




CAPACITY SHARING STRATEGY WITH SUSTAINABLE REVENUE-SHARING CONTRACTS

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Abstract. This paper develops a duopoly model to analyse capacity sharing strategy and the optimal revenue-sharing contract under a two-part tariff and examines the effects of capacity sharing, cost, and sharing charges in three scenarios. The paper uses the two-part tariff method and adds a more realistic assumption of incremental marginal costs to improve the research on capacity sharing strategies. The results show that capacity constraints affect the sustainable development of firms. A sustainable revenue-sharing contract can create a win-win situation for both firms and promote capacity sharing. Capacity sharing, cost, and the revenue-sharing rate have different impacts in different scenarios; the optimal revenue-sharing rate and fixed fee can be determined to maximise the profits of firms that share capacity. However, capacity sharing may not improve social welfare.

Keywords: duopoly, capacity sharing strategy, sustainable revenue-sharing contracts, two-part tariff, win-win development.

JEL Classification: D24, L13.

Introduction

The productive capacity is related to the sustainable development and competitiveness of an enterprise (González-Blanco et al., 2019). A mismatch between supply and demand is a common problem in many markets. Excessive or insufficient productive capacity in off-peak and peak seasons is a major barrier to sustainable development. This problem is particularly prominent in traditional manufacturing, manifesting in issues such as labour shortages and insufficient production capacity in peak seasons, surplus labour in off-seasons, and idle production machinery. One way to solve the problem is to share capacity among firms, which is conducive to revitalising idle resources and realising the sharing of production factors from

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the perspective of sustainable development concepts. The capacity sharing strategy is applied in the manufacturing and aviation fields (Sun et al., 2020; Zhao & Han, 2020; Xie & Han, 2020; Silva et al., 2019). For example, some firms that provide air transportation services have the dual characteristics of product homogeneity and low marginal costs. To address a mismatch between supply and demand before flight departure, competing airlines may share their aircraft seats. The capacity sharing has been seriously discussed in the literature, as evidenced by extensive research conducted from both theoretical and empirical perspectives (Dussauge et al., 2010; Hu et al., 2013; Zhang et al., 2010).

The paper mainly solves the problem of insufficient capacity or overcapacity faced by traditional manufacturing firms. It addresses the mismatch between production capacity supply and demand in both off-peak and peak seasons. Furthermore, it proposes solutions for formulating sustainable revenue-sharing contracts for two firms with different levels of production capacity. A sustainable revenue-sharing contract can create a win-win situation for both firms and allow the contract to continue. If a firm's interests are damaged, the contract is unsustainable. In sustainable revenue-sharing contracts, the profits obtained by both firms after entering the contract will be higher than before. To solve the problem of resource mismatch between insufficient capacity and overcapacity in the manufacturing industry, this study designed revenue-sharing contracts. For example, owing to short-term overcapacity, the automaker Renault shared production capacity with Nissan, thereby achieving a win-win situation. In China, many small firms faced with short-term difficulties have not realised sharing capacity for various reasons, such as underdeveloped sharing strategies. To solve similar problems, most existing models consider simple fixed-charge capacity sharing contracts. However, the model in this study uses the two-part tariff method. In addition, it adds a more realistic assumption of incremental marginal cost to increase the authenticity and availability of the capacity sharing strategy. In practice, it is conducive to the sustainable development of firms in volatile markets.

Driven by these real cases and problems, this study examines sustainable revenue-sharing contracts for capacity sharing among competitors under capacity constraints. We mainly focus on the following issues:

- (1) Under capacity constraints, should two firms choose a capacity sharing strategy?
- (2) If firms adopt a capacity sharing strategy, how should they establish a sustainable revenue-sharing contract, that is, the sharing of charges?
- (3) How does the revenue-sharing contract influence the firms' profits, consumer surplus, and social welfare?

1. Literature review

Numerous studies on capacity sharing have been conducted which provide useful references for this study.

First, there is a large amount of relevant literature on the role of capacity sharing. Nunes et al. (2014) discussed the need for capacity sharing in User-Centric Networks. Moghaddam and Nof (2014) combined demand and capacity sharing with the best matching decisions used during enterprise collaboration. Yu et al. (2015) studied the benefits of sharing produc-

tion capacity for a group of independent companies. Li and Zhang (2015) studied capacity sharing, which can be realised under various market conditions, between pairs of freight forwarders in the shipping market, and find that it can benefit both operators and agents. Roels and Tang (2016) discussed a strategic alliance in which a company shared its manufacturing capabilities with another company. Padilla Tinoco et al. (2017) studied the benefits of collaborative shipping, whereby two shippers bundled their shipments to share the same vehicle. Feng et al. (2017) discussed capacity sharing for an integrated secondary supply chain and then developed an effective algorithm to find the optimal solution. Qi et al. (2019) combined sourcing from a shared supplier and capacity reservation and analysed the effects. Melo et al. (2019) concluded that municipalities can take advantage of a sharing economy context. Gorelick et al. (2019) explored treatment and capacity sharing agreements between water utilities. Fang and Wang (2020) investigated competition strategies adopted under two widely used capacity sharing contracting mechanisms. Wei and Zhang (2021) analysed the relationship between capacity sharing and postponement flexibility. Chen et al. (2021c) revealed the technology sharing and sharing fees in a Stackelberg market.

Second, there are many related studies on the formulation of capacity sharing contracts. For example, Cachon and Lariviere (2005) analysed supply chain coordination of revenue-sharing contracts. Wu and Chang (2007) presented a capacity trading method that have established a capacity-sharing partnership. Seok and Nof (2014) developed a collaborative demand and capacity sharing protocol that addressed long-term profitability for each manufacturer through distributed decision making. Yang et al. (2017) designed different sharing contracts and concluded that such contracts are best applied under different scenarios. They also analysed how to choose capacity decision strategies for each type of capacity sharing contract. Guo and Wu (2018) discussed horizontal capacity sharing strategies under competition. They studied the market equilibrium and optimal enterprise strategy of production capacity sharing between two competing firms in a horizontally differentiated market. Zhao et al. (2019) examined channel selection and pricing decisions considering three modes of production capacity sharing platforms. Zhao et al. (2020) compared the effectiveness of the fixed and quality-based transaction fee strategies stipulated by a manufacturing capacity sharing platform in a capacity sharing supply chain. They also proposed a quality target contract and introduced a two-part tariff contract. Shao (2020) investigated the timing of capacity sharing agreements and contracts between two firms. Qin et al. (2020) chose the horizontal capacity-sharing strategy with a revenue-sharing contract. Liu et al. (2021) studied unmanned vehicle distribution capacity sharing with demand surge under option contracts.

Third, the oligopoly game model has been widely adopted in many decision-making research fields, including capacity selection (Ignatius et al., 2018; Nie et al., 2018). Tomaru et al. (2011) analysed capacity selection considering management delegation. Ben Elhadj et al. (2012) characterized the subgame perfect equilibrium for the homogenous market case. Nakamura (2014) examined capacity selection between a consumer-friendly firm and a standard absolute profit maximisation firm. Tao et al. (2018) studied the applicability of the duopoly model. Ueda (2019) investigated the effect of information asymmetry on a dynamic Cournot duopoly game with bounded rationality. Chen et al. (2021b) considered corporate social responsibility under capacity sharing and analysed equilibrium results. Chen

et al. (2020a) analysed capacity sharing decisions made under different oligopoly competition types and government supervision. Chen et al. (2020b) studied capacity sharing under Cournot competition. Choi and Lee (2020) analysed capacity choices adopted in upstream and downstream markets. Chen et al. (2019) established a capacity decision model with corporate social responsibility and product differentiation, derives the market equilibrium outcomes under Cournot and Bertrand competitions. Chen et al. (2021a) construct a duopoly model and analyze the optimal level of corporate social responsibility.

Our work contributes to the literature in three ways. First, we comprehensively evaluate the impact of capacity sharing on stakeholders, such as consumers, firms, and society. We analyse the multiple effects of capacity sharing and then explore the boundaries of its implementation. Many previous studies have assumed that the cost is zero or constant to simplify the analysis (Xie & Han, 2020; Zhao & Han, 2020; Qi et al., 2019). However, in this study, we introduce incremental marginal costs to the analysis of capacity sharing. Second, we study changes in profits, consumer surplus, and social welfare that result from entering a revenue-sharing contract. In this analysis, we find that capacity sharing enables both firms to obtain higher profits, but this is not necessarily beneficial to social welfare. If the approach benefits social welfare, the revenue-sharing rate needs to be controlled within a smaller range. Third, in the existing literature, few studies have applied a two-part tariff to capacity sharing charges. We apply the two-part tariff method and analyse the range of fixed fees and the optimal revenue-sharing rate, thus enriching the research on capacity sharing charges.

The remainder of this paper is organised as follows. Section 2 illustrates our model construction, including one with no capacity limitation or capacity sharing, one with capacity limitation but no capacity sharing, and one with capacity limitation and capacity sharing. Sections 3 to 5 present the equilibrium outputs, profits, consumer surplus, and social welfare of the three different models. In addition, we analyse the impact of cost and the revenue-sharing rate for profit, consumer surplus, and social welfare. Section 6 compares prices, profit, consumer surplus, and social welfare across different situations and analyses them. Finally, the conclusions are presented.

2. Model setup

We consider a market with two competing firms (Firm 1 and Firm 2) that sell a homogeneous product (or service) under Cournot competition. In a duopoly market, the two firms have the same demand because the products are homogeneous. We let q_1 and q_2 be the sales of Firm 1 and Firm 2, respectively. Thus, the inverse demand function is written as $p = a - q_1 - q_2$, where a is a constant that does not consider the effect of demand.

We assume that Firm 1's capacity is constrained to k_1 and Firm 2's capacity to k_2 . Based on the assumption of increasing marginal cost (Chen & Nie, 2014; Dong & Barcena-Ruiz, 2021; Jain & Pal, 2012), we denote the total production cost for Firm 1 as $c_1 = \frac{dq_1^2}{2}d > 0$ and that for Firm 2 as $c_2 = \frac{dq_2^2}{2}d > 0$, where d is the variable that affects the cost and is hence called the cost factor. We analyse equilibrium decisions, profits, consumer surplus, and social welfare under the three scenarios. The respective profits can be expressed as $\pi_i = pq_i - c_i$.

Consumer surplus can be expressed as $cs = u(q_1, q_2) - p_1 q_1 - p_2 q_2 = \frac{(q_1 + q_2)^2}{2}$ when the products are homogeneous, where the representative consumer utility $u(q_1, q_2) = a(q_1 + q_2) - \frac{q_1^2 + q_2^2 + 2q_1 q_2}{2}$.

Social welfare can be expressed as $sw = \pi_1 + \pi_2 + cs$.

First, we discuss the scenario of firms without limited capacity constraints and a revenue-sharing contract as a benchmark model (Model NN). Second, we analyse the scenario of firms with limited capacity constraints without a revenue-sharing contract (Model CN). Third, we further investigate the scenario of firms with limited capacity under a revenue-sharing contract (Model CS). We assume that if the profit remains unchanged after sharing, Firm 1 will still accept sharing. Firm 2 designates and proposes a revenue-sharing contract, and Firm 1 considers whether to accept it.

For the capacity sharing scenario, we consider voluntary capacity sharing between competing firms with a revenue-sharing contract. The sequences of the movements made under the contracting schemes are detailed as follows: First, before engaging in market competition, the firms agree and commit to a two-part tariff scheme, including fixed fee F and revenue-sharing rate δ . Second, firms set their production and fulfill their respective demands using their capacity and shared capacity. A revenue-sharing contract is exercised at committed sharing charges. The notation used in the models is listed in Table 1.

Table 1. The notation used in the models

i	Index for Firm i , $i = 1, 2$
q_i	Deterministic production for Firm i
\bar{q}	Production shared by Firm 1 to Firm 2
p	Price in the market
k_i	The capacity quantity owed by Firm i
c_i	The total cost of Firm i
δ	revenue-sharing rate
F	Fixed fee in the revenue-sharing contract
NN	No capacity constraint and no capacity sharing
CN	Capacity constraint and No capacity sharing
CS	Capacity constraint and capacity sharing
CSA	Capacity constraint and capacity Sharing, Firm 2 has Adequate capacity
CSI	Capacity constraint and capacity Sharing, Firm 2 has Inadequate capacity

3. Benchmark model

First, we study the benchmark case with no capacity constraints and no capacity sharing for both firms. The profits of Firm 1 and Firm 2, respectively, are as follows:

$$\pi_1^{NN} = (a - q_1 - q_2)q_1 - \frac{dq_1^2}{2}, \quad \pi_2^{NN} = (a - q_1 - q_2)q_2 - \frac{dq_2^2}{2}. \quad (1)$$

Both firms make their production decisions at the same time. We solve the Cournot model and obtain the equilibrium production, profits, consumer surplus, and social welfare from Lemma 1.

Lemma 1 For the unlimited capacity model, the equilibrium productions, profits, consumer surplus, and social welfare are as follows:

$$q_1^{NN*} = \frac{a}{d+3}, q_2^{NN*} = \frac{a}{d+3}, p^{NN*} = \frac{a(d+1)}{d+3}, \pi_1^{NN*} = \frac{a^2(d+2)}{2(d+3)^2}, \pi_2^{NN*} = \frac{a^2(d+2)}{2(d+3)^2},$$

$$cs^{NN} = \frac{2a^2}{(d+3)^2}, sw^{NN} = \frac{a^2(d+4)}{(d+3)^2}.$$

To determine the influence of cost on the decisions and profits of both participants, consumer surplus, and social welfare, we obtain Corollary 1.

Corollary 1: The effects of cost factors on the profits of the participants, consumer surplus, and social welfare are as follows:

$$\frac{\partial \pi_1^{NN*}}{\partial d} < 0, \frac{\partial \pi_2^{NN*}}{\partial d} < 0, \frac{\partial cs^{NN}}{\partial d} < 0, \frac{\partial sw^{NN}}{\partial d} < 0.$$

Proof: see Appendix A.

Evidently, as the cost increases, the profits of both firms, consumer surplus, and social welfare decrease in the unlimited capacity model, which has a practical guiding significance for the two firms. d is the cost; therefore, Corollary 1 shows that reducing costs is conducive to increasing the profits of the two firms while improving consumer surplus and social welfare. Consequently, in this case, each firm should strive to reduce costs regardless of whether it has insufficient capacity or excess capacity; thus, consumer surplus and social welfare can be improved. However, in reality, we cannot solve the problem of a mismatch between supply and demand by reducing costs to obtain greater profits.

4. No capacity sharing with constrained capacity

In this scenario, Firm 1's capacity is constrained to k_1 and Firm 2's to k_2 . Both firms use their capacity to meet the market demand. Additionally, the firms make production decisions at the same time. Therefore, we can obtain the firms' profit functions to describe the scenario involving limited capacity for both firms in Model CN (constrained and no sharing).

$$\max \pi_1^{CN} = (a - q_1 - q_2)q_1 - \frac{dq_1^2}{2}, \max \pi_2^{CN} = (a - q_1 - q_2)q_2 - \frac{dq_2^2}{2} \quad (2)$$

$$s.t. k_1 < q_1^{NN*}, k_2 \geq q_2^{NN*}$$

In Model CN, information on production costs and capacity constraints is complete for both firms. We obtain equilibrium production, profits, consumer surplus, and social welfare by solving the no-sharing Cournot model.

In this section, we focus on the case of $k_1 < q_1^{NN*}$ and $k_2 \geq q_2^{NN*}$, which indicates that Firm 1 does not have an adequate capacity to meet the desired demand generated in the benchmark model. Thus, we can obtain the profit functions under Model CN.

$$\pi_1^{CN} = (a - k_1 - q_2)k_1 - \frac{dk_1^2}{2}, \quad \pi_2^{CN} = (a - k_1 - q_2)q_2 - \frac{dq_2^2}{2}. \quad (3)$$

The equilibrium productions, profits, consumer surplus, and social welfare are given by Lemma 2.

Lemma 2 When $k_1 < q_1^{NN*}$ and $k_2 \geq q_2^{NN*}$, the equilibrium solutions in the constrained no-sharing model are as follows:

$$q_1^{CN*} = k_1, \quad q_2^{CN*} = \frac{a - k_1}{d + 2}, \quad \pi_1^{CN*} = \frac{k_1(2ad - 2k_1d + 2a - 2k_1 - d^2k_1 - 2dk_1)}{2(d + 2)},$$

$$p^{CN*} = \frac{(a - k_1)(d + 1)}{d + 2}, \quad \pi_2^{CN*} = \frac{(a - k_1)^2}{2(d + 2)}, \quad cs^{CN} = \frac{(dk_1 + k_1 + a)^2}{2(d + 2)^2},$$

$$sw^{CN} = \frac{a^2d + 3a^2 + 2ad^2k_1 + 6adk_1 + 2ak_1 - d^3k_1^2 - 5d^2k_1^2 - 7dk_1^2 - k_1^2}{2(d + 2)^2}.$$

Based on Lemma 2, we can easily obtain the influence of cost factor on solutions, profits, consumer surplus, and social welfare from Corollary 2.

Corollary 2: When $k_1 < q_1^{NN*}$ and $k_2 \geq q_2^{NN*}$, the effects of the cost factor on solutions, profits, consumer surplus, and social welfare are as follows:

$$\text{If } d < \sqrt{\frac{2(a - k_1)}{k_1}} - 2 \text{ then } \frac{\partial \pi_1^{CN*}}{\partial d} > 0; \text{ if } d > \sqrt{\frac{2(a - k_1)}{k_1}} - 2, \text{ then } \frac{\partial \pi_1^{CN*}}{\partial d} < 0.$$

$$\frac{\partial \pi_2^{CN*}}{\partial d} < 0, \quad \frac{\partial cs^{CN}}{\partial d} < 0, \quad \text{and} \quad \frac{\partial sw^{CN}}{\partial d} < 0.$$

Proof: see Appendix B.

Corollary 2 indicates a situation in which one firm has constrained capacity, while the other has sufficient capacity. As the cost increases, Firm 2's profit, consumer surplus, and social welfare decrease. However, as the cost increases, Firm 1's profit first decreases and then increases. Thus, when the cost is low, the cost increase does not harm Firm 1's profits. However, Firm 2's profit decreases at a higher cost. When Firm 1 is constrained by its capacity, it may not expect a cost reduction, and Firm 2 expects this to increase its profits. The reasoning here is that the increase in cost will reduce Firm 2's output, which leads to a higher market price, while Firm 1 is less directly affected by the increase in cost and thus obtains higher profits.

This capacity constraint model shows that insufficient capacity will affect profits. In addition, although the firm with sufficient capacity will not be affected, the mismatch between supply and demand will affect social welfare and consumer surplus. To improve sustainable development, it is necessary to solve this problem and propose a win-win strategy.

5. Revenue-sharing contracts with constrained capacity

Considering the condition in which $k_1 < q_1^{NN*}$ and $k_2 \geq q_2^{NN*}$, when Firm 1 has constrained capacity, yet Firm 2 has overcapacity, there is the opportunity for Firm 2 to share its capacity with Firm 1. We analyse the capacity sharing strategy and possible improvements in profits

for both firms. When both firms profit from the capacity sharing strategy, they negotiate sharing charges for shared capacity.

Under these schemes, the firms contract and commit to sharing charges before engaging in market competition. Then, they simultaneously set their respective production and consider the constraints of their capacity and possible shared capacity. The revenue-sharing contract is exercised at a committed revenue-sharing rate. Finally, the demand is realised. Both firms obtain profits. We use backward induction to solve this problem.

Thus, we can obtain Model CS to characterise the revenue-sharing contract model with constrained capacity.

$$\begin{aligned}\pi_1^{CS} &= (a - q_1 - q_2)q_1 - \frac{dq_1^2}{2} - F - \delta\bar{q}, \\ \pi_2^{CS} &= (a - q_1 - q_2)q_2 - \frac{dq_2^2}{2} + F + \delta\bar{q}, \quad q_1 = k_1 + \bar{q}.\end{aligned}\quad (4)$$

Under the conditions of $k_1 < q_1^{NN*}$ and $k_2 \geq q_2^{NN*}$, considering the constraints and shared production \bar{q} , we have the following cases:

(1) Case 5-1 $k_2 \geq q_2^{NN*}$ and $q_1^{CS*} + q_2^{CS*} < k_1 + k_2$ (Model CSA).

In this case, Firm 2 has adequate capacity, and the total demand of Firms 1 and 2 can be satisfied by their total capacity. Firm 2's demand is not constrained by its capacity. Therefore, Firm 2 uses its capacity to meet its desired market demand. Firm 1's demand can be fully satisfied using its capacity and shared capacity.

(2) Case 5-2 $k_2 \geq q_2^{NN*}$ and $q_1^{CS*} + q_2^{CS*} \geq k_1 + k_2$ (Model CSI).

In this case, Firm 2 has adequate capacity, but the total demand of both firms cannot be satisfied by their total capacity. Firm 2's demand is not constrained by its capacity. Therefore, Firm 2 can use its capacity to meet its desired market demand. Firm 1's demand cannot be fully satisfied by its capacity and shared capacity.

5.1. Case 5-1 $k_2 \geq q_2^{NN*}$ and $q_1^{CS*} + q_2^{CS*} < k_1 + k_2$

In this case, Firm 2 has adequate capacity, and the total demand of Firms 1 and 2 can be satisfied by their total capacity. Here, we can obtain the equilibrium production, profits, consumer surplus, and social welfare from Lemma 3.

Lemma 3 In the capacity sharing model of Case 5-1, the equilibrium results are:

(1) The productions for the capacity sharing model are as follows:

$$q_1^{CSA*} = \frac{a - 2\delta + ad - d\delta}{d^2 + 4d + 3}, \quad q_2^{CSA*} = \frac{a + \delta + ad}{d^2 + 4d + 3}.$$

(2) The shared capacity quantity is $\bar{q}^{CSA*} = \frac{a - 2\delta + ad - 4dk_1 - d\delta - d^2k_1 - 3k_1}{d^2 + 4d + 3}$.

(3) The equilibrium profits of the capacity sharing model are as follows:

$$\pi_1^{CSA*} = \frac{a^2d^3 + 4a^2d^2 + 5a^2d + 2a^2 - 2ad^3\delta - 10ad^2\delta - 16ad\delta - 8a\delta + 2k_1d^4\delta - 2Fd^4 + d^3\delta^2 + 16k_1d^3\delta - 16Fd^3}{2(d^2 + 4d + 3)^2},$$

$$\pi_2^{CSA*} = \frac{a^2d^3 + 4a^2d^2 + 5a^2d + 2a^2 + 2ad^3\delta + 12ad^2\delta + 20ad\delta + 10a\delta - 2k_1d^4\delta + 2Fd^4 - 2d^3\delta^2 - 16k_1d^3\delta + 16Fd^3 - 12d^2\delta^2 - 44k_1d^2\delta + 44Fd^2 - 21d\delta^2 - 48k_1d\delta + 48Fd - 10\delta^2 - 18k_1\delta + 18F}{2(d^2 + 4d + 3)^2}.$$

(4) The pricing, consumer surplus, and social welfare are as follows:

$$p^{CSA*} = \frac{a + 3k_1 + \delta + ad + dk_1}{d + 3}, \quad cs^{CSA} = \frac{(2a - \delta)^2}{2(d + 3)^2},$$

$$sw^{CSA} = \frac{2a^2d^3 + 12a^2d^2 + 18a^2d + 8a^2 - 2ad^2\delta - 4ad\delta - 2a\delta - d^3\delta^2 - 5d^2\delta^2 + 7d\delta^2 + \delta^2}{2(d^2 + 4d + 3)^2}.$$

Lemma 3 shows the equilibrium solutions for Case 5-1. Based on Lemma 3, we can obtain Corollary 3 to discuss the effects of the cost factors and revenue-sharing rate on the equilibrium solutions.

Corollary 3: In the capacity sharing model of Case 5-1 with $\delta > 0$, the effects of cost factor and revenue-sharing rate on the equilibrium solutions are as follows:

- (1) If $\delta < \frac{-9k_1 + 4a + 8ad - 24dk_1 + 5ad^2 + ad^3 - 22d^2k_1 - 8d^3k_1 - d^4k_1}{d^3 + 6d^2 + 12d + 8}$, then $\frac{\partial \pi_1^{CSA*}}{\partial \delta} < 0$;
 if $\delta > \frac{-9k_1 + 4a + 8ad - 24dk_1 + 5ad^2 + ad^3 - 22d^2k_1 - 8d^3k_1 - d^4k_1}{d^3 + 6d^2 + 12d + 8}$, then $\frac{\partial \pi_1^{CSA*}}{\partial \delta} > 0$.
- (2) If $\delta < \frac{-9k_1 + 5a + 10ad - 24dk_1 + 6ad^2 + ad^3 - 22d^2k_1 - 8d^3k_1 - d^4k_1}{2d^3 + 12d^2 + 21d + 10}$, then $\frac{\partial \pi_2^{CSA*}}{\partial \delta} > 0$;
 if $\delta > \frac{-9k_1 + 5a + 10ad - 24dk_1 + 6ad^2 + ad^3 - 22d^2k_1 - 8d^3k_1 - d^4k_1}{2d^3 + 12d^2 + 21d + 10}$; then $\frac{\partial \pi_2^{CSA*}}{\partial \delta} < 0$.
- (3) If $\delta < 2a$, then $\frac{\partial cs^{CSA}}{\partial \delta} < 0$; if $\delta > 2a$, then $\frac{\partial cs^{CSA}}{\partial \delta} > 0$.
- (4) $\frac{\partial sw^{CSA}}{\partial \delta} < 0$.
- (5) $\frac{\partial \pi_1^{CSA*}}{\partial d} < 0$, $\frac{\partial \pi_2^{CSA*}}{\partial d} < 0$, $\frac{\partial cs^{CSA}}{\partial d} < 0$, $\frac{\partial sw^{CSA}}{\partial d} < 0$.

Proof: see Appendix C.

If the two firms want to enter a revenue-sharing contract, they must ensure that the profits after sharing will be greater than before. We compare the production of the capacity sharing model in Case 5-1 to the no-capacity sharing model. Then, we obtain Proposition 1: It can be seen that the two firms follow their profit maximisation principles when setting the revenue-sharing rate. However, social welfare decreases as the revenue-sharing rate increases. Therefore, the government can guide firms to reduce the revenue-sharing rate on the premise of increasing profits as much as possible to minimise the negative impact on social welfare.

Proposition 1. The range of the revenue-sharing rate is as follows:

If the two firms' profits after sharing are greater than when sharing is not applied, the revenue-sharing rate must satisfy $x_2^1 < \delta < x_1^1$, where

$$x_1^1 = \frac{(d+1) \left[4a - 9k_1 - 15k_1d + 4ad + (d+3) \sqrt{k_1^2 + 2Fd^3 + 12Fd^2 + 24Fd + 16F - 7k_1d^2 - k_1d^3 + ad^2} \right]}{(d+2)^3}$$

and $x_2^1 = \frac{(d+1) \left(5a - 9k_1 - 15k_1d + 5ad + (d+3) \sqrt{\sigma_1 - 7k_1d^2 - k_1d^3 + ad^2} \right)}{(d+2)^3}$.

Proof: see Appendix D.

Based on the range of revenue-sharing rate, both firms negotiate the sharing charges under the capacity sharing strategy, which means that in the case of limited capacity, partners with the revenue-sharing contract for capacity sharing obtain more profits than those without it. The two firms' profits satisfy the following inequality: $\pi_1^{CSA^*} > \pi_1^{CN^*}$ and $\pi_2^{CSA^*} > \pi_2^{CN^*}$.

Within the range of this revenue-sharing rate, we further study the optimal charges that fit the contract. We assume that this revenue-sharing rate is determined by the firm that shares its capacity (Firm 2). Therefore, Firm 2 will set a revenue-sharing rate that maximises its profit. From Corollary 3(2), we know that if we take δ as a variable, its profit function will first increase and then decrease. Then, we can determine the optimal δ_0^{CSA} as follows:

$$\delta_0^{CSA} = \frac{-9k_1 + 5a + 10ad - 24dk_1 + 6ad^2 + ad^3 - 22d^2k_1 - 8d^3k_1 - d^4k_1}{2d^3 + 12d^2 + 21d + 10}. \quad (5)$$

In addition, we find the range of fixed-fee F . For Firm 2, there is no doubt that the larger F is, the more profit Firm 2 will make. Therefore, F has no upper limit. However, for Firm 1, if F is too large, it may be impossible for Firm 1 to make a profit regardless of how low the revenue-sharing rate is. From the perspective of Firm 1, F has an upper limit. We assume the extreme case when the revenue-sharing rate δ is 0 (all the profits obtained from shared capacity belong to Firm 1). When the revenue-sharing rate is zero, F is maximum. When F exceeds this number, Firm 1 cannot make more profit regardless of how the rate of return is adjusted. We then find the optimal F as follows:

$$F^{CSA} = \frac{k_1^2d}{2} + \frac{a^2(d+2)}{2(d+3)^2} + \frac{k_1(k_1-a)(d+1)}{d+2}. \quad (6)$$

Of course, F^{CSA} is the largest possible fixed fee. Only with a fixed fee which is lower than F^{CSA} , Firm 1 obtains higher profits than before. Firm 2's profit increases with an increase in F . Therefore, the two firms should flexibly set the fixed fee in the contract within this upper limit.

5.2. Case 5-2 $k_2 \geq q_2^{NN^*}$ and $q_1^{CS^*} + q_2^{CS^*} \geq k_1 + k_2$

In this case, Firm 2 has sufficient capacity, but the total demand of both firms cannot be satisfied by their capacity. Firm 2's demand is not constrained by its capacity. Thus, Firm 2 uses its capacity to meet its desired market demand. However, Firm 1's desired demand cannot be fully satisfied by its capacity and shared capacity because Firm 2's capacity is inadequate for total demand. That is $q_1^{CS^*} + q_2^{CS^*} \geq k_1 + k_2$, where $q_2 = k_2 - \bar{q}$ and $q_1 = k_1 + \bar{q}$.

Thus, we can obtain Model CSI to characterise the model of the revenue-sharing contract with constrained capacity, and Firm 2's capacity is inadequate.

$$\pi_1^{CSI} = (a - k_1 - k_2)q_1 - \frac{dq_1^2}{2} - F - \delta\bar{q}, \quad \pi_2^{CSI} = (a - k_1 - k_2)q_2 - \frac{dq_2^2}{2} + F + \delta\bar{q}. \quad (7)$$

Lemma 4. In the capacity sharing models of Case 5-2, the equilibrium results are

(1) The equilibrium productions for the capacity sharing model are as follows:

$$q_1^{CSI*} = \frac{a - k_1 - k_2 - \delta}{d}, \quad q_2^{CSI*} = \frac{a - k_1 - k_2}{d}.$$

(2) The shared capacity quantity is $\bar{q}^{CSI*} = \frac{a - k_1 - k_2 - \delta - dk_1}{d}$.

(3) The equilibrium profits of the capacity sharing model are as follows:

$$\pi_1^{CSI*} = \frac{2k_1k_2 - 2ak_1 - 2ak_2 - 2a\delta - 2dF + 2k_1\delta + 2k_2\delta + a^2 + k_1^2 + k_2^2 + \delta^2 + 2dk_1\delta}{2d},$$

$$\pi_2^{CSI*} = F + \frac{a - k_1 - k_2^2}{2d} - \frac{\delta - a + k_1 + k_2 + \delta + dk_1}{d}.$$

(4) Consumer surplus and social welfare are as follows:

$$p^{CSI*} = \frac{ad + \delta}{d + 2}, \quad cs^{CSI} = \frac{(k_1 + k_2)^2}{2},$$

$$sw^{CSI} = \frac{4k_1k_2 - 4ak_1 - 4ak_2 + dk_1^2 + dk_2^2 + 2a^2 + 2k_1^2 + 2k_2^2 - \delta^2 + 2dk_1k_2}{2d}.$$

Lemma 4 shows the equilibrium solutions for Case 5-2. Based on Lemma 4, we can obtain Corollary 4 to discuss the effects of cost factors and the revenue-sharing rate on the equilibrium solutions.

Corollary 4: In the capacity sharing models of Case 5-2 with $\delta > 0$, the effects of cost factors and the revenue-sharing rate on the equilibrium solutions are as follows:

If $\delta < a - k_1 - k_2 - dk_1$, then $\frac{\partial \pi_1^{CSI*}}{\partial \delta} < 0$; if $\delta > a - k_1 - k_2 - dk_1$, then $\frac{\partial \pi_1^{CSI*}}{\partial \delta} > 0$.

If $\delta < \frac{a - k_1 - k_2 - dk_1}{2}$, then $\frac{\partial \pi_2^{CSI*}}{\partial \delta} > 0$; if $\delta > \frac{a - k_1 - k_2 - dk_1}{2}$, then $\frac{\partial \pi_2^{CSI*}}{\partial \delta} < 0$.

$$\frac{\partial cs^{CSI}}{\partial \delta} = 0, \quad \frac{\partial sw^{CSI}}{\partial \delta} < 0.$$

$$\frac{\partial \pi_1^{CSI*}}{\partial d} < 0, \quad \frac{\partial \pi_2^{CSI*}}{\partial d} < 0, \quad \frac{\partial cs^{CSI}}{\partial d} = 0, \quad \frac{\partial sw^{CSI}}{\partial d} < 0.$$

Proof: see Appendix E.

If the two firms want to use the revenue-sharing contract, they must ensure that the profits generated after sharing are greater than before. We compare the productions of the capacity sharing model in Case 5-2 to the no-capacity sharing model. Then, we obtain Proposition 2. The difference from Case 5-1 is that when Firm 2's capacity is sufficient to share with Firm 1 and it can meet all market demands, consumer surplus is not affected by the cost and revenue-sharing rate. The utility of consumers does not change because of changes in the revenue-sharing rate. These results have some enlightenment for government-guiding firms to reach a win-win sustainable revenue-sharing contract.

Proposition 2. The range of the revenue-sharing rate is:

If the two firms' profits are greater with sharing than without, the revenue-sharing rate must satisfy $y_2^1 < \delta < y_1^1$, where $y_1^1 = a - k_1 - k_2 - dk_1 - \sqrt{2d} \sqrt{\frac{2F + dF - ak_1 + 2k_1k_2 + k_1^2 + dk_1k_2}{d(d+2)}}$, and $y_2^1 = \frac{a - k_1 - k_2 - dk_1}{2} - \sqrt{\frac{\sigma_2}{4(d+2)}}$.

Proof: see Appendix F.

Based on the range of revenue-sharing rate, both firms negotiate the sharing charges under the capacity sharing strategy, which means that in the limited capacity case, partners should obtain more profits with the revenue-sharing contract for capacity sharing than without the revenue-sharing contract. The two firms' profits satisfy the following inequality: $\pi_1^{CSI*} > \pi_1^{CN*}$ and $\pi_2^{CSI*} > \pi_2^{CN*}$. Similar to the analysis of Case 1, we also assume that Firm 2 sets the final revenue-sharing rate. Then, we find the optimal δ_0^{CSI} and the maximum F^{CSI} .

$$\delta_0^{CSI} = \frac{a - k_1 - k_2 - dk_1}{2}, \quad F^{CSI} = \frac{k_1^2 d}{2} + \frac{(k_1 + k_2 - a)^2}{2d} + \frac{k_1(k_1 - a)(d+1)}{d+2}. \quad (8)$$

6. Model comparisons

Based on the above three models, we can compare the three equilibrium solutions. Through these comparisons, we provide a reference for decision makers in the two firms about whether to share capacity and how to predict market changes. For example, when a firm's capacity is constrained, are consumer surplus and social welfare reduced? When two firms share their capacity, how will the price change?

Based on the previous analysis, we identified three situations. We compare the prices, profits, consumer surplus, and social welfare in these situations.

Corollary 5: The comparison results are as follows:

- (1) $p^{CN*} > p^{NN*}$, $p^{CSA*} > p^{NN*}$,
 - if $\delta < -\frac{9k_1 - a - ad + 9dk_1 + 2d^2k_1}{d+2}$, then $p^{CSA*} < p^{CN*}$,
 - if $\delta > -\frac{9k_1 - a - ad + 9dk_1 + 2d^2k_1}{d+2}$, then $p^{CSA*} > p^{CN*}$,
 - if $\delta < \frac{2a}{d+3}$, then $p^{CSI*} < p^{NN*}$,
 - if $\delta > \frac{2a}{d+3}$, then $p^{CSI*} > p^{NN*}$,
 - if $\delta < a - k_1 - dk_1$, then $p^{CSI*} < p^{CN*}$,
 - if $\delta > a - k_1 - dk_1$, then $p^{CSI*} > p^{CN*}$,
 - if $\delta < k_1d^2 + 5k_1d + 2a + 6k_1$, then $p^{CSI*} < p^{CSA*}$,
 - if $\delta > k_1d^2 + 5k_1d + 2a + 6k_1$, then $p^{CSI*} > p^{CSA*}$.

(2) $\pi_1^{CN*} < \pi_1^{NN*}$, $\pi_2^{CN*} > \pi_2^{NN*}$,

if $x_2^1 < \delta < x_1^1$, then $\pi_1^{CSA*} > \pi_1^{CN*}$ and $\pi_2^{CSA*} > \pi_2^{CN*}$,

if $y_2^1 < \delta < y_1^1$, then $\pi_1^{CSI*} > \pi_1^{CN*}$ and $\pi_2^{CSI*} > \pi_2^{CN*}$,

$$x_1^1 = \frac{(d+1) \left[4a - 9k_1 - 15k_1d + 4ad + (d+3) \sqrt{k_1^2 + 2Fd^3 + 12Fd^2 + 24Fd + 16F - 7k_1d^2 - k_1d^3 + ad^2} \right]}{(d+2)^3},$$

$$x_2^1 = \frac{(d+1) \left(5a - 9k_1 - 15k_1d + 5ad + (d+3) \sqrt{\sigma_1 - 7k_1d^2 - k_1d^3 + ad^2} \right)}{(d+2)^3},$$

and $y_1^1 = a - k_1 - k_2 - dk_1 - \sqrt{2d} \sqrt{\frac{2F + dF - ak_1 + 2k_1k_2 + k_1^2 + dk_1k_2}{d(d+2)}}$,

$$y_2^1 = \frac{a - k_1 - k_2 - dk_1}{2} - \sqrt{\frac{\sigma_2}{4(d+2)}}.$$

σ_1 and σ_2 : see Appendix K.

(3) $cs^{CN} < cs^{NN}$, $cs^{CSA} < cs^{NN}$,

if $\delta < \frac{ad - 4dk_1 + a - d^2k_1 - 3k_1}{d+2}$, then $cs^{CSA} > cs^{CN}$,

if $\delta > \frac{ad - 4dk_1 + a - d^2k_1 - 3k_1}{d+2}$, then $cs^{CSA} < cs^{CN}$,

if $\delta < \frac{2a}{d+3}$, then $cs^{CSI} > cs^{NN}$,

if $\delta > \frac{2a}{d+3}$, then $cs^{CSI} < cs^{NN}$,

if $\delta < a - (d+1)k_1$, then $cs^{CSI} > cs^{CN}$,

if $\delta > a - (d+1)k_1$, then $cs^{CSI} < cs^{CN}$,

$cs^{CSI} > cs^{CSA}$.

(4) $sw^{CN} < sw^{NN}$ and $sw^{CSA} < sw^{NN}$,

if $\delta < \frac{(d+1)(a - 3k_1 - dk_1)}{d+2}$, then $sw^{CSA} > sw^{CN}$,

if $\delta > \frac{(d+1)(a - 3k_1 - dk_1)}{d+2}$, then $sw^{CSA} < sw^{CN}$,

if $\delta < \frac{\sqrt{\sigma_3}}{d+3}$, then $sw^{CSI} > sw^{NN}$,

if $\delta > \frac{\sqrt{\sigma_3}}{d+3}$, then $sw^{CSI} < sw^{NN}$,

if $\delta < \frac{\sqrt{\sigma_5}}{d+3}$, then $sw^{CSI} > sw^{CN}$,

if $\delta > \frac{\sqrt{\sigma_5}}{d+3}$, then $sw^{CSI} < sw^{CN}$,

$$\delta < \frac{(d+1)(ad - (3+d)\sqrt{\sigma_7} + ad^2)}{3d^3 + 15d^2 + 23d + 9}.$$

$$\text{If } \delta < \frac{(d+1)(ad - (3+d)\sqrt{\sigma_7} + ad^2)}{3d^3 + 15d^2 + 23d + 9}, \text{ then } sw^{CSI} > sw^{CSA},$$

$$\text{if } \delta > \frac{(d+1)(ad - (3+d)\sqrt{\sigma_7} + ad^2)}{3d^3 + 15d^2 + 23d + 9}, \text{ then } sw^{CSI} < sw^{CSA}.$$

Proof: see Appendix G

The comparison between Model NN and Model CN analyses the impact of capacity constraints on firms, consumers, and society. We conclude that when Firm 1 faces capacity constraints, its profit decreases, Firm 2's profit increases, and consumer surplus and social welfare decrease.

The comparison between Model NN and Model CS determines whether the negative effects of capacity constraints can be completely offset by capacity sharing. We conclude that although profits increase after sharing, the negative effects of capacity constraints on consumer surplus and social welfare cannot be eliminated.

The comparison between Model CN and Model CS analyses the impact of sharing on firms, consumers, and society when there is a capacity constraint. For firms, the profits increase. For consumers, when the revenue-sharing rate is lower than a critical value, sharing can increase consumer surplus; when the rate exceeds this critical value, sharing reduces consumer surplus. A critical value exists for social welfare as well. When the revenue-sharing rate exceeds this critical value, it reduces social welfare.

7. Discussion

This study examines the capacity sharing strategies of two firms under Cournot competition, revealing the conditions for realising capacity sharing and the factors that affect sharing behaviour. The discussion of the results is as follows:

First, the choice of sharing behaviour is affected by production costs, capacity constraints, and sharing charges. If production costs are too high, or firms with constrained capacity can produce too few products, firms with sufficient capacity are less likely to choose to share. Capacity sharing is seen to be a complex game process affected by many factors which must be fully considered when formulating a capacity sharing contract.

Second, under different capacity constraints, charges affect the sharing decisions of firms. If the capacity constraint is small, two firms with different capacities can reach a capacity sharing contract with a lower sharing charge (including a lower fixed fee and lower revenue-sharing rate). If a firm's capacity is very limited, or the costs are high, capacity sharing can only be achieved under the condition of increasing fixed fees and revenue-sharing rate. In the case of increased sharing costs, firms with constrained capacity within a certain range can still obtain higher profits than before. Therefore, if firms want to achieve higher profits, they can reduce costs by improving production technology and other methods. Firms that

face insufficient capacity for a long time can flexibly adjust their negotiation strategies and look for firms with sufficient capacity for long-term and stable cooperation. Firms with sufficient capacity, in addition to producing products that meet market demand, can also obtain additional profits by sharing capacity. In addition, the government should create a good environment for capacity sharing, popularise sharing policies, and enable more firms to adopt this means of increasing profits. Further, the government can establish a sharing platform to reduce the problem of finding firms for cooperation due to information asymmetry.

Third, when a firm faces capacity constraints, its profits are affected. At this time, the profits of the firm with sufficient capacity will increase. Capacity sharing will increase the profits of both firms and increase social welfare when the revenue-sharing rate is low, but it cannot completely offset the reduction in social welfare due to capacity constraints. Therefore, even if the government cannot solve the capacity constraints, it can improve social welfare by guiding firms to set a lower sharing rate.

The above results have a certain guiding significance for capacity sharing and cooperation between firms of different sizes. In reality, a large number of small and medium-sized firms are faced with insufficient capacity and can achieve sustainable development through capacity sharing with large firms. In addition, the study shows that it is necessary to pay attention to stakeholders, especially to protect the rights and interests of consumers. If the result of capacity sharing is harmful to the interests of consumers, it will cause consumers to have a bad experience and negative impressions of the firm, which will be detrimental to the future of the firm's sustainable development. Finally, the research results are also applicable to other resource-sharing behaviours between firms, especially a certain extent of technological asset sharing, which is conducive to promoting production intensification and achieving economies of scale.

Conclusions

This study examines the capacity sharing strategy between two firms under Cournot competition and analyses the optimal revenue-sharing contract. In terms of innovation, we use the two-part tariff method and consider the assumption of increasing marginal costs. We derive the equilibrium outputs, profit, consumer surplus, and social welfare under different scenarios and analyse the impacts of capacity sharing, revenue-sharing rate, and cost. In reality, the contract is sustainable only if both firms are profitable. A sustainable capacity sharing strategy is conducive to the realisation of a win-win arrangement.

First, the adoption of capacity sharing requires both firms to be profitable. We designed a feasible range for revenue-sharing rate and fixed fees. Within this range, the two firms choose to apply a revenue-sharing contract to obtain higher profits. Therefore, to promote capacity sharing cooperation between them, a contract that can achieve a win-win outcome should be designed.

Second, the cost and revenue-sharing rate have different impacts on firms, consumers, and society under different scenarios. When there is no capacity constraint or sharing, as costs increase, the profits, consumer surplus, and social welfare of the two firms decrease.

When there are capacity constraints but no sharing, as costs increase, the profit of the firm with capacity constraints first increases and then decreases. The profit of the firm with overcapacity increases, and consumer surplus and social welfare increase. When there are capacity constraints and sharing, as costs increase, the profits of the two firms, consumer surplus, and social welfare decrease, and as the revenue-sharing rate increases, the profit of the firm with capacity constraints first decreases and then increases. The profit of the firm with overcapacity first increases and then decreases, and consumer surplus and social welfare decrease.

Third, according to the conditions for maximising the profit of the firm that is sharing its capacity, the optimal revenue-sharing rate and fixed fee (i.e. the optimal sharing charges and revenue-sharing contract), which are affected by the constrained capacity and cost, can be determined.

Fourth, capacity sharing impacts firms, consumers, and social welfare, and these impacts are further affected by the revenue-sharing rate and cost. For firms, capacity sharing increases profits. When the revenue-sharing rate is lower than a critical value, sharing can increase consumer surplus, and when this critical value is exceeded, sharing reduces consumer surplus. When another critical value is exceeded, capacity sharing reduces social welfare.

Based on these conclusions, the government should formulate policies conducive to business cooperation that achieve sustainable development and establish an efficient platform that enables firms to seek cooperation. Firms should adopt various measures and try their best to achieve sustainable revenue-sharing contracts.

In the future, capacity sharing contracts among multiple enterprises should be considered, along with more types of oligopoly competition (e.g. Stackelberg competition and Bertrand competition). In addition, to simplify the study, we assumed that there is no product differentiation; therefore, considering product differentiation will be more practical and interesting for future studies.

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Author contributions

Junlong Chen and Jiali Liu conceived the study and contributed to the design of model. Liu Jiali was responsible for the topic selection of the article. Junlong Chen and Jiayan Shi were responsible for the derivation and analysis of model. Jiayan Shi and Junlong Chen wrote the first draft of the article. Junlong Chen and Liu Jiali checked and revised the draft.

Disclosure statement

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APPENDIX

A. Proof of Corollary 1

$$\frac{\partial \pi_1^{NN^*}}{\partial d} = -\frac{a^2(d+1)}{2(d+3)^2} < 0, \quad \frac{\partial \pi_2^{NN^*}}{\partial d} = -\frac{a^2(d+1)}{2(d+3)^2} < 0, \quad \frac{\partial cs^{NN}}{\partial d} = -\frac{4a^2}{(d+3)^3} < 0,$$

$$\frac{\partial sw^{NN}}{\partial d} = -\frac{a^2(d+5)}{(d+3)^3} < 0.$$

B. Proof of Corollary 2

$$\frac{\partial \pi_1^{CN^*}}{\partial d} = \frac{k_1(a-k_1)}{(d+2)^2} - \frac{k_1^2}{2}, \text{ if } d < \sqrt{\frac{2(a-k_1)}{k_1}} - 2 \text{ then } \frac{\partial \pi_1^{CN^*}}{\partial d} > 0,$$

$$\text{if } d > \sqrt{\frac{2(a-k_1)}{k_1}} - 2, \text{ then } \frac{\partial \pi_1^{CN^*}}{\partial d} < 0.$$

$$\frac{\partial \pi_2^{CN^*}}{\partial d} = -\frac{(a-k_1)^2}{2(d+2)^2} < 0, \quad \frac{\partial cs^{CN}}{\partial d} = -\frac{(a-k_1)(a+k_1+dk_1)}{(d+2)^3} < 0,$$

$$\frac{\partial sw^{CN}}{\partial d} = \frac{k_1(a-k_1)}{(d+2)^2} - \frac{k_1^2}{2} - \frac{(a-k_1)^2}{2(d+2)^2} - \frac{(k_1 + \frac{a-k_1}{d+2})(a-k_1)}{(d+2)^2} < 0.$$

C. Proof of Corollary 3

$$(1) \frac{\partial \pi_1^{CSA^*}}{\partial \delta} = \frac{18k_1 - 8a + 16\delta - 16ad + 48dk_1 + 24d\delta - 10ad^2 - 2ad^3 + 44d^2k_1 + 16d^3k_1 + 2d^4k_1 + 12d^2\delta + 2d^3\delta}{2(d^2 + 4d + 3)^2}.$$

$$\text{If } \delta < \frac{-9k_1 + 4a + 8ad - 24dk_1 + 5ad^2 + ad^3 - 22d^2k_1 - 8d^3k_1 - d^4k_1}{d^3 + 6d^2 + 12d + 8}, \text{ then } \frac{\partial \pi_1^{CSA^*}}{\partial \delta} < 0;$$

$$\text{if } \delta > \frac{-9k_1 + 4a + 8ad - 24dk_1 + 5ad^2 + ad^3 - 22d^2k_1 - 8d^3k_1 - d^4k_1}{d^3 + 6d^2 + 12d + 8}, \text{ then } \frac{\partial \pi_1^{CSA^*}}{\partial \delta} > 0.$$

$$(2) \frac{\partial \pi_2^{CSA^*}}{\partial \delta} = \frac{18k_1 - 10a + 20\delta - 20ad + 48dk_1 + 42d\delta - 12ad^2 - 2ad^3 + 44d^2k_1 + 16d^3k_1 + 2d^4k_1 + 24d^2\delta + 4d^3\delta}{2(d^2 + 4d + 3)^2}.$$

$$\text{If } \delta < \frac{-9k_1 + 5a + 10ad - 24dk_1 + 6ad^2 + ad^3 - 22d^2k_1 - 8d^3k_1 - d^4k_1}{2d^3 + 12d^2 + 21d + 10}, \text{ then } \frac{\partial \pi_2^{CSA^*}}{\partial \delta} > 0;$$

$$\text{if } \delta > \frac{-9k_1 + 5a + 10ad - 24dk_1 + 6ad^2 + ad^3 - 22d^2k_1 - 8d^3k_1 - d^4k_1}{2d^3 + 12d^2 + 21d + 10}; \text{ then } \frac{\partial \pi_2^{CSA^*}}{\partial \delta} < 0.$$

$$(3) \frac{\partial cs^{CSA}}{\partial \delta} = -\frac{4a - 2\delta}{2(d+3)^2}. \text{ If } \delta < 2a, \text{ then } \frac{\partial cs^{CSA}}{\partial \delta} < 0; \text{ if } \delta > 2a, \text{ then } \frac{\partial cs^{CSA}}{\partial \delta} > 0.$$

$$(4) \frac{\partial sw^{CSA}}{\partial \delta} = -\frac{2a + 2\delta + 4ad + 14d\delta + 2ad^2 + 10d^2\delta + 2d^3\delta}{2(d^2 + 4d + 3)^2} < 0.$$

$$(5) \frac{\partial \pi_1^{CSA^*}}{\partial d} = -\frac{a^2d^4 + 4a^2d^3 + 6a^2d^2 + 4a^2d + a^2 - 2ad^4\delta - 12ad^3\delta - 30ad^2\delta - 36ad\delta - 16a\delta + d^4\delta^2 + 8d^3\delta^2 + 27d^2\delta^2 + 44d\delta^2 + 28\delta^2}{2(d^2 + 4d + 3)^3} < 0,$$

$$\frac{\partial \pi_2^{CSA^*}}{\partial d} = -\frac{a^2d^4 + 4a^2d^3 + 6a^2d^2 + 4a^2d + a^2 + 2ad^4\delta + 16ad^3\delta + 42ad^2\delta + 48ad\delta + 20a\delta - 2d^4\delta^2 - 16d^3\delta^2 - 45d^2\delta^2 - 52d\delta^2 - 17\delta^2}{2(d^2 + 4d + 3)^3} < 0,$$

$$\frac{\partial \pi_{CS}^{CSA}}{\partial d} = -\frac{(2a - \delta)^2}{(d + 3)^3} < 0,$$

$$\frac{\partial \pi_{sw}^{CSA}}{\partial d} = -\frac{-2a^2d^4 - 16a^2d^3 - 36a^2d^2 - 32a^2d - 10a^2 + 4ad^3\delta + 12ad^2\delta + 12ad\delta + 4a\delta + d^4\delta^2 + 6d^3\delta^2 + 12d^2\delta^2 + 2d\delta^2 - 13\delta^2}{2(d^2 + 4d + 3)^3} < 0.$$

D. Proof of Proposition 1

Based on Model CN, we have $\pi_1^{CN^*} = \frac{k_1(2ad - 2k_1d + 2a - 2k_1 - d^2k_1 - 2dk_1)}{2(d + 2)}$

and $\pi_2^{CN^*} = \frac{(a - k_1)^2}{2(d + 2)}$.

Based on Model CSA, we have $\pi_1^{CSA^*} = \frac{\sigma_6}{2(d + 2)}$ and $\pi_2^{CSA^*} = \frac{\sigma_7}{2(d + 2)}$.

Then, $\pi_1^{CSA^*} - \pi_1^{CN^*} = \frac{\sigma_6}{2(d + 2)} + \frac{dk_1^2}{2} - \frac{k_1(a - k_1)(d + 1)}{d + 2}$,

$\pi_2^{CSA^*} - \pi_2^{CN^*} = \frac{\sigma_7}{2(d + 2)} + \frac{(a - k_1)^2}{2(d + 2)}$.

Let the solution of $\pi_1^{CSA^*} - \pi_1^{CN^*} = 0$ be x_1^1 and x_1^2 .

$$x_1^1 = \frac{(d + 1)\left(4a - 9k_1 - 15k_1d + 4ad + (d + 3)\sqrt{k_1^2 + 2Fd^3 + 12Fd^2 + 24Fd + 16F - 7k_1d^2 - k_1d^3 + ad^2}\right)}{(d + 2)^3},$$

$$x_1^2 = -\frac{(d + 1)\left(9k_1 - 4a + 15k_1d - 4ad + (d + 3)\sqrt{k_1^2 + 2Fd^3 + 12Fd^2 + 24Fd + 16F + 7k_1d^2 + k_1d^3 - ad^2}\right)}{(d + 2)^3}.$$

If $\delta < x_1^1$ or $\delta > x_1^2$, $\pi_1^{CSA^*} > \pi_1^{CN^*}$. If $x_1^1 \leq \delta \leq x_1^2$, $\pi_1^{CSA^*} < \pi_1^{CN^*}$.

Let the solution of $\pi_2^{CSA^*} - \pi_2^{CN^*} = 0$ be x_2^1 and x_2^2 .

$$x_2^1 = \frac{(d + 1)\left(5a - 9k_1 - 15k_1d + 5ad + (d + 3)\sqrt{\sigma_1} - 7k_1d^2 - k_1d^3 + ad^2\right)}{(d + 2)^3},$$

$$x_2^2 = -\frac{(d + 1)\left(9k_1 - 5a + 15k_1d - 5ad + (d + 3)\sqrt{\sigma_1} + 7k_1d^2 + k_1d^3 - ad^2\right)}{(d + 2)^3}.$$

σ_1 : see Appendix K

If $\delta < x_2^1$ or $\delta > x_2^2$, $\pi_2^{CSA^*} < \pi_2^{CN^*}$. If $x_2^1 \leq \delta \leq x_2^2$, $\pi_2^{CSA^*} > \pi_2^{CN^*}$.

If two firms want to satisfy $\pi_1^{CSA^*} > \pi_1^{CN^*}$, $\pi_2^{CSA^*} > \pi_2^{CN^*}$.

δ must satisfy $\delta < x_1^1$, $x_2^1 \leq \delta \leq x_2^2$, or $\delta > x_1^2$, $x_2^1 \leq \delta \leq x_2^2$.

Comparing x_1^1 , x_1^2 , x_2^1 and x_2^2 , we find that $x_2^1 < x_1^1 < x_2^2 < x_1^2$.

Finally, $x_2^1 < \delta < x_1^1$.

E. Proof of Corollary 4

$$(1) \frac{\partial \pi_1^{CSI^*}}{\partial \delta} = -\frac{a-k_1-k_2-\delta-dk_1}{d}. \text{ If } \delta < a-k_1-k_2-dk_1, \text{ then } \frac{\partial \pi_1^{CSI^*}}{\partial \delta} < 0;$$

$$\text{if } \delta > a-k_1-k_2-dk_1, \text{ then } \frac{\partial \pi_1^{CSI^*}}{\partial \delta} > 0. \frac{\partial \pi_2^{CSI^*}}{\partial \delta} = \frac{a-k_1-k_2-2\delta-dk_1}{d}.$$

$$\text{If } \delta < \frac{a-k_1-k_2-dk_1}{2}, \text{ then } \frac{\partial \pi_2^{CSI^*}}{\partial \delta} > 0; \text{ if } \delta > \frac{a-k_1-k_2-dk_1}{2}, \text{ then } \frac{\partial \pi_2^{CSI^*}}{\partial \delta} < 0.$$

$$\frac{\partial cs^{CSI}}{\partial \delta} = 0, \quad \frac{\partial sw^{CSI}}{\partial \delta} = -\frac{\delta}{d} < 0.$$

$$(2) \frac{\partial \pi_1^{CSI^*}}{\partial d} = -\frac{(a-k_1-k_2-\delta)^2}{2d^2} < 0,$$

$$\frac{\partial \pi_2^{CSI^*}}{\partial d} = -\frac{\delta(a-k_1-k_2-\delta-dk_1)}{d^2} - \frac{k_1\delta}{d} - \frac{(a-k_1-k_2)^2}{2d^2} < 0, \quad \frac{\partial cs^{CSI}}{\partial d} = 0,$$

$$\frac{\partial sw^{CSI}}{\partial d} = \frac{a-k_1-k_2-\delta}{d} < 0.$$

F. Proof of Proposition 2

Based on Model CN, we have $\pi_1^{CN^*} = \frac{k_1(2ad-2k_1d+2a-2k_1-d^2k_1-2dk_1)}{2(d+2)}$ and $\pi_2^{CN^*} = \frac{(a-k_1)^2}{2(d+2)}$.

Based on Model CSA, we have $\pi_2^{CSI^*} = F + \frac{a-k_1-k_2^2}{2d} - \frac{\delta-a+k_1+k_2+\delta+dk_1}{d}$ and $\pi_1^{CSI^*} = \frac{2k_1k_2-2ak_1-2ak_2-2a\delta-2dF+2k_1\delta+2k_2\delta+a^2+k_1^2+k_2^2+\delta^2+2dk_1\delta}{2d}$.

Then, $\pi_2^{CSI^*} - \pi_2^{CN^*} = F - \frac{(a-k_1)^2}{2(d+2)} + \frac{a-k_1-k_2^2}{2d} - \frac{\delta-a+k_1+k_2+\delta+dk_1}{d}$,

$$\pi_1^{CSI^*} - \pi_1^{CN^*} = \frac{dk_1^2}{2} + \frac{2k_1k_2-2ak_1-2ak_2-2a\delta-2dF+2k_1\delta+2k_2\delta+a^2+k_1^2+k_2^2+\delta^2+2dk_1\delta}{2d} - \frac{k_1(a-k_1)(d+1)}{d+2}.$$

Let the solution of $\pi_1^{CSI^*} - \pi_1^{CN^*} = 0$ be y_1^1 and y_1^2 .

$$y_1^1 = a-k_1-k_2-dk_1 - \sqrt{2d} \sqrt{\frac{2F+dF-ak_1+2k_1k_2+k_1^2+dk_1k_2}{d(d+2)}},$$

$$y_1^2 = a-k_1-k_2-dk_1 + \sqrt{2d} \sqrt{\frac{2F+dF-ak_1+2k_1k_2+k_1^2+dk_1k_2}{d(d+2)}}.$$

If $\delta < y_1^1$ or $\delta > y_1^2$, $\pi_1^{CSI^*} > \pi_1^{CN^*}$. If $y_1^1 \leq \delta \leq y_1^2$, $\pi_1^{CSI^*} < \pi_1^{CN^*}$.

Let the solution of $\pi_2^{CSI^*} - \pi_2^{CN^*} = 0$ be y_2^1 and y_2^2 .

$$y_2^1 = \frac{a-k_1-k_2-dk_1}{2} - \sqrt{\frac{\sigma_2}{4(d+2)}}, \quad y_2^2 = \frac{a-k_1-k_2-dk_1}{2} + \sqrt{\frac{\sigma_2}{4(d+2)}}.$$

σ_2 : see Appendix K

If $\delta < y_2^1$ or $\delta > y_2^2$, $\pi_2^{CSI^*} < \pi_2^{CN^*}$. If $y_2^1 \leq \delta \leq y_2^2$, $\pi_2^{CSI^*} > \pi_2^{CN^*}$.

If two firms want to satisfy $\pi_1^{CSI^*} > \pi_1^{CN^*}$, $\pi_2^{CSI^*} > \pi_2^{CN^*}$.

δ must satisfy $\delta < y_1^1$, $y_2^1 \leq \delta \leq y_2^2$, or $\delta > y_1^2$, $y_2^1 \leq \delta \leq y_2^2$.

Comparing y_1^1 , y_1^2 , y_2^1 and y_2^2 , we find that $y_2^1 < y_1^1 < y_2^2 < y_1^2$.

Finally, $y_2^1 < \delta < y_1^1$.

G. Proof of Corollary 5

(1) In Model CN, $k_1 < \frac{a}{d+3}$.

Therefore, $p^{CN^*} = \frac{(a-k_1)(d+1)}{d+2} > \frac{\left(a - \frac{a}{d+3}\right)(d+1)}{d+2} = \frac{a(d+1)}{d+3} = p^{NN^*}$.

Because $p^{NN^*} = \frac{a(d+1)}{d+3}$ and $p^{CSA^*} = \frac{a+3k_1+\delta+ad+dk_1}{d+3}$, it is obvious that $p^{NN^*} < p^{CSA^*}$.

The threshold of δ is calculated by solving the following equation: $p^{CSA^*} - p^{CN^*} = 0$.

We then obtain $\delta = -\frac{9k_1 - a - ad + 9dk_1 + 2d^2k_1}{d+2}$. The function is monotonic, so if $\delta < -\frac{9k_1 - a - ad + 9dk_1 + 2d^2k_1}{d+2}$, then $p^{CSA^*} < p^{CN^*}$. If $\delta > -\frac{9k_1 - a - ad + 9dk_1 + 2d^2k_1}{d+2}$, $p^{CSA^*} > p^{CN^*}$.

In Model CSI, the retail price is $p^{CSI^*} = a - k_1 - k_2$. In this model, after sharing, the total capacity of two firms can be sold. Then we have $k_1 + k_2 = q_1^{CSI^*} + q_2^{CSI^*}$, which means that

$k_1 + k_2 = \frac{2a - \delta}{d+2}$. From these equations, we know that $p^{CSI^*} = \frac{ad + \delta}{d+2}$ (s.t. $a > (d+3)k_1$, $d > 0$, $a > \frac{2a - \delta}{d+2}$).

The threshold of δ is calculated by solving the following equation: $p^{CSI^*} - p^{NN^*} = 0$. We then obtain $\delta = \frac{2a}{d+3}$. The function is monotonic, so if $\delta < \frac{2a}{d+3}$, then $p^{CSI^*} < p^{NN^*}$. If $\delta > \frac{2a}{d+3}$, $p^{CSI^*} > p^{NN^*}$.

The threshold of δ is calculated by solving the following equation: $p^{CSI^*} - p^{CN^*} = 0$. We then obtain $\delta = a - k_1 - dk_1 > 0$. The function is monotonic, so if $\delta < a - k_1 - dk_1$, then $p^{CSI^*} < p^{CN^*}$, if $\delta > a - k_1 - dk_1$, then $p^{CSI^*} > p^{CN^*}$.

The threshold of δ is calculated by solving the following equation: $p^{CSI^*} - p^{CSA^*} = 0$.

We then obtain $\delta = k_1d^2 + 5k_1d + 2a + 6k_1 > 0$. The function is monotonic, so if $\delta < k_1d^2 + 5k_1d + 2a + 6k_1$, then $p^{CSI^*} < p^{CSA^*}$, if $\delta > k_1d^2 + 5k_1d + 2a + 6k_1$, $p^{CSI^*} > p^{CSA^*}$.

(2) In Model CN, $k_1 < \frac{a}{d+3}$.

Therefore, $q_1^{CN^*} + q_2^{CN^*} = \frac{a+dk_1+k_1}{d+2} < \frac{\frac{a}{d+3}(d+1)+a}{d+2} = \frac{2a}{d+3} = q_1^{NN^*} + q_2^{NN^*}$. It means that $cs^{CN} < cs^{NN}$.

Because $q_1^{NN^*} + q_2^{NN^*} = \frac{2a}{d+3}$ and $q_1^{CSA^*} + q_2^{CSA^*} = \frac{2a - \delta}{d+3}$, it is obvious that $q_1^{CSA^*} + q_2^{CSA^*} < q_1^{NN^*} + q_2^{NN^*}$. That is $cs^{CN} < cs^{NN}$.

The threshold of δ is calculated by solving the following equation: $q_1^{CSA^*} + q_2^{CSA^*} - q_1^{CN^*} - q_2^{CN^*} = 0$. We then obtain $\delta = \frac{ad - 4dk_1 + a - d^2k_1 - 3k_1}{d + 2} > 0$. The function is monotonic, so if $\delta < \frac{ad - 4dk_1 + a - d^2k_1 - 3k_1}{d + 2}$, then $cs^{CSA} > cs^{CN}$, if $\delta > \frac{ad - 4dk_1 + a - d^2k_1 - 3k_1}{d + 2}$, $cs^{CSA} < cs^{CN}$.

In Model CSI, the retail price is $q_1^{CSI^*} + q_2^{CSI^*} = k_1 + k_2$. In this model, after sharing, the total capacity of two firms can be sold. Then we have $k_1 + k_2 = q_1^{CSI^*} + q_2^{CSI^*}$ (s.t. $a > (d + 3)k_1$, $d > 0$, $a > \frac{2a - \delta}{d + 2}$).

The threshold of δ is calculated by solving the following equation: $q_1^{CSI^*} + q_2^{CSI^*} - q_1^{NN^*} - q_2^{NN^*} = 0$. We then obtain $\delta = \frac{2a}{d + 3}$. The function is monotonic, so if $\delta < \frac{2a}{d + 3}$, then $cs^{CSI} > cs^{NN}$, if $\delta > \frac{2a}{d + 3}$, $cs^{CSI} < cs^{NN}$.

The threshold of δ is calculated by solving the following equation: $q_1^{CSI^*} + q_2^{CSI^*} - q_1^{CN^*} - q_2^{CN^*} = 0$. We then obtain $\delta = a - k_1 - dk_1 > 0$. The function is monotonic, so if $\delta < a - k_1 - dk_1$, then $cs^{CSI} > cs^{CN}$, if $\delta > a - k_1 - dk_1$, then $cs^{CSI} < cs^{CN}$.

$q_1^{CSI^*} + q_2^{CSI^*} = \frac{2a - \delta}{d + 2} > \frac{2a - \delta}{d + 3} = q_1^{CSA^*} + q_2^{CSA^*}$. It means that $cs^{CSI} > cs^{CSA}$.

(3) In Model CN, $k_1 < \frac{a}{d + 3}$.

Therefore,

$$sw^{CN} = \frac{a^2d + 3a^2 + 2ad^2k_1 + 6adk_1 + 2ak_1 - d^3k_1^2 - 5d^2k_1^2 - 7dk_1^2 - k_1^2}{2(d + 2)^2} < \frac{a^2(d + 4)}{(d + 3)^2} = sw^{NN}.$$

$$sw^{CSA} - sw^{NN} = -\frac{\delta(2a + \delta + 4ad + 7d\delta + 2ad^2 + 5d^2\delta + d^3\delta)}{2(d^2 + 4d + 3)^2} < 0. \text{ It means that } sw^{CSA} < sw^{NN}.$$

The threshold of δ is calculated by solving the following equation: $sw^{CSA} - sw^{CN} = 0$. We then obtain $\delta = \frac{(d + 1)(a - 3k_1 - dk_1)}{d + 2} > 0$. The function is monotonic, so if $\delta < \frac{(d + 1)(a - 3k_1 - dk_1)}{d + 2}$, then $sw^{CSA} > sw^{NN}$, if $\delta > \frac{(d + 1)(a - 3k_1 - dk_1)}{d + 2}$, $sw^{CSA} < sw^{NN}$.

The threshold of δ is calculated by solving the following equation: $sw^{CSI} - sw^{NN} = 0$. We then obtain $\delta = \frac{\sqrt{\sigma_3}}{d + 3}$. The function is monotonic, so if $\delta < \frac{\sqrt{\sigma_3}}{d + 3}$, then $sw^{CSI} > sw^{NN}$, if $\delta > \frac{\sqrt{\sigma_3}}{d + 3}$, $sw^{CSI} < sw^{NN}$.

The threshold of δ is calculated by solving the following equation: $sw^{CSI} - sw^{CN} = 0$. We then obtain $\delta = \frac{\sqrt{\sigma_5}}{d + 3} > 0$. The function is monotonic, so if $\delta < \frac{\sqrt{\sigma_5}}{d + 3}$, then $sw^{CSI} > sw^{CN}$, if $\delta > \frac{\sqrt{\sigma_5}}{d + 3}$, $sw^{CSI} < sw^{CN}$.

The threshold of δ is calculated by solving the following equation: $sw^{CSI} - sw^{CSA} = 0$.

We then obtain $\delta = \frac{(d+1)(ad - (3+d)\sqrt{\sigma_7} + ad^2)}{3d^3 + 15d^2 + 23d + 9} > 0$. The function is monotonic,

so if $\delta < \frac{(d+1)(ad - (3+d)\sqrt{\sigma_7} + ad^2)}{3d^3 + 15d^2 + 23d + 9}$, then $sw^{CSI} > sw^{CSA}$,

if $\delta > \frac{(d+1)(ad - (3+d)\sqrt{\sigma_7} + ad^2)}{3d^3 + 15d^2 + 23d + 9}$, $sw^{CSI} < sw^{CSA}$.

σ_3 , σ_4 and σ_5 : see Appendix K.

K.

$$\sigma_1 = k_1^2 d^4 + 8k_1^2 d^3 + 20k_1^2 d^2 + 16k_1^2 d + 4k_1^2 - 2k_1 a d^3 - 8k_1 a d^2 - 4k_1 a d + a^2 d^2 + 4F d^3 + 24F d^2 + 42F d + 20F;$$

$$\sigma_2 = a^2 d + 6a^2 - 2ad^2 k_1 - 6adk_1 - 6adk_2 - 12ak_1 - 12ak_2 + d^3 k_1^2 + 4d^2 k_1^2 + 2d^2 k_1 k_2 + 4Fd^2 + 5dk_1^2 + 10dk_1 k_2 + 3dk_2^2 + 8Fd + 6k_1^2 + 6k_2^2 + 12k_1 k_2;$$

$$\sigma_3 = 4a^2 d + 18a^2 - 4ad^2 k_1 - 4ad^2 k_2 - 24adk_2 - 36ak_1 - 36ak_2 + d^3 k_1^2 + 2d^3 k_1 k_2 + d^3 k_2^2 + 8d^2 k_1^2 + 16d^2 k_1 k_2 + 8d^2 k_2^2 + 21dk_1^2 + 21dk_2^2 + 42dk_1 k_2 + 18k_1^2 + 18k_2^2 + 36k_1 k_2;$$

$$\sigma_4 = a^2 d + 5a^2 d + 8a^2 - 2ad^3 k_1 - 10ad^2 k_1 - 4ad^2 k_2 - 18adk_1 - 16adk_2 - 16ak_1 - 16ak_2 + d^4 k_1^2 + 6d^3 k_1^2 + d^3 k_2^2 + 2d^3 k_1 k_2 + 12d^2 k_1 k_2 + 13d^2 k_1^2 + 6d^2 k_2^2 + 13dk_1^2 + 12dk_2^2 + 24dk_1 k_2 + 8k_1^2 + 8k_2^2 + 16k_1 k_2;$$

$$\sigma_5 = 13a^2 d^2 + 38a^2 d + 18a^2 - 12ad^3 k_1 - 12ad^3 k_2 - 60ad^2 k_1 - 60ad^2 k_2 - 92adk_1 - 92adk_2 - 36ak_1 - 36ak_2 + 3d^4 k_1^2 + 6d^4 k_1 k_2 + 3d^4 k_2^2 + 21d^3 k_1^2 + 42d^3 k_1 k_2 + 21d^3 k_2^2 + 53d^2 k_1^2 + 53d^2 k_2^2 + 106d^2 k_1 k_2 + 55dk_1^2 + 55dk_2^2 + 110dk_1 k_2 + 18k_1^2 + 18k_2^2 + 36k_1 k_2;$$

$$\sigma_6 = a^2 d^3 + 4a^2 d^2 + 5a^2 d + 2a^2 - 2ad^3 \delta - 10ad^2 \delta - 16ad\delta - 8a\delta + 2k_1 d^4 \delta - 2Fd^4 + d^3 \delta^2 + 16k_1 d^3 \delta - 16Fd^3 + 6d^2 \delta^2 + 44k_1 d\delta - 44Fd^2 + 12d\delta^2 + 48k_1 d\delta - 48Fd + 8\delta^2 + 18k_1 \delta - 18F;$$

$$\sigma_7 = a^2 d^3 + 4a^2 d^2 + 5a^2 d + 2a^2 + 2ad^3 \delta + 12ad^2 \delta + 20ad\delta + 10a\delta - 2k_1 d^4 \delta + 2Fd^4 - 2d^3 \delta^2 - 16k_1 d^3 \delta + 16Fd^3 - 12d^2 \delta^2 - 44k_1 d^2 \delta + 44Fd^2 - 21d\delta^2 - 48k_1 d\delta + 48Fd - 10\delta^2 - 18k_1 \delta + 18F.$$