct by which grievances are to be settled. Critically, not all labour markets are alike, and their differences are not inconsequential.

This thesis canvass the Continental European Labour Markets, particularly those of the Southern European and from French based origin, by resorting to the Portuguese case - one of its representative designs. Those markets are structured in a centralized way, where collective bargaining assumes a primal role. The architecture of contracts are bargained between unions and employer's associations at sector or industry level, with firm and worker left with the ability to bargain over marginal improvements on the collectively bargained preset. Jointly with a comprehensive labour code, this preset effectively constitutes the core of the worker's contract, prescribing labour conditions, hours of work, rules of absences and holidays. Particularly in permanent contracts, which are the dominant type, its clauses determines restrictive conditionality to dismissal and stipulates an increasing severance compensation in its occurrence. It further conditions pay, with collective bargaining prescribing a detailed wage floor table, and often minimum values of wage supplements, that maps each job assignment. In fact, as overall principle, pay is considered an *acquired right* which shall not be reduced.

In the first chapter, I overview the presence of unions in the market and the dynamics of collective bargaining throughout the Great Recession, inspecting how a severe downturn affects the ability of unions and employer's associations to bargain. Instead of a shift of the institutional paradigm, the reduced frequency of collective bargaining agreements and extension revisions is attributed to downward nominal wage rigidity in low-inflation regimes. In severe temporary hardship, the regular conduct is merely stalled, not changed, with the sides temporarily freezing contract revisions. Unions acknowledge that nominal improvements on current contracts are not feasible through bargaining, and firms realize contracts are largely legally protected against nominal wage cuts.

Despite its resilience, the institutional setting has witness substantial structural changes. Noteworthy, trade unions have recorded an intense erosion of membership, while keeping its perks in the apparatus of collective bargaining. These dynamics raise the question about the ability of trade unions to influence wages, and subsequently about the importance to consider its presence in modelling the way wages are defined. Have they become an inconsequential *pro forma*? Have collective bargaining became a specter whose prescriptions do not bind or constraint? The second chapter intends to analyse those pertaining questions. It examines the association between union membership and wages, measures the union density wage gap, and attributes such gap to the heterogeneity of worker, firm, and job-title or 'occupational' characteristics. Even with almost universal representation, unions are found to casually increase wages as they increase workplace and/or sector membership, until the majority of the workforce is unionized - a clear sign that unions remain relevant beyond its mere statutory presence.

This evidence raised the challenge addressed in the third chapter, namely how can we describe the way wages are set in this type of markets, while acknowledging and taking advantage of its institutional framework? Consequently, the chapter proposes a structural model capable to be empirically implemented to estimate bargaining powers of the actors in the negotiations, the link between the value of the worker-firm relationship and the worker's wages, the passthrough of changes in collectively bargained wages and changes in actual worker's wages, the worth of worker and firm characteristics in explaining the wage dispersion, and the extend to which higher quality workers match with higher productive firms. The findings obtained in a consistent framework conform with the literature, and provides a sound perspective to understand the wage setting protocol in these markets.

Structurally, I witness a secular deterioration of worker's bargaining powers at the top and the middle of the wage distribution, while at the bottom I recorded a broad stability, in a process that should not be oblivious to the union membership erosion. In distressed times, the bargaining powers are remarkably stable, signalling a significant wage setting resilience. Thus, the eventual adjustment of real wages occur fundamentally due to the changes in the value of real outside options and of the real value of the productivity generated by the current worker-firm match, and not because the worker obtains less from the labour relationship.

With the wage setting defined and overviewed, the fourth chapter incorporates its findings and studies the ability of the labour market to cope with downturns of varying magnitudes, in the context of a macroeconomic model with consumers possessing heterogeneous wealth and productivity. There, consumers work and save to insure themselves against unemployment. The contracts are either temporary or permanent, with the latter implying displacement costs when they breakdown. The wages can not be nominally reduced, and firms enter in financial distress whenever their leverage reaches substantially high levels. Such distress makes firms more likely to succumb due to the pull-off of financial institutions, leading to a costly process of destruction of inter-temporally viable jobs.

By the end of this thesis, a comprehensive and consistent overview of the labour market has been performed, particularly during distressed periods. While the institutions adjust to downturns by temporarily shutting down, the pre-crisis agreements are by rule not overturned. Consequently, the potential realignment of real wages to new market conditions is constrained by the impossibility of nominal wage reductions, thus taking place at the rhythm that inflation allows. This constraint consumes firm liquidity. If such rhythm, jointly with permanent worker's (mostly) voluntary exits, as involuntary becomes very costly, the cut down on hiring, and the dismissal of often the few existing temporary workers are not sufficient to avoid the firm's liquidity exhaustion, a catastrophic job destruction takes place, led by firms exiting the market, often through bankruptcy. That has been the tale of the Great Recession in some countries which at least partially justifies the Great Unemployment Divergence witnessed in Continental Europe in those times.

If we knew what we were doing, it would not be called research, would it?
Albert Einstein

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Chapter I

Unions and Collective Bargaining in the Wake of the Great Recession: Evidence from Portugal

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Abstract

Against the backdrop of its industrial relations architecture, characteristic of the 'southern European group' and intimately linked to the recommendations of the Troika, this chapter examines four key aspects of Portuguese collective bargaining. First, it provides definitive estimates of private sector union density for that nation. Second, it models the determinants of union density at firm level. Third, it yields estimates of the union wage gap for different ranges of union density. The final issue examined is contract coverage. The received notion that the pronounced reduction in the number of industry-wide agreements and extension ordinances of late is to be equated with a fall in coverage is shown to be a chimera, the number of workers covered by new and existing agreements remaining largely unaffected by the economic crisis. The reduced frequency of new agreements and extensions is instead attributed to downward nominal wage rigidity in low-inflation regimes.

1 Introduction

Portugal is one of a number of countries within the Euro zone to have experienced a major external balance problem, leading to financial support by the *Troika* (the European Commission, the European Central Bank, and the International Monetary Fund) in return for the joint therapy of fiscal consolidation and structural reforms. Portugal's external problem may be seen as the outcome of a competitiveness crisis stemming from a decade of misalignment between (rising) wages and (lagging) productivity growth. (For a full diagnosis, see Blanchard (2007)). One mechanism underlying this misalignment was the country's wage setting architecture that failed to deliver the necessary responsiveness of wages to their underlying determinants. This failure was exacerbated by the conjunction of a low inflation regime with severe nominal wage rigidity, resulting in burdensome employment losses (Carneiro et al. (2014)).

Now the role of collective bargaining systems in achieving efficiency – to be equated with the ability of the economy to maintain a low average unemployment rate and to limit fluctuations in the unemployment rate in response to economic shocks – has been the topic of considerable policy interest in recent years (see Addison et al. (2014)), and in this regard the Portuguese model has been allied to labour market rigidity and heightened reliance on quantity adjustment. But Portugal does not stand alone: it is not atypical of the group of southern European nations in terms of its industrial relations infrastructure and in particular high degree of collective bargaining coverage and social dialogue, albeit accompanied by union fragmentation, lack of articulation in collective bargaining, and material informality in the labour market. An additional commonality has been the historical importance of the *favor laboris* principle, which in the Portuguese case has traditionally meant that successor collective agreements can only improve on the terms of the immediate past agreement. More generally, Portugal is representative of continental European nations – indeed in the vanguard – in respect of its extension (of collective agreement) procedures, employment protection legislation, and decline in union density from its peak circa 1980.

In consequence, Portugal was one of three southern (and other) European nations placed under the scrutiny of the *Troika* and pressured to push through material reforms to its collective bargaining system (inter al.) in the wake of their economic difficulties/sovereign debt crises. As we shall see, the quid pro quo for financial assistance included sharp curbs on extension agreements that were perceived to produce downward wage rigidity – for example, Martins (2014) argues that extension ordinances promote nominal wage rigidity through the setting of around 30,000 wage floors – and greater unemployment.

Notwithstanding the importance of the great recession and *Troika* policies to contemporary industrial relations in Portugal, therefore, is important to discuss the pre-existing architecture of collective bargaining to understand the behavioural shift sought by the *Troika* and also to properly address the nature of the so-called crisis in collective bargaining which is widely perceived to be more acute for southern European nations in general and Portugal in particular than elsewhere in Europe (see, for example, Cruces et al. (2015)). To this end, we have first to describe the bargaining framework in Portugal and its evolution both before and in response to the economic crisis. This will enable us to address the functionality of the wage bargaining system and to speak of a standstill rather than an overt crisis in collective bargaining.

Having outlined the bargaining framework, measurement of overall union density is a key component of any discussion of there being a developing crisis in collective bargaining. Unfortunately, countries often lack official statistics on union density. Portugal is one such country. The consequence is that prior estimates of private-sector union density for that nation are little more than informed guesstimates. Fortunately, a solution to this problem has recently become available with the availability of the *Relatório Único*, or Single Report, conducted by the Office for Strategy and Planning (in the Ministry of Employment, Solidarity and Social Security). That is, we can now derive accurate estimates of union density. We provide the first estimates for Portugal using these new data for the years 2010 through 2013. Even if there has certainly occurred a steady erosion in union density since 1980, we report broad stability over this four-year period with average density a little over 10 per cent. For its part, collective agreement coverage approximates 90 per cent over the same interval. The extension phenomenon that helps explain this disparity between density and coverage will be addressed throughout the chapter.

Again using the *Relatório Único*, and following on the provision of economy-wide statistics on density, our cet. par. analysis then examines the determinants of density at firm level. Among other things, we report that union density is highest in those sectors and activities sheltered from product market competition. More generally, it is firms with higher union density that are observed to drive the bargaining process.

The next part of our analysis presents a stylized version of a recent investigation into the union premium (see Addison et al. (2021)). That study explores a high-dimensional fixed effects regression model and then offers a smoothed nonlinear function relating wages to union density at firm level. Our modification simply collapses that nonlinear relationship into five union density intervals. We find evidence that our measure of the union wage gap is sharply increasing in firm density. Changes in the premium are investigated as a further check on the stability of the collective bargaining system.

Returning to the theme of European crisis management, the chief structural reforms sought by the Troika included greater decentralization of collective bargaining, derogations from industry agreements at workplace level, close attention to what countries can afford in setting national minimum wages (i.e. linking increases to broad competitiveness and efficiency audits), and cutting back on extension agreements. In its dealings with the agency in 2011, as documented in the next section, the Portuguese government agreed to a number of such measures. Most notably from the perspective of the present chapter, extension agreements were initially frozen and the procedures for such extension subsequently reset by the government. Additionally, any increase in the nation-wide minimum wage was meant to be made conditional on economic and labour market developments.

The data seemingly suggest that the government measures on extension agreements have had a distinct chilling effect on collective bargaining. Thus, the number of new agreements has fallen dramatically and with them the number of extension ordinances. It is just a short step to argue – as have a number of influential observers – that Portuguese collective bargaining is in crisis mode. But we choose instead to focus on the fact that, although new agreements have stalled, coverage under existing agreements (and operational extensions) has remained basically unchanged. Furthermore, since new agreements cannot generally be less favourable to employees than the agreements they succeed, the decline in new agreements is less to be construed as structural (i.e. a crisis in collective bargaining) than as indicative of the difficulty in securing increases in nominal wages (that are admittedly less likely to be extended) in a deflationary environment. Vulgo: real wages (or wage costs) are not declining enough. In short, we interpret fewer agreements and extensions as associated with downward nominal wage rigidity combined with a severe recession rather than the 'upward nominal wage rigidity' experienced in more normal times.[1] This interpretation does not mean that the Portuguese collective bargaining system is fit for purpose or for that matter to deny that structural changes are in the offing.

The outline of the chapter is as follows. First, we review the structure of Portuguese collective bargaining. Second, we describe the main datasets used in this inquiry and the

[1]Upward nominal wage rigidity refers to the notion that in normal times many firms are oblidged to increase the wages of their workers either through the extension of collective wage agreements or hikes in the minimum wage (see Martins et al. (2017)).

base elements involved in modelling the determinants of union density and the wage gap. Third, we present some introductory descriptives followed by the main findings of the chapter. A summary concludes.

2 The Bargaining Framework

Portuguese law makes provision for three types of collective bargaining at national, regional, and local level, although it is the case that contemporary wage setting has been dominated by the widespread use of administrative instruments such as government extensions of agreements entered into by the bargaining parties. First, there is a modicum of firm-level bargaining signed by an individual company and one or more unions, resulting in so-called *Acordos de Empresa* (or AEs) that are important in the oil sector and transport and communications. Somewhat more important in terms of direct impact are collective agreements signed by several employers that are not part of an employers' association and one or more trade unions, known as *Acordos Colectivos de Trabalho* (or ACTs), that are significant in the financial sector and utilities. However, it is industry-level/branch or sectoral agreements, so-called *Contratos Colectivos de Trabalho* (CCTs), negotiated between one of more employers' associations and one or more unions, that predominate. As a result, CCTs in conjunction with extension agreements that are very largely based on them (the other option is to extend ACTs) explain levels of collective bargaining coverage in the order of 90 per cent of workers despite a marked decline in union density that extends back over 30 years (see Visser (2015)) and that we now estimate for the private sector at around 10 per cent. The vast majority of agreements are signed by unions linked to the two major union confederations: the CGPT-IN or General Confederation of Portuguese Workers (*Confederação Geral dos Trabalhadores Portugueses - Intersindical Nacional*), and the UGT or General Workers' Union (*União Geral de Trabalhores*). The gaps in coverage are largely in personal and other services, and in public administration where, despite centralized negotiations between the government and the trade unions, wages are decided upon unilaterally by the government. The wages of employees in publicly-controlled companies, such as public transportation and the postal service, are collectively bargained in the normal way.

The industry-level or sectoral agreements may cover a range of industry-specific occupations but as the system does not rule out parallelism or overlapping collective agreements a single enterprise may be covered by two or more agreements depending on the union affiliation of the workers (although as a practical matter the content of most of the agreements is similar, the respective tables of wages tending to be the same). The situation may be further stratified if the firm in question straddles more than one line of economic activity, thereby belonging to one or more employer associations. As a result of fragmentation, therefore, several agreements may coexist for the same region, occupation, and firm. Horizontal or occupation-based agreements are also possible, although they are infrequent largely because the law gives precedence to vertical sectoral agreements many of which are signed by a large number of primary unions that may include occupation-based unions.

Portuguese collective agreements are at once both extensive and general. They are extensive insofar as they cover many categories of worker. They are general in that they set only minimum conditions of which the most important is the base level monthly wage. Other terms and conditions most frequently covered are working time, night work, overtime, shift rates, and additional social benefits (see Addison et al. (2021)). The focus is upon wage floors rather than anticipated wage growth that in some centralized bargaining regimes such as Sweden have been incorporated directly into sectoral agreements. In a typical year, about 300 agreements are in place, determining around 30,000 wage floors for the corresponding job titles or professional categories (*Categorias Profissionais*), one consequence of which is that employers in principle have freedom of manoeuvre to tailor remuneration to their prevailing economic circumstances.[2]

Until recently, it has been the case that Portuguese collective agreements remain in place until a new agreement is signed. This important feature of collective agreements is statutory, as in Spain, whereas in Italy it is established by collective agreement. Coupled with the *favor laboris* principle that new agreements should yield more favourable conditions that those they are replacing,[3] this provision has meant that collective agreements have tended to be revised regularly only insofar as wages are concerned, their other terms and conditions often being left untouched for many years. Recent changes in the labour code mean that collective agreements can now expire if they are not renewed. Although more sweeping changes had been envisaged under the labour code of 2003, the status quo ante until late 2014 was as follows. Collective agreements expired five years after they had last been agreed, or five years after one of the parties had indicated its intention to renegotiate their terms and conditions. That said, the collective agreement would continue to apply for at least 18 months after this, to allow negotiations to take place. Indeed, either of the parties had a period of 12 months during which to request the appointment of an arbi-

[2]On the determinants of the contractual wage and the wage cushion, or difference between actual wages and the contractual wage set for each worker category, see Cardoso and Portugal (2005).

[3]For changes in the law on the suspension of the norms of collective agreements and of labour contracts as well as the terms of the suspension of collective bargaining in situations of company crisis, see Ramalho (2014) and Martins (2014a), respectively.

trator to draw up new terms and conditions. New rules for the extended validity period (*sobrevigência*) of collective agreements came into force on 1 September 2014, under the seventh amendment to the 2009 labour code. Specifically, the expiry period was reduced from five years to three years, and the period of continuation upon expiry reduced from 18 months to 12 months. In addition, whenever there is an interruption of negotiations, including conciliation, mediation or voluntary arbitration for more than 30 days, the period of 'survival' is suspended and the period of negotiation with suspension cannot exceed 18 months.[4]

The most potent mechanism shaping the formation of wages has traditionally been the systematic generalization via extension ordinances or *Portarias de Extensão* of industry-wide agreements (and occasionally ACTs) by the Ministry of Employment, following a request from either or both of the parties to the agreement.[5] The upshot of this near automatic procedure is that even agreements reached by trade unions and employers' associations with very low representation have had a strong impact in setting wage floors, with roughly 70 to 80 per cent of the labour force benefiting from collective agreements without being members of the signatory organizations. Finally, in the absence of one of the representatives, or in the presence of strategic delays in negotiations/refusals to negotiate, the Ministry of Employment can regulate the sector directly through an Ordinance of Working Conditions, or *Portarias de Condições de Trabalho*.[6] The extension mechanism in conjunction with the large number of job titles set down in the typical sectoral agreement together explain the 30,000 (informal) minimum wages identified in Martins (2014) analysis of the employment and wage bill consequences of collective bargaining extensions referred to earlier. Observe that among the firmament of European extension arrangements, or

[4]Since contracts can now lapse or lose their validity, it is of interest to determine the vintage of Portuguese contracts. For the year 2009, the frequency and per cent of all workers with contracts of up to 10 years in length were as follows:

Year	Number of Workers	Percentage of Workforce
2000	41,192	1.93%
2001	46,995	2.21%
2002	64,729	3.04%
2003	135,502	6.36%
2004	87,629	4.11%
2005	107,979	5.07%
2006	157,031	7.37%
2007	170,233	7.99%
2008	496,709	23.32%
2009	636,396	29.89%

In other words, some 8.71% of all workers were employed under contracts that were more than 10 years old.

[5]Voluntary extensions are also common, while employers who sign an agreement with a trade union(s) usually extend its application to the entire workforce, irrespective of the worker's union status.

[6]An arbitration process, either mandatory or voluntary, may be set in motion to unfreeze 'blockages'.

their functional equivalents, those in Portugal, Spain, Greece, and Italy have traditionally been among the most far reaching (see Visser (2013): Table 4).

In addition to the extension procedure, wage floors are also set under national minimum wage machinery, set up in 1974. The minimum wage can exceed that set under sectoral bargaining. This guaranteed monthly minimum wage (*Retribuição Mínima Mensal Garantida* or RMMG) was virtually stagnant in real terms between 2002 and 2006, leading to an agreement between the social partners (government, the trade union confederations, and the employers' confederation) in 2006 allowing for an increase of almost 30 per cent, to be phased in over five years and setting a medium-term target value €500 by 2011. It has been estimated that the share of minimum wage earners among full-time workers aged 18 to 61 years rose dramatically from 6.7 per cent of total employment in 2006 to 16.6 per cent in 2010 (Carneiro et al.'s (2012)).

Both systems of minimum wages – nation-wide and collectively bargained/extended – were disrupted by the economic crisis of 2011/12. As part of the *Memorandum of Understanding*[7] concluded between the Portuguese government and *Troika* in May 2011 it was agreed among other things that the procedures for extending collective agreements would be changed, even prior to which the government committed to restrict the extension of collective agreements.[8] In October 2012 the government announced new criteria for the administrative extension of collective agreements taking into account the representativeness of the negotiating organizations and the implication of such extension for non-affiliated firms. Most importantly, agreements could only be extended if at least one union and one employers' organization requested it and the wider signatory organizations employed more than one-half of the employees in the relevant industry. However, in June 2014 this Resolution was modified: **by way of dilution**, extensions were exempted from the 50 per cent rule if more than 30 per cent of firms affiliated to the employer association consisted of micro, small, and medium-sized companies (employing up to 250 employees). As far as the national minimum wage was concerned, the *Memorandum* proposed to make any increase in the minimum wage conditional on economic and labour market developments. The minimum wage was duly frozen and in 2012 and 2013 it stood at the level of 2011 (viz. €485). It was not uprated to €505 – a little above the medium-term target – until October 2014. However, with effect from January 1, 2016, the minimum wage was raised to €530.

[7]The terms of the *Memorandum* are available at https://infoeuropa.eurocid.pt/registo/000046743/.

[8]Other changes under the *Memorandum* included revisions to the unemployment insurance system in terms of the level and duration of benefits, a dimunition in employment protection via a reduction in severance payments and the relaxation of the protection against individual dismissals, and an expansion of flexible working time arrangements in the form of working time accounts at individual and plant level.

The economic crisis and the response of the public authorities to it are credited with some fairly dramatic changes in Portuguese collective bargaining. In particular, the decline in the number of Portuguese collective agreements and worker coverage have been the subject of several critical European Observatory of Working Life reports (e.g. EurWORK (2014)), while the topic of so-called radical decentralization affecting all crisis EU nations and leading to the "destruction of national collective bargaining systems" has been identified as a key paradigm shift by Schulten (2013).

Nevertheless, in addition to the reversals noted above, there has been little evolution in *atypical collective agreements*, by which is meant the ability of works councils and other representative bodies of workers at plant level to conclude collective agreements at the workplace. The current legal position is that works councils can negotiate at this level in firms with a minimum of 150 employees but only in circumstances where this is authorized by the trade union. The latter provision reflects the constitutional provision that competence to conclude collective agreements is the exclusive preserve of the trade unions. Reform proposals favoring so-called *Acordo Geral de Empresa* that can be signed independently of the trade union had to be abandoned in the face of strong union opposition in the Standing Council for Social Concertation (*Comissão Permanente da Concertação Social* or CPCS).[9]

3 The Datasets

The main dataset used in the present inquiry is the *Relatório Único* for the four years 2010 through 2013. The data are collected through a mandatory questionnaire to every establishment with at least one wage earner, and the survey is conducted by the Office for Strategy and Planning in the Ministry of Employment, Solidarity and Social Security. The *Relatório Único* replicates its precursor the *Quadros de Pessoal* (Personnel Tables) other than in one important respect. The new dataset for the first time contains a question on unionism. Specifically, the union question contained in the survey, and asked of the manager respondent, is as follows: "Indicate the number of workers for whom you have knowledge of their membership in a union (because they are union officials, because you

[9]Space constraints preclude other than passing reference to the process of social concertation/social dialogue. Suffice to say here that although the last pact establishing reference values for nation-wide wage increases was in 1996, a number of agreements have been reached in the tripartite CPCS since 1996 of which 2006 social accord on minimum wages is a practical example. Moreover, many of the proposals in the *Memorandum* relied heavily on another tripartite agreement in March 2011, while some subsequent labour market new reforms had a basis in another such agreement in January 2012. By the same token, modern concertation has been practised without the participation of the largest trade union.

deduct membership dues from their salary, or because the worker informed you about his/her membership so as to determine which particular collective regulation is applicable to their case)." This information allows the researcher to measure the union density of the firm, rather than establish the union membership or otherwise of individual workers. It is likely to provide a more objective and consistent measure of unionism than union sources because of the pecuniary and legal underpinning of the survey question – the only source of understatement arising where the employee wishes to hide his or her union affiliation from the employer. We note parenthetically that there is a large gap between the figures provided by the unions and the union density estimates of the government.[10]

Data are given on each establishment (its location, economic activity, employment, number of temporary employees, legal structure, and sales) and for each of its employees (their gender, age, education, skill, occupation, tenure, earnings, and work schedule). The earnings data are both detailed and accurate. The information pertains to the gross wage for normal hours of pay (or base wage) together with both regular and irregular benefits, overtime pay and hours, and wage bargaining mechanism/type of contract. The full-wage, or total compensation, is the wage variable used in estimating the union wage gap or premium. The accuracy of this information is assured by its administrative nature and the fact that by law the wage and all other information in the survey has to be made publicly available at the establishment.

The restrictions imposed on the raw dataset were as follows. First, we limited our analysis to full-time workers in mainland Portugal. Part-timers are excluded primarily because trade unions and employers bargain over full-time wages. Furthermore, their inclusion would require us explicitly to model the labour supply decision over the number of hours worked. Second, we excluded workers from agriculture, forestry, and fishing as well as those in public administration whose wages are not collectively bargained. Third, individuals aged under 16 years and above 65 years at survey date were excluded.[11] The final restriction was that the worker's monthly wage had to be at least 80 per cent of the mandatory minimum wage, which here corresponds to the lowest admissible wage for apprentices. Our final sample for the density computations comprises some 612,336 year-firm observations, which correspond to some 301,724 firms matched by identifying number, representing 2,758,197 individuals matched by identifying number, gender, and year of birth, and 48,913 jobs matched by code of the collective agreement occupational

[10] See http://www.worker-participation.eu/National-Industrial-Relations/Countries/Portugal/Trade-Unions.

[11] The former individuals are those in reserved professions, such as theatre and sports activities, otherwise exempted from the legal minimum working age, while the latter individuals constitute another special group of those above retirement age.

category. Computation of the union wage premium will also use data from the *Relatório Único* , the sample now comprising 2,697,114 workers, 298,185 firms, and 48,222 job-titles. However, our analysis of downward nominal wage rigidity in high and low inflation regimes, in which we effect comparisons between matched samples of workers across the years 1985, 2009, and 2013, will perforce draw on the *Quadros de Pessoal* for the former two years.

The third principal dataset used in this inquiry lists all new agreements and extensions of collective agreements between January 2008 and December 2014, and is maintained by the Ministry of Economics from the texts of both the individual collective agreements and extension orders. We take Martins (2014) estimates based on these data as indicators of the *flow* of collective agreements, and update and supplement this information with data from union and official sources, respectively the UGT (2014) and BTE (2015) (*Boletim do Trabalho e Emprego*), on the flow of agreements and extension ordinances. These data will be juxtaposed against stock data from the Relatório Único (and the Quadros de Pessoal) to examine the validity of the assertion that profound changes in the numbers of extensions and new agreements imply sharp falls in coverage.

4 Modelling

We address in turn the models used to estimate union density and the union wage gap. The context of both constructs are expanded upon in section 6. Given that the remaining components of our analysis are based on simple tabulations of collective agreements/extensions and presentation of nominal wage change distributions, they too are not addressed here.

To estimate the determinants of firm unionism we deploy a count regression model and, in recognition that many firms in the sample do not have any unionized workers (287,056 firms, or 95.1 per cent of the total), we elect to use a zero inflated count model. More specifically, since the non-zero observations may be over-dispersed, we use a zero inflated negative binomial model (ZINB) after Lee et al. (2001).

The zero inflated negative binomial model can be written:

$$Pr(y|x) = \begin{cases} \rho + (1-\rho)e^{-\lambda(x,u)} & \text{for} \quad y = 0 \\ (1-\rho)\frac{e^{-\lambda(x,u)}\lambda(x,y)^y}{y!} & \text{for} \quad y \geq 1 \end{cases} , \tag{I.1}$$

where y denotes the count of the expected number of union workers at the firm, x are the covariates influencing unionization, u is an error term, and $\rho \in [0, 1]$ is a zero-inflated parameter obtained through a simple logit parameterization:

$$\rho = \frac{e^{\tau}}{1 + e^{\tau}}. \tag{I.2}$$

Observed data often display empirical variances larger than their means, implying the existence of an "overdispersion" problem. This may reflect the existence of firm unobserved heterogeneity. The problem can be circumvented by adding the random variable u to the vector of explanatory variables. This will allow for the expected value to differ from the variance. But conditional on the error u, y follows a Poisson distribution. If we assume that u follows a gamma distribution, then the unconditional distribution of y is a negative binomial. In other words, $\lambda(x, u) = e^{x'\beta + log(u)}$, with u following a gamma distribution, and where $E(u|x) = 1$ and $Var(u|x) = 1/\delta$. Furthermore, we included an artificial covariate, $logN$, reflecting the size of the workforce to account for exposure to the "risk" of being unionized. That is, $\lambda(x, u) = e^{x'\beta + log(N) + log(u)}$, where N is number of workers in the firm. Proceeding this way, the specification lends itself to a convenient interpretation as a fractional regression model.

The dependent variable in the model is the number of unionized workers at the firm. The independent variables in the model are (average) worker characteristics and firm characteristics. The former comprise a continuous measure in age, and the proportions of females, foreign nationals, and individuals in various educational categories. The firm characteristics are the share of public equity, and dummies for establishment size (6), industry (30) and time because of our use of all waves of the *Relatório Único*.

Our second model links union density to the union wage gap. It takes into account the results given by Addison et al. (2021) showing that it is unwise to assume that union density impacts linearly on (log) wages. That study seeks to decompose the union wage gap in terms of the contributions of worker, firm, and job title heterogeneity in the framework of a high-dimensional fixed-effects regression model. Its main finding is that the wage gap largely reflects the allocation of workers to firms with distinct wage policies due to the heterogenous presence of unions at firm/sector level rather than unobserved worker quality or differences among job titles, and we will take this interpretation as the main driving force behind the union wage gap. Observe that the union wage gap in this model – that is, prior to the decomposition exercise – gives the cet. par. average difference in wages at a given firm resulting from the specific union density rate of that firm via a two-stage estimation. Essentially the first step of the procedure is a wage regression with standard controls but a separate fixed effect corresponding to each level of union density in the database. That is, there is a different wage intercept capturing the impact of the firm's particular density rate on a worker's compensation. In a second step, a kernel regression is used to smooth these fixed effects. The outcome of this exercise shows that the impact of union density is highly nonlinear. Specifically, it is miniscule up to 20 per cent density but increases steeply thereafter, reaching a maximum at around 70 per cent density. As an approximation to the flexible functional form of the full model, the worker earnings function that we deploy here substitutes four union dummies (and the omitted category) for the 4,828 individual union fixed effects. Our estimating equation is thus specified as:

$$log(w_{ift}) = \beta x_{ift} + \sum_{k=1}^{4} \eta_k ud_{ft}^k + \epsilon_{ift} \tag{I.3}$$

where $log(w_{ift})$ is the log of monthly gross compensation for the individual worker i in firm f at year t and, in addition to the union dummies, x_{ift} is a vector of explanatory variables and ϵ_{ift} denotes an idiosyncratic error term.

The four union density dummies (ud_{ft}^k) are as follows: greater than zero but less than or equal to 25 per cent, greater than 25 per cent but less than or equal to 50 per cent, greater than 50 per cent but less than or equal to 75 per cent; and greater than 75 per cent. The base controls to be used at the level of the worker are age, age squared, tenure, tenure squared and dummy variables for gender, and education level; for the firm, they are industry and size dummies. In addition, there are time dummies.

5 Introductory Descriptives

We preface our cet. par. findings and estimates of the stock of workers covered by collective bargaining instruments with some aggregate statistics on union density and mostly secondary flow data on worker coverage by collective agreement. Table 1 provides three sets of estimates of trade union density covering the period 1980 to 2013. The first are taken from Blanchflower and Bryson (2003) and point to a sharp reduction (a little under 50 per cent) in membership over the final two decades of the last century. The next set of estimates are broadly comparable in construction and are taken from the OECD (2015). They now show a much reduced rate of decline over the first decade of the present century and even a slight uptick as of 2012. As noted earlier, our own estimates shown in the last row of the table rely on administrative data from the *Relatório Único* – rather than union sources – and exclude the public sector if not public enterprises. Perhaps more important than the difference between our data and those of the OECD, only partly

Table I.1: Trade Union Density in Portugal, 1980-2013

Year	Study		
	Blanchflower and Bryson (2003)	OECD (2015)[a]	This study[b]
1980	52		
1990	40		
1995	30		
1998	25		
2000		21.6	
2005		21.3	
2008		20.5	
2010		19.3	10.8
2011		19.5	10.2
2012		20.5	10.4
2013			10.1

Notes: (*a*) Values extracted on 4 March 2015 from OECD.StatExtracts (available at http://stats.oecd.org/Index.aspx?DataSetCode=UN_DEN); (*b*) Computed from the *Relatório Único* 2010 - 2013.

occasioned by this exclusion,[12] is the broad stability of private sector union density over the last four years for which the data are available. That is, union density held at a little above 10 per cent in the face of economic crisis and retrenchment.

Table 2 plots the *flow* of collective agreements and their coverage from 2008 to 2014. This is where the real drama unfolds. Between 2008 and 2013 there occurred a precipitous fall in the number of new sectoral agreements (84.3%), multi-company agreements (33.3%), and single-firm agreements (49.5%), albeit with indications of an uptick in the coverage of all types of new agreement as well as some churning in agreement type in 2014. On aggregate, the number of workers covered by new collective agreements fell from 1,894,846 to 242, 676 (or by 87.2%) between 2008 and 2013, followed by a very modest increase of 1.5 per cent in the combined flows in 2014. The last column of the table reveals an even more dramatic fall off in the number of extension agreements of 93.1 (90.1) per cent between 2008 and 2013 (2014). The table does not give the number of workers affected by these extension agreements as the authorities do not collect information on the number of affiliated workers in each signatory union and the universe of workers potentially covered by a given bargained instrument of collective bargaining. As a result, the available information simply reports the workers covered by each instrument independently of its origin, either by affiliation or extension.

[12]Data provided by Visser (2015), while mostly consistent with OECD data in the years following 2000, if not before then, suggest lower density rates of 18.3 and 18.5 per cent in 2011 and 2012, respectively.

Table I.2: The Flow of Collective Agreements by Type, and Extensions, 2008-2014

Year	Type of Collective Agreement				Extension
	Sectoral (CCTs)	Multi-Employer (ACTs)	Company Agreements (AEs)	Total	(PEs)
2008	172 (1,778,216)	27 (47,232)	97 (69,398)	296 (1,894,846)	131
2009	142 (1,299,371)	22 (59,902)	87 (37,952)	251 (1,397,225)	103
2010	141 (1,309,267)	25 (64,455)	64 (33,344)	230 (1,407,066)	113
2011	93 (1,160,080)	22 (52,737)	55 (24,102)	170 (1,236,919)	17
2012	36 (291,068)	9 (26,645)	40 (9,009)	85 (327,662)	12
2013	27	18	49	94 (242,676)	9
2014	49	23	80	152 (246,388)	13

Notes: Numbers of collective agreements, and workers covered (in parentheses). Earlier values are reported by Dias and da Conceição Cerdeira (2011); and, for slightly different values, see EurWORK (2014). The data contained in the last two rows of the table are consistent with but not strictly comparable to those reported by Martins (2014). *Sources:* Martins (2014) for the years 2008-2012; UGT's (2014) (2014, Figures 2 and 4) for 2013-2014; BTE (2015) for extension ordinances.

The latter data have been interpreted as indicating a major rupture of the industrial relations system in Portugal that has excluded large numbers of workers from collective agreements. But the popular notion that the Portuguese reforms have resulted in a major overall reduction in the coverage of collective agreements is to confuse flows with stocks, namely the number of workers covered by new agreements and extension orders and those covered by *existing* agreements and ordinances. In short, Table 2, dealing with flows, has to be considered alongside stock data. To anticipate our subsequent findings, we shall report that the number of workers covered by collective agreements has, in common with union density, evinced broad stability in recent years and that there is little evidence in the flow data to herald the disappearance of collective bargaining.

6 Main Findings on Union Density, the Wage Gap, and Stock Data on Coverage

Table 3 presents the regression estimates from the negative binomial model discussed in section 4. Observe firstly the positive relationship between average age and the fraction of unionized workers in the firm, indicating the increased propensity on the part of the worker to become unionized with age. The negative association between membership and nationality (here the share of foreign workers) is also familiar. Less anticipated is positive association between the share of females in the firm and union density, although it is only marginally significant. (The coefficient estimate of 0.0710 means that increasing the share of females by 10 percentage points increases union density by 0.07 per cent.) Evidently, traditional gender-based supply and demand have become moot with the increased labour market exposure of females. For its part education is strongly associated with union membership: the relationship is near monotonic, the omitted category being those with no schooling.

Perhaps the clearest association of all is the monotonic relation between firm size and union membership, most likely reflecting scale economies to unions in the supply of union services, as well as potential collective voice benefits. Note, too, the sharply higher membership rates in companies with greater public equity. It is frequently argued that the 'dispersed' nature of property rights in such circumstances together with implicit guidelines on collective bargaining operating in public administration (not included in the sample), where government assumes the position of employer, provides encouragement to higher union density in publicly-owned firms.

Finally, among the sectoral dummies (not separately identified in the table), the cases of Finance and Insurance Services and Transportation and Storage Services are important. The former branch is the sole private sector industry where the labour unions offer a system of private healthcare benefits to workers.[13] The latter industry is well known for its inelastic demand, small share of labour costs in total costs, and pervasive featherbedding. Other important sectors are Oil Refined Products and Electricity and Water, oligopolistic sectors with an historical prominence of public equity, which despite its erosion under successive privatization schemes likely has legacy effects on collective bargaining arrangements.

[13]Note the analogy here with modern cross-country union studies linking the presence of a Ghent system to the growth in union density and its attenuation – as a result of the emergence of independent unemployment insurance funds providing such insurance without requiring union membership – to the decline in union membership/density; see, respectively, Schnabel (2013); Böckerman and Uusitalo (2006).

Table I.3: Determinants of Union Density in Portugal. Dependent Variable: Number of Unionized Employees at the Firm

Variable	Coefficient (s.e)
Average age (in decades)	0.374***
	(0.0152)
Proportion of females	0.0710*
	(0.0379)
Proportion of foreigners	-0.544***
	(0.107)
Proportion of workers with elementary schooling	0.738***
	(0.0665)
Proportion of workers with preparatory schooling	0.708***
	(0.0589)
Proportion of workers with completed high school	0.774***
	(0.0654)
Proportion of workers with college degree	0.893***
	(0.0719)
Proportion of public equity in the firm	1.086***
	(0.0703)
Firm with 10 to 49 employees	0.943***
	(0.0215)
Firm with 50 to 99 employees	1.820***
	(0.0306)
Firm with 100 to 499 employees	2.219***
	(0.0288)
Firm with 500 to 999 employees	2.431***
	(0.0631)
Firm with 1,000 to 4,999 employees	2.615***
	(0.0668)
Firm with more than 5,000 employees	2.905***
	(0.0966)
Year dummies	Yes
Industry dummies	Yes
Constant	-7.073***
	(0.113)
Inflation parameter	-18.830***
	(0.0261)
$\ln(\lambda)$	3.010***
	(0.0108)
Log likelihood	-164908.5
No. of observations	612,336

Notes: Robust standard errors are in parentheses. ***, **, and * denote statistical significance at the 0.01, 0.05, and 0.10 levels, respectively. *Source: Relatório Único*

Table I.4: Estimated Union Wage Gap (%) for Different Levels of Unionization

Specification	Firm density rate			
	1 - 25%	25 - 50%	50 - 75%	75 - 100%
Without controls	17.59	28.29	63.66	61.11
Controls for (1)	12.31	16.34	38.00	34.00
Controls for (1)+(2)	4.41	6.66	20.27	14.95
Controls for (1)+(2)+(3)	6.61	7.74	21.07	15.26
(Distribution of unionized workers)	(61.67%)	(15.84%)	(9.97%)	(12.53%)

Notes: The following coding is used: (1) worker's characteristics; (2) firm characteristics; and (3) occupations. *Source: Relatório Único.*

We turn next to the association between union density and the union wage gap, using the modified (first-stage) procedure described earlier, in which four density dummies substitute for individual union fixed effects in each worker's wage equation. Table 4 summarizes the model in presenting regression coefficients for the density dummies ([>0%, ≤25%]; [>25%, ≤ 50%]; (>50%, ≤75%]; and [>75%,∞)), the omitted dummy being union density = 0%. Results from four separate specifications are reported. The first contains just the union density dummies; the second adds a set of worker controls; the third adds firm controls; and the fourth includes a single additional control for occupation. It will be recalled that in addition to the occupational control, the worker controls consist of age and age squared, tenure and tenure squared, and dummy variables for gender and educational level while the firm arguments are industry and size dummies.

As can be seen from Table 4, once density climbs above one-quarter of the firm's workforce, the union premium is sizeable and survives the incorporation of worker, firm, and occupational controls. For example, absent controls, in those workplaces where more than 75 per cent of the workforce is unionized the average value of the premium is 61.11 per cent. With the addition of worker and firm controls, the corresponding values are less at 34.0 and 14.95 per cent, respectively, indicating that union workers are more highly educated and the importance of a firm's location and size. Finally, adding in occupation controls causes a no appreciable change in the premium, indicating that for this dataset at least the controversy over the inclusion of the occupational controls in augmented Mincerian wage equations is perhaps overdrawn.

These estimates of the union premium for Portugal are really rather substantial. First of all, they exceed findings based on individual wage and membership data reported for Britain and even the United States. For example, Blanchflower and Bryson (2003) cross-country estimates point to a union wage gap for Portugal of 19.6 per cent in 1998/99, well before the onset of economic crisis. This is not to downplay the difficulties involved in estimating the union premium, such as misclassification and selection biases in conventional studies using membership data or to minimize the problems arising in comparing estimates in which the unit of observation is typically the individual rather than as here the firm. Indeed, where most workers are covered by collective agreements focusing on union membership effects is not particularly valuable and investigation of union wage effects should instead address the question of the effects of collective bargaining arrangements on wages. This emphasis is perhaps clearest in case of German research where studies have investigated the impact of collective agreements on the hourly wage of an average worker employed in an average firm, distinguished between individual worker coverage and firm-level shares of covered employees, sought to estimate the causal effect of sectoral collective bargaining on the wage structure, and provided selectivity-adjusted estimates controlling for the nonrandom selection of workers with unobservable skills into the various contractual regimes (see the summary in Addison et al. (2014)). But there is nothing in this literature to suggest that *contemporaneous* estimates of the Portuguese wage gap are other than very substantial.

Recently, however, some plant-based estimates of the union wage gap more in the spirit of those provided here have been reported for the United States. In particular, DiNardo and Lee (2004) use a regression discontinuity design to estimate the effects of being in a unionized environment relative to a nonunionized environment, using the narrow margin between union success and failure in representation elections. Evidence of a discontinuous relationship between the vote share and wages is deemed to be the true effect of unionization by eliminating any confounding election and omitted variable biases. DiNardo and Lee find small and mostly negative union effects – the largest positive wage effect within two standard errors of the point estimate being just 0.014. As a practical matter, however, this methodology captures only union effects at the margin. This is made clear in an events study analysis by Lee and Mas (2012) of the effect of new unionization on publicy-traded firms' equity values, 1961-1999. The authors use a long panel – of up to 4 years before and after the representation election – of high frequency data on stock market returns. This events study analysis reveals substantial losses in a firm's market value following a union election of $40,500 per unionized worker, which value can be equated with a union premium of around 10 per cent. In addition to addressing the issue of how equity values respond to certification elections, Lee and Mas also estimate events study models for elections with varying degrees of union support. Their results indicate a clear negative association between abnormal returns and vote share. Although there is no discernible discontinuity at the 50 per cent union vote threshold, a greater than 60 per cent share for example is associated with negative cumulative average returns in the range 20-30 per cent while a formal discontinuity estimate of union victory is indistinguishable from zero, allowing these findings to be reconciled with the DiNardo-Lee result without vindicating the regression discontinuity design. The conclusion is that our estimates for Portugal are consistent with but continue to exceed U.S. plant-based estimates based on material union victories in representation elections. Second, our results are obtained in a framework that recognizes, as have other studies (e.g. Bryson (2014)), that wage gains are a function of localized bargaining power. But in Portugal there is an additional wrinkle, namely the substantial disparity between the number of unionized workers and the number of workers covered by collective agreements. This disparity varies significantly among industries and sizes of firm. In this context, the generalized use of extension ordinances that extend to the entire sector agreements reached between unions and employer federations with weak or very weak representation is especially problematic. Such unions and employers' federations will likely represent larger firms and better paid workers. These, then, are the likely leaders in the wage updating exercise and the process one of cartelization. And in circumstances where the use of extension regulations serves to erode representation through time, the problem of a misalignment between bargained wages and wages that are feasible is compounded and reflected in higher unemployment. Finally, when adverse economic conditions impede further increase in wages, given low inflation and the need for real wage realignment, and without an ability to pass through wage revisions to the entire sector, the result is bargaining standstill.

As a secondary exercise we estimated the average union wage gap regression model separately for each year of the 2010-2013 sample period. We failed to detect that the union premium had changed with the severity of the recession, irrespective of the union density interval considered. If anything, the results pointed to a slight increase in the union wage gap over time. While this outcome might suggest that nominal wage rigidity may be stronger in higher union density firms, the more general reading would be that inertia of the wage structure plays a dominant role and that the crisis has yet to produce a major shift in the impact of collective bargaining.

This narrative returns us to the issue of coverage. In Table 2 we reported the dramatic fall-off in new agreements and new extension ordinances after 2010, drawing on Martins (2014) inter al. Other observers have misleadingly concluded from these data that the changes since 2008 have left a little over 1.5 million workers without coverage and that the decline in collective bargaining has reached crisis point. (EurWORK (2014)). In Table 5,

Table I.5: Workers covered by an Instrument of Collective Bargaining in Portugal

Year	Union Coverage (%)
2008	90.5
2009	90.5
2010	92.0
2011	91.4
2012	89.8
2013	89.2

Sources: Quadros de Pessoal, 2008-2009; *Relatório Único*, 2010-2013.

using data from the Relatório Único for 2010-2013 and from the Quadros de Pessoal for 2008-2009, we report that the number of workers covered by an existing or new instrument of collective bargaining – either the collective agreement or the extension ordinance based on it – has declined more modestly from 90.5 per cent of all workers in 2008 to 89.2 per cent of all workers in 2013.

Figure I.1: The Stock of Collective Agreements by Type, 2008 - 2013

Notes: Percentage values indicate the employment coverage share of each respective instrument/all instruments. Figures in parentheses indicate the number of each respective instrument/all instruments. Figures at base of each column in brackets above year indicate the total numbers of workers covered in thousands.*Sources: Quadros de Pessoal*, 2008-2009; *Relatório Único*, 2010-2013.

We supplement the material in Table 5 with information on the component instruments in Figure 1 which also gives information on the total number of workers affected. By way of clarification, the category referred to as 'other' picks up the other non-bargained instruments such as Regulations of Working Conditions and arbitration where the Ministry of Employment, or an independent third party, regulates the sector directly and not just orders that simply enlarge the franchise of a bargained instrument. Clearly, the bulk of extension agreements are assigned here to branch agreements or CCTs. The most notable feature of Figure 1 is the broad-based stability in the coverage of the various types of agreements over the sample period. The contrast with the information contained in Table 2 is sharp. That said, the number of covered workers does appear to have fallen by 562,578 or a little over 20 per cent from 2008 to 2013. Although not contesting that there has been a decline – if not the cataclysmic fall suggested by an incorrect reading of Table 2 – we would caution that some of decline has to do with the changeover from the *Quadros de Pessoal* to the *Relatório Único* in 2010.

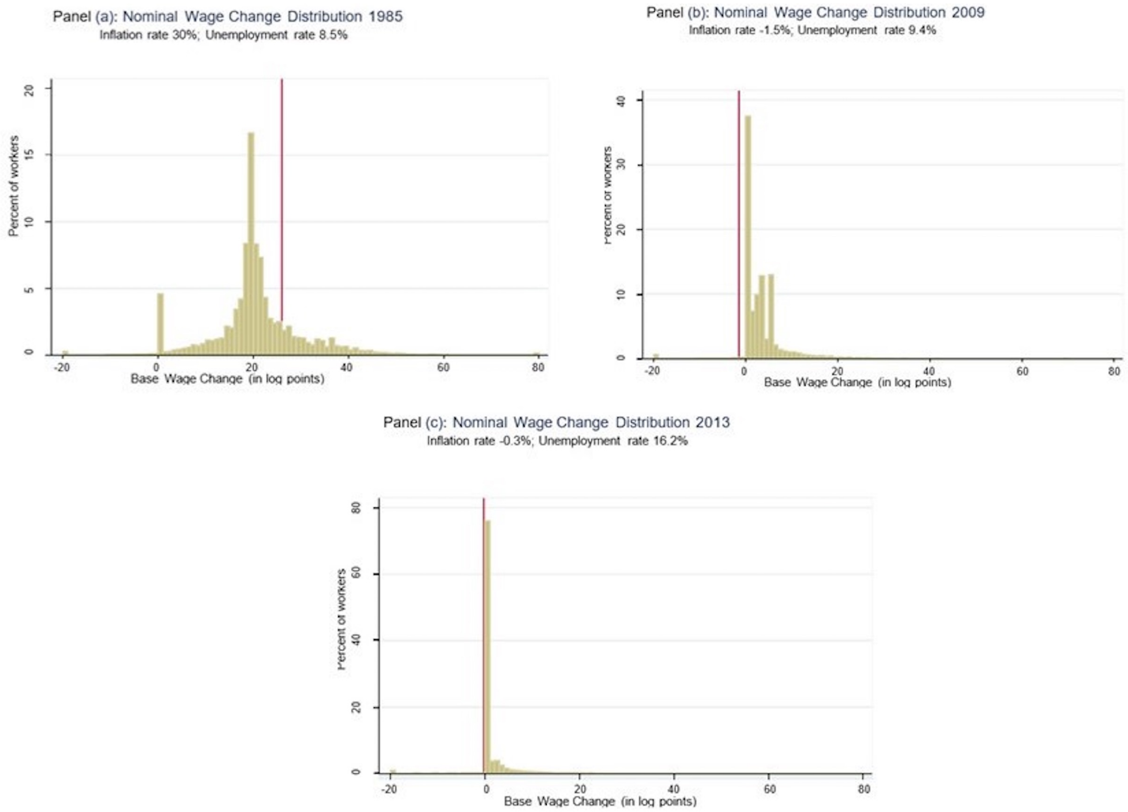
The bottom line with respect to coverage is that reports of the death of Portuguese collective bargaining have greatly been exaggerated by outside observers. This interpretation is underscored by the subsequent dilution of the restrictions on extension agreements, noted earlier, and an uptick in their number from 13 in 2014 to 36 in the first 10 months of 2015. We next proceed to offer a different explanation for the decline in new agreements and extension ordinances.

7 Collective Bargaining Coverage once More (or Downward Nominal Wage Rigidity in High and Low Inflation Regimes)

The notable feature of wage setting in recent years, and in particular at the end of our sample period, is the evidence of extreme nominal wage rigidity. This outcome is the result of a conflation of severe economic contraction and a low rate of inflation. In these circumstances, a revealing exercise is to contrast the nominal wage change distribution in 2013 with that of 1985 when another interval of economic recession was accompanied by high inflation. We will also identify another recession year – 2009 – that was accompanied by lower inflation to establish whether there is anything anomalous about 2013. The respective wage change distributions (of job stayers) for 1985, 2009, and 2013 are provided in panels (a), (b), and (c) of Figure 2. In each case the inflation rate is given by the solid vertical line.[14]

[14]We focus here on the base wage since this measure is more closely related to the theoretical notion of a negotiated wage rate. Furthermore, it is less subjected to measurement error than other components of

Figure I.2: Downward Nominal Wage Rigidity in High and Low Inflation Regimes



Sources: Quadros de Pessoal, 2008-2009; *Relatório Único*, 2010-2013.

Beginning with panel (a) of the figure, it can be seen that a tiny proportion (1.4%) of workers faced nominal wage cuts and a somewhat larger share (4.4%) of workers experienced a wage freeze. Although a large majority of workers had nominal wage increases (94.3%), only 18.4 per cent of workers enjoyed real wage increases; the latter being those individuals located in the wage change distribution to the right of the inflation rate (30%) identified by the vertical line. The share of workers located to the right of the vertical line at zero per cent wage change and to the left of the vertical line corresponding to 30 per cent wage change/inflation of 74 per cent, that is, the fraction of worker with nominal wage increases and real wage decreases, provides a rough indication of real wage contractual flexibility facilitated by a high inflation rate. On net, the average decline in real wages was 5.7 per cent in 1985 which was contemporaneous with the stagnation in GDP (0% growth). Concurrently, the unemployment rate was stable at 8.5 per cent.

The drama of *contemporary* wage adjustment is illustrated in panel (c) of the figure. The wage change distribution nearly collapses at zero per cent nominal wage change. Fully 74.5 per cent of workers had nominal wage freezes – an outcome without parallel in other

labour remuneration.

developed nations – and 21.4 per cent nominal wage gains. Only 3.6 per cent of workers experienced real wage cuts: because the inflation rate was negative, even the small share of workers with nominal wage cuts (4.1 per cent) modestly exceeded the proportion with real wage cuts (3.6%). Overall, real wages rose by 1.2 per cent on average in 2013, while GDP declined by 2.7 per cent. Meantime, the unemployment rate increased somewhat from 15.5 per cent to 16.2 per cent.

There is no indication that these contemporary developments for 2013 are anomalous in low inflation (actually, deflationary) regimes. Panel (b) of Figure 2 illustrates the corresponding situation in 2009 when the inflation rate was -1.5 per cent as compared with -0.3 per cent in 2013. As can be seen, with higher deflation, almost all workers now enjoyed a real wage increase (97.9 per cent), despite the large fraction of nominal wage freezes (36.5%). Nominal wage cuts were experienced by just 2.5 per cent of workers, some of whom nevertheless saw an increase in real wages. The overall increase in real wages was 4.8 per cent. Meantime, unemployment rose from 7.6 per cent to 9.4 per cent, while GDP declined by 3.2 per cent.

In low inflation regimes, the margin for downward real wage adjustment that would not imply nominal wage cuts is highly circumscribed.[15] The problem of real wage adjustment is of course exacerbated in deflationary times since even material cuts in nominal wages may be consistent with real wage increases. In both sets of circumstances, a large fraction of workers may be expected to experience nominal wage freezes, which is what we observe in Portugal. Apart from this perfect storm, as it were, other reasons why nominal wages have been frozen have been discussed earlier. They reflect the mechanisms that generate automatic nominal wages increases, namely a sharp decline in new collective agreements, the legal limits placed on the extension of such agreements, and a freeze on minimum wage hikes. In short, contemporary Portugal is properly characterized by aggravated nominal wage rigidity. The consequences are presumably to be felt in the future. They include job destruction since the incidence of wage freezes is associated with lower hiring rates and more intensive firm closures (see Carneiro et al. (2014)), and pent-up wage deflation (or repressed deflation) because firms that are forced to avoid nominal wage cuts will tend to delay future wage increases.[16]

[15] We might also note the situation in 2012 when inflation was a modest 2.1 per cent and hence closer to the 2013 (and 2009) situation than that of 1985. Here, roughly the same share of workers experienced wage freezes as in 2013 (76%) although fully 86 per cent of workers now experienced real wage decreases, much as in 1985. This was an interval of rapidly rising unemployment (up from 12.7% in 2011 to 15.5% in 2012) and sharply lower GDP (-3%).

[16] According to Nunes (2016), in 2013 the level of wage freezes in the Portuguese labour market prevented real wages from declining by 6 to 7 per cent.

It could be argued that downward nominal wage rigidity may act as a mechanism that, during an economic recession, would smooth income and consumption fluctuations and thereby attenuating the impact of the slowdown on the economy. In the absence of a structural macroeconomic model, however, it is not possible to properly disentangle this stabilization effect from the labour demand effect mentioned above. Be as it may, the dramatic decline of employment during the Portuguese adjustment program seems to suggest that the wage rigidity channel played a very significant role driving job destruction.

8 Conclusions

This inquiry has uncovered what to many will be some surprising facts about collective bargaining in Portugal. This is largely the result of our being able to use a new dataset that contains reliable data on union membership in the private sector. We provide not merely the first accurate estimates of union density in Portugal, 2010-2012, but also demonstrate evidence of a sizeable density-related union premium in an industrial relations regime of near-universal coverage. As with other such estimates, however, causality remains an issue because unions may locate in firms with more generous compensation policies or that are more 'permeable' to union wage demands.

Finally, using the same dataset, despite an unambiguous shift in bargaining momentum that has led to far fewer collective agreements and extensions in the wake of the economic crisis, we report that coverage by collective agreement is largely unaffected once one accounts for the stock of existing contracts. Although the bargaining milieu has changed, we argue that this is best seen as a consequence of a low inflation regime in conjunction with a severe economic downturn. Whereas in the past the wage setting system was largely driven by what has been termed 'upward nominal wage rigidity', the present environment is one in which downward nominal wage rigidity has become truly binding.

In short, we have yet to observe a sea-change in Portuguese collective bargaining practice. This is of course not to deny that since the 1990s and during the years preceding the economic crisis there have been changes in the law and industrial relations in Portugal, or that further changes have been made since then in response to the euro crisis with the implementation of the Memorandum of Understanding. Indeed, we have addressed number of these changes. Rather, our point would be that the effect of these changes have been exaggerated by certain influential observers in their diagnosis of the deregulation in the institutional framework. We refer here in particular to the claim that restrictions on the life of expired collective agreements and the extension principle *have involved a significant reduction in the number of workers benefiting from and covered by collective agreements*. This assertion is to confuse changes in flows with changes in stocks. That said, the decreases in employment that are implied by maintenance of the status quo may be expected increasingly to impact union density, in which circumstance free riding may become more of an issue to unions than heretofore, leading them to be less supportive of the extension mechanism and to focus instead on those groups for whom the unemployment risk is already attenuated. Assuming that unions are able to push for higher wages, therefore, serious declines in coverage may ultimately result.

In the interstices, the main lesson to be carried over from the Portuguese experience charted here to other European economies is that low inflation regimes are likely to lead not only to less frequent wage changes but also to inadequate real wage adjustments if the latter imply nominal wage cuts. Although observers have pointed to a pattern of 'unilateral state interference' in these circumstances (Molina (2014)), a more positive approach might be a policy of fiscal devaluation. That is, a reduction in social security contributions financed through higher consumption taxes may provide a practical way of adjusting labour costs in the short run without a continued haemorrhaging of employment.

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Chapter II

Union Membership Density and Wages: The Role of Worker, Firm, and Job-Title Heterogeneity

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Abstract

We examine the association between union density and wages in Portugal where just 10 percent of all workers are union members but nine-tenths of them are covered by collective agreements. Using a unique dataset on workers, firms, and collective bargaining agreements, we examine the union density wage gap in total monthly wages and its sources - namely, worker, firm, and job-title or 'occupational' heterogeneity - using the Gelbach decomposition. The most important source of the mark-up associated with union density is the firm fixed effect, reflecting the differing wage policies of more and less unionized workplaces, which explains two-thirds of the wage gap. Next in importance is the job-title fixed effect, capturing occupational heterogeneity across industries. It makes up one-third of the gap, the inference being that the unobserved skills of workers contribute at most only trivially to the union density wage gap. In a separate analysis based on disaggregations of the total wage, it is also found that employers can in part offset the impact of the bargaining power of unions on wages through firm-specific wage arrangements in the form of the wage cushion. Finally, union density is shown to be associated with a modest reduction in wage inequality as the union wage gap is highest among low-wage workers. This result is driven by the job-title fixed effect, low-wage workers benefiting more from being placed in higher paying 'occupations.'

1 Introduction

In this chapter we examine the association between union density and wages in Portugal and the role played by worker, firm, and job-title heterogeneity in that relationship. Both are interesting if challenging questions. Thus, for example, in continental Europe, the architecture of wage bargaining systems typically leads to bargaining coverage rates that exceed union density rates, and often by a considerable margin. The pervasive presence of extension mechanisms that generalize union wage agreements throughout industry is responsible for this asymmetry. One might easily be led to conclude in these circumstances that unions have an indistinguishable effect on wages across firms. Vulgo: how can a union premium arise when all workers are covered?

Clearly union membership has a different meaning in continental Europe than in the United States. In the first place, the union contract does not determine the wages of covered workers, but rather the wage floor for specific job titles. Actual wages may differ from bargained wages. Moreover, being a union member in Europe is less likely to correspond to a discrete event associated with strong union power via a (successful) union representation election at the firm, as is the case for the United States. That said, union density rates at the firm or contractual level may still convey relevant information regarding the strength of unions, union power elevating the worker share of the firm's economic rents in all theoretical models of collective bargaining. Indeed, union density may even prove a more adequate measure of the degree of dispersion in union power than in the United States. The case for measuring union power is further underscored by the existence different layers of wage negotiations in continental European nations.

With respect to what lies behind the union wage gap, a second interesting feature of the chapter is that it quantifies the importance of sorting in unionized labor markets. In particular, we examine the allocation of heterogeneous union workers across firms with heterogeneous wage policies and granular job-titles that are differently remunerated. That is, we seek answers to the following questions. Is union density systematically related to heterogeneous firm wage policies? Do we consistently observe more higher-paying job-titles (or more generous promotion policies) in more highly unionized firms? Do unionized workers possess unobserved characteristics that make them more productive? While clearly very much in the spirit of the contemporary interest in the process of allocation of heterogeneous workers to firms, this examination of the manner in which union members are assigned to firms and detailed 'occupations' has to our knowledge never before been examined in the union literature.

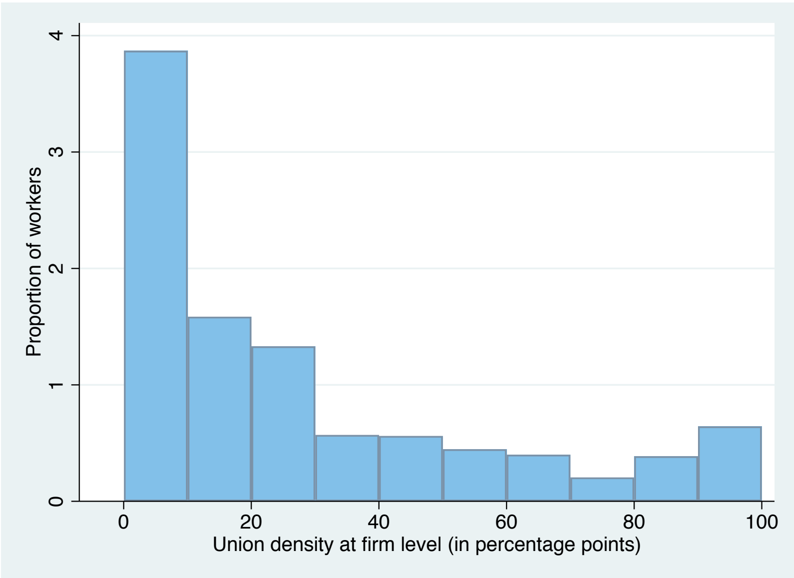
The innovations contained in the chapter are as follows. Having obtained conventional linear regression estimates of the union wage gap, we then decompose the union density effect for total monthly wages using a high-dimensional fixed effects wage regression model. Specifically, using the Gelbach (2016) procedure, we exploit the omitted variable bias formula to decompose the union density rate regression coefficient estimate into its worker heterogeneity, firm heterogeneity, and job-title heterogeneity components. We extend the procedure by replacing the single union density coefficient estimate by a smooth nonlinear function characterization of the union effect. We then seek to peer more closely inside the mechanisms of wage setting, distinguishing between the bargained wage determined at sectoral level and the wage cushion, where the latter is defined as the difference between the bargained wage and total wages. The wage cushion is made up of the payment at firm level of a base wage above the bargained wage (which is widely held to offset the impact of union power on the latter) and various supplements received by the worker in the form of overtime/shift pay and fringes not related to working time. Finally, to the best of our knowledge, we estimate for the first time a three-way high-dimensional quantile regression model, using the Machado and Santos Silva (2018) method of moments estimator to address the impact of union power on wage inequality and, no less important, its component sources.

Our main findings with respect to total wages are fivefold. First, the union wage gap is around 16 log points on average. Second, the effect of union density is not linear. Some critical mass of union density - around 30 percent - is required to have any material influence on wages, while beyond some level (approximating 70 percent) further increases in union density detract from the peak premium. Third, the union wage gap would change profoundly were workers randomly assigned to firms and job titles; that is, firm heterogeneity accounts for two-thirds of the gap. Job-title heterogeneity, whereby unionized workers are assigned to higher paying job titles in the compensation tables of collective agreements, accounts for the remaining one third, so that compensation for their unobserved ability plays a negligible role in determining the premium enjoyed by union workers. Fourth, the impact of union density on wage dispersion though modest is inequality reducing. Thus, workers in the 10th percentile record a higher wage gap than their counterparts in the 90th percentile (17.2 and 15.0 log points, respectively). After taking into account sorting and worker heterogeneity, it emerges from our extension of the Gelbach procedure that the wage compressing effect of union density mainly stems from low-wage workers benefiting more from being placed (or promoted) into higher paying job titles than high-wage workers. Finally, returning to the overall picture, we conclude with all necessary caution that the union wage gap is likely to be between 6 and 16 log points. We base this interpretation on the notion that job-title fixed effects largely capture the effect of trade unions. A postscript from our work on the wage cushion is that in sectors where unions are weaker, relatively low bargained wages offer scope for compensatory adjustment at firm level.

2 The Bargaining Framework

Collective bargaining in Portugal occurs mainly at the sectoral or industry level, where wage ordinances only set wage floors. Further, although in a strict sense these union negotiated wage agreements only formally bind the members of the employer's association and the affiliated workers of the signatory trade union(s), there is in practice a widespread and well anticipated practice of legal extensions that dramatically broaden the incidence of the agreements. The upshot of this near automatic procedure is that every firm within the sector in which the agreement is extended, and every worker within those firms, are no less covered than the original signatories themselves.[1]

Figure II.1: The Distribution of Union Density Conditional on Positive Union Density



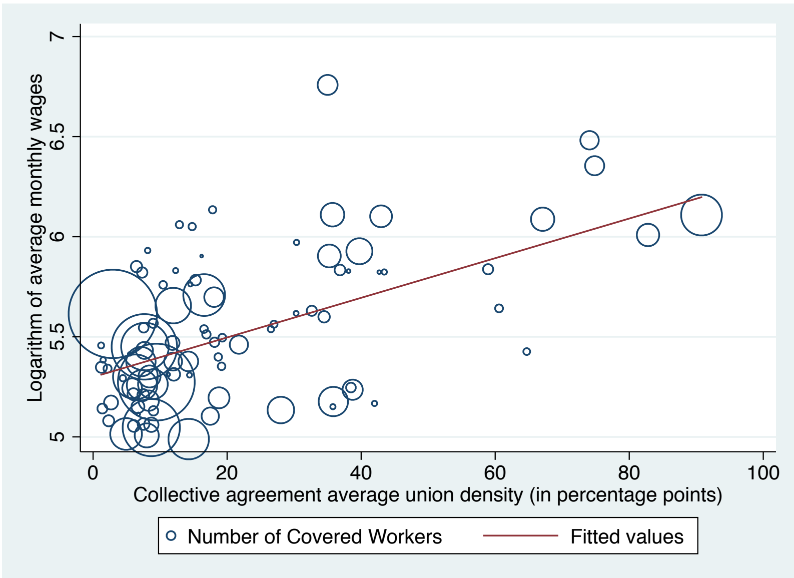
Source: Relatório Único, 2010-2013.

This decoupling of coverage and membership, both within and between firms, translates into levels of collective bargaining coverage in the order of 90 percent of workers at a time

[1]Individual companies and one or more unions may sign firm-level agreements, known as *Acordos de Empresa* (or AEs), which represent a better-paying alternative to the nearest-neighbor sectoral agreement. Their prevalence is limited to less than 5 percent of the private sector workforce, and their existence is mostly related to idiosyncrasies in the firms' shareholder structures. A similarly modest proportion of workers is covered by agreements established between a group of firms and one or more unions under *Acordos Colectivos de Trabalho* (or ACTs).

when union density is around 10 percent. That is, around 80 percent of the labor force have benefited from collective agreements without being members of a trade union. It further explains the finding that around 62 percent of workers are in firms with zero union density. And among those in firms with positive union density, its distribution is highly skewed toward low values. As shown in the histogram of Figure 1, close to two-fifths of these observations record less than 10 percent density.[2]

Figure II.2: Collective Agreement Size, Average Total Wage at Collective Agreement Level, and Average Union Density at Firm Level



Source: Relatório Único, 2010-2013.

As a result, while individual membership of unions and employers' associations becomes inconsequential for collective bargaining agreement coverage purposes, it does not follow that membership is inconsequential. It is unlikely that a union representing 2 percent of the workers in an industry will have the same success as a union representing 40 percent of the workers, even if both contracts record 100 percent coverage. In other words, heterogeneity in union power (as proxied by union density) should after all be reflected in wage differentials. This is indeed the indication of Figure 2 showing that union density rates comove with wages. In summary, union membership in continental Europe is best seen as a continuous process measuring union power rather than a discrete event always identifying high union densities.

In a more granular setting, the strength of unions can be expressed in negotiations con-

[2]The average union density rates changed from 10.3 percent in 2010, to 9.7 percent in 2011 and 2012, and to 9.9 percent in 2013. During the 2010-2013 period, the between component of the union density variance is 0.0416 and for the within component is 0.0057. More generally, for analysis of the effect of union decline on wage inequality see inter al., DiNardo et al. (1996), Card et al. (2004), and Farber et al. (2021).

Table II.1: The Hospitality Sector Collective Agreement, 2017

Rank	Job Description	Minimum Monthly Base Wage (in Euros)
13	General Manager	1,515
12	Board Assistant; Commercial Manager; Service Manager; Human Resources Manager; Technical Manager	1,240
11	Head of Department; Head of Division; Head of Services; Nutrition Technician 1st Class	1,018
10	Head of Section (office); Head of Sales; Inspector; Board Secretariat officer; Nutrition Technician 2nd Class	898
9	Administrative; Head of Cafeteria; Head of Purchases; Head of Kitchen; Head of Pastries; Head of Storage; Head of Dinning Room; Inspector of Sales	808
8	Cashier; Head of Preparation Room; Controller; Cook of 1st Class; Sub-Head of Dinning Room; Administrative Assistant; Pastry Cook; Sales Technician	771
7	Driver of Heavy Vehicles; Storage Keeper; Polyvalent Worker	716
6	Driver of Non-heavy Vehicles; Administrative Assistant 2nd Class; Pastry Cook 2nd Class; Sub-Head of Dinning Room 2nd Class; Sales Representative	700
5	Cook 2nd Class; Controller of Balcony; Controller of Bar; Controller of Storage and packing; Admin. Assistant 3rd Class	629
4	Head of Copa; Cook of 3rd Class; Packing Worker; Storage Worker	582.50
3	Controller Cashier; Storage Worker; Bar Worker; Balcony Worker 1st Class; Distribution Handler;	570
2	Balcony Worker 2nd Class; Admin. Intern; Hospitality Assistant	562
1	Driver Assistant; Distribution Assistant; Barman Intern (1 year); Cook Intern (1 year); Pastry Intern (1 year); Cleaning Worker; Dining room Employer	557

Notes: Table extracted from the Sectoral agreement of April 22, 2017, between *AHRESP* (the Association (of Employers) in Hotels, Restaurants and Similar) and *SITESE* (the Union of Workers and Technicians of Services, Commerce and Hospitality). *Source: Boletim do Trabalho e Emprego, 2017.*

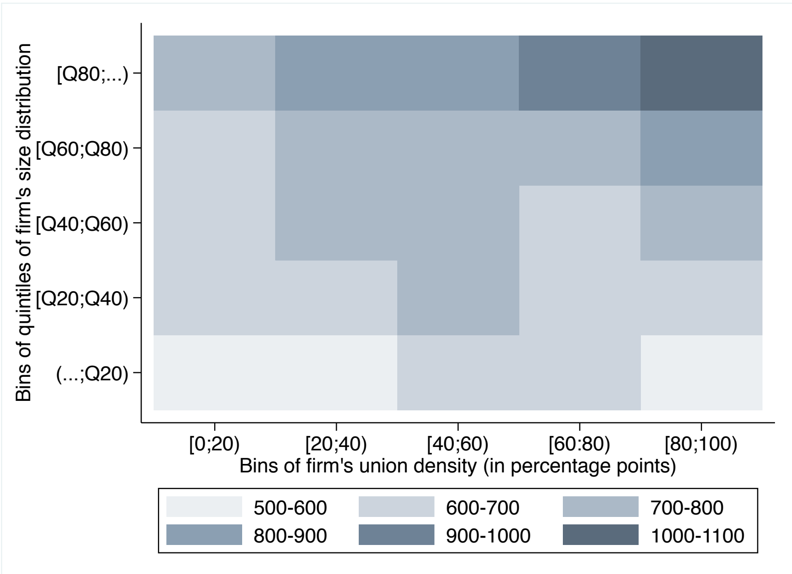
ducted at the industry level mainly through the determination of industry wide wage floors, or bargained wages, for specified job titles ("categorias profissionais"); the definition of job titles themselves over which wage floors are negotiated; and the promotion rules that automatically place workers into higher job titles. The partitioning of the workforce into these occupational categories, whose specificity is sufficient to justify bargaining over, takes into consideration the complexity of the tasks involved, the hierarchical standing of the worker, and the demands of the associated working conditions. In a typical year, the firmament of 300 sectoral or industry level agreements, each containing fully specified tables of job titles, constitutes the core outcome of collective bargaining.[3] Some 30,000 dif-

[3] For a detailed review of recent developments in collective bargaining and extension arrangements in

ferent job titles/bargained wages are prescribed market wide. Table 1 illustrates by means of a representative agreement with its associated complement of job titles and bargained wages.[4]

While exhaustive, these sectoral wage tables neither fully determine wages nor completely define their application to individual workers. At firm level, the power of trade unions may be manifested in higher compliance with collective wage agreements; in negotiated wages exceeding those agreed at the industry level; in more favorable promotion rules; and by new or more generous productivity or tenure related premia. Consequently, firms frequently pay more than bargained wages, particularly larger and more unionized firms. In sum, wage negotiations at firm level are expected to generate wage gains above and beyond those obtained at industry level. Indeed, this is the suggestion of Figure 3 which shows that firm level union density rates impact differently in firms of different sizes. More concretely, union wage effects are higher among larger firms.

Figure II.3: Heatmap of Union Density, Firm Size, and Average Real Total Wages (in Euros) at Firm Level



Source: Relatório Único, 2010-2013.

3 The Datasets

The data sources used in this exercise are the *Quadros de Pessoal* (Personnel Tables), from its inception in 1986 until 2009, and the successor *Relatório Único* (Single Report) from

Portugal, see Hijzen and Martins (2016), Addison et al. (2017), and Card and Cardoso (2021).

[4]Appendix Table 1 provides some details on the largest 15 collective agreements, including the number of job titles specified per agreement and the average union density of the constituent firms.

2010-2013.[5] For our purposes, each longitudinal matched employer-employee-contract-job title database is identical other than in one main respect: the follow-up survey contains data on the union density of the firm that for the first time permit accurate estimates of union density to be obtained. We will therefore not distinguish between the two other than to address the major, albeit temporary, innovation in the successor survey, and to note that while the union wage gap can only be computed using the *Relatório Único*, both surveys are instrumental in computing the conditional decomposition of estimates of the union wage gap in to its component firm, worker, and job-title fixed effects.

The surveys are mandatory in nature and are administered by the Ministry of Employment and Social Security on an annual basis for all establishments with at least one wage earner. All workers employed by the firm in the reference month (March of each year until 1993, October thereafter) are reported, although civil servants and workers in domestic service are not covered while the coverage of agriculture is necessarily uneven because of the importance of the informal sector/low share of wage earners in this sector. In short, the entire population of private-sector firms in manufacturing and services with wage earners is covered. Further, by virtue of their mandatory nature, the high response rate in the surveys ensures that problems commonly associated with panel data such as panel attrition are much attenuated. Confidence in the coverage and reliability of the surveys is further underscored by the requirement that the data be made publicly available at the place of work.

The databases report the location, industry, employment, sales, ownership, and legal basis of the establishment/firm. Worker information includes gender, age, skill, broad occupation, schooling completed, starting date with the firm, earnings, and working hours. In addition, the surveys also record the collective bargaining arrangement and the specific job title held by the worker defined in the collective agreement.[6] As noted earlier, this job classification variable goes beyond a fine definition of occupations to encompass the complexity of the task performed, the skill level of the job, the required labor market experience, and the hierarchical standing of the worker. The wage variable records the worker's gross monthly earnings (the actual or total wage as of the survey month), which sum is split into the following four components: the base wage (i.e. the gross pay for

[5]For the years 1990 and 2001 the *Quadros de Pessoal* survey was not administered. Although the union question is ongoing, data on union density in the *Relatório Único* have not been made available to researchers since 2013.

[6]Those workers not covered by any collective agreement are coded as such (i.e. "non-covered workers"). Such workers often have higher paying individual contracts. A significant portion of them are in the IT industries. In Appendix Table 2 it is shown that the inclusion of such workers does not impact significantly the regression coefficient estimate for the union density in the base model. Estimation of the full model is, of course, not possible for "non-covered workers."

normal hours of work), overtime pay, and regularly and irregularly paid supplements. Normal monthly hours worked and overtime hours are also reported.

The following restrictions were placed on the data. First, the analysis was confined to full-time employed workers in receipt of what was contractually defined for the survey reporting month. Second, workers from the agriculture, fisheries, and energy products/extraction sectors were excluded. Third, workers aged less than 18 years and greater than 65 years were excised, as also were those whose monthly wages were less than 80 percent of the mandatory minimum wage, corresponding to the lowest admissible wage for apprentices. Finally, observations not belonging to the largest connected group were dropped, amounting to some 1 percent of the total number of observations.[7]

This brings us to the distinguishing feature of the successor survey - the *Relatório Único* - namely that it allows us to construct a measure of union density at firm level. Specifically, the survey asks of the manager respondent: *Indicate the number of workers for whom you have knowledge of their membership in a union (because they are union officials, because you deduct membership dues from their salary, or because the worker informed you about his/her membership so as to determine which particular collective regulation is applicable to their case).* The sum total of such workers - whose personal union status is unknown, thereby precluding use of an individual union membership variable - divided by the number of workers employed by the firm provides our measure of union density.

Overall, the joint dataset includes 36,616,379 observations of worker-year pairs, of which 6,218,777 are from the *Relatório Único*. The joint dataset has a basis in the records of 6,042,315 workers matched by identifying social security number, 652,487 firms matched by identifying number, and 132,908 job-titles matched by the code of the collective agreement occupational category. Appendix Table 3 provides the descriptive statistics for observations from both the *Quadros de Pessoal* and the *Relatório Único*.

4 Modelling

We next describe the procedures used: firstly, to estimate the union wage gap; and, secondly, to account for the component contributions of firm compensation policies, worker ability, and detailed occupational premiums via the estimation of firm, worker, and job-

[7]A connected group links the job title and the firm to the rest of the group such that all the fixed effects are connected. Restricting the analysis to this subset of the data ensures that the estimates of the fixed effects are comparable (see Guimarães and Portugal (2010)).

title fixed effects, respectively.

4.1 Estimation of the Union Wage Gap

We begin with a standard Mincerian wage equation, augmented to include union density, as follows:

$$w_{it} = \boldsymbol{x'_{it}}\boldsymbol{\beta_0} + \delta_{0_t} + \gamma_0 U_{F(i,t)} + \epsilon_{0_{it}}, \tag{II.1}$$

where w_{it} is the natural logarithm of worker i monthly compensation at year t, $\boldsymbol{x'_{it}}$ is a vector of observed characteristics of the worker and his/her employer, $\boldsymbol{\beta_0}$ is a vector of coefficients for the observed characteristics of workers and firms, $U_{F(i,t)}$ is the level of union density of employer F in year t, γ_0 is the coefficient associated with the level of union density, δ_{0_t} are calendar year fixed effects included to capture the macroeconomic environment (business cycle), and $\epsilon_{0_{it}}$ is an error term, assumed to be uncorrelated with the covariates. The explanatory variables (or observed characteristics) of workers and firms are age, age squared, seniority, seniority squared, and dummies for gender, education, firm size, and industry.[8]

To allow for a non-proportional impact of union density on wages we will consider both parametric and non-parametric approaches. The parametric version employs a third-degree polynomial in union density:

$$w_{it} = \boldsymbol{x'_{it}}\boldsymbol{\beta_0} + \delta_{0_t} + \gamma_{0_1} U_{F(i,t)} + \gamma_{0_2} U^2_{F(i,t)} + \gamma_{0_3} U^3_{F(i,t)} + \epsilon_{0_{it}}. \tag{II.2}$$

In the non-parametric version, the impact of union density is captured by the presence of fixed effects $\psi_{0u_{F(i,t)}}$ (one for each different level of union density U):

$$w_{it} = \boldsymbol{x'_{it}}\boldsymbol{\beta_0} + \delta_{0_t} + \psi_{0u_{F(i,t)}} + \epsilon_{0_{it}}. \tag{II.3}$$

Information contained in the union fixed effects while necessarily complete is rather noisy and "staccato." Thus, in a second step, we will estimate a kernel regression linking the estimates of the union density fixed effects and actual union density at firm level, as

[8] The subscript **0** denotes the base regression model specification.

follows:

$$\widehat{\psi_u} = K(U) + \upsilon_u, \tag{II.4}$$

where $\widehat{\psi_u}$ is the union density fixed effect estimate obtained from the first step, U is the prevailing union density of the firm, υ_u is the disturbance term, and K is a standard Epanechnikov kernel function (Silverman (1986)).

The estimation of local weighted union wage gaps, as well as the third-degree polynomial specification, result in smoothed estimates of a union wage gap curve in actual earnings. To facilitate interpretation of the results, a convenient normalization in the nonparametric case requires that the fixed effect in the absence of workplace unionism be set equal to zero. No further restrictions are implied by this assumption as the union wage gap represents the relative difference in wages for workers at firms with different levels of union density, controlling for the observed characteristics of workers and firms.

4.2 Estimation of the Sources of the Union Wage Gap

Given the estimate of the union wage gap it is useful to decompose this outcome measure into its constituent mechanisms; that is, to identify the contributions of worker, firm, and job-title time-invariant heterogeneity. To this end, we adapt the conditional decomposition of Gelbach (2016).

For the full-specification model, we include in equation (3) the sources of time-invariant heterogeneity, namely the worker fixed effect (α_{1_i}), the firm fixed effect ($\lambda_{1_{F(i,t)}}$), and the job-title fixed effect ($\theta_{1_{J(i,t)}}$), exploiting the methodology first introduced in Carneiro et al. (2012). The model thus becomes:[9]

$$w_{it} = \boldsymbol{x'_{it}\beta_1} + \delta_{1_t} + \gamma_1 U_{F(i,t)} + \alpha_{1i} + \lambda_{1_{F(i,t)}} + \theta_{1_{J(i,t)}} + \epsilon_{1_{it}}. \tag{II.5}$$

In general the identification of the worker, firm, and job title fixed effects is assured by the restriction that the sample identifies the largest connected set. A connected set is defined when at least one element of a worker, firm, and job title combination links the rest of the group (Abowd et al. (1999)). The largest connected group represents more than 99

[9] The subscript **1** denotes the full model specification.

percent of the sample.

At this stage, we calculate the independent contribution of each fixed effect to the union wage gap. For this purpose we adapt the methodology developed in Gelbach (2016), which appeals to the omitted variables bias formula to compute a detailed decomposition. Departing from a baseline specification to which covariates are added, Gelbach's procedure allows us to compute the contribution of each new covariate to the change in the estimate of the coefficient of the variable under scrutiny. In our case, it allows us to unambiguously disentangle the contribution of each excluded variable (each fixed effect) to the variation of the coefficient estimate(s) of the union density variable(s).

To better understand our decomposition exercise it is useful to present the benchmark wage regression equation, emphasising the union effects, in the following matrix formulation:

$$\mathbf{W} = \mathbf{X}\boldsymbol{\beta_0} + \mathbf{U}\boldsymbol{\gamma_0} + \boldsymbol{\epsilon_0}, \tag{II.6}$$

where $\mathbf{W}$ stands for vector of wages, $\mathbf{X}$ denotes the matrix of control variables, including the year dummies, $\boldsymbol{\beta_0}$ is a vector of regression coefficients, $\mathbf{U}$ collects the union density variable(s), $\boldsymbol{\gamma_0}$ represents the union wage gap, and $\boldsymbol{\epsilon}$ is the vector containing the error terms.

Making use of the Frisch-Waugh-Lovell theorem, we can express the OLS estimate of $\boldsymbol{\gamma_0}$ by running a regression of $\boldsymbol{W}$ on $\mathbf{U}$ after partialing out the effect of $\mathbf{X}$ on both variables. That is,

$$\widehat{\gamma_0} = (\mathbf{U}'\mathbf{M_X}\mathbf{U})^{-1}\mathbf{U}'\mathbf{M_X}\mathbf{W}, \tag{II.7}$$

where, $\mathbf{M_X} = I - X(X'X)^{-1}X'$ is the residual-maker, or "annihilator" matrix.

More compactly, we can write:

$$\widehat{\gamma_0} = \mathbf{A_X}\mathbf{W}, \tag{II.8}$$

and introduce the definition of the matrix $\mathbf{A_X} = (\mathbf{U}'\mathbf{M_X}\mathbf{U})^{\mathbf{-1}}\mathbf{U}'\mathbf{M_X}$, which will be instrumental in the application of the omitted variable bias formula. In general, if we pre-multiply any variable by $\mathbf{A_X}$ we will always obtain the corresponding regression coefficient estimate of the union density variable, after controlling for the variables included in $\mathbf{X}$.

We now define the the full regression model, where we incorporate the worker effects (identified via the matrix $\mathbf{D}$), the firm effects (identified via $\mathbf{F}$), and the job-title effects (identified via $\mathbf{J}$). The estimated full regression can be now expressed as:

$$\boldsymbol{W} = \mathbf{X}\widehat{\boldsymbol{\beta}_\mathbf{1}} + \mathbf{U}\widehat{\boldsymbol{\gamma}_\mathbf{1}} + \mathbf{D}\widehat{\boldsymbol{\alpha}_\mathbf{1}} + \mathbf{F}\widehat{\boldsymbol{\lambda}_\mathbf{1}} + \mathbf{J}\widehat{\boldsymbol{\theta}_\mathbf{1}} + \widehat{\boldsymbol{\epsilon}_\mathbf{1}}, \tag{II.9}$$

where $\widehat{\boldsymbol{\alpha}_\mathbf{1}}$, $\widehat{\boldsymbol{\lambda}_\mathbf{1}}$, and $\widehat{\boldsymbol{\theta}_\mathbf{1}}$ denote the worker, firm, and job-title fixed effects, respectively.

At this stage, we build on the approach suggested by Gelbach (2016), which makes use of the OLS omitted variable bias formula, to decompose the union wage gap in terms of individual self-selection in unionized firms (the worker component) and sorting across firms with different wage policies and differently remunerated job titles. This can be achieved by multiplying both terms of the full regression by $\mathbf{A_X}$, leading to:

$$\widehat{\boldsymbol{\gamma}_\mathbf{0}} - \widehat{\boldsymbol{\gamma}_\mathbf{1}} = \mathbf{A_X}\mathbf{D}\widehat{\boldsymbol{\alpha}_\mathbf{1}} + \mathbf{A_X}\mathbf{F}\widehat{\boldsymbol{\lambda}_\mathbf{1}} + \mathbf{A_X}\mathbf{J}\widehat{\boldsymbol{\theta}_\mathbf{1}} = \widehat{\boldsymbol{\tau}_{\boldsymbol{\alpha}_\mathbf{1}}} + \widehat{\boldsymbol{\tau}_{\boldsymbol{\lambda}_\mathbf{1}}} + \widehat{\boldsymbol{\tau}_{\boldsymbol{\theta}_\mathbf{1}}} \tag{II.10}$$

as, by construction, $\mathbf{A_X}\widehat{\boldsymbol{\epsilon}_\mathbf{1}} = \mathbf{0}$.

Equation (10) yields an exact, unambiguous and conditional decomposition of the union wage gap. The interpretation of this equation is that we can split the wage gap into three components: a worker component ($\widehat{\boldsymbol{\tau}_{\boldsymbol{\alpha}_\mathbf{1}}}$), a firm component($\widehat{\boldsymbol{\tau}_{\boldsymbol{\lambda}_\mathbf{1}}}$), and a job-title component ($\widehat{\boldsymbol{\tau}_{\boldsymbol{\theta}_\mathbf{1}}}$). In practice, all we need to do is to run a regression for each type of fixed effect on all regressors of the benchmark regression ($\mathbf{X}$ and $\mathbf{U}$) and extract the union regression coefficient estimates.

Mutatis mutandis, we can apply the same principle of the Gelbach decomposition to the union wage gap curve given in equation (2) and to the union fixed effect specification in equation (3). In the latter, the difference between the union density fixed effects of the full and base models can be decomposed into three fixed effects:

$$\widehat{\boldsymbol{\psi}_{\mathbf{0}_u}} - \widehat{\boldsymbol{\psi}_{\mathbf{1}_u}} = \mathbf{A_{X_u}}\mathbf{D}\widehat{\boldsymbol{\alpha}_{\mathbf{1}_u}} + \mathbf{A_{X_u}}\mathbf{F}\widehat{\boldsymbol{\lambda}_{\mathbf{1}_u}} + \mathbf{A_{X_u}}\mathbf{J}\widehat{\boldsymbol{\theta}_{\mathbf{1}_u}} = \widehat{\boldsymbol{\tau}_{\boldsymbol{\alpha}_u}} + \widehat{\boldsymbol{\tau}_{\boldsymbol{\lambda}_u}} + \widehat{\boldsymbol{\tau}_{\boldsymbol{\theta}_u}}, \tag{II.11}$$

where the subscript u is used to emphasize that we are decomposing the changes in the union density fixed effects. In practice, and as before, the decomposition is achieved by estimating three auxiliary regressions in which the worker, firm, and job-title fixed effects become the dependent variables and the regressors match those of equation (3). Then, by smoothing these estimates via a kernel function, we can provide a graphical representation of the components of the union wage gap.

Before turning to our empirical results, however, we should resist the notion that the union density fixed effect is simply to be equated with a firm fixed effect. Even if union density were not to change over time, the union density fixed effect is to be seen as subsumed in the firm fixed effect in the same way that the gender fixed effect is subsumed in the worker fixed effect. Contrary to intuition, this fact does not preempt the decomposition of the union density effect along its worker, firm, and job-title dimensions for the same reason that the gender effect can be disentangled along the worker, firm, and job-title dimensions (Cardoso et al. (2016)).[10] A clear indication that the dominant role of the firm is not mechanically implied by the fact that the union density variable is computed at the firm level will be given by the decomposition exercise below.

5 Main Findings

5.1 The Union Wage Gap Curve for Monthly Wages

In Portugal evidence of sizable wage differentials associated with a firm's degree of unionization is unmistakable, even though nearly every worker benefits from union bargaining. For example, the heuristic distributions of the logarithm of total hourly wages shown in Figure 4 display meaningful differences in both shape and mean when unionized and non-unionized workplaces are considered.[11]

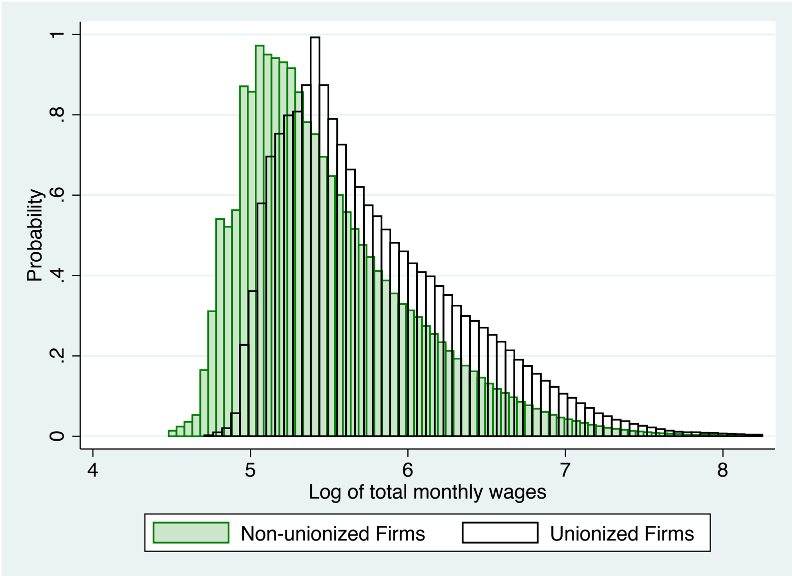
These stylized facts are confirmed by our benchmark results presented in Table 2 which chart the impact of union density on wages after having controlled for the full set of observed worker and firm characteristics, as described in section 4. For the linear specification given in the first column of the table, the estimated union wage gap is 17.6 percent $[(e^{0.1619}-1)\times 100]$; a sizable union wage differential that is either on a par with or exceeds U.S. estimates.[12] This wage gap is to be interpreted in the following way: it represents the

[10] Cardoso et al. (2016) extend the Gelbach procedure and prove that it is applicable even in settings where the omitted variables are fixed effects and the coefficient under scrutiny refers to a variable (viz. gender) that is subsumed in one of the fixed effects.

[11] The values of zero and greater than zero were chosen because of the large majority of covered firms without union members.

[12] See the early studies of Blanchflower and Bryson (2003), and Hirsch (2004); and, especially, the more

Figure II.4: Distribution of Total Monthly Wages by Union Status



Source: Relatório Único, 2010-2013.

wage difference between two observationally identical workers, one of whom is employed in a fully unionized firm and the other in an otherwise identical non-unionized firm.

The preceding methodology implies that the value of the union wage gap for each point in the continuum of union density is determined by and conforms to a linear relationship. However, an important issue is whether the marginal change in the union wage gap is in fact the same when a newly unionized worker joins a union-free workforce as opposed to a situation in which, say, a plurality of workers is already organized. In seeking to estimate a union wage gap without assuming constant marginal effects throughout, we shall follow the two procedures described earlier to estimate the union wage gap curve.

These estimates are shown in Figure 5. Clearly, the linear approach is misleading; in particular, unions need some critical mass (of unionized workers) in order to materially influence wages. Panel A of the figure is based on a third-degree polynomial function and indicates that union density attains statistical significance at around 30 percent, with a maximum wage gap of 17 log points being achieved once union density reaches approximately 70 percent (see also the first three rows of the second column of Table 2). Panel B of the figure, which gives the more flexible kernel smoother, shows that although the polynomial is a sensible parsimonious approximation to the wage gap curve it understates the peak premium (now in excess of 24 log points) and overstates the decline in the premium thereafter.

recent plant-level studies of Frandsen (2012) and Lee and Mas (2012).

Table II.2: Determinants of Union Density in Portugal. Dependent Variable: Number of Unionized Employees at the Firm

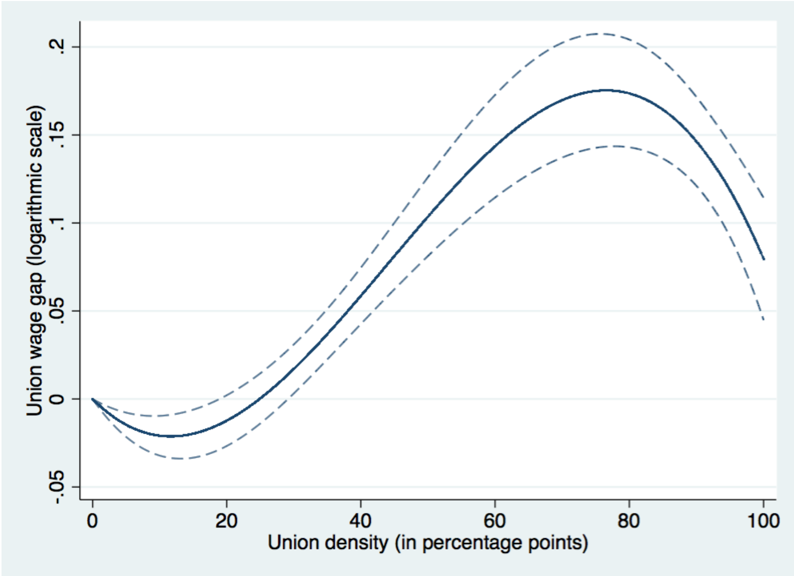
Variable	Linear Specification	Polynomial Specification
Union density	0.1619	−0.3844
	(0.0244)	(0.1474)
Union density squared	-	1.9030
		(0.5533)
Union density cubed	-	−1.4390
		(0.4290)
Worker's age	0.0281	0.0282
	(0.0006)	(0.0006)
Worker's age squared	−0.0002	−0.0002
	(0.0000)	(0.0000)
Tenure of the worker	0.0156	0.0156
	(0.0005)	(0.0005)
Tenure of the worker squared	−0.0002	−0.0002
	(0.0000)	(0.0000)
Female	−0.2109	−0.2108
	(0.0035)	(0.0034)
Primary school	0.0548	0.0550
	(0.0058)	(0.0056)
Basic school	0.1586	0.1581
	(0.0062)	(0.0060)
Elementary school	0.2670	0.2662
	(0.0068)	(0.0065)
Secondary school	0.4269	0.4249
	(0.0091)	(0.0086)
Post-secondary school	0.5569	0.5550
	(0.0140)	(0.0139)
University attendance	0.8143	0.8128
	(0.0117)	(0.0111)
College degree	0.9025	0.8995
	(0.0104)	(0.0099)
Firms with 50 to 99 employees	0.1316	0.1372
	(0.0041)	(0.0042)
Firms with 100 to 499 employees	0.1708	0.1817
	(0.0066)	(0.0069)
Firms with 500 to 999 employees	0.1929	0.2043
	(0.0190)	(0.0190)
Firms with 1000 to 4999 employees	0.1561	0.1700
	(0.0203)	(0.0202)
Firms with more than 5000 employees	0.1171	0.1400
	(0.0478)	(0.0475)
R^2	0.4847	0.4859

Notes: Dependent variable: total monthly wages (in logs). The controls also include 25 sector of activity dummies, and 3 year dummies. The number of observations is 6,218,777. Robust firm clustered standard errors are in parentheses: all coefficients are statistically significant at the 0.01 confidence level. *Source: Relatório Único*, 2010-2013.

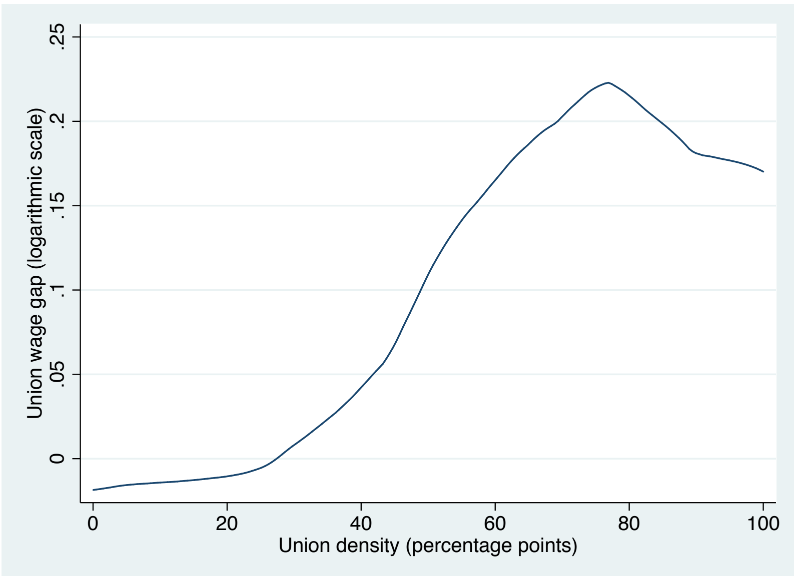
The explanation for the robust evidence on the importance of union density to wage setting has to do with the intensive margin of representation. The mere realization that a bargaining instrument covers a given worker does not seem to shed sufficient light on

Figure II.5: The Union Wage Gap Curve for Total Monthly Wages

Panel A



Panel B



Notes: The base model includes as regressors a quadratic term in age, a quadratic term in tenure, schooling dummies (7), a gender dummy, firm size dummies (5), year dummies (3), and sector dummies (25). Robust clustered firm-year standard errors were used. In Panel A, the 95 percent confidence interval is indicated. *Source: Relatório Único*, 2010-2013.

the properties of an agreement, namely the specific environment in which it was agreed. Moreover, wage setting is not identical for every covered worker, and firms do not have a homogeneous approach to compensation policies, irrespective of union presence, either at firm or sectoral level.

The importance of considering the intensive margin is also implicit in the shape of our union wage gap curves. A plausible explanation for the configuration of the fitted curves relies on the idea that the bargaining power of a union is a function of its ability to credibly threaten the employer through a withdrawal of labor (e.g. Farber (1986)). It is reasonable to assume that unions need some minimum complement of unionized workers to effectively impose costs on the employer in the event of a failure to agree. With a preponderance of the workforce organized, the capability to impose a total shutdown is implied, such that further increases in union density are not to be equated with higher union wage premia.

5.2 The Sources of the Union Wage Gap for Total Monthly Wages

The union wage gap coefficient estimates for total earnings provide an average differential between the wages of two observationally identical workers in two observationally identical firms with distinct levels of unionization. What are the potential sources of this sizable union wage gap? As leading contenders, we next consider the contributions of heterogeneity in the compensation policies of firms, the rules governing how the workforce is assigned to the compensation tables of the collective agreement, and the allocation of workers of different unobserved ability.

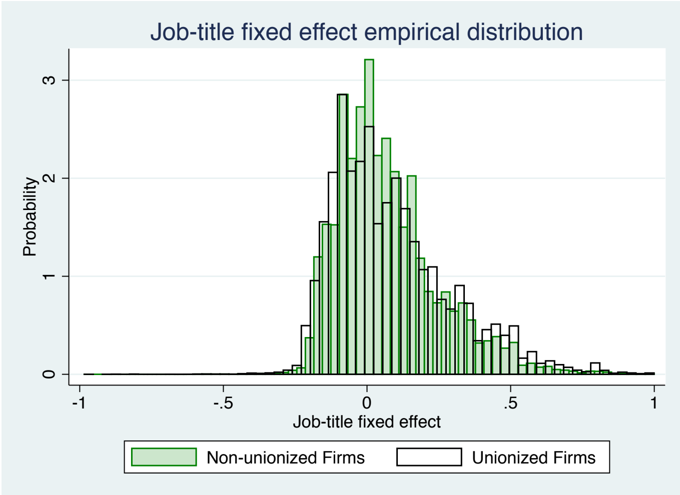
In principle, the conditional influence of unions on earnings compensation can arise from other sources than these. However, to anticipate one of our key findings, we report that after accounting for firm, job-title, and worker fixed effects the portion of the union wage gap remaining to be explained is vestigial. This is the case for both the linear approach and the fitted union wage gap curve. In decomposing the union wage gap, therefore, our focus will be upon the contributions of each of these three sources of unobserved heterogeneity. In what follows, the major difference between the two (decomposition) approaches resides in the flexibility of the estimates, namely the improved estimation offered by the union wage gap curve over the restrictive linear approach.

Job title refers to the worker's assigned role at the firm, as explicitly defined in the collective sectoral agreement governing the employment relationship. This defined "occupation" most importantly determines a floor for the base wage that a worker is legally entitled to receive. Note that the base wage set at sectoral level (which we call the *bargained wage*) does not necessarily equal the actual base wage paid by the firm. Indeed, a majority of firms pay more than the bargained wage. The difference between the base wage and the base wage floor or bargained wage is essentially determined at the discretion of the firm.

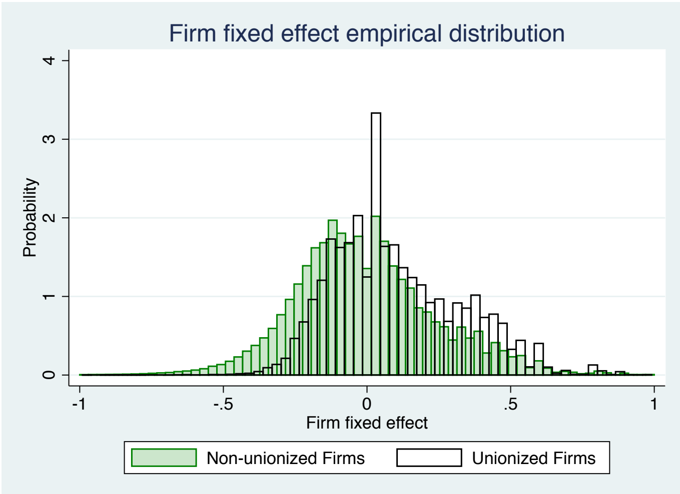
Thus, the job-title fixed effect summarizes the influence of the compensation floor defined for each worker. Each job title is collective agreement specific, such that two workers

Figure II.6: Distribution of Worker, Firm, and Job-title Fixed Effects by Union Status

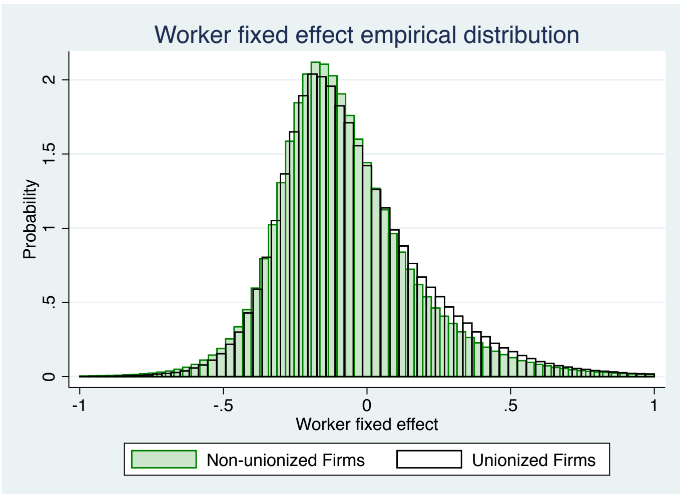
Panel A



Panel B



Panel C



Notes: In addition to the fixed effects, the model includes as regressors a quadratic term in age, a quadratic term in tenure, schooling dummies (7), a gender dummy, firm size dummies (5), and sector dummies (25). *Sources: Quadros de Pessoal* 1986-2009; *Relatório Único*, 2010-2013.

performing the same task and having the same responsibilities covered by different collective bargaining arrangements will be assigned different job titles. Overall, in a given year there are around 30,000 collective agreement/occupational category combinations to which workers are assigned. The inclusion of job-title fixed effects may be viewed as building upon a first generation Mincerian wage equation that recognizes only a broad definition of job descriptions.

Panel A of Figure 6 shows the empirical distribution of the job-title fixed effects, contrasting workers in 'union' firms with their counterparts in 'nonunion' firms. For the former, the distribution of job-title fixed effects is visibly displaced to the right. The implication is that better-paid job titles tend to be more heavily populated by workers of unionized firms (after taking into account firm and worker heterogeneity).

For its part, the firm fixed effect captures the (constant) wage policy of the firm, including the relative standing of the firm's wage tables, after having controlled the placement of workers into the distinct job categories presented in such tables which fully captures the previously discussed job-title fixed effect. Firms with generous compensation policies will exhibit positive firm fixed effects, while firms with compensation policies close to the bargained wage will generate negative fixed effects. In Panel B of Figure 6 we contrast the distribution of the firm fixed effects for workers in union and nonunion firms.[13] Clearly, unionized workers disproportionately populate high-paying firms.

Finally, the empirical distribution of the worker fixed-effects is presented in Panel C of Figure 6. The worker fixed effects capture the influence of the constant characteristics of individuals on their wages. They are essentially a proxy for the portable human capital (or productivity) of the worker. The pattern revealed is one in which more unionized firms seemingly employ relatively more skilled individuals. This outcome can be the result of observed characteristics (such as schooling or gender) or unobserved factors (ability), and we shall subsequently address the specific role of the latter.

Table 3 presents the results of the Gelbach decomposition for the linear specification.[14] The coefficient estimate contained in the first column of the table simply recalls the estimated union wage gap (of 16.2 log points) obtained from equation (1) and reported earlier in Table 2. The estimated union wage gap, after the inclusion of the three high dimensional fixed effects (equation (5)), is no longer statistically significant and is given in the second

[13]Observe, however, that in this comparison the influence of variables such as industry or firm size is still subsumed in the firm fixed effect. The subsequent Gelbach decomposition will enable us to filter out the impact of the firm fixed effect on the wage gap from the variable included in the benchmark specification.

[14]In Appendix Table 4 we present the decomposition results for alternative sets of observable controls.

Table II.3: The Conditional Decomposition of the OLS Estimation of the Union Wage Gap for Total Monthly Wages

Variables	Base Model (γ_0)	Full Model (γ_1)	Gelbach Decomposition
Union Wage Gap	0.1619***	-0.0029	–
	(0.0244)	(0.0086)	
τ_{α_1} (Worker FE)	–	–	-0.0026
			(0.0055)
τ_{θ_1} (Job-title FE)	–	–	0.0571***
			(0.0198)
τ_{λ_1} (Firm FE)	–	–	0.1103***
			(0.0134)
R^2	0.5373	0.8801	

Notes: Decomposition based on Gelbach (2016). Robust firm clustered standard errors are in parentheses: *** denotes statistical significance at the 0.01 level. The base model includes as regressors a quadratic term in age, a quadratic term in tenure, schooling dummies (7), a gender dummy, firm size dummies (5), and sector dummies (25). The number of observations is 36,616,379. *Sources: Quadros de Pessoal* 1986-2009; *Relatório Único*, 2010-2013.

column of the table (-0.3 log points). To attenuate the bias arising from the incidental parameter problem in the estimation of high-dimensional fixed effect regression models, we employ the full sample, 1986-2013. To guarantee that the results are comparable with those provided in Table 2, we interact all the explanatory variables with a time dummy identifying the period before and after 2010.[15]

The third column provides the contribution of each fixed effect to the change in the estimated union wage gap (equation (10)).[16] The differences in firms' compensation policies explain a large fraction of the union wage gap. After accounting for the observable characteristics of the worker and the firm, the constant unobserved characteristics of workers, and the process of job-title placement, the worker compensation policies of firms are responsible for 11 log points of the union wage gap of some 16.2 log points. Put differently, if every worker was faced with a neutral stance of his or her firm regarding its compensation policies, the union wage gap would be reduced by about 68 percent. Next, consistent with

[15] Notice that the use of these interaction terms, in the case of the base model, guarantees that the regression coefficient estimates for the 2010-2013 period are exactly the same as those given in Table 2.

[16] In practice, as presented before, the application of the Gelbach decomposition in the current framework amounts to running three auxiliary OLS regressions identical to equation (1), but where the dependent variable is successively replaced by the corresponding estimated fixed effects. By construction, this decomposition is both exact (being the sum of the contributions corresponding to the difference between the two union wage gap estimates) and unambiguous.

the evidence provided in Panel A of Figure 6, we find that the allocation of workers into job titles - either directly, or indirectly through promotion decisions - contributes 5.7 log points (or another 35 percent) of the estimated union wage gap. Implicitly, therefore, trade unions achieve real success in either creating or in placing their members into higher paying job categories.[17]

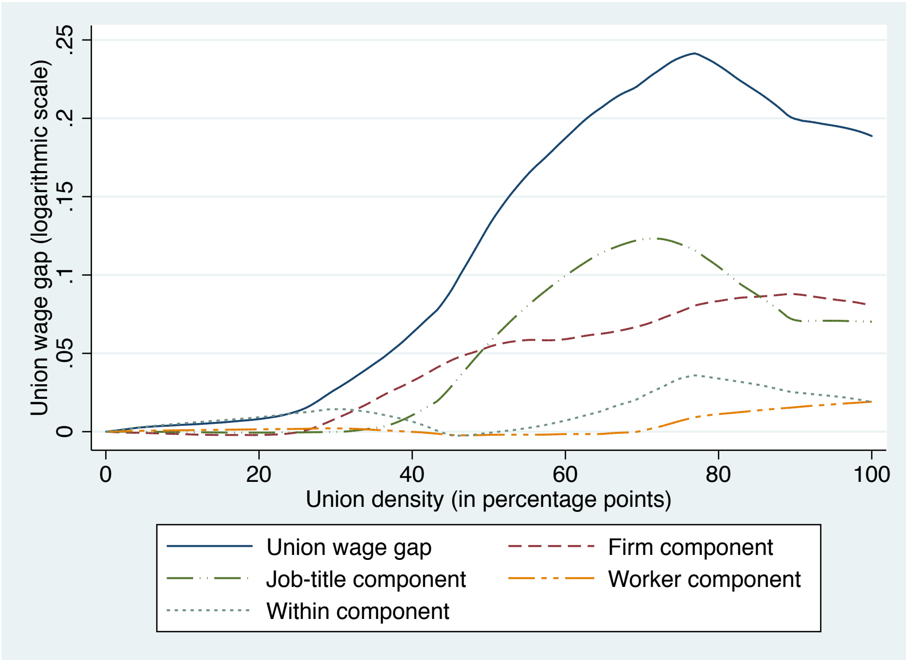
Turning to the worker dimension, and after accounting for observable traits as well as their sorting into firms and job-titles, it can be seen that individuals working in a fully-unionized firm receive compensation for their permanent unobserved characteristics that is estimated to be just 0.3 log points lower than in the case of a non-unionized firm. But to all intents and purposes there are no statistically significant differences between unionized and non-unionized workers in terms of (unobserved) ability.

In the pooled OLS estimation, the sources of identification of the union wage effect have both a cross-section and a time-varying dimension. The cross-section or between-effect arises simply from the comparison of wages of workers with different average union density rates. In the following decomposition exercise, we will explore the three sources of this cross-sectional variation: worker heterogeneity, firm heterogeneity, and job-title heterogeneity. This objective is achieved via the inclusion of their corresponding fixed (hence, cross-sectional) effects. The time-varying effects stem from two obvious sources: workers moving across firms with different union densities, and workers remaining in firms with changing union densities. The inclusion of a worker fixed effect necessarily removes the cross-sectional variation. The inclusion of the firm fixed effect precludes any variation arising from worker movement to other firms. The inclusion of the job-title fixed effect will restrict the source of wage variation to that which is not accounted by job-title moves. Thus, in our full model, identification of the union wage gap comes solely from the wage effects of changing union density rates, net (i.e. after accounting for) of job-title changes. Not surprisingly, therefore, the union wage gap is close to zero in the full model. The advantage of the full model, however, is that it allows us to obtain an unambiguous decomposition of the sources of heterogeneity and, by implication, to measure the relative importance of unobserved skills and sorting across firms and job-titles as drivers of the union wage premium.

Estimates of the Gelbach decomposition of the union wage gap curve obtained from the kernel regression (equations (3) and (11)) are depicted in Figure 7. The figure broadly

[17] In Appendix Table 5 we use the ESCO occupation definition instead of job-title. Not surprisingly, the contribution of occupation fixed effects is much attenuated (1.6 log points), suggesting that the informational content of the job-title fixed effects is more relevant in associating union density with wages.

Figure II.7: The Gelbach Decomposition of the Union Wage Gap Curve for Total Monthly Wages



Sources: Quadros de Pessoal, 1986-2009; *Relatório Único*, 2010-2013.

confirms the principal result of the linear approach, namely the leading roles reserved for job titles and the compensation policies of firms (viz. the job-title and firm fixed effects). The flexibility of this approach indicates that the major source of non-linearity stems from the job-title component. The figure also provides a more informative picture of the sources of the union wage gap. In particular, the assignment of workers to job titles reveals that there is a zone or a relevant region of union densities (roughly between 50 and 85 percent) where its contribution to the wage gap is elevated, reaching almost 12.5 log points. Worker unobserved heterogeneity once again plays no role, irrespective of union density. Note, finally, that the part of the union wage gap remaining after having allowed for the three high-dimensional fixed effects, and identified in the figure as the 'within component,' is essentially zero up to 70 percent density after which it increases to only a little over 2 log points at 100 percent density.

5.3 On the Endogeneity of Union Membership

The main purpose of this study is to investigate what lies behind the union wage gap rather than claiming a causal interpretation of the union density regression coefficient. Nevertheless, a number of endogeneity issues do need to be addressed. A widespread concern is that of individual self-selection into union status (Lee (1978); Card et al. (2020)).

It may well be the case that more skilled workers disproportionately select into unionization, and in so doing drive the union wage gap. This concern can be directly assessed within the framework of our decomposition exercise. Thus, there is no indication that union members have unobserved characteristics that render them more productive, given the very weak (near null) impact of the worker fixed effect on the union wage gap.

Another potential source of ambiguity is the possibility that trade unions tend to organize in industries and firms where economic rents are higher. In this case, we face a problem of reverse causality. Although we certainly cannot rule out this possibility, we can nonetheless determine an upper-bound for this source of endogeneity. Specifically, the contribution of the firm fixed effect to the union wage gap - of around two-thirds of the union wage gap - is our upper-bound for the reverse effect. In other words, the indication that higher union density rates coincide with more generous firm wage policies may arise from 'pure' causal effects (unions pushing for higher wages) or from the tendency of trade unions to be organizationally stronger in those activities where economic rents are higher.

According to our methodology, the remaining one third of the union wage gap is driven by the finding that union members are more likely to occupy better-paid job titles. This effect corresponds to the impact of job-title fixed effects on the union wage gap. Because the wage floors for the job titles (i.e. bargained wages) and promotion policies are negotiated by the trade unions, this channel of wage determination is inherently linked to the actions and power of unions. In this sense, we would argue that the association between higher union density rates and better paid job titles can convincingly be regarded as causal. With some caution, therefore, we summarize this discussion by suggesting that the causal effect of union density on wages is likely to be contained within a 6 to 16 log points range.

Taking this source of causality one step further, one can think of the job-title fixed effect(s) as a suitable instrumental variable for union density. Using this set-up, the first-stage regression for union density will include the covariates of the full model. In the second stage, the coefficient estimate of predicted union density is exactly identified by the presence of job title in the first stage. On the basis of this alternative IV approach at least, it transpires that the two-stage coefficient estimate of the union density variable is a sturdier 0.126, or 12.6 log points (see Appendix Table 6).

6 Inside the Mechanisms of Wage Setting

In a complementary exercise, so as to better understand the role of trade unions in Portugal, it is instructive to decompose total wages into two parts: the bargained wage and a wage mark-up (Π). So the total wage is given by:

$$W_{Total} = W_{Bargained} \times \underbrace{\left[\frac{W_{Bargained} + \Pi}{W_{Bargained}}\right]}_{=1+\mu}, \tag{II.12}$$

where μ corresponds to the mark-up in relative terms. Through a straightforward logarithm transformation, one obtains:

$$\ln W_{Total} = \ln W_{Bargained} + \underbrace{\ln(1+\mu)}_{\text{Wage cushion}} \ . \tag{II.13}$$

The bargained wage is the base wage floor as defined in the relevant collective agreement for the worker job title. For its part, the wage cushion corresponds to the difference between the total compensation and the bargained wage. As information on bargained wages is not contained in the dataset, we follow the methodology proposed in Cardoso and Portugal (2005), and define the bargained wage as the mode of the actual base wage within each year and job-title.[18] The wage cushion has two components. The first is simply the difference between the actual base wage and the bargained wage, as firms often pay a base wage above the bargained wage. The second component is the sum of the wage supplements received by the worker, to include seniority-indexed pay, overtime, fringe benefits such as the daily meals allowance, and irregularly paid components.

We shall estimate two separate wage regressions, one for the (log) bargained wage and the other for the (log) wage cushion, where the wage cushion is expressed in relative terms. A useful way to look at the bargained wage regression is to think of an artificial exercise in which all workers collect the enacted wage floor, corresponding to their job titles, as signed in the applicable collective bargaining, and no more. On this basis, and as shown in Table 4, the union wage gap would amount to 28.36 log points.

At first glance, this may seem a puzzling result, as the union wage gap for total monthly wages is only 16.19 log points, while that for the wage cushion is negative at -12.18

[18] Having documented contractual wages in three industries employing around 10 percent of full-time workers in manufacturing and services, these authors show that the mode of the wage distribution of the base wage for each worker category within each collective agreement matches quite well the mandatory floors for each job-title at collective bargaining level.

Table II.4: Estimation of the Union Wage Gap for Total Monthly Wages, the Bargained Wage, and the Wage Cushion

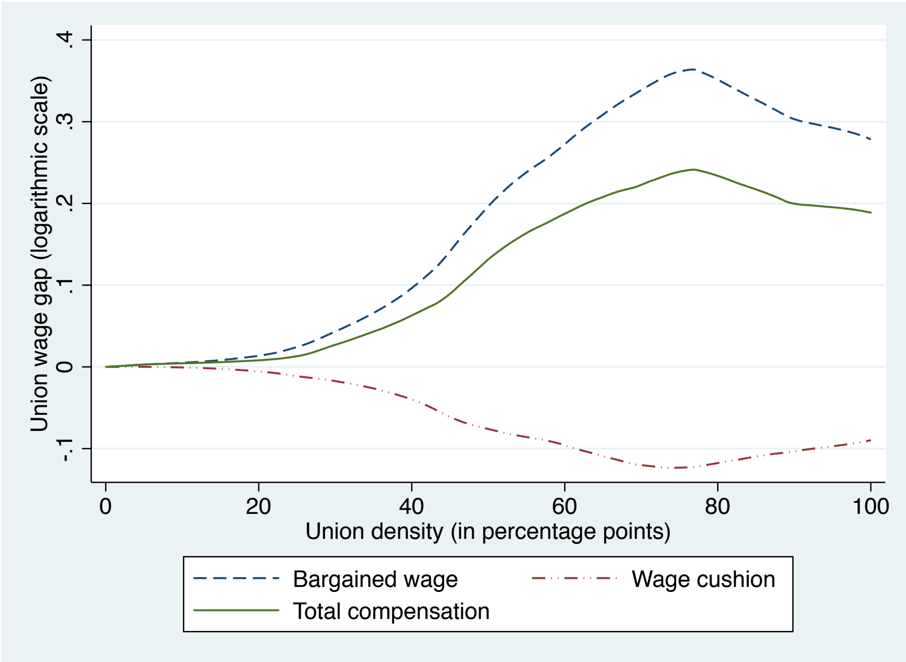
Dependent variable	Base Model	R^2
Total Compensation	0.1619	0.5373
	(0.0244)	
Bargained Wage	0.2836	0.5095
	(0.0283)	
Wage Cushion	−0.1218	0.0994
	(0.0184)	

Notes: Robust firm clustered standard errors are in parentheses: all coefficients are statistically significance at 0.01 confidence level. The base model includes as regressors a quadratic term in age, a quadratic term in tenure, schooling dummies (7), a gender dummy, firm size dummies (5), and sector dummies (25). The number of observations is 36,616,379. *Sources: Quadros de Pessoal*, 1986-2009; *Relatório Único*, 2010-2013.

log points. Similar results are obtained when union wage gap curves are estimated, as demonstrated in Figure 8. Here the union wage gap curve for the bargained wage attains levels of more than 30 log points when the large majority of the workforce is unionized, while the corresponding curve of the wage cushion declines up to around -12 log points. What is going on here is that the union wage gap for total compensation hides the opposing effects of the two wage components.

The sizable union wage gap of the bargained wage is not surprising, as unions naturally seek to lock in a significant share of their gains through the mandatory dispositions of collective agreements. Thus, where collective agreements are signed in sectors with highly unionized firms, signifying enhanced union bargaining power, unions may be expected to succeed in securing higher base wage floors (bargained wages). Our results in the form of a negative union gap for the wage cushion do indeed suggest that the wage cushion is deployed by firms to attenuate the bargained wage gap. It follows that the union wage gap for total compensation is lower than that for the bargained wage. This compression may result from either lower wage supplements or by smaller drift between the actual base wage and the bargained wage floor. In other words, in high union density environments, union success in raising the bargained wage limits the ability of firms to pay base wages in excess of bargained wages. In branches where trade unions are weaker, however, relatively low bargained wages offer scope for local improvement.

Figure II.8: The Union Wage Gap Curve for Total Monthly Wages, the Bargained Wage, and the Wage Cushion



Source: Relatório Único, 2010-2013.

7 A Glimpse into the Impact of Unions on Wage Inequality

In this section we take advantage of conditional quantile regression methods to explore the effect of union density on the entire wage distribution. The usefulness of conditional quantile regression to study the impact of unionization was first demonstrated by Gary Chamberlain in his influential address to the 1990 Word Congress of the Econometric Society. To the best of our knowledge, only the method of moments quantile regression estimator (MM-QR), recently proposed by Machado and Santos Silva (2018), is able to accommodate multiple high-dimensional fixed effects that can affect the whole distribution instead of just a location shift. Accordingly, we will use this estimator to analyze the impact of union density on wage inequality. A critical advantage of the estimator is that it is based on the OLS solutions to two high-dimensional fixed effects regression equations, one corresponding to the location function and the other to the scale function. Apart from its ease of implementation, the MM-QR estimator lends itself in a straightforward way to the decomposition of the bias arising from the omission of the high-dimensional effects.

We will first specify the conditional quantile regression version for a model without fixed effects:

$$Q_w(\tau|(x_{it}) = \alpha^l + \boldsymbol{x'_{it}\beta^l} + \sigma(\alpha^s + \boldsymbol{z'_{it}\beta^s})q(\tau), \tag{II.14}$$

where $Q_w(\tau|(x_{it})$ denotes the conditional quantile of w corresponding to percentile τ. z is a vector of transformed covariates x. σ identifies the scale function. α^l and $\boldsymbol{\beta^l}$, and α^s and $\boldsymbol{\beta^s}$, correspond to the intercept and regression coefficients in the location and scale functions, respectively. $q(\tau)$ is simply $F^{-1}(\tau)$.

The full model can be written:

$$Q_w(\tau|(x_{it}) = \alpha_i^l + \lambda_f^l + \theta_j^l + \boldsymbol{x'_{it}\beta^l} + \sigma(\alpha_i^s + \lambda_f^s + \theta_j^s + \boldsymbol{z'_{it}\beta^s})q(\tau), \tag{II.15}$$

where α_i^l (α_i^s), λ_f^l (λ_f^s), and θ_j^l (θ_j^l) are the worker, firm, and job-title fixed effects in the location (scale) function, respectively.

Considering that $\boldsymbol{z = x}$ and that $\sigma(.)$ is the identity function, the model simplifies to:

$$Q_w(\tau|(x_{it}) = [\alpha_i^l + \alpha_i^s q(\tau)] + [\lambda_f^l + \lambda_f^s q(\tau)] + [\theta_j^l + \theta_j^s q(\tau] + \boldsymbol{x'_{it}[\beta^l + \beta^s}q(\tau)]. \tag{II.16}$$

Estimation of this model can be achieved in four simple steps.

(1) Obtain the parameters of the location function minimizing the sum of the squares of the residuals from the model:

$$w = \alpha_i^l + \lambda_f^l + \theta_j^l + \boldsymbol{x'_{it}\beta^l} + \epsilon^l, \tag{II.17}$$

using, for example, the algorithm of Guimarães and Portugal (2010). This procedure corresponds exactly to the OLS solution which was estimated before;

(2) Compute the estimation residuals from this model, $\hat{R_{it}}$, and calculate $|\hat{R_{it}}|$;

(3) Obtain the parameters of the scale function minimizing the sum of the squares of the residuals from the model:

$$|\hat{R_{it}}| = \alpha_i^s + \lambda_f^s + \theta_j^s + \boldsymbol{x'_{it}\beta^s} + \epsilon^s; \tag{II.18}$$

(4) Obtain $q(\tau)$ as the τ-th sample quantile from a standardized residual $\hat{R_{it}}/(|\hat{R_{it}}|)$.

Table II.5: Method of Moments Quantile Regression Estimation

Variables	Percentiles		
	10	50	90
Base Model			
γ_τ (Union Wage Gap)	0.1720	0.1632	0.1498
	(0.0012)	(0.0012)	(0.0034)
Full Model			
γ_τ (Union Wage Gap)	-0.0052	-0.0031	-0.0003
	(0.1941)	(0.0996)	(0.1288)
Decomposition			
Worker FE	0.0001	-0.0024	-0.0056
Job-title FE	0.0710	0.0610	0.0476
Firm FE	0.1136	0.1068	0.1006
Residual	-0.0076	0.0009	0.0004

Notes: The base model includes as regressors a quadratic term in age, a quadratic term in tenure, schooling dummies (10), a gender dummy, firm size dummies (5), and sector dummies (25). All regression coefficients in the base model are statistically significant at the 0.01 confidence level. The regression coefficients in the full model are not statistically different from zero. The contributions of the fixed effects add the location and scale the components. The number of observations is 36,616,379. *Source: Quadros de Pessoal 1986-2009; Relatório Único, 2010-2013.*

Results for the location-scale representation of the quantile regression model are given in Table 5. The first line of the table presents the union coefficient estimates corresponding to the base specification for the 10th, 50th, and the 90th percentiles.[19] The impact of union density at the conditional mean and at the conditional median are virtually identical (16.2 log points for the mean and 16.3 log points for the median). Low-wage workers (say those in the 10th percentile) record a higher union wage gap (17.2 log points). For their part, high-wage workers (those in the 90th percentile) have a somewhat lower wage gap (15.0 log points) than the median wage worker. It appears that union membership mildly compresses the wage distribution by improving the wages of low-wage workers more than those of high-wage workers.

The full model specification, in which we take into account unobserved worker heterogeneity and sorting across firms with distinct wage policies and job titles is given in the second line of the table. Consistent with the previous linear regression model results is the suggestion that the union wage gap is close to zero across all three percentiles. Nevertheless, worker, firm, and job-title heterogeneity fully account for the union wage gap in the base model. Moreover, the three high-dimensional fixed effects account for the dispersion in

[19]The Stata procedure *mmqreg* provided by Santos Silva was used to obtain the method of moments quantile regression estimates.

the union coefficient estimates. In the same vein, we can extend the Gelbach procedure used before to decompose the change in the quantile regression estimates coefficient estimates (from the base model to the full model). It is clear that we can apply the Gelbach decomposition to both the location and the scale functions. For the location function, the procedure is exactly the same (and, in turn, produces exactly the same results) as that followed in section 5.2. For the scale function, we obtain the contribution of each fixed effect again using the omitted variable bias formula and multiply it by $q(\tau)$ in order to differentiate them by percentile. There is, of course, a fourth (residual) component arising from the nonlinear change in $|\hat{R}_{it}|$ when we move from the base model to the full model.

Three indications emerge from the decomposition exercise given in Table 5. First, the contribution of of high-dimensional fixed effects is largely driven by the location function, which can be seen by comparing Table 5 with Table 3. Second, the wage compressing effect of union density is concentrated in the job-title dimension. Low-wage workers benefit more from being placed (or promoted) into higher paying job titles than do high-wage workers. Finally, the residual component of the decomposition is small, or even negligible, at higher percentiles.

8 Conclusions

This chapter has shown that in a regime of near-universal collective bargaining coverage one may nevertheless discern pronounced union wage gaps due to the heterogeneous influence of unions in covered settings. Our baseline estimates of the union density wage gap for total monthly wages top out at approximately 24 log points. Wage gaps are not proportional to union density. Rather, some critical mass (circa 30 percent density) is required to materially influence wages, while beyond some level (70 percent) further increases in union density detract from the peak premium.

Having obtained these sizable values of union wage gaps based on estimates of average wage differentials between the wages of two observationally identical workers in two observationally identical firms with distinct levels of unionization, we turned to consider the potential sources of the gaps. Our fixed-effects model included in addition to worker and firm fixed effects a job-title or finely drawn occupational fixed-effect. We then applied Gelbach's (2016) decomposition which allows for the quantification of the share of the union wage gap accounted for by each of these three fixed effects. That is, we sought to account for the separate contributions to the union wage gap of the targeted wage policies of firms (i.e. versus a neutral stance which would imply that unionized workers are randomly assigned to firms); of differential rules, both direct and indirect, whereby unionized workers are assigned to higher paying job titles in the compensation tables of collective agreements; and of the manner of the allocation of workers of different unobserved ability to firms. The wage policies of firms were found to account for approximately two-thirds of the union wage gap in monthly wages, a profound change over a situation in which workers are randomly assigned to firms. Also of importance was the allocation of unionized workers into better paying job titles. This source contributed another one-third of the union wage gap. Accordingly, compensation for unobserved ability played little or no role in determining the premium enjoyed by union workers even if, as was demonstrated, their observed heterogeneity would patently produce a higher wage gap were it not controlled for.

We also distinguished between the components of total earnings to gain a fuller understanding of the role of trade unions. Specifically, we split total earnings into the bargained wage and the wage cushion. The bargained wage establishes the rate for the highly detailed job classifications or job titles that are contained in each collective bargaining agreement and sets a floor to the base wage obtaining at firm level. For its part, the wage cushion is the gap between the bargained wage and the actual base wage determined at firm level plus the difference between the latter and total earnings. We found that the size of the union wage gap is considerably larger for the bargained wage than for total earnings - peaking at more than 30 log points once some three-quarters of the workforce is unionized - and is actually negative for the wage cushion given its specification as the logarithmic ratio of actual earnings to the bargained wage. In other words, the union wage gap would be much higher (and wage dispersion lower) were all workers remunerated according to the bargained wage. The disparity between the premia attaching to the bargained wage and total earnings, and the negative gap for the wage cushion, reflect firm-specific arrangements that partly offset collective bargaining. In those sectors where trade unions are weaker, the relatively low bargained wages offer scope for local adjustments as when employers experience market opportunities or face constraints.

In a final contribution we sought to determine union impact on wage inequality, using conditional quantile regressions. Here, we used an approach that can accommodate multiple high-dimensional fixed effects, using the methods of moments quantile regression estimator to explore the impact of union density on the whole wage distribution. Estimation of the baseline model suggested that union membership mildly narrowed the wage distribution. For the full model containing the three fixed effects, application of the Gelbach procedure indicated that the narrowing effect of union density previously noted was concentrated in the job-title dimension where low wage workers benefited more from being placed or promoted into higher-paying job titles than other groups.

In sum, the present treatment can report progress along several dimensions. It uses an innovative procedure to determine the union density premium in circumstances, again by no means unique to Portugal, where almost all workers are covered by a collective agreement but few are union members. It offers a unique attribution of the union wage gap to three types of heterogeneity that 'explain' almost all wage variation. These high-dimensional fixed effects were also directly incorporated in a new application investigating the effect of union density on the wage distribution. They also contributed to our discussion of endogeneity, leading us to conclude that the causal effect of union density on wages likely fell within a 6 to 16 log points range. For the future, more work is clearly needed on the firm fixed effect. Reflecting recent developments in the literature, a logical extension would be to examine the contribution of firm monopsony power and the countervailing influence of union density.

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Chapter III

A Saga of Wage Resilience: Like a Bridge over Troubled Water

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Abstract

This chapter proposes a dynamic search and matching model adjusted to the Southern European and French based labour markets, where collective bargaining assumes a key role. Using the Portuguese employer-employee matched data for the last two decades, and its institutionally defined categories of workers, we reach a consistent and unified framework where we estimate an average worker bargaining power of 20%; an elasticity of quasi-rents of 0.062; an average passthrough of bargained wages of 44.8%; and a degree of assortative matching of 44.1%. These findings conform with the literature developed in each of these dimensions. Throughout the period, we witness a secular deterioration of worker's bargaining powers at the top and the middle of the wage distribution, while in the bottom we recorded a broad stability. Throughout the Great Recession, these findings are remarkably stable, signalling a significant wage setting resilience. Accordingly, the considerable real wage distribution adjustment throughout the downturn was led by job and firm flows, and for the staying workers through the valuation of the quasi-rents of the worker-firm match and of the worker's outside options.

1 Introduction

This chapter addresses two closely related questions. First, how can the microeconomic wage setting mechanism be structurally described and empirically estimated for a Southern European and French based labour market? Second, how does the resulting bargaining powers evolved overtime, and particularly throughout the Great Recession?

Wage setting mechanisms, and in particular the evolution of bargaining powers, have received a significant recent interest due to its pervasive implications to macroeconomic dynamics. Since the 1980s, a significant strand of papers focusing in the U.S. have recorded substantial increases in corporate profitability and firm markup trends, while the labour share has been in consistent decline. Gutiérrez and Philippon (2017), Farhi and Gourio (2018) and De Loecker et al. (2020) have linked these trends to the rise of market power, focusing in the rise of monopoly (or monopsony) power, but concurrent developments have broaden the perspective. Krueger (2018) influentially noted in his *2018 Jackson Hole* address, that the evolution of labour market practices have not only enhanced monopsony power, but has significantly weaken worker bargaining powers. Following his line, Stansbury and Summers (2020) provides a significant case for a relevant (if not leading) role of bargaining power trends in explaining those macroeconomic dynamics, and Lombardi et al. (2020) has theoretically linked a weakening of worker's bargaining powers with an abatement of inflation dynamics and an amplification of employment adjustments over the business cycle.

In a related literature, the wage setting and bargaining powers are also particularly relevant in the context of an imperfectly competitive labour market due to its role on wage inequality. Since the 1980s, cross sectional data unveiled a significant increasing trend of wage inequality in the United States.[1] Then, the skill biased technological change was coined as the leading cause (see Krueger (1993), Berman et al. (1994) and Acemoglu (2002)). Later, in a more microeconomic perspective, Autor et al. (2003) influentially highlighted job polarization, due to the idiosyncratic share of automatable work across occupations (see Acemoglu and Autor (2011) for a recent overview). However, from the outset of the debate, several developed economies like France, Japan or Germany were not displaying the same wage inequality trend, leading several authors to focus on the role of 'institutions' (Freeman and Katz (1995)).[2] Amidst those, different degrees of unionization,

[1] For the U.S., Katz and Murphy (1992), Levy and Murnane (1992) Bound and Johnson (1992) correspond to a sample of studies that decisively contributed to the consensus.

[2] A strand of the literature has focused on top income inequality (see Piketty and Saez (2003)), and more recently, in the context of the Great Recession, confirmed markedly different trends along those geographical lines (see Piketty and Saez (2013)).

wage setting structures and minimum wage policies lead the most studied.[3]

In Southern European and French based labour markets, while each country's labour market has institutional idiosyncrasies, the blueprint is largely analogous, and derived from few common ancestral roots (see Botero et al. (2004)). A brief wage setting synopsis would present a strong union coverage despite lower membership rates, significant employment protection, and centralized sector wage bargaining as its institutional cornerstones. Accordingly, among the existing labour markets, we select Portugal for our empirical implementation due to three convenient reasons. Firstly, as reinforced by Visser (2016), Boeri and van Ours (2013) and Card and Cardoso (2021), Portugal is in most wage setting mechanisms a representative of the Southern European and French based labour markets. Secondly, the country was particularly hit by the Great Recession. Finally, we have access to high quality data about the characteristics of the privately employed workers, including detailed information on wages, and of the balance sheet and income statement of firms.[4]

This chapter makes four contributions. First, it develops a dynamic search and matching model, which presents a wage setting mechanism accounting for the most relevant Southern European and French based labour market institutions. Second, this chapter explores the role of those institutions by resorting to the collective bargaining administrative ranking of workers, which allow us to identify the value of outside options and the value of quasi-rents, while relying on a significantly parsimonious parameter dimensionality when compared with a typical high dimensional fixed effect wage regression. It does not rely on worker mobility patterns across firms, or in general on firm effects. Third, differently from most of the rent-sharing literate we do not need to directly rely on firm-side measures of quasi-rents or value added (see Card et al. (2018) for an overview of these approaches), and we will abstain from following a substantive structural perspective as we will not specify a production function.[5] Fourth, this framework translates into a unified and consistent empirical setting capable to estimate time-varying bargaining powers, quasi-rents' elasticities, collective bargaining pass-through in wages, wage variance decompositions, and the degree of assortative matching in the economy, which lie within the ranges of the recent developments in those literatures.

Our proposed model entails: (i) firms composed by a hierarchical occupational struc-

[3]See DiNardo et al. (1996), Lee (1999), Card and DiNardo (2002), Dickens and Manning (2004) and Autor et al. (2008) for evidence on the influence of these institutional settings on wage inequality.

[4]See appendix A1 for a detailed description of the dataset used in this study.

[5]See Mortensen et al. (2010) and Bagger et al. (2014) for representative examples of a more structural approach, with a fully specified production function.

ture; (ii) worker-firm multi-layered high dimensional heterogeneity; (iii) the possibility of on-job-search in the classic Roy (1951) model approach; (iv) existence of binding firing taxes as in Boeri (2011); and (v) the adoption of the intra-firm bargaining apparatus of Stole and Zwiebel (1996a) and Stole and Zwiebel (1996b), unfolding between firms and a representative union, which however is unable to cause an *hold-up problem* in case of a bargaining breakdown, as considered in Dobbelaere and Luttens (2016). Altogether, the core modelling infrastructure was crafted in the intersection between Acemoglu and Hawkins (2014), Cahuc et al. (2008), Mortensen (2009), and under small twists in the assumption framework it becomes isomorphic to them.[6]

The empirical implementation of the resulting wage equation resorts to the collective bargaining administrative ranking, which is consistently produced for decades, covering directly around 85 percent of the labour contracts, without any *opt-out* possibility. As noted by Schulten (2016) and Card and Cardoso (2021), rather than a Portuguese idiosyncrasy the existence of these rankings is a Southern European and French based labour markets' cornerstone, which can be equally seen in countries like Belgium, France, Italy, Netherlands and Spain. This ranking corresponds to the core of the legal framework of the labour relationships, defining strictly the job descriptions, the clauses of the contracts, and the bargained tables of wage floors.[7] In detail, the ranking is composed of a granular system of around 30,000 institutionally defined sector-occupation ranks of the workers. We estimate the wage floors compatible with such ranking, and we follow the model's postulate that those coincide with the union and firm belief about the valuation of the workers' outside options, which incidentally do not depend on the employing firm alone.[8] Further, those ranks also provide a framework to identify the *type* of workers, and thus enhance the identification of the marginal products of the match.

Our use of collective bargaining rankings to estimate outside options consists in an alternative to recent developments in this literature. Caldwell and Harmon (2019) uses past co-workers job movements as a measure of the value of the social network of the worker, Schubert et al. (2020) resorts to the analysis of worker's job histories, and Caldwell and Danieli (2020) implements a sufficient statistic that assesses the supply of jobs in the area

[6]Lise et al. (2016) is in several theoretical aspects close in spirit to the exercise we propose, but focused on the U.S. economy and henceforth without the theoretical and empirical contribution of the European institutional setting. Tschopp (2017) also mostly abstracts the institutional setting, but implements an empirical strategy to identify key parameters in a search and matching model that in spirit is close to ours, however with less degrees of heterogeneity.

[7]See Cardoso and Portugal (2005), Martins (2014), Addison et al. (2017), and Card and Cardoso (2021) for further detail about the referred ranking.

[8]For Italy, Card et al. (2014) uses bargained wages to precisely control for outside options in a rent-sharing analysis, but does so at sector level, without the degree of granularity presented here.

and the worker's flexibility to take them. In a broader perspective, the proper use of rankings of workers and/or firms has received great attention in the mincerian wage equation estimation. Bonhomme et al. (2017), Bonhomme et al. (2019) and Lentz et al. (2018) resort to a two-step algorithm where in the first step either firms or workers (or both) are classified in categories by using a *k-means clustering algorithm*; Sorkin (2018) resorts to a replica of the *Google's PageRank* algorithm to rank firms based on revealed preference, and thus identify the value of compensating differentials; and Hagedorn et al. (2017) presents the classical *Kemeny-Young rank aggregation* algorithm as a way to rank workers and then firms based on the worker's ranking.[9] Our approach, instead of resorting to indirect measures to estimate the value of outside options or statistical algorithms to identify types of workers or firms, relies on firms and unions being capable to assess the worker's worth in the market, which translates into a credible and consistent administrative ranking.

In the last two decades, the wage setting mechanism synchronized a notable stability of workers' bargaining positions at the bottom of the wage distribution, with a perennial erosion at the middle and the top. On average, we estimate the worker's bargaining powers at 20 percent for the entire economy, with higher levels for managers, and very identical values for the remainder of the workforce. Those translate into some responsiveness of wages to exogenous shocks in the level of the quasi-rents of the labour relationship, concretely with an average estimated elasticity of 0.062, within the 0.05-0.15 most referred interval in the literature (see Card et al. (2018)). Likewise, we find levels of positive assortative matching on the range 38-48 percent, and an average passthrough of changes in bargained wages into changes of total wages of around 44.8 percent, in line with Card and Cardoso (2021). In respect to wage dispersion, the workplace heterogeneity justifies around 60 percent of the overall wage dispersion, conforming with an imperfectly competitive labour market perspective, while the outside option effectively contributes for a compression of wages relatively to the implied productivity distribution.

The structural slow paced erosion of bargaining powers at the top and middle of the skill distribution, unveil potential future productivity hazards, particularly if amplified by a considerable progressive income taxation. With compressed wage differentials across skill groups and the increasing orthogonality between wages and firm productivity levels, which naturally arises when the worker has a lower take on the quasi-rents of the match, the matching efficiency in the economy may be degraded, the incentives for training may be dulled, and the alignment of worker and firm incentives may be laxed.

[9] See Kemeny (1959) for the first treatment of this algorithm.

The macroeconomic context of our analysis has been particularly turbulent in a significant part of the sample, when the Great Recession emerged. Within the recessionary shock, for those that were capable to remain employed throughout the rough waters, the wage setting laid out a sound bridge, displayed through a significant resilience of bargaining powers, which contained the propagation of the sluggish macroeconomic outlooks to temporary and moderate real wage losses, arising from the fall of outside option real values and real quasi-rents of the match.

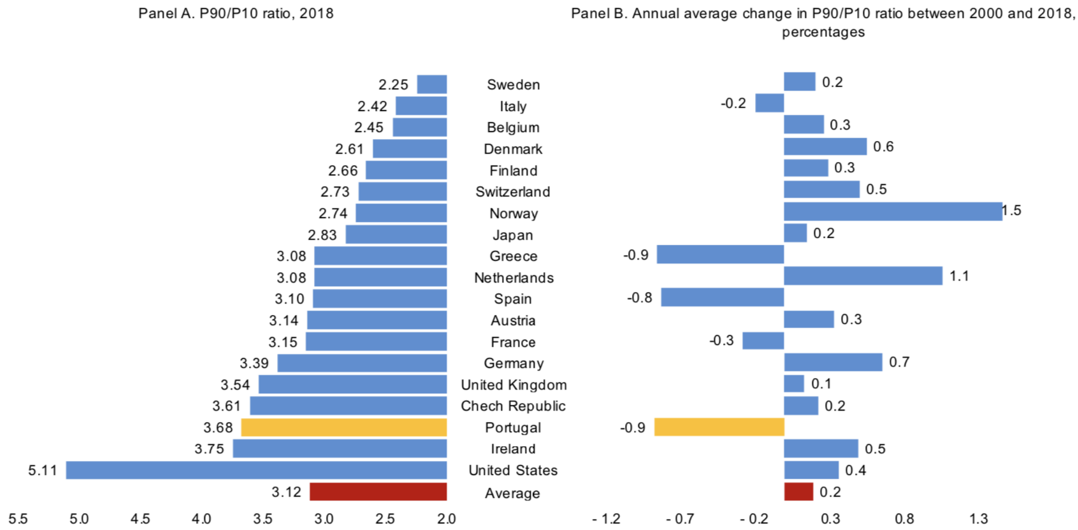
The chapter is organized as follows. Section 2 presents the main features of the considered labour market and the way those are subsequently introduced in the model. Section 3 presents the dynamic search and matching model proposed in the chapter, and section 4 presents a straightforward way to empirically estimate the resulting wage equation. Section 5 presents the empirical results of the model, the estimates of the level of firm-worker sorting in the economy, of the elasticity of wages to the quasi-rents and of the average passthrough of bargained wages. It further analyses the variance of wages in the proposed wage setting perspective. Section 6 discusses the resilience of the wage setting mechanisms in the wake of the Great Recession. Section 7 concludes.

2 Wage Setting and Labour Market Institutions

The cross-country comparison of labour market institutions literature, classically featuring an *U.S. - Europe comparison*,[10] has developed around *countries' institutional clusters*. Indicatively, Boeri (2011) partitions Europe in a *Continental cluster*, a *Southern European cluster*, a *Nordic cluster*, and an *Anglo-Saxon cluster*. Alternatively, Botero et al. (2004) presents a partition of taxonomies based on ancestry, grouping labour markets in the *French*, *German*, *Anglo-Saxon* and *Socialist* systems. Given our modelling emphasis in the wage setting process, we follow a mixed version of those taxonomies, with our analysis being particularly suitable for *Southern European* and *French* based systems. Noteworthy, as presented in figure 1, a primer assessment of recent trends of wage inequality presents divergent paths among OECD countries, with this particular cluster decoupling from others by displaying a reduction of wage inequality.

[10] See Nickell and Layard (1999), Bertola (1999), Blau and Kahn (1999), Boeri (2011) for chapters of the *Handbook of Labour Economics* featuring a classical analysis.

Figure III.1: Inequality of Gross Earnings of Full-Time Dependent Employees



Source: OECD.

2.1 Institutional Setting and its Dynamics

As presented in table 1, the wage setting has been considerably more centralized and/or coordinated with a leading role of trade unions in the continental European labour markets than in their Anglo-Saxon counterparts. The former is dominated by industry, sector or even national level agreements, whereas the latter unfolds recurrently at firm or even plant level (see Boeri and van Ours (2013)). While such structural institutional differences could have little influence in market outcomes, the literature has strongly suggested otherwise, with these asymmetries suspected to play a role in explaining the degree of wage compression across economies.[11]

In the last decades, the OECD indicators on labour market institutions and regulations points to a detrimental evolution to workers, as presented in the synthetic indicator of Lombardi et al. (2020) in figure 2. While in the Anglo-Saxon markets we have witnessed a co-movement of de-unionization and the fall in collective bargaining coverage, rendering a greater subsidiary role of unions in the wage setting mechanism, that is not the case in the continental European labour markets, where despite the strong fall in membership, union coverage remains remarkably high.[12] As claimed by Booth (2014), this persistence justifies the importance of modelling the behaviour of trade unions when, in general, one

[11]The suggestion of different degrees of trade union influencing the wage setting and wage compression has been established in Freeman (1980), Blau and Kahn (1996), Aidt and Tzannatos (2002) and Card et al. (2003), among others.

[12]See Blanchflower (1996), Blanchflower and Bryson (2004), Bryson et al. (2011), and Visser (2016) which document this process. In the Anglo-Saxon case, several papers, among which DiNardo et al. (1996), Fortin and Lemieux (1997), Firpo et al. (2009), Frandsen (2012) and Farber et al. (2018) correspond to a sample, have studied the relationship between the fall in membership and collective bargaining coverage and the increase in wage inequality.

Table III.1: Average Collective Bargaining Indicators by Decade

Countries:	Level of Bargaining		Bargaining Centralization		Union Density (%)		Union Coverage (%)		Single Employer Bargaining (%)	
	2000's	2010's	2000's	2010's	2000's	2010's	2000's	2010's	2000's	2010's
Portugal	3	3	2.8	2.6	18.4	17.4	80.5	75.5	3	5
Spain	3	3	2.52	2.17	19.3	18.9	70.9	70.5	7.5	5.9
France	3	3	2.4	2.3	10.7	10.9	98	94.2	3.6	3.8
Italy	3	3	2.59	2.41	33.6	35.5	80	80	-	-
Germany	3	3	2.2	2.2	21.7	17.7	65	57.1	8.5	8.1
Austria	3	3	2.29	2.28	33.3	27.6	78.5	78.5	3	3
Netherlands	3.4	3	2.59	2.19	20.4	18.2	82	83.1	10.1	7.6
Belgium	4.5	4.6	4.09	4.14	54.9	53.6	94.6	92	10	-
Denmark	3	3	2.34	2.3	70.4	67.4	77	79.3	22	-
Sweden	3	3	2.49	2.39	74.5	67	92	89.6	8.3	5.6
Norway	3.2	3	2.57	2.39	54.3	52.3	73.7	71	-	-
Finland	3.65	3.67	3.06	3.06	69.2	63.45	86.2	89.6	9	9
Switzerland	3	3	2.49	2.39	74.5	67	91.9	89.6	8.3	5.6
Greece	3.9	2.4	3.6	1.3	27.75	21.07	100	57.1	8.2	15
Poland	1	1	0.96	0.9	17.1	16.9	20.9	17.9	15.4	14.8
Czech Republic	2	2	1.8	1.8	23	13.3	27.9	33.1	27.9	33.1
United Kingdom	1	1	1	1	28.6	24.9	34.6	28.3	29.4	27.5
Ireland	4.6	1	4.05	0.99	33.8	27.2	42.1	34	32.4	-
United States	1	1	1	1	12.2	10.7	13.4	11.9	-	-

Notes: Level of bargaining takes the values: (5) central or cross-industry level bargaining; (4) alternating between central and industry bargaining; (3) sector or industry bargaining; (2) sector or company bargaining; (1) company bargaining. Bargaining Centralization is a measure created by ICTWSS, ranging between 1 and 5, with 5 being the highest level of centralization. *Source: ICTWSS*, version 6.1, 1960-2018.

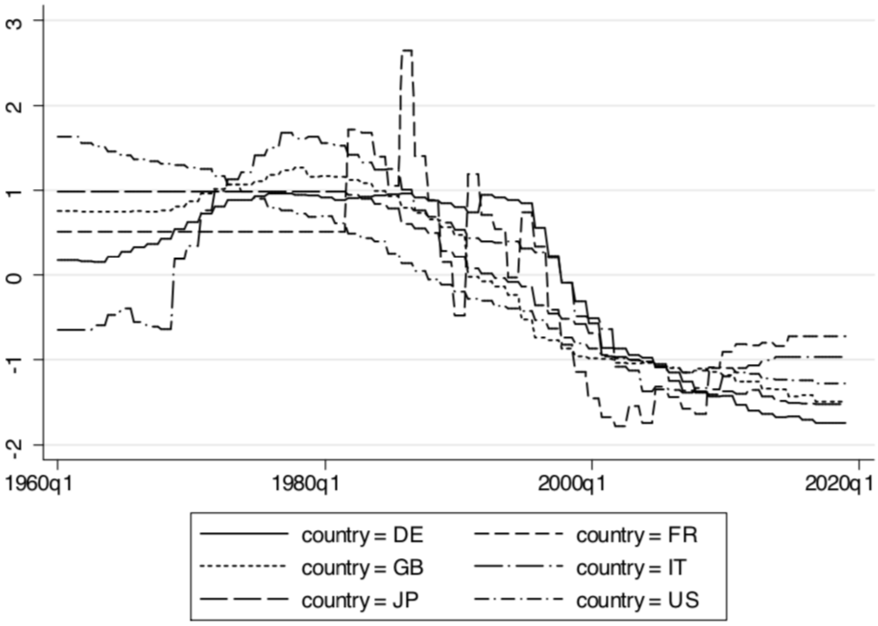
assesses the wage setting in these economies.[13] Moreover, employment protection remains relevant in those latitudes (See OECD (2016) for a data overview).

2.2 Modelling the Labour Market Institutional Setting

In our framework we account for a significant employment protection and a preeminent union presence both in bargaining wages and ensuring the enforcement of implicit clauses of contracts, in the spirit of Hogan (2001). Consequently, our model considers the firm defines employment, and then with the *right-to-manage* union bargain sequentially the contract of every worker first at sector/industry level through an employer's association, and then at local level. Further, the modelled union has universal coverage, but lacks the ability to force a full lockdown of production when a bargaining process breaks down. Instead,

[13] See Pissarides (1986) for the first inclusion of unions in a search and matching framework, and Bauer and Lingens (2010) and Dobbelaere and Luttens (2016) for recent treatments with an approach closer to ours. Finally, also see Abowd and Lemieux (1993) for a classical and mainstream treatment, and Krusell and Rudanko (2016) which revisits the question in the context of a defined frictional labour market in a macroeconomic perspective.

Figure III.2: Synthetic Indicator of Worker's Bargaining Powers



Note: Each line represents their synthetic indicator of the worker's bargaining power, by using the first principal components of several indicators, including union density, union coverage, employment protection indexes, and coverage of collective agreements.
Source: Lombardi et al. (2020)

we assume the Dobbelaere and Luttens (2016) *gradual collective bargaining* structure.[14] If negotiations break down, the parties reach stalemate until one of the workers in that contract leaves the firm without any firing tax being levied. Then, both sides restart bargaining every contract aiming at unlocking the stalemate, with the process unfolding with the same gradualism until a full simultaneous agreement is reached.[15]

Moreover, we assume bargaining takes place under three additional requirements building the implicit foundation of the wage bargaining contract. Concretely, the sides define: (i) the accurate belief about the level of outside options; (ii) a fair ladder of outside options' beliefs in the market when compared with productivity of the workers, so that no worker is unfairly and consistently degraded; and (iii) that wages are settled under the principle of match stability, implying that both parties bargain over the split of the match surplus assuming ex-ante that neither side will exert their at-will option to dissolve

[14] Dobbelaere and Luttens (2016) justifies the discarding of this mainstream prior, due to the almost lack of empirical evidence of such event, with the sole exception of the *Ronald Reagan and the air traffic controllers* case in 1981, which they argue was political. Holden (1988) and Holden (1989) resort to the nordic *peace-clause* to also abstain from a *full-lockdown assumption*.

[15] Under this bargaining protocol, the presence of unions by itself does not deviate the equilibrium wages from the one that would emerge through individual bargaining, as in Cahuc et al. (2008) or Acemoglu and Hawkins (2014). See appendix B1 for further insights about the relationship of our modelling approach and other canonical union-firm models.

that match.[16] Finally, we complement the bargaining apparatus with an employment protection framework, which notably translates into the existence of firing costs/taxes, in the spirit of Bentolila and Bertola (1990), Bertola and Caballero (1994) and Boeri (2011).[17]

2.3 The Wage Setting and Outside Options

The debate about how to model outside options in the context of wage determination has been prolific. The work of Postel-Vinay and Robin (2002) for the French labour market sparked attention to the implications of different modelling choices for the bargaining framework of wages. The authors adopted sequential bargaining where firms bid for worker's services, creating an enforceable link between the prescribed value of the worker's outside option and the history of his past job offers while employed.

This link has been disputed due to the empirical rarity of a sequential bidding in defining wages, and the predictable lack of enforceability of incumbent firm - individual worker promises.[18] Barron et al. (2006) highlights the implications of the Postel-Vinay and Robin (2002) apparatus on the co-workers' contracts, and thus define as theoretically reasonable the existence of at most a selective counter-offer policy. Cullen and Pakzad-Hurson (2021) reinforces the limited use of sequential bidding if there is transparent pay. They empirically find a very moderate decline of wages when transparent pay is introduced in some US locations, which is consistent with an ex-ante limited application of sequential bidding. Concurrently, Di Addario et al. (2021) finds that the impact of previous firm match is moderate in the wage setting, particularly in low-middle skilled jobs, making such empirical results also difficult to conciliate with sequential bidding.

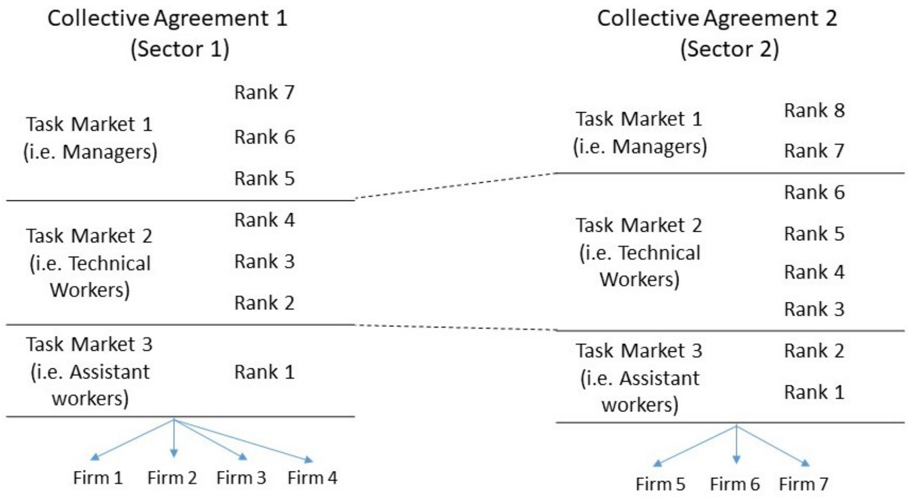
Altogether, our modelling choice follows the principle of nonexistence of counter-offers. Rather, we assume unions and firms predict the value of outside options for each type of worker, and those predictions are enforced in the collective bargaining. If a worker receives

[16] The ability of unions to have more accurate information than isolated workers, and to have the ability to coordinate actions of workers to enforce state-contingent actions or implicit clauses in the contract is precisely at the core of Hogan (2001) analysis. As a matter of fact, the dataset on workers' characteristics was created on the purpose that unions could inspect and monitor firm's behaviour. Created by law in 1976, our workers' characteristics dataset - *Quadros de Pessoal* - were mandatorily sent by firms to the Ministry, and posted in a visible place in each establishment, with every relevant employment characteristic, including the wages and the worker's position.

[17] While Elsby and Michaels (2013) presents evidence on the identical fitting properties of a macroeconomic search and matching model with kinked employment adjustment costs on hiring, on firing or on both sides, we opted to follow the modelling option of Bentolila and Bertola (1990) and Bertola and Caballero (1994), and the evidence presented by Lazear (1990), about the relevance of considering firing costs.

[18] See Pissarides (1994), Mortensen (2005), Shimer (2006), and Dolado et al. (2008) for examples of studies resorting to this set of arguments.

Figure III.3: The Collective Bargaining System



a beneficial proposal, he leaves the current match.

2.4 The Institutional Data on Outside Options

In Portugal, as in the majority of the Southern European and French based labour markets, the centralization of wage bargaining is clear and engraved in existing collective bargaining agreements. Those are signed without any *opt-out* possibility for a firm in a covered industry or sector, and overwhelmingly by unions linked to the two major union confederations. While there exist a fringe of tailored agreements at firm level, those represent less than 4 percent of the workforce, being the sector agreements, negotiated between employers' associations and unions, or the agreements only entangling some employers in a sector, that predominate. Jointly with their subsequent administrative extensions either to other similar sectors, or to an entire sector when the initial coverage was reduced to some employers, the total coverage of collective bargaining reaches more than 85 percent of the private sector workforce. This quite centralized apparatus provides strengthened coherence to the bargained outcomes, particularly across comparable labour market contexts.[19]

[19] Legally, the firm-level agreements signed between an individual company and one or more unions are designated *Acordos de Empresa* or AEs. Those are important in the oil sector and transport and communications. The collective agreements signed by several employers that are not part of an employers' association and one or more trade unions are known as *Acordos Colectivos de Trabalho* or ACTs. Those are significant in the financial sector and utilities. Finally, the industry-level or sectoral agreements, the so-called *Contratos Colectivos de Trabalho* or CCTs, are the ones negotiated between one or more employers' associations and one or more unions. The administrative instruments that extend the agreements are either *Portarias de Extensão* or *Portarias de Condições de Trabalho*. See further details in Addison et al. (2017). See appendix A2 for an example of a representative wage floor table.

In toto, the enacted agreements set a substantive set of rules on working conditions, and a system of wage floors for detailed categories of workers. As presented in figure 3, we have that those wage floors are defined based on the firm's sector and the worker's career rank, or category, within a given task market (i.e. senior manager, junior manager, and so on). Those ranks are then aggregated in a comparable set of types of workers, or task markets across the sectors of activity (i.e. managers, group leaders, senior technical workers, non-technical workers, and so on). As a matter of example, this labour market functions in resemblance to the organization of the armed forces. You have a hierarchy composed by groups of ranks or task markets (i.e. generals, commissioned officers and unlisted grades), arguably compared across the three branches of the armed forces (i.e. army, air force and navy), and within each task market you have a plethora of ranks (i.e. field marshall, general, brigadier, captain, and so on). In total, the sectoral agreements and their extensions explain the claim that Portugal has no less than 30,000 minimum wages (see Martins (2014)).[20]

In our framework, we assume this definition corresponds to an estimate of the worker's outside option value, or *fire-sale*, assuming the match stability principle. Therefore, this estimate corresponds to the worker's value if he becomes suddenly and unexpectedly unemployed. Empirically, we only have access to the ranks of the workers, and therefore we establish the minimum paid regular wage in each identified rank as the proxy of the legally binding minimum. In a nutshell, the system of sector-occupation ranks provides a comparable measure of the outside option value of the workers across sectors, and individual histories. This empirical choice is not at the odds of the literature (see Card et al. (2014) for an identical empirical choice).

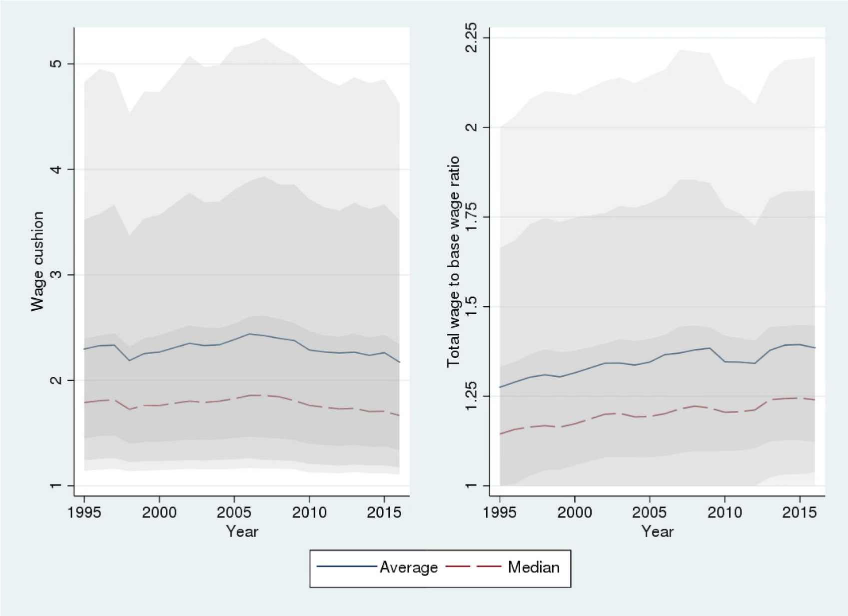
2.5 Bargained Wages and Wage Cushion

As presented in figure 4, while this *bargained wage* sets the minimum wage conditions of each labour relationship it does not correspond to the actual wage of the worker, as the latter results from the proper wage bargaining dynamics at firm level. As seen by the *wage cushion* measure, corresponding to the ratio between the total wage and this proxy of the bargained wage, it is extremely common, and a stable feature of the market, to see firms paying above the minimum condition.[21] By the same token, the base wage

[20] We abstract the possibility of several collective agreements covering the same firm, as either they tend to be coincidental, or the workers tend to pick the most advantageous one.

[21] The term *wage cushion* was proposed in Cardoso and Portugal (2005), corresponding to the difference on the levels of the bargained wage and the base wage actually paid to the worker. Noteworthy, while Cardoso and Portugal (2005) and Card and Cardoso (2021) assesses the difference between bargained and

Figure III.4: Evolution of the Wage Cushion



Notes: The wage cushion is calculated as the ratio $wage_{drift} = \frac{wage_{total}}{wage_{bargained}}$. The fading grey shades corresponds, from the darker to the lighter respectively, to (a) 75^{th}-25^{th} percentile range; (b) $90^{th} - 10^{th}$ percentile range; and (c) $95^{th} - 5^{th}$ percentile range. *Sources:* Quadros de Pessoal and Relatório Único, 1995-2016.

does not match the total compensation of the workers, as the proper institutional setting often determines the mandatory existence of several supplements, as the meals subsidy or even tenure related regular payments. Moreover, this distinction confers different degrees of future enforceability among types of pay, with some other regular compensation supplements, as shift subsidies or availability supplements being in *de-jure* temporary or partially temporary.

2.6 Heuristic Reduced Form Approaches to Estimate Wage Setting

A widely accepted way to portray the wage determination in the context of imperfect labour markets is given by:[22]

$$w_{ift} = Out_{it} + \beta QR_{ift} \tag{III.1}$$

base wages of the worker, we assess the difference between bargained and total wages. See Addison et al. (2021) for other analysis resorting to the *wage cushion* concept and unions. Further, this concept differs from the *wage drift* which assesses minimum wage changes versus actual wage changes. See Holden (1988) and Holden (1989) for treatments of this concept for the Nordic countries.

[22]This style of wage determination equation is for example compatible with: (a) bargaining in single worker firm models as in Pissarides (2000); (b) search with multiple workers and Stole and Zwiebel (1996a) bargaining as in Cahuc et al. (2008), Acemoglu and Hawkins (2014) and this chapter; (c) efficient unions bargaining as in Abowd and Lemieux (1993); and (d) monopsonistic wage posting as in Manning (2011) and Card et al. (2018).

where Out_{it} corresponds to the outside wage of worker i in moment t, and QR_{ift} corresponds to the non-competitive quasi-rent the worker obtains at firm f in time t.

In this context, the leading way to identify the parameter β, or its related elasticity resides in using proxies of the value of the quasi-rent. This path can be seen in Card et al. (2014), and its extensively reviewed in Card et al. (2018). The potential limitation of this approach resides in the proper availability of credible measures of productivity, as the value added. Also, even when it exists, it often doesn't enable the possibility to adopt an identification that secures within firm heterogeneity of the quasi-rent component.

The advent of matched employer-employee administrative datasets represented a turning point on the study of labour market dynamics. Empirically, the major drive to rely on this data lies on its detailed, and overtime consistent, description of the relationship network established among market actors (i.e. workers, firms, unions, households, among others). Such interest on this type of data became most salient after the seminal contribution of Abowd et al. (1999) – the widely known AKM model. Their model owes its popularity to the pleasant empirical properties of the within estimator, which it naturally extends, the easiness of its empirical implementation, its high explanatory power, the apparent intuitive interpretation of its results in a canonical wage setting model, and even the natural relationship with the traditional schooling and experience frameworks of Mincer (1974) and Griliches (1977). Consistently, Card et al. (2013) highlighted the importance of considering network data to identify unobservables. Contrary to past wage inequality surge episodes, their assessment of the recent rise in inequality for West Germany found that the most relevant drivers were not easily observable worker and firm characteristics. Identically, Bloom et al. (2018) records long-lasting unobservable wage differentials with firm size.

Typically, as presented in Card et al. (2018), an AKM approach to estimate the parameter β (or its implied elasticity) would entail:

$$\begin{aligned} ln(w_{ift}) =& ln(Out_{it}) + \tilde{\beta} ln(QR_{ift}) \\ ln(Out_{it}) =& \alpha_i^O + \phi_t^O + \mathbf{X}_{it}^O \gamma^O \\ ln(QR_{ift}) =& \alpha_i^{VA} + \psi_f^{VA} + \phi_t^{VA} + \mathbf{Z}_{ift}^{VA} \gamma^{VA} \end{aligned} \tag{III.2}$$

which results in a AKM type model as:

$$\begin{aligned} ln(w_{ift}) = & \underbrace{\alpha_i}_{=\alpha_i^O+\tilde{\beta}\alpha_i^{VA}} + \underbrace{\phi_t}_{=\phi_t^O+\tilde{\beta}\phi_t^{VA}} + \underbrace{\psi_f}_{=\tilde{\beta}\psi_f^{VA}} + \mathbf{X}_{it}^O\gamma^O + \mathbf{Z}_{ift}^{VA}\gamma^{VA}\tilde{\beta} + \epsilon_{ift}, \\ \widehat{\psi_f} = & \xi + \tilde{\beta}E_f[ln(QR_{ift})] + v_f, \end{aligned} \tag{III.3}$$

where the identification lies on the idea that the firm fixed effect only affects the non competitive quasi-rent, and consists in a measure of the link between different compensation policies across firms after partialling out the value of individual specific traits, and time effects.

However, from those outset contributions, the debate acknowledged empirical hazards in both the interpretation and use of the fixed effect estimates, and in its proper estimation, which may suffer from several econometric impairments.[23] Within the latter group, limited mobility bias leads the suspects (see Abowd and Kramarz (2004) for the seminal treatment of the question),[24] endogenous mobility closely trails (see Card et al. (2016) and Card et al. (2018) for classical discussions), and finite-sample bias and measurement error complete the quartet of the most referred potential econometric faults. Even if validity is set aside, the functional form of the canonical AKM setting has been questioned.[25]

Those empirical *crossroads* encouraged researchers to enlarge their strategies. Some attempted to increase the parameter set to account for misspecification, even though it implies even stronger identification restrictions when compared with the canonical AKM (see Woodcock (2015) for a recent discussion). In more encouraging paths, some attempted to devise bias correction methods for the limited mobility issue (see Andrews et al. (2008), Borovicková and Shimer (2017) for two examples), while others defined different estimators theoretically capable to address some of the identified issues without restricting the modelling environment (see Kline et al. (2019) for a leave-out estimator of quadratic form). Alternatively, several studies, among which Bonhomme et al. (2019) is a prime example, tried to perform rankings of workers and/or firms, before the proper estimation of the wage equation, with the aim of reducing the parameter set to be estimated while

[23]A leading example of an interpretation an use of fixed effects issue concerns the identification of age and experience profiles in the presence of worker fixed effects (see Card et al. (2018)).

[24]One of the most relevant consequences of limited mobility bias concerns the validity of assessing the level of sorting of workers and firms through the empirical correlation of worker and firm effects. See Eeckhout and Kircher (2011) and Jochmans and Weidner (2019) for theoretic discussions; and Abowd and Kramarz (2004), Andrews et al. (2008), Lopes de Melo (2008), Andrews et al. (2012), Hagedorn et al. (2017), Borovicková and Shimer (2017), Torres et al. (2018) and Bonhomme et al. (2020) for a sample of relevant studies on this debate.

[25]Allowing the value of the worker-firm match to unveil in more convoluted ways than its additive specification, and considering dynamic mechanisms composing a richer and arguably more robust longitudinal analysis than its static environment have been two leading courses of action (see Bonhomme et al. (2019), Card et al. (2018) and Lachowska et al. (2020) for recent analysis).

maintaining the sound statistical explanatory power of AKM.[26]

In the spirit of these last developments, and given the collective bargaining administrative ranking, a simplified version of our approach will empirically implement:[27]

$$\begin{aligned} w_{ift} =& Out_{it} + \beta QR_{ift} \\ QR_{ift} =& TS_{ift} - Out_{it} \\ Out_{it} =& \varphi_t + f(rank_{it}, age_{it}, gender_i) \\ TS_{ft} =& \psi_{ft} + \mathbf{X}_{it}\gamma + \alpha_{it} \\ \mathbf{X}_{it}\gamma + \alpha_{it} =& E[\mathbf{X}_{it}|rank_{it}]\gamma. \end{aligned} \tag{III.4}$$

where TS_{ft} corresponds to the total surplus of the match worker-firm. Notice that the use of the collective bargaining administrative ranking will allow to identify bargaining powers in a more parsimonious and flexible model as compared with the AKM, and in a more flexible approach than the one usually available to the rent-sharing approach. In the next two sections we will present the model and its implied assumption framework that postulates this presented system of equations.

3 A Search and Matching Model of Labour Tasks

In a nutshell, this model corresponds to a search and matching archetype at workplace level (*id est* firm/occupation/time partition), with gradual collective bargaining, intra-firm wage bargaining, severance payments and on-job search. Under several twists in its assumptions it becomes isomorphic to the models presented in Mortensen (2009), Cahuc et al. (2008), Acemoglu and Hawkins (2014) or Pissarides (1990).

In this section we focus our description of the model in the required ingredients to obtain dynamic equilibrium wages. The treatment of the remainder conditions describing the full dynamic equilibrium, an alternative steady-state equilibrium, and the theoretical links of this model and the literature of search and matching and union-firm bargaining are

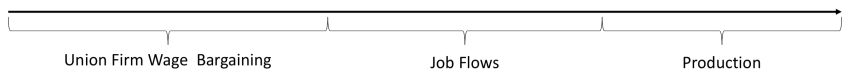
[26] In this broad class of papers, one can consider: (a) Bonhomme et al. (2017), Bonhomme et al. (2019) and Lentz et al. (2018) that resort to a two-step algorithm where in the first step either firms or workers (or both) are classified in categories by using a *k-means clustering algorithm*; (b) Sorkin (2018) which resorts to a replica of the *Google's PageRank* algorithm to rank firms based on revealed preference to then identify compensating differentials; and (c) Hagedorn et al. (2017) which, building on theoretic foundations of Shimer and Smith (2000) and Eeckhout and Kircher (2011), presents the classical *Kemeny-Young rank aggregation* algorithm as a way to rank workers and then firms based on the worker's ranking (see Kemeny (1959) for the first treatment of this algorithm).

[27] In the actual empirical implementation we will add a task market dimension - j - as we will subsequently introduce.

presented in appendix B.[28]

Labour market structure. The proposed model unfolds in each period t according with figure 5, where each period is decomposed in three hypothetical moments, with wage bargaining happening before the job flow decisions take place.

Figure III.5: Structure of the Model in each Moment t



In this context, consider an economy with a *numeraire* good sold under perfectly competitive conditions, and produced by a unit measure of large firms. In each period t, each firm employ multiple workers, from the available pool $i \in \{1, \ldots, \aleph\}$, with each specializing in one of the available labour tasks, $j \in \{1, \ldots, J\}$. Workers sell their tasks exclusively to a single firm. Time is continuous, and workers, the union, and firms discount time at rate $r \geq 0$.

The labour market is assumed to be frictional, as firms are required to post vacancies to hire workers, and pay a cost γ_j per vacancy posted in task market j. Workers, either employed or unemployed, do direct search at task market level, by selecting the task market they are willing to perform search. They incur in the search cost c_j if they search, and then meet firms following a random search process within the task market.

The flow of worker-firm meets in task market j is determined by a typical constant returns to scale aggregate matching function, $M(u_j(t)+e_j(t), \bar{V}_j(t))$, where $u_j(t)$ is the measure of unemployed workers searching for a job in task market j at moment t, $e_j(t)$ is the measure of employed workers searching in market j at time t, and $\bar{V}_j(t)$ is the measure of vacancies in such task market. Correspondingly, the market tightness is given by:

$$\theta_j(t) = \frac{\bar{V}_j(t)}{u_j(t) + e_j(t)}, \tag{III.5}$$

and

$$\theta_j(t)q(\theta_j(t)) = \frac{M(u_j(t) + e_j(t), \bar{V}_j(t))}{\bar{V}_j(t)} \tag{III.6}$$

[28] In detail, appendix B provides further technical details on the model, namely: (i) the conditions under which the model become isomorphic to several search and matching models in the literature; (ii) the relationship between the gradual collective bargaining apparatus and other canonical union-firm bargaining solutions; (iii) the derivation of the remainder conditions describing the dynamic equilibrium and of equilibrium wages; (iv) the definition and properties of the dynamic equilibrium and the steady state equilibrium.

represents the Poisson rate at which a worker, either employed or unemployed meets a firm. Further, $q(\theta_j(t))$ is the Poisson rate at which a firm meets a candidate, per vacancy posted. For notational ease, we often just write θ_j, u_j, e_j or $q(\theta_j)$, omitting its time dependence.

On the other side, matches are dissolved due to one of four reasons namely: (a) a bargaining breakdown in the wage negotiation; (b) a termination exogenous shock, representing reasons beyond the control of workers and firms, which happens with probability $\bar{s}$; (c) a successful on-job-search of a worker; and (d) the decision of the firm to fire the worker at will, which may be triggered after the firm pays a severance payment given by S.

Description of Market Agents. Firms to be productive employ a J dimensional vector of workers, $\mathbf{N}$, resort to an exogenously predetermined capital input, K, whose rental cost, $I(K)$, is considered to be fixed and sunk, and implement the available homogeneous production function $F(\mathbf{N}, K)$.[29] Moreover, firms bargain with a representative union over wages with heterogeneous bargaining strength, with $I - \beta$ representing the firm's bargaining power vector - a $[J \times 1]$ dimensional vector - implying heterogeneous bargaining powers across types of tasks. Altogether, the firm exogenous heterogeneity is captured in the two dimensional tuple $\{K, \beta\}$.[30] The cumulative distribution of the firm's types in each moment is given by $\Gamma(K, \beta)$.

Workers are potentially infinitely lived, but may suffer a death shock with a constant hazard rate δ, and new workers arrive at the market at the same rate. Each worker is exogenously endowed with an initial generic training and ability, whose stock is given by $a(0)$ extracted from the distribution $\Psi_0(a) = N(\mu_0, \Sigma_0)$. Then, the worker develops skill through a stationary and invariant process with the Markov property, so that the transitions are described by the cumulative distribution function

$$\begin{aligned} \Psi(a'|a) &= Prob(a_i(t+1) \leq a'|a_i(t) = a), \\ \psi(a'|a) &= \frac{d}{da'}\Psi(a'|a) \sim N(B_0 a, CC'), \end{aligned} \tag{III.7}$$

and accordingly, the density over the history of the worker $a^t = [a(t), a(t-1), \ldots, a(0)]$

[29]The firm's production function is continuous at all arguments, concave, with constant returns to scale, infinitely differentiable for all positive arguments. As will be clear in our identification strategy, the adoption of an homogeneous production function is taken for exposition purposes, and do not constraint our empirical environment.

[30]Notice that in describing the model, we present β as a scalar, so that we ease notation burden. When pertinent, we present the implied differences. Further, we are assuming the agents while are forward looking, they assume $\{K, \beta\}$ will be stable, so that any future change in firm's fundamentals are fully unexpected, when bargaining takes place.

corresponds to:

$$\psi(a^t) = \psi[a(t)|a(t-1)] \dots \psi[a(1)|a(0)]\psi_0[a(0)], \tag{III.8}$$

with the unconditional invariant distribution given by:

$$\psi(a') = \int_a \psi(a'|a)\psi(a)da. \tag{III.9}$$

In these regards, in each period, firms are required to incur in a operating cost per employed worker dependent on the training and ability of the worker, and the task market, otherwise he will become fully unproductive. The operating cost, corresponds to the flow:[31]

$$A(j,a) = \omega_j(a). \tag{III.11}$$

Conditional on the firm's characteristics, the function $G_{j,t}(a|K,\beta)$ represents the number of workers, employed in task market j in moment t, with at most a as level of operating cost, which is assumed to be, in each moment t, common knowledge.[32] Consistently, the economy pool of workers is given by:

$$\aleph = \int_a d\aleph(a,t)da = \sum_{j=1}^{J} \int_a \int_K \int_\beta dG_{j,t}(a|K,\beta)d\Gamma(K,\beta)da + \int_a dU_t(a)da, \tag{III.12}$$

where: (a) $U_t(a)$ corresponds to the number of unemployed workers with at most an operating cost of a; and (b) $\aleph(a,t)$ consists in the number of available workers with at most an estimated operated cost a in period t.[33]

The last notable agent in our model is the representative union. We assume the union fully represents the workforce in the wage bargaining, independently of the actual workforce

[31] Technically, assume that $\omega_j(x) > \omega_l(x), \forall x \in [0,\bar{A}], \forall j > l$ due to the increasing complexity of task market. Further $\omega_j(x)$ is strictly convex and holds $\lim_{x\to 0^+}\omega(x) = \bar{A}$, $\lim_{x\to\infty}\omega(x) = 0$. The use of a Markov process in this context is classical. Bonhomme et al. (2017) uses a Markovian process to describe earnings directly, whereas we adopt a Markovian process in skill, which allow for dynamics to be treated in a slightly different angle. Jointly, the operating cost function and the skill acquisition can be represented by a linear state-space system, as:

$$\begin{aligned} a(t) =& B_0 a(t-1) + Ce(t) \\ A(j,a) =& \omega_j(a(t)), \end{aligned} \tag{III.10}$$

with $e(t) \sim N(0,I)$. See Ljungqvist and Sargent (2012) for further details of this process.

[32] In the process of matching in the labour market, we critically assume that a hiring firm only acquires knowledge about a after the hiring is completed.

[33] Notice that $\aleph(a,t)$ unfolds according with:

$$d\aleph\left(a,t\right) = \int_a d\aleph\left(a,t-\epsilon\right)\psi(a'|a)da. \tag{III.13}$$

Further $\aleph$ is exogenous and fixed. Then given $\Gamma(K,\beta)$ and an initial distributions $dG_{j,0}(a|K,\beta)$ and $d\aleph(a,0)$ the distribution $dU_0(a)$ is identified, and given the dynamics of the former distributions the dynamics of the latter is equally identified. The dynamics of $dG_{j,t}(a|K,\beta)$ are described latter.

membership status, while employment decisions are left to firms. In representing workers, the objective of this utilitarian union is to maximize the workforce value given by:

$$W_t = \sum_{j=1}^{J} \int_a \int_K \int_\beta \Xi_{j,t}(a|K,\beta) dG_{j,t}(a|K,\beta) d\Gamma(K,\beta) da + \int_a Out(a) dU_t(a) da, \tag{III.14}$$

where $\Xi_{j,t}(a|K,\beta)$ is the value of a worker of type a conditional on being in a firm of type $\{K,\beta\}$, and $Out(a)$ is the value of the outside option of the worker of type a.

Value functions.[34] The profit of a firm with fundamentals $\{K,\beta\}$ is assumed to be strictly concave and twice continuously differentiable in employment. It is given by:

$$\begin{aligned} r\Pi(K,\beta) - \frac{\partial \Pi(K,\beta)}{\partial t} &= \underbrace{F(\mathbf{N}(K,\beta);K)}_{\text{Production}} - \underbrace{\sum_{j=1}^{J} \int_a w_j(a|K,\beta) dG_j(a|K,\beta) da}_{\text{Wage Bill}} \\ &- \underbrace{\sum_{j=1}^{J} \int_a A(j,a) dG_j(a|K,\beta) da}_{\text{Operating Cost Bill}} - \underbrace{\sum_{j=1}^{J} \int_a s_j(a|K,\beta) J_j(a|K,\beta) dG_j(a|K,\beta) da}_{\text{Cost of Displacement}} - \underbrace{I(K)}_{\text{Sunk Cost}} \\ &- \underbrace{\sum_{j=1}^{J} \int_a \tilde{s}_j(a|K,\beta) S dG_j(a|K,\beta) da}_{\text{Exp. firing tax}} + \underbrace{\sum_{j=1}^{J} \max_{V_j(K,\beta)} \Big\{ -\gamma_j V_j(K,\beta) + V_j(K,\beta) q(\theta_j) J_j^R(K,\beta) \Big\}}_{\text{Value of the Hiring Policy}}, \end{aligned} \tag{III.15}$$

where $\tilde{s}_j(a|K,\beta)$ corresponds to the probability that the firm (K,β) fires at will the worker of type a paying in such event a firing tax of S, and $s_j(a|K,\beta)$ corresponds to the probability that the match (a,K,β) is dissolved. The intuition of equation (15) is standard in the models of this type (see Cahuc et al. (2008)). Accordingly, profit of a firm $\{K,\beta\}$ accounts for: (a) the output of the firm; (b) the firm expenditure in the wages of the employed workers; (c) the firm expenditure with operating costs; (d) expected firing taxes; (e) the sunk cost related to the capital input; (f) the firm losses due to the separation shock; and (g) the proceeds of the firm's optimal vacancy posting behaviour (i.e. $V_j(K,\beta)$), considering the probability the firm meets a candidate, the cost of creating a vacancy (i.e. γ_j), and $J_j^R(K,\beta)$ the firm's expectation about the marginal profit obtained with a new hire.

[34]For the sake of simplicity, and with a slight abuse of notation, we drop the time subscript in the remainder of this modelling section and in most of appendix B, excepting when the expression is dynamic.

The corresponding HJB equation of the marginal profit of a worker is given by:

$$
\begin{aligned}
rJ_j(a|K,\beta) - \frac{\partial J_j(a|K,\beta)}{\partial t} &= \frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - w_j(a|K,\beta) - A(j,a) \\
&- \underbrace{\sum_{l=1,l\neq j}^{J} \int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)} dG_l(a|K,\beta)da}_{\text{Employment effect on wages of other task markets}} - \underbrace{s_j(a|K,\beta)J_j(a|K,\beta)}_{\substack{\text{Expected value loss of}\\ \text{dissolving the match}}} - \underbrace{\tilde{s}_j(a|K,\beta)rS}_{\text{Firing tax}} + \\
&+ \underbrace{\sum_{l=1}^{J} \left\{ y_l(K,\beta)V_l(K,\beta) - \int_a s_l(a|K,\beta)dG_l(a|K,\beta)da \right\} \frac{\partial J_j(a,K,\beta)}{\partial N_l(K,\beta)}}_{\text{Impact on value of the job of hiring and firing policies of the firm}}.
\end{aligned}
\tag{III.16}
$$

Altogether, the value function of a filled job in firm (K,β) by a worker a can be described as the match marginal productivity discounting the value of the worker's wage, the impact of the marginal hiring on the wages set in the other task markets, the loss inherent to the future dissolution of this match, potentially including a firing tax if the dissolution happens to be a firm at-will decision, and lastly the impact of the firm's hiring and firing decisions in other task markets on the value of the filled job.

Regarding the unemployed worker, we have that his HJB equation is given by:

$$
\begin{aligned}
rOut(a) - \frac{\partial Out(a)}{\partial t} &= \underbrace{b}_{\substack{\text{Unemployment}\\ \text{benefit}}} + \\
&+ \underbrace{\sum_{j=1}^{J} \xi_j^o(a) \left\{ \theta_j q(\theta_j) \frac{\int_K \int_\beta \Xi_l(a|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K \int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a) - c_j(a) \right\}}_{\text{Expected value of searching for a job}},
\end{aligned}
\tag{III.17}
$$

where $\xi_j^o(a)$ corresponds to an indicator function being 1 if the unemployed is searching in task market j, and zero otherwise.[35] By the same token, the corresponding value function for the employed worker in the match with fundamentals $\{a,K,\beta\}$ is given by:

$$
\begin{aligned}
r\Xi_j(a|K,\beta) - \frac{\partial \Xi_j(a|K,\beta)}{\partial t} &= w_j(a|K,\beta) + \bar{s}\underbrace{\Big(Out(a) - \Xi_j(a|K,\beta)\Big)}_{\text{Value loss of losing the job}} + \\
&+ \underbrace{\sum_{l=1}^{J} \xi_l^{\Xi}(a|K,\beta) \left\{ \theta_l q(\theta_l) \frac{\int_K \int_\beta \mathbf{1}[\Xi_l(a|K,\beta) > \Xi_j(a|K,\beta)]\Xi_l(a|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K \int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a) - c_j(a) \right\}}_{\text{Value of searching for a job while employed}} \\
&+ \underbrace{\sum_{l=1}^{J} \Big[y_l(K,\beta)V_l(K,\beta) - s_l(K,\beta)N_l(K,\beta) \Big] \frac{\partial \Xi_j(a|K,\beta)}{\partial N_l(K,\beta)}}_{\text{Impact of hiring and firing policy of the firm in the value of the employment}},
\end{aligned}
\tag{III.18}
$$

with $\mathbf{1}[\Xi_l(a|K,\beta) > \Xi_j(a|K,\beta)]$ representing an indicator function equal to 1 if the value in

[35] It is assumed that the unemployed worker only searches in the task market that maximizes is expected value of search. The same takes place in the case of the employed worker. Further details in appendix B2. Moreover, the unemployment benefits - b - are independent of a merely for exposition purposes. It will be clear that considering it dependent on a, i.e. $b(a)$, will not affect our identification strategy.

alternative match is greater than the current, and $\xi_l^{\Xi}(a|K,\beta)$ corresponding to an indicator function being 1 if the employed worker is searching in task market j, and zero otherwise.

Collective Bargaining Protocol. In wage bargaining, which occurs in every period t, the union and firms follow bilateral bargaining protocols, with a system of offers and counter-offers in the spirit of Rubinstein (1982) and Brügemann et al. (2018).[36] Further, as previously described, the sides will firstly bargain binding minimum wages at aggregate level, and subsequently will bargain actual wages at firm level.

In both stages, the union and the firm, or employer's association, bargain wages assuming *the principle of match stability*, which is algebraically translated into:

$$\tilde{s}_l(a|K,\beta) = \xi_l^{\Xi}(a) = 0, \forall l \in \{1,\ldots,J\}, \forall \{a,K,\beta\}. \tag{III.19}$$

Notice that precisely match stability implies that neither side is ex-ante considering the other will dissolve the match at-will.

At aggregate level, we assume the association of firms and the union bargain the minimum binding wage which is compatible with the lowest surplus viable match, namely the match that generate a zero expected quasi-rent. Notice that the level of the expected quasi-rent of the match of a worker of type a with the average firm in the bargaining corresponds to:

$$\begin{aligned} E_{K,\beta}[QR_j(a,K,\beta)] =& E_{K,\beta}\left[\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}\right] - A(j,a) \\ &- E_{K,\beta}\Bigg[\underbrace{\sum_{l=1,l\neq j}^{J}\int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)} dG_l(a|K,\beta)da}_{\text{Employment effect on wages of other task markets}}\Bigg] - rOut(a) - \frac{\partial Out(a)}{\partial t}. \end{aligned} \tag{III.20}$$

Altogether, the aggregate bargaining solves the axiomatic constrained Nash bargaining, considering the Dobbelaere and Luttens (2016) proposition, and thus that the disagreement points are given by the loss of one match with a worker of type-a without the

[36] In particular, the insight of the Brügemann et al. (2018) allows for the ordering at which the contracts for each match fundamentals are bargained do not influence the outcome of the bargaining.

existence of side payments. Consequently:

$$\begin{aligned}
&w_{j,t}^{MIN}(a) = argmax_w \Big\{ E_{K,\beta}[\Xi_{j,t}(a|K,\beta)] - Out(a) \Big\}^{\beta} \Big\{ E_{K,\beta}[J_{j,t}(a|K,\beta)] \Big\}^{1-\beta} \\
&\text{subject to:} \\
&\tilde{s}_l(a|K,\beta) = \xi_l^{\Xi}(a|K,\beta) = 0, \qquad \textbf{(match stability)} \\
&E_{K,\beta}[QR_j(a,K,\beta)] = 0, \qquad \textbf{(No quasi-rent condition)} \\
&\forall l \in \{1,\dots,J\}, \forall \{a,K,\beta\}.
\end{aligned} \tag{III.21}$$

The solution of the aggregate bargaining problem is given by:

$$w_{j,t}^{MIN}(a) = rOut(a) - \frac{\partial Out(a)}{\partial t}, \tag{III.22}$$

which precisely defines the minimum wage at aggregate level for type-a worker, i.e. the bargained wage of type-a worker, as the level of his outside option.[37]

Considering the bargained wages of each type-a worker, the firm level bargaining takes place, with the union and each firm bargaining contracts for each match fundamentals $\{a, K, \beta\}$. In case of a full bargaining breakdown one of the matches, with match fundamentals as $\{a, K, \beta\}$, is expected to be dissolved without the existence of side payments among the involved market actors.

Accordingly, the wage of a match with fundamentals $\{a, K, \beta\}$ is obtainable by solving an axiomatic generalized Nash bargaining as:

$$\begin{aligned}
&w_{j,t}(a|K,\beta) = argmax_w \Big\{ \Xi_{j,t}(a|K,\beta) - Out(a) \Big\}^{\beta} \Big\{ J_{j,t}(a|K,\beta) \Big\}^{1-\beta} \\
&\text{subject to:} \\
&\tilde{s}_l(a|K,\beta) = \xi_l^{\Xi}(a|K,\beta) = 0, \qquad \textbf{(match stability)} \\
&w_{j,t}(a|K,\beta) \geq w_{j,t}^{MIN}(a) = rOut(a) - \frac{\partial Out(a)}{\partial t} \qquad \textbf{(agg. bargaining constraint)} \\
&\forall l \in \{1,\dots,J\}, \forall \{a,K,\beta\}.
\end{aligned} \tag{III.23}$$

Equilibrium Wages in the Dynamic Equilibrium. Considering the assumptions

[37] The solution of the aggregate bargaining entails that the worker's bargaining power does not directly influence the bargained wage of type a worker, but it influences the outside option through the worker's bargaining powers of the expected potential offers of the worker. Thus aggregate movements of bargaining powers affect the level of the outside option and the bargained wage, while idiosyncratic movements of bargaining powers of the bargained contract do not.

referred in the previous sub-section, and the generalization of the bargaining power parameter set to be heterogeneous at task-market-firm level, the unique solution of the equilibrium wages, which is compatible with the dynamic equilibrium presented, is given by:

$$w_j(a|K,\beta) = \\ = \begin{cases} (1-\beta_j)\underbrace{rOut(a) - \frac{\partial Out(a)}{\partial t}}_{Out^\star(a)} + \underbrace{\underbrace{\int_0^1 z^{\frac{1-\beta}{\beta}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} dz}_{\substack{\textit{Perturbed marginal productivity} \\ \textit{of the worker in the workplace (f,j,t)}}} - \beta_j \underbrace{A(j,a)}_{\text{Op. Cost}}}_{\text{Idiosyncratic Surplus of the Match } \{a,K,\beta\}}, \\ \qquad \text{if } w_j(a|K,\beta) \geq w_{j,t}^{MIN}(a) \\ w_{j,t}^{MIN}(a), \qquad \text{if otherwise.} \end{cases} \tag{III.24}$$

The expression of the idiosyncratic surplus of the match in the interior solution has a *perturbed marginal productivity of the worker in the workplace*, affected by the heterogeneity in the bargaining powers across task markets, which is fully consistent with Cahuc et al. (2008). Critically, notice that this term is invariant within the workplace (f, j, t).[38]

Moreover, in the absence of corner solutions, the difference between the average wage within the workplace, and individual wages, is determined by two fundamental factors, namely: (i) the differences in the level of outside options of the workers in the workplace; and (ii) the heterogeneity in the level of operating costs of the workers. This conclusion is easily reached if one represents equation (24) as a function of the average wage within the workplace. Accordingly, we have that:

$$w_j(a|K,\beta) = w_j(K,\beta) + (1-\beta_j)\underbrace{\Big[Out^\star(a) - E[Out^\star(a)|K,\beta]\Big]}_{\substack{\text{Diff. in outside options=} \\ \Delta Out^\star(a,K,\beta)}} + \beta_j \underbrace{\Big\{E[A(j,a)|K,\beta] - A(j,a)\Big\}}_{\text{Diff. in Op. Costs}=\Delta A_j(a,K,\beta)}. \tag{III.26}$$

Finally, notice that the aggregate bargaining constraint is assumed to not be binding in

[38]Technically, we refer to a workplace as the combination of worker-observations that share (f, j, t) dimensions. Intuitively, the workers that in moment t are in firm f in task market j. Additionally, note that the average wage in each workplace in the absence of corner solutions is given by:

$$w_j(K,\beta) = (1-\beta_j)E[Out^\star(a)|K,\beta] + \int_0^1 z^{\frac{1-\beta_j}{\beta_j}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} dz - \beta_j E[A(j,a)|j,K,\beta]. \tag{III.25}$$

the dynamic and steady state equilibria.[39] We assume that the worker will pull off at will from any match he is involved into that offers him a wage equal to the flow value of his unemployment. Consequently, in the empirical implementation we will consider just interior solutions.

4 Empirical Implementation

The main objective of the empirical implementation of the model is to estimate the actual wage equation of the worker. According to the presented model, wages are given by:[40]

$$w_j(a_t|K_t,\beta_t) = (1-\beta_{j,t})Out^\star(a_t) + \int_0^1 z^{\frac{1-\beta_{j,t}}{\beta_{j,t}}} \frac{\partial F(\mathbf{Q}_{j,t}(z)\mathbf{N}(K_t,\beta_t),K_t)}{\partial N_j(K_t,\beta_{j,t})} dz - \beta_{j,t}A(j,a_t). \tag{III.27}$$

In estimating equation (27), we will take advantage of the information about the workplace definition to infer the average wages, the outside option of each worker, provided by the referred workers' ranks, and the context of the union-firm bargaining, which will inform the behaviour of operating costs.

Empirical Outside Options. Given our theoretical framework, we have an estimate of the outside options of each worker (i.e. $Out^\star(a_t)$ is known), which coincides with the estimate of the wage floor of the rank in which the worker is placed. Thereafter, assuming that: (i) the outside options' valuation within each task market evolve, holding a parallel trends assumption, as $\lambda(j,t)$; and (ii) a first order Taylor approximation of the worker's outside option value around the average outside option of each workplace is reasonable, we have that the outside option can be estimated as:

$$ln[w_{j,t}^{MIN}(a_t)] = ln[Out^\star(a_t)] = \lambda(j,t) + \psi[rank(i,t), t-\tau_0(i), female_i] + v_{i,t}, \tag{III.28}$$

where λ is a task market and period effect, ψ is the contract characteristics effect. The latter accounts for: (a) the experience of the worker (i.e. $t-\tau_0(i)$), (b) the current rank of the worker (i.e. $rank(i,t)$), and (c) different career prospects by gender.[41]

[39] See Haanwinckel and Soares (2020) for an analysis of binding minimum wages in a wage bargaining setting close to ours, and the consequences for the wage setting. They assume an arbitrary minimum wage is set, and thus may become binding, while we assume the bargaining that results in the minimum wage.

[40] Notice that in the empirical description we recover the index t to avoid any confusion on the dimensions of heterogeneity in the empirical model.

[41] The derivation details of this functional form are provided in appendix C. While in theory we could resort directly to our proxy of $Out^\star(a)$, we use the insight of Pei et al. (2018) to minimize the potential impact of measurement error arising from the definition of the proxy.

Empirical Operating Costs To fully estimate equation (27) one needs to consider the functional form of the operating costs. In general, those correspond to the individual characteristics of the workers, which influence the marginal productivity of the match worker-workplace. For estimation purposes, we assume an additive functional form as:

$$A(j, a_t) = \underbrace{\xi(i,t) + \mathbf{X}_{i,t}\zeta_j}_{\text{Worker Characteristics}}, \tag{III.29}$$

where $\xi(i,t)$ corresponds to time-varying unobservable characteristics of the worker, and $\mathbf{X}_{i,t}$ to a vector of k observable characteristics of the worker. Notice that ξ would translate in a very high dimensional parameter set, which ordinarily is beyond identification capabilities of ordinarily implemented models.

In our approach we do not directly estimate the individual operating costs. Rather, we assume unions and firms have an estimate of these operating costs, and consequently use it to place the worker in the ranks of the firm. Consequently, we have that:

$$A(j, a_t) \approx E[A(j, a_t)|rank(i,t)] = E[\mathbf{X}_{i,t}|rank(i,t)]\zeta_j + E[\xi(i,t)|rank(i,t)]. \tag{III.30}$$

Equation (30) results from a *revealed preference argument.* Concretely, *if a worker is placed in a given rank within the firm, it can only be because the union and the firm agree he has a level of operating cost that is compatible with that rank.*

Moreover, we assume that the expected value of the unobserved characteristics of operating costs is the same within each task-market. Thus:

$$E[\xi(i,t)|rank(i,t)] = E\left[E[\xi(i,t)|rank(i,t)]\middle|j,t\right], \forall \;\; rank \in j. \tag{III.31}$$

This assumption implies that idiosyncratic characteristics of the workers, either observable or unobservable, contribute to their career path, i.e. their placement on a given rank, as the measure of operating costs of each worker is assumed to be equal to the workers' average of operating cost in the rank he is enlisted on - $E[A(j,a)|rank(i,t)]$. Moreover, the unobserved component of this average operating cost, $E[\xi(i,t)|rank(i,t)]$, is common to every rank in the corresponding task market of the relevant collective agreement. Intuitively, this assumption implies that the ranks are differentiated based on observable characteristics of their respective workforces, which we find a natural assumption given the bargaining takes place between unions and firms.[42]

[42] Notice that we are not fully excluding idiosyncratic pay to a given characteristic of the worker *vis-à-vis* the remuneration in the corresponding rank, as long as such payment is performed by resorting to bonuses,

Given the equation (31), and our interest in the difference in operating cost between the average worker in workplace and the worker's rank, we have that:

$$\begin{aligned}\Delta A_j(a_t, K_t, \beta_t) = &\underbrace{\left\{E\left[E\left[\mathbf{X}_{i,t}\middle|rank(i,t)\right]\middle|K_t, \beta_t\right] - E\left[\mathbf{X}_{i,t}\middle|rank(i,t)\right]\right\}}_{\text{Diff. in workplace versus rank on time-varying observables}}\zeta_j \\ =&\Delta E[\mathbf{X}_{i,t}|rank(i,t), K_t, \beta_t]\zeta_j,\end{aligned} \tag{III.32}$$

as the unobservable components cancel out. Notice that the *ceteris paribus* interpretation of empirical marginal effects of any of the variables in matrix $E[\mathbf{X}_{i,t}|rank(i,t)]$ is equivalent to the interpretation of a change in $\mathbf{X}_{i,t}$ in terms of wage change. Mechanically, one expects that the change in operating costs changes the rank of the worker, implying that the worker gets promoted. Altogether, equation (29) holds for that same worker, but on a different rank the worker was then assigned to.[43]

The logarithm of actual wages. Considering equation (26), and the described behaviour of outside options and operating costs, we have that the log of actual wages corresponds to:

$$\begin{aligned}ln[w_j(a_t|K_t, \beta_{j,t})] =&ln\Big[w_j(K_t, \beta_{j,t}) + (1-\beta_{j,t})\Delta Out^\star(a_t, K_t, \beta_t) \\ &+ \beta_{j,t}\Delta E\Big(\mathbf{X}_{i,t}|rank(i,t), (f,j,t)\Big)\zeta_j\Big] + \epsilon_{i,f,j,t},\end{aligned} \tag{III.33}$$

where $\epsilon_{i,f,j,t}$ corresponds to a disturbance. Accordingly, the wage is decomposed into: (i) the average wage in the workplace of the worker; (ii) the differences in the outside options, properly weighted by the worker's bargaining power; and (iii) the differences in the operating cost observables and the average observable operating cost in the workplace, weighted by the firm's bargaining power.

The estimation procedure. The first step deals with potential measurement error in our proxy of outside option values, and consists in resorting to a high dimensional heterogeneous slope model as:

$$ln[Out^\star(a_t)] = \lambda(j,t) + \psi[rank(i,t), t - \tau_0(i)] + v_{i,t}. \tag{$\mathbf{1^{st}}$ \textbf{Step}}$$

While this first step estimates a large number of parameters, due to ψ term, it is significantly more parsimonious than a model that resorts to worker and/or firm effects.

or irregular compensation policies.

[43] Noteworthy, implicit on the ceteris paribus analysis, we are assuming that the change in operating costs is not affecting the outside option, which may be an unrealistic assumption. To fully study *general equilibrium* marginal effects, one would have to estimate the impact of the change in that observable characteristic both on operating costs and outside options. That is beyond the purpose of this chapter.

Then we resort to the predicted outside option value - $\widehat{Out^{\star}(a_t)}$ to feed the estimation of the actual wage empirical model, which corresponds to:

$$\begin{aligned} ln[w_j(a_t|K_t,\beta_{j,t})] =& ln\Bigg[w_j(K_t,\beta_{j,t}) + (1-\beta_{j,t})\Delta \widehat{Out^{\star}(a_t,}K,\beta) + \\ &+ \beta_{j,t}\zeta_j \Delta E[\mathbf{X}_{i,t}|rank(i,t),(f,j,t)]\Bigg] + \epsilon_{i,f,j,t}. \end{aligned} \qquad (\mathbf{2^{nd}\ Step})$$

We use a non linear least squares, as:

$$\hat{\Theta} = argmax_{\theta\in\Theta} \sum_{i}^{N} \left\{ ln[w_j(a_t|K_t,\beta_{j,t})] - f\left(\theta, \mathbf{X}_{i,t}, w_j(K_t,\beta_{j,t}), \Delta \widehat{Out^{\star}(a_t,}K,\beta)\right)\right\}^2, \qquad \text{(III.34)}$$

where $\theta = \theta(\beta_{j,t},\zeta_j)$ is the parameter vector.

Conjointly, the workings of section 3 and section 4 presented a search and matching model, with the sufficient ingredients to consist in a credible environment for empirical implementation. While not every ingredient, particularly on job flows, is strictly necessary to establish the identification of equilibrium wage equations, their inclusion allows to understand how the modelled collective bargaining environment interacts with them, in a setting of a multi-employee and multi-occupational firm. In appendix D, we present a Toy model version of the sections 3 and 4 presenting the minimal components considered in a search and matching model capable to deliver the empirical identification presented.

5 Empirical Results

5.1 Highlights from a Parsimonious Wage Setting Equation

The outset of our empirical analysis adopts the most parsimonious and static version of our model. For this purpose, over the presented framework we assume that the workers' bargaining powers are constant overtime. In this stylized version, the model preserves a within-workplace structural approach, accounting for workplace time-varying effects and workers' observed and unobserved time-varying characteristics. The analogous reduced-form implementation would correspond to:

$$ln[w_{ifjt}] = ln\Bigg[\phi_{f,j,t} + (1-\beta_j)\widehat{Out(a)} - \beta_j[\xi(i,t) + \mathbf{X}_{i,t}\zeta_j]\Bigg] + \epsilon_{i,f,j,t}, \qquad \text{(III.35)}$$

where $E[\mathbf{X}_{i,t}|rank(i,t)] = \xi(i,t) + \mathbf{X}_{i,t}\zeta_j$.

While apparently straightforward, this reduced form would frame a conundrum in the absence of the model design of data usage. Either from a fixed or random effects perspectives, the estimation of parameters and/or implied distributional features for $\phi_{f,j,t}$, $\xi(i,t)$ and

$Out(a)$ would become computationally unattainable; would be largely unidentifiable given any largest connected set requirement; would imply strictly unrealistic orthogonality conditions; or even would result in a preposterous overparameterized model. The suggested relationship between workplace hourly wage and worker's hourly wage; the use of bargained wages; and the presented structure of individual and rank operational costs pave the way out of the riddle.

The estimation results are presented in table 2. Forthwith, the goodness-of-fit of the model is convincing, even in the context of a very favourable number of estimated parameters versus number of observations trade-off, particularly if one uses the AKM standpoint. Notice that in the first and second stages, the model resorts to 606,759 and 10 parameters, respectively, while the number of observations are above 29 million in each stage, with an average number of workers per year of around 1.4 million.[44]

Among the presented results, while the operating cost coefficients will be discussed at a later stage of this section, we promptly highlight the worker's bargaining power. Our estimate for the entire economy is around 20%, which is consistent with several other studies in the literature. For instance, for the Veneto region of Italy, Card et al. (2014) find a reduced form coefficient of the outside option (i.e. $(1-\beta)$) of 80%, when using sector minimum wage as the proxy of the outside option, in both OLS and IV within spell models of rent sharing. For France, Cahuc et al. (2006) estimate bargaining powers mostly in the range between 0 and 38% depending on the task-market.[45] For Germany, Hirsch and Schnabel (2011) implement a right-to-manage model and estimate yearly bargaining powers between 11%-18%, for the years 1992-2009. For Denmark, resorting to a structural model with some commonalities with our theoretical approach, Bagger et al. (2014) estimates an average workers' bargaining power of around 30 percent, and while Mortensen et al. (2010) matches that empirical estimate for the same dataset it further presents sectoral heterogeneity, ranging from 7-61 percent. Discordantly, Dumont et al. (2012) presents higher workers' bargaining power estimates for Belgium, between 45-71 percent depending on the sector in analysis.

A breviary of the empirical results of the supra-cited studies and of table 2 highlights the importance of bargaining power and operating cost heterogeneity over the time. Accordingly, we consider the time dimension of heterogeneity in our empirical analysis.

[44]In our sample, a typical worker-firm-time AKM specification entangles 3,660,238 worker fixed effects, 127,333 firm effects and 21 yearly dummies. So we estimate around 16 percent of the number of parameters a typical AKM would use in the first stage.

[45]The sole exception of that range is 98% for managers in the construction sector. Their partition of the task markets is identical to ours, but they have 4 categories. Their two top categories (i.e. 1 and 2) are condensed in our 1^{st} category.

Table III.2: Non-linear Least Squares with Common Slopes

Panel A: First-stage on Outside Option

Heterogeneous Slopes and Intercepts	Number of Parameters
Rank - 3^{rd} order polynomial age function - Female	606,696
Task Market - Year	63
Adjusted R^2	0.9371
N. Obs.	29,586,448

Panel B: Second-stage NLS Estimation

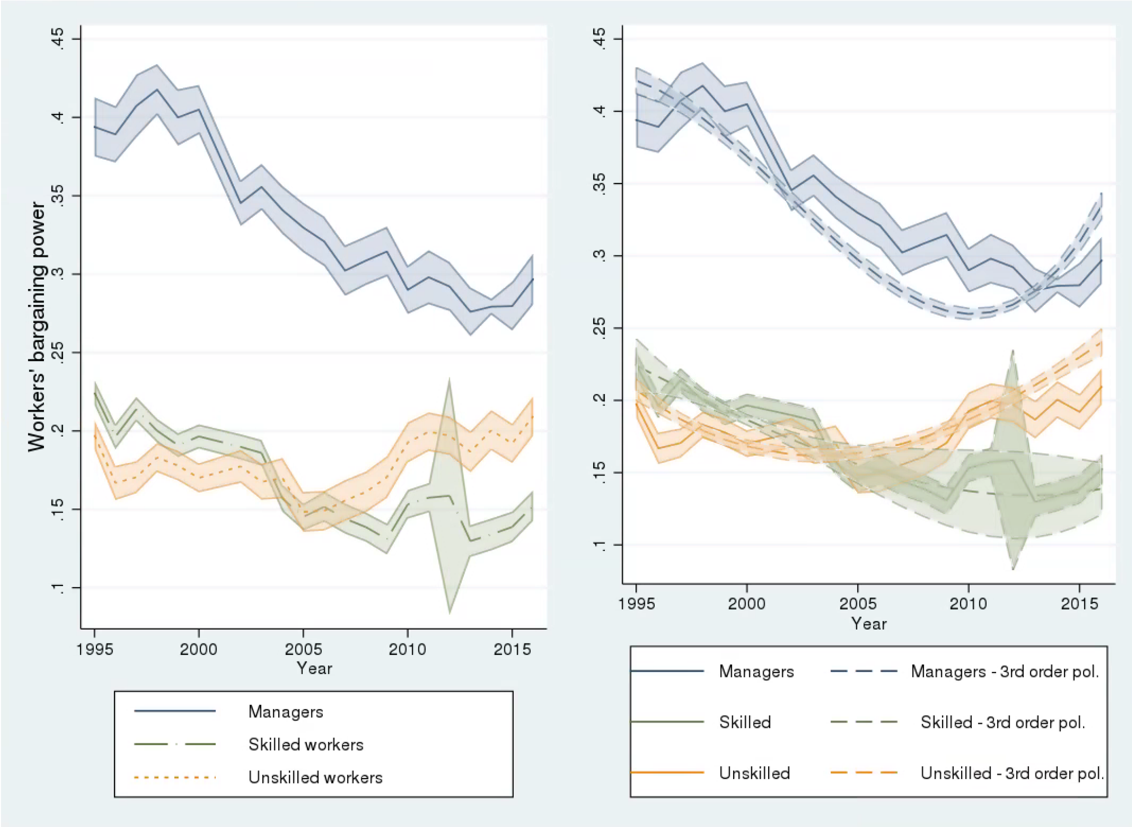
Variable	Coefficient (s.e.)		
	Managers	Skilled workers	Unskilled workers
β	0.3547***	0.1761***	0.1733***
	(0.0021)	(0.0002)	(0.0013)
$\zeta_{\text{tenure}}/10$	-0.0037	-0.0640***	-0.2681***
	(0.0130)	(0.0055)	(0.0045)
$\zeta_{\text{tenure}^2}/100$	0.1457***	0.0159***	0.0555***
	(0.0092)	(0.0046)	(0.0036)
$\zeta_{\text{tenure}^3}/1000$	-0.0290***	-0.0040***	-0.0006
	(0.0018)	(0.0009)	(0.0008)
$\zeta_{\text{age}}/10$	2.291***	-1.1753***	-1.4556***
	(0.6120)	(0.0413)	(0.0230)
$\zeta_{\text{age}^2}/100$	-0.6283***	0.2289***	0.3414***
	(0.1531)	(0.0106)	(0.0061)
$\zeta_{\text{age}^3}/1000$	0.0477***	-0.0086***	-0.0258***
	(0.0124)	(0.0009)	(0.0005)
$\zeta_{\text{education}}$	-0.0312***	0.0916***	-0.0452***
	(0.0028)	(0.0015)	(0.0012)
$\zeta_{\text{education}^2}$	-0.0040***	-0.0139***	0.0009***
	(0.0002)	(0.0001)	(0.0001)
ζ_{female}	0.0453***	-0.3189***	-0.2211***
	(0.0054)	(0.0025)	(0.0023)
Adjusted R^2	0.9034	0.8567	0.7288
Obs.	4,797,735	14,937,457	9,851,256

Notes: Robust clustered workplace standard errors used in first-stage. Robust standard errors use in the second-stage. ***, **, * denote significance at 0.01, 0.05, and 0.10 levels, respectively. *Sources:* Quadros de Pessoal, 1995-2009 and Relatório Único, 2010-2016.

5.2 The Prime Empirical Wage Setting Equation

In our benchmark empirical setting, we explicitly model an idiosyncratic time evolution of bargaining powers for each task market, through two alternative functional form specifications, expressing different smoothness degrees ex-ante assumed for such time progression. Ergo, in a more involving specification, we relax any smoothness prerequisite and estimate bargaining powers resorting to a vector of year-task market dummies, while in a more frugal alternative, we impose a type-task specific third order degree polynomial functional form of the time trend. Figure 6 presents the resulting workers' bargaining powers estimates.[46]

Figure III.6: Estimated Workers' Bargaining Power per Task Market



Notes: The fading shades correspond to the $95^{th} - 5^{th}$ confidence interval, using robust standard errors. *Sources:* Quadros de Pessoal and Relatório Único, 1995-2016.

At first glance, figure 6 reaffirms the existence of relevant bargaining power heterogeneity at task market level, and confirms the relevance of accounting for time evolution. Technically, even though the polynomial specification doesn't fully match the curves resulting from the best fitting polynomial trend of the corresponding dummies' series, it provides

[46]In the estimation, we adopted robust standard errors instead of clustering at any dimension. Abadie et al. (2017) advocate the absence of clustering in the presence of a fixed effect specification when there is homogeneous treatment effects within the cluster formed at the level of the fixed effect. We assume such homogeneity by design as the workplace heterogeneity arises solely from the heterogeneity in worker's characteristics and not from the valuation of their characteristics. Moreover, the use of average real hourly wage, i.e. $w(K, \beta)$, approximates our setting to the fixed effect setting.

The time variation of bargaining powers will be modelled in two alternative specifications. Firstly, we will consider

$$\beta_{j,t} = \mathbf{D}'\tilde{\beta}, \tag{III.36}$$

where $\mathbf{D}$ is a $[(J \times T) \times 1]$ vector of year-task market dummies, and $\tilde{\beta}$ is the corresponding vector of parameters. Alternatively, we will assume sufficient smoothness of the time series of bargaining powers, and consequently fit a polynomial approximation as:

$$\beta_{j,t} \approx b_0 + b_{1,j} \times t + b_{2,j} \times t^2 + b_{3,j} \times t^3. \tag{III.37}$$

a close approximation, and thus consists in a reliable modelling option particularly in the presence of smaller samples. Moreover, both sets of estimates are consistent with the results from table 2, and consequently confirm higher bargaining power levels for managers, and very identical levels for skilled and unskilled workers. While some studies present an higher bargaining power at the bottom of the wage table (for example Dumont et al. (2012)), others present either broad monotonicity between wage tables and bargaining powers, or even very identical, U-shaped, or mixed results depending on the sector of activity under analysis (see Cahuc et al. (2006), Mortensen et al. (2010) or even Bagger et al. (2014)).

On the time dimension, figure 6 records a downward trend in the bargaining powers unfolding in-tandem for skilled workers and managers, broadly up to the Great Recession, while for the unskilled workers we record a broad stability. These findings are consistent with the estimates of Hirsch and Schnabel (2011) for the entire German economy in the 2002-2009 period, where the authors resorting to a right-to-manage model record a one-third decline, which compares with 19 percent fall for managers, 20 percent for skilled workers and rough stability for the unskilled workers, as displayed in table 3. For the preceding period, i.e. 1995-2000, the cited authors find a stable environment whereas we record a decreasing trend in every task market. Moreover, our findings also broadly concurs with the evolution of the synthetic indicator of collective bargaining of Lombardi et al. (2020) in figure 2.

Table III.3: Average Predicted Change in Bargaining Powers Implied by Best Fitting 3rd Order Polynomial Trend to the Dummy Series

Period	Dates	Ratio $\beta_{final}/\beta_{initial} - 1$		
		Managers	Skilled workers	Unskilled workers
The Boom	1995-2000	-4.44%	-12.96%	-11.95%
The Slump	2000-2008	-17.98%	-20.58%	0.39%
Financial Crisis	2008-2011	-7.69%	-6.23%	5.77%
Euro Crisis	2011-2014	-3.43%	-0.89%	8.00%
Timid Recovery	2014-2016	1.78%	3.54%	6.04%
Overall	1995-2016	-28.89%	-33.48%	7.08%
β_{1995}		40.30%	21.76%	18.99%
β_{2016}		28.66%	14.48%	20.34%

Notes: The 3rd order polynomial best fitting curve to the dummy series of bargaining powers is used to avoid the over-influence of any transitory fluctuation. The used periods are collected from Blanchard and Portugal (2017) which outlines a detailed macroeconomic analysis of the Portuguese Economy. *Sources:* Quadros de Pessoal and Relatório Único, 1995-2016.

At the outbreak of the Great Recession, we estimate that bargaining powers were around historical minimums. As presented in both table 3 and figure 6, the path of bargaining powers was thereafter markedly idiosyncratic to each task market. For managers and skilled workers, throughout the crisis we record a consistent fall in bargaining powers, but at a less than halved pace relatively to the previous period. For the unskilled workers the bargaining drainage is halted, and we witnessed a substantial re-surge, which visibly commenced in the 2007-2010 period, coinciding with the most sizable surge in the minimum wage for the entire time-frame of the study, of around 18 percent.[47] In the post recession period, namely marked by the end of the adjustment program in July 2014, every task market observed an increase in the bargaining powers which was more pronounced the lower the position of the task markets at the wage tables. By the finale of our analysis, while for bottom of the distribution the bargaining drainage in the earlier period was fully recovered, for the managers and skilled workers the bargaining bygones are estimated of around 30 percent. Such abatement is a consequence of a long-lasting trend which has unfolded with notable stability.

The erosion of bargaining powers, for managers and skilled workers, particularly if amplified by a significant progressive income taxation, may spark a plethora of effects. Namely, the reduction in the wage differentials across skill groups and the increasing decoupling of the worker's wage from the firm productivity levels may degrade the incentives for skill acquisition and on job training, and the alignment of worker and firm incentives may be reduced.

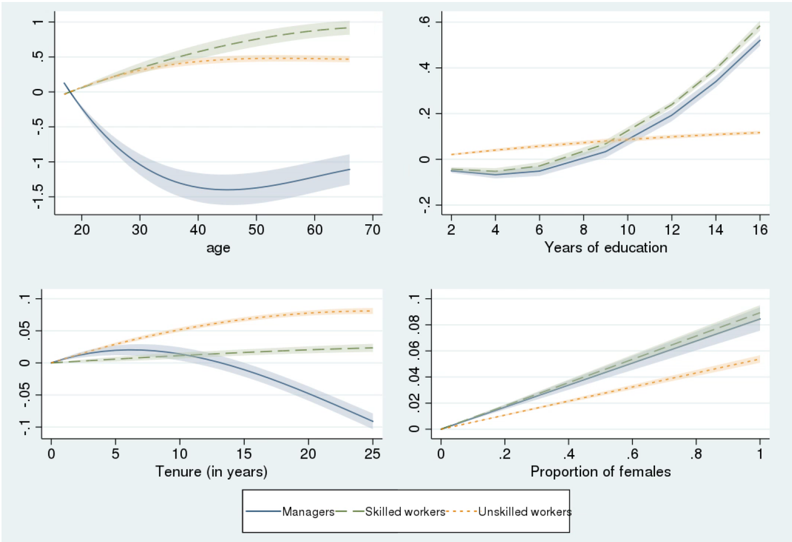
Regarding the remainder of the model, figure 7 displays the total effect of education, tenure, age and female proportions at the rank on the profile of real productivity of the match per hour, as well as some of its distributional features on those dimensions. As a general remark, the estimated behaviour of the productivity is consistent with the broad literature on wage determination, and as well with Postel-Vinay and Robin (2002), where the contribution of the individual characteristics become more relevant for higher skill level task markets.

In broad brushstrokes, education has little effect on the unskilled workers, while it increases exponentially the productivity of the remainder workforce, confirming in our model its role as primary choice-based productivity enhancer when the job can benefit from the proceeds of schoolwork. Also, it conforms with the idea of potential over-education when the worker doesn't land on a suitable rank position given her education (see Hartog (2000) for further

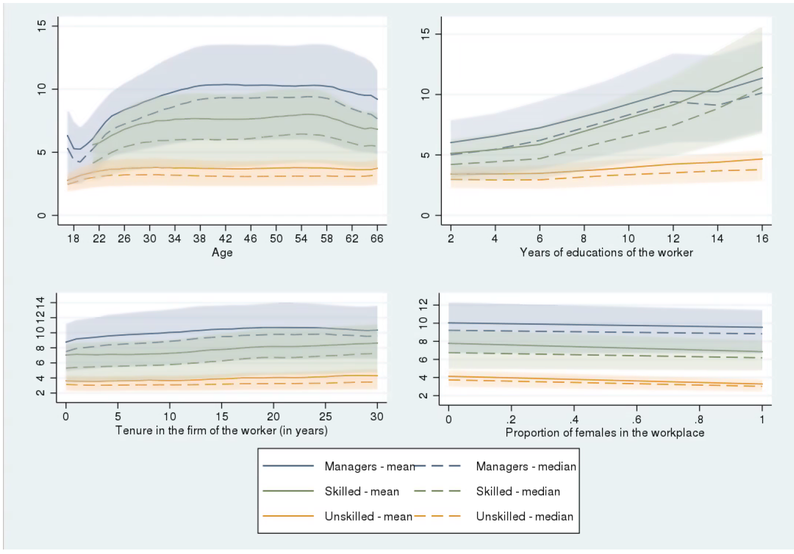
[47]The evolution of the monthly minimum wage is presented in figure 11.

Figure III.7: The Operating Cost Estimates

Panel A: Estimated total effect on real productivity per hour.



Panel B: Distributional features of the real productivity per hour.



Notes: On panel A, the fading grey shades corresponds to the $95^{th} - 5^{th}$ confidence interval. On panel B, the fading shades corresponds to the $75^{th} - 25^{th}$ percentile range of the implied distribution. *Sources:* Quadros de Pessoal and Relatório Único, 1995-2016.

details on over-education for Portugal).

Differently, the tenure on the firm albeit displaying lower effects when compared with education, present a reversed relationship. The lower fragments of the wage tables, primarily unskilled workers, but also at a lower magnitude skilled ones, benefit the most from the permanence at the firm, with a stable monotonic relationship, even for very long tenures. In a different archetype, the productivity gains for managers while increasing more sizably at early stages of the worker-firm match fade after the first five years, becoming increasingly detrimental for very long tenures. Overall, such profiles could be consistent with

Acemoglu and Pischke (1999), where increases in minimum wages, more likely to affect the bottom of the wage tables, induce higher levels of on-job training sponsored by firms.

Regarding the age profiles, the gains for skilled and unskilled workers are increasing with the wage tables, though ceasing to further accrue earlier in the career the lower the position of the worker on those wage tables. Relatively to managers, increases in experience translates in consistent real productivity gains at a latter stage in their careers, broadly after around the 40 years of age, which however coincides with the most sizable mass of existing managers.[48] Before the cited threshold, the younger the manager the higher his real productivity, ceteris paribus. Tentatively, their appointment encloses relevant traits that make them particularly productive, as it is likely to be linked with an entrepreneurial/family-owned business perspective - the ideal, and perhaps unique greenway to land on a managerial position so early in their careers (see Blanchflower and Oswald (2007) for an analysis on the traits of young entrepreneurs).

5.3 Elasticity of Wages to Exogenous Changes in the idiosyncratic Quasi-Rents

One of the most pursued measures when studying wage setting mechanisms consists in the elasticity of wages to exogenous changes in the idiosyncratic quasi-rents of the worker-firm match, or in general the response of wages to exogenous productivity shocks.[49] As noted by Card et al. (2018), such measures provide a succinct description of the link between productivity heterogeneity and wage inequality and it constitutes a cornerstone indicator in the rent-sharing literature. Beyond its own merits, the estimation of this elasticity, implied by the results of our model, contributes for the external validity of our results given the proliferation of studies with different empirical techniques, inclusively resorting to the Portuguese labour market.

In detail, the average elasticity of wages to exogenous changes in the quasi-rents is given

[48] In the period of analysis, 90 percent of the managers are older than 28 years, and 75 percent are older than 33.

[49] See Garin and Silvério (2019) for a compatible theory review of the difference in changes in quasi-rents which are: (a) idiosyncratic to the match; and (b) general to the relevant portions of the labour market. The major difference is due to the presence of feedback effects on the outside options. We analyze option (a) in this section.

by:[50]

$$\epsilon_t^{QR}(K,\beta) = \frac{\partial w_j(a|K,\beta)}{\partial QR_j(K,\beta)} \frac{QR_j(K,\beta)}{w_j(a|K,\beta)} = \beta_{j,t} \frac{QR_j(K,\beta)}{w_j(a|K,\beta)}. \tag{III.39}$$

Moreover, we implement the insight of Card et al. (2014) and compute the quasi-rents estimate as:

$$E_{ft}[QR_j(K,\beta)] = VAB_{ft}(K,\beta) - 0.1K - Out(a), \tag{III.40}$$

where $VAB_{ft}(K,\beta)$ is the value added per hour, K corresponds to the level of assets presented in the balance sheet of each firm, and 10% the considered costs of capital, as in Card et al. (2014). Given equations (39) and (40), we resort to a two-step GMM, with the logarithm of sales at firm level as instrument (i.e. $ln[F(K,\beta)]$), to estimate the average of the referred elasticity of the economy per year.

Table 4 presents our average result, 0.062, and a sample of comparable studies about the Portuguese labour market, which highlights the consistency of our findings with a significant branch in the literature that locates this elasticity estimates in the range between 0.05-0.15.[51] Regarding the time evolution, as presented in figure 8, the average elasticity is generally within the 0.05-0.15 bounds, and present a downward trend as in the bargaining powers, with a particular fall during the *Euro Crisis*.

Table III.4: Estimates of the Elasticity of Wages to Exogenous Changes in the Idiosyncratic Quasi-Rents for Portugal

Study	Dates	Elasticity (s.e.)
Our study	2005-2016	0.062 (0.0090)
Card et al. (2018)	2005-2009	0.056 (0.016)
Card et al. (2016)	2002-2009	Males - 0.14-0.16 Females - 0.04-0.05
Cardoso and Portela (2009)	1991-2000	0.00 (transitory shock) 0.09 (permanent shock)
Martins (2009)	1993-1995	0.03-0.05
Garin and Silvério (2019)	2005-2013	0.15 (0.066)

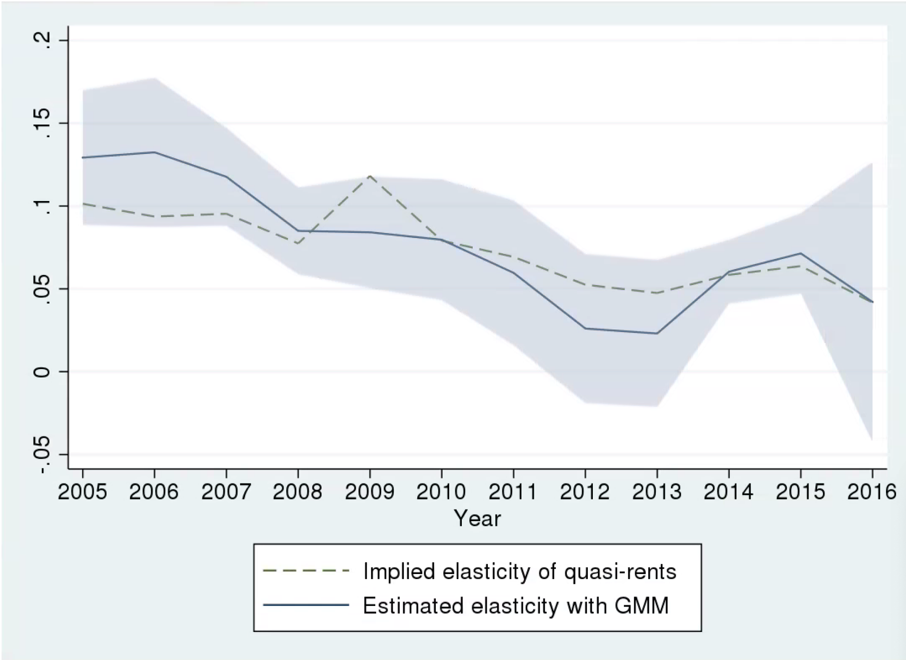
Sources: Quadros de Pessoal and Relatório Único, 1995-2016; SCIE 2005-2016; and the referred articles.

[50]The structure of wages relates with the quasi-rents implied by our model as:

$$w_j(a|K,\beta) = Out(a) + \beta_{j,t} QR_t(K,\beta). \tag{III.38}$$

[51]See Card et al. (2018) for a more extended review of the literature covering 22 different studies including for several European countries and the U.S.

Figure III.8: Estimated Average Elasticity of Wages to an Exogenous Change in the Quasi-Rents



Notes: The fading shades correspond to the $95^{th} - 5^{th}$ confidence interval range, using clustered standard errors at collective bargaining level. The implied elasticity of quasi-rents corresponds to the raw measure of equations (39) and (40). *Sources:* Quadros de Pessoal and Relatório Único, 1995-2016.

5.4 Passthrough Rate of Bargained Wages

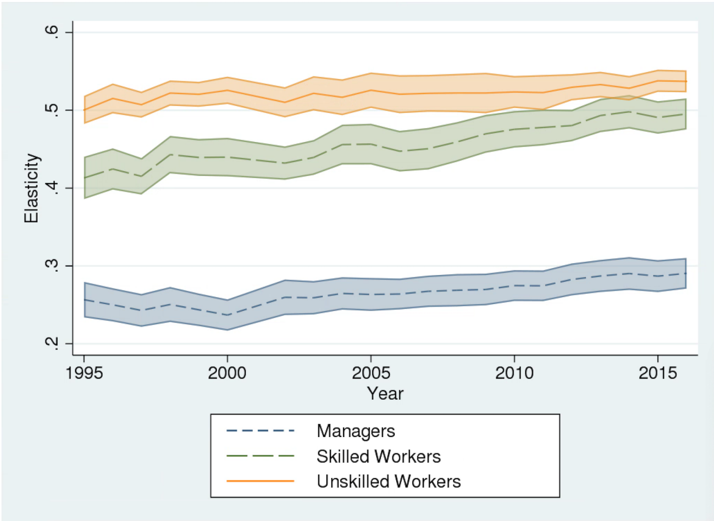
In the analysis of the wage setting in the continental European labour markets, the passthrough rate of wage floors constitutes a particularly relevant market measure. It displays how the settlement of bargained wages translates to total wages, thus broadly displaying the influence of collective bargaining wage setting on the total wages. In our setting, the average passthrough per year and task market, i.e. the average elasticity of the total wage to the wage floor, is given by:

$$\tau(j,t) = E\Big[(1-\beta_{j,t})\frac{Out(a)}{w(a|K,\beta)}\Big]. \tag{III.41}$$

Our results are obtained through a standard two-step GMM with clustered standard errors at collective bargaining level. Those support a classical continental European wage setting feature, where the raise of bargained wages correspond to a shrinkage of the wage cushion, due to an imperfect passthrough. We estimate the average passthrough at 44.8 percent, implying that a 10 percent increase of bargained wages translates into a 4.5 percent increase on total wages. Our findings concur with Card and Cardoso (2021) which estimates an average passthrough of about 50 percent.

Moreover, our finding displays the relevance of collective bargaining, as changes in wage floors are associated to meaningful changes in total wages. According to our results in figure 9, the link between wage floors and total wages is ordered, with the lowest task markets displaying the strongest link, in a relationship which is significantly stable overtime. This finding is also consistent with Card and Cardoso (2021) which presents evidence of an ordered passthrough rates with skill groups.

Figure III.9: Estimated Average Passthrough



Notes: The fading shades correspond to the $95^{th} - 5^{th}$ confidence interval range, using clustered standard errors at collective bargaining level. *Sources:* Quadros de Pessoal and Relatório Único, 1995-2016.

5.5 An Analysis of Heterogeneity

A classical heuristic method to assess wage differentials and the contribution of its constituents to mounting inequalities is a covariance decomposition assessment. This section assesses the contribution of each of the components of our primal wage setting equation, accounting for the adopted formulation of the operating costs. Consequently, we have:

$$ln[w_j(a_t|K_t,\beta_t) \times \text{hours}_{it}] = ln[\text{hours}_{it}] + ln\Bigg[\underbrace{(1-\beta_{j,t})Out^{\star}(a_t)}_{\text{Out. option component}} + \underbrace{\int_0^1 z^{\frac{1-\beta_{j,t}}{\beta_{j,t}}} \frac{\partial F(\mathbf{Q}_{j,t}(z)\mathbf{N}(K_t,\beta_t),K_t)}{\partial N_j(K_t,\beta_{j,t})}dz}_{\text{Real productivity per hour of the match}}$$
$$- \underbrace{\beta_{j,t}\mathbf{X}_{i,t}\zeta_j}_{\text{Obs. operating costs comp.}} + \underbrace{\beta_{j,t}[E[E(\xi(i,t)|rank(i,t))|j,t]] - \beta_{j,t}\xi(i,t)}_{\text{Unobservable operating costs comp.}} + \underbrace{\epsilon_{i,f,j,t}}_{\text{residual}}\Bigg]. \tag{III.42}$$

with the covariance decomposition components obtained as:

$$\mathbf{\Gamma}(n) = \frac{cov\Big(ln[w_j(a_t|K_t,\beta_t) \times \text{hours}_{it}]; \Gamma_{i,f,j,t}(n)\Big)}{var(ln[w_j(a_t|K_t,\beta_t) \times \text{hours}_{it}])}, \quad \text{with} \sum_{n=1}^{6} \mathbf{\Gamma}(n) = 1 \tag{III.43}$$

with $\Gamma_{i,f,j,t}(n)$ representing each of the components of equation (42). Inspired on the same structural view of decomposing the components an equivalent mean decomposition is performed. While we could perform an yearly decomposition, we present the results for the average decomposition given we find a broad stability of the contributes and of the overall wage inequality over the period of analysis.[52] Differently, Card et al. (2014) and Song et al. (2019) link the evolution of heterogeneity with perceived growing inequality in the US and Germany, which in their case is noticeable.

Our average covariance and mean decomposition results are presented in table 5. Our first finding corresponds to the peripheral role of the hours worked in wage dispersion, as it even reduces the magnitude of the heterogeneity of monthly wages *vis-á-vis* the hourly counterpart, implying a negative correlation between hourly wages and hours worked. Within the hourly wages, the major drivers of wage differentials resides at workplace level, which explains around 63 percent of the overall heterogeneity in monthly wages, while the components attributed to the worker, namely the level of the outside options and operating cost components, contribute around 22 percent.

Within the later dimension, it is noteworthy the different weight of the outside option in the covariance and mean decompositions of table 5. While the outside option component only weights around 7 percent of the variability of wages, it represents around 41 percent of the average wage paid in the economy. This finding highlights the property of the outside option, and the role of labour market institutions, in compressing wages relatively to the implied distribution of the real productivity of the match.

Holistically, table 5 indicates the workplace level productivity as the core engine of wage dispersion, and consequently for the prevalence of heterogeneous labour market histories of otherwise identical workers, as measured by the level of their outside options and operating costs. In an experimental approach, this analysis signals that two unemployed workers with identical ex-ante valuations for their work may end up with significantly different wages based on the way they (re-)enter the labour market. This finding stresses the importance of studying the mechanisms, and their underlying efficiency, by which workers pair with a given level of occupation, and subsequently a given firm. It gives credit to the

[52] See figure 11 for the evolution of the percentiles of the wage distribution.

literature branch that verses on the imperfectly competitive nature of the labour market, either from a more monopsony perspective as in Manning (2011) and Card et al. (2018), or a bargaining in a frictional labour market as we adopted.

Table III.5: Average Covariance and Mean Decompositions of Monthly Wages, over the Period 1995-2016

Panel A: Average Variance Decomposition of Monthly Wages

Dimension	Detailed Components	Contributions	
Extensive Margin	Hours worked		-2.66%
Worker	Outside Option	6.92%	21.84%
	Observed Operating Costs	4.95%	
	Unobserved Operating Costs	9.97%	
Workplace	Workplace Real Productivity		62.61%
Residual			18.21%

Panel B: Average Mean Decomposition of Monthly Wages

Dimension	Detailed Components	Contributions	
Worker	Outside Option	40.67%	29.5%
	Observed Operating Costs	-3.71%	
	Unobserved Operating Costs	-7.46%	
Workplace	Workplace Real Productivity		71.29%
Residual			-0.8%

Sources: Quadros de Pessoal and Relatório Único, 1995-2016.

In general, the literature has seen a sizable proliferation of variance decompositions of wages, particularly since the advent of fixed effects spurred in the Abowd et al. (1999) model (i.e. AKM). Typically, those, as in Card et al. (2014) for Germany, Torres et al. (2018) for Portugal, and Song et al. (2019) for the US, attributes a leading role of the worker dimension, which can be perceived at odds with our findings. However that might not be the case. Firstly, note that we perform covariance decompositions and not variance-covariance decompositions.[53] Secondly, our central results from table 5 are not based on firm dimension, but workplace dimension which is substantially different and more atomistic, as it combines firm and task market. Thirdly, the studies resorting to ranking algorithms or new estimator designs capable of solving the limited mobility bias issue present mixed results. Borovicková and Shimer (2017) for Austria indicates that wage heterogeneity is attributable more to the firm side than the worker side, while Bonhomme et al. (2019) for Sweden, Bonhomme et al. (2020) for U.S. and several European countries, and Kline et al. (2019) for Italy point for a leading role of the worker dimension with a much marginal role for firms relatively to the AKM designs. Finally, in a more structural

[53]The results could be comparable by considering our covariances correspond to the relevant variance plus half of the covariance term where the relevant term is displayed.

and rent-sharing perspective, Mortensen et al. (2010) for Denmark conforms with our findings by attributing a leading role to the *rent-sharing* component of wages *vis-á-vis* the labour heterogeneity. As noted by both Borovicková and Shimer (2017) and Bonhomme et al. (2020), the different specifications for modelling earnings and the processes of worker and firm heterogeneity can be a leading cause for the disparity of results.

Amidst this debate, we partial out the main components of table 5 decomposition into worker, task market, firm and time dimensions. This procedure approaches our decomposition to an AKM setting, given the intuition of equations 2 and 3 of section 2. Then, using the same dataset, we resort to a typical covariance decomposition of 3 different AKM based strategies, which correspond to: (a) a firm-worker-year fixed effect formulation, in the spirit of Card et al. (2014) and Song et al. (2019); (b) a firm, worker, year and task market decomposition in the spirit of Torres et al. (2018); and (c) a workplace-worker decomposition, where in the workplace fixed effect, measured as firm-task-market-year cells, those three dimensions and their complementarities are captured.

In this exercise, we could perform bias correction for the covariances estimated in the AKM, following recent trends in the literature. However, we take the approach of Song et al. (2019), which abstains from going in this direction by noticing: (a) the computational cost of the solution of Kline et al. (2019); and (b) the additional assumption framework required in Andrews et al. (2012) and Borovicková and Shimer (2017). If anything the bias is likely to affect equally the AKM decompositions and our AKM detailed decomposition, thus establishing a comparable ground between our model and the most canonical empirical approach. Noteworthy, our aggregate decomposition in table 5 does not suffer from limited mobility bias, given it is based on a very small set of parameters.

The findings reported in table 6, particularly the comparison of the workplace-worker AKM and our methodology presents a bridge between both approaches, as the workplace dimension weights roughly around 60 percent of the entire variability of real hourly wages in both methodologies. The differences between both lie on the worker and residual components, with the latter sizably reduced in the AKM approach, and as a consequence the formed fully absorbing the proceeds of the higher fit of the model. As one moves towards more time invariant formulations of the firm side, and account for a partition of the labour market into types of tasks, the worker fixed effect is largely capable to absorb the *leftovers* of the variation which was previously enclosed in the workplace definition, with the types of tasks acquiring a peripheral role.

Altogether, beyond modelling differences in earnings and in the processes of worker and

Table III.6: Comparison of Covariance Decomposition between AKM and our Methodology of the Logarithm of Real Hourly Wages

Dimension	Components	Our methodology: Aggregate	Our methodology: Detailed	AKM: Workplace Worker	AKM: Firm Worker Task market Year	AKM: Firm Worker Year
Worker	Worker attributes	21.27%	10.60%	34.37%	38.06%	41.42%
	Task market FE		6.06%	-	-	-
	Generic time FE		2.49%	-	-	-
	Within residual		2.12%	-	-	-
Workplace	Firm FE	60.99%	37.59%	58.16%	35.60%	39.02%
	Task market FE		9.35%		-	-
	Generic time FE		2.68%		-	-
	Within residual		11.37%		-	-
Year effects FE		-	-	-	6.91%	7.68%
Task market FE		-	-	-	7.93%	-
Residual		17.74%	17.74%	7.47%	11.46%	11.88%

Notes: The detailed decomposition consists in resorting to AKM models to decompose the worker and workplace components, namely: $(1-\beta_{j,t})Out^{*}(a_t) - \beta_{j,t}[\mathbf{X}_{i,t}\zeta_j + \xi(i,t)] + \beta_{j,t}E[E(\xi(i,t)|rank(i,t))|j,t] = \mathbf{X}_{i,t}\tilde{\zeta} + \alpha_i + \delta_j + \chi_t + \epsilon_t$, and $\int_0^1 z^{\frac{1-\beta_{j,t}}{\beta_{j,t}}} \frac{\partial F(\mathbf{Q}_{j,t}(z)\mathbf{N}(K_t,\beta_t),K_t)}{\partial N_j(K_t,\beta_{j,t})} dz - \beta_{j,t}E[E(\xi(i,t)|rank(i,t))|j,t] = \alpha_i + \gamma_f + \delta_j + \chi_t + \epsilon_t$, respectively. *Sources:* Quadros de Pessoal and Relatório Único, 1995-2016.

firm heterogeneity, this analysis raises the question of the boundaries of the worker effect realm in such estimations as to precisely which components of wages are encompassed in such set of parameters. Further, it also reinforces the importance to further assess to what extend such worker's dimension primary role could be a data based idiosyncrasy, a mechanical consequence of the relative dimensionality of the worker's parameter set versus the firm, the consequence of any statistical shrinkage of such dimensionality to conform workers and firms into classes, and/or the biases generated from the proper mobility patterns of the workers - known to particularly affect AKM designs. In this context, our approach provides a novel decomposition which links each covariance decomposition component to the structural component of the equilibrium wage equation. While, a comparative scrutiny of the referred empirical methods would be interesting, it is beyond the scope of this chapter.

5.6 Assortative Matching

In another but related domain, the literature has embodied the proliferation of studies with measures of assortative matching, canonically defined as the correlation between the worker and firm components of wages. This empirical turf signalled a plausible defect of the fixed effect designs based on Abowd et al. (1999), as it consistently estimated unrealistically small or even negative correlations. These unheralded results soon crystallized as one of the most ad rem *puzzles* of the referred approach. In our setting, a measure of this class is given by:

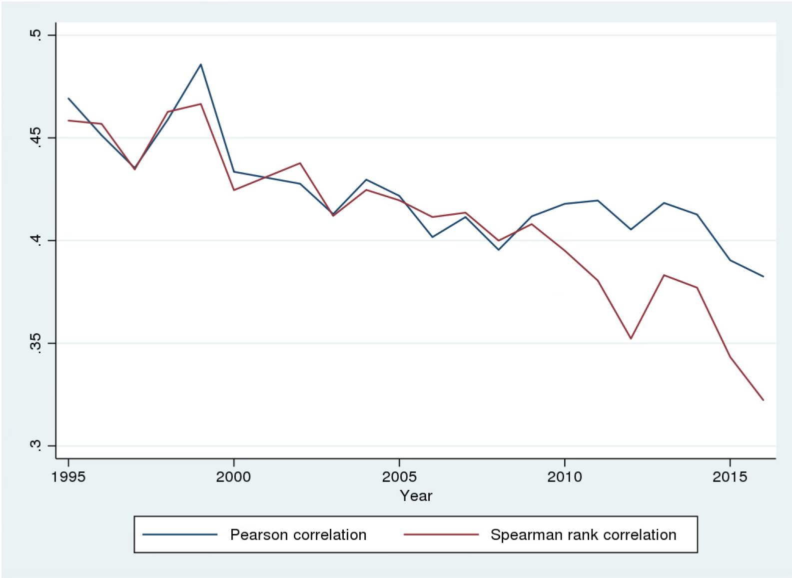
$$AM_t = corr\left\{ \underbrace{ln\left(E\left[\frac{1}{\beta_{j,t}}\int_0^1 z^{\frac{1-\beta_{j,t}}{\beta_{j,t}}}\frac{\partial F(\mathbf{Q}_{j,t}(z)\mathbf{N}(K_t,\beta_t),K_t)}{\partial N_j(K_t,\beta_{j,t})}dz\middle| f,t\right] - E[A(j,a_t)|f,t]\right)}_{\text{firm component}};\ \underbrace{ln[Out^{\star}(a_t)]}_{\text{worker component}}\right\}, \tag{III.44}$$

where intuitively the worker component is given by his outside option value, while the firm component is composed by the average real productivity in the firm.

The empirical estimates are translated in figure 10, where we resort to both the Pearson and Spearman correlations to estimate equation (44). Our findings locate the Pearson correlation in the range 0.38-0.48, with an average over the period of 44.14%, which is broadly consistent with the Spearman rank correlation, despite its moderate decline in the latter years of the sample. In detail, our time evolution do not present the upward trend of Card et al. (2014), which the authors link with the increase of the wage inequality in Germany; it does not record the significant fall as in Torres et al. (2018); and it conforms with a broad stability as observed in Lentz et al. (2018) for Denmark.

Regarding the level, which has been the hotspot of scrutiny for the AKM approach, our findings are consistent with the most recent developments on this specific literature, which are resumed in table 7. Noteworthy, our approach seems not to suffer from the downward bias that limited mobility allegedly causes to AKM, and conforms with the findings of Bonhomme et al. (2019) for Sweden, Borovicková and Shimer (2017) for Austria, and are somewhat higher than Kline et al. (2019) for Veneto, Bonhomme et al. (2020) for a range of European countries and the US, and Lentz et al. (2018) for Denmark.

Figure III.10: Measure of Assortative Measure



Sources: Quadros de Pessoal and Relatório Único, 1995-2016.

Table III.7: Breviary on Recent Results on Assortative Matching

Study	Dates	Country	Method	Estimate
Our study	1995-2016	PT	Our methodology AKM	0.4414 0.1797
Torres et al. (2018)	1986-2013	PT	Firm Measures AKM	0.308 0.08
Card et al. (2016)	2002-2009	PT	AKM	0.152-0.162
Kline et al. (2019)	1984-2001	IT (Veneto)	Leave-out	0.283
Borovicková and Shimer (2017)	1972-2007	AUS	Non-parametric measure	Within 0.4-0.6
Hagedorn et al. (2017)	1993-2007	GER	Ranking Algorithm AKM	0.75 0.055
Card et al. (2013)	1985-2009	GER	AKM	0.034-0.249
Andrews et al. (2012)	1998-2007	GER	Turnover correction	0.2-0.3
Andrews et al. (2008)	1993-1997	GER	High turnover plants	0.224
Bonhomme et al. (2019)	1997-2008	SWE	Bonhomme et al. (2017)	0.4913 (static) 0.419 (dynamic)
Lentz et al. (2018)	1985-2013	DEN	Bonhomme et al. (2017)	0.28
Lachowska et al. (2020)	2002-2014	Washington	AKM Leave-out	-0.03-0.1 0.1-0.2
Abowd et al. (2019)	1999-2003	US	AKM Structural	0.1 0.0
Woodcock (2015)	1990-1999	US	Match effects	-0.1
Bonhomme et al. (2020)	1996-2015	US, IT, AUS, NOR, SWE	Woodcock (2015) Bonhomme et al. (2019)	0.24-0.34
Lopes de Melo (2008)	1972-2007	BRA	Co-workers AKM	0.3 -0.08

6 The Resilience of the Wage Setting Mechanisms: Like a Bridge over Troubled Water

One of the advantages of studying Portugal in the last years resides precisely in the severe magnitude of the Great Recession shock witnessed by this continental European labour market. Moreover, the mishaps of the economic performance of the country, which records the last period of sizable growth ending by 2001, being followed by a long slump, two geminated crisis, and a modest recovery from 2014 onwards, presents an economy and a labour market in a severe frail standing (see figure 11 and Blanchard (2007) and Blanchard and Portugal (2017) for two analysis of the Portuguese economy over the timeline of our study). Altogether, it presents the framework of assessing a Southern European and French based labour market in distress.

Within these troubled times, the two crisis, most notably the European debt crisis, decisively stroke the Portuguese economy, sparking the need for a significant loan from the European Institutions and IMF to avoid an immediate sudden stop by 2011, backed by significant economic conditionality under a *Financial Assistance Program.* On the labour market side, while the collective bargaining apparatus kept its kinetics in the the wake of the financial crisis shock, potentially even witnessing a more wage prone policy, it came to a complete stall during the European debt crisis.[54]

Despite the regulatory stall, the labour market entropy was by 2013 evident, with the unemployment rate skyrocketing for record highs, in a phenomena coined by Carneiro et al. (2014) as *a catastrophic job destruction.* As noted in figure 12 and confirmed by Carneiro et al. (2014), the dramatic surge in unemployment was largely fuelled by very low job creation dynamics, and a stunning and unique surge of bankruptcies and firms exiting the market.

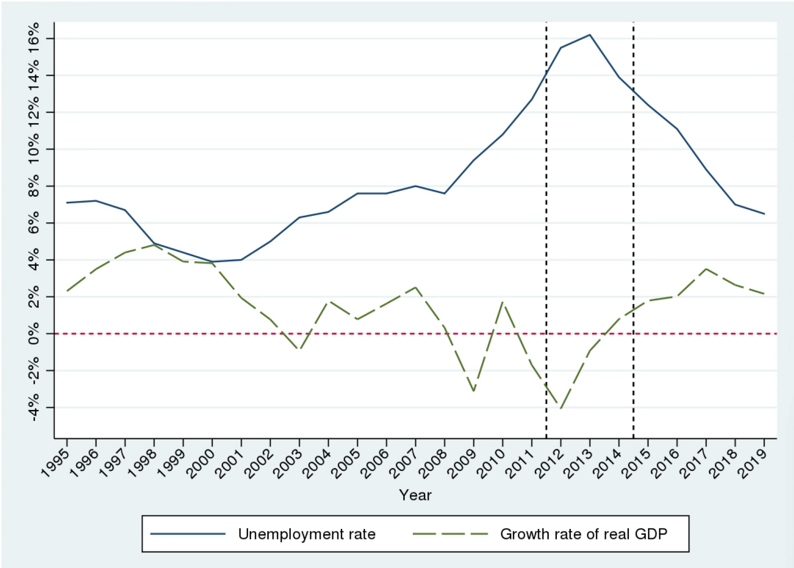
Simultaneously, as noted in figure 11, the wages endured the shock with some signs of downward nominal wage rigidity, evidenced by a sizable proportion of base wages, around 70 percent in our sample for 2013, frozen, while the distribution of real total wages shifted rightwards, in a more sizable drift in the higher percentiles of the distribution, thus evidencing a downward real adjustment capable of preserving, or even shrinking the pre-crisis wage inequality level.

By the end of the adjustment program in 2014, and at the outbreak of the recovery path,

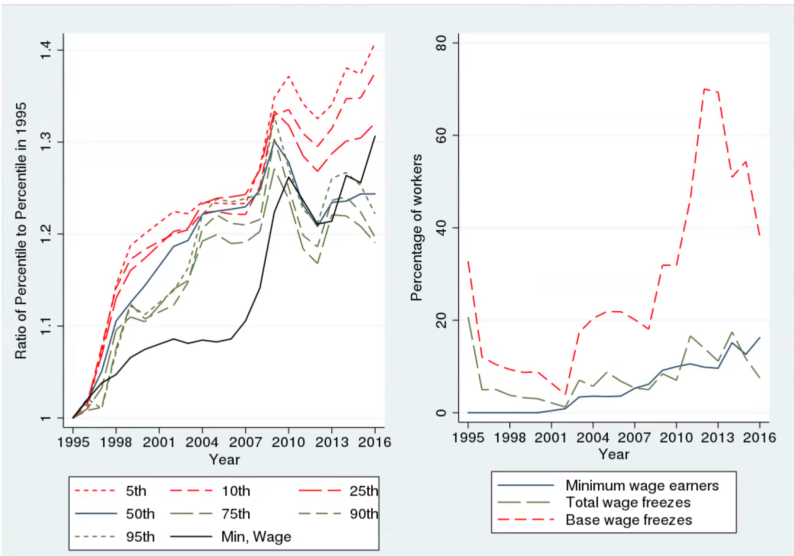
[54] As presented in figure 11, the financial crisis coincided with the most sizable surge on the minimum wage in our timeframe, as a policy intended to boost internal demand in the wake of the financial crisis.

Figure III.11: The Labour Market Dynamics

Panel A: The the real GDP growth and the unemployment rate.



Panel B: The evolution of the wage distribution in the labour market.

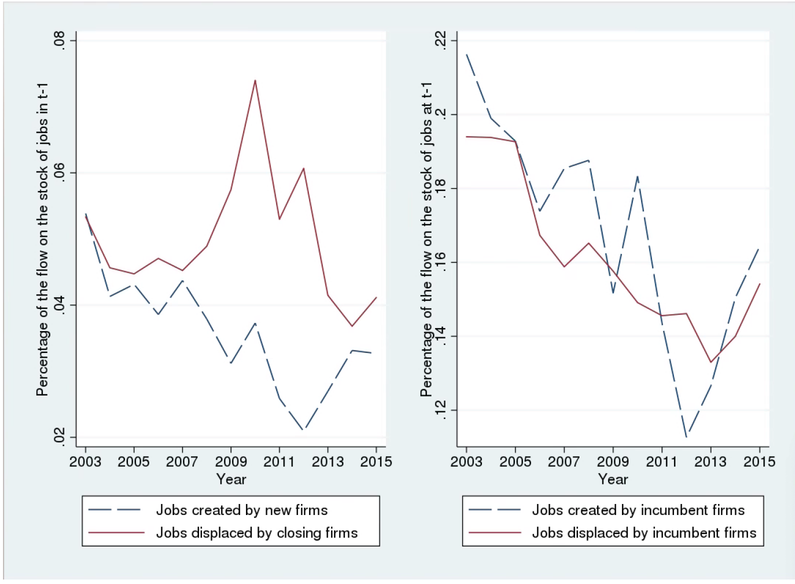


Notes: In the graph the two vertical black dashed lines delimit the period of *Financial Assistance Program* with the ECB, the IMF and the European Commission, namely between 7th April 2011 and 30 June 2014. *Sources:* Quadros de Pessoal and Relatório Único, 1995-2016; Pordata website.

the capability of the labour market to follow the recovery dynamics of the economy, and the timespan required for it to recoup pre-crisis levels, both on wages and unemployment was the quintessential question. Alongside, the labour attachment of the newly created employment, the potential regime shifts on the wage setting mechanisms, and the conceivable alterations on the mechanisms of labour market matching, capable of shifting the quality of newly created matches were pertaining queries.

Regarding the core question, figure 11 briefly provide the answer. In the first two years

Figure III.12: The Evolution of the Job Flows in the Labour Market



Sources: Quadros de Pessoal and Relatório Único, 1995-2016.

of recovery, the labour market exhibited a strong and steady recovery path, affirming its capability to absorb large chunks of unemployment in a relatively short period of time, while keeping moderate real wage evolution, particularly at the top of the distribution.

Precisely, regarding the wage dynamics, this chapter provides answers for the latter two questions, pertaining the wage setting mechanisms and the assortativeness of the economy. Throughout the entire timeframe of analysis, and particularly during the European debt crisis, the resilience of the wage setting mechanisms and of the matching quality in the labour market is remarkable. For those incumbent workers that were spared from unemployment, a freeze of base wages, coupled with at most a 10 percent fall in real wages was momentarily imposed, but the underlying collective bargaining mechanisms of wage setting were not destroyed, thus hastening the wage recovery once the crisis fade out. Beyond significant doubts, those workers that were capable to sail out the crisis while employed faced a solid bridge over troubled waters, particularly at the bottom of the wage distribution. Their real wage losses broadly represented the depreciation of their real outside option value and the real value of quasi-rents. For those that faced the prospects of unemployment or tried to enter the tormented market, they encountered dire prospects of employability during the crisis, but witnessed a swift strong recovery, that in large proportions absorbed them into employment. In toto, the assortativeness of the market, if something, recorded a moderate decline and the bargaining powers kept their trends virtually unchanged.

7 Final Remarks

The analysis of the wage setting mechanisms in a typical Southern European and French based labour market is the focal point of this chapter. For this purpose, we develop an empirically implementable microeconomic search and matching model with a collective bargaining apparatus, and we implement it in the Portuguese labour market using data from 1995 until 2016.

The proposed model has the convenience of discipline the use of data about the characteristics of the placement of the worker-firm match on the collective bargaining wage tables - the most perennial and comparable characterization of the labour relationships. Consequently, our empirical identification do not rely on the mobility of workers across firms, or on the definition and estimation of a production function or marginal product. Despite such flexibility, the framework provided allows for the estimation of bargaining powers, elasticities of quasi-rents, the passthrough rate of bargained wages, assortative matching and variance decomposition of wages in a fully unified framework.

The macroeconomic context of our analysis has been particularly turbulent, which underscores the displayed resilience of the wage setting mechanism, particularly at the bottom of the wage distributions, as a noteworthy feature of the market. For those that were capable to remain employed throughout the rough waters, the wage setting laid out a sound bridge, where the potential impact of the macroeconomic outlooks was circumscribed to temporary and moderate wage losses, particularly at the top of the wage distribution. Those reflected the temporary decline in the valuation of the real quasi-rents of the worker-firm match and of the worker's real outside options.

Finally, the structural slow paced continuous erosion of bargaining powers, for managers and skilled workers, highlights potential future productivity hazards, particularly if amplified by a significant progressive income taxation. In the absence of wage differentials across skill groups, and the increasing decoupling of wages from firm productivity levels, the sorting in the economy may be degraded, the incentives for skill acquisition and on job training may be abated, and the alignment of worker and firm incentives may be reduced. Any of these endanger future productivity levels.

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Chapter IV

The Last Dance? Credit Cycles and Labour Market Adjustments throughout Downturns

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Abstract

The Great Recession in Continental Europe sparked a Great Unemployment Divergence, led by job destruction in some countries. I resort to a Bewley-Huggett-Aiyagari incomplete-markets model with an indirect search frictional labour market, financial constraints, labour market dualism, and downward nominal wage rigidity. I show that a catastrophic job destruction phenomena can take place in the presence of significant aggregate negative productivity shock, leading to a costly process of destruction of inter-temporally viable jobs - *a last dance episode.* For such outcome, the leverage of firms and the management of liquidity along the downturn are critical. In a calibration to portray the Portuguese economy, no intervention would imply an elasticity between the initial downturn and the unemployment rate of close to 1 for downturns of more than 5-6 percent. In such context, several balanced-budget policies targeting the liquidity in permanent contracts prove to be capable to alleviate sizably the job destruction mechanism.

1 Introduction

The Great Recession in the European Union was defined by its *Great Unemployment Divergence*, particularly between neighbouring countries as France, Portugal and Spain, or Italy and Germany (see Boeri and Jimeno (2015) for an overview). Despite similar starting points, and often identical institutional designs (see Botero et al. (2004)), some sailed through without significant unemployment spikes, while others recorded *catastrophic job destruction spells*, defined by a strong spike in job destruction that led to a significant rise in unemployment, as presented by Carneiro et al. (2014). What could justify such divergent paths?

A standard agency problem between borrowers and lenders imply the existence of credit constraints, which weaken the liquidity of firms in the presence of a negative MIT type shock (see Gertler and Gilchrist (2018) for a broad overview). Even from an aggregate perspective, Holmstrom and Tirole (2011) highlights that the constant need of firms to manage and forecast liquidity and the *limited pledgeability* of the entrepreneur, as only part of his income or wealth can be borrowed upon or used as collateral, provides a conceptual framework where liquidity provision would be insufficient in the presence of aggregate liquidity shocks. Empirically, among a plethora of studies in the context of the Great Recession, Benmelech et al. (2011), Bentolila et al. (2013), Berg (2016) and Chodorow-Reich (2014) shows a significant quantitative importance of financial constraints in the employment decisions of firms, and Boeri et al. (2012) reports a clear link between firm leverage and job destruction. In the latter, more leveraged firms may see their liquidity being suddenly called back by the lender, thus affecting their ability to run and manage existing jobs, forcing them to destroy some. In toto, if the external financing channel shuts, and the shareholders do not inject further equity, firms can only turn to the adjustment of input utilization, and their prices, to correct liquidity shortages, and alleviate the risk of failure.

To address the ability of firms to resort to this *internal financing channel*, one needs to define the institutional framework of the labour market and the financing of the economy, as those shape the magnitude and the intensive-extensive margin calibration of labour and capital adjustments. Regarding the labour market, I define the continental labour market institution, where there is downward nominal wage rigidity, and following the insights of Bentolila et al. (2013) and Boeri and Jimeno (2015), contract duality, with permanent jobs enjoying higher employment protection, expressed by the existence of firing taxes and higher predicted job tenures. Regarding the financing of the economy, while studying the shape of policies devised to alter it, such as monetary policy, is worth pursuing, in this study I focus on the role of labour market policies. Accordingly, a stable and comparable environment of external financing provides an orderly benchmark, and thus, I keep the external financing of firms constant as percentage of assets.

Altogether, I propose a Bewley-Huggett-Aiyagari incomplete-markets setting with an indirect search frictional labour market type model, in the spirit of Krusell et al. (2010).[1] Then, I extend it to display: (a) increasing productivity with wealth, as proposed by Eeckhout and Sepahsalari (2020);[2] (b) financial constraints on the firm side in the spirit of Kiyotaki and Moore (1997);[3] (c) the referred permanent-temporary contract duality; and (d) the downward nominal wage rigidity, in the spirit of Schmitt-Grohé and Uribe (2016). The heterogeneity of the model, coupled with its realistic wage setting, the considered labour market institutions, and the analysis of the workers' savings problem allows for a just sufficiently rich environment to analyse what conditions spark a catastrophic burst of employment in the presence of stable financing of the economy, and which are the most targeted workers.

The first contribution of this chapter is to clearly show that a model of this class is capable to display catastrophic job destruction spells, for a model reasonably calibrated to match the Portuguese economy. Indeed, for medium to severe aggregate temporary downturns in productivity, above 5-6 percent of the stationary equilibrium levels, the unemployment rate responds on a 1-to-1 basis to the initial productivity shock. This process is led by job destruction of existing jobs, leading to a reallocation process that is more costly than ensuring the continuity of most of the job assignments, in what I coin as a *last dance phenomena*, as most of those jobs are inter-temporally viable.[4]

The second contribution of this chapter is to clearly establish a comparison between the downward nominal wage rigidity environment and a benchmark wage flexibility environment, while preserving the other characteristics of the model. This provides a useful comparison, where one can decouple the job creation weakening in the outcome of the productivity downturn, and the job destruction mechanism. From this comparison, this

[1] See de Almeida Vilares and Reis (2021) for an empirical analysis of the suitability of a search and matching structure of the wage setting for Portugal.

[2] See Bernstein et al. (2021) for an empirical analysis of the effect of negative wealth shocks on the productivity of U.S. innovative workers. It confirms the wealth-productivity relationship proposed.

[3] Holmstrom and Tirole (2011) focuses in the ability of the *private sector* to generate enough liquidity to fund aggregate liquidity needs to implement a second best production plan. Here we follow Kiyotaki and Moore (1997) in defining financial constraints at firm level.

[4] *The Last Dance* corresponds to a documentary describing the decision of the Chicago Bulls to dismantle its current winning team and undergo into a deep restructure. It may find parallel in the phenomena to dissolve inter-temporally viable firms due to instantaneous leverage concerns.

chapter shows that the job destruction mechanism becomes particularly acute for medium to severe downturns and coincides with the sizable spike in unemployment rate levels the model prescribes. Consequently, it highlights the initial levels of leverage and its the management along the adjustment path as decisive in determining the path of the unemployment rate, and potentially in explaining the diverging paths undertaken by apparently similar labour markets.

The third contribution of the chapter is to explore a set of budget balanced labour market policies focused on permanent contracts, and their ability to alleviate the liquidity constraint, and thus the job destruction phenomena. Concretely, I consider: (a) a cash transfer to firms balanced with the saved unemployment benefits' proceeds; (b) a mandated reduction of wages; (c) a cash transfer to firms balanced with firms' future payments; and (d) temporary wage flexibility. Implemented for a year, with a quarter of implementation lag, I found that those policies can alleviate job destruction in the presence of aggregate productivity shocks, thus reducing the unemployment peak in the trough. Moreover, those improve worker's welfare along the worker's wealth distribution and preserve higher levels of capital stocks, while prescribing substantially different wage profiles along the adjustment paths. Consequently, the chapter puts forward a wide set of policies that could be adopted in the presence of substantial aggregate downturns.

The chapter is organized as follows. Section 2 presents an analysis of the Continental European Labour market framework and some stylized facts on the divergent paths among the economies sharing such taxonomy. Section 3 takes a closer look on the stylized facts of the Portuguese labour market adjustment. Section 4 presents the model. Section 5 explains the calibration method and the most relevant stationary equilibrium results. Section 6 presents the transitional dynamics of the model following a aggregate productivity downturn. Section 7 relaxes the wage rigidity component and compares the transitional dynamics with the benchmark economy. Section 8 prescribes several labour market policies towards permanent contracts and their ability to alleviate job destruction. Section 9 concludes.

2 Continental European Labour Market Institutions and the Great Recession

The cross country differences in labour market institutions have been at the core of the discussion of economic performance for decades. Traditionally, the Continental European countries have significant higher levels of labour market regulations and more generous labour policies than the U.S. or the United Kingdom, ranging from higher levels of employment protection and more centralization of collective bargaining to more generous unemployment benefit systems and a more frequent use of active labour market policies.

Even within Continental Europe, there is a high institutional heterogeneity. Boeri (2011) singles out the *Scandinavian*, the *Continental* and the *Mediterranean* clusters as the sensible partition. The first is characterized by a strong emphasis on fiscal policy rather than employment protection, thus resorting to a generous unemployment benefit system, substantial use of active labour market policies and significant tax wedges instead of preventing unemployment spells. The continental cluster, formed by France, Germany, Austria and Belgium, presents a more balanced use of both types of instruments, resorting jointly to high levels of employment protection and a substantial system of unemployment benefits and active labour market policies. Finally, the Mediterranean concept of Portugal, Spain, Italy and Greece relies fundamentally on very substantial employment protection laws and wage rigidity, while the unemployment insurance is relatively low.

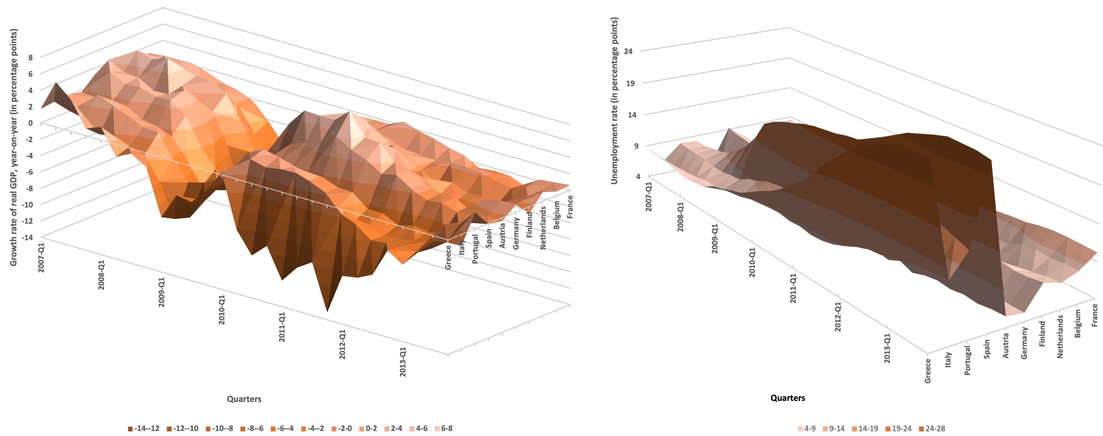
Table IV.1: Labour Market Dualism in European Continental Labour Markets

Countries:	Strictness of EPL Dismissals of Perm. Workers		Strictness of EPL Hiring Temp. Workers		Temporary employees (%)		Wage Premia Perm. Contracts (%)
	2007	2015	2007	2015	2007	2015	2010
Portugal	4.42	3.14	2.56	1.81	17.1	18.3	13.8
Spain	2.36	1.96	3.00	2.47	25.2	20.7	10.5
Greece	3.13	2.45	2.75	2.25	7.1	7.8	-
Italy	3.02	2.93	2.00	1.63	9.5	10.6	11.4
France	2.71	2.50	3.13	3.13	10.8	13.3	7.5
Germany	2.60	2.60	1.00	1.13	10.6	10.1	17.3
Austria	2.29	2.29	1.31	1.31	4.5	5.4	10.1
Belgium	1.69	2.07	2.25	2.06	6.8	7.3	8.3
Netherlands	3.30	3.24	0.94	0.94	12.9	14.0	16.5
Finland	2.08	2.08	1.56	1.56	12.7	12.1	9.4
Sweden	2.45	2.45	1.44	0.81	13.5	13.5	9.8
Denmark	1.47	1.53	1.38	1.63	7.4	6.9	3.4
U.S.	0.09	0.09	0.25	0.25	-	-	-
U.K.	1.51	1.35	0.38	0.38	4.4	4.6	12.1

Notes: On columns (1) and (2), the higher the index indicator the more stringent it is to dismiss permanent workers or resort to hiring temporary workers respectively. The wage premia for temporary contracts is the unexplained component of an Oaxaca-Blindler decomposition using data of 2010 European Structure of Earnings Survey, as estimated by Silva and Turrini (2015). *Sources: OECD* for employment protection legislation indexes (EPL). *Eurostat* for percentage of temporary workers as percentage of all employees. Silva and Turrini (2015) for the wage premia of temporary contracts.

Despite different policy mixes, a notable common feature of every Continental European labour market is duality (see Boeri and van Ours (2013) and Boeri (2011) for recent overviews on the topic). In every geography coexists permanent and temporary contracts, where the former are highly protected from dismissal, have a clearly defined career path, and enjoy a sizable wage premium. The latter have hardly any protection, and are designed to fill temporary necessities of firms. Table 1 presents a snapshot of duality indicators, before and after the Great Recession. Beyond its across board prevalence, duality seems more intense among the Southern European economies, where employment protection of permanent contracts is particularly stringent, the use of temporary contracts is frequent and the wage premia is sizable.

Figure IV.1: Real GDP Growth and Unemployment across Continental Europe



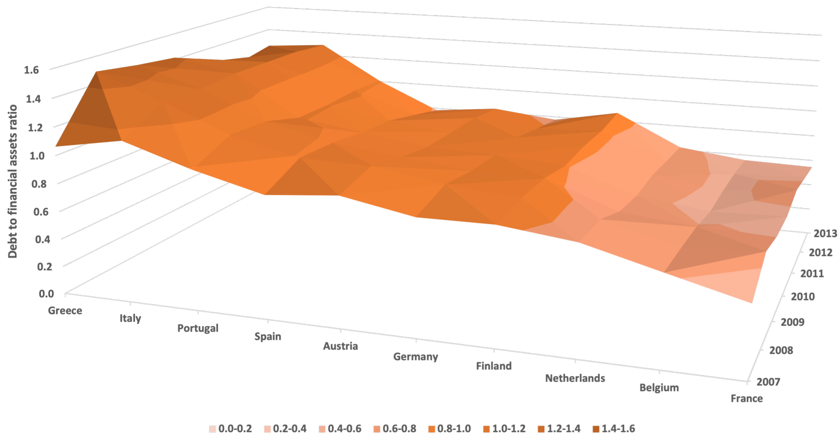
Notes: Every country, excluding Luxembourg, that adopted the Euro before 2005 are included. *Sources: Eurostat.*

The institutional framework of the market is particularly scrutinized during downturns. Among those, the Great Recession triggered by the financial turmoil in 2007-2008 sparked a severe recession across the board. As showed in figure 1, the initial shock was broadly symmetric, while the aftermath was influenced by an asymmetric sovereign debt crisis, which particularly affected the Southern European countries. Altogether, one of the most distinctive features of the Great Recession in Continental Europe was its *unbearable unemployment divergence* which cannot be explained by the size or the specific nature of the shock, or even by region-sector idiosyncrasies, as noted by Boeri and Jimeno (2015). Bentolila et al. (2012)) in a comparison between the Spanish and the French labour market highlights the role of labour market duality in mounting two remarkably different unemployment realities across a single border, reinforcing the evidence of Boeri and Garibaldi (2007), Costain et al. (2010) and Sala et al. (2012) that dual labour markets are more prone to employment volatility.

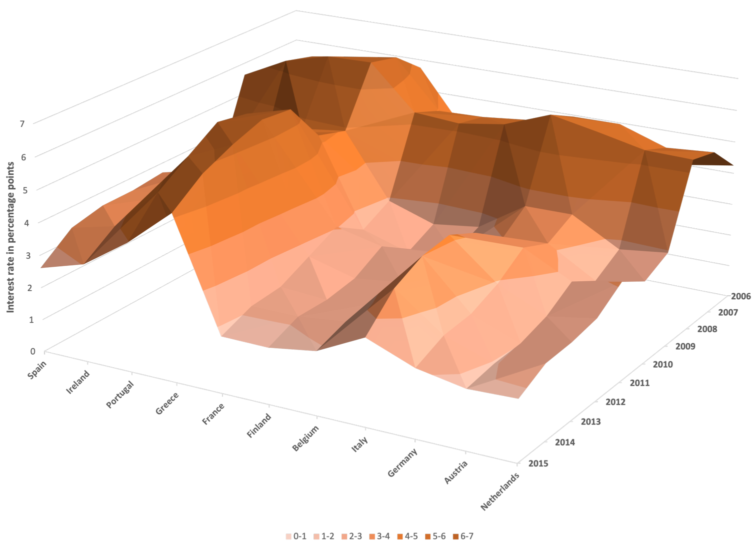
In a complementary view, the pre-existing cross country financial asymmetries both on the financial standing of firms and on the financial system resilience also played significant roles. Resorting to the U.S., Duygan-Bump et al. (2015) established the link between higher financing needs and unemployment particularly for small firms. For Italy, Barone et al. (2016) negatively links outstanding loans and credit supply and find a significant effect of a credit crunch on productivity and employment, particularly for small firms and in areas dependent of external financing. Bentolila et al. (2013) refers to the Spanish credit crunch and confirm the existence of significant job losses on firms that were particularly linked to frailer banks. Identical findings are found in Chodorow-Reich (2014) for the US. For Portugal, Blattner et al. (2017) extends this evidence by presenting the role of weaker financial institutions in misallocating resources towards stressed firms, rather than recognizing further losses.

Figure IV.2: Financing Indicators

Panel A: Debt to financial assets ratio of non financial corporations



Panel B: Annual non financial corporations cost of borrowing for new business



Sources: OECD and *ECB*.

In this context, figure 2 presents key stylized facts on the financial standing of firms in several Continental European economies. First, there is a high asymmetry in the leverage of firms at the start of the Great Recession, with southern European economies presenting significant higher leverage levels, notably Portugal, Italy, Greece or even Spain. Second, beyond preexisting spreads across countries, some didn't witness the same sizable reduction in the cost of borrowing for non financial corporations as the majority of the Eurozone. Thus, particularly for those countries that witnessed a severe repercussion of the sovereign debt crisis component of the Great Recession, namely Portugal, Greece or Ireland, the costs of borrowing didn't witness a significant and durable reduction, thus allegedly muting the effect of a potential financing ease along the downturn on the liquidity of firms. Altogether, for an analysis of the Portuguese case, such stylized facts are compatible with a stable financing environment as defined in our assumption framework.[5]

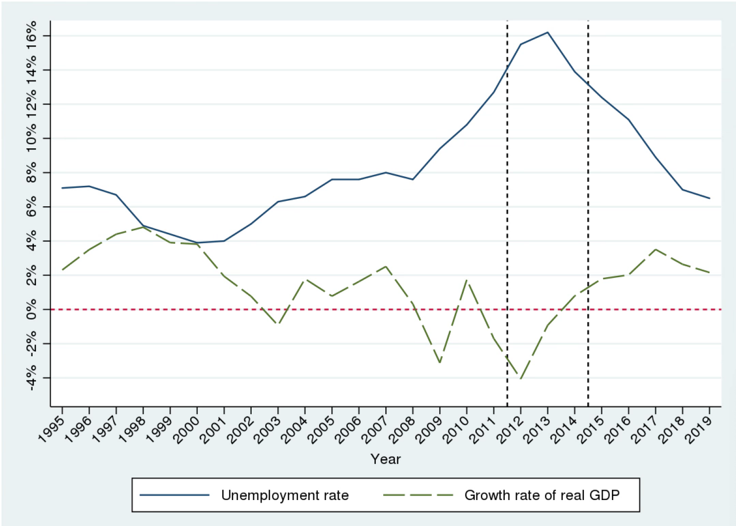
3 Stylized Facts of the Labour Market Adjustment in Portugal throughout the Great Recession

The analysis of this chapter will adopt a calibration that is focused in the Portuguese case. Beyond the accessibility to administrative datasets to inform the calibration, another relevant advantage of calibrating the model to Portugal resides in the severe magnitude of the Great Recession shock to the economy and the labour market (see Blanchard (2007) and Blanchard and Portugal (2017) for context macroeconomic analysis of the Portuguese economy). In detail, the Portuguese economy was severely hit by two germane crisis, namely the 2008 Financial crisis, and the subsequent European debt crisis. By 2013 the Portuguese economy was in such a frail standing that a significant loan from the European institutions and IMF, coupled with an economic conditionality program was needed. From 2008 to just before the start of the *Financial Assistance Program*, the Portuguese economy recorded a 7 percent fall in real GDP.

From a labour market perspective, as displayed in figure 3, the most notable consequence was the *catastrophic job destruction*, as coined by Carneiro et al. (2014), that saw the unemployment rate jump from around 8 percent in 2008 to 16 percent in 2013, displaying an unemployment rate growth that surpassed the real gdp fall in the same period. In inspecting closely the drivers of such dramatic unemployment surge, the market was severely affected in its flows. Leading the effects, figure 4 presents the displacement caused

[5] See Carneiro et al. (2014) for an empirical analysis of the influence of the credit channel in the job destruction process.

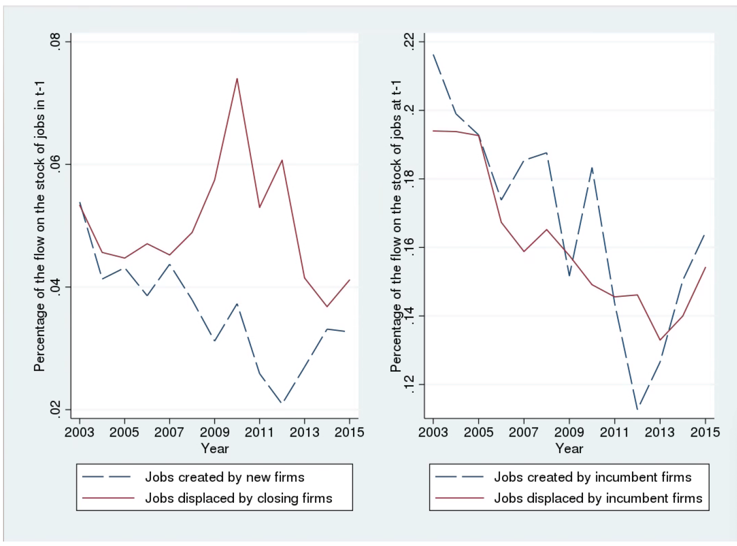
Figure IV.3: The Real GDP Growth and the Unemployment Rate in Portugal



Sources: INE.

by the closing of firms, many of which bankrupted over the period, or simply closed due to striking financial constraints. Simultaneously, the creation of new firms more than halved in the trough in 2012, confirming a significant adverse conditions on firm survival and creation. For the staying firms over the period, the job flows are significantly reduced, with the reduction on job displacement not being enough to overturn the supra-cited adverse dynamics.

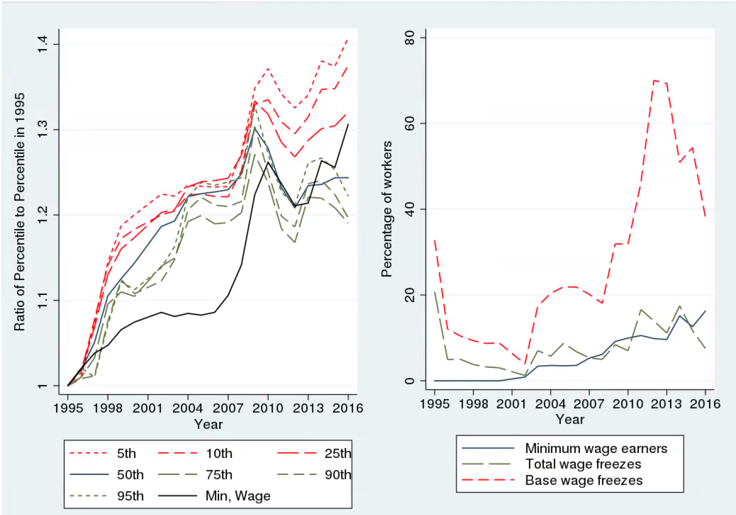
Figure IV.4: The Evolution of Job Flows in Portugal



Sources: Quadros de Pessoal.

Beyond flows, if one assesses the behaviour of wages, the existence of some degree of wage rigidity is clear, as seen in figure 5. In the acute downturn moments, the base wage freezes

Figure IV.5: The Evolution of the Wage Distribution in Portugal



Notes: The percentiles of wages consider real wages. The year-on-year wage freeze calculations consider nominal wages.*Sources: Quadros de Pessoal.*

significantly spiked to record highs of around 70 percent of the existing labour contracts in 2012 and 2013, the total wage freezes increased to levels close to 20 percent, and the number of minimum wage recipients continuously increased.[6] If one measures the fall in real wages, it is around 7.5 percent for median wages, having a monotonic increasing relationship with the percentile of the 2007 initial wage distribution.

In a general sketch of the Portuguese adjustment from 2008 to 2013, the Great Recession meant a downturn that reached around 7 percent, leading to a significant burst of employment, with the unemployment rate doubling to around 16 percent, in a process significantly led by the reduction of flows in surviving firms, the reduction in the creation of firms and a significant firm destruction. In the process wages displayed some degree of wage rigidity, even though real wages sizably adjusted, with a fall of 7,5 percent of median wages. Consequently, compositional effects played a significant role. A model produced to describe the adjustment should be capable to mimic these findings.

4 The Model

Population and technology. Time is continuous. There is a continuum of consumers in the economy, with measure 1, whom have standard time-additive preferences with discount factor ρ. The consumers do not value leisure and they may be either unemployed

[6]The difference between total and base wages regards wage supplements, which typically may include regular bonuses, meals subsidies, overtime compensation, and other fringe benefits.

or employed. When employed, their contract may be either permanent (P) or temporary (T).

Production is decentralized. There is a large mass of potential single job firms, which operate in a perfectly competitive environment. Each firm produces $z(a, j, t)F[k(a, j, t)]$, with $j \in \{P, T\}$ corresponding to the type of contract. $k(a, j, t)$ is the capital stock used by the worker and $z(a, j, t)$ represents the productivity level of the match, which is assumed to depend positively on the level of the worker's assets - a, and it is heterogeneous by types of contracts.[7] $F(\cdot)$ is increasing and strictly concave. Capital depreciates geometrically at rate δ. The output is either consumed, invested in further capital or in vacancy creation, as I will subsequently detail.

The frictional financial market. Workers are at risk of unemployment, which is assumed to be uninsurable. However, workers can save to smooth out the impact of such adverse shock. For that purpose, workers hold an heterogeneous amount of assets a, which they lend to a banking institution, and obtain a deposit rate $r(t)$.

The financial system is composed by an investment institution and a banking institution. The former manages a portfolio composed by the equity of the mass of firms ($Q(t)$), whose flow dividends ($D(t)$) accrue from the difference between the firms' flow profits and the costs with labour flows, namely the investment in job creation and the taxes of firing workers. The latter lends capital to firms, obtaining a lending rate $r_l(t)$, so that $r_l(t) - \delta = r(t)$. For parsimony purposes, we assume there is no seniority between loans and capital in case of firm insolvency. Therefore, the arbitrage condition in the allocation of the resources available to the financial system entails:

$$r_l(t) - \delta = \frac{Q(t) + D(t)}{Q(t)}. \tag{IV.1}$$

The lending market is frictional. The investment institution may withdraw activity or declare bankruptcy of specific firms. Aware of such risk, the lender, in an attempt to insure itself, may establish lending limits or tighten the surveillance of firm's activity when the EBITDA flows of each firm, or prospective firms, are sufficiently low. Such enhanced surveillance affects the firms' decisions regarding hiring, promotion and/or dismissal of

[7]Intuitively, we imply that a wealthier worker is inherently more productive. In the context of direct search models, Eeckhout and Sepahsalari (2020) rationalizes an identical relationship.

workers, as we will see. The EBITDA is defined by:

$$\xi(a,j,t) = z(a,j,t)F[k(a,j,t)] - w(a,j,t), \quad j \in \{T,P\}, \tag{IV.2}$$

where $w(a,i,t)$ corresponds to the wage of a given worker with contract type i, which, as will be evident in this section, depends on the asset level of the worker.

On top of the resources obtained from the wealth of domestic consumers, the banking institution may may obtain resources from abroad, under the same conditions as the domestic resources. Accordingly, ϑ corresponds to the ratio of domestically owned assets to total assets in the economy, and the stock of assets owned by domestic consumers is given by:

$$A = \vartheta[e_T(t)K_T(t) + e_P(t)K_P(t) + Q(t)], \tag{IV.3}$$

where: (a) $e_T(t)$ and $e_P(t)$ are respectively the stock of temporary and permanent contract employed workers; (b) $K_T(t)$ corresponds to the aggregate capital in temporary contracts; and (c) $K_P(t)$ corresponds to the aggregate capital in permanent contracts. The aggregate capital by type of contract will be defined later. Further, we assume that external funding is stable, thus ϑ is constant.

The frictional labour market. The labour market is assumed to be frictional. Firms are required to post vacancies to hire workers, and pay a cost γ per vacancy posted. The flow of worker-firm meets is determined by a typical constant returns to scale aggregate matching function, $M(u(t),v(t))$, where $u(t)$ is the measure of unemployed workers searching for a job, and $v(t)$ is the measure of open vacancies. Correspondingly, $\theta(t) = v(t)/u(t)$ is the market tightness; $\theta(t)q[\theta(t)] = M(u(t),v(t))/v(t)$ represents the Poisson rate at which an unemployed worker meets a firm; $q[\theta(t)]$ is the Poisson rate at which a firm meets a candidate, per vacancy posted; and $\gamma v(t)$ corresponds to the vacancy costs of the investment institution.

At the hiring moment, any worker is hired on a temporary contract (T). Once the firm and the worker meet, the probability of a successful completion of the match, $h[\xi(a,T,t)]$, follows a logistic decreasing function $h(\cdot)$, which is dependant of the EBITDA of the newly formed match, $\xi(a,T,t)$. While on a temporary contract, the match may be dissolved with probability $\sigma_T[\xi(a,T,t)]$, where $\sigma_T(\cdot)$ corresponds to a logistic decreasing function. In the event of a dissolution it occurs costlessly, i.e. without any firing tax involved ($S_T = 0$).

Simultaneously, the temporary worker can be promoted into a permanent contract (P).

Such event occurs with probability $\varphi[\xi(a,P,t)]$, where $\varphi(\cdot)$ is an increasing logistic function, and $\xi(a,P,t)$ corresponds to the EBITDA of the firm after the potential promotion of the worker. Once on a permanent contract, the match may be dissolved with probability $\sigma_P[\xi(a,P,t)]$, where $\sigma_P(\cdot)$ is a decreasing logistic function. However, contrary to a temporary contract, the dissolution entangles a positive firing tax - S_P - paid by the firm.

Notation note. For notation simplicity, in the remainder of this section, consider $j = \{P,T\}$ and: (a) $\ddot{z} = z(a,j,t)$; (b) $\ddot{k} = k(a,j,t)$; (c) $\ddot{\xi}_P = \xi(a,P,t)$, (d) $\ddot{c} = c(a,j,t)$, and (e) $\ddot{\xi}_T = \xi(a,T,t)$.

Consumers. Following the insight of Kaplan et al. (2018), beyond the wage, the worker is also entitled to a percentage of the flow profit of the firm, ψ, in a form of a *bonus*.[8] Consequently:

$$\eta(a,j,t) = \psi\Big[\ddot{z}F[\ddot{k}] - r_l(t)\ddot{k} - w(a,j,t)\Big], \;\; j \in \{T,P\}. \tag{IV.4}$$

In a recursive form, the employed workers in temporary contracts choose their consumption by resorting to the following HJB equation and state-constraint boundary condition:

$$\begin{aligned}\rho W(a,T,t) = \max_{c} \; \Bigg\{ & u(\ddot{c}) + \partial_a W(a,T,t)\Big[w(a,T,t) + \eta(a,T,t) + (r_l(t)-\delta)a - \ddot{c}\Big] + \\ & + \sigma_T(\ddot{\xi}_T)\Big(W(a,U,t) - W(a,T,t)\Big) + \varphi(\ddot{\xi}_P)\Big(W(a,P,t) - W(a,T,t)\Big) + \partial_t W(a,T,t)\Bigg\}\end{aligned}$$

subject to:

$$\partial_a W(\underline{a},P,t) \geq u'\Big(w(\underline{a},P,t) + \eta(\underline{a},T,t) + (r_l(t)-\delta)\underline{a}\Big), \tag{IV.5}$$

where: (a) the utility function, $u(\bullet)$, is assumed to be increasing and concave; (b) the $W(a,U,t)$ corresponds to the value function of the unemployed worker; (c) $W(a,T,t)$ represents the value function of an employed worker in a temporary contract; (d) $W(a,P,t)$ corresponds to the value function of a worker in a permanent contract; and (e) $\underline{a}$ represents the lowest admissible level of wealth.

Beyond the proper structure of the career, which predicts no possible promotion for permanent workers, the fundamental difference between a temporary contract and a permanent contract is the existence of the positive firing tax S_P in case of a termination of the match

[8] Intuitively, in this model the term ψ controls the percentage of the firms' profits that are distributed as a lump sum to the population, versus the share that is distributed directly to the involved workers.

of the latter type. Consequently, the recursive problem of permanent workers is given by:

$$\rho W(a,P,t) = \max_{c} \Bigg\{ u(\ddot{c}) + \partial_a W(a,P,t)\Big[w(a,P,t) + \eta(a,P,t) + (r_l(t)-\delta)a - \ddot{c}\Big] + \\ + \sigma_P(\ddot{\xi}_P)\Big[W(a,U,t) - W(a,P,t)\Big] + \partial_t W(a,P,t)\Big]\Bigg\}$$

subject to:

$$\partial_a W(\underline{\mathrm{a}},P,t) \geq u'\Big(w(\underline{\mathrm{a}},P,t) + \eta(\underline{\mathrm{a}},P,t) + (r_l(t)-\delta)\underline{\mathrm{a}}\Big). \tag{IV.6}$$

Consistently with Krusell et al. (2010), the unemployed workers earn some exogenous income of level b. Their recursive problem is given by:

$$\rho W(a,U,t) = \max_{c} \Bigg\{ u(\ddot{c}) + \partial_a W(a,U,t)\Big[b + r(t)a - \ddot{c}\Big] + \theta(t)q[\theta(t)]h(\ddot{\xi}_T)\Big[W(a,T,t) - W(a,U,t)\Big] + \\ + \partial_t W(a,U,t)\Big]\Bigg\}$$

subject to:

$$\partial_a W(\underline{\mathrm{a}},U,t) \geq u'\Big(b + (r_l(t)-\delta)\underline{\mathrm{a}}\Big). \tag{IV.7}$$

The first order conditions of the described recursive problems corresponds to the respective policy functions of consumption and savings, which due to symmetry can be represented as:

$$\ddot{c} = u'^{-1}\Big(\partial_a W(a,i,t)\Big),$$

$$\dot{a}(a,i,t) = y(a,i,t) + (r_l - \delta)a - \ddot{c}, \quad \text{where } y(a,i,t) = \begin{cases} w(a,P,t) + \eta(a,P,t) & \text{if } i = P \\ w(a,T,t) + \eta(a,T,t) & \text{if } i = T \\ b & \text{if } i = U \end{cases}. \tag{IV.8}$$

The savings policy functions are then considered in Kolmogorov forward equations defined as:

$$\begin{aligned} \partial_t g(a,P,t) =& -\partial_a[\dot{a}(a,P,t)g(a,P,t)] + \varphi(\ddot{\xi}_P)g(a,T,t) - \sigma_P(\ddot{\xi}_P)g(a,P,t), \\ \partial_t g(a,T,t) =& -\partial_a[\dot{a}(a,T,t)g(a,T,t)] + h(\ddot{\xi}_T)\theta(t)q[\theta(t)]g(a,U,t) - [\sigma_T(\ddot{\xi}_T) + \varphi(\ddot{\xi}_P)]g(a,T,t), \\ \partial_t g(a,U,t) =& -\partial_a[\dot{a}(a,U,t)g(a,U,t)] + \sigma_T(\ddot{\xi}_T)g(a,T,t) + \sigma_P(\ddot{\xi}_P)g(a,P,t) \\ & - h(\ddot{\xi}_T)\theta(t)q[\theta(t)]g(a,U,t). \end{aligned} \tag{IV.9}$$

Equation (9) describes the dynamics of the distribution of workers over states, $g(a,i,t)$, where $i \in \{P, T, U\}$. Consequently, given the presented structure, the labour market transitions are described by the following system:

$$\begin{aligned}
\dot{u} &= \int_{\underline{\text{a}}}^{\infty} \sigma_T(\ddot{\xi}_T) g(a,T,t) da + \int_{\underline{\text{a}}}^{\infty} \sigma_P(\ddot{\xi}_P) g(a,P,t) da - \int_{\underline{\text{a}}}^{\infty} h(\ddot{\xi}_T)\theta(t) q[\theta(t)] g(a,U,t) da \\
\dot{e}_T &= -\int_{\underline{\text{a}}}^{\infty} [\sigma_T(\ddot{\xi}_T) + \varphi(\ddot{\xi}_P)] g(a,T,t) da + \int_{\underline{\text{a}}}^{\infty} h(\ddot{\xi}_T)\theta(t) q[\theta(t)] g(a,U,t) da \\
\dot{e}_P &= -\int_{\underline{\text{a}}}^{\infty} \sigma_P(\ddot{\xi}_P) g(a,P,t) da + \int_{\underline{\text{a}}}^{\infty} \varphi(\ddot{\xi}_P) g(a,T,t) da \\
1 &= \underbrace{\int_{\underline{\text{a}}}^{\infty} g(a,T,t) da}_{\equiv e_T(t)} + \underbrace{\int_{\underline{\text{a}}}^{\infty} g(a,P,t) da}_{\equiv e_P(t)} + \underbrace{\int_{\underline{\text{a}}}^{\infty} g(a,U,t) da}_{\equiv u(t)}.
\end{aligned} \tag{IV.10}$$

Firms. The firms are owned by the investment institution, rent capital from the bank, create jobs and produce. The firms maximize the present value of the profits, or identically, the value of equity, which is not zero due to the existence of labour market frictions. Further, to create jobs, firms are required to post vacancies and search for workers. The time-dependent HJB equation for the value of posting a vacancy, $V(t)$, is given by:

$$(r_l(t) - \delta)V(t) = -\gamma + q(\theta) \int_{\underline{\text{a}}}^{\infty} J(a,T,t) \frac{g(a,U,t)}{u(t)} da, \tag{IV.11}$$

where $J(a,T,t)$ corresponds to the value of a temporary contract job performed by a worker with asset level a; and $g(a,U,t)/u(t)$ represents the density function of the unemployed workers. Noteworthy, due to labour market frictions, the firm posting a vacancy is unable to *target* potential workers based on their level of assets.

The free-entry condition in the market implies that firms will post vacancies until $V(t) \equiv 0$. Consequently, the number of vacancies in equilibrium is implicitly given by:[9]

$$\gamma = q[\theta(t)] \int_{\underline{\text{a}}}^{\infty} J(a,T,t) \frac{g(a,U,t)}{u(t)} da. \tag{IV.12}$$

Once a vacancy is filled, the value of the corresponding filled temporary contract job, given

[9] Notice that new firms enter the market based on the comparison between the expected inter-temporal value of the job and the cost of the vacancy. It is possible that the firm would initially be formed with very high leverage. In this situation, the job is not created, even though the firm undertakes the cost of the vacancy creation.

the assets owned by the worker, results from the following recursive problem:

$$(r_l(t)-\delta)J(a,T,t) = \max_{k} \Big\{\ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} - w(a,T,t) - \eta(a,T,t) + \partial_a J(a,T,t)\dot{a}(a,T,t) + \\ [\sigma_T(\ddot{\xi}_T) + \varphi(\ddot{\xi}_P)][V(t) - J(a,T,t)] + \partial_t J(a,T,t)\Big\}. \tag{IV.13}$$

Notice I consider that a job that gets promoted means the current temporary job is dissolved. For a permanent contract job, its value corresponds to the solution of the respective problem given by:

$$(r_l(t)-\delta)J(a,P,t) = \max_{k} \Big\{\ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} - w(a,P,t) - \eta(a,P,t) + \partial_a J(a,P,t)\dot{a}(a,P,t) + \\ \sigma_P(\ddot{\xi}_P)[V(t) - S_P - J(a,P,t)] + \partial_t J(a,P,t)\Big\}. \tag{IV.14}$$

Notice that independently of the type of contract, the investment institution will borrow capital optimally, and therefore we have that:

$$r_l(t) = \ddot{z}F'_k(\ddot{k}), \tag{IV.15}$$

which implicitly defines the capital stock available to each match.[10] The dividends collected by the investment institution are given by:

$$\begin{aligned} d = &(1-\psi)\int_{\underline{a}}^{\infty}\Big[\ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} - w(a,T,t)\Big]g(a,T,t)da + \\ &+ (1-\psi)\int_{\underline{a}}^{\infty}\Big[\ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} - w(a,P,t)\Big]g(a,P,t)da - \gamma v, \end{aligned} \tag{IV.16}$$

and the aggregate capital stock by each type of contract is:

$$K_j(t) = \int_{\underline{a}}^{\infty} \ddot{k}g(a,j,t)da, \ \ j \in \{T,P\}. \tag{IV.17}$$

Wage Setting Mechanism and Worker's Bonus. The wage is determined considering: (a) a rent-sharing rule, subjected to a downward wage rigidity constraint;[11] and (b) the parties set wages disregarding the subsequent attribution of bonuses, that are only defined once profits are determined. Therefore, for wage setting purposes, the parties consider

[10] Noteworthy, independently of a firm's EBITDA, I assume each firm will acquire the optimal level of capital. Consequently, in the proposed model the potential financial constraints affect the probability of dissolution of the firm, and not the capital available to it.

[11] An alternative approach is to consider a (Generalized) Nash Bargain as in Krusell et al. (2010). However, l'Haridon et al. (2013) shows that the difference is negligible in models without precautionary savings, and the derivation of wages in a Nash Bargain arrangement in continuous time raises computational issues regarding the marginal propensity to consume.

$\eta(a,i,t)=0$.[12] Accordingly, the rent sharing rule is given by:

$$\beta\Big[J(a,j,t)+S_j-V(t)\Big]=(1-\beta)\Big[W(a,j,t)-W(a,U,t)\Big],\ \text{ s. to } \eta(a,i,t)=0,\ \ j\in\{T,P\}, \tag{IV.18}$$

where β corresponds to the worker's bargaining power, $J(a,j,t)$ value function of the job, $W(a,j,t)$ the value function of the employed worker, and $W(a,U,t)$ the value function of the unemployed. The path of wages is subjected to downward wage rigidity, in the spirit of Schmitt-Grohé and Uribe (2016), so that:

$$w(a,j,t)\geq \lambda w(a,j,t-\epsilon),\ \ j\in\{T,P\}, \tag{IV.19}$$

where λ corresponds to the maximum quarterly downward adjustment of real wages.

Considering the solutions of the presented recursive problems and the downward wage rigidity constraint, the wage functions are defined as:

$$w(a,j,t)=\begin{cases}\tilde{w}(a,j,t) & \text{if } \tilde{w}(a,j,t)\geq \lambda w(a,j,t-\epsilon)\\ w(a,j,t-\epsilon) & \text{if } \tilde{w}(a,j,t)<\lambda w(a,j,t-\epsilon)\end{cases},\ \ j\in\{T,P\}, \tag{IV.20}$$

where $\tilde{w}(a,T,t)$ corresponds to the unconstrained wages, resulting from: (a) the rent-sharing rule; and (b) the lack of firm commitment, so that those unconstrained wages are set on a period by period basis. However, when the resulting unconstrained nominal wage is lower than the existing nominal wage in moment $t-\epsilon$, the wage is not lowered. Accordingly, the unconstrained wages are given by:[13]

$$\begin{aligned}\tilde{w}(a,T,t)=&\frac{\beta\Big[\rho+\varphi(\ddot{\xi}_P)+\sigma_T(\ddot{\xi}_T)\Big]\Big\{\ddot{z}F(\ddot{k})-r_l(t)\ddot{k}+\partial_a J(a,T,t)\Big(r(t)a-\ddot{c}\Big)+\partial_t J(a,T,t)\Big\}}{[1-\beta][r(t)+\varphi(\ddot{\xi}_P)+\sigma_T(\ddot{\xi}_T)]\partial_a W(a,P,t)+\beta[\rho+\varphi(\ddot{\xi}_P)+\sigma_T(\ddot{\xi}_T)][1-\partial_a J(a,T,t)]}\\ &-\frac{(1-\beta)\Big[r(t)+\varphi(\ddot{\xi}_P)+\sigma_T(\ddot{\xi}_T)\Big]\Big\{u(\ddot{c})+\partial_a W(a,P,t)\Big(r(t)a-\ddot{c}\Big)+\varphi(\ddot{\xi}_P)W(a,P,t)}{[1-\beta][r(t)+\varphi(\ddot{\xi}_P)+\sigma_T(\ddot{\xi}_T)]\partial_a W(a,P,t)+\beta[\rho+\varphi(\ddot{\xi}_P)+\sigma_T(\ddot{\xi}_T)][1-\partial_a J(a,T,t)]}\\ &\frac{-[\rho+\varphi(\ddot{\xi}_P)]W(a,U,t)+\partial_t W(a,P,t)\Big\}}{[1-\beta][r(t)+\varphi(\ddot{\xi}_P)+\sigma_T(\ddot{\xi}_T)]\partial_a W(a,P,t)+\beta[\rho+\varphi(\ddot{\xi}_P)+\sigma_T(\ddot{\xi}_T)][1-\partial_a J(a,T,t)]};\end{aligned} \tag{IV.21}$$

$$\begin{aligned}\tilde{w}(a,P,t)=&\frac{\beta\Big[\rho+\sigma_P(\ddot{\xi}_P)\Big]\Big\{\ddot{z}F(\ddot{k})-r_l(t)\ddot{k}+\partial_a J(a,P,t)\Big(r(t)a-\ddot{c}\Big)+r(t)S_P+\partial_t J(a,P,t)\Big\}}{[1-\beta][r(t)+\sigma_P(\ddot{\xi}_P)]\partial_a W(a,P,t)+\beta[\rho+\sigma_P(\ddot{\xi}_P)][1-\partial_a J(a,P,t)]}-\\ &\frac{(1-\beta)\Big[r(t)+\sigma_P(\ddot{\xi}_P)\Big]\Big\{u(\ddot{c})+\partial_a W(a,P,t)\Big(r(t)a-\ddot{c}\Big)-\rho W(a,U,t)+\partial_t W(a,P,t)\Big\}}{[1-\beta][r(t)+\sigma_P(\ddot{\xi}_P)]\partial_a W(a,P,t)+\beta[\rho+\sigma_P(\ddot{\xi}_P)][1-\partial_a J(a,P,t)]}.\end{aligned}$$

[12] Intuitively, the wage corresponds to the fixed component of worker's compensation. Then, once the parties determine the profit of the firm and split the profit according to the profit-sharing rule, the variable compensation component is determined.

[13] See appendix A for further details in deriving the unconstrained wage equations.

Notice that by the proper definition of a stationary recursive equilibrium, the downward wage rigidity constraint would not bind in computing such equilibrium.

5 Calibration, Computation and Stationary Equilibrium Results

In this section, the objective is to analyze the ability of the stationary equilibrium of the model, and its proposed calibration strategy, to mimic a steady state for the Portuguese economy. Once performed, such calibration strategy will serve as the basis for the transitional dynamics analysis of the subsequent sections, and the impact of the labour market policy alternatives considered.

Broadly, the three novel features of the model are the integration of a detailed description of the labour market institutional setting, the prevalence of endogenous job destruction associated with the level of leverage of firms, and the link between productivity and wealth. Those will be key in the assessment of the adjustment paths of the economy during downturns, and thus particular emphasis will be placed in comparing key untargeted moments of the calibrated model and their empirical counterparts.

Computation. I formally define the recursive stationary equilibrium of this economy in appendix B. The proposed model corresponds to a Krusell et al. (2010) type model in a continuous time setting, solved through the finite-difference method of Achdou et al. (2017). Differently from the traditional one-dimensional fixed-point problem of the Bewley-Huggett-Aiyagari standard paradigm, this model corresponds to a functional fixed-point problem, given the need to find the endogenous wage schedule.

Compared to the standard Krusell et al. (2010) model in continuous time, as presented in Bardóczy (2017), our model presents two additional layers of complexity: (a) the existence of two categories of workers, translating into two sets of HJB equations for employed workers and firms, and a transition matrix defined for employed workers for both types of contracts and the unemployed; and (b) the existence of endogenous job destruction functions.

Despite the additional computational intensity, the algorithm I use is standard and works as follows: (1) set up a grid for assets; (2) guess the market tightness, θ, the interest rate, $r(t)$, and define the wage schedule for temporary and permanent workers, $w(a, T, t)$ and

$w(a, P, t)$; (3) compute the leverage of each type of worker along the grid and compute the transition matrix; (4) solve the contraction mapping of the 3 types of HJB equations for the workers; (5) compute the stationary distribution of workers; (6) given the distribution solve the contraction mapping of the 2 types of HJB equations for the firms; (7) evaluate the free entry condition and the asset market clearing condition; (8) perform the rent-sharing rule and update the wage function; and (9) update r_l and θ until there is convergence from step 2 onwards in (w, r, θ). Appendix C presents further details on the algorithm used.

Benchmark calibration. The calibration follows two main objectives. First, the calibration should present a consistent mapping between the model and several labour market measures. Second, it should present a realistic representation of the financial standing of firms and of households. For this purpose, I make the mapping between the model parameters and quarterly data from various sources, and take the years of 2016-2017 as the benchmark steady state of the economy.

Regarding functional form assumptions, the utility function is $u(\ddot{c}) = ln(\ddot{c})$. The production function $\ddot{z}F(\ddot{k})$ corresponds to $\ddot{z}F(\ddot{k}) = \ddot{z}\ddot{k}^{\alpha}$, where $\ddot{z} = \tilde{z}_t z(a, j)$, where $j\{P, T\}$. In the stationary equilibrium $\tilde{z}_t = 1$, and $z(a, j)$ is set as a logistic type function:

$$z(a, j) = \frac{\overline{z}}{1 + e^{-\kappa_z [a - z^{inv}]}} + \tilde{z}_j, \quad j \in \{T, P\}. \tag{IV.22}$$

The matching function $M(u, v)$ is set as $M[u(t), v(t)] = \chi u(t)^{\eta} v(t)^{1-\eta}$. The labour flows also follow logistic functions of the interest payment to EBITDA ($\frac{r_l \ddot{k}}{\ddot{\xi}}$), so that the displacement, promotion and hiring of workers is given by:

$$\begin{aligned} \sigma_j(\ddot{\xi}_j) &= \frac{\overline{\sigma_j}}{1 + e^{\kappa_\sigma [\frac{r_l(t)\ddot{k}}{\ddot{\xi}} - \sigma_j^{inv}]}} + \tilde{\sigma_j}, \quad \varphi(\ddot{\xi}_P) = \frac{\overline{\varphi}}{1 + e^{\kappa_\varphi [\varphi^{inv} - \frac{r_l(t)\ddot{k}}{\ddot{\xi}_P}]}}, \\ h(\ddot{\xi}_T) &= \frac{1}{1 + e^{\kappa_h [h^{inv} - \frac{r_l(t)\ddot{k}}{\ddot{\xi}_T}]}}, \quad j \in \{T, P\}. \end{aligned} \tag{IV.23}$$

The benchmark parameter calibration is presented in table 2, and the mapping between the empirical counterparts and several endogenous outcomes of the model is presented in table 3. As in Shimer (2005), home production is commensurate with the average level of unemployment insurance. Accordingly, the flow of home production is set at $b = 0.85$, which corresponds to 47% of the average wage of the economy, matching the 5-year average of the OECD net replacement ratio indicator. It is significantly higher than the 40% in Shimer's work, thus representing the more beneficial unemployment insurance protection prevalent in Southern European economies, and particularly in Portugal.

Table IV.2: Proposed Calibration

Parameter		Expression	Value
Consumers	Discount factor	ρ	0.005
	Home Production	b	0.85
	Minimum asset level	$\underline{a}$	0
Labour Market	Matching Efficiency	χ	0.65
	Worker Bargaining Power	β	0.4
	Share of EBIT Paid as Variable Component	$\eta(a,j,t)$	0.3
	Matching Elasticity of Unemployed	η	0.4
	Firing Tax	S_P	0.4
	Vacancy Cost	γ	0.772
	Downward rigidity parameter	λ	0.995
	Steepness of flow logistic functions	$\kappa_\sigma = \kappa_\varphi = \kappa_h$	200
	Inversion of logistic functions	$\sigma_j^{inv} = h^{inv}$	0.95
	Non-stressed promotion rate	$\overline{\varphi}$	0.11
	Non-stressed displacement rate of temp. workers	$\tilde{\sigma_T}$	0.03
	Non-stressed displacement rate of perm. workers	$\tilde{\sigma_P}$	0.12
	Fully-stressed displacement rate of temp. workers	$\overline{\sigma_T}$	0.4
	Fully-stressed displacement rate of perm. workers	$\overline{\sigma_P}$	0.15
Financial Sector	Ratio of domestically owned assets	ϑ	0.625
	Share of Capital	α	0.225
	Depreciation rate	δ	0.018
Prod. Function	Steepness Parameter	κ_z	0.03
	Inversion Parameter	z^{inv}	12
	Location Parameter (Permanent Contracts)	$\tilde{z}_P$	1.03
	Location Parameter (Temporary Contracts)	$\tilde{z}_T$	0.97
	Maximum Spread of Productivity	$\overline{z}$	0.20

The firing tax of permanent workers is set at $S_P = 0.4$, it represents 22.5% of the average wage, and it is consistent with survey data for Oslington and Freyens (2005). Notice that following Boeri (2011), a firing tax should be distinguished from a severance payment, which can be fully offset with a compensating wage adjustment in this framework (Lazear (1990)). Thus, the firing tax corresponds to a pure deadweight loss, accruing from the cost on the resources required to obtain legal advice, make the decision, gather the case for the dismissal decision, meet with the worker, and potentially go to court. On the other side, the cost of post a vacancy is set so that the labour market tightness is calibrated at $\theta = 1$. It corresponds to 51.6% of the average wage which is similar to the calibrations of Krusell et al. (2010) and Hagedorn and Manovskii (2008) whom set the vacancy cost at 60% and 58.4%, respectively.

The *non-stressed* separation rates of unemployment, $\tilde{\sigma}_P = 0.03$ and $\tilde{\sigma}_T = 0.12$, are set considering the average tenure for each type of contract, the model's postulated career path, and the fact that job-to-job transitions are particularly noticeable in temporary jobs, which increases the effective spell of employment of workers vis-á-vis firm tenure. The promotion rate, $\overline{\varphi} = 0.1$, and the matching efficiency parameter, $\chi = 0.65$, are set so that conditional on separation rates the calibration approximates: (a) the empirical

Table IV.3: Endogenous Outcomes

	Moment	Expression	Value	Target	Period	Source
Labour Market	Labour Market Tightness	θ	1	1	-	Internally calibrated
	Unemployment Rate	$100 \times u\%$	7.59%	8%	-	*Eurostat*$^{(a)}$
	Net Replacement Ratio	$100 \times \frac{h}{E[w]}\%$	46.55%	48%	2016-2017	*OECD*$^{(b)}$
	Proportion of Temporary contracts	$100 \times \frac{e_T}{(e_P+e_T)}\%$	21.4%	17.85%	2016-2017	*Eurostat*
	Permanent contract wage gap	$100 \times E[\frac{w(a,P,t)-w(a,T,t)}{w(a,T,t)}]\%$	18%	13.8%	-	Silva and Turrini (2015)
	Minimum to average wage ratio	$100 \times [\frac{E[w]}{\underline{W}}]\%$	122.9%	136.3%	2016-2017	*Quadros de Pessoal*$^{(c)}$
	Non-distressed average displacement rate	$100 \times \frac{e_T\sigma_T(\xi_T)+e_P\sigma_P(\xi_P)}{(e_P+e_T)}\%$	4.93%	5.16%	2016-2017	*Eurostat*$^{(d)}$
	Annual maximum real wage decline	$100 \times (1-e^{-4\lambda})$	1.98%	2%	-	*Inflation Benchmark*
	Coefficient of variation of wages	$\frac{\sigma_w}{E[w]}$	0.81	1.51	2016	*Quadros de Pessoal*
	Average tenure of temporary contracts	$\frac{1}{4} \times \frac{1}{\tilde{\sigma}_T}$	2.08	1	2016	*Quadros de Pessoal*
	Average tenure on permanent contract	$\frac{1}{4} \times \frac{1}{\tilde{\sigma}_P}$ - $\frac{1}{4} \times \frac{1}{\tilde{\sigma}_T}$	8.33	9	2016	*Quadros de Pessoal*
Financial Sector	Quarterly gross interest rate on capital	$100 \times [1-e^{-4(r_l)}]$	1.99%	2.64%	-	Card et al. (2014) (Italy)
	Investment-output ratio	$100 \times \delta k^{1-\alpha}$	21.29%	17.15%	2016-2017	*Eurostat*
	Average Leverage	$100 \times E[\frac{r_l k}{\xi}]\%$	83.7%	90.23%	2016-2017	*SCIE*
	Annual deposit rate	$100 \times [1-e^{-4(r_l-\delta)}]$	0.72%	0.54%	2016-2017	*Banco de Portugal*$^{(e)}$
	Average savings' rate	$100 \times E[\dot{a}]$	-7.6%	-1.775%	2016-2017	*OECD*
	Net External Debt to output	$100 \times \frac{(1-\vartheta)K}{z_t k(e_T+e_P)}$	111.92%	94.6%	2016-2017	*Bank of Portugal*
	Consumers net worth as perc. of output	$100 \times \frac{A}{z_t k(e_T+e_P)}$	186.54%	183.63%	2016-2017	*OECD*$^{(f)}$

Notes: (a) the indicator corresponds to unemployment as percentage of total population for individuals between 20-64 years; (b) Indicator in *2009 OECD Emplooyment Outlook*; (c) the indicator considers the average net average wage and net minimum wage; (d) the indicator resorts to the labour market transition database and include transitions from job-to-unemployment and job-to-inactivity; (e) the indicator corresponds to the average annual deposit rate of term deposits; (f) the indicator corresponds to the product of the household net worth and the household proportion of financial assets. Implicitly, as a simplifying assumption I assume the liquidity of assets as a proxy of the liquidity of the net worth.
Sources: Eurostat, *OECD*, *Banco de Portugal*, *SCIE 2016-2017*, *Quadros de Pessoal 2016*, Silva and Turrini (2015), Card et al. (2014).

counterpart of the percentage of temporary workers in the economy; (b) the unemployment rate; and (c) the duration of a temporary contract until promotion, $\frac{1}{4}\frac{1}{\varphi}$, which is set at 2.5 years.[14]

Then, as the interest payment to EBITDA ratio approaches 95 percent, the displacement probabilities increase logistically, and the hiring and promotion probability decrease, also logistically, as presented in figure 6. The intuition behind the calibration is that a very indebted firm will neither hire nor promote a worker. For a temporary worker in a stressful firm, the hazard of being dismissed becomes 52 percent, translating into an additional expected job duration of 5.8 months, while for a permanent worker in an equally stressed firm, the additional expected job duration is shortened to around 1,4 years.

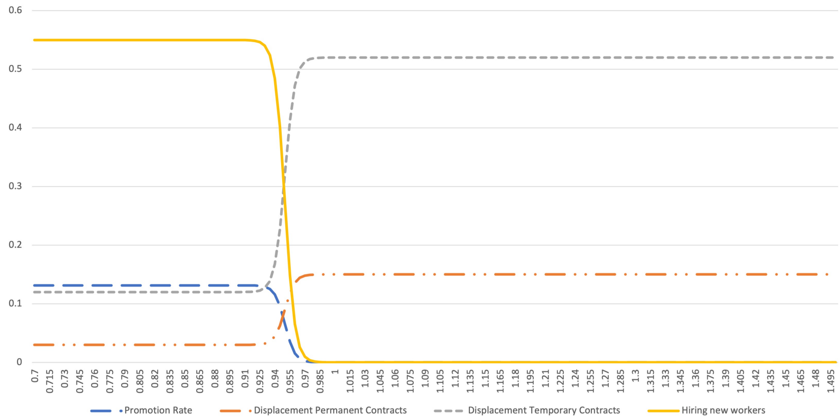
Regarding the productivity function, two features are noteworthy. First, I assume the insight of Eeckhout and Sepahsalari (2020). The productivity of the match is an increasing function of the wealth of the worker, as presented in figure 6.[15] Second, the permanent

[14]The maximum allowed number of successive temporary contracts in a firm is 3. Given the typical temporary contract is of 1 year, 2.5 years of average length until promotion, conditional on not being dismissed before, is deemed adequate.

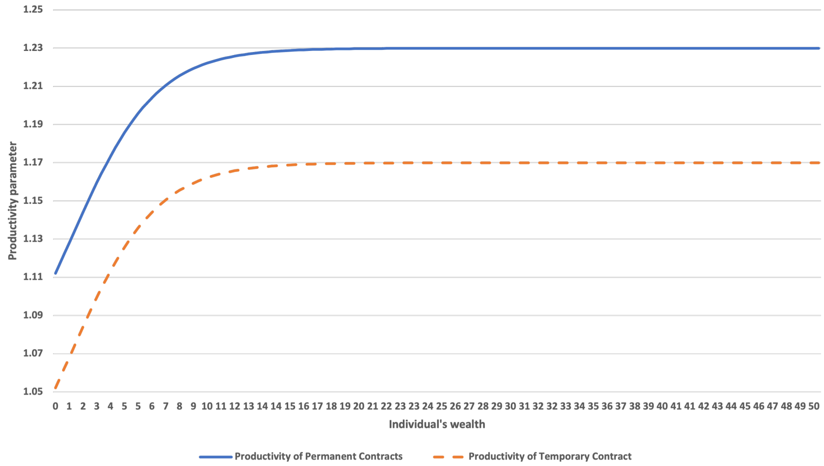
[15]The model presented by Eeckhout and Sepahsalari (2020) is a direct search model, where the wealthier

Figure IV.6: Calibrated Functions

Panel A: Functions for Labour market transitions



Panel B: Productivity function



jobs are more productive, following a standard reasoning of positive productivity effects of tenure, which is fully consistent with the structure of the adopted model.

The discount rate and the deposit rate influences savings decisions of consumers. The discount rate is set at $\rho = 0.005$, which corresponds to a 2% annual discounting. The annual deposit rate of assets is endogenously determined at $0,7\%$, leading to an average savings rate of -7.6%, which is lower than the *OECD* estimate of the savings of households. Further, the ratio of domestically owned assets, $\vartheta = 0.625$, pin-points the net external debt, endogenously defined as 112% and the consumers net financial worth of around 187%. Both are not substantially different from the empirical counterparts from *Bank of Portugal* and *OECD*.[16]

The deposit rate jointly with the depreciation rate, leads to a gross annual cost of capital

worker is more selective when searching for a job, thus landing on average on a more productive firm. Here we take such relationship as exogenous.

[16]The OECD indicator used for the net financial worth of the agents corresponds to the product of the household net worth and the household proportion of financial assets. Implicitly, as a simplifying assumption I assume the liquidity of assets as a proxy of the liquidity of the net worth.

of 7.59%, which is consistent with Card et al. (2014) for Italy. Then supplemented with the capital share leads to an investment share of 21.29%, which is not at the odds with the *Eurostat* data. Noteworthy in the financial side, particularly given the mechanism of job destruction proposed, the average leverage of firms, of 83.7% is broadly in line, or being slightly more conservative, when compared with the information of the administrative panel data of firms (*SCIE*).

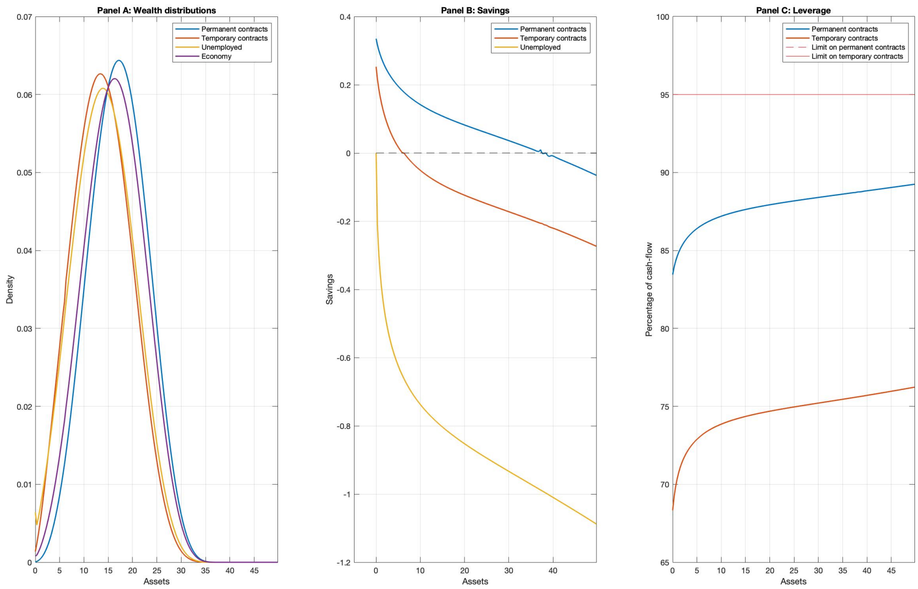
An important feature of the model is its ability to generate wage and wealth dispersion. When compared with the benchmark, the model generates around 54% of the empirical standardized variability of wages, as directly measured in *Quadros de Pessoal* for 2016. This lack of dispersion is consequence of the simplified nature of the productivity functions. While it moderately depends on the wealth of the worker and its type of contract, it does not vary exogeneously, as I do not assume the existence of inherently different *firm types*.

In a more detailed analysis, if one focuses in the wage variation associated with worker's components of a typical AKM wage decomposition, the benchmark empirical coefficient of variation of wages becomes around 0.92, which implies that the model generates 88% of such empirical within variability of wages.[17] Altogether, I abstain from undertaking adjustments to the model in order to allow it to generate wage variability compatible with its empirical counterpart. That would require the adoption of a distribution of *firm types* that would significantly increase computational burden. Given the match of the model's variability with the within empirical results, one should interpret the results as representing the adjustment of a representative firm. The implied stationary wealth distribution is given in figure 7.

One of the key model mechanisms of job destruction is associated with the leverage of firms, namely the interest payment to EBITDA ratio. In figure 7, it is clear that in steady state the firms are not financially stressed, and the permanent contract jobs are around 20 percent more leveraged than the temporary jobs. In what concerns savings, the shape of the savings functions are consistent with the results of the standard Krusell et al. (2010) model, and establishes a reasonable ranking of types of workers, where the permanent contract workers are the ones saving the most.

[17] See Torres et al. (2018) for an empirical analysis of the sources of wage variability in Portugal, resorting to the same dataset. I use their findings in calculate the within variability of wages, as the sum of the variability associated with the worker fixed effect and time-varying covariates, in their two high dimensional fixed effect model.

Figure IV.7: Steady State Worker's Wealth Distribution, Savings and Firm Leverage

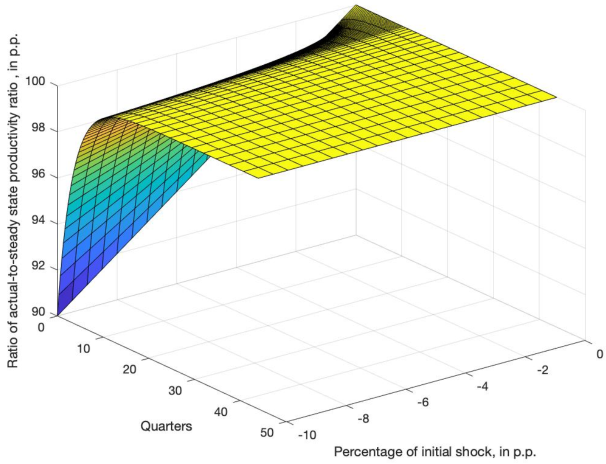


Productivity shock. With the presented calibration, the core of the exercise of the chapter is to study the adjustment paths of the economy subjected to a negative, aggregate, unexpected and one time productivity shock, with a subsequent productivity path until the steady state values. In this process, instead of a defined magnitude of the shock, I build a grid of different magnitudes of an initial shock, z_0, from zero to 10% of the steady state productivity levels. The function of the shock is given by:

$$z_t = 1 + (z_0 - 1)e^{-0.3t}. \tag{IV.24}$$

Thus, the shape of the calibration of those shocks is presented in figure 8. As we will inspect, the adoption of an interval of shocks allows to further highlight the mechanisms generating labour market adjustments, particularly under different wage rigidity regimes and policy options. The algorithm to analyse the transitional dynamics of the proposed economy throughout the downturns implied by the grid of productivity shocks is presented in appendix D.

Figure IV.8: Productivity Shock



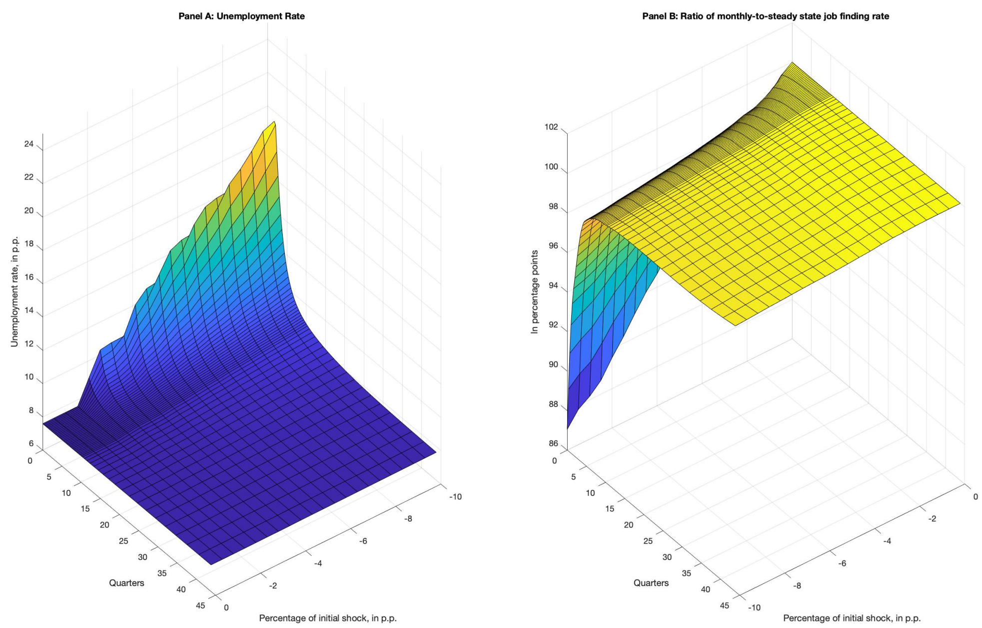
6 Transitional Dynamics with Downward Nominal Wage Rigidity

In the standard description of the modelled labour market, wages are rigid in nominal terms, there persist a significant contract duality, where 21 percent of the workers have their labour relationship governed by a more precarious temporary contract, and firms are at hazard of financial distress if their leverage increases substantially, thus increasing the odds of an episode of match dissolution or insolvency. Such dissolution episodes, when affecting a permanent contract firm consumes a share of the capital available to the insolvent firm in the form of a firing tax.

Under these cornerstone conditions, the model generates substantial unemployment spikes in the outcome of sufficiently negative productivity shocks, particularly at the higher end of the shock magnitude, in absolute terms. As presented in figure 9, the unemployment growth in the first quarters of the downturn, consistently exceeds the initial size of the downturn for initial shocks of 3.5% or higher, reaching a job destruction of 11.8% for the maximum considered downturn of 10%. By definition, such unemployment behaviour corresponds to the aggregate of job creation and destruction dynamics. In our model it intimately relates with the leverage standing of firms. As presented in figure 10, a sizable productivity shock creates significant liquidity constraints at firm level. Those substantially increases the odds of firm dissolution.

The results presented partially follows Mortensen and Pissarides (1994), where endogenous job destruction generates increased volatility of employment when compared to the standard textbook search and matching model. However, in our model, the destruction mechanism is not directly determined by an idiosyncratic productivity shock coupled with a general exogenous threshold, but rather due to an increased hazard rate of bankruptcy of highly indebted firms, which increases significantly the odds of a terminal financial disruption.

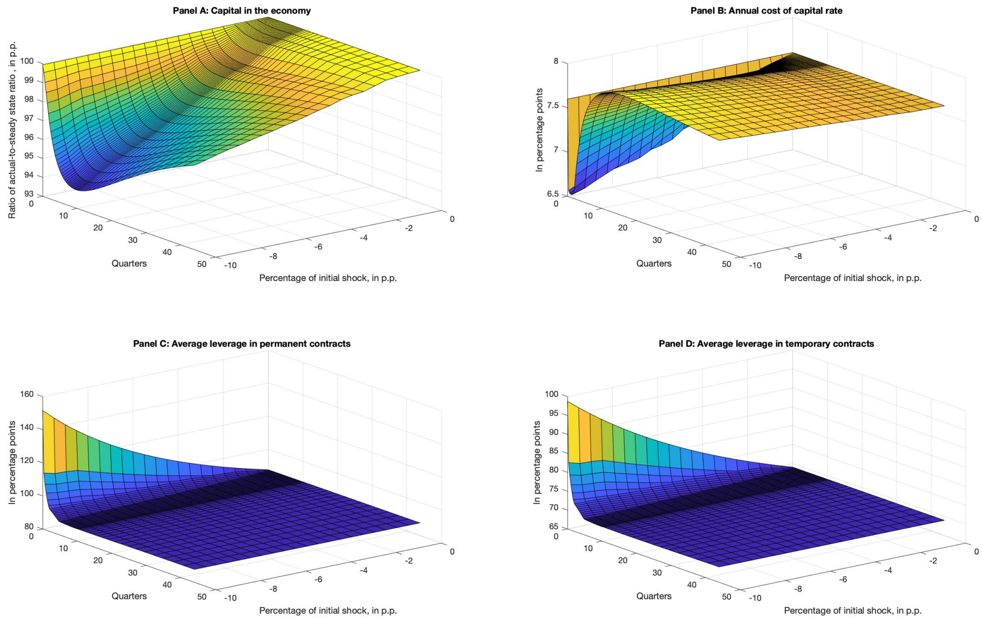
Figure IV.9: Unemployment and Job Finding Rate Transitional Dynamics



Additionally, differently from the standard mechanism, our model establishes a firm liquidity channel to job creation. As presented in figure 6, in this labour market, no highly leveraged firm is ever created, as firms cease to hire temporary workers, and promotions into highly indebted firms is also ceased, thus stopping the steady state dynamic of contract graduation. Consequently, the leverage also refrains the job creation dynamics, over and above the standard receding caused by the fall of the textbook inter-temporal job value *vis-à-vis* the cost of posting a vacancy.

The increased leverage of firms with impact on the firm survival rate also impacts capital, over the standard prescribed mechanism. Traditionally, the fall in productivity reduces the capital utilization of firms, reducing the annual cost of capital rate and the deposit rate, thus creating a pull of investment into consumption. In this model, on top of the traditional mechanism, when firms become highly leveraged, some of then bankrupt, which leads to a capital loss due to increased firing taxes, which further weightens the decline in capital, while on this part not transforming it into consumption.

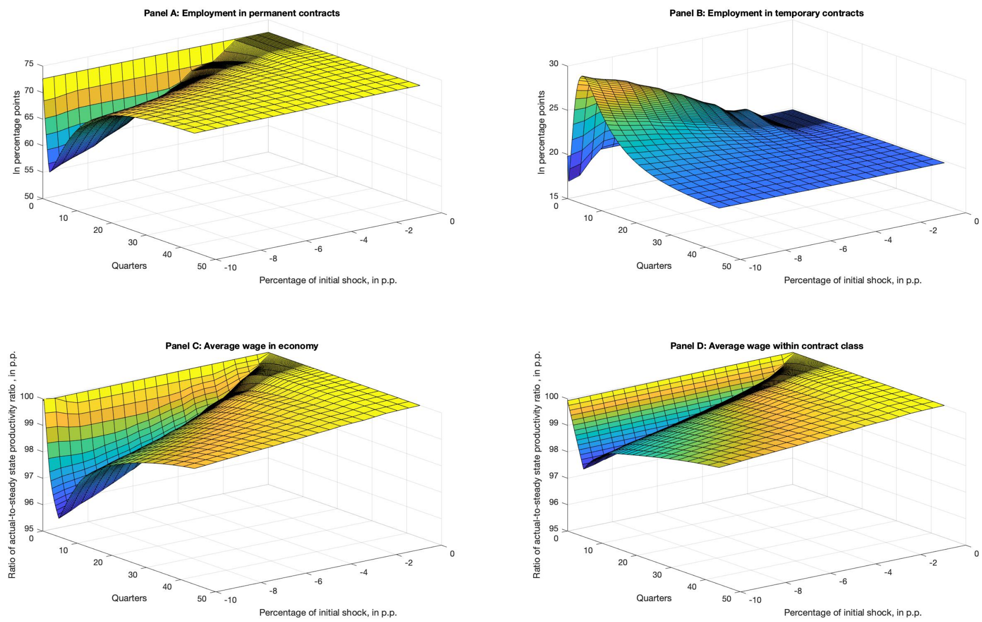
Figure IV.10: Firm Level Mechanisms of Adjustment



Then as the productivity starts to recover, leverage starts to fall, firms start employing more capital, the distress in the labour market flows recedes, with direct impacts on job creation and job destruction, and the unemployment starts to converge to the steady state values. Altogether, the aggregate dynamics highlight a strong propagation mechanism of productivity downturns into unemployment, which can be described as a *catastrophic job destruction*, as the empirical study of Carneiro et al. (2014) as coined.

To further understand the mounting of the described adverse mechanisms, one should assess the behaviour of wages and contract types, which are displayed in figure 11. Generally, in search and matching models, the instantaneous bargaining of wages can constitute an internal financing of the firm in the presence of downturns, as wages are automatically reduced in the response to the downturn. However, in this setting the fall in wages is anchored to a maximum rate of allowed decline, which is compatible with a calibrated year inflation rate of 2%. In these regards, it is assumed that any potential new hires are also subjected to the same degree of rigidity of their wages in relation to steady state wages. Beyond computational concerns, such assumption seems reasonable, as the hiring during the acute phase of downturn is particularly hampered thus not substantially altering the implied wage rigidity, there are tabled wages and compensation policies sizably bargained at sector bargaining level, and there are equal pay equal job concerns. Consequently, with the wage financing channel impaired, firms witness a stronger leverage increase that sparks the described labour market distress.

Figure IV.11: Segmented Wage and Employment Transitional Dynamics

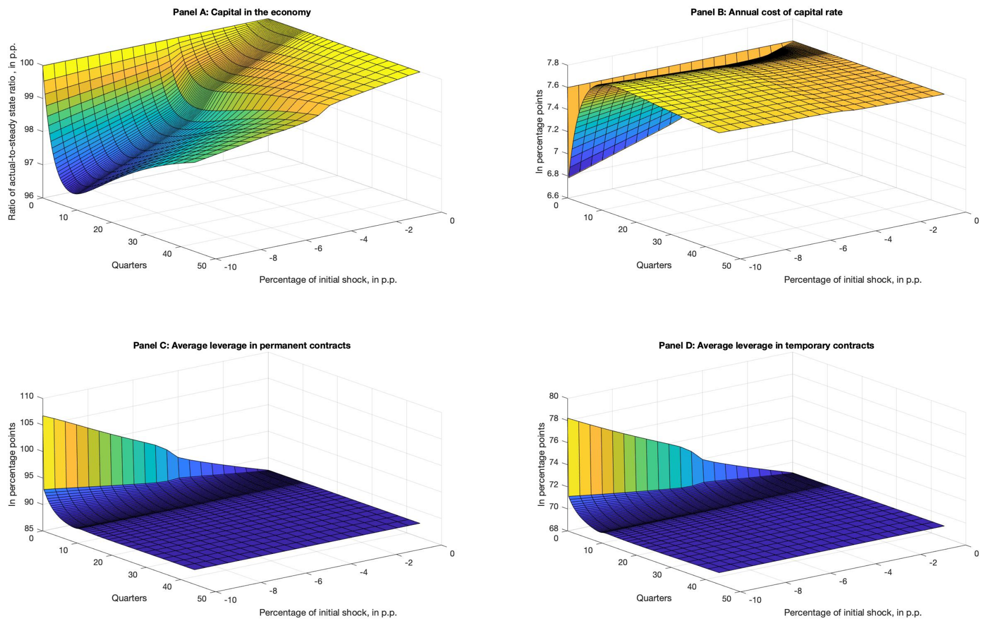


When one partials out the employment dynamics by type of contract, the initial response is lead by a fall in both temporary and permanent employment. Then, as the leverage of actual and potential temporary contract firms is reduced, from an significantly lower level when compared with permanent contract firms, both the current temporary workers see the survival rate of their jobs increase substantially, and the job creation recovers substantially, even though still adversely affected by free entry condition fundamentals. Thus, one starts witnessing an increase in temporary contract employment soon after the initial shock. However, the possibility of graduation into permanent contracts is still impaired while the potential permanent contracts would result into highly leveraged firms. Consequently, the adjustment is marked by a cleansing of permanent contract firms which are dissolved, and the mounting of temporary contract firms which do not graduate during the first phase of the transition. All in all, such adjustment in flows leads to compositional effects that imply a more sizable economy wide average wage adjustment, than the ones witness at contract level, which may potentially mask at aggregate level the degree of wage rigidity.

7 Flexible Wages Paradigm

In this section I start from the same benchmark economy description, but the wages are assumed to be flexible, so that they are continuously determined through the rent-sharing rule, disregarding the wage rigidity rule.[18] Beyond the usefulness of analysing transitional dynamics under this policy change on the standard environment, this analysis allows to further highlight the consequences of the shutdown of the wage financing channel present in the previous section.

Figure IV.12: Firm Level Mechanisms of Adjustment under a Flexible Wage Regime



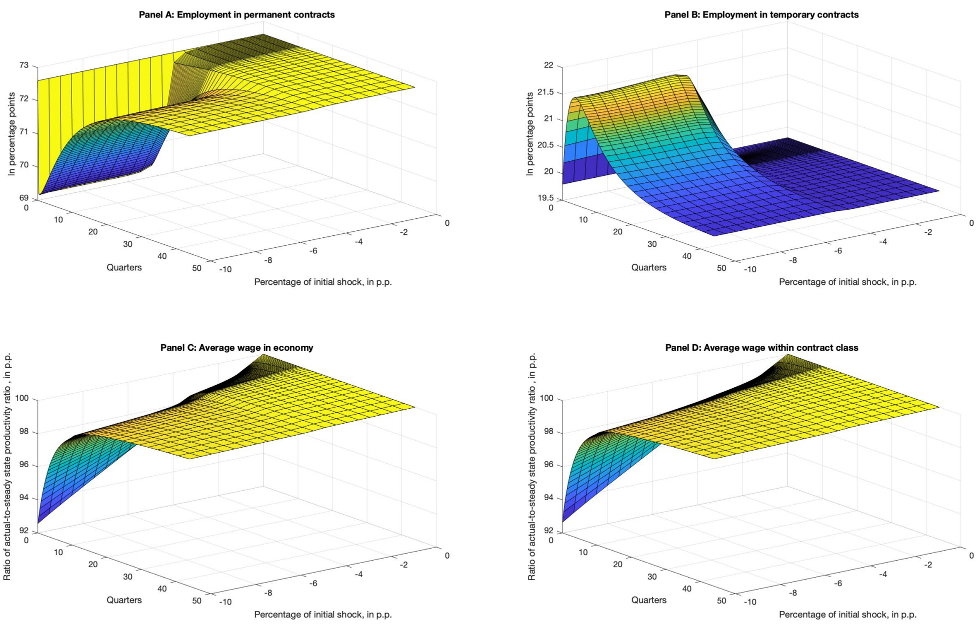
As presented in figure 12, the first stark difference between the considered environments concerns leverage. As wages immediately adjust in response to a fall in productivity, and thus of the match surplus, the leverage of firms increase by a substantially less degree when compared to the rigid-wage scenario. Consequently, firms are less financially distressed, with temporary job firms not even trespassing to a region of substantial increase in the dissolution hazard rate. Thus, the cleansing effect on permanent contract firms is more short-lived and less sizable. The reduction of hiring dynamics is fundamentally driven by the traditional free entry condition dynamics, rather than by the influence of leverage on hiring, and the promotion hold-up lasts fewer periods. Altogether, it implies a lower reduction of the capital stock, due to less firing taxes paid, and less adverse job dynamics.

[18] See appendix E for further details on implementing the flexible wage transitional dynamics algorithm.

The annual cost of capital thus fall by less.

The underlying mechanism that hampers the rise in leverage is displayed in the behaviour of wages in figure 13. When compared with the rigid-wage case, wages fall substantially more, for example by more than the triple in a 10% initial negative shock in productivity. Such adjustment immediately provides the referred internal financing mechanism, which translates in significantly less employment adjustments across types of contracts, thus leading to less powerful composition changes between temporary and permanent contracts. Consequently, the fall in wages in both contract types and economy wide are identical. Noteworthy, while the fall in wages is more pronounced the recovery path is also stronger, implying a return of wages to the steady state levels at a much faster pace.

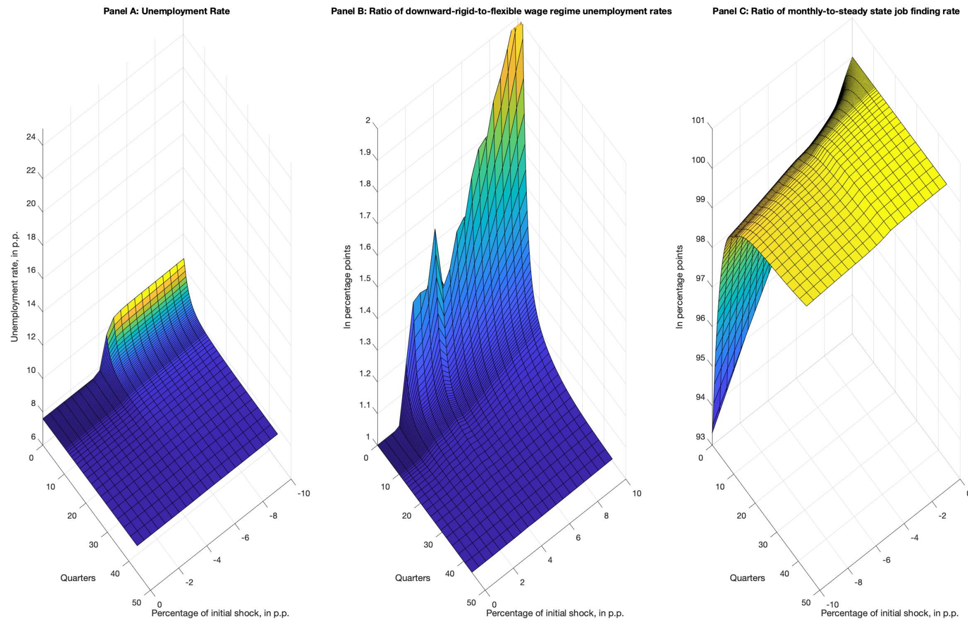
Figure IV.13: Segmented Wage and Employment Transitional Dynamics under a Flexible Wage Regime



The less impactful financial distress also affects the job creation dynamics, as seen in figure 14, which are now solely driven by the dynamics of the inter-temporal free entry condition. Altogether, the difference in unemployment is very sizable. With the presence of flexible wages, the unemployment rate increase is fairly moderate and never surpasses, or comes close, to the amount of the initial productivity downturn. For example for a 10% downturn the unemployment rate increase reaches a maximum of 3%.

Notice that the *defusal* of a catastrophic job destruction is not operated by assuming a textbook drop of an endogenous job destruction mechanism, thus assuming a standard

Figure IV.14: Unemployment and Job Finding Rate Transitional Dynamics under a Flexible Wage Regime



search and matching model unable to generate significant unemployment spikes, or in a more broad discussion a significantly high elasticity of productivity to labour market tightness (see Shimer (2005) for further details). In this environment, I keep the endogenous job destruction mechanism untouched, in the sense that equally high leverage still causes financial distress with the previously described consequences. However, the simple relaxation of wage rigidity, with the consequent opening of the wage bill internal financing channel of firms, substantially mutes the adverse effects witnessed in the downward rigid wage environment.

From a comparative perspective, the adoption of downward nominal rigid wages does not have a substantial impact on unemployment behaviour for minor or moderate productivity downturns, namely in the range of 0% to 2%. For more sizable downturns the labour market disruption is increasingly more noticeable in downward nominal rigid wages. In a nutshell, this comparison highlights the potential nefarious consequence of the job destruction leverage based mechanism coupled with downward nominal rigid wages. Indeed, such mechanism is capable to generate a costly process of destruction of inter-temporally viable jobs, particularly permanent jobs, due to the bite of the instantaneous leverage effect on firm survival. This constitutes what I coin as *a last dance phenomena*. The subsequent policy section of the chapter will discuss different alternatives of aggregate policy designs in trying to defuse this catastrophic job destruction.

In a welfare based perspective, we adopt an adapted continuous-time version of the welfare measure proposed by Krusell et al. (2010). I follow the percentiles of the distribution of workers' asset holdings for each relevant situation of the workers, under the relevant transition path, and record his consumption path. Then, I compare the consumption path of the worker's percentile with the consumption path of the equivalent worker in the equivalent percentile of asset holdings under the stationary distribution. Denote this welfare measure as π. The welfare measure is then calculated holding the following condition:

$$E_0\left[\int_0^{\infty} e^{-\rho t} log[(1+\pi)c(a,j,t)]dt\right] = E_0\left[\int_0^{\infty} e^{-\rho t} log[c^s(a,j,t)]dt\right], \quad j \in \{P,T\} \quad \text{(IV.25)}$$

where $c^s(a,j,t)$ represents the consumption level in the steady state.[19] Accordingly, I will focus in assessing π to measure welfare losses or gains.

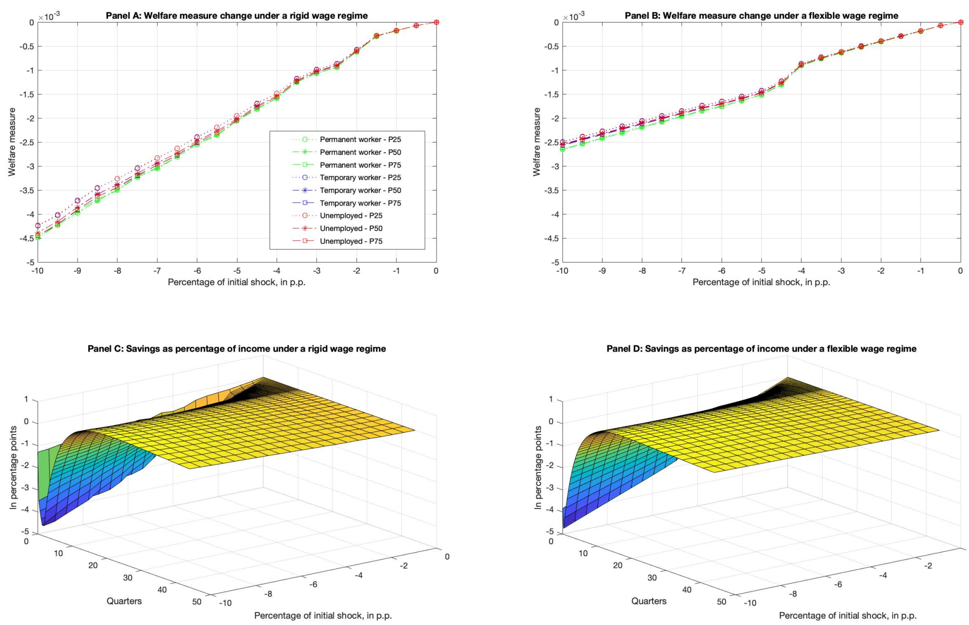
Figure 15 resumes the findings for the downward nominal wage rigidity and wage flexible regimes, coupled with an analysis of the path of average savings in both environments. Regarding the welfare measure, the results broadly confirm the insight of a direct unemployment analysis. Until 2%-3%, the welfare loss associated with either regime is broadly identical, whereas for stronger downturns, the downward nominal wage rigidity regime significantly penalizes the workers, in an across board effect, namely targeting the considered percentiles for every condition the worker finds himself into.

A notable characteristic is the strong consistency of the results for the different labour market groups, and broadly for percentiles. This implies that in the presented model, with either wage adjustment regime, an aggregate productivity shock affects the different groups of workers in a broadly identical percentage of the stationary equilibrium initial welfare state. In terms of percentiles, while in the flexible wage regime there is a substantial homogeneity in the percentage effect, in the case of downward nominal wage rigidity there is a slightly lower impact for the lower percentiles of temporary and unemployed workers. For example, in such regime in the presence of a 10 percent downfall, the ratio of the fall in the welfare measure between the 25^{th} and the 50^{th} percentiles of those workers is on average 1.06. Noteworthy, an identical fall in consumption paths, signalled by π, imply a loss in the utility levels that is decreasing in percentage terms as the initial consumption

[19] Notice that the relevant welfare measure, π, can be calculated as:

$$\pi = e^{-\rho\left\{E_0\left[\int_0^{\infty} e^{-\rho t} log[c^s(a,j,t)]dt\right] - E_0\left[\int_0^{\infty} e^{-\rho t} log[c(a,j,t)]dt\right]\right\}} - 1. \quad \text{(IV.26)}$$

Figure IV.15: Welfare and Savings Comparison across Different Wage Regimes



level increases. Consequently, the utility loss is greater for lower levels of assets, which predominantly populates the unemployed and temporary worker groups.

Regarding the average savings, those are fundamentally influenced by the earnings, and the relationship between the discount rate and the deposit rate, with the latter being indexed to the annual cost of capital. The differences in the trough of the savings adjustment path, across regimes are not substantial. In the downward nominal rigid wage case, while the wage rigidity binds, and the job destruction does not sufficiently kicks in, the adjustment in savings happens at a slower pace, and the trough is attained later, when compared with the flexible wage regime. There, the immediate fall in earnings entices the consumers to immediately resort to savings in order to smooth out consumption. Overall, the rigid wage regime delays slightly the path of adjustment, but do not substantially change it, neither in terms of magnitude, nor in terms of shape. Consequently, it adds additional evidence that the potential intent of downward rigid wages to secure income of the consumers is *trampled* by the job destruction dynamics.

8 Labour Market Policies on Permanent Contracts

While the previous section provides a useful benchmark to assess the mechanisms underlying the adjustment of a more typical Continental European labour market as modelled, it is often the case that wage flexibility is not a feasible policy option, even on dire downturns. By standard, these labour markets have been defined around the notion of some degree of either nominal or real wage security, which is often enshrined in the law. Its relaxation frequently requires either clear popular support, or a broad collective bargaining consensus. Those have not been reached in an aggregate, or even large sector sense in past downturns as the Great Recession. Consequently, in this section I will explore other policy alternatives, focusing in defusing the mounting of very high leverage levels in permanent contract firms.[20]

Simultaneously, as the stance of the fiscal policy in the response to a downturn, and particularly a sizable downturn is often uncertain, and frequently constrained, I will abstain from assuming any degree of fiscal expansion, or any acceptable level of public debt. Thus, the policies considered will be budget balanced, either inter-temporally or instantaneously. Moreover, I will assume that a policy response takes time. To ensure a comparable basis, every policy entangles one quarter of policy lag implementation. The policy objective will be labour market stabilization, with the policy maker willing to fasten the closure of the gaps *vis-à-vis* the stationary equilibrium levels, in terms of employment in both types of contracts and of unemployment.[21] Consequently, when selecting the optimal calibration of the policy, the policy maker will minimize:

$$\int_0^\infty e^{-\rho t}\Bigg\{\Big|e_T(t) + e_P(t) - e_T^s - e_P^s\Big| + \Big|u(t) - u^s\Big|\Bigg\}dt. \tag{IV.27}$$

Finally, also to ensure comparability, and given the nature of the policies to be considered, I will assume their liquidity provision phase will last for 1 year in the calibration proposed. Altogether, the core of this section is to explore a set of policies that are comparable at face value.

Policy #1: Unemployment benefit transfer. The first policy considered starts with

[20]See appendix E for the details of the algorithms to estimate the transitional dynamics implied by the productivity shocks, while accounting for each of the proposed policies.

[21]Alternative, one could consider a policy maker that intents to maximize a welfare function. Here, I take the approach to assume the policy maker intents to act over a visible indirect measure of welfare and functioning of the labour market, and by the end of the section we will make a detailed analysis of the welfare implications of such approach.

a slight change in the assumption framework. If one assumes b is home production that is provided by an unemployment benefit transfer, a natural policy is to provide a transfer to firms by *front-loading* the resources that latter, throughout the adjustment path, would be spent in unemployment benefits if the front-loading does not take place. Consequently, the policy maker pools such resources and provides a transfer to firms in form of a *ad-valorem* subsidy on the wage of the worker - τ^{ub}. Accordingly, from the end of the 1^{st} quarter and until the 5^{th} (i.e. $\Upsilon = [\upsilon; \bar{\upsilon}]$), while the wage of the worker is not altered, the profit of the firm, before the payment of bonuses, becomes:

$$\tilde{\eta}(a, P, t) = \left[\ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} - (1 - \tau^{ub})w(a, P, t) \right], \text{ if } t \in \Upsilon. \tag{IV.28}$$

The referred budget balance condition is given:

$$\int_{\upsilon}^{\infty} b \times [u^{NP}(t) - u(t)]dt = \int_{t \in \Upsilon} [\tau^{ub} w(a, P, t)]dt, \tag{IV.29}$$

where u_t^{NP} correspond to no-policy scenario unemployment paths, which are naturally the ones recorded in the downward nominal rigid wage environment. Given conditions (28) and (29), a calibration of such transfer is attainable, and it will be performed numerically.

Policy #2: Income Tax and firm subsidy transfer.[22] The second policy represents a direct attempt to unmute the wage internal financing channel of firms, through an *ad-valorem* tax on wages and the equivalent subsidy to firms - τ^{ts}. From the side of permanent workers, their period income becomes:

$$y(a, P, t) = (1 - \tau^{ts})w(a, P, t) + \eta(a, P, t), \text{ if } t \in \Upsilon. \tag{IV.30}$$

From the side of the firm, the flow profit before bonuses becomes:

$$\tilde{\eta}(a, P, t) = \left[\ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} - (1 - \tau^{ts})w(a, P, t) \right], \text{ if } t \in \Upsilon. \tag{IV.31}$$

Differently from policy #1, this policy requires an objective function, as it is instantaneously balanced by nature. Thus the policy maker will select τ^{ts}, holding conditions (30) and (31), that minimizes condition (27).

[22] An identical policy based on a reduction of the social security contribution paid by the firms, and an increase of the social contribution paid by workers was proposed in Portugal, with a calibration of 7 percent, during the Great Recession in 2012. Due to strong public resistance it was abandoned.

Policy #3: Moratorium in cost of capital transfer. This policy entails a temporary administrative reduction in the cost of capital, which is subsequently balanced by a future increase in the cost of capital. After the initial policy lag in the period $[0, v]$, the reduction covers the period Υ, which is calibrated to cover 1 year. Then the repayment period is calibrated as 5Υ, in the period $[\Upsilon, 5\Upsilon]$.

The present policy does not consist in a pure loan. Rather, it is drafted as a loan with an adjusted interest rate which accounts for the entry and exit of firms along the policy implementation and repayment periods, and assumes that every firm is subjected to the policy at any given moment, independently of its history. Consequently, for instance firms that enter the market in the repayment period are always subjected to the increased cost of capital, whereas a firm that bankrupts in the middle of the policy implementation period does not have a repayment requirement. The interest rate considered in the policy will be calibrated to ensure the policy is balanced in aggregate flows.

Given this structure, defining $LC(a, P, t)$ as the function that perturbs the standard cost of capital, the flow profit before bonuses becomes:

$$\tilde{\eta}(a, P, t) = \ddot{z}F(\ddot{k}) + LC(a, P, t) - r_l(t)\ddot{k} - w(a, P, t), \tag{IV.32}$$

with $LC(a, P, t)$ being positive in the implementation period, and negative in the repayment period. The relevant interest rate applied to the profile of the moratorium, r_{loan}, is adjusted so that on aggregate, the policy is budget balanced. Therefore, it holds:

$$\int_{t\in\Upsilon}\int_a [LC(a, P, t)g(a, P, t)e^{-r_l(t)t}]dadt = 0. \tag{IV.33}$$

As in the case of a tax-subsidy policy 2, the pin-point of the calibration requires the policy maker to minimize the problem in equation (27), as any design policy entangling equations (32) and (33) is by definition inter-temporally balanced on aggregate terms.

Policy #4: Joint transfer policy. This policy combines the policy instruments of policies 1, 2 and 3. The policy has a balanced unemployment benefit transfer component, a income tax - firm transfer component and a cost of capital moratorium transfer component. Accordingly, the flow profit of the firm, before the payment of bonuses is given by:

$$\tilde{\eta}(a,P,t) = \begin{cases} \ddot{z}F(\ddot{k}) + LC(a,P,t) - r_l(t)\ddot{k} - (1-\tau^{ub,j}-\tau^{ts,j})w(a,P,t), & \text{if } t \in \Upsilon. \\ \\ \ddot{z}F(\ddot{k}) + LC(a,P,t) - r_l(t)\ddot{k} - w(a,P,t), & \text{if } t \in [\Upsilon, 5\Upsilon]. \end{cases} \tag{IV.34}$$

The balancing of the policy entangles the moratorium and income tax and firm subsidy transfer components. Accordingly, the following conditions hold:

$$\begin{gathered} \int_{t\in\Upsilon}\int_a [LC(a,P,t)g(a,P,t)e^{-r_l(t)t}]dadt = 0; \\ \int_{\upsilon}^{\infty} b \times [u^{NP}(t) - u(t)]dt = \int_{t\in\Upsilon}[\tau^{ub,j}w(a,P,t)]dt. \end{gathered} \tag{IV.35}$$

Finally, the calibration of the policy is achieved by the minimization of the objective function in equation (27).

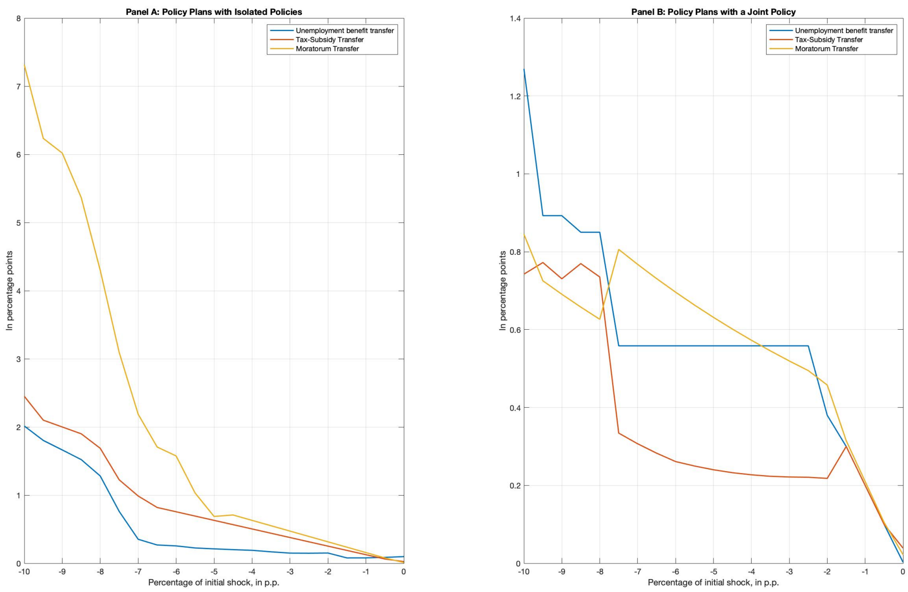
Policy #5: Temporary wage flexibility transfer. Differently from the previous policies, this policy will not be based on transfers and taxes. Rather, it will impose a temporary wage flexibility of wages, which could be implemented by forcing a renegotiation of the collective agreements in the market to define temporary paying clauses that will bind in the period Υ. So while the wages will follow a flexible wage path in the period Υ, they will recover the value they would have under the downward nominal rigid wage regime afterwards. Quantitatively, instead of equation (20), we have:

$$w(a,P,t) = \begin{cases} \tilde{w}(a,P,t) & \text{if } t \in [0,\upsilon] \cup (\bar{\upsilon},\infty),\ \ \tilde{w}(a,T,t) \geq \lambda w(a,P,t-\epsilon) \\ w(a,P,t-\epsilon) & \text{if } t \in [0,\upsilon] \cup (\bar{\upsilon},\infty),\ \ \tilde{w}(a,P,t) < \lambda w(a,P,t-\epsilon) \\ \tilde{w}(a,P,t) & \text{if } t \in \Upsilon, \\ \tilde{w}(a,P,t) & \text{if } t = \bar{\upsilon},\ \ e^{-\lambda(t-t_0)}w(a,P,t_0) < \tilde{w}(a,P,t) \\ e^{-\lambda(t-t_0)}w(a,P,t_0) & \text{if } t = \bar{\upsilon},\ \ e^{-\lambda(t-t_0)}w(a,P,t_0) \geq \tilde{w}(a,P,t) \end{cases} \tag{IV.36}$$

where t_0 is the moment of the downturn shock. This policy will be different from the previously considered flexible wage regime in two fundamental characteristics. First, I assume that the market is initially functioning as a downward nominal rigid wage regime. When faced with the downturn, the policy maker assesses the situation and during the policy lag implementation forces the collective bargaining sides to bargain temporary wage relief clauses. Second, the policy is temporary. The reintroduction of the wage rigidity environment takes place at moment $\bar{\upsilon}$. In that moment, the wage rigidity constraint for that period is compatible with the wage rigidity that would have bound if the downward nominal rigidity constraint was not temporarily relaxed.

Calibration and policy effects. The calibration of the policies is presented in figure 16. For policies #1-#3, the calibration prescribes an increasing path of intervention, particularly for downturns of higher magnitude, in which the growth of the rates assumes an exponential path. There, the policies that are balanced through transfers from the market, namely the income tax firm subsidy, has higher rates, when compared with a policy that is back-up by future savings in unemployment benefits. Also notice that the rate of the moratorium policy is not directly comparable with the other two, given that the rate focuses in the cost of capital flow instead of wages.

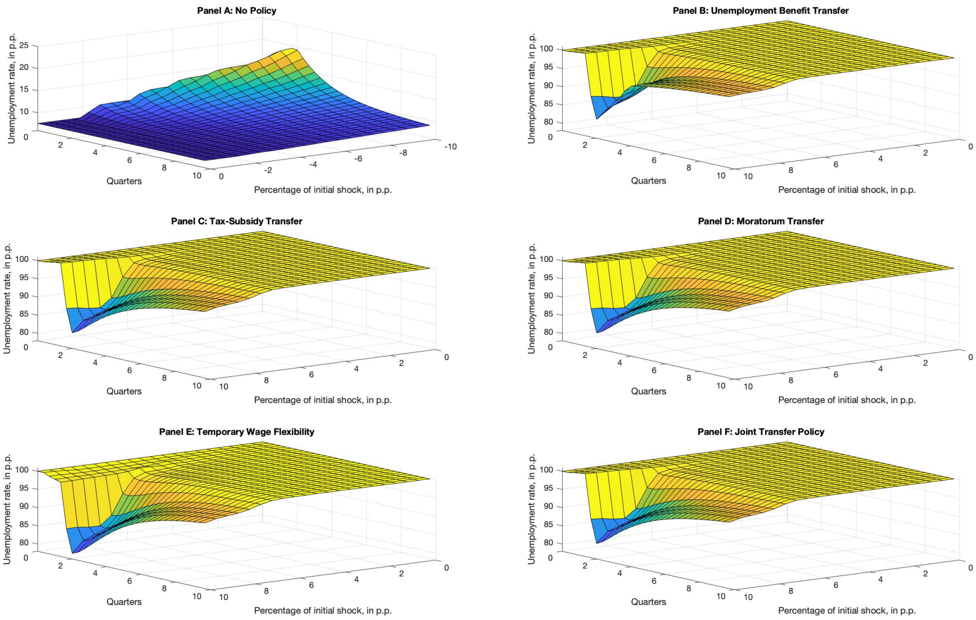
Figure IV.16: Optimal Policy Calibration



Regarding the joint policy approach #4, the broad trend of rates follows the findings of the isolated policies #1-#3, but there is some degree of substitution of the magnitudes of each component of the policy. This interaction is expected, as in theory there may exist trade offs among the components of the policy. For example, the opening of the firm's wage bill financing channel through the income tax firm subsidy component could reduce the front-loading of future savings in the unemployment benefits component.

The main policy maker priority is to contribute to the stabilization of labour market flows. In figure 17, the comparison of the unemployment rate dynamics of the market under a

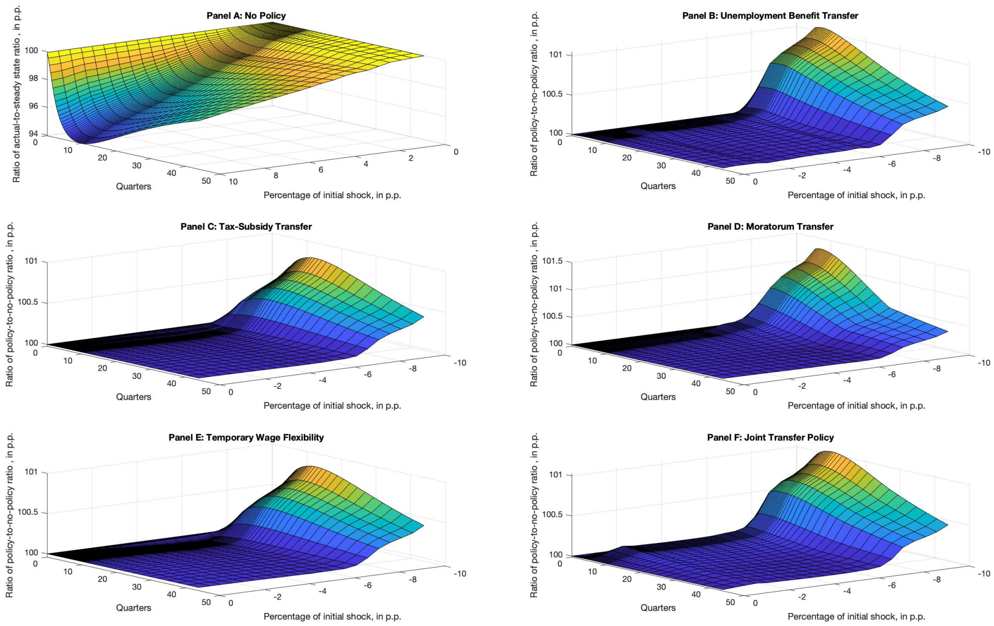
Figure IV.17: Unemployment Effects



no policy approach *vis-à-vis* each considered policy is presented. The first notable finding is that, in presence of a small to moderate downturns, namely below 6 percent, every policy is largely ineffective in enhancing a faster convergence to the stationary equilibrium flow values. For those downturn levels, the leading mechanism for the dynamics of job creation and destruction is rather inter-temporal, focused in the free entry condition and non-distressed hazard rates, as the period financial distress due to very high leverage levels is not triggered. Consequently, the temporary short-lived policies are fundamentally transfers into the profits before bonuses of the firms and wages, rather than having a sizable effect on job creation and job destruction. In a nutshell, given the budget balance nature of the policies, they are simply not sufficiently powerful to translate into significant gains in the dynamics of job creation, and thus their effects are very moderate.

However, in the presence of significant downturns, namely with an initial shock above 6 percent, the policies become significantly effective, reaching a reduction in unemployment in the peak of around 20 percent for a 10 percent downturn. In levels that would translate in a reduction of around 4 percentage points in unemployment rate, which is notable given that every policy does not entail a change in the stance of the fiscal policy. Moreover, the bucket of considered policies present an identical unemployment effect along the considered paths of productivity shocks. These findings highlight the ability of the policies to reduce the sharp increase of the leverage of firms, thus stimulating a quicker reduction in its levels from the disruptive region that, as presented before, constitutes a powerful mechanism of job destruction and weakening of job creation. Thus, by attenuating the adverse contribution of the leverage mechanism, the policy maker effectively attenuates the catastrophic job destruction spell.

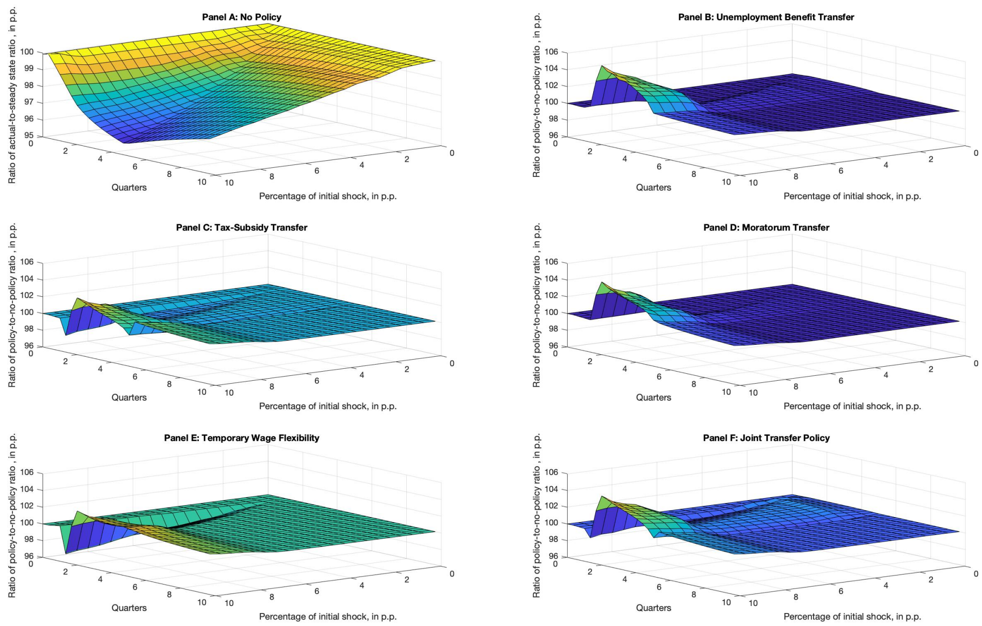
Figure IV.18: Capital Effects



The referred attenuation translates into visible capital stock effects, as presented in figure 18. As the job destruction is reduced, the loss in the capital stock associated with firing taxes is also attenuated. Here, while the effects are broadly shape identical across policies, it is visible the moderate adverse effects of the moratorium policy in the repayment period, where the capital gains are attenuated when compared to the other policy alternatives, and inclusively when compared with the joint policy alternative that resorts to a less sizable moratorium component.

Regarding the effects of the policies in the income profiles along the adjustment path, figure 19 presents the path of the average income. Here, significantly different paths co-exist. On one hand, the policies that focus in reposing some degree of wage flexibility, namely the income tax firm subsidy and the temporary wage flexibility, imposes an initial fall in real terms on the income of workers, when compared with the no policy path. Further, those declines are prescribed even for moderate and small downturn magnitudes. Consequently, these policies confirm the ability to be imperfect substitutes to a wage flexible regime. Differently, the policies that do not directly affect the wages of workers, namely the unemployment benefit subsidy and the moratorium policies, do not entangle an initial real income decline, prescribing immediate sizable income gains.

Figure IV.19: Average Income Effects



These different income profiles offers are a very relevant finding in this discussion. Those constitute alternatives that enable the policy maker to select a preferred shape of income profiles along the adjustment path, while attaining identical unemployment and capital effects. For instance, an initial decline of real income may encounter strong political and public opposition, as for instance was the case in several continental European countries during the Great Recession.

The final point of analysis of this section focuses in the welfare, along the percentiles of the wealth distribution for the different types of workers. The approach is identical to the welfare discussion undertaken before, but the benchmark policy to compare with is the standard model with downward nominal rigid wages without any policy intervention. Consequently, the welfare measure is now given by:

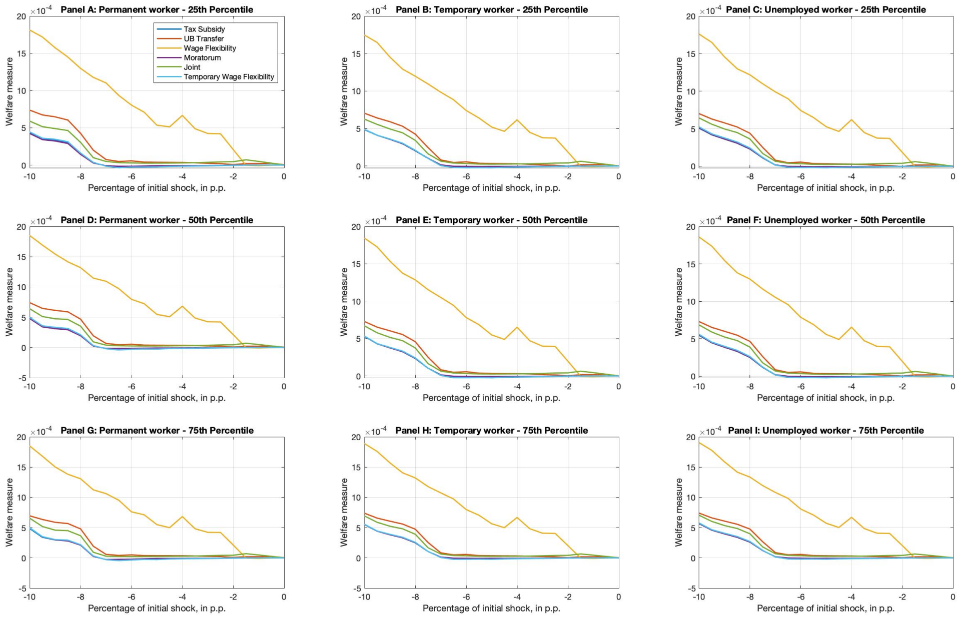
$$E_0\Big[\int_0^{\infty} e^{-\rho t} log[(1+\pi)c(a,j,t)]dt\Big] = E_0\Big[\int_0^{\infty} e^{-\rho t} log[c^{NP}(a,j,t)]dt\Big], \ \ j \in \{T,P\} \tag{IV.37}$$

where $c^{NP}(a,j,t)$ are the consumption paths under a no policy intervention.

The results are summarized in figure 20, and supplemented with a comparison between the pure wage flexible regime and the downward nominal wage rigidity regime. Noteworthy, in the presence of aggregate productivity shocks, the welfare effects are quite identical in shape and magnitude across workers in different positions in the labour market, concretely permanent and temporary contracts and unemployed, and among the considered wealth percentiles.

A fully flexible wage regime has sizable welfare gains for any moderate to strong downturn when compared to any of the adopted policy interventions in the downward nominal wage rigidity environment. In fact, confirming previous findings, those policies only have visible effects for strong downturns. In a ranking, the consumers would prefer a policy based on a subsidy backed by future savings in the unemployment benefit system, followed by the joint policy alternative, and then the remainder policies, concretely the temporary wage flexibility, the moratorium and the income tax firm subsidy alternatives, which attain identical welfare results.

Figure IV.20: Welfare Analysis of Policy Interventions



Notes: The presented welfare measure is the ratio of the standard welfare measure under the policy intervention to the standard welfare measure under the no policy intervention regime.

Altogether, this section highlights several key findings. First, when facing severe aggregate productivity downturns in an environment where there are downward nominal rigid wages and instantaneous financial distress in the presence of high firm leverage, there are budget-balanced policy interventions that can significantly attenuate the spike in unemployment, reduce the loss in capital stock arising from labour market disruption, and provide across board welfare gains to consumers. Those are particularly ineffective for lower magnitude shocks. Second, the bucket of policy alternatives is sufficiently rich to provide an ample set of alternatives to the policy maker concerned in stabilizing labour market flows. Third, those alternatives are visibly different in the wage profiles they imply, with some potentially collecting more public and political support than others. Fourth, either current financing constraints of the fiscal policy, or time-consistency constraints in the implementation of some of the prescribed policy alternatives can be surpassed by alternatives that, while not entailing the same type of policy structure, produces similar unemployment and capital effects. All in all, in our environment, when facing a severe downturn there is a high cost of not intervening.

9 Conclusions

I have analyzed the effect of negative aggregate productivity shocks varying from 0.5 to 10 percent of the benchmark productivity levels, and the subsequent labour market adjustments, in the context of a Bewley-Huggett-Aiyagari incomplete markets model with search frictions, financial constraints, and a labour market of the Continental European type, namely one that combines contract duality and downward nominal wage rigidity. With a benchmark calibration to portray the Portuguese case, I analyze the adjustment paths of the main labour market indicators.

Along the grid of negative aggregate productivity shocks, job creation is increasingly negatively affected, due to the fall of the inter-temporal value of a job, which restricts the creation of new vacancies. However, this mechanism alone is not sufficiently powerful to cause a burst in employment, as the decline in the inter-temporal value of a job facing a short-lived productivity downturn is not sizable enough to generate the required and sufficiently permanent fall in job creation. Even if it was, that would not mimic the stylized empirical facts of the market in the Great Recession, where a job destruction spike carried a sizable role.

Moreover, under standard wage flexibility, the model does not predict any other sizable adverse mechanism to employment. When one allows wages to instantaneously adjust to the value of the worker-firm surplus, firms facing productivity downturns resort to internal financing to face liquidity shortages, as they can reduce the wage bill. Intuitively, they share with the workers, as per the magnitude of the relative bargaining powers plus the fall in bonuses, the costs of the adjustment and largely ensure their survival, even in the presence of severe downturns. However, this mechanism contradicts the stylized facts of wage adjustments, where one sees a significant rise of wage freezes, and few to none wage reductions. Thus, I propose a mechanism that hampers the internal financing channel of firms and places them at greater risk of failure.

The mechanism consists in combining downward nominal wage rigidity, which limits downward real wage adjustments, with instantaneous liquidity constraints, where firms with very high leverage levels, measured as the comparison of the period cost of capital versus the EBITDA of the firm, significantly increases the firm's failure rate when it reaches very sizable levels, of above 0.95. Indeed, as the magnitude of the shock becomes severe, namely above 5 to 6 percent of the stationary equilibrium levels, the model with this mechanism predicts a sizable unemployment surge, with an around 1-to-1 elasticity between the shock magnitude and the unemployment surge, lead by the failure of firms. This finding is compatible with a *catastrophic job destruction phenomena* described in the empirical literature during the Great Recession for Portugal.

Noteworthy, even in the presence of a temporary productivity shock, this mechanism predicts a sizable phenomenon of firms' failure even if they are solvent, as the critical binding constraint becomes liquidity. Thus, the market enters in turmoil with a significant number of workers, particularly permanent workers, facing unemployment with their reemployment dynamics being led by the job creation dynamics. This process is more costly than ensuring the continuity of their job assignments, and can be coined as a *last dance effect* of terminating inter-temporally viable jobs.

The *last dance effect* is critically determined by the leverage level at the dawn of the productivity shock, and the liquidity of firms along the adjustment path. While the managing of the former is fundamentally to be performed prior to the shock, the managing of the latter can be made once the shock has materialized. Accordingly, I devise several balanced-budget policies to enhance the internal financing channel in the trough of the downturn, in order to alleviate the last dance effect. Options entangle instantaneously transferring resources from permanent workers to firms, inter-temporally changing the profiles firm's interest rate payments when they have permanent workers, relaxing them in the trough of the downturn, or even by resorting to future savings in unemployment benefits to subsidize permanent jobs. For severe downturns above 7 percent, every policy, or combination of policies, proved to be effective in reducing by almost 20 percent the maximum recorded unemployment rate under a no policy scenario. Those also translates into less capital destruction, and they have positive welfare effects across the board. Interestingly, they prescribe different wage profiles along the adjustment, opening the possibility of the policy maker to select the most acceptable path, also considering potential public financing constraints.

All in all, in the presence of a market where wage flexibility is impaired and there are instantaneous financing constraints, the labour market adjustment would benefit from policies devised to partially open the internal financing channel of firms, when the downfall is predicted to be strong or severe. The lack of action can spark catastrophic job destruction spells, with significant costs in terms of unemployment and welfare. In this chapter, I addressed the question resorting to aggregate shocks and homogeneous policies targeting permanent jobs. While this proves to be effective under the conditions of the proposed model, it also opens the question of how much more powerful could be targeted policies that takes into account the much richer information about each firm's financial standing that policy makers have access to. Altogether, it reinforces the merit of relaxing institutionally defined constraints in extraordinary periods, to minimize costly *last dance effects*.

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Epilogue

This manuscript have overviewed the conduct of the Portuguese labour market, an example of a Southern European and French based labour market, particularly during the Great Recession. In this analysis, the institutions, as the features that define the market, the contracts, and the relationship of the market actors with those, took a primal part.

In this archetype of market, sectoral bargaining assumes a key role, with trade unions and employer's associations defining the preset of labour contract that firms and workers are bound to follow, or to improve upon. Dynamically, the market is set around the notion of *acquired rights*, largely implying that nominal pay may not be reduced, and dismissals may only take place in a limited set of events, often entangling sizable severance payments. The upshot of this central organization is the enhanced role of trade unions. Differently from other latitudes, trade unions do not need to justify their legitimacy to bargain, as they have their role in collective bargaining legally enshrined. Thus, despite a severe membership erosion through the last decades, they still cover an almost universal set of contracts. Their presence in the wage setting has not abated, but their link to workers have been continuously lessened. Despite this lessening, trade unions still secure higher wages in workplaces and/or sectors in which they represent more workers.

In this treatment, I interpret the wage bargaining as constituting two phases, in a periodic process, with the trade unions incapable to cause complete shutdowns of activity in case of disagreement. At sector level, trade unions and employer's try to argue the worth of each type of workers in the market, and inscribed their findings as collective bargaining tables of nominal wage floors, with sufficient detail to be tailored for each type of worker and job performed. Then, at firm level, either a trade union or the worker, and the respective firm, bargain for potential improvements upon the established minima. Once an agreement is reached, the nominal wage often becomes an enduring right of the worker, which shall not receive less than it for the remainder of the job spell.

Consequently, when the market goes into turmoil due to a severe downturn, the sides temporarily shutdown bargaining, as trade unions understand that further nominal gains are impossible to agree upon, and firms realize that trade unions have locked up their already acquired nominal wage levels. For staying workers, the realignment of real wages with productivity levels takes place as inflation degrades the previously agreed nominal wages. Then, the bargaining process eventually reopens once both sides reacquire incentives to bargain, leading to a very resilient wage setting in real terms.

This internal devaluation is a process that is more liquidity intensive to firms than the re-bargaining of wages without nominal constraints. Moreover, often firms have limited ability to adjust the workforce without substantial costs, resulting in dismissals of permanent contracts being often circumscribed to voluntary exits, and the dismissal of temporary contract workers plus the pause on hiring not being enough to secure a sizable adjustment. If liquidity is sufficiently exhausted and leverage increases sizably the firm risks bankruptcy as the financial institutions pull off.

Critical to avoid a disorderly adjustment in these markets, with the breakdown of intertemporally viable worker-firm matches, is inflation, which fastens the process of real wage adjustment. It is not by chance that many of these economies have hold higher inflation levels when managing their own currencies. Alternatively, in contemporary low inflation regimes, the management of firms' liquidity, and of its leverage, becomes critical during the slump. If neither is sufficiently effective, a catastrophic job destruction takes place, largely fueled by bankruptcies. I argue this process was behind the Great Unemployment Divergence recorded during the Great Recession.

Yet, the fate is not mechanical. It is possible to navigate identically rough waters with substantially different outcomes. For such, the management of leverage and liquidity of firms along the downturn is critical, and relief may pour from a combination of monetary and fiscal policy. Even considering temporary fiscal budget balanced policies, or extraordinary collective agreement flexibility, it is possible to sizably alleviate the pressure. Perhaps, that has been the difference between the approach undertaken in the Great Recession, and so far in the current COVID shock, where unemployment didn't increase sizably.

After severe tribulations that these labour market institutions have endured and prevailed, one should find ways to fit policy to alleviate its propensity to spark catastrophic job destruction, particularly in a low inflation regime, rather than attempting to dismantle the institutional framework, that has always witness substantial popular support.

Success is not final; failure is not fatal: it is the courage to continue that counts.
Winston Churchill

Appendix to Chapter 2

Appendix Table 1: The Most Representative Collective Agreements, 2010-2013

Rank	Collective Agreement description	Number of Covered workers per year	Average Union Density	Number of Job Titles
1	Building and Construction	254,840	2.9%	461
2	Office Workers	143,941	2.9%	62
3	Metalwork and Metallurgy Industry	126,300	10.4%	34
4	Building and Construction II	109,115	2.6%	421
5	Supermarkets and Hypermarkets	108,632	9.4%	125
6	Private Social Security Institutions	103,563	5.8%	386
7	Car Industry and Trade	78,897	7.7%	412
8	Building and Construction III	69,662	2.2%	407
9	Textile Industry	65,065	5.6%	233
10	Hospitality Sector - Center and South Region - Restaurants	63,072	3.0%	259
11	Retail - Lisbon Region	61,141	2.9%	493
12	Cleaning Services	60,240	8.7%	105
13	Hospitality Sector - North Region - Restaurants	45,364	1.6%	390
14	Public Transport and Road Transport of Goods	43,470	3.4%	106
15	Private Education	42,196	3.2%	115

Source: Relatório Único, 2010-2013.

Appendix Table 2: OLS Estimation of the Union Wage Gap for Total Monthly Wages including Non-Covered Workers

Variable	Coefficient (s.e.)	
Union density	0.1370	0.1387
	(0.0197)	(0.0197)
Non-Covered worker		0.0605
		(0.0088)
R^2	0.5499	0.5503

Notes: Dependent variable: total monthly wages (in logs). The controls also include a quadratic term in age, a quadratic term in tenure, a gender dummy, schooling dummies (7), firm size dummies (5), sector of activity dummies (25), and year dummies (24). The number of observations is 43,455,566. Robust firm clustered standard errors are in parentheses: all coefficients are statistically significant at the 0.01 confidence level. *Sources: Quadros de Pessoal* 1986-2009; *Relatório Único*, 2010-2013.

Appendix Table 3: Summary Statistics

Variable	Period	
	1986-2009	2010-2013
Union density	–	9.90%
	–	(21.77)
Logarithm of Total Monthly Wages	5.47	5.60
	(0.55)	(0.53)
Logarithm of Bargained Wages	5.22	5.31
	(0.46)	(0.44)
Logarithm of Wage Cushion	0.25	0.29
	(0.40)	(0.43)
Worker's age (in years)	36.85	40.01
	(10.93)	(10.37)
Tenure of the worker (in years)	8.18	8.79
	(8.43)	(8.56)
Proportion of Females	40.4%	45.7%
Education:		
Less than primary school	2.76%	0.68%
Primary school	35.59%	15.52%
Basic school	21.94%	18.51%
Elementary school	17.05%	27.05%
Secondary school	16.39%	24.2%
Post-secondary school	0.23%	0.41%
University attendance	1.58%	1.90%
College degree	4.47%	11.72%
Workers in firms with:		
less than 50 employees	47.23%	51.94%
50 to 99 employees	11.73%	11.51%
100 to 499 employees	21.20%	18.92%
500 to 999 employees	6.00%	4.55%
1000 to 4999 employees	8.93%	8.13%
more than 5000 employees	4.91%	4.95%
Worker-Year Observations	$30,397,602$	$6,237,187$

Notes: For the continuous variables, the mean is presented, with the standard deviation displayed in parenthesis. *Sources: Quadros de Pessoal*, 1986-2009; *Relatório Único*, 2010-2013.

Appendix Table 4: Conditional Decomposition of the OLS Estimation of the Union Wage Gap for Different Model Specifications

Variables	Base Model (γ_0)	Full Model (γ_1)	Gelbach Decomposition
Standard Model			
Union Wage Gap	0.1619***	−0.0029	–
	(0.0244)	(0.0086)	
τ_{α_1} (Worker FE)	–	–	−0.0026
			(0.0055)
τ_{θ_1} (Job-title FE)	–	–	0.0571***
			(0.0198)
τ_{λ_1} (Firm FE)	–	–	0.1103***
			(0.0134)
R^2	0.5373	0.8801	
Standard Model without Firm Size and Sector Dummies.			
Union Wage Gap	0.4492***	−0.0222*	–
	(0.0333)	(0.0115)	
τ_{α_1} (Worker FE)	–	–	0.0031
			(0.0063)
τ_{θ_1} (Job-title FE)	–	–	0.0637**
			(0.0281)
τ_{λ_1} (Firm FE)	–	–	0.4047***
			(0.0208)
R^2	0.4442	0.8799	
Standard Model without Education, Firm Size and Sector Dummies.			
Union Wage Gap	0.6643***	−0.0178	–
	(0.0572)	(0.0123)	
τ_{α_1} (Worker FE)	–	–	0.0744***
			(0.0140)
τ_{θ_1} (Job-title FE)	–	–	0.1491***
			(0.0337)
τ_{λ_1} (Firm FE)	–	–	0.4586***
			(0.0261)
R^2	0.2044	0.8797	

Notes: Decomposition based on Gelbach (2016). Robust firm clustered standard errors are in parentheses: ***, **, * denote statistical significance at the 0.01, 0.05, and 0.10 levels, respectively. The standard model includes as regressors a quadratic term in age, a quadratic term in tenure, schooling dummies (7), a gender dummy, firm size dummies (5), year dummies (24), and sector dummies (25). The number of observations is 36,616,379. *Sources: Quadros de Pessoal*, 1986-2009; *Relatório Único*, 2010-2013.

Appendix Table 5: The Conditional Decomposition of the OLS Estimation of the Union Wage Gap for Total Monthly Wages, using ESCO Occupation Definition

Variables	Base Model (γ_0)	Full Model (γ_1)	Gelbach Decomposition
Union Wage Gap	0.1619***	0.0091	–
	(0.0244)	(0.0095)	
τ_{α_1} (Worker FE)	–	–	0.0072
			(0.0086)
τ_{θ_1} (Occupation FE)	–	–	0.0164**
			(0.0077)
τ_{λ_1} (Firm FE)	–	–	0.1292***
			(0.0126)
R^2	0.5373	0.8801	

Notes: Decomposition based on Gelbach (2016). Robust firm clustered standard errors are in parentheses: ***, ** denote statistical significance at the 0.01 and 0.05 levels, respectively. The base model includes as regressors a quadratic term in age, a quadratic term in tenure, schooling dummies (7), a gender dummy, firm size dummies (5), and sector dummies (25). The number of observations is 35,764,082. *Sources: Quadros de Pessoal* 1986-2009; *Relatório Único*, 2010-2013.

Appendix Table 6: 2SLS Estimation of the Union Wage Gap for Total Monthly Wages, Using the Job-title Fixed Effects as an Instrument for Union Density

Variable	2SLS
Union density	0.1257
	(0.0658)
Worker FE	Yes
Firm FE	Yes
R^2	0.8683

Notes: Dependent variable: total monthly wages (in logs). The second stage regression also includes as controls a quadratic term in age, a quadratic term in tenure, a gender dummy, schooling dummies (7), firm size dummies (5), sector dummies (25), year dummies (24), firm fixed effect and worker fixed effect. The first stage regression has union density on the left-hand side and the job-title fixed effect plus the controls presented in the second stage on the right-hand side. The number of observations is 36,616,379. Robust firm clustered standard error is in parentheses. The coefficient for predicted union density is statistically significant at the 0.10 confidence level. *Sources: Quadros de Pessoal* 1986-2009; *Relatório Único*, 2010-2013.

Appendix to Chapter 3

A The Dataset and Collective Bargaining Structure in Portugal

A1 Quadros de Pessoal, Relatório Único, and Sistema de Contas Integradas das Empresas (SCIE).

The data sources of this study comprises information about the balance sheet and income statement of the firms obtained from SCIE - *Sistema de Contas Integradas das Empresas*, for the years 2005-2016, and information about the characteristics of the workers and of their employment relationship, provided in *Quadros de Pessoal* (Personnel Tables) for the years 1986-2009, and *Relatório Único* (Single Report), for the years 2010-2016.[1]

The three datasets are fully matched for the common years of the data. Consequently, for each year from 1986 to 2016, we have a longitudinal matched employee-employer-contract-job-title database, which from 2005 is supplemented with matched information at firm level about the balance sheet and income statement of those firms.

The *Quadros de Pessoal* and the *Relatório Único* datasets are recorded by the Ministry of Employment and Social Security and correspond to a mandatory survey on an annual basis for all establishments with at least one wage earner. In this survey all workers employed in October of each year are reported, although civil servants and workers in domestic service are not covered. Therefore, the dataset covers the entire population of workers of private-sector firms in manufacturing and services. Further, the long-lived requirement of the information to be published at establishment level, ordinarily at the door of the establishment ensures greater validity.

The dataset reports the firm's location, industry, employment, sales, ownership, and legal

[1]For the year 1990 and 2001, the survey was either not administered, or not digitized.

basis. Worker information includes gender, age, skill, occupation, schooling completed, starting date at the firm, earnings, and working hours. In addition, the survey also records the collective bargaining arrangement and the specific job-title held by the worker under collective agreement, which is of particular importance for this study.

In these datasets the following restrictions were applied: (a) we only consider full-time employers in receipt of what is contractually defined for the reporting month; (b) we exclude workers from agriculture, fisheries, and energy products/extraction sectors; (c) we exclude workers aged less than 18 years or greater than 65 years; and (d) we exclude workers earning less than 80 percent of the minimum wage[2]. A significant array of descriptive statistics on this matched dataset is provided in table A1 and A2.

As a matter of empirical implementation, we do not have the wage floor for each job-title, and we just have the identification of each category for each worker. However, we are aware that each job-title is subjected to a minimum base wage. Therefore we estimate the minimum base wage for each job-title as the minimum hourly wage reported for the workers working with that category in the entire economy. Implicitly, we are assuming that the minimum base wage for each job-title is at least binding for one worker.

Then we combine the established matched dataset with the SCIE dataset, with a coverage of the match above 97 percent. The SCIE dataset is managed by National Statistics Institute, which provides a unified survey system. Its reporting is mandatory for the universe of registered firms operating in Portugal, including those with no employees. This dataset has a vast array of accounting information, namely with detailed information about every entry of the balance sheet and income statement legally required for accounting purposes under the *SNC - Sistema Normalização de Contas*. Among the information provided, one has access to the level of assets, liabilities and equity, and its typical accounting partitions, as well as profits, output value, value-added, payroll, purchase of intermediate goods, investment levels, service of debt among others.

A2 Example of a Wage Table of a Collective Agreement of the Portuguese Labour Market

In the Portuguese Labour Market, each collective agreement and its wage table is published in the *Boletim do Trabalho e Emprego* (i.g. Work and Employment Bulletin). As a matter of example, table A3 presents one of the wage tables existing in 2017. It corresponds

[2]Corresponding to the lowest admissible wage in the case of apprendiceships.

to a CCT, or sector agreement, between *AHRESP* (i.g. the association of employers of hospitality and similar), and *SITESE* (i.g. the union of workers and technicians of services, commerce and hospitality), signed in April 22, 2017.[3]

Interestingly, and common to the vast majority of the agreements in place, the presented agreement is incredibly detailed with more than 60 clauses and a vast number of appendices. For example, there are specific rules for: (a) working conditions; (b) minimum payments for meal subsidy among other side payments; (c) specific rules that even includes clothing; and (d) the definition of minimum ranks based on observables, for example education and tenure.

Our claim in the text is that each of these agreements correspond to a branch, each ranking the workers, in this case with 13 ranks, and classifying the workers, within the ranks, according to their job titles. These classifications often exist in more than one agreement. It is noteworthy the sector nature of these agreements. While it may be perceived as a breach in the ranking, in our interpretation it provides adjustment of the outside options to sector preferences and sector labour market outcomes governing the outside option. Such argument is reinforced by the proper structure of unions, which are aggregated in two major confederations (i.e. UGT and CGTP) - clearly a centralized system.

Our interpretation is reinforced by the idea that those wages are pledged and legally binding for the future, and tie the firm to pay those values given the worker's rank, whom cannot be demoted. Simultaneously, the firm and the union may still at firm level bargain, with the latter pushing for better working conditions and compensation, which for the former will be more flexible to withdraw in the presence of adverse economic conditions. It is this reasoning that lead us to consider that those bargained wages are indeed the level of the immediate outside option of the worker.

[3] The translations presented are freely derived by the authors, and thought as faithful to the original meaning in Portuguese. This corresponds to a revision of a previous sector agreement between the parties as it is often the case in these agreements.

Table A1 - Descriptive Statistics on the Dataset Dimmensions

Variables:	Percentiles			Mean	Standard	Skweness	Kurtosis	Total
	5	50	95		Deviation			Number
Observations	-	-	-	-	-	-	-	29,115,656
Per Year	1,305,735	1,391,557	1,474,118	1,386,460	-	-	-	-
Workers characteristics								
Age (years)	22	38	57	38.46	10.87	0.26	2.23	-
Education (years)	4	9	16	8.56	3.97	0.38	2.01	-
Tenure in the Firm (years)	0	6	23	8.7	9.09	1.26	4	-
Female (perc./year)	39.91	41.89	46.04	42.4	0.03	0.25	1.73	-
Mover to the Firm (perc./year)	15.96	22.77	27.08	22.81	4.89	1.20	5.96	-
Duration of Spells (years)								
Workers	1	4	14	6.04	5.29	1.03	3.07	4,816,652
Worker-Firm	1	2	10	3.92	4.09	1.86	6.16	7,436,538
Worker-Task Market	1	2	10	3.88	3.91	1.80	6.01	7,513,257
Worker-Rank	1	1	3	1.27	0.68	2.14	5.59	22,946,436
Firms								
Workforce Size	6	13	55	33.84	187.13	53.28	4,488.52	1,139,819
Workplace Size	1	3	15	8.9	58.36	70.69	7,894.81	4,309,996
Collective agreements								
Agreements-Year	477	536	764	574	108	1.02	2.46	-
Task Markets-Year	3	7	8	6.11	2.12	-0.93	2.70	-
Ranks in Agreement-Year	2	25	113	48.43	75.59	5.07	47.31	-
Firms-Year	1	6	164	82.55	350.89	12.54	224.98	-
Workers-Year	9	260	4,599	2,455.2	9,073.7	8.34	93.09	-
Years	1	5	21	8.47	7.45	1.26	3.64	-

Notes: There are 21 years in the dataset. The year 2001 is missing from the dataset as it was not recorded. *Source: Quadros de Pessoal, 1995-2009 and Relatório Único, 2010-2013.*

Table A2 - Descriptive Statistics on Worker's Wages

Variables:	Percentiles					Mean	Standard Deviation	Skweness	Kurtosis
	5	25	50	75	95				
Log of Nominal Monthly Wages									
Total Wages	5.30	6.07	6.48	6.93	7.76	6.51	0.72	0.29	3.63
Base Wages	5.20	5.92	6.26	6.63	7.44	6.29	0.65	0.41	3.99
Bargained Wages	4.78	5.52	5.92	6.18	6.71	5.85	0.56	-0.02	3.99
Wage cushion $\left(\frac{w_{base}}{w_{base\|barg.}}\right)$	1	1.20	1.42	1.85	3.49	1.77	1.61	57.34	11,407.95
Base Wage Ratio $\left(\frac{w_{base}}{w_{total}}\right)$	0.44	0.72	0.85	0.95	1	0.81	0.19	-1.98	36.16
Bargained Wage Earners (%)	0.32	0.58	0.64	0.86	0.98	0.68	0.23	-0.20	2.05
Hours Worked	152	168	173	194	193	171.8	11.62	-0.25	4.32

Source: Quadros de Pessoal, 1995-2009 and Relatório Único, 2010-2013.

Table A3 - Representative Wage Table, with Bargained Wages.

Rank	Job Description	Minimum Base Wage
13	General Manager	1,515
12	Board Assistant; Commercial Manager; Service Manager; Human Resources Manager; Technical Manager	1,240
11	Head of Department; Head of Division; Head of Services; Nutrition Technician 1st Class	1,018
10	Head of section (office); Head of Sales; Inspector; Board Secretariat officer; Nutrition Technician 2nd Class	898
9	Administrative; Head of Cafeteria; Head of Purchases; Head of Kitchen; Head of Pastries; Head of Storage; Head of Dinning Room; Inspector of Sales	808
8	Cashier; Head of Preparation Room; Controller; Cook of 1st Class; Sub-Head of Dinning Room; Administrative Assistant; Pastry Cook; Sales Technician	771
7	Driver of Heavy Vehicles; Storage Keeper; Polyvalent Worker	716
6	Driver of Non-heavy Vehicles; Administrative Assistant 2nd Class; Pastry Cook 2nd Class; Sub-Head of Dinning Room 2nd Class; Sales Representative	700
5	Cook 2nd Class; Controller of Balcony; Controller of Bar; Controller of Storage and packing; Admin. Assistant 3rd Class	629
4	Head of Copa; Cook of 3rd Class; Packing worker; Storage Worker	582.50
3	Controller cashier; Storage worker; Bar Worker; Balcony Worker 1st Class; Distribution Handler;	570
2	Balcony Worker 2nd Class; Admin. Intern; Hospitality Assist.	562
1	Driver Assistant; Distribution Assistant; Barman Intern (1 year); Cook Intern (1 year); Pastry Intern (1 year); Cleaning Worker; Dining room Employer	557

Notes: Amounts in euros, and corresponding to monthly payment. Table extracted from the Sector agreement signed on the 22^{nd} of April 2017, between *AHRESP* (e.g. the association of employers of hospitality and similar) and *SITESE* (e.g. the union of workers and technicians of services, commerce and hospitality). *Source: Boletim do Trabalho e Emprego, 2017.*

B Technical Details on the Model and Links with the Literature

In this appendix we present: (i) the relationship of the model with the search and matching literature and the union-firm bargaining literature, in part B1; (ii) the derivation of additional relevant conditions to fully describe both the dynamic and the steady state equilibria, in part B2; (iii) the derivation of equilibrium wages for both types of equilibria, in part B3; and (iv) the description of the dynamic and steady state equilibria and their properties, concretely regarding existence and uniqueness, in part B4.

B1 The Links between the Model and the Search and Matching Literature and the Firm-Union Bargaining Literature

Bridging the Model of Labour Tasks and some Canonicals of the Search and Matching Literature. The apparatus of our model was constructed so it can be easily reshaped as several canonical models in the literature of search, mostly resorting to: (b) a wise choice of parameters, and (c) variations of the modelling of the choice of capital.

Firstly, the mutation of our model into the model of Acemoglu and Hawkins (2014) critically requires that: (a) we set $J = 1$, as they abstain from modelling different types of tasks; (b) $\xi^{\Xi}(a|K, \beta) = 0, \forall\{a, K, \beta\}$, as they don't consider a model with on job search; (c) $\{K\}$ to be relabelled as z their idiosyncratic firm's productivity parameter; (d) $\beta = \bar{\beta}$ so there is homogeneous bargaining power in the economy; (e) $a = \bar{a}$ and known by workers and firms; and (f) $\tilde{s}(a|K, \beta) = 0$ as there is no firing at-will.

Further, in the model of Acemoglu and Hawkins (2014) consider $\gamma(V)$ to be linear and not strictly convex as presented. The fundamental implication of this deviation is that instead of having a growth path of each firm, which is pivotal in the author's analysis, we assume a firm can immediately attain its optimal scale without incurring in further costs due to simultaneous hiring.[4] Closely related with this deviation, we do not model entry and exit of firms, and thus in their model consider $FC_t = \infty$ - no entry of new firms, and $\delta = 0$ - no exit of firms.[5]

[4] The extension to allow the incorporation of their class of vacancy costs increases the complexity of the model by some degrees, given one is required to keep track of the history of firms.

[5] This deviation allows for notation simplicity, and given the intention to focus on optimal scale, comes without further implication.

Secondly, one can also adapt this model to resemble Cahuc et al. (2008), which models a representative firm. For this purpose, consider: (a) $\xi^{\Xi}(a|K,\beta) = 0, \forall\{a,K,\beta\}$, so there is no on job search; (c) $A(j,a) = 0, \forall j, a$ so that operating costs are fully neglectful; (d) there is no firing at will so $\tilde{s}(a|K,\beta) = 0, \forall\{a,K,\beta\}$; and either (e1) $\{K,\beta\} = \{k,\bar{\beta}\}, \forall\{K,\beta\}$, where k is a given constant, and β is homogeneous across firms, but potentially different across task markets, representing a $J \times 1$ vector. Therefore, firms do not have any heterogeneity arising from capital or bargaining powers, and we follow the most restrictive version of their model, without capital; or (e2) consider that capital is also chosen optimally *ex-ante* to the task decisions and thus add the following condition to our equilibrium:[6]

$$\frac{\partial F(\mathbf{N},K)}{\partial K} = r + d + \int_0^1 \sum_{j=1}^{J} N_j \pi^{\frac{1-\beta_j}{\beta_j}} \frac{\partial^2 \tilde{F}(\mathbf{N}\mathbf{A_j}(\pi),K)}{\partial N_j \partial K} d\pi, \tag{.1}$$

where d is the depreciation rate, which we have abstracted in our model formulation, and $\mathbf{A_j}(\pi)$ is identical to equation (41) of appendix B2.

Thirdly, resorting to the insights of Cahuc and Wasmer (2001), the model can also be translated to the large firm version of the matching model of Pissarides (1990). For that purpose, we have: either (a1) $J = 1$ so that there is only one type of tasks; or (a2) $J > 1$, but the types of tasks are perfect substitutes; (c) $\xi^{\Xi}(a|K,\beta) = 0, \forall\{a,K,\beta\}$, so no on job search; (d) $a = \bar{a}, \forall j, a$, so that the worker heterogeneity is fully neglectful for equilibrium purposes; and (e) perfect capital markets, and simultaneous decision of labour and capital so that the capital stock of the firm becomes a function of employment ($K_t(\mathbf{N})$), and the following condition hold:[7]

$$\frac{\partial F(\mathbf{\Phi}_t, K_t)}{\partial K_t} = r + d. \tag{.2}$$

Fourthly, the model would mimic Mortensen (2009) if we consider: (a) there is not entry or exit of firms (i.e. $\delta = 0$ and $FC_f = \infty$); (b) $J = 1$ so that there is only one type of tasks; (c) $a = \bar{a}, \forall j, a$ so that operating costs are fully neglectful; (d) K is constant overtime, and represents the idiosyncratic productivity of the firms, presented in the chapter as $p(x)$, and (e) $\beta = \bar{\beta}$ is constant for every firm and in every task market.

In a nutshell, our model is isomorphic to a wide range of standard search and matching models. Mainly that is attainable by: (a) sufficiently restricting the parameterization of the model (i.g. heterogeneity); (b) considering alterations of the capital allocation

[6]See Cahuc et al. (2008) to the details on how to obtain this expression from our model under this set of assumptions. Further notice that if one considers optimal choice of capital, *ex-ante* to task allocation, and fully neglects the firm heterogeneity arising from bargaining powers, then we are in an environment of a representative firm.

[7]See Cahuc and Wasmer (2001) for the specific details about this equivalence.

mechanism, whose implications for our modelling objectives are fairly minor; and (c) considering the dynamics of vacancy costs and entry and exit of firms.

Finally, Krause and Lubik (2007) provides relevant insight about the macroeconomic implications for the empirical fitting of a search and matching model with intra-firm bargaining in the spirit of Stole and Zwiebel (1996a). Firstly, intra-firm bargaining causes firms to expand employment in order to bolster their bargaining position relatively to workers, as analyzed by Cahuc et al. (2008). Theoretically, such movement causes not only an expansion in employment, but also an increase in wages, due to the lower unemployment and higher vacancies posted, which raises outside option values in a general equilibrium framework. While this could be perceived as an important effect, whose implications are worth to study, Krause and Lubik (2007) provides evidence of a meaningless effect in the fitting of a proper macroeconomic model. Our choice of modelling is thus rooted not by claims it increases the macroeconomic fitting, but by the microeconomic appeal it has in fitting with the workings of the market.

The Relationship of the model with some Canonical Firm-Union Bargaining Models. Particularly since the 1970's and 1980's, there was a growing interest in developing and assessing different approaches to incorporate trade unions in regular labour economics models in general, and in the search and matching context in particular.[8]

From this debate four stylized approaches have settled, concretely: (i) the monopoly model of Dunlop (1944); (ii) the right-to-manage model of Nickell and Andrews (1983); (iii) the efficient bargaining model of McDonald and Solow (1981); and (iv) the sequential models of wage-employment bargaining of Manning (1987). The fundamental theoretic divergences among those classes of models lie on the structure of the bargaining and the *bargainable* variables, rather than the profit maximization objective of the firm, and the members' welfare maximization of unions. Those objective functions are presented in equations 15 and 14 in section 3.

While there is a *consensus* on the objectives of the parties, the notion of union's members has been debated. Some approaches consider as union members the employed workforce, while others consider the entire available labour force as *potential* union members.

Regarding the structure of bargaining, each of the four presented categories of models have

[8]See Johnson (1975), Parsley (1980), Oswald (1985) and Manning (1992) for comprehensive surveys of the literature of theory models of union and firm wage and employment setting. Further, see Pissarides (1986) which corresponds to the first paper to introduce a union in a search and matching model.

different implications, which however can be compared. The monopoly model considers that the union maximizes its welfare by choosing the wage, and firms are left to define the employment. Consequently we would have:

$$\begin{aligned} \max_{w(a|K,\beta)} \quad & W_t \\ \text{s. to} \quad & J_j^R(K,\beta) = \frac{\gamma_j}{q(\theta_j)} \end{aligned} \tag{.3}$$

assuming an interior solution in equilibrium.

The right to manage model is a generalization of the monopoly model, where instead of unions defining wages, union and the firm bargain over the wage, resorting to an axiomatic generalized bargaining as in Nash Jr (1950), or a game theoretic approach as in Rubinstein (1982) and Binmore et al. (1986). In our model that would translate into:

$$\begin{aligned} \max_{w(a|K,\beta)} \quad & \beta ln\Big(W_t - \bar{W}_t\Big) + (1-\beta)\Big(\Pi(K,\beta) - \bar{\Pi}\Big) \\ \text{s. to} \quad & J_j^R(K,\beta) = \frac{\gamma_j}{q(\theta_j)}, \end{aligned} \tag{.4}$$

where $\bar{W}_t$ and $\bar{\Pi}$ would correspond to union's members welfare and profits in the case of a full bargaining breakdown, respectively.

In a different paradigm, in the efficient bargain model, unions and firms bargain resorts to the propositions of either Nash Jr (1950) or Rubinstein (1982) and Binmore et al. (1986), but the parties bargain simultaneously about wages and employment. Consequently, we have:

$$\max_{w(a|K,\beta),\mathbf{N}(\mathbf{K},\beta)} \quad \beta ln\Big(W_t - \bar{W}_t\Big) + (1-\beta)\Big(\Pi(K,\beta) - \bar{\Pi}\Big). \tag{.5}$$

While apparently different, the work of Manning (1987) bridges the previously presented models, by considering a sequential bargaining, where first unions and firms bargain over wages, and subsequently they bargain over employment. Thus solving backwards, we have that the second stage corresponds to:

$$\max_{\mathbf{N}(\mathbf{K},\beta)} \quad \beta ln\Big(W_t - \bar{W}_t\Big) + (1-\beta)\Big(\Pi(K,\beta) - \bar{\Pi}\Big). \tag{.6}$$

and the first stage to:

$$\max_{w(a|K,\beta)} \quad \tilde{\beta} ln\Big(W_t(\mathbf{N}^\star(\mathbf{K},\beta)) - \bar{W}_t\Big) + (1-\tilde{\beta})\Big(\Pi(K,\beta,\mathbf{N}^\star(\mathbf{K},\beta)) - \bar{\Pi}\Big). \tag{.7}$$

Noteworthy, this model boils into: (i) the monopoly model if $\tilde{\beta} = 1$ and $\beta = 0$; (ii) the right to manage model if $\beta = 0$; and (iii) the efficient bargain model if $\tilde{\beta} = \beta$. Moreover, Manning (1987) shows that if employment is decided before wages the sequential model always yield the efficient bargain model solution, regardless of the relative strength of bargaining powers. Altogether, in the canonical firm-union environment the models' equilibria can be graphically compared, as in figure B1.

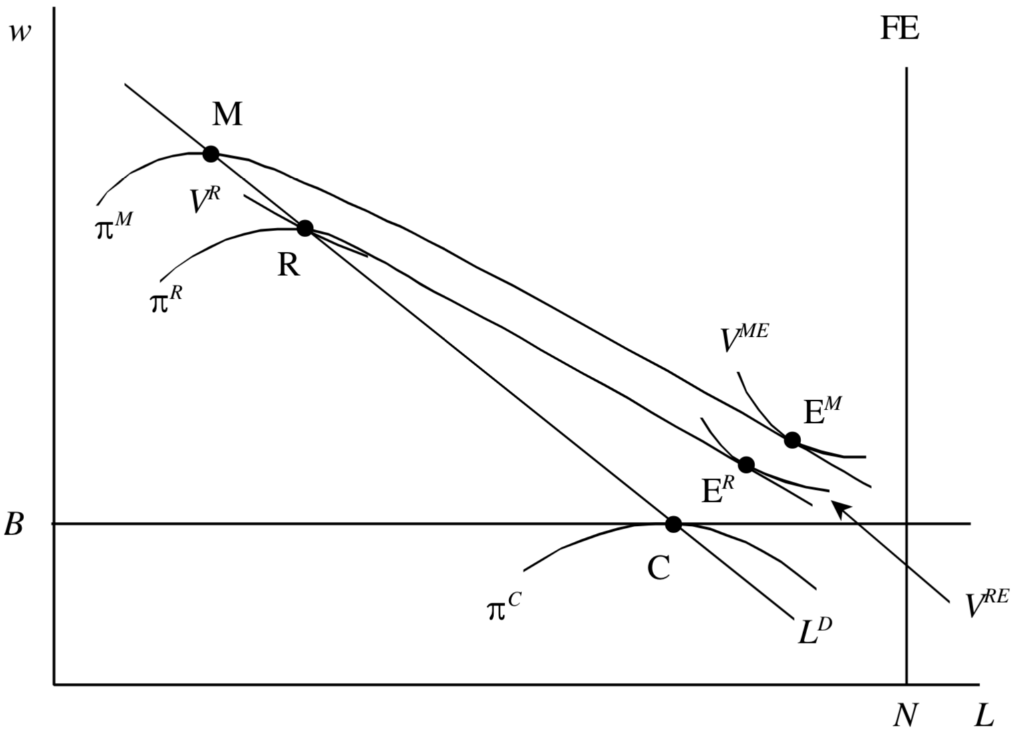
The way the representative union is integrated in our model relates to this debate, subjected to the remainder assumption framework of the model. Firstly, given in continental Europe collective bargaining coverage is extremely high, independently of membership, we adopt a fully representative union, which therefore considers the welfare of the entire available labour force, as presented in equation 14. Secondly, we consider the insight of the right-to-manage and monopoly models by considering the union bargains over wages alone. Thirdly, in the dynamic formulation of our model, we adopt a sequential modelling where firms and the union bargain over wages, and then firms set employment.

Finally, we supplement our model with another instrumental assumption. Most of the models in the literature assume that if there is a bargaining breakdown the entire workforce is displaced at least temporarily. In our setting, we adopt a gradual collective bargaining setting as in Dobbelaere and Luttens (2016), where the union bargains sequentially in the name of each worker, and does not impose a full lockdown in case of bargaining reaching a standstill. Rather, in the specific contract that originated the stalemate, a worker leaves the firm without being entitled to a severance payment. Then, the parties restart the bargain again, for every contract. This process unfolds until the deadlock may be surpassed.

Altogether, our *kind* of union will determine that the outcome of the bargaining corresponds to the competitive environment, as in other search and matching models with individual intra-firm bargaining, whose equilibrium conditions perfectly match the ones of our model (for example Cahuc et al. (2008) and Acemoglu and Hawkins (2014)). In a more stylized model would therefore be identifiable as point C in figure B1.

Alternatively, one could consider the proposition of Bauer and Lingens (2010), where in a setting close to ours the authors consider a union with power to impose lockdowns, and firms with the ability to displace the entire workforce. The major change actually comprises equation (23) of the model. Instead of bargaining sequentially for each worker,

Figure B1: Graphical Analysis of the Union's Models Solutions.



Notes: The M point represents the Monopoly model solution, and C point represents the competitive solution, where in the canonical environment $\beta = 0$. The right to manage model solution takes place anywhere on the demand curve (L^D), between points M and C, as it entails bargaining with an intermediate bargaining power of unions, compared to the presented extremes. Then point E^R represents the efficient bargain solution, when β would be compatible with a right-to-manage solution in point R.

the union bargains for the entire mass of workers simultaneously. Thus, we would have:

$$w_{j,t}(a|K,\beta) =$$

$$argmax_w \Big\{ \sum^J \int_\beta \int_K \int_a [\Xi_{j,t}(a|K,\beta) - Out(a)] dG_j(a|K,\beta) dK d\beta da \Big\}^\beta \Big\{ \Pi(K,\beta) \Big\}^{1-\beta} \quad (.8)$$

s. to: same constraints.

In this case we would end up with a solution somewhere in the locus of points $[C, M]$.

Our modelling choice relies on the lack of empirical evidence that a bargaining breakdown leads to either: (i) the dismissal of the entire workforce of a firm, or (ii) a prolonged strike that locks down the firm. Regarding the former, the only known case of an entire dismissal of workers following the breakdown of negotiations happened in the *Ronald Reagan and the air traffic controllers case* in 1981, which as presented by Dobbelaere and Luttens (2016) was predominantly political. Regarding the latter, Boeri and van Ours (2013) presents evidence that strikes are relatively rare, and prolonged strikes even more in Continental European labour markets. For the years 2000-2003, Spain had the most lost working days to strikes among continental European countries, representing 0.6 working days per year and worker. For the same sample, the average duration of strikes was at most 5.1 days, recorded for Ireland and Poland.

B2 Complementary Definitions on Job-Flows

Firm's Hiring Policy. The profit of the firm with fundamentals $\{K, \beta\}$ is assumed to be strictly concave and twice continuously differentiable in employment. It is given, as in equation (15) of the text, by:

$$\begin{aligned} r\Pi(K,\beta) - \frac{\partial \Pi(K,\beta)}{\partial t} &= \underbrace{F(\mathbf{N}(K,\beta);K)}_{\text{Production}} - \underbrace{\sum_{j=1}^{J}\int_a w_j(a|K,\beta)dG_j(a|K,\beta)da}_{\text{Wage Bill}} \\ &- \underbrace{\sum_{j=1}^{J}\int_a A(j,a)dG_j(a|K,\beta)da}_{\text{Operating Cost Bill}} - \underbrace{\sum_{j=1}^{J}\int_a s_j(a|K,\beta)J_j(a|K,\beta)dG_j(a|K,\beta)da}_{\text{Cost of Displacement}} \\ &- \underbrace{\sum_{j=1}^{J}\int_a \tilde{s}_j(a|K,\beta)SdG_j(a|K,\beta)da}_{\text{Exp. firing tax}} - \underbrace{I(K)}_{\text{Sunk Cost}} + \\ &+ \underbrace{\sum_{j=1}^{J}\max_{V_j(K,\beta)}\Big\{ -\gamma_j V_j(K,\beta) + V_j(K,\beta)q(\theta_j)J_j^R(K,\beta)\Big\}}_{\text{Value of the Hiring Policy}}. \end{aligned} \tag{.9}$$

Solving for the optimal vacancy policy, we obtain first order conditions as:[9]

$$J_j^R(K,\beta) = \begin{cases} \frac{\gamma_j}{q(\theta_j)} & \text{if } V_j(K,\beta) > 0 \\ \\ \left[\underline{J}_j^R(K,\beta); \frac{\gamma_j}{q(\theta_j)}\right] & \text{if } V_j(K,\beta) = 0 \end{cases}. \tag{.10}$$

Firm's Firing Policy. Apart from the hiring policy, firms also define their firing policy. Firms, may fire at-will any worker, as long as they pay a corresponding exogenous firing

[9]The considered corner solution exists due to the impossibility of costless firing at will, as motivated in Bertola and Caballero (1994). Further we assume $\underline{J}_j^R(K,\beta) > -\infty$, so it is bounded from below.

tax, given as S. Therefore, the decision of fire at-will is given by:[10]

$$\tilde{s}_j(a|K,\beta) = argmax_{\tilde{s}_j(a|K,\beta)\in\{0,1\}} \sum_{j=1}^{J}\int_a \Bigg[\underbrace{\Big(1-\tilde{s}_j(a|K,\beta)\Big)J(a,K,\beta)}_{\text{Value of not firing}} + \underbrace{\tilde{s}_j(a|K,\beta)\Big(-J_j(a,K,\beta) - S + \underbrace{\max\Big\{-e^{-r\Delta t}\gamma + e^{-r\Delta t}q(\theta_j)J_j^R(K,\beta);0\Big\}}_{\text{Optimally decide if replace or not the fired worker}}\Big)}_{\text{Value of Firing the worker}}\Bigg] \times \times dG_j(a|K,\beta)da, \tag{.11}$$

where the firm balances the option of keeping the worker, against the options to firing him, and subsequently either replace or not replace him in the following period. Notice the firm considers a consistent policy for its entire workforce due to the linkages between the marginal values of the jobs across workers. The maximand function $\tilde{s}_j(a|K,\beta)$ corresponds to a threshold function as:

$$\tilde{s}_j(a|K,\beta) = \begin{cases} 1 & \text{if } a \leq a_j(K,\beta) \\ 0 & \text{if } a > a_j(K,\beta) \end{cases}. \tag{.12}$$

Beyond a potential bargaining breakdown with the union, and its decision of firing a given worker, the firm is also subjected to displacement due to an exogenous shock, for example due to the death of a worker, which happens with probability $\bar{s}$, and a successful outcome of the on-job-search of its employed worker, given by the function $m_j(a|K,\beta)$. The displacement rate function is given by:

$$s_j(a|K,\beta) = \begin{cases} 1 & \text{if } a \leq a_j(K,\beta) \\ \bar{s} + m_j(a|K,\beta) & \text{if } a > a_j(K,\beta). \end{cases} \tag{.13}$$

Altogether, we have defined the behaviour of firm's (K,β) type in managing its workforce.

Job Search. Synchronously, both workers and the unemployed have the option to search for jobs. For that purpose, when they decide to search, they do so in the task market which yields the most expected return, and in searching they incur in a cost given by $c_j(a)$, dependent on the task market they intend to search for, and their type a. For an

[10]The operator Δt represents a time lag. $-e^{-r\Delta t}$ represents a discount factor, where (1) $\lim_{\Delta t\to 0^+} -e^{-r\Delta t} = 1$, and (2) $\lim_{\Delta t\to\infty} -e^{-r\Delta t} = 0$.

employed worker, he solves his search problem as:

$$\xi_j^{\Xi}(a|K,\beta) = argmax_{\xi_l^{\Xi}\in\{0,1\},\sum_{l=1}^{J}=\{0,1\}} \sum_{l=1}^{J}\left[\xi_l^{\Xi}(a|K,\beta)\left\{\theta_l q(\theta_l)\times \right.\right.$$
$$\left.\left.\times\left(\frac{\int_K\int_\beta \mathbf{1}[\Xi_l(a|x,y) > \Xi_j(a|K,\beta)]\Xi_l(a|x,y)V_l(x,y)d\Gamma(x,y)}{\int_K\int_\beta V_l(x,y)d\Gamma(x,y)} - \Xi_j(a|K,\beta)\right) - c_l(a)\right\}\right], \tag{.14}$$

where $\mathbf{1}[\Xi_l(a|x,y) > \Xi_j(a|K,\beta)]$ is an indicator function being 1 if the potential offer $\Xi_l(a|x,y)$ provides an higher value than $\Xi_j(a|K,\beta)$.

The unemployed solves a similar problem as:

$$\xi^u(a) = argmax_{\xi_l^u\in\{0,1\},\sum_{l=1}^{J}=1} \sum_{l=1}^{J}\left[\xi_l^u(a)\left\{\theta_l q(\theta_l)\times\right.\right.$$
$$\left.\left.\times\left(\frac{\int_K\int_\beta \Xi_l(a|x,y)V_l(x,y)d\Gamma(x,y)}{\int_K\int_\beta V_l(x,y)d\Gamma(x,y)} - Out(a)\right) - c_l(a)\right\}\right],, \tag{.15}$$

where $Out(a)$ is the unemployed value, given his type, and ξ are indicator functions, displaying the selected task market of the worker, given his context. We assume that: (i) $\theta_j q(\theta_j)\frac{\int_K\int_\beta \Xi_l(a|x,y)V_l(x,y)d\Gamma(x,y)}{\int_K\int_\beta V_l(x,y)d\Gamma(x,y)} - Out(a) - c_l(a) > 0$ for at least one task market j, so that the unemployed always search in some market; (ii) $\Xi_j(a|x,y) > Out(a)\ \forall\{x,y\}$, so that after they decide to search and after paying the search cost, which becomes sunk, an unemployed worker will always prefer to work;[11] (iii) $c_j(a)$ is differentiable, convex, strictly decreasing in a, holds $\lim_{x\to 0} c_j(x) = \bar{C} > 0$ and $\lim_{x\to\infty} c_j(x) = 0$; and (iv) $c_j(a) < c_l(a)\ \ \forall l > j$. Altogether, the optimal search behaviour for the unemployed follows a system of threshold rules. Thus, the vector of search choices becomes:

$$\xi^u(a) = \Big[1(l=1, a > a_1), \ldots, 1(l=j, a_{l-1} > a > a_l), \ldots, 1(j=J, a < a_J)\Big] \tag{.17}$$

implying that unemployed perfectly segment across the task markets accordingly to their type. Furthermore, given the search behaviour of an unemployed, we have that:

$$\xi_j^{\Xi}(a|K,\beta) \in \{\mathbf{0}, \xi^u(a)\}, \tag{.18}$$

so that an employed worker if he searches, he does so in the task market he would search if he was unemployed. Concretely, given $\Xi_l(a|K,\beta) > Out(a), \forall l \in \{0,\ldots,J\}$, the worker may eventually decide not to search, when he would do so if unemployed. Therefore, the worker's type, a, is a sufficient statistic of task market choice in on-job search, conditional on searching.

[11] Further, we also assume that:

$$\lim_{N_j\to\infty}\frac{\partial F(\mathbf{N},K)}{\partial N_j} < b, \tag{.16}$$

where b stands for the unemployment benefit, so that eventually a firm shall not grow indefinitely.

As a reference, this behaviour of the agents regarding employment flows and selection reproduces the behaviour of the classical selection model of Roy (1951).[12]

Probability of successful on-job-search. Following this structure, the probability of a success on-job-search for a worker with match fundamentals (a, K, β) in task market j is given by:

$$m_j(a|K,\beta) = \sum_{l=1}^{J} \xi_l^{\Xi}(a|K,\beta)\, \underbrace{\theta_l q(\theta_l)\Big[1 - D_l(\Xi_j(a|K,\beta))\Big]}_{\text{Prob. the worker accepts a job in } j} \tag{.19}$$

where $D_l(w(a|K,\beta))$ is the distribution of wage offers in task market j.[13] The expectation of the marginal profit of a new hire to be given by:

$$\begin{aligned} J_j^R(K,\beta) = & \underbrace{\frac{1}{u_j}\int_a \xi_j^u(a) J(a|K,\beta) dU(a)}_{\substack{\text{Expected value of a job} \\ \text{when unemployed is hired}}} + \\ & + \underbrace{\int_a \frac{\sum_{l=1}^{J}\int_K\int_\beta \xi_j^{\Xi}(a|x,y) 1\Big(\Xi_j(a|K,\beta) > \Xi_l(a|x,y)\Big) J_j(a|K,\beta) dG_l(a|x,y) d\Gamma(x,y)}{\sum_{l=1}^{J}\int_K\int_\beta \xi_j^{\Xi}(a|x,y) dG_l(a|x,y) d\Gamma(x,y)} da}_{\text{Expected value of a job when a employed searching at task } j \text{ is hired}} . \end{aligned} \tag{.20}$$

Market tightness and vacancy yield. Given the search behaviour of workers and unemployed, the labour market tightness of task market j is also given by:

$$\theta_j = \frac{\int_K\int_\beta V_j(K,\beta) d\Gamma(K,\beta)}{\int_a \xi_j^u(a) dU(a) da + \sum_{l=1}^{J}\int_a\int_K\int_\beta \xi_j^{\Xi}(a|K,\beta) dG_l(a|K,\beta) d\Gamma(K,\beta) da}, \tag{.21}$$

where: (i) $e_j = \sum_{l=1}^{J}\int_a\int_K\int_\beta \xi_j^{\Xi}(a|K,\beta) dG_l(a|K,\beta) d\Gamma(K,\beta) da$ represents the number of workers performing on-job-search in task market j; (ii) $u_j = \int_a \xi_j^u(a) dU(a) da$ gives the number of unemployed searching for a job in the task market j; and (iii) the number of vacancies are obtained as $\int_K\int_\beta V_j(K,\beta) d\Gamma(K,\beta)$.

Given this market structure, the probability of a firm of type $\{K, \beta\}$ to find a worker of

[12]Notice that in this framework, theoretically our results will not be plagued by endogenous mobility conditional on the described behaviour. A different outcome would potentially be achieved if for instance one would consider a generalized Roy model (see Heckman and Vytlacil (2007) for an example). For modelling ease we do not consider such generalization, as we do not empirically explore the potential selection model that would emerge from this behaviour.

[13]The presented distribution is given by:

$$D_j(\Xi_j(a|K,\beta)) = \frac{\int_K\int_\beta V_l(x,y) 1\Big(\Xi_j(a|K,\beta) > \Xi_l(a|x,y)\Big) d\Gamma(x,y)}{\int_K\int_\beta V_l(K,\beta) d\Gamma(x,y)}.$$

type a in task market j corresponds to:

$$y_j(a|K,\beta) = q(\theta_j)\frac{\frac{\partial u_j}{\partial a} + \frac{\partial e_j}{\partial a} X_j^{-}(w(a|K,\beta))}{\frac{\partial uj}{\partial a} + \frac{\partial e_j}{\partial a}}, \tag{.22}$$

where $X_j^{-}(w(a|K,\beta)) = lim_{x \uparrow w(a|K,\beta)} X_j(x)$ is the distribution of wages that employed workers which are searching in task market j are receiving. Notice, as typical in these type of models we assume workers do not move to a worse paying match.[14] Accordingly, the vacancy yield of a firm of type (K,β), i.e. the probability of firm of type $\{K,\beta\}$ to hire a worker, is given by:

$$y_j(K,\beta) = \int_a y_j(a|K,\beta)da. \tag{.23}$$

Evolution of workforce composition. Finally, the expected evolution of the workforce composition of a firm of type (K,β) is then given by:[15]

$$\begin{aligned} \frac{\partial dG_{j,t}(a|K,\beta)}{\partial t} = & \underbrace{-dG_{j,t-\epsilon}(a|K,\beta)}_{\text{Workers of type } a \text{ at period } t-\epsilon} + \underbrace{y_{j,t}(a|K,\beta)V_{j,t}(K,\beta)}_{\text{Prob. hiring worker of type } a} \\ & + \underbrace{\int_{a''}\Big(1 - s_{j,t-\epsilon}(a''|K,\beta)\Big)\psi(a|a'')dG_{j,t-\epsilon}(a''|K,\beta)da''}_{\text{Prob. that incumbent workforce at firm develops into workforce of type } a}, \end{aligned} \tag{.24}$$

where: (i) the probability that a worker to keep his operational cost fixed from a period to another is zero so the workforce in a at period $t-\epsilon$ will not be in a at period t; (ii) $y_{j,t}(a|K,\beta)V_{j,t}(K,\beta)$ represents the probability of hiring a worker of precisely operating cost a per vacancy posted (i.e. $V_j(K,\beta)$); and (iii) the third term consider, from the workers that have not left the firm of type $\{K,\beta\}$, those whose skill acquisition process leave them precisely at operating cost level a, where $\psi(a|a'')$ is the probability distribution function of the random component of the skill acquisition process, from previous period a'' to current period a.

[14] The distribution presented is given by: $X_j(w) = \frac{\sum_{l=1}^{J}\int_K\int_\beta \xi_j^{\Xi}(a|x,y)1\Big(\Xi_j(a|K,\beta) > \Xi_l(a|x,y)\Big)dG_l(a|x,y)d\Gamma(x,y)}{\sum_{l=1}^{J}\int_K\int_\beta \xi_j^{\Xi}(a|x,y)dG_l(a|x,y)d\Gamma(x,y)}$.

[15] For notation clarity, in this equation, we refer to a'' as the skill stock in the previous period, and a as the skill process in the current period.

B3 Derivation of Equilibrium Wages

On the heterogeneity of bargaining powers. The derivation of the interior solution of the dynamic equilibrium wages follows closely the steps considered in Acemoglu and Hawkins (2014) and Cahuc et al. (2008). In this derivation, we will assume that β is a vector of bargaining powers, implying instead of a common bargaining power for every task market within the firm, the existence of heterogeneous bargaining powers per task market, i.e. $\beta = [\beta_1, \dots, \beta_j, \dots, \beta_J]$.

System of differential equations for equilibrium wages. Consider the equation (18) of the text:

$$r\Xi_j(a|K,\beta) - \frac{\partial \Xi_j(a|K,\beta)}{\partial t} = w_j(a|K,\beta) + \underbrace{\bar{s}\Big(Out(a) - \Xi_j(a|K,\beta)\Big)}_{\text{Value loss of losing the job}} +$$
$$+ \underbrace{\sum_{l=1}^{J} \xi_l^{\Xi}(a|K,\beta)\left\{\theta_l q(\theta_l)\frac{\int_K \int_\beta \mathbf{1}[\Xi_l(a|K,\beta) > \Xi_j(a|K,\beta)]\Xi_l(a|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K \int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a) - c_j\right\}}_{\text{Value of searching for a job while employed}} \quad (.1)$$
$$+ \underbrace{\sum_{l=1}^{J}\Big[y_l(K,\beta)V_l(K,\beta) - s_l(K,\beta)N_l(K,\beta)\Big]\frac{\partial \Xi_j(a|K,\beta)}{\partial N_l(K,\beta)}}_{\text{Impact of hiring and firing policy of the firm in the value of the employment}},$$

At this stage we impose the assumption that both parties bargain under the assumption of *match stability*, i.e. no party, at-will, will dissolve the match, implying the parties believe, for wage bargaining purposes, that $\xi_l^{\Xi}(a|K,\beta) = 0, \forall l \in \{1,\dots,J\}$, and $\tilde{s} = 0$. Thus equation (1) becomes:

$$\Big(r + \bar{s}\Big)\Big(\Xi_j(a|K,\beta) - Out(a)\Big) - \left[\frac{\partial \Xi_j(a|K,\beta)}{\partial t}\right] = w_j(a|K,\beta) - rOut(a) +$$
$$+ \sum_{l=1}^{J}\left[y_l(K,\beta)V_l(K,\beta) - \int_a s_l(a|K,\beta)dG_l(a|K,\beta)da\right]\frac{\partial \Xi_j(a|K,\beta)}{\partial N_l(K,\beta)}. \quad (.2)$$

Given the bargaining arrangement, expressed in equation (23) of the text, assuming the match stability condition, and that the aggregate bargaining constraint is not binding, we have:

$$\Big(1 - \beta_j\Big)\Big(\Xi_j(a|K,\beta) - Out(a)\Big) = \beta_j\Big(J_j(a|K,\beta)\Big). \quad (.3)$$

In addition, considering that the outside option bargained between the parties is not affected by changes in firm's employment, given the presence of a large number of firms, i.e.:

$$\frac{\partial Out(a)}{\partial N_j(K,\beta)} = 0, \ \ \forall j \in [1,\dots,J], \quad (.4)$$

we have that:

$$\Big(1-\beta_j\Big)\Big(\frac{\partial \Xi_j(a|K,\beta)}{\partial t}-\frac{\partial Out(a)}{\partial t}\Big)=\beta_j\Big(\frac{\partial J_j(a|K,\beta)}{\partial t}\Big), \tag{.5}$$

$$\Big(1-\beta_j\Big)\Big(\frac{\partial \Xi_j(a|K,\beta)}{\partial N_j(K,\beta)}\Big)=\beta_j\Big(\frac{\partial J_j(a|K,\beta)}{\partial N_j(K,\beta)}\Big). \tag{.6}$$

Using the result of equation (6) with equation (1), we have:

$$\begin{aligned}&\sum_{l=1}^{J}\Bigg[\Bigg\{y_l(K,\beta)V_l(K,\beta)-\int_a s_l(a|K,\beta)dG_l(a|K,\beta)da\Bigg\}\frac{\partial J_j(a|K,\beta)}{\partial N_l(K,\beta)}\Bigg]=\\&=\frac{1-\beta_j}{\beta_j}\sum_{l=1}^{J}\Bigg[\Bigg\{y_l(K,\beta)V_l(K,\beta)-\int_a s_l(a|K,\beta)dG_l(a|K,\beta)da\Bigg\}\frac{\partial \Xi_j(a|K,\beta)}{\partial N_l(K,\beta)}\Bigg]=\\&=\frac{1-\beta_j}{\beta_j}\Bigg[\Big(r+\bar{s}\Big)\Big(\Xi_j(a|K,\beta)-Out(a)\Big)-\frac{\partial \Xi_j(a|K,\beta)}{\partial t}-w_j(a|K,\beta)+rOut(a)\Bigg].\end{aligned} \tag{.7}$$

Moreover, resorting to equation (16) of the text, under the assumption of match stability, we have:

$$\begin{aligned}&rJ_j(a|K,\beta)-\frac{\partial J_j(a|K,\beta)}{\partial t}=\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}-w_j(a|K,\beta)\\&-\sum_{l=1,l\neq j}^{J}\int_a\frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)}dG_l(a|K,\beta)da-A(j,a)\\&-\bar{s}J_j(a|K,\beta)+\sum_{l=1}^{J}\Bigg\{y(l|K,\beta)V(l|K,\beta)-\int_a s_l(a|K,\beta)G_l(a|K,\beta)da\Bigg\}\frac{\partial J_j(a,K,\beta)}{\partial N_l(K,\beta)}.\end{aligned} \tag{.8}$$

and together with equation (7), one obtains:

$$\begin{aligned}&\beta_j(r+\bar{s})J_j(a|K,\beta)-\beta_j\frac{\partial J_j(a|K,\beta)}{\partial t}=\beta_j\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}-\beta_j w_j(a|K,\beta)\\&-\beta_j\sum_{l=1,l\neq j}^{J}\int_a\frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)}dG_l(a|K,\beta)da-\beta_j A(j,a)+\\&+(1-\beta_j)\Bigg[\Big(r+\bar{s}\Big)\Big(\Xi_j(a|K,\beta)-Out(a)\Big)-\frac{\partial \Xi_j(a|K,\beta)}{\partial t}-w_j(a|K,\beta)+rOut(a)\Bigg].\end{aligned} \tag{.9}$$

Incorporating equations (3) and (5), and simplifying the resulting equation, we obtain a system of differential equations governing the equilibrium wages for each match with fundamentals $\{a,K,\beta\}$. Such system is given by:

$$\begin{aligned}&w_j(a|K,\beta)=(1-\beta_j)\Big(rOut(a)-\frac{\partial Out(a)}{\partial t}\Big)\\&+\beta_j\Big(\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}-\sum_{l=1,l\neq j}^{J}\int_a\frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)}dG_l(a|K,\beta)da-A(j,a)\Big).\end{aligned} \tag{.10}$$

Further, we have that the average wage per workplace - $w_j(K,\beta)$ - is given by:

$$w_j(K,\beta)=\frac{1}{N_j(K,\beta)}\int_a w_j(a|K,\beta)dG_j(a|K,\beta)da, \tag{.11}$$

and consistently with equation (10), becomes:

$$\begin{aligned} w_j(K,\beta) = (1-\beta_j)E\Big(rOut(a) - \frac{\partial Out(a)}{\partial t}\Big|K,\beta\Big) + \\ + \beta_j\Big(\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - \sum_{l=1,l\neq j}^{J}\frac{\partial w_l(K,\beta)}{\partial N_j(K,\beta)}N_l(K,\beta) - E(A(j,a)|K,\beta)\Big). \end{aligned} \tag{.12}$$

Further, given equation (11), the proper wage for the match fundamentals $\{a,K,\beta\}$ is given by:

$$\begin{aligned} w_j(a|K,\beta) = (1-\beta_j)\Big(rOut(a) - \frac{\partial Out(a)}{\partial t}\Big) + \\ + \beta_j\Big(\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - \sum_{l=1,l\neq j}^{J}\frac{\partial w_l(K,\beta)}{\partial N_j(K,\beta)}N_l(K,\beta) - A(j,a)\Big). \end{aligned} \tag{.13}$$

Solving the system of differential equations for the partial equilibrium wages
To solve the system of differential equations of equation (12), we follow the insight of Cahuc et al. (2008). Thus, take the partial derivative of the average wages, $w_j(K,\beta)$, with respect to employment in another task market $N_l(K,\beta)$, $l \neq j$, given the difference between any $w_j(a|K,\beta)$ and $w_j(K,\beta)$ is based, in the moment of the bargaining of prices within the firm, on the exogenous values, i.d. (a) $Out(a)$ versus $E[Out(a)|K,\beta]$; and (b) $E[A(j,a)|K,\beta]$ and $A(j,a)$. Thus:

$$\frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)} + \beta_j\frac{\partial w_l(K,\beta)}{\partial N_j(K,\beta)} = \beta_j\Big[\frac{\partial^2 F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)\partial N_l(K,\beta)} - \sum_{k=1}^{J}\frac{\partial^2 w_k(K,\beta)}{\partial N_j(K,\beta)\partial N_l(K,\beta)}N_k(K,\beta)\Big] \tag{.14}$$

which yields second-order differential equation as:

$$(1-\beta_j)\frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)} = \beta_j\frac{\partial^2}{\partial N_j(K,\beta)\partial N_l(K,\beta)}\Big[F(\mathbf{N}(K,\beta),K) - \sum_{j=1}^{J} w_k(K,\beta)N_k(K,\beta)\Big]. \tag{.15}$$

Further, given the equality of second-order cross derivatives, one can also infer that:

$$\frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)} = \underbrace{\frac{\beta_j}{1-\beta_j}\frac{1-\beta_l}{\beta_l}}_{\chi_{jl}}\frac{\partial w_l(K,\beta)}{\partial N_j(K,\beta)}, \tag{.16}$$

and:

$$\sum_{j=1}^{J} N_j(K,\beta)\frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)} = \sum_{j=1}^{J}\chi_{lj}N_j(K,\beta)\frac{\partial w_l(K,\beta)}{\partial N_j(K,\beta)}. \tag{.17}$$

Jointly, this allows to write equation (12) as:

$$
\begin{aligned}
w_j(K,\beta) &= (1-\beta_j)E\Big(rOut(a) - \frac{\partial Out(a)}{\partial t}\Big| K,\beta\Big) + \\
&+ \beta_j\Big(\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - \sum_{l=1,l\neq j}^{J} \chi_{j,l}\frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)} N_l(K,\beta) - E(A(j,a)|K,\beta)\Big).
\end{aligned} \tag{.18}
$$

The case of homogeneous β at firm level. At this stage let us first assume that $\beta_j = \beta$ - homogeneous bargaining power at firm level so that $\chi_{jl} = 1, \forall\{l,j\}$. Considering the generalized spherical coordinates $\iota, \omega_1, \dots, \omega_{J-1}$, where ι is the distance to the origin such that $\sum_{j=1}^{J} N_j(K,\beta)^2 = \iota^2$, where ω_j are angles of projection in different subplanes, one can write:

$$
\begin{aligned}
N_1(K,\beta) &= \iota cos\omega_1 \dots cos_{J-2}cos_{J-1} \\
N_2(K,\beta) &= \iota cos\omega_1 \dots cos_{J-3}sin_{J-2} \\
N_2(K,\beta) &= \iota cos\omega_1 \dots cos_{J-2}sin_{J-3} \\
&\dots \\
N_{J-1}(K,\beta) &= \iota cos\omega_{f1t} sin\omega_{f2t} \\
N_J(K,\beta) &= \iota sin\omega_{f1t},
\end{aligned} \tag{.19}
$$

and with such coordinates, using the notation $\omega = (\omega_1, \dots, \omega_J)$, one writes:

$$
\sum_{l=1}^{J} N_l(K,\beta)\frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)} = \iota\frac{\partial w_j(\iota,\omega,K,\beta)}{\partial \iota}, \tag{.20}
$$

where ι is the scale of use of labour tasks, and ω reflects the proportions in which the different types of labour tasks are used. $\omega = (0,\dots,0)$ means that firm only employ workers in the first task market. Then equation (12) reads as:

$$
\begin{aligned}
\beta\frac{\partial w_j(\iota,\omega,K,\beta)}{\partial \iota} + w_j(\iota,\omega,K,\beta) = (1-\beta)E\Big[rOut(a) - \frac{\partial Out(a)}{\partial t}\Big| K,\beta\Big] + \\
+\beta\Big[\frac{\partial F(\iota,\omega,K,\beta)}{\partial N_j(K,\beta)}\Big] - \beta E[A(j,a)|K,\beta].
\end{aligned} \tag{.21}
$$

Notice that given the exogeneity of: (1) $(1-\beta)\Big[rOut(a) - \frac{\partial Out(a)}{\partial t}\Big]$; and (2) $A(j,a)$, we can drop it and thus we have:

$$
\frac{dw_j(\iota,\omega,K,\beta)}{d\iota} + \frac{w_j(\iota,\omega,K,\beta)}{\beta\iota} - \frac{\partial F(\iota,\omega,K_{ft})}{\partial \phi_{fjt}}\frac{1}{\iota} = 0. \tag{.22}
$$

Notice that the solution of the homogeneous equation $\frac{\partial w_j(\iota,\omega,K,\beta)}{\partial \iota} + \frac{w_j}{\beta\iota} = 0$ is given by:

$$
w_j(\iota,\omega,K,\beta) = C\iota^{-\frac{1}{\beta}} \tag{.23}
$$

and thus derivating it towards ι, while assuming C depends on ι, one obtains:

$$\frac{dw_j(\iota,\omega,K,\beta)}{d\iota} = \frac{dC}{d\iota}\iota^{\frac{-1}{\beta}} - \frac{1}{\beta}C\iota^{-1-\frac{1}{\beta}} \tag{.24}$$

which plugging back (24) and (23) in equation (22), one obtains:

$$\frac{dC}{d\iota}\iota^{\frac{-1}{\beta}} - \frac{1}{\beta}C\iota^{-1-\frac{1}{\beta}} + \frac{C\iota^{-\frac{1}{\beta}}}{\beta\iota} - \frac{\partial F(\iota,\omega,K,\beta)}{\partial N_j(K,\beta)}\frac{1}{\iota} = 0 \tag{.25}$$

and simplifying one obtains:

$$\frac{dC}{d\iota} = \iota^{\frac{1-\beta}{\beta}}\frac{\partial F(\iota,\omega,K,\beta)}{\partial N_j(K,\beta)}, \tag{.26}$$

and through integration one gets:

$$C_j(\omega,K,\beta) = \int_0^{\iota} z^{\frac{1-\beta}{\beta}}\frac{\partial F(z,\omega,K,\beta)}{\partial N_j(K,\beta)}dz + D, \tag{.27}$$

where D is the constant of integration. Given the property that

$\lim_{\iota\to 0^+} \iota w_j(\iota,\omega,K,\beta) = 0$, we have that the constant D is identically equal to zero. Therefore the solution to equation (21) satisfies:

$$\begin{aligned} w_j(\iota,\omega,K,\beta) =& (1-\beta)E\left[rOut(a) - \frac{\partial Out(a)}{\partial t}\middle|K,\beta\right] + \\ &+ \iota^{\frac{-1}{\beta}}\left(\int_0^{\iota} z^{\frac{1-\beta}{\beta}}\frac{\partial F(z,\omega,K,\beta)}{\partial \phi_{fjt}}dz\right) - \beta E[A(j,a)|K,\beta]. \end{aligned} \tag{.28}$$

Further, notice that if $\mathbf{N}(K,\beta) = (\iota,\omega)$, then $(z\iota,\omega) = [zN_1(K,\beta), zN_2(K,\beta),\ldots,zN_J(K,\beta)] = z\mathbf{N}(K,\beta)$, one can turn equation (28) in:

$$\begin{aligned} w_j(K,\beta) =& (1-\beta)E\left[rOut(a) - \frac{\partial Out(a)}{\partial t}\middle|K,\beta\right] + \\ &+ \int_0^1 z^{\frac{1-\beta}{\beta}}\frac{\partial F(z\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}dz - \beta E[A(j,a)|K,\beta], \end{aligned} \tag{.29}$$

and by doing so fully eliminating the spherical coordinates, which results in the solution of the system of differential equations in equation (12).

Further, from equation (29), one can infer that the equilibrium wages, and the solution of the system of differential equations in equation (13) is given by:

$$\begin{aligned} w_j(a|K,\beta) =& (1-\beta)\left[rOut(a) - \frac{\partial Out(a)}{\partial t}\right] + \\ &+ \int_0^1 z^{\frac{1-\beta}{\beta}}\frac{\partial F(z\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}dz - \beta A(j,a), \end{aligned} \tag{.30}$$

The case of heterogeneous β at firm level. It is helpful to consider a new variable, as does Cahuc et al. (2008). Accordingly, define $\mathbf{M}_j(K,\beta) = \{M_{j,1}(K,\beta), M_{j,2}(K,\beta), \dots, M_{j,J}(K,\beta)\}$, such that:

$$\sum_{l=1}^{J} M_{j,l} \frac{\partial v_l(\mathbf{M}_j, K)}{\partial M_{j,l}} = \sum_{l=1}^{J} \chi_{jl} N_l(K,\beta) \frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)}, \tag{.31}$$

with $v_j[M_{j,j}(K,\beta), K] = w_j(K,\beta)$. Also, we assume it holds:

1. $G(\mathbf{M}_j, K) = F(\mathbf{N}(K,\beta), K)$;
2. $M_{j,l} = M_{j,l}(N_l(K,\beta))$;
3. $\frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)} = \frac{\partial v_l(\mathbf{M}_j)}{\partial M_{j,l}} \frac{dM_{j,l}}{dN_l(K,\beta)}$.

For equation (31) to hold it suffices that the following equation to hold:

$$M_{j,l} \frac{\partial v_l(\mathbf{M}_j, K)}{\partial M_{j,l}} = \chi_{jl} N_l(K,\beta) \frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)}. \tag{.32}$$

Given property (3), one obtains a differential equation for $M_{j,l}$, which is given by:

$$M_{j,l} = \chi_{jl} N_l(K,\beta) \frac{dM_{j,l}}{dN_l(K,\beta)}. \tag{.33}$$

One feasible solution, not necessarily the only one, corresponds to:

$$M_{j,l} = N_l(K,\beta)^{\frac{1}{\chi_{jl}}} = N_l(K,\beta)^{\chi_{lj}} \tag{.34}$$

given $\chi_{lj} = \frac{1}{\chi_{jl}}$. Considering that the mapping between notations, and properties (1) and (2), we have:

$$\frac{\partial F(\mathbf{N}(K,\beta), K)}{\partial N_l(K,\beta)} = \chi_{lj} N_l(K,\beta)^{\chi_{lj}-1} \frac{\partial G(\mathbf{M}_j, K)}{\partial M_{j,l}}, \tag{.35}$$

and concretely,

$$\frac{\partial F(\mathbf{N}(K,\beta), K)}{\partial N_j(K,\beta)} = \chi_{jj} N_j(K,\beta)^{\chi_{jj}-1} \frac{\partial G(\mathbf{M}_j, K)}{\partial M_{j,j}} = \frac{\partial G(\mathbf{M}_j, K)}{\partial M_{j,j}}, \tag{.36}$$

since $\chi_{jj} = 1$. The system in equation (12) can be expressed as:

$$\begin{aligned} v_j(\mathbf{M}_j, K) = (1-\beta_j) E\left[rOut(a) - \frac{\partial Out(a)}{\partial t} \middle| K, \beta \right] + \\ + \beta_j \left(\frac{\partial G(\mathbf{M}_j, K)}{\partial M_{j,j}} - \sum_{l=1}^{J} M_{j,l} \frac{\partial v_j(\mathbf{M}_j, K)}{\partial M_{j,l}} \right) - \beta_j E[A(j,a)|K,\beta], \end{aligned} \tag{.37}$$

which is identical to equation (18). Therefore, following the procedure explained for identical β's, one obtains:

$$\begin{aligned} v_j(\mathbf{M}_j, K) = (1-\beta_j)E\Big[rOut(a) - \frac{\partial Out(a)}{\partial t}\Big| K, \beta\Big] + \\ + \int_0^1 z^{\frac{1-\beta_j}{\beta_j}} \frac{\partial G(z\mathbf{M}_j, K)}{\partial M_{j,j}}, K)dz - \beta_j E[A(j,a)|K,\beta] \end{aligned} \tag{.38}$$

and translating the transformed variables in the initial notation variables, one realizes equation (38) becomes:

$$\begin{aligned} w_j(K,\beta) =& (1-\beta_j)E\Big[rOut(a) - \frac{\partial Out(a)}{\partial t}\Big| K, \beta\Big] + \\ & + \int_0^1 z^{\frac{1-\beta_j}{\beta_j}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta), K)}{\partial N_j(K,\beta} dz - \beta_j E[A(j,a)|K,\beta] \end{aligned} \tag{.39}$$

Then, given the definition of the wages for each match fundamentals $\{a, K, \beta\}$, and namely that the heterogeneity arises in operating costs and outside options only, one realize that:

$$\begin{aligned} w_j(a|K,\beta) =& (1-\beta_j)\Big[rOut(a) - \frac{\partial Out(a)}{\partial t}\Big] + \\ & + \int_0^1 z^{\frac{1-\beta_j}{\beta_j}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta), K)}{\partial N_j(K,\beta)} dz - \beta_j A(j,a), \end{aligned} \tag{.40}$$

where the matrix $\mathbf{Q}_j(z)$ is a diagonal matrix of the shape:

$$\mathbf{Q}_j(z) = \begin{bmatrix} z^{\frac{\beta_{f1t}}{1-\beta_{f1t}}\frac{1-\beta_{fjt}}{\beta_{fjt}}} & 0 & \dots & 0 \\ 0 & \dots & \dots & 0 \\ 0 & \dots & z^{\frac{\beta_{flt}}{1-\beta_{flt}}\frac{1-\beta_{fjt}}{\beta_{fjt}}} & 0 \\ \dots & \dots & \dots & 0 \\ 0 & \dots & 0 & z^{\frac{\beta_{fJt}}{1-\beta_{fJt}}\frac{1-\beta_{fJt}}{\beta_{fjt}}} \end{bmatrix}. \tag{.41}$$

As one can notice, considering heterogeneous β fundamentally change the calculus of the relevant marginal product of labour, and the portion of the idiosyncratic surplus that the worker is capable to extract.

General equilibrium wages in a dynamic equilibrium with heterogeneous β. To obtain the equilibrium wages in a dynamic equilibrium, we need to consider the HJB equation of the unemployed worker in equation (17) of the main text. Thus, we have that equation (40) becomes:

$$w_j(a|K,\beta) = (1-\beta_j)\sum_{j=1}^{J}\xi_j^u(a)\Bigg\{b+\theta_j q(\theta_j)\frac{\int_K\int_\beta \mathbf{1}[\Xi_l(a|K,\beta) > Out(a)]\Xi_l(a|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K\int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a)\Bigg) - c_l\Bigg\} + \int_0^1 z^{\frac{1-\beta_j}{\beta_j}}\frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}dz - \beta A(j,a), \tag{.42}$$

where $\mathbf{1}[\Xi_l(a|K,\beta) > Out(a)]$ is an indicator function being 1 if the offer has a higher value than the outside option. Notice that this result confirms the intuition that under the assumption of *match stability* the outside option do not depend on the current match of the worker. Moreover, notice that by assumption we have that $\Xi_l(a|K,\beta) > Out(a)\forall\{a,K,\beta\}$, so that a worker is not unemployed by choice. Thus we have that equation (42) becomes:

$$w_j(a|K,\beta) = (1-\beta_j)\sum_{j=1}^{J}\xi_j^u(a)\Bigg\{b+\theta_j q(\theta_j)\frac{\int_K\int_\beta \Xi_l(a|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K\int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a)\Bigg) - c_l\Bigg\} + \int_0^1 z^{\frac{1-\beta_j}{\beta_j}}\frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}dz - \beta A(j,a), \tag{.43}$$

and the average wage per workplace becomes:

$$w_j(K,\beta) = (1-\beta_j)\sum_{j=1}^{J}\xi_j^u(a)\Bigg\{b+\theta_j q(\theta_j)E_a\left[\frac{\int_K\int_\beta \Xi_l(a|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K\int_\beta V_l(K,\beta)d\Gamma(K,\beta)}\right] - Out(a)\Bigg) - c_l\Bigg\} + \int_0^1 z^{\frac{1-\beta_j}{\beta_j}}\frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}dz - \beta A(j,a), \tag{.44}$$

Given the assumptions of: (1) bounded expression; (2) smoothness of production function; and (3) match stability in bargaining, the unique solution for wages in the dynamic equilibrium is given by equation (41) and (43). This expression is identical to wage expressions using intra-firm bargaining of Stole and Zwiebel (1996a). In that stream of studies, one can refer to Cahuc et al. (2008), Bauer and Lingens (2010), Elsby and Michaels (2013), Acemoglu and Hawkins (2014) and Dobbelaere and Luttens (2016).

The solution of the aggregate bargaining. Consider the solution of the firm level bargaining, as provided in equation (24) of the text. Accordingly, the average wage for a type a worker is given by:

$$\begin{aligned} E_{K,\beta}[w_j(a|K,\beta)] &= (1-\beta_j)\Big(rOut(a) - \frac{\partial Out(a)}{\partial t}\Big) \\ &+ \beta_j\Bigg[E_{K,\beta}\Big(\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - \sum_{l=1,l\neq j}^{J}\int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)} dG_l(a|K,\beta)da\Big) - A(j,a)\Bigg], \end{aligned} \tag{.45}$$

and therefore, we have that:

$$\begin{aligned} E_{K,\beta}[w_j(a|K,\beta)] &= \Big(rOut(a) - \frac{\partial Out(a)}{\partial t}\Big) + \beta_j\Bigg[E_{K,\beta}\Big(\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} \\ &- \sum_{l=1,l\neq j}^{J}\int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)} dG_l(a|K,\beta)da\Big) - A(j,a) - \Big(rOut(a) - \frac{\partial Out(a)}{\partial t}\Big)\Bigg], \end{aligned} \tag{.46}$$

$$E_{K,\beta}[w_j(a|K,\beta)] = \Big(rOut(a) - \frac{\partial Out(a)}{\partial t}\Big) + \beta_j E_{K,\beta}[QR(a)], \tag{.47}$$

and consequently, once we impose the constraint of zero expected quasi-rents, we have:

$$w_{j,t}^{MIN}(a) = rOut(a) - \frac{\partial Out(a)}{\partial t}. \tag{.48}$$

Therefore, the solution of the aggregate bargaining in equation (21) of the text is equation (48). Notice, that the key to identify this solution is to realize that an identical derivation to the one presented in equations (1) to (10) of this appendix can be easily computed for the average firm in the market. Indeed if equation (10) holds for every single firm, it also holds on average for each type a worker. Then one needs just to consider the additional constraint presented in the aggregate bargaining versus the firm bargaining, namely the absence of quasi-rents.

B4 The Dynamic and Steady State Equilibria

Additional Assumptions for dynamic equilibrium. At this stage we consider two additional technical assumptions, as follows:

- **Bounded expression** - $\lim_{N_j \to} w_j(a|K,\beta)N_j = 0$;
- **Smoothness of Production Function** - $F(\mathbf{N}(K,\beta),K)$ is continuous for all $N_j > 0$, and infinitely differentiable for all $N_j > 0$. Further, $N_j \frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j}$, and the quantity $N_1^{m_1} \ldots N_J^{m_J} \frac{\partial^2 F}{\partial N_1^{m_1} \partial \ldots} N_J^{m_J}$, or simply $\mathbf{N^m F^{(m)} N}(N,\beta)$, with $\bar{m} \sum_{j=1}^{J} m_j$ is continuous at zero.

These assumptions are fairly technical to ensure there exists an equilibrium wage function that is smooth in all $N_j > 0$, and unique. Altogether, the dynamic equilibrium of the model is defined as follows:

Theorem 1 (Dynamic Equilibrium)**.** *A tuple*

$$\Big\{\theta_j(t), Out(a), G(a|K,\beta), dG(a|K,\beta), J(a|K,\beta), \Xi(a|K,\beta), w_j(a|K,\beta), \xi_j^u(a),$$
$$\xi^{\Xi}(a|K,\beta), s_j(a|K,\beta), m_j(a|K,\beta), y_j(a|K,\beta), V_j(K,\beta), d\aleph(a)\Big\} \tag{.1}$$

is a dynamic equilibrium if for all t, the following statements are jointly satisfied:

- *$J(\cdot)$, $Out(\cdot)$ and $\Xi(\cdot)$ satisfy HJB equations (16), (17) and (18) of the text;*
- *Vacancy Posting is optimal so it holds equation (9) and equation (20) of appendix B2;*
- *$G(a|K,\beta)$ has a density $dG(a|K,\beta)$ satisfying equation (24) of appendix B2;*
- *Job search is optimal so it solves the problems in equations (14) and (15) of appendix B2;*
- *$s_j(a|K,\beta)$ holds equation (13) and $m_j(a|K,\beta)$ holds equation (19) of appendix B2;*
- *The vacancy yield holds equations (22) and (23) of appendix B2;*
- *The market tightness hold equation (21) of appendix B2, and equation (5) of the text;*
- *The unemployed distribution $dU(a)$ and the distribution of workers $d\aleph(a)$ follow equations (12) of the text;*

- *The equilibrium wage satisfies the problem in equation (21) and equation (23) of the text.*

Core definitions and assumptions of the steady state equilibrium. In the model one can define a steady state equilibrium where all aggregate variables are constant over time, and where wages and the vacancy-posting strategies of firms depend only on firm's fundamentals (K, β). Let us define a level a^R such that:

$$J_j^R(K, \beta) = J_j(a^R|K, \beta), \tag{.2}$$

so that it is the level of skill that is compatible with the expected marginal profit profit of the firm with fundamentals (K, β). Thus consistent with equation (16) of the text, we have:

$$\begin{aligned} rJ_j(a^R|K,\beta) - \frac{\partial J_j(a^R|K,\beta)}{\partial t} &= \frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - w_j(a^R|K,\beta) - A(j,a^R) \\ - \sum_{l=1,l\neq j}^{J} \int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)} dG_l(a|K,\beta)da &- s_j(a^R|K,\beta)J_j(a^R|K,\beta) + \\ + \sum_{l=1}^{J} \left\{ y_l(K,\beta)V_l(K,\beta) - \int_a s_l(a|K,\beta)dG_l(a|K,\beta)da \right\} &\frac{\partial J_j(a^R,K,\beta)}{\partial N_l(K,\beta)}. \end{aligned} \tag{.3}$$

Given the steady state equilibrium imposes stability of aggregate variables, there is stability of the workforce, namely:

$$y_l(K,\beta)V_l(K,\beta) = \int_a s_l(a|K,\beta)dG_l(a|K,\beta)da, \tag{.4}$$

and, also consider that:

$$\frac{\partial J_j(a^R|K,\beta)}{\partial t} = \frac{\partial \Xi_j(a^R|K,\beta)}{\partial t} = \frac{\partial Out(a^R)}{\partial t} = 0. \tag{.5}$$

Through a process identically presented in *The system of Differential equations for equilibrium wages* part of appendix B3, we have:

$$\begin{aligned} (r+\bar{s})J_j(a^R|K,\beta) &= \frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - w_j(a^R|K,\beta) \\ &- \sum_{l=1,l\neq j}^{J} \int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)} dG_l(a|K,\beta)da - A(j,a^R). \end{aligned} \tag{.6}$$

The Vacancy Curve. Given by assumption $\bar{s} > 0$, then we have that $V(K, \beta) > 0$ in a steady state equilibrium. Thus, given equation (10) of appendix B2, we have:

$$J_j^R(K, \beta) = \frac{\gamma_j}{q(\theta_j)}. \tag{.7}$$

Consequently:

$$\begin{aligned}\underbrace{\frac{\partial F(\mathbf{N}(K, \beta), K)}{\partial N_j(K, \beta)} - A(j, a^R)}_{\text{Marginal Product net of op. costs}} &= \underbrace{w_j(a^R|K, \beta)}_{\text{Wage}} + \underbrace{(r + \bar{s})\frac{\gamma_j}{q(\theta_j)}}_{\text{Turnover Costs}} + \\ &+ \underbrace{\sum_{l=1, l \neq j}^{J} \int_a \frac{\partial w_l(a|K, \beta)}{\partial N_j(K, \beta)} dG_l(a|K, \beta) da}_{\text{Employment effect on wages}}.\end{aligned} \tag{.8}$$

This result is typical in steady-state equilibria of search and matching models, and intuitively entails that the expected marginal worker produces on the margin the value of the cost of hiring such worker.[16] Following similar steps to the ones presented to solve this system, we have:

$$\underbrace{\frac{\int_0^1 \frac{1}{\beta} z^{\frac{1-\beta_j}{\beta_j}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} dz}{\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}}}_{\text{Overemployment Effect - } OE_j(K,\beta)} \frac{\partial F(\mathbf{N}(K, \beta), K)}{\partial N_j(K, \beta)} - A(j, a^R) = \underbrace{w_j(a^R|K, \beta) + (r + \bar{s})\frac{\gamma_j}{q(\theta_j)}}_{\text{Labour costs}} \tag{.9}$$

$$OE_j(K, \beta)\frac{\partial F(\mathbf{N}(K, \beta), K)}{\partial N_j(K, \beta)} - A(j, a^R) = w_j(a^R|K, \beta) + (r + \bar{s})\frac{\gamma_j}{q(\theta_j)}.$$

Considering the wage equation with the assumptions identified in equations (5) and (6), and the definition of employment effect, we have:

$$\begin{aligned}w_j(a^R|K, \beta) &= (1 - \beta_j)\sum_{j=1}^{J} \xi_j^u(a)\left\{b + \theta_j q(\theta_j)\left(\frac{\int_K \int_\beta \Xi_l(a^R|K, \beta)V_l(K, \beta)d\Gamma(K, \beta)}{\int_K \int_\beta V_l(K, \beta)d\Gamma(K, \beta)} - Out(a^R)\right) - c_l\right\} \\ &+ \int_0^1 z^{\frac{1-\beta_j}{\beta_j}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K, \beta), K)}{\partial N_j(K, \beta)} dz - \beta A(j, a^R) \\ w_j(a^R|K, \beta) &= (1 - \beta_j)\sum_{j=1}^{J} \xi_j^u(a)\left\{b + \theta_j q(\theta_j)\left(\frac{\int_K \int_\beta \Xi_l(a^R|K, \beta)V_l(K, \beta)d\Gamma(K, \beta)}{\int_K \int_\beta V_l(K, \beta)d\Gamma(K, \beta)} - Out(a^R)\right) - c_l\right\} \\ &+ \beta OE_j(K, \beta)\frac{\partial F(\mathbf{N}(K, \beta), K)}{\partial N_j(K, \beta)} - \beta A(j, a^R).\end{aligned} \tag{.10}$$

Joining equations (9) and (10) so that one eliminates wages, we have:

$$\begin{aligned}&OE_j(K, \beta)\frac{\partial F(\mathbf{N}(K, \beta), K)}{\partial N_j(K, \beta)} - A(j, a^R) = \\ &\sum_{j=1}^{J} \xi_j^u(a)\left\{b + \theta_j q(\theta_j)\left(\frac{\int_K \int_\beta \Xi_l(a^R|K, \beta)V_l(K, \beta)d\Gamma(K, \beta)}{\int_K \int_\beta V_l(K, \beta)d\Gamma(K, \beta)} - Out(a^R)\right) - c_l\right\},\end{aligned} \tag{$VC_j(K,\beta)$}$$

[16] For instance, Cahuc et al. (2008) finds an identical equation in their equation (9).

and equivalently:

$$\int_0^1 \frac{1}{\beta_j} z^{\frac{1-\beta_j}{\beta_j}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} dz - A(j,a^R) = \sum_{j=1}^{J} \xi_j^u(a) \left\{ b + \theta_j q(\theta_j) \left(\frac{\int_K \int_\beta \Xi_l(a^R|K,\beta) V_l(K,\beta) d\Gamma(K,\beta)}{\int_K \int_\beta V_l(K,\beta) d\Gamma(K,\beta)} - Out(a^R) \right) - c_l \right\}. \tag{$VC_j(K,\beta)$}$$

Those equations corresponds to the vacancy curves of the firm with fundamentals (K,β) - $VC_j(K,\beta)$.[17] Notice that the right-hand side of the vacancy curve is unambiguously increasing in θ.

Lemma on Profit of firms and wages. The flow profit

$$\begin{aligned} r\Pi(K,\beta) - \frac{\partial \Pi(K,\beta)}{\partial t} = & \underbrace{F(\mathbf{N}(K,\beta);K)}_{\text{Production}} - \underbrace{\sum_{j=1}^{J} \int_a w_j(a|K,\beta) dG_j(a|K,\beta) da}_{\text{Wage Bill}} \\ & - \underbrace{\sum_{j=1}^{J} \int_a A(j,a) dG_j(a|K,\beta) da}_{\text{Operating Cost Bill}} - \underbrace{I(K)}_{\text{Sunk Cost}} - \underbrace{\sum_{j=1}^{J} \int_a s_j(a|K,\beta) J_j(a|K,\beta) dG_j(a|K,\beta) da}_{\text{Cost of Displacement}} + \\ & + \underbrace{\sum_{j=1}^{J} \max_{V_j(K,\beta)} \left\{ -\gamma_j V_j(K,\beta) + V_j(K,\beta) q(\theta_j) J_j^R(K,\beta) \right\}}_{\text{Value of the Hiring Policy}}. \end{aligned} \tag{.12}$$

is continuous, strictly concave and satisfies:

$$\lim_{\mathbf{N} \to \mathbf{0}^+} \Pi(K,\beta) = 0. \tag{.13}$$

Then, given:

$$\lim_{N_j(K,\beta) \to \infty} \frac{\partial F(\mathbf{N}(K,\beta))}{\partial N_j(K,\beta)} < b, \tag{.14}$$

assumed in the model, it implies that:

$$\lim_{N_j(K,\beta) \to \infty} \Pi(K,\beta) = -\infty, \tag{.15}$$

implying that the optimal workforce size vector $\mathbf{N}(K,\beta)$ is finite in every task market for every firm. Further notice that

[17] Notice that as typically we have:

$$\begin{aligned} &\lim_{x \to 0^+} q(x) = \infty; \quad \lim_{x \to \infty} q(x) = 0; \quad \frac{\partial q(\theta)}{\partial \theta} < 0 \\ &\lim_{x \to 0^+} xq(x) = 0; \quad \lim_{x \to \infty} xq(x) = \infty; \quad \frac{\partial \theta q(\theta)}{\partial \theta} > 0. \end{aligned} \tag{.11}$$

$$\lim_{N_j(K,\beta)\to\infty} w_j(a^R|K,\beta) = \sum_{j=1}^{J}\xi_j^u(a)\left\{b+\theta_j q(\theta_j)\left(\frac{\int_K\int_\beta \Xi_l(a^R|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K\int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a^R)\right) - c_l\right\}+$$
$$+\lim_{N_j(K,\beta)\to\infty}\int_0^1 z^{\frac{1-\beta_j}{\beta_j}}\frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}dz - \beta_j A(j,a^R) \tag{.16}$$
$$\lim_{N_j(K,\beta)\to 0^+} w_j(a^R|K,\beta) = \sum_{j=1}^{J}\xi_j^u(a)\left\{b+\theta_j q(\theta_j)\left(\frac{\int_K\int_\beta \Xi_l(a^R|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K\int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a^R)\right) - c_l\right\}+$$
$$+\lim_{N_j(K,\beta)\to 0^+}\int_0^1 z^{\frac{1-\beta_j}{\beta_j}}\frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}dz - \beta_j A(j,a^R)$$

is identical to

$$\lim_{N_j(K,\beta)\to\infty} w_j(a^R|K,\beta) = \sum_{j=1}^{J}\xi_j^u(a)\left\{b+\theta_j q(\theta_j)\left(\frac{\int_K\int_\beta \Xi_l(a^R|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K\int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a^R)\right) - c_l\right\}+$$
$$+\lim_{N_j(K,\beta)\to\infty}\frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - \beta_j A(j,a^R) \tag{.17}$$
$$\lim_{N_j(K,\beta)\to 0^+} w_j(a^R|K,\beta) = \sum_{j=1}^{J}\xi_j^u(a)\left\{b+\theta_j q(\theta_j)\left(\frac{\int_K\int_\beta \Xi_l(a^R|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K\int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a^R)\right) - c_l\right\}+$$
$$+\lim_{N_j(K,\beta)\to 0^+}\frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - \beta_j A(j,a^R).$$

Therefore, given the production function is strictly concave and displaying decreasing returns to scale, one concludes, given

$$\sum_{j=1}^{J}\xi_j^u(a)\left\{b+\theta_j q(\theta_j)\left(\frac{\int_K\int_\beta \Xi_l(a^R|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K\int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a^R)\right) - c_l\right\} > 0, \tag{.18}$$

by assumption, that wages are strictly positive and strictly decreasing with firm size. As noted by Acemoglu and Hawkins (2014), given

$$\lim_{N_j(K,\beta)\to 0^+} w_j(a^R|K,\beta) = +\infty \tag{.19}$$

the level of employment that maximizes the flow profit is strictly positive.

Considerations on steady-state equilibrium. Given the shape of the profit function of a firm, precisely: (a) $\lim_{\mathbf{N}\to\mathbf{0}^+}\Pi(K,\beta)=0$; (b) $\Pi(K,\beta)$ is strictly concave on employment; and (c) $J_j^R(K,\beta)$ is strictly decreasing in employment, and given

$$J_j^R(K,\beta) = \frac{\gamma_j}{q(\theta_j)}, \tag{.20}$$

then equation (20) has a unique vector of employment $\mathbf{N}(K,\beta)$ conditional on the values of the endogenous variables.

The steady-state condition The steady state equilibrium concept offers a greater simplification to our framework, which arises from imposing stability of aggregate flows at firm level. Thus, the stability of aggregate flows, equation (24) of appendix B2, and equation (5) jointly yield:

$$\begin{aligned} 0 = &- dG_j(a|K,\beta) + y_j(a|K,\beta)V_j(K,\beta) + \\ &+ \int_{a''} \Big(1 - s_j(a''|K,\beta)\Big)\psi(a|a'')dG_j(a''|K,\beta)da'' \end{aligned} \tag{.21}$$

which after further simplification becomes:

$$dG_j(a|K,\beta) = \frac{\int_{a''} \Big(1 - s_j(a''|K,\beta)\Big)\psi(a|a'')dG_j(a''|K,\beta)da''}{1 - s_j(a|K,\beta)} \tag{.22}$$

Accordingly, in the steady state equilibrium we hold that:

$$G_j(a|K,\beta) = \int_{-\infty}^{a} \frac{\int_{a''} \Big(1 - s_j(a''|K,\beta)\Big)\psi(a|a'')dG_j(a''|K,\beta)da''}{1 - s_j(a|K,\beta)} da. \tag{.23}$$

Given the stability of the workforce in each workplace economy-wide, we therefore also can hold that:

$$\int_K \int_\beta y_j(K,\beta)V_j(K|\beta)dKd\beta = \int_K \int_\beta \int_a s_j(a|K,\beta)dG_j(a|K,\beta)dadKd\beta. \tag{.24}$$

Steady state equilibrium description. Therefore, the steady-state equilibrium is a specialization of the dynamic equilibrium presented with the following properties:

Theorem 2 (Steady-State Equilibrium)**.** *A tuple*

$$\begin{aligned} &\Big\{\theta_j(t), Out(a), G(a|K,\beta), dG(a|K,\beta), J(a|K,\beta), \Xi(a|K,\beta), w_j(a|K,\beta), \xi_j^u(a), \\ &\xi^{\Xi}(a|K,\beta), s_j(a|K,\beta), m_j(a|K,\beta), y_j(a|K,\beta), V_j(K,\beta), d\aleph(a)\Big\} \end{aligned} \tag{.25}$$

is a steady state equilibrium if for $q(\theta) > 0$ and $\bar{s} > 0$, the following statements are jointly satisfied:

- *$J(\cdot)$, $Out(\cdot)$ and $\Xi(\cdot)$ satisfy HJB equations (16), (17) and (18) of the text;*
- *Vacancy Posting is optimal so it holds equation (9) and equation (20) of appendix B2;*
- *$G(a|K,\beta)$ has a density $dG(a|K,\beta)$ satisfying equation (24) of appendix B2, and equation (23) of this appendix;*
- *Job search is optimal so it solves the problems in equations (14) and (15) appendix B2;*

- *$s_j(a|K,\beta)$ holds equation (13), and $m_j(a|K,\beta)$ holds equation (19) of appendix B2;*
- *The vacancy yield holds equations (22), and (23) of appendix B2;*
- *The market tightness hold equation (21) of appendix B2, and equation (5) of the text;*
- *The unemployed distribution $dU(a)$ and the distribution of workers $d\aleph(a)$ follow equations (12) of the text;*
- *The equilibrium wage satisfies the problem in equation (21) and (23) of the text;*
- *The steady state conditions of equation (4) and equation (24);*
- *The stability of expectations of HJB functions in equation (5).*

Note that the the distribution of skill within workplaces satisfies an ergodicity condition and thus $G_j(a|K,\beta)$ is unique. So there is no loss of generality to apply such distribution which is assumed to be continuously differentiable. Note that $\bar{s} > 0$, which is partially justified by the death and birth shocks d. Note, that to ease the technical explanation on ergodicity, one can reason such shocks as a massive destructive shock on skill, which leads the worker to have his skill extracted from $\Psi_0(a)$, rather than having the worker dying and a new worker entering the market. Thus, we arive at the uniqueness of the invariant distribution through Theorem 11.9 of Stokey et al. (1989).[18]

[18] An identical argument is used in Acemoglu and Hawkins (2014).

C Derivation of the Empirical Model for Outside Options

The equilibrium wage expression with heterogeneous bargaining powers is given by:

$$w_j(a|K,\beta) = (1-\beta_j)Out^{\star}(a) + \int_0^1 z^{\frac{1-\beta}{\beta}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta), K}{\partial N_j(K,\beta)} dz - \beta_j A(j,a). \qquad (.1)$$

Let us focus in the dynamic behaviour of $Out^{\star}(a)$ function.

The time effect on outside Options. Notice that we have that the outside option is given by:

$$Out(a) = E_t[Out(a)] + \Big[Out(a) - E_t[Out(a)]\Big], \qquad (.2)$$

where $E_t[Out(a)]$ is the expected outside option of worker with skill level a. With standard algebraic manipulations one obtains:

$$Out(a) = E_t[Out(a)]\Big[1 + \frac{Out(a) - E_t[Out(a)]}{E_t[Out(a)]}\Big], \qquad (.3)$$

and considering a first order Taylor approximation, we have:

$$ln[Out(a)] = ln\Big[E_t[Out(a)]\Big] + \frac{Out(a) - E_t[(Out(a)]}{E_t[Out(a)]}. \qquad (.4)$$

Inside the expected value of the outside option. We have that the expected value of the outside option of the worker is given by:

$$ln\Big[E_t(Out(a))\Big] = ln\Bigg\{E_t \sum_{j=1}^{J} \xi_j^u(a)\Big\{b + \theta_j q(\theta_j)\Big(\frac{\int_K \int_\beta \Xi_l(a^R|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K \int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a^R)\Big) - c_l\Big\}\Bigg\}. \qquad (.5)$$

At this point, we consider a first order Taylor approximation around the initial value of a for each worker. Thus:

$$E_t\Big(Out(a)\Big) = E_t\Big(Out(a_{\tau_0(i)})\Big) + \frac{\partial E_t(Out(\tilde{a}))}{\partial a}\Big|_{\tilde{a}\in[a_{\tau_0(i);a}]}\Big(a - a_{\tau_0(i)}\Big), \qquad (.6)$$

where $a_{\tau_0(i)}$ represents the skill value of the worker in the moment he enters the labour market.

Moreover, let us consider the expected value of skill a worker with a in the current period t should have had in the first period of her current contract - $E_t[a_{\tau_0(i)}|a]$. Consequently, equation 6 becomes:

$$E_t\Big(Out(a)\Big) = E_t\Big(Out(a_{\tau 0(i)})\Big) + \frac{\partial E_t(Out(\tilde{a}))}{\partial a}\Big|_{\tilde{a}\in[a_{\tau 0(i);a}]}\Big(a - E_t[a_{\tau 0(i)}|a]\Big) + \\ + \frac{\partial E_t(Out(\tilde{a}))}{\partial a}\Big|_{\tilde{a}\in[a_{\tau 0(i);a}]}\Big(E_t[a_{\tau 0(i)}|a] - E_t[a_{\tau 0(i)}]\Big), \tag{.7}$$

and after a first order Taylor approximation around the logarithm of expected value of outside option, one obtains:

$$ln\Big[E_t\Big(Out(a)\Big)\Big] \approx ln\Big[E_t\Big(Out(a_{\tau 0(i)})\Big)\Big] + \\ + \frac{1}{E_t[Out(a_{\tau 0(i)})]}\frac{\partial E_t(Out(\tilde{a}))}{\partial a}\Big|_{\tilde{a}\in[a_{\tau 0(i);a}]}\Big(a - E_t[a_{\tau 0(i)}|a]\Big) + \\ + \frac{1}{E_t[Out(a_{\tau 0(i)})]}\frac{\partial E_t(Out(\tilde{a}))}{\partial a}\Big|_{\tilde{a}\in[a_{\tau 0(i);a}]}\Big(E_t[a_{\tau 0(i)}|a] - E_t[a_{\tau 0(i)}]\Big). \tag{.8}$$

Functional form of empirical model for outside options. Combining equations (4) and (8), we have:

$$ln[Out(a)] = \underbrace{\frac{Out(a) - E_t[Out(a)]}{E_t[Out(a)]}}_{\text{Task-market-time effect - }\lambda(j,t)} \\ + \underbrace{ln\Big[E_t\Big(Out[a_{\tau 0(i)}]\Big)\Big] + \frac{1}{E_t[Out(a_{\tau 0(i)})]}\frac{\partial E_t(Out(\tilde{a}))}{\partial a}\Big|_{\tilde{a}\in[a_{\tau 0(i);a}]}\Big(a - E_t[a_{\tau 0(i)}|a]\Big)}_{\substack{\text{expected contract profile -} \\ \psi(t-\tau_0(i),rank(i,t),female_i)}} + \\ + \underbrace{\frac{1}{E_t[Out(a_{\tau 0(i)})]}\frac{\partial E_t(Out(\tilde{a}))}{\partial a}\Big|_{\tilde{a}\in[a_{\tau 0(i,f);a}]}\Big(E_t[a_{\tau 0(i)}|a] - E_t[a_{\tau 0(i)}]\Big)}_{\text{disturbance - }v^S_{i,t}}. \tag{.9}$$

where we explicitly introduce a parallel trend assumption, namely:

$$\frac{Out(a) - E_t[Out(a)]}{E_t[Out(a)]} = \frac{Out - E_t[Out]}{E_t[Out]}(j,t) = \lambda(j,t). \tag{.10}$$

Intuitively, one is assuming that the evolution of outside option value of every worker of a given type a within a task market j is identical.

The reduced form of Outside Options and measurement error. Altogether, we therefore have that the reduced form representation, as presented in equation (28) of the text, is given by:

$$ln[w^{\star}_{bargain}] = \underbrace{\lambda(j,t) + \psi(t-\tau_0(i), rank(i,t), female_i)}_{\widehat{Out(a)}} + v^S_{i,t}. \tag{.11}$$

Moreover, following the insight of Pei et al. (2018), notice that in the case of existence of a classical measurement error in $w_{bargain}$, so that:

$$w_{bargain} = w^{\star}_{bargain} + \eta(i,t), \tag{.12}$$

where $\eta(i,t)$ corresponds to the classical measurement error, with the following properties:

1. $E[\eta(i,t)] = 0$;
2. $E[\lambda(j,t)\eta(i,t)] = 0$;
3. $E[\psi(t-\tau_0(i), rank(i,t))\eta(i,t)] = 0$;
4. $E[v^S_{i,t}\eta(i,t)] = 0$.

Consequently:

$$ln[w_{bargain}] = \underbrace{\lambda(j,t) + \psi(t-\tau_0(i), rank(i,t), female_i)}_{\widehat{Out(a)}} + \underbrace{v^S_{i,t} - \eta(i,t)}_{v_{i,t}}, \tag{.13}$$

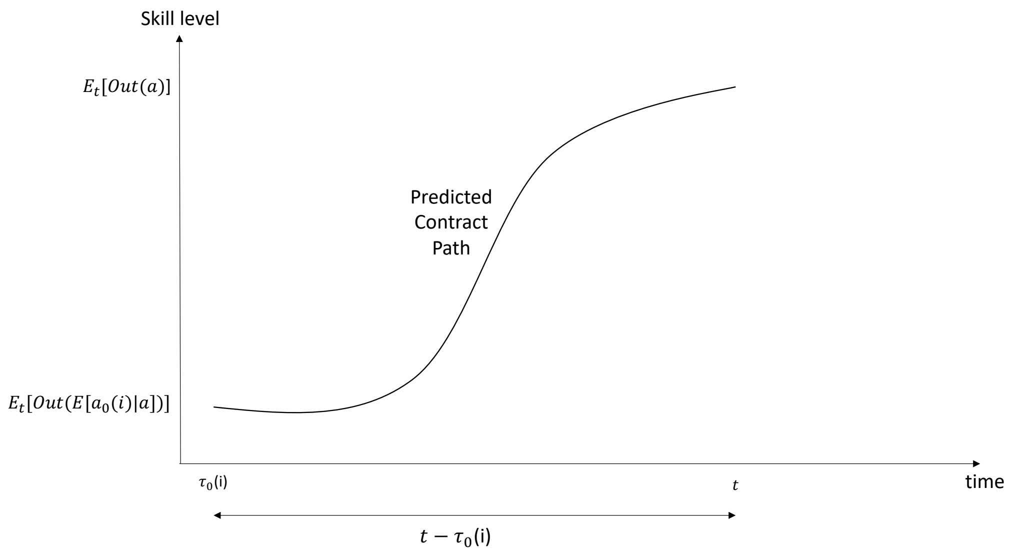
with $v_{i,t}$ corresponding to the composite error term.

In a nutshell, the first stage of our empirical implementation, beyond providing empirical structure to our estimation, also provides relevant answer to the existence of measurement error, particularly given $w_{bargain}$ corresponds to a proxy. As long as the measurement error is classical, it only has efficiency impacts, and not on the consistency of the estimates. Given the high dimensionality of our data, naturally efficiency of the estimator does not lie in the top of our priorities.

Intuition on the expected contract profile. We take advantage of the knowledge of: (i) the actual rank of the worker, which is linked with $E_t[Out(a)]$, apart from the trend behaviour; (ii) the experience of the worker, given by $t - \tau_0(i)$, so that we are capable to estimate the predicted contract path of each worker; and (iii) we allow for heterogeneous contract profiles by gender.

The identification of the predicted contract path enables the estimation of a time-task market effect, so that it controls for any time trend. Altogether, we are bunching the information of the workers sharing the same contract at collective agreement level (i.e. experience, actual rank and gender), and thus we improve our position to better value the

Figure C1: Structure of the Model in each Moment t.



individual-task market effect. Accordingly our identification follows the intuition of figure C1.

D A Toy Model

In this appendix we present a toy version of the model presented in the text. This toy version abstracts from several key ingredients of the main model, but preserves the link between the model and the identification strategy implemented.

Labour market structure. The labour market is frictional, workers search for jobs and get randomly matched with vacancies posted by firms.

Consider a standard constant returns to scale matching function as M(u,v), and define the labour market tightness as $\theta = \frac{v}{u}$, where v is the number of vacancies in the market posted by firms, and u is the number of workers unemployed and searching for a job. Accordingly, the poisson rate at which workers meet a vacancy is given by $\theta q(\theta)$, and the poisson rate at which a vacancy is filled corresponds to $q(\theta)$.

Worker and firm heterogeneity. Further assume the economy is populated by exogenously capital heterogeneous firms, and exogenously skill heterogeneous workers, with

$K \in [0,1]$ and $A \in [0,1]$, respectively. The production function of a job filled in a type-K firm by a worker of type-A, is given by:

$$F(K, A) = A + f(K). \tag{.1}$$

Value Functions. Considering the setting, the value function of a vacancy is given by:

$$rV(K) = -\gamma + q(\theta)\Big[E_A[J(A,K)] - V(K)\Big], \tag{.2}$$

where r is the discount rate, γ the cost of posting a vacancy and $E_A[J(A,K)]$ the expected value of a vacancy filled in a type-K firm.

The value function of a filled vacancy constituting a match between a type-A worker and a type-K firm corresponds to:

$$rJ(A,K) = A + f(K) - (r-\delta)K - w(A,K) - s(A,K)\Big[J(A,K) - V(K)\Big], \tag{.3}$$

where δ is the depreciation rate, s is the probability of termination of the match and $w(K, A)$ is the wage earned by the type-A worker in such filled vacancy. Moreover, the value function of a type-A employed worker in a type-K firm is:

$$r\Xi(A,K) = w(A,K) + s(A,K)\Big[U(A) - \Xi(A,K)\Big], \tag{.4}$$

with $U(A)$ corresponding to the value function of a type-A worker which is unemployed. Such value function is in detailed given by:

$$rU(A) = b + \theta q(\theta)\Big[E_A[\Xi(A,K)|K \in \Theta] - U(A)\Big], \tag{.5}$$

with b corresponding to the flow of unemployment benefits, Θ consisting in the set of type-K firms which are expected to post vacancies, and $E_A[J(A,K)|K \in \Theta]$ representing the expected value of filled vacancies by a type-A worker, given the type-K firms posting vacancies.

Free Entry. It is assumed that there is free entry for posting vacancies. Accordingly, it holds that $V(K) = 0$ for all type-K firms, and consequently:

$$E_A[J(A,K)] = \frac{\gamma}{q(\theta)}. \tag{.6}$$

Termination rate. It is assumed that a filled vacancy can be terminated either due to exogenous reasons happening at rate s, or due to economic reasons. Accordingly, it holds that:

$$s(A,K) = \begin{cases} 1 \text{ if } A + f(K) - (r-\delta)K - rU(A) \geq 0 \\ s \text{ if } A + f(K) - (r-\delta)K - rU(A) < 0. \end{cases} \tag{.7}$$

Wage bargaining and equilibrium wages. The wage is set through a 2 stage bargaining. In detail:

1. The first takes place at national level between a fully representative right-to-manage union and a fully representative employers' association;
2. The second stage happens subsequently at firm level, between the referred union and the firm.

At national level, the union and the firm association bargain the minima wage requirement for each type-A worker, with the union committing to abstain from a full national lockdown. Moreover, the sides agree the minima wage requirement by bargaining an average national wage for type-A worker and then consider the least minimum viable match, namely the wage that would arise in the absence of an average positive quasi-rent at vacancy-filled level - $E_K[QR(A,K)]$. Notice that the quasi-rent corresponds to:

$$QR(A,K) = A + f(K) - (r-\delta)K - rU(A). \tag{.8}$$

Therefore, the bargaining at national level solves:

$$\begin{aligned} w_{MIN}(A) =& argmax_{E_K[w(A,K)]} \Big[E_K[\Xi(A,K)] - U(A)\Big]^{\beta} \Big[E_K[J(A,K)] - V(K)\Big]^{1-\beta} \\ & \text{subject to} \\ & A + E_K[f(K)] - (r-\delta)E(K) - rU(A) = 0 \quad \text{(Quasi-Rent Constraint)}, \end{aligned} \tag{.9}$$

where β corresponds to the union's bargaining power. Altogether, the solution of optimization problem in equation (9) corresponds to:

$$w_{MIN}(A) = rU(A). \tag{.10}$$

At firm level, the union and a single firm bargain over wages, with the union committing to abstain from a full lockdown, and considering the minimum wage requirement set. So it holds:

$$\begin{aligned} w(A,K) = argmax_{w(A,K)} &\Big[\Xi(A,K) - U(A)\Big]^{\beta}\Big[J(A,K) - V(K)\Big]^{1-\beta} \\ &\text{subject to} \\ &\qquad w(A,K) \geq w_{MIN}(A) \qquad\qquad \text{(National bargaining constraint).} \end{aligned} \tag{.11}$$

Accordingly, the solution corresponds to:

$$w(A,K) = \begin{cases} (1-\beta)rU(A) + \beta\Big[A + f(K) - (r-\delta)K\Big] & \text{if } A + f(K) - (r-\delta)K - rU(A) \geq 0 \\ w_{MIN}(A) & \text{if } A + f(K) - (r-\delta)K - rU(A) < 0, \end{cases} \tag{.12}$$

which given the termination rate, translates to:

$$w(A,K) = (1-\beta)rU(A) + \beta\Big[A + f(K) - (r-\delta)K\Big]. \tag{.13}$$

Empirical Identification. Consider the difference between the equilibrium wage defined in equation (13), and the average wage of type-K firm:

$$w(K,A) - E_A[w(K,A)|K] = (1-\beta)\Big[rU(A) - rE_A[U(A)|K]\Big] + \beta\Big[A - E[A|K]\Big], \tag{.14}$$

which can be represented as:

$$\ln w(A,K) = \ln\Bigg[E_A[w(K,A)|K] + (1-\beta)\Big[rU(A) - rE_A[U(A)|K]\Big] + \beta\Big[A - E[A|K]\Big]\Bigg]. \tag{.15}$$

Given the solution of national wide bargain, equation (15) can be translated to:

$$\ln w(A,K) = \ln\Bigg[E_A[w(K,A)|K]+(1-\beta)\Big[w_{MIN}(A)-E_A[w_{MIN}(A)|K]\Big]+\beta\Big[A-E[A|K]\Big]\Bigg]. \tag{.16}$$

Equation (14) corresponds to the empirical model to be implemented resorting to a Nonlinear Least Squares algorithm.

Derivation of the set of minimum wage requirement. Given the optimization problem presented in equation (9), the value functions in equations (2)-(5), the free entry condition and the termination rate, we have that the minimum wage requirement set for viable filled vacancies solves the following system:

$$\begin{cases} (1-\beta)\Big[E_K[\Xi(A,K)] - U(A)\Big] = \beta\Big[E_K[J(A,K)]\Big] \\ A + E_K[f(K)] - (r-\delta)E(K) - rU(A) = 0 \\ rE_K[\Xi(A,K)] = E_K[w(A,K)] + s\Big[U(A) - E_K[\Xi(A,K)]\Big] \\ rE_K[J(A,K)] = A + E_K[f(K)] - (r-\delta)E[K] - E_K[w(A,K)] - s\Big[E_K[J(A,K)] - E_K[V(K)]\Big]. \end{cases}$$

Consequently, we have that the system can be reduced to:

$$\begin{cases} (1-\beta)\Big[E_K[w(A,K)] - rU(A)\Big] = \beta\Big[A + E_K[f(K)] - (r-\delta)E[K] - E_K[w(A,K)]\Big] \\ A + E_K[f(K)] - (r-\delta)E(K) - rU(A) = 0, \end{cases}$$

and the set of wage minimum is given by:

$$w_{MIN}(A) = E_K[w(A,K)] = (1-\beta)rU(A) + \beta\Big[A + E_K[f(K)] - (r-\delta)E(K)\Big]$$

which can be reduced to:

$$w_{MIN}(A) = rU(A).$$

Derivation of the equilibrium wage. Given the optimization problem presented in equation (11), the value functions in equations (2)-(5), the free entry condition and the termination rate, the system that solves the equilibrium wages is given by:

$$\begin{cases} (1-\beta)\Big[\Xi(A,K) - U(A)\Big] = \beta\Big[J(A,K)\Big] \\ r\Xi(A,K) = w(A,K) + s\Big[U(A) - \Xi(A,K)\Big] \\ rJ(A,K) = A + f(K) - (r-\delta)K - w(A,K) - s\Big[J(A,K) - V(K)\Big]. \end{cases}$$

The referred system can be reduced to:

$$w(A,K) = (1-\beta)rU(A) + \beta\Big[A + f(K) - (r-\delta)K\Big],$$

which corresponds to the equilibrium wage equation in the interior solution. Thus:

$$w(A,K) = \begin{cases} (1-\beta)rU(A) + \beta\Big[A + f(K) - (r-\delta)K\Big] & \text{if } A + f(K) - (r-\delta)K - rU(A) \geq 0 \\ w_{MIN}(A) & \text{if } A + f(K) - (r-\delta)K - rU(A) < 0. \end{cases}$$

However, notice given the termination rate, the equilibrium wage holds in the interior solution.

Appendix to Chapter 4

A Derivation of Unconstrained Wages

As referred in the text, the wage setting follows a rent-sharing rule assuming: (a) the potential promotion of a temporary job into a permanent job entails the extinguishing of the temporary job position, requiring the firm to potentially post a new vacancy; and (b) the wage setting do not consider the subsequent attribution of bonuses, that is only defined once profits are determined. Hence for wage setting purposes $\eta(a, j, t) = 0$ with $j \in \{T, P\}$.

Permanent contract wages. Given the assumption framework, the wages are defined considering the value functions and the rent-splitting rule. Accordingly, consider the set of equations:

$$\begin{aligned}
&\beta\Big[J(a,P,t) + S_P\Big] = (1-\beta)\Big[W(a,P,t) - W(a,U,t)\Big];\\
&r(t)J(a,P,t) = \ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} - \tilde{w}(a,P,t) + \partial_a J(a,P,t)\dot{a}(a,P,t) + \sigma_P(\ddot{\xi}_P)[-S_P - J(a,P,t)] + \partial_t J(a,P,t);\\
&\rho W(a,P,t) = u(\ddot{c}) + \partial_a W(a,P,t)\dot{a}(a,P,t) + \sigma_P(\ddot{\xi}_P)\Big[W(a,U,t) - W(a,P,t)\Big] + \partial_t W(a,P,t)\\
&\dot{a}(a,P,t) = \tilde{w}(a,P,t) + (r_l - \delta)a - \ddot{c}
\end{aligned} \tag{.1}$$

Solving this set of equations, I have:

$$\begin{aligned}
\tilde{w}(a,P,t) = &\frac{\beta\Big[\rho + \sigma_P(\ddot{\xi}_P)\Big]\Big\{\ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} + \partial_a J(a,P,t)\Big(r(t)a - \ddot{c}\Big) + r(t)S_P + \partial_t J(a,P,t)\Big\}}{[1-\beta][r(t) + \sigma_P(\ddot{\xi}_P)]\partial_a W(a,P,t) + \beta[\rho + \sigma_P(\ddot{\xi}_P)][1 - \partial_a J(a,P,t)]} -\\
&\frac{(1-\beta)\Big[r(t) + \sigma_P(\ddot{\xi}_P)\Big]\Big\{u(\ddot{c}) + \partial_a W(a,P,t)\Big(r(t)a - \ddot{c}\Big) - \rho W(a,U,t) + \partial_t W(a,P,t)\Big\}}{[1-\beta][r(t) + \sigma_P(\ddot{\xi}_P)]\partial_a W(a,P,t) + \beta[\rho + \sigma_P(\ddot{\xi}_P)][1 - \partial_a J(a,P,t)]}.
\end{aligned} \tag{.2}$$

Then notice that in the case of a stationary equilibrium, by definition $\partial_t W(a,P,t) = \partial_t J(a,P,t) = 0$.

Temporary contract wages. The wages for the temporary contracts solve the following set of equations:

$$
\begin{aligned}
\beta\Big[J(a,T,t)\Big] =& (1-\beta)\Big[W(a,T,t)-W(a,U,t)\Big];\\
r(t)J(a,T,t) =& \ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} - \tilde{w}(a,T,t) + \partial_a J(a,T,t)\dot{a}(a,T,t) + [\varphi(\ddot{\xi}_P)+\sigma_T(\ddot{\xi}_T)][-J(a,T,t)]+\\
&+\partial_t J(a,T,t);\\
\rho W(a,T,t) =& u(\ddot{c}) + \partial_a W(a,T,t)\dot{a}(a,T,t) + \sigma_T(\ddot{\xi}_T)\Big[W(a,U,t)-W(a,T,t)\Big] + \varphi(\ddot{\xi}_P)\Big[W(a,P,t)-W(a,T,t)\Big]+\\
&+\partial_t W(a,T,t);\\
\dot{a}(a,T,t) =& \tilde{w}(a,T,t) + (r_l-\delta)a - \ddot{c};
\end{aligned}
\tag{.3}
$$

Solving this set of equations, I have:

$$
\begin{aligned}
\tilde{w}(a,T,t) =& \frac{\beta\Big[\rho+\varphi(\ddot{\xi}_P)+\sigma_T(\ddot{\xi}_T)\Big]\Big\{\ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} + \partial_a J(a,T,t)\Big(r(t)a-\ddot{c}\Big) + \partial_t J(a,T,t)\Big\}}{[1-\beta][r(t)+\varphi(\ddot{\xi}_P)+\sigma_T(\ddot{\xi}_T)]\partial_a W(a,P,t) + \beta[\rho+\varphi(\ddot{\xi}_P)+\sigma_T(\ddot{\xi}_T)][1-\partial_a J(a,T,t)]}\\
&-\frac{(1-\beta)\Big[r(t)+\varphi(\ddot{\xi}_P)+\sigma_T(\ddot{\xi}_T)\Big]\Big\{u(\ddot{c}) + \partial_a W(a,P,t)\Big(r(t)a-\ddot{c}\Big) + \varphi(\ddot{\xi}_P)W(a,P,t) - [\rho+\varphi(\ddot{\xi}_P)]W(a,U,t) + \partial_t W(a,P,t)\Big\}}{[1-\beta][r(t)+\varphi(\ddot{\xi}_P)+\sigma_T(\ddot{\xi}_T)]\partial_a W(a,P,t) + \beta[\rho+\varphi(\ddot{\xi}_P)+\sigma_T(\ddot{\xi}_T)][1-\partial_a J(a,T,t)]}.
\end{aligned}
\tag{.4}
$$

Then notice that in the case of a stationary equilibrium, by definition $\partial_t W(a,T,t) = \partial_t J(a,T,t) = 0$.

B The Definition of the Stationary Equilibrium

In this section we provide a calibration for the model and present its consequent stationary equilibrium. Formally, the stationary recursive equilibrium consists of (a) set of value functions $\{J(a,P,t), J(a,T,t), W(a,P,t),\ W(a,T,t), V(t), W(a,U,t)\}$; (b) a set of policy functions $\{c(a,P,t), c(a,T,t), c(a,U,t), \dot{a}(a,P,t), \dot{a}(a,T,t), \dot{a}(a,U,t)\}$; (c) a distribution over assets and employment $\{g(a,P,t), g(a,T,t), g(a,U,t)\}$; (d) a capital stock per firm $k(a,T,t)$ and $k(a,P,t)$; (e) a labour market tightness $\theta(t)$; (f) the firm's dividents $D(t)$ and (g) a set of prices $\{r(t), r_l(t), Q(t)\}$ such that:

- the workers solve the problems in equations (5), (6) and (7) given prices, the capital stock and labour market tightness;
- the firms defines the capital stock by solving the problems in equations (13) and (14), given prices and the labour market tightness;
- the job finding and filling probabilities are as explained in the frictional labour market description part;
- free entry condition holds as presented in equation (12);
- the stationary distributions over assets and employment satisfy the standard Kol-

mogorov forward equations;

- the wage setting satisfies the rent-sharing rule;
- the asset market clears according to equation (3), the price of equity corresponds to the present value of the dividend string as in equation (16), and the arbitrage condition as in equation (1) holds;
- The final good is produced in firms and at home and it is spent in consumption and investment in physical capital, vacancy creation and firing taxes:

$$\int_a^\infty \sum_{i=\{P,T,U\}} c(a,i,t)g(a,i,t)da + \int_a^\infty \sum_{i=\{P,T,U\}} [\dot{k}(a,i,t) - \delta k(a,i,t)]g(a,i,t)da + \xi v = \int_a^\infty \sum_{i=\{P,T\}} [z_t(a,i)F(k(a,i,t)]g(a,i,t)da + u_t h \tag{.1}$$

C Algorithm of Stationary Equilibrium

The stationary equilibrium results from a fixed-point algorithm over $\{\theta, r(t)\}$. The HJB equations are solved via the finite difference method as described in Achdou et al. [2017]. This section follows closely the work of Bardóczy [2017], with the required adjustments to the concrete model implemented. Accordingly, the algorithm unfolds through the following steps:

Step 1. Set up the grid for assets. In this computations, I have $a \in [0, 150]$, with 501 points. The choice reflects: (a) the resulting stationary equilibrium estimate, which is well covered in the grid; (b) the idea that the upper bound is quite irrelevant for the agent's decisions in the relevant range; and (c) the computational requirements to efficiently run the algorithm.

Step 2. Guess the tightness θ in the market. The calibration implemented seeks a $\theta = 1$ in equilibrium. Accordingly, the cost of posting a vacancy, γ, is adjusted to attain such objective. Then, given tightness, one obtains:

$$\theta q(\theta) = \chi\theta^{1-\eta}; \quad q(\theta) = \chi\theta^{-\eta}; \tag{.1}$$

Step 3. Guess the capital per worker k for each type of contract, and the cost

of capital $\mathbf{r_l(t)}$. So we use as initial guess:

$$k(a,T,t) = \left[\frac{\alpha z(a,T,t)}{r_l(t)}\right]^{\frac{1}{1-\alpha}}; \quad k(a,P,t) = \left[\frac{\alpha z(a,P,t)}{r_l(t)}\right]^{\frac{1}{1-\alpha}}; \quad r_l(t) = r(t) + \delta; \tag{.2}$$

where $r(t)$ is initially set at 0.004.

Step 4. Define the wage schedules $\mathbf{w(a,T,t)}$ and $\mathbf{w(a,P,t)}$. I start with an initial guess where we have:

$$w(a,j,t) = \beta\Big[z(a,j,t)k(a,j,t)^\alpha - r_l(t)k(a,j,t)\Big], \quad j \in \{T,P\}. \tag{.3}$$

Step 5. Define the leverage of firms $\xi(\mathbf{a,T,t})$ and $\xi(\mathbf{a,P,t})$, and define the flow probabilities. Accordingly, the leverage of the firms is defined as:

$$\xi(a,j,t) = \frac{r_l(t)k(a,j,t)}{z(a,j,t)k(a,j,t)^\alpha - w(a,j,t)}, \quad j \in \{T,P\}. \tag{.4}$$

Then, define the flow probabilities $\sigma_j[\xi(a,j,t)]$ with $j \in \{T,P\}$, $h[\xi(a,T,t)]$ and $\varphi[\xi(a,P,t)]$.

Step 6. Solve the worker's problem. The discretized HJB equations become:

$$\begin{aligned} \rho W(a,i,t) =& u[c(a,i,t)] + \underbrace{\frac{W(a+\epsilon,i,t) - W(a,i,t)}{\Delta a}}_{\partial_a W^F(a,i,t)} \dot{a}(a,j,t)^+ \\ &+ \underbrace{\frac{W(a,i,t) - W(a-\epsilon,i,t)}{\Delta a}}_{\partial_a W^B(a,i,t)} \dot{a}(a,j,t)^- + \lambda_{i,i'}[W(a,i',t) - W(a,i,t)], \end{aligned} \tag{.5}$$

with $i \in \{T,P,U\}$. The consumption associated with increasing, decreasing or unchanging assets is defined as:

$$c^F(a,i,t) = (\partial_a W^F(a,i,t))^{-1}, \quad c^B(a,i,t) = (\partial_a W^B(a,i,t))^{-1}, \quad c^0(a,i,t) = y(a,i,t) + r(t)a. \tag{.6}$$

Accordingly, the savings decisions become:

$$\dot{a}^F(a,i,t) = y(a,i,t) - r(t)a - c^F(a,i,t), \quad \dot{a}^B(a,i,t) = y(a,i,t) - r(t)a - c^B(a,i,t). \tag{.7}$$

By concavity of the value function,

$$\partial_a W^F(a,i,t) < \partial_a W^B(a,i,t) \to \dot{a}^F(a,i,t) < \dot{a}^B(a,i,t). \tag{.8}$$

As standard, one defines 3 cases:

$$I^B = \{(a,i,t) : \dot{a}^B < 0\}; \quad I^F = \{(a,i,t) : 0 < \dot{a}^F\}; \qquad I^0 = \{(a,i,t) : \dot{a}^F < 0 < \dot{a}^B\}, \tag{.9}$$

representing a certain decreasing in assets, a certain increasing in assets of an ambiguous situation. Then, I implement the upwinding scheme, with the final policies as:

$$\begin{aligned} c(a,i,t) =& \mathbb{1}_{\{I^F\}} c^F(a,i,t) + \mathbb{1}_{\{I^B\}} c^B(a,i,t) + \mathbb{1}_{\{I^0\}} c^0(a,i,t), \\ \dot{a}(a,i,t) =& y(a,i,t) - r(t)a - c(a,i,t). \end{aligned} \tag{.10}$$

Let l index to represent the iterations of the value function. In matrix form, the HJB equation can be written as:

$$\frac{W^{l+1} - W^l}{\Delta} = u^l + (\mathbf{A}^l + \mathbf{\Lambda}^l) W^{l+1} - \rho W^l \tag{.11}$$

where $\mathbf{A}^l$ represents the transition matrix along the asset dimension. It is defined as:

$$\mathrm{A}^l = \begin{bmatrix} \mathbf{A_P} & 0 & 0 \\ 0 & \mathbf{A_T} & 0 \\ 0 & 0 & \mathbf{A_U} \end{bmatrix}, \quad \mathbf{A_i} = \begin{bmatrix} y(1,i,t) & z(1,i,t) & 0 & 0 & \dots & 0 \\ x(2,i,t) & y(2,i,t) & z(2,i,t) & 0 & \dots & 0 \\ 0 & x(3,i,t) & y(3,i,t) & z(3,i,t) & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & x(150,i,t) & y(150,i,t) \end{bmatrix}, \quad \begin{cases} x(a,i,t) = \frac{-[\dot{(a)}(a,i,t)^-]}{\Delta a} \\ z(a,i,t) = \frac{-[\dot{(a)}(a,i,t)^+]}{\Delta a} \\ y(a,i,t) = -x(a,i) - z(a,i,t). \end{cases} \tag{.12}$$

The matrix $\mathbf{\Lambda}^l$ represents the transition matrix on the employment dimmension, and it is defined as:

$$\Lambda^l = \begin{bmatrix} \mathbf{\Lambda_{P,P}} & \mathbf{0} & \mathbf{\Lambda_{P,U}} \\ \mathbf{\Lambda_{T,P}} & \mathbf{\Lambda_{T,T}} & \mathbf{\Lambda_{T,U}} \\ \mathbf{0} & \mathbf{\Lambda_{U,T}} & \mathbf{\Lambda_{U,U}} \end{bmatrix} \tag{.13}$$

where $\mathbf{\Lambda_{i,i'}}$ is a diagonal matrix with the flow probability of transitioning from i to i' along the diagonal, given the level of leverage of the particular assignment. Accordingly, the set of flow probabilities are given by the functions:

$$\begin{cases} \mathbf{\Lambda_{P,P}}(m,m) = -\sigma_P[\xi(m,P,t)] \\ \mathbf{\Lambda_{P,U}}(m,m) = \sigma_P[\xi(m,P,t)] \end{cases}, \quad \begin{cases} \mathbf{\Lambda_{T,P}}(m,m) = \varphi[\xi(m,P,t)] \\ \mathbf{\Lambda_{T,T}}(m,m) = -\varphi[\xi(m,P,t)] - \sigma_T[\xi(m,T,t)] \\ \mathbf{\Lambda_{T,U}}(m,m) = \sigma_T[\xi(m,T,t)] \end{cases}, \\ \begin{cases} \mathbf{\Lambda_{U,T}}(m,m) = \theta q(\theta) h[\xi(m,T,t)] \\ \mathbf{\Lambda_{U,U}}(m,m) = -\theta q(\theta) h[\xi(m,T,t)] \end{cases}, \tag{.14}$$

with m representing the positioning in the grid of assets.

Step 7. Stationary distribution of workers. I solve the discretized version of the Kolmogorov forward equations as:

$$(\mathbf{A}^l + \mathbf{\Lambda}^l)' g = 0. \tag{.15}$$

Given this equation has infinitely many solutions, the solution is pinned down by the fact that g is a density and thus integrate to 1. So as standard, I fix the first row of $(\mathbf{A}^l + \mathbf{\Lambda}^l)'$ to be $[1, 0, \ldots, 0]$ and then rescale the density g to integrate to 1.

Step 8. Solve the firm's problem. The discretized HJB equation for the firm is solved through the system given by:

$$\begin{aligned} \frac{J^{l+1} - J^l}{\Delta} =& \ddot{z} F(\ddot{k}) - r_l(t)\ddot{k} - w(a,j,t) - \sigma_j(\xi(m,j,t)) S_j + \\ & + \mathbf{A}^l_j - (r(t) + \tilde{\sigma}[\xi(m,j,t)] J^{l+1}, \quad j \in \{T, P\}, \\ \tilde{\sigma}[\xi(m,j,t)] =& \begin{cases} \tilde{\sigma}[\xi(m,P,t)] = \sigma_P[\xi(m,P,t)]; \\ \tilde{\sigma}[\xi(m,T,t)] = \sigma_T[\xi(m,T,t)] + \varphi[\xi(m,P,t)]. \end{cases} \end{aligned} \tag{.16}$$

Step 9. Evaluate the Free Entry condition:

$$FE = -\gamma + q[\theta(t)] \sum_a J(a,T,t) \frac{g(a,U,t)}{u(t)} \Delta a. \tag{.17}$$

If FE is positive, more firms enter the market of temporary jobs and thus θ will have to increase. If negative, more firms leave or not enter the market and thus θ will be decreased. Accordingly, the function that adjusts the guess is given by:

$$\theta^{l+1} = \theta^l + \Delta_\theta FE^l, \tag{.18}$$

where Δ_θ regulates the adjustment of the guess.

Step 10. Evaluate the asset clearing condition. The asset clearing condition is given by:

$$\begin{aligned} AD =& \sum_a \sum_{i \in \{T,P,U\}} a g(a,i,t) \Delta a - \vartheta \Big[\sum_a \sum_{i \in \{T,P,U\}} k(a,i,t) g(a,i,t) \Delta a + \\ & \frac{\sum_a \sum_{j \in \{T,P\}} [\ddot{z} F(\ddot{k}) - r_l(t)\ddot{k} - w(a,j,t)] g(a,i,t) \Delta a - \gamma v(t)}{r(t)} \Big], \end{aligned} \tag{.19}$$

where $v(t)$ is calculated given $\theta(t) = \frac{v(t)}{u(t)}$, and:

$$u(t) = \sum_a [g(a, T, t) + g(a, P, t)]\Delta a. \tag{.20}$$

If AD^l is positive, then the guess on $r(t)$ is too low and $r(t)$ should be increased. If AD^l is negative then the guess on $r(t)$ is too high. Accordingly, I adjust it following:

$$r(t)^{l+1} = r(t)^l + \Delta r.AD^l, \tag{.21}$$

with Δr regulating the speed of adjustment of the guess.

Step 11. Update the wages. Update wages following the results of appendix A.

Step 12. Update θ and $r(t)$. Update θ and $r(t)$ until there is convergence from step 6 onward.

D Algorithm of Transitional Dynamics

The algorithm to describe the transitional dynamics given the aggregate productivity shock is standard, as described in Bardóczy [2017]. The economy starts from the stationary equilibrium and returns to it. The algorithm iterates backwards the forward looking variables as prices and value functions, and iterates states forwards. The fixed point theorem algorithm is implemented over p_0 and the sequences of $\{r(t), \theta(t)\}_t$. The algorithm needs to guess the equity price on impact, p_0 as it implies a revaluation of the assets of the agents in the economy. In detail, the algorithm entails solving the following steps:

Step 1. Solve the stationary equilibrium. The stationary equilibrium described in appendix C will provide the initial and terminal description of the economy.

Step 2. Set up a grid for time steps. The algorithm defines $N - 1$ time steps from N time points where the economy will be evaluated. Accordingly, we have:

$$t_n = \begin{cases} 0, & n = 1 \\ \sum_{m=1}^{N-1} \Delta t_m & n = 2, \dots, N \end{cases} . \tag{.1}$$

The grid is defined as a non-uniform grid, that gets wider as n increases. As standard, the objective is to have a dense map of points right after the initial shock, which then gets sparser as the time elapses and consequently the transition becomes smoother towards the stationary equilibrium.

Step 3. Define the aggregate shock process. The aggregate shock and the subsequent transition path of productivity towards stationary equilibrium levels is given by:

$$z_t = 1 + (z_0 - 1)e^{-0.3t}. \tag{.2}$$

Step 4. Guess the path for tightness $\theta(\mathbf{t})$ and the interest rate $\mathbf{r}(\mathbf{t})$. The guess for theta allows for the definition of:

$$\theta(t)q[\theta(t)] = \chi\theta(t)^{1-\eta}; \quad q[\theta(t)] = \chi\theta(t)^{-\eta}. \tag{.3}$$

With the interest rate guess, the capital variables are also defined as:

$$k(a,T,t) = \left[\frac{\alpha z(a,T,t)}{r_l(t)}\right]^{\frac{1}{1-\alpha}}; \quad k(a,P,t) = \left[\frac{\alpha z(a,P,t)}{r_l(t)}\right]^{\frac{1}{1-\alpha}}. \tag{.4}$$

Step 5. Iterate the HJB equations. The idea is to iterate the HJB equations backwards. So start from the terminal conditions, $W^N = W$ and $J^N = J$, where J and W corresponds to the HJB of the stationary equilibrium. Then, the method is analogous to the stationary equilibrium algorithm and solves the following system:

$$\begin{aligned}
\frac{W^n - W^{n+1}}{\Delta t_n} =& u^{n+1} + (\mathbf{A}^{n+1} + \mathbf{\Lambda}^l)W^n - \rho W^{n+1} \\
\frac{J^n - J^{n+1}}{\Delta} =& [\ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} - w(a,j,t) - \sigma_j(\ddot{\xi}_j)S_j]^{n+1} + \mathbf{A}_j^{n+1} - [(r(t) + \tilde{\sigma}(\ddot{\xi}_j)]^{n+1}J^n, \quad j \in \{T,P\}, \\
\tilde{\sigma}[\ddot{\xi}_j]^{n+1} =& \begin{cases} [\tilde{\sigma}(\ddot{\xi}_P)]^{n+1} = [\sigma_P(\ddot{\xi}_P)]^{n+1}; \\ [\tilde{\sigma}(\ddot{\xi}_T)]^{n+1} = [\sigma_T(\ddot{\xi}_T) + \varphi(\ddot{\xi}_P)]^{n+1}. \end{cases}
\end{aligned} \tag{.5}$$

Step 6. Adjust wages. In the standard transitional dynamics algorithm, wages are subjected to downward nominal wage rigidity. Accordingly, we have that wages are adjusted given:[1]

[1] As in step 5 the $\ddot{\bullet}$ variables refer to period n+1. For notation ease I don't make it explicit in the formulae as it was done for step 5 with $[\bullet]^{n+1}$.

$$\begin{aligned}\tilde{w}(a,T,n+1) =& \frac{\beta\Big[\rho+\varphi(\ddot{\xi}_P)+\sigma_T(\ddot{\xi}_T)\Big]\Big\{\ddot{z}F(\ddot{k})-r_l(n+1)\ddot{k}+\partial_a J(a,T,n+1)\Big(r(n+1)a-\ddot{c}\Big)+\partial_t J(a,T,n+1)\Big\}}{[1-\beta][r(n+1)+\varphi(\ddot{\xi}_P)+\sigma_T(\ddot{\xi}_T)]\partial_a W(a,P,n+1)+\beta[\rho+\varphi(\ddot{\xi}_P)+\sigma_T(\ddot{\xi}_T)][1-\partial_a J(a,T,n+1)]} \\ &- \frac{(1-\beta)\Big[r(n+1)+\varphi(\ddot{\xi}_P)+\sigma_T(\ddot{\xi}_T)\Big]\Big\{u(\ddot{c})+\partial_a W(a,P,n+1)\Big(r(n+1)a-\ddot{c}\Big)+\varphi(\ddot{\xi}_P)W(a,P,n+1)}{[1-\beta][r(n+1)+\varphi(\ddot{\xi}_P)+\sigma_T(\ddot{\xi}_T)]\partial_a W(a,P,n+1)+\beta[\rho+\varphi(\ddot{\xi}_P)+\sigma_T(\ddot{\xi}_T)][1-\partial_a J(a,T,n+1)]} \\ & \frac{-[\rho+\varphi(\ddot{\xi}_P)]W(a,U,n+1)+\partial_t W(a,P,n+1)\Big\}}{[1-\beta][r(n+1)+\varphi(\ddot{\xi}_P)+\sigma_T(\ddot{\xi}_T)]\partial_a W(a,P,n+1)+\beta[\rho+\varphi(\ddot{\xi}_P)+\sigma_T(\ddot{\xi}_T)][1-\partial_a J(a,T,n+1)]} \end{aligned} \tag{.6}$$

$$\begin{aligned}\tilde{w}(a,P,n+1) =& \frac{\beta\Big[\rho+\sigma_P(\ddot{\xi}_P)\Big]\Big\{\ddot{z}F(\ddot{k})-r_l(n+1)\ddot{k}+\partial_a J(a,P,n+1)\Big(r(t)a-\ddot{c}\Big)+r(n+1)S_P+\partial_t J(a,P,n+1)\Big\}}{[1-\beta][r(n+1)+\sigma_P(\ddot{\xi}_P)]\partial_a W(a,P,n+1)+\beta[\rho+\sigma_P(\ddot{\xi}_P)][1-\partial_a J(a,P,n+1)]} - \\ & \frac{(1-\beta)\Big[r(n+1)+\sigma_P(\ddot{\xi}_P)\Big]\Big\{u(\ddot{c})+\partial_a W(a,P,n+1)\Big(r(n+1)a-\ddot{c}\Big)-\rho W(a,U,n+1)+\partial_t W(a,P,n+1)\Big\}}{[1-\beta][r(n+1)+\sigma_P(\ddot{\xi}_P)]\partial_a W(a,P,n+1)+\beta[\rho+\sigma_P(\ddot{\xi}_P)][1-\partial_a J(a,P,n+1)]}.\end{aligned}$$

$$w(a,j,n+1) = \begin{cases}\tilde{w}(a,j,n+1) & \text{if } \tilde{w}(a,j,n+1) \geq \lambda w(a,j,n) \\ w(a,j,n) & \text{if } \tilde{w}(a,j,n+1) < \lambda w(a,j,n)\end{cases}, \quad j \in \{T,P\}, \tag{.7}$$

If in a particular exercise wages are not subjected to downward nominal wage rigidity then it is simply a matter to relax the constraint imposed in equation (7).

Step 7. Revalue assets in the after the initial shock. Let p represent the price of equity in the stationary equilibrium and p_0 the price on impact. An agent that owns s_i share of the total assets in the economy, $a_i = s_i(K+p)$, will have its assets updated on impact as:

$$a_i' = s_i(K+p_0) = a_i\Big[1+\frac{p_0-p}{K+p}\Big]. \tag{.8}$$

Further, assume that agents can't do anything about the revaluation, and thus $g_0(a',i) = g(a,i)$, where $i = \{T,P,U\}$. The g_0 density at old grid points $\{a_i\}$ is found be interpolation. I follow the insight of Bardóczy [2017] and interpolate directly g by resulting to MATLAB's *pchip* method, and then rescale it.

Step 8. Iterate the Kolmogorov forward equations. Given the revalued stationary distribution, after the reassessment of prices, I iterate the Kolmogorov forward equations. Accordingly, using the implicit method, it consists in solving:

$$\frac{g_{n+1}-g_n}{\Delta t_n} = [\mathbf{A}^n+\mathbf{\Lambda}^n]' g_{n+1}. \tag{.9}$$

Step 9. Calculate unemployment and vacancies. So given the iterated distributions, we have:

$$u(n) = \sum_a g(a, U, n)\Delta a, \quad v(n) = \theta(n)u(n). \tag{.10}$$

Step 10. Evaluate the free entry condition. The set of free entry conditions along the transitional dynamics is given by:

$$FE_n = -\gamma + q[\theta(t)]_n \sum_a J(a, T, n)\frac{g(a, U, n)}{u(n)}\Delta a. \tag{.11}$$

Step 11. Evaluate the asset market clearing condition. The arbitrage condition implies that one can solve the price of equity backwards. Thus one obtains:

$$\frac{p_{n+1} - p_n}{\Delta t_n} = p_n(r(n) - d(n)), \tag{.12}$$

where $d(n)$ corresponds to the dividends, given by:

$$\begin{aligned} d(n) =& (1-\psi)\sum_a \Big[\ddot{z}F(\ddot{k}) - r_l(n)\ddot{k} - w(a, T, n)\Big] g(a, T, n)\Delta a + \\ &+ (1-\psi)\sum_a \Big[\ddot{z}F(\ddot{k}) - r_l(n)\ddot{k} - w(a, P, n)\Big] g(a, P, n)\Delta a - \gamma v. \end{aligned} \tag{.13}$$

Given the path of prices, the capital in each firm and the distribution of assets, we have:

$$\begin{aligned} AD_n =& \sum_a \sum_{i\in\{T,P,U\}} a g(a, i, n)\Delta a - \vartheta\Big[\sum_a \sum_{i\in\{T,P,U\}} k(a, i, n) g(a, i, n)\Delta a + \\ & \frac{\sum_a \sum_{j\in\{T,P\}}[\ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} - w(a, j, n)] g(a, i, n)\Delta a - \gamma v(n)}{r(n)}\Big]. \end{aligned} \tag{.14}$$

Step 12. Adjust $\mathbf{p_0}$ and $\{\mathbf{r(n)}, \theta(\mathbf{n})\}_{\mathbf{n}}^{\mathbf{N-1}}$. Accordingly, adjust: (a) the price on impact based on the no-arbitrage condition; (b) the labour market tightness based on the free entry condition; and (c) the interest rate based on the excess demand for assets condition. Consequently, consider:

$$\begin{aligned} p_0^{l+1} =& \Delta_p p_0 + (1-\Delta_p)p_0^l \\ r(n)^{l+1} =& r(n)^l + \Delta_r AD_n \\ \theta(n)^{l+1} =& \theta(n)^l + \Delta_{FE} FE_n. \end{aligned} \tag{.15}$$

The iterate from step 5 onward, until reaching convergence.

E Details on the algorithms for policy analysis.

Algorithm for transitional dynamics under flexible wages. The algorithm to estimate the transitional dynamics under flexible wages is identical to the algorithm in appendix D, with the sole exception residing in relaxing equation (7), which has no applicability.

Altogether, the transitional dynamics are therefore estimated in a fully comparable way with the downward nominal rigid wages. Paramount to this approach is the fact that the stationary equilibrium under both scenarios is identical, as the downward nominal wage rigidity is not binding in the equilibrium.

Algorithm for transitional dynamics under the unemployment benefit transfer policy. This algorithm differs from the one presented in appendix D in two major ways. First, during the implementation period of the subsidy, wages, leverage and profits are adjusted to include the subsidy. Secondly, the algorithm needs to ensure the budget balancing of the policy.

Notice that the discretization of the algorithm implies that the time steps are matched to quarters. Accordingly, the subsidy is provided in the periods $t \in [1;5]$, corresponding to 4 quarters of implementation and 1 quarter of implementation lag.

The profits become:

$$\tilde{\eta}(a,P,t) = \left[\ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} - (1-\tau^{ub})w(a,P,t)\right], \text{ if } t \in [1,5], \tag{.16}$$

and the leverage of the firm becomes:

$$\xi(a,P,t) = \frac{r_l(t)}{z(a,P,t)F[k(a,P,t)] - (1-\tau^{ub})w(a,P,t)}, \text{ if } t \in [1,5]. \tag{.17}$$

Accordingly, the step 5 of the standard algorithm is duly adjusted, namely the HJB equations for the value of the job. Noteworthy, the HJB equations for the workers do not change.

The set of equations defining the permanent contract wages during the implementation period becomes:

$$\begin{aligned}
&\beta\Big[J(a,P,t)+S_P\Big]=(1-\beta)\Big[W(a,P,t)-W(a,U,t)\Big];\\
&r(t)J(a,P,t)=\ddot{z}F(\ddot{k})-r_l(t)\ddot{k}-(1-\tau^{ub})\tilde{w}(a,P,t)+\partial_a J(a,P,t)\dot{a}(a,P,t)+\\
&+\sigma_P(\ddot{\xi}_P)[-S_P-J(a,P,t)]+\partial_t J(a,P,t);\\
&\rho W(a,P,t)=u(\ddot{c})+\partial_a W(a,P,t)\dot{a}(a,P,t)+\sigma_P(\ddot{\xi}_P)\Big[W(a,U,t)-W(a,P,t)\Big]+\partial_t W(a,P,t)\\
&\dot{a}(a,P,t)=\tilde{w}(a,P,t)+(r_l-\delta)a-\ddot{c}
\end{aligned} \tag{.18}$$

with the resulting permanent contract wages, relevant for step 6 defined as:

$$\begin{aligned}
\tilde{w}(a,P,t)=&\frac{\beta\Big[\rho+\sigma_P(\ddot{\xi}_P)\Big]\Big\{\ddot{z}F(\ddot{k})-r_l(t)\ddot{k}+\partial_a J(a,P,t)\Big(r(t)a-\ddot{c}\Big)+r(t)S_P+\partial_t J(a,P,t)\Big\}}{[1-\beta][r(t)+\sigma_P(\ddot{\xi}_P)]\partial_a W(a,P,t)+\beta[\rho+\sigma_P(\ddot{\xi}_P)][(1-\tau^{ub})-\partial_a J(a,P,t)]}-\\
&\frac{(1-\beta)\Big[r(t)+\sigma_P(\ddot{\xi}_P)\Big]\Big\{u(\ddot{c})+\partial_a W(a,P,t)\Big(r(t)a-\ddot{c}\Big)-\rho W(a,U,t)+\partial_t W(a,P,t)\Big\}}{[1-\beta][r(t)+\sigma_P(\ddot{\xi}_P)]\partial_a W(a,P,t)+\beta[\rho+\sigma_P(\ddot{\xi}_P)][(1-\tau^{ub})-\partial_a J(a,P,t)]},\ \text{ if } t\in[1,5].
\end{aligned} \tag{.19}$$

The temporary contract wages are unchanged.

The remainder of the steps from step 7 to step 12 are unchanged, other than for the adjustment of dividends during the implementation period, where it becomes:

$$\begin{aligned}
d(n)=&(1-\psi)\sum_a\Big[\ddot{z}F(\ddot{k})-r_l(n)\ddot{k}-w(a,T,n)\Big]g(a,T,n)\Delta a+\\
&+(1-\psi)\sum_a\Big[\ddot{z}F(\ddot{k})-r_l(n)\ddot{k}-(1-\tau^{ub})w(a,P,n)\Big]g(a,P,n)\Delta a-\gamma v,\ \text{ if } t\in[1,5].
\end{aligned} \tag{.20}$$

In step 12, the convergence needs to be supplemented with the balanced budget criteria. The savings and costs from the policy are given by:

$$\begin{aligned}
ub^{sav}&=\sum_{n=1}^{N}\Big(u(n)-u(n)^{DRWR}\Big)b;\\
ub^{exp}&=\sum_{n \text{ if } t\in[1,5]}\sum_a\tau^{ub}\Big(w(a,P,n)g(a,P,n)\Big)\Delta a.
\end{aligned} \tag{.21}$$

The convergence of the fixed point algorithm is supplemented with the balanced budget condition as:

$$\begin{aligned}
B^{ub,l}=&ub^{sav,l}-ub^{exp,l};\\
\tau^{ub,l+1}=&\tau^{ub,l}+\Delta_{ub}B^{ub,l}.
\end{aligned} \tag{.22}$$

Intuitively, notice that if the policy is generating savings, then the *ad-valorem* rate of the subsidy can be increased, whereas if the policy is generating deficits the rate should be reduced.

The algorithm iterates from step 5, with the referred changes until a convergence is reached.

Algorithm for transitional dynamics under the income tax and firm subsidy transfer. In this policy the budget is permanently balanced, as the policy consists in an instantaneous transfer between workers and firms. However, given this property, just ensuring budget balance do not pin point a single calibration of the policy. For that purpose, as presented in the text, the policy maker attempts to select the *ad-valorem* rate of transfer that minimizes the disturbance in employment and unemployment flows.

Consequently, the algorithm to run this policy consists in an inner loop algorithm, which consists in an adjusted version of the one proposed in appendix D, whereas the outer loop consists in a minimization algorithm of the policy objective, by selecting the *ad-valorem* rate of transfer.

As in every policy algorithm, the discretization of the algorithm implies that the time steps are matched to quarters. Accordingly, the subsidy is provided in the periods $t \in [1;5]$, corresponding to 4 quarters of implementation and 1 quarter of implementation lag. Thus, by comparison with the algorithm in appendix D, the inner algorithm is adjusted to incorporate the policy. The profits become:

$$\tilde{\eta}(a,P,t) = \Big[\ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} - (1-\tau^{ts})w(a,P,t)\Big], \ \text{ if } \ t \in [1,5], \tag{.23}$$

and the leverage of the firm becomes:

$$\xi(a,P,t) = \frac{r_l(t)k(a,P,t)}{z(a,P,t)F[k(a,P,t)] - (1-\tau^{ts})w(a,P,t)}, \ \text{ if } \ t \in [1,5]. \tag{.24}$$

Accordingly, the step 5 of the standard algorithm is duly adjusted, namely the HJB equations for the value of the job. Noteworthy, the HJB equations for the workers do not change, even though the income of the individuals is adjusted so that:

$$y(a,i,t) = \begin{cases} (1-\tau^{ts})w(a,P,t) + \eta(a,P,t) & \text{if } i = P \\ w(a,T,t) + \eta(a,T,t) & \text{if } i = T \\ b & \text{if } i = U \end{cases} \cdot \tag{.25}$$

The set of equations defining the permanent contract wages during the implementation period becomes:

$$
\begin{aligned}
&\beta\Big[J(a,P,t)+S_P\Big]=(1-\beta)\Big[W(a,P,t)-W(a,U,t)\Big];\\
&r(t)J(a,P,t)=\ddot{z}F(\ddot{k})-r_l(t)\ddot{k}-(1-\tau^{ts})\tilde{w}(a,P,t)+\partial_a J(a,P,t)\dot{a}(a,P,t)+\\
&+\sigma_P(\ddot{\xi}_P)[-S_P-J(a,P,t)]+\partial_t J(a,P,t);\\
&\rho W(a,P,t)=u(\ddot{c})+\partial_a W(a,P,t)\dot{a}(a,P,t)+\sigma_P(\ddot{\xi}_P)\Big[W(a,U,t)-W(a,P,t)\Big]+\partial_t W(a,P,t)\\
&\dot{a}(a,P,t)=(1-\tau^{ts})\tilde{w}(a,P,t)+(r_l-\delta)a-\ddot{c}
\end{aligned}
\tag{.26}
$$

with the resulting permanent contract wages, relevant for step 6 defined as:

$$
\begin{aligned}
\tilde{w}(a,P,t)=&\frac{\beta\Big[\rho+\sigma_P(\ddot{\xi}_P)\Big]\Big\{\ddot{z}F(\ddot{k})-r_l(t)\ddot{k}+\partial_a J(a,P,t)\Big(r(t)a-\ddot{c}\Big)+r(t)S_P+\partial_t J(a,P,t)\Big\}}{[1-\beta][r(t)+\sigma_P(\ddot{\xi}_P)](1-\tau^{ts})\partial_a W(a,P,t)+\beta[\rho+\sigma_P(\ddot{\xi}_P)](1-\tau^{ts})[1-\partial_a J(a,P,t)]}-\\
&\frac{(1-\beta)\Big[r(t)+\sigma_P(\ddot{\xi}_P)\Big]\Big\{u(\ddot{c})+\partial_a W(a,P,t)\Big(r(t)a-\ddot{c}\Big)-\rho W(a,U,t)+\partial_t W(a,P,t)\Big\}}{[1-\beta][r(t)+\sigma_P(\ddot{\xi}_P)](1-\tau^{ts})\partial_a W(a,P,t)+\beta[\rho+\sigma_P(\ddot{\xi}_P)](1-\tau^{ts})[1-\partial_a J(a,P,t)]},\quad \text{if}\ \ t\in[1,5].
\end{aligned}
\tag{.27}
$$

The temporary contract wages are unchanged.

The remainder of the steps from step 7 to step 12 are unchanged, other than for the adjustment of dividends during the implementation period, where it becomes:

$$
\begin{aligned}
d(n)=&(1-\psi)\sum_a\Big[\ddot{z}F(\ddot{k})-r_l(n)\ddot{k}-w(a,T,n)\Big]g(a,T,n)\Delta a+\\
&+(1-\psi)\sum_a\Big[\ddot{z}F(\ddot{k})-r_l(n)\ddot{k}-(1-\tau^{ts})w(a,P,n)\Big]g(a,P,n)\Delta a-\gamma v,\quad \text{if}\ \ t\in[1,5].
\end{aligned}
\tag{.28}
$$

Given these adjustments to the inner algorithm, one obtains the employment and unemployment sequences for each level of τ^{ts}. Then, resorting to the *Fminsearch* procedure of Matlab, I pinpoint the rate that minimizes the objective function of the policy maker, given by:

$$
\tau^{ts}\in[0,1]\sum_{n-1}^{N}e^{-\rho n}\Big\{\Big|e_T(n)+e_P(n)-e^s_T-e^s_P\Big|+\Big|u(n)-u^s\Big|\Big\}dn. \tag{.29}
$$

where e^s_P, e^s_T and u^s correspond to the stationary equilibrium levels of employment and unemployment.

Altogether, this last step solves the outer loop ensuring a policy that is budget balanced by definition, and implemented with the objective to minimize employment fluctuations over the adjustment. Intuitively, it consists in computing the inner algorithm sufficient times to search over the grid of admissible *ad-valorem* rates the one that most fulfills the objective.

Algorithm for temporary wage flexibility transfer. This policy consists in relaxing the downward nominal wage rigidity for the periods $t \in [1, 5]$. Accordingly, the algorithm is analogous to the one in appendix D with the replacement of equation (7) of the appendix D with equation (36) of the text.

Algorithm for Moratorium in cost of capital transfer. This policy consists in an adjustment of the cost of capital. Critically, during part of the downturn, the policy will temporarily reduce the loan size to the banking institution, providing relief in the leverage ratio perceived by such institution. Then, in the repayment period, the policy will increase the cost of capital to ensure the repayment of the subsidy provided. Given the discretization implemented, the policy is implemented with a quarter of implementation lag, and it is calibrated to consist in a 1-year subsidy, with a 5-year repayment. The interest rate of the loan is set to ensure the budget-balance of the policy.

As the policy is budget-balanced by construction, it requires the calibration of the size of the initial of the subsidy to be selected given the objective of the policy maker. As in the income tax firm subsidy transfer case, I will consider the policy maker intends to minimize the employment fluctuations over the adjustment.

Accordingly, the algorithm of this policy is as defined in appendix D with the required changes to ensure the incorporation of the subsidy, the definition of the interest rate associated with the repayment, and the outer loop to define the optimal size of the credit line to ensure the minimization of employment fluctuations. The step 4 of the algorithm is enlarged to calculate the specificity of the subsidy and repayment profits. For the implementation period, the firms receive a flow as:

$$LC(a, P, t) = \tau^m r_l^s K(a, P)^s, \quad \text{if} \quad t \in [1, 5], \tag{.30}$$

where $r_l^s K(a, P)^s$ corresponds to the cost of capital the firm was incurring in the stationary equilibrium. By the end of the implementation period, the total amount provided for the firm corresponds to:

$$TL(a, P) = \tau^m r_l^s K(a, P)^s \left[\frac{1 - e^{-4r_{loan}}}{r_{loan}} \right] e^{4r_{loan}}, \tag{.31}$$

with r_{loan} corresponding to the quarterly interest rate considered to ensure the budget balance of the policy, and the factor 4 maps into the idea that the cost of capital ease takes place for 4 quarters. Then the loan is paid back in installments for 5 years. Accordingly, the installments become:

$$LC(a,P,t) = -\left[\tau^m r_l^s K(a,P)^s \left[\frac{1-e^{-4r_{loan}}}{r_{loan}}\right] e^{4r_{loan}}\right]\left[\frac{1-e^{-20r_{loan}}}{r_{loan}}\right]^{-1} \quad \text{if} \quad t \in (5,25]. \tag{.32}$$

Notice that for $t > 25$ then $LC(a,P,t) = 0$. Given the policy details as defined, the capital variables are unchanged, as it results from first order condition of the capital. The profit becomes:

$$\tilde{\eta}(a,P,t) = \left[\ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} + LC(a,P,t) - w(a,P,t)\right], \tag{.33}$$

and the leverage of the firm becomes:

$$\xi(a,P,t) = \frac{r_l(t)k(a,P,t) - LC(a,P,t)}{z(a,P,t)F[k(a,P,t)] - w(a,P,t)}. \tag{.34}$$

The guess of tightness and interest rate are as defined in step 4. Further, step 5 is unchanged, other than for the adjustment of profits in the HJB equation of the value of the job, and the changes in the leverage of the permanent contract jobs.

Regarding wages, the wages of the permanent contracts are adjusted to consider the existence of the credit line. Accordingly one have the relevent set of equations as:

$$\begin{aligned}
&\beta\left[J(a,P,t) + S_P\right] = (1-\beta)\left[W(a,P,t) - W(a,U,t)\right];\\
&r(t)J(a,P,t) = \ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} + LC(a,P,t) - \tilde{w}(a,P,t) + \partial_a J(a,P,t)\dot{a}(a,P,t) +\\
&+ \sigma_P(\ddot{\xi}_P)[-S_P - J(a,P,t)] + \partial_t J(a,P,t);\\
&\rho W(a,P,t) = u(\ddot{c}) + \partial_a W(a,P,t)\dot{a}(a,P,t) + \sigma_P(\ddot{\xi}_P)\left[W(a,U,t) - W(a,P,t)\right] + \partial_t W(a,P,t)\\
&\dot{a}(a,P,t) = \tilde{w}(a,P,t) + (r_l - \delta)a - \ddot{c};
\end{aligned} \tag{.35}$$

which results into permanent contract wages as:

$$\begin{aligned}
\tilde{w}(a,P,t) = &\frac{\beta\left[\rho + \sigma_P(\ddot{\xi}_P)\right]\left\{\ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} + LC(a,P,t) + \partial_a J(a,P,t)\left(r(t)a - \ddot{c}\right) + r(t)S_P + \partial_t J(a,P,t)\right\}}{[1-\beta][r(t) + \sigma_P(\ddot{\xi}_P)]\partial_a W(a,P,t) + \beta[\rho + \sigma_P(\ddot{\xi}_P)][1 - \partial_a J(a,P,t)]} -\\
&\frac{(1-\beta)\left[r(t) + \sigma_P(\ddot{\xi}_P)\right]\left\{u(\ddot{c}) + \partial_a W(a,P,t)\left(r(t)a - \ddot{c}\right) - \rho W(a,U,t) + \partial_t W(a,P,t)\right\}}{[1-\beta][r(t) + \sigma_P(\ddot{\xi}_P)]\partial_a W(a,P,t) + \beta[\rho + \sigma_P(\ddot{\xi}_P)][1 - \partial_a J(a,P,t)]}.
\end{aligned} \tag{.36}$$

From steps 7 to step 11 the algorithm remains analogous to the one presented in appendix D, other than for the adjustment of dividends during the implementation period, where it becomes:

$$d(n) =(1-\psi)\sum_a\Big[\ddot{z}F(\ddot{k})-r_l(n)\ddot{k}-w(a,T,n)\Big]g(a,T,n)\Delta a+$$
$$+(1-\psi)\sum_a\Big[\ddot{z}F(\ddot{k})-r_l(n)\ddot{k}+LC(a,P,n)-w(a,P,n)\Big]g(a,P,n)\Delta a-\gamma v,\ \text{ if } t\in[1,5]. \tag{.37}$$

Regarding step 12, one needs to enlarge the convergence adjustment conditions to include the definition of the interest rate of the policy. Accordingly, consider that:

$$ADT(r_{loan}) = \sum_{n \text{ if } t\in[1,5]}\sum_a \tau^m r_l^s K(a,P)^s e^{-r_{loan}t} g(a,P,t)\Delta a, \tag{.38}$$

corresponds to the aggregate level of the subsidy provided to firms, where as:

$$ADR(r_{loan}) = \sum_{n \text{ if } t\in(5,25]}\sum_a \left[\tau^m r_l^s K(a,P)^s \left[\frac{1-e^{-4r_{loan}}}{r_{loan}}\right]e^{4r_{loan}}\right]\left[\frac{1-e^{-20r_{loan}}}{r_{loan}}\right]^{-1} e^{-r_{loan}t} g(a,P,t)\Delta a, \tag{.39}$$

coorresponds to the aggregate level of repayments. Accordingly, the policy balance corresponds to:

$$BP(r_{loan}) = ADR(r_{loan}) - ADT(r_{loan}). \tag{.40}$$

Given this balance, the r_{loan} is adjusted according to:

$$r_{loan}^{l+1} = r_{loan}^{l} - \Delta_r BP^l. \tag{.41}$$

Notice that when the policy results in savings, the interest rate of the policy can be reduced, whereas when it results in expenditure, the interest rate of the policy needs to be increased. The resulting interest rate will therefore include the impact of the employment and unemployment flows in the revenues and expenditure of the policy.

Given these adjustments to the inner algorithm, one obtains the employment and unemployment sequences for each level of τ^m - a measure of the magnitude of the flow subsidy in the level of loans of the permanent job firms. Then, resorting to the *Fminsearch* procedure of Matlab, I pinpoint the rate that minimizes the objective function of the policy maker, given by:

$$\tau^m\in[0,1] \sum_{n-1}^{N} e^{-\rho n}\Big\{\Big|e_T(n)+e_P(n)-e_T^s-e_P^s\Big|+\Big|u(n)-u^s\Big|\Big\}dn. \tag{.42}$$

where e_P^s, e_T^s and u^s correspond to the stationary equilibrium levels of employment and unemployment.

Altogether, as in the case of the income tax firm subsidy transfer, this last step solves the outer loop ensuring a policy that is budget balanced by definition, and implemented with the objective to minimize employment fluctuations over the adjustment.

Algorithm for joint transfer policy. This policy combines the moratorium policy, the income tax and firm subsidy transfer policy and the unemployment benefit transfer policy. Accordingly, the algorithm combines the features described in the previous algorithms for the relevant policies.

Accordingly, for the implementation period of the policy, the moratorium policy implies the firms receive a flow as:

$$LC(a, P, t) = \tau^m r_l^s K(a, P)^s, \quad \text{if} \quad t \in [1, 5], \tag{.43}$$

where $r_l^s K(a, P)^s$ corresponds to the cost of capital the firm was incurring in the stationary equilibrium. By the end of the implementation period, the total amount of the loan provided for the firm corresponds to:

$$TL(a, P) = \tau^m r_l^s K(a, P)^s \left[\frac{1 - e^{-4r_{loan}}}{r_{loan}}\right] e^{4r_{loan}}, \tag{.44}$$

with r_{loan} corresponding to the quarterly interest rate considered to ensure the budget balance of the policy, and the factor 4 maps into the idea that the cost of capital ease takes place for 4 quarters. Then the subsidy is paid back in installments for 5 years. Accordingly, the installments become:

$$LC(a, P, t) = -\left[\tau^m r_l^s K(a, P)^s \left[\frac{1 - e^{-4r_{loan}}}{r_{loan}}\right] e^{4r_{loan}}\right] \left[\frac{1 - e^{-20r_{loan}}}{r_{loan}}\right]^{-1} \quad \text{if} \quad t \in (5, 25]. \tag{.45}$$

Notice that for $t > 25$ then $LC(a, P, t) = 0$. Given the policy details as defined, the capital variables are unchanged, as it results from first order condition of the capital.

Given the moratorium policy block, the profits also incorporate the other two policies. Thus, the profit becomes:

$$\tilde{\eta}(a, P, t) = \left[\ddot{z}F(\ddot{k}) - r_l(t)\ddot{k} + LC(a, P, t) - (1 - \tau^{ub} - \tau^{ts})w(a, P, t)\right], \tag{.46}$$

and the leverage of the firm becomes:

$$\xi(a, P, t) = \frac{r_l(t)k(a, P, t) - LC(a, P, t)}{z(a, P, t)F[k(a, P, t)] - (1 - \tau^{ub} - \tau^{ts})w(a, P, t)}. \tag{.47}$$

Regarding wages, the wages of the permanent contracts are adjusted to consider the existence of the joint policy. Accordingly one have the relevant set of equations as:

$$\begin{aligned}
&\beta\Big[J(a,P,t)+S_P\Big]=(1-\beta)\Big[W(a,P,t)-W(a,U,t)\Big];\\
&r(t)J(a,P,t)=\ddot{z}F(\ddot{k})-r_l(t)\ddot{k}+LC(a,P,t)-(1-\tau^{ub}-\tau^{ts})\tilde{w}(a,P,t)+\partial_a J(a,P,t)\dot{a}(a,P,t)+\\
&+\sigma_P(\ddot{\xi}_P)[-S_P-J(a,P,t)]\\
&+\partial_t J(a,P,t);\\
&\rho W(a,P,t)=u(\ddot{c})+\partial_a W(a,P,t)\dot{a}(a,P,t)+\sigma_P(\ddot{\xi}_P)\Big[W(a,U,t)-W(a,P,t)\Big]+\partial_t W(a,P,t)\\
&\dot{a}(a,P,t)=(1-\tau^{ts})\tilde{w}(a,P,t)+(r_l-\delta)a-\ddot{c};
\end{aligned} \tag{.48}$$

which results into permanent contract wages as:

$$\begin{aligned}
\tilde{w}(a,P,t)=&\frac{\beta\Big[\rho+\sigma_P(\ddot{\xi}_P)\Big]\Big\{\ddot{z}F(\ddot{k})-r_l(t)\ddot{k}+LC(a,P,t)+\partial_a J(a,P,t)\Big(r(t)a-\ddot{c}\Big)+r(t)S_P+\partial_t J(a,P,t)\Big\}}{[1-\beta][r(t)+\sigma_P(\ddot{\xi}_P)](1-\tau^{ts})\partial_a W(a,P,t)+\beta[\rho+\sigma_P(\ddot{\xi}_P)][(1-\tau^{ts}-\tau^{ub})-\partial_a J(a,P,t)(1-\tau^{ts})]}-\\
&\frac{(1-\beta)\Big[r(t)+\sigma_P(\ddot{\xi}_P)\Big]\Big\{u(\ddot{c})+\partial_a W(a,P,t)\Big(r(t)a-\ddot{c}\Big)-\rho W(a,U,t)+\partial_t W(a,P,t)\Big\}}{[1-\beta][r(t)+\sigma_P(\ddot{\xi}_P)](1-\tau^{ts})\partial_a W(a,P,t)+\beta[\rho+\sigma_P(\ddot{\xi}_P)][(1-\tau^{ts}-\tau^{ub})-\partial_a J(a,P,t)(1-\tau^{ts})]}.
\end{aligned} \tag{.49}$$

From steps 7 to step 11 the algorithm remains analogous to the one presented in appendix D, other than for the adjustment of dividends during the implementation period, where it becomes:

$$\begin{aligned}
d(n)=&(1-\psi)\sum_a\Big[\ddot{z}F(\ddot{k})-r_l(n)\ddot{k}-w(a,T,n)\Big]g(a,T,n)\Delta a+\\
&+(1-\psi)\sum_a\Big[\ddot{z}F(\ddot{k})-r_l(n)\ddot{k}+LC(a,P,n)-w(a,P,n)\Big]g(a,P,n)\Delta a-\gamma v,\ \ \text{if}\ \ t\in[1,5].
\end{aligned} \tag{.50}$$

Regarding step 12, one needs to enlarge the convergence adjustment conditions to include the definition of the interest rate of the moratorum and the definition of the τ^{ub} parameter. Accordingly, consider that:

$$ADT(r_{loan})=\sum_{n\ \text{if}\ t\in[1,5]}\sum_a \tau^m r_l^s K(a,P)^s e^{-r_{loan}t}g(a,P,t)\Delta a, \tag{.51}$$

corresponds to the aggregate level of loans provided to firms, where as:

$$ADR(r_{loan})=\sum_{n\ \text{if}\ t\in(5,25]}\sum_a\Bigg[\tau^m r_l^s K(a,P)^s\Bigg[\frac{1-e^{-4r_{loan}}}{r_{loan}}\Bigg]e^{4r_{loan}}\Bigg]\Bigg[\frac{1-e^{-20r_{loan}}}{r_{loan}}\Bigg]^{-1}e^{-r_{loan}t}g(a,P,t)\Delta a. \tag{.52}$$

Accordingly, the policy balance corresponds to:

$$BP(r_{loan})=ADR(r_{loan})-ADT(r_{loan}). \tag{.53}$$

Given this balance, the r_{loan} is adjusted according to:

$$r_{loan}^{l+1}=r_{loan}^{l}-\Delta_r BP^l. \tag{.54}$$

Notice that when the policy results in savings, the interest of the loan can be reduced, whereas when it results in expenditure, the interest of the loan needs to be increased. The resulting interest rate of the loan will therefore include the impact of the employment and unemployment flows in the revenues and expenditure of the credit line.

Regarding the savings and costs from the unemployment benefit transfer policy are given by:

$$\begin{aligned} ub^{sav} &= \sum_{n=1}^{N} \Big(u(n) - u(n)^{DRWR} \Big) b; \\ ub^{exp} &= \sum_{n \text{ if } t\in[1,5]} \sum_{a} \tau^{ub} \Big(w(a,P,n) g(a,P,n) \Big) \Delta a. \end{aligned} \tag{.55}$$

The convergence of the fixed point algorithm is supplemented with the balanced budget condition as:

$$\begin{aligned} B^{ub,l} =& ub^{sav,l} - ub^{exp,l}; \\ \tau^{ub,l+1} =& \tau^{ub,l} + \Delta_{ub} B^{ub,l}. \end{aligned} \tag{.56}$$

Intuitively, notice that if the policy is generating savings, then the *ad-valorem* rate of the subsidy can be increased, whereas if the policy is generating deficits the rate should be reduced.

Altogether, the convergence criteria entails the adjustment of: (a) the price on impact based on the no-arbitrage condition; (b) the labour market tightness based on the free entry condition; (c) the interest rate based on the excess demand for assets condition; (d) the unemployment benefit subsididy transfer τ^{ub}; and (e) the interest rate of the moratorium, τ^{m}. Consequently, consider:

$$\begin{aligned} p_0^{l+1} =& \Delta_p p_0 + (1-\Delta_p) p_0^l, \\ r(n)^{l+1} =& r(n)^l + \Delta_r AD_n, \\ \theta(n)^{l+1} =& \theta(n)^l + \Delta_{FE} FE_n, \\ \tau^{ub,l+1} =& \tau^{ub,l} + \Delta_{ub} B^{ub,l}, r_{loan}^{l+1} = \quad r_{loan}^{l} - \Delta_r BP^l. \end{aligned} \tag{.57}$$

The iteration of the inner loop of appendix D, from step 5 onward, with the prescribed adjustments in this appendix, allows for the convergence of the inner loop.

The given the convergence of the inner loop algorithm, one obtains the employment and unemployment sequences for each level of $\{\tau^{m}, \tau^{ts}\}$. Then, resorting to the *Fminsearch* procedure of Matlab, I pinpoint the rate that minimizes the objective function of the policy maker, given by:

$$\tau^m\in[0,1],\tau^{ts}\in[0,1] \sum_{n-1}^{N} e^{-\rho n}\left\{\left|e_T(n)+e_P(n)-e_T^s-e_P^s\right|+\left|u(n)-u^s\right|\right\}dn. \qquad (.58)$$

where e_P^s, e_T^s and u^s correspond to the stationary equilibrium levels of employment and unemployment.

Altogether, as in the relevant isolated policies, this last step solves the outer loop ensuring a policy that is budget balanced by definition, and implemented with the objective to minimize employment fluctuations over the adjustment.