Digital Object Identifier 10.1109/IECON.2019.8926952

IECON 2019-45th Annual Conference of the IEEE Industrial Electronics Society

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Suggested Citation

F. Cecati, S. Pugliese, R. Zhu and M. Liserre, "Integration and Optimization of Voltage Active Filtering Functionality in a PV Park," IECON 2019 - 45th Annual Conference of the IEEE Industrial Electronics Society, Lisbon, Portugal, 2019.

Integration and Optimization of Voltage Active Filtering Functionality in a PV Park

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***Abstract*—The stringent regulations on the power quality declared in the standard IEEE 519-2014 push the companies and the power producers to install active filters to compensate the voltage harmonics distortion in the Point of Common Coupling (PCC). However, in the case of a Photovoltaic (PV) park, the cost for an additional active filter converter can be saved by using the PV converters themselves as active filter. This solution is very attractive, but reserves several challenges. In fact, the harmonic current injection by the PV converter can bring a ripple in the DC link which increases the stress on the converter components and effects the MPPT. Moreover, the limit on the nominal current for the PV converters must be taken into consideration. In this paper a centralized optimized strategy to share the harmonic current injection among all the converters in a PV park is investigated. The optimization is formulated as a Quadratic Programming (QP) problem: the active power consumed by the PV park for the active filtering and the DC link ripple of the PV converters caused by the harmonic currents injection are minimized. The limit on the maximum injectable harmonic current by each PV converter are respected.**

***Index Terms*—Photovoltaic Energy, Optimization, DC link ripple**

I. Introduction

In the last decades, a great effort has been made towards the development and the integration of more renewable energy sources in the electrical grid. The fast development of Photovoltaic (PV) cell technologies, the continuous cost reduction of PV modules, and the constant upgrades in the power electronics technologies have been the main driving forces for the intensive deployment of PV power generation plants [1]. The presence of a Voltage Source Inverter (VSI) as interface between the PV modules and the AC bus in a PV farm makes possible, beside the fundamental active current injection, also several ancillary services [2]. Active filtering has gained particular attention because of the more and more stringent regulations regarding the voltage harmonics in the Point of Common Coupling (PCC), as described in the standard IEEE 519-2014 [3]. Many decentralized strategies which realize the active filtering through the introduction of additional control loops inside the converter have been studied [4], [5]. However, when dealing with several distributed generators like in a PV park, a centralized approach which enables the optimal coordination between the single units can result in a higher efficiency. In [6] a centralized control architecture for harmonic voltage suppression operated with a low-bandwidth communication technique is studied

The grid impedance in the Point of Common Coupling (PCC) and the grid voltage harmonic components can be exploited for the design of the active filter. Several techniques for the measurements of the grid impedance have been developed and are in literature [7], [8]. From the knowledge of the grid impedance, the current able to induce a voltage drop in the PCC which compensates the voltage harmonics can be derived. However, the active power consumption due to the compensation current injection must be limited as much as possible and the injected reactive power must meet the inverter limitations represented by the overcurrent protection [9]. Many example of optimization of active and reactive power are present in literature [10], [11]. Nevertheless, when dealing with harmonic currents injection, an oscillatory component in the power arises as well and must be mitigated. In fact, the fluctuations of the active power produce undesired ripples in the DC link voltage which increases the stress on the DC link capacitor of the converters and have negative effects on the MPPT [12]. Instantaneous power theory offers powerful mathematical tools to analyze and quantify the oscillatory component of the active power due to the harmonic injection [13], [14].

In this paper a PV farm with multiple parallel-connected converters is considered [15]. The whole PV park acts as an active filter to compensate, through an appropriate current injection, the voltage harmonic distortion in the PCC. The main contribution of this paper is the optimal sharing of the harmonic compensation current among the PV converters. The problem is formulated as a constrained quadratic programming (QP) problem [16]: the centralized optimization algorithm computes the harmonic currents set-points for each PV converter, in order to achieve the desired current injection at the PCC level. An optimal choice of the amplitudes and phase angles of those currents is made in order to minimize both the total active power consumed by the active filtering and DC link ripple in the PV converters. The limits on the maximum injectable current of each converter are taken into account in the optimization.

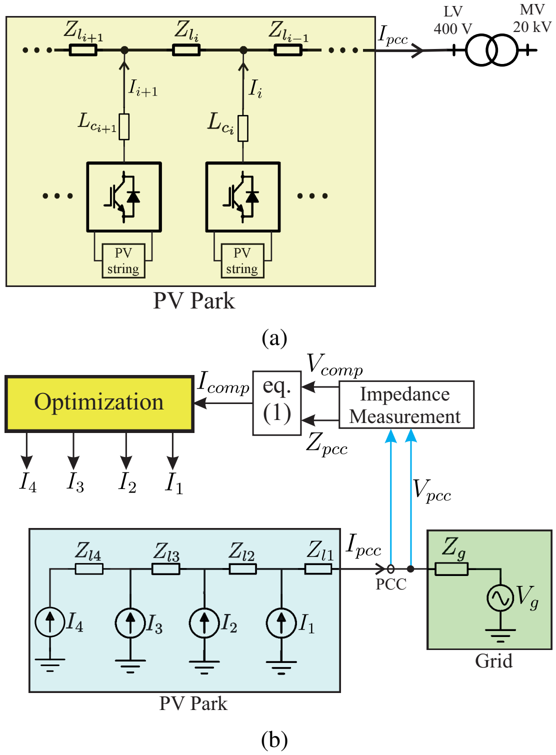


Fig. 1: (a) The scheme of a PV farm with multiple parallel-connected PV three-phase converters. (b) The steady state model of the PV farm with the optimization algorithm.

The paper is divided as follow: Section II describes the PV park model, Section III analyzes the problem of the DC link ripple, Section IV deals with the QP problem formulation, Section V shows the simulation results and Section VI summarizes the achievements of the paper.

II. System Description

The simplified scheme of the PV farm at system level is shown in Fig. 1a: each PV three-phase converter is connected to a common AC bus and injects a current I. The whole PV park acts as a voltage active filter, by injecting harmonic currents able to reduce the PCC voltage distortion until it complies with the grid code [3]. In this case, less active power is available for the fundamental current injection to supply the facilities. The PV farm is connected to the Medium Voltage (MV) through a step-up transformer. A scenario with the presence of the 5^{th} and the 7^{th} harmonic voltages in the grid is examined. Line impedances Z_l are considered in order to model the different distance of each PV converter from the PCC. For the optimization problem formulation, a steady state model of the system in Fig. 1a with four identical PV converters has been used; the PV converters have been modeled as ideal current sources as shown in Fig. 1b. This kind of modeling for the PV converters is particularly suitable for this application, since the optimization algorithm updates the current set-points in a time interval of few seconds, much bigger respect to the time constants of the converter dynamics. The current set-points are then sent to each converter through a low bandwidth communication system. The tracking of the harmonic currents is realized through PR controllers [14]. The impedance measurement algorithm, shown in Fig. 1b, estimates the equivalent grid impedance at the harmonic frequencies and the amplitude and phase of the harmonics in the grid voltage, according to [7]. The current I_{comp} to be injected in the PCC in order to mitigate the PCC voltage distortion presents a 5^{th} and a 7^{th} harmonic component and is computed through the ratio

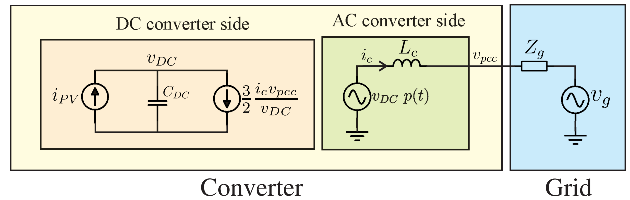


Fig. 2: The average model of a power converter considering the DC and AC part.

$$\dot{I}^{5th}_{comp} = \frac{\dot{V}^{5th}_{comp}}{\dot{Z}^{5th}_{pcc}} \qquad \dot{I}^{7th}_{comp} = \frac{\dot{V}^{7th}_{comp}}{\dot{Z}^{7th}_{pcc}} \tag{1}$$

where $\dot{V}_{comp}$ is the voltage drop to be induced in the PCC in order to comply with the grid code requirements. According to the equivalent circuit Fig. 1b, the current I_{pcc} is given by the sum of all the currents injected by the PV converters, thus:

$$\dot{I}_{pcc} = \dot{I}_1 + \dot{I}_2 + \dot{I}_3 + \dot{I}_4 \tag{2}$$

where $\dot{I}_{pcc} = \left(\dot{I}^{5^{th}}_{pcc} \quad \dot{I}^{7^{th}}_{pcc}\right)^T$ and the notation $(\cdot)^T$ refers to the vector transposition. The optimization algorithm computes the current set-points $I_{1,2,3,4}$ in order to obtain an optimal sharing of the compensation current I_{comp} among the converters. The minimization of the active power consumed for the active filtering and the DC link ripple are set as optimization goal. In Section III, a theoretical analysis of the phenomenon of the DC link ripple is carried out.

III. DC link ripple analysis

In Fig. 2, the model of a DC/AC converter used to study the DC link ripple is shown. The DC link voltage behavior is described by the energy conservation law between the DC and the AC part of the converter [14], thus:

$$i_{PV} v_{DC} = v_{DC} \frac{1}{C_{DC}} \frac{dv_{DC}}{dt} + \frac{3}{2}\left(i_c \cdot v_{PCC}\right) \tag{3}$$

where '·' represents the dot product, and i_c and v_{PCC} are expressed in the dq frame. Considering a scenario with the 5^{th} and the 7^{th} harmonic in the PCC voltage, and a harmonic compensation by the converter, the phasors of i_c and v_{PCC} can be written as

$$\begin{cases} \dot{I}_c = I_{1st} e^{j\omega t + \delta_1} + I_{5th} e^{-j5\omega t + \delta_5} + I_{7th} e^{j7\omega t + \delta_7} \\ \dot{V}_{PCC} = V_{1st} e^{j\omega t} + V_{5th} e^{-j5\omega t} + V_{7th} e^{j7\omega t} \end{cases} \tag{4}$$

considering that the fifth harmonic in a three-phase system is negative sequence and rotate at -5ω. It is assumed that the harmonics of the voltage v_{PCC} have the same initial phase of the fundamental. The active power can be computed as the real part of the product $\dot{V}_{PCC}\dot{I}_c^*$ This product, expressed in equation returns both constant terms and oscillatory terms at frequency 6ω. The latter are responsible for the DC link ripple, and must be minimized. The expression of the oscillatory component of the active power, neglecting the terms which do not play an important role in the optimization, is:

$$\dot{\tilde{p}} = V_{1st}\left(I_{7th}e^{j\delta_7} + I_{5th}e^{-j\delta_5}\right)e^{-j6\omega t} \tag{5}$$

which gives the condition on the amplitude and initial angle of the the 5^{th} and the 7^{th} injected current harmonic, in order to minimize the DC link ripple

$$I_{5th}e^{-j\delta_5} = -I_{7th}e^{j\delta_7} \tag{6}$$

in term of amplitude and phase:

$$\begin{cases} |\dot{I}_{5th}| = |\dot{I}_{7th}| \\ \underline{/\dot{I}_{5th}} = \pi - \underline{/\dot{I}_{7th}} \end{cases} \tag{7}$$

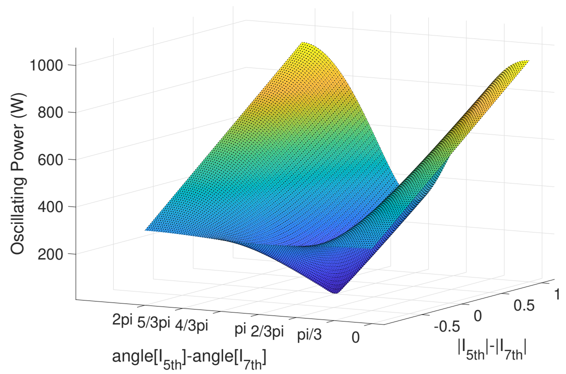


Fig. 3: The amplitude of the oscillating power depending on the harmonic current injection.

Fig. 3 describes graphically the analytic result obtained in equation (5). The surface shows the dependence of the oscillating power on the phase displacement and the amplitude difference of the 5^{th} and 7^{th} harmonic currents. As declared in equation (7), the condition of null power oscillation is met when the amplitude of the 5^{th} and 7^{th} harmonic currents are equal and their phase displacement is π.

IV. The Optimization Problem Formulation

In the previous section, the average model of the converter has been used to mathematically characterize the oscillatory component of the active power. The derived expression is exploited in this section for the formulation of the optimization problem.

Several kinds of optimization problems exist, and in order to use an optimization algorithm for a real application, at first it is necessary to identify which kind of optimization problem is the most suitable for the case under study. One of the main difference between one optimization problem and the other one, is how the cost function and the constraints are expressed. In the PV park optimization, as it will be shown, the cost function is quadratic and the constraints are linear: the optimization problems with those characteristics are modeled as Quadratic Programming (QP) problems [16]. The general expression of a QP problem is

$$\begin{cases} \min\limits_{x}\left(\frac{1}{2}x^T Hx + f^T x\right) \\ Ax \leq b \end{cases} \tag{8}$$

where

- x is the vector of the variables to optimize, in this case the harmonic currents injected by each PV panel. The optimization algorithm gives as output the optimal value of the vector x
- $\frac{1}{2}x^T Hx + f^T x$ is the quadratic cost function to minimize, in this case the active power reserved to the active filtering and the oscillatory component of the power which is responsible of the DC link ripple
- $Ax \leq b$ are the constraints that the vector x must respect. In this case the constraint are given by the maximum injectable current by each PV converter, and from the fact that the total current injected in the PCC must be exactly the current required for the voltage harmonic compensation

In the next paragraph, the definition and the calculation of the vectors and the matrices of equation (8) needed to set up the optimization algorithm is carried out.

A. The vector x of the variables to optimize

In the case of the PV farm, the state variables to optimize are the current references for each converter, as shown in Fig. 1b. Thus, the vector x is defined as

$$\begin{cases} x = \begin{pmatrix} I_1^T & I_2^T & I_3^T & I_4^T \end{pmatrix}^T \\ I_k = \begin{pmatrix} I_k^{5^{th}} \\ I_k^{7^{th}} \end{pmatrix}, k = 1, 2, 3, 4 \end{cases} \tag{9}$$

The currents $I_k^{5^{th}}$ and $I_k^{7^{th}}$ are the references for the 5^{th} and 7^{th} harmonic of the *k-th* converter, expressed in the dq synchronous frame of the corresponding harmonic. The dq frames of harmonic currents rotate at the same speed of the corresponding harmonic, thus at -5ω for the 5^{th} and 7ω for the 7^{th} harmonic current. The values of the current are expressed in p.u. respect to the rated harmonic currents.

B. The cost function

As cost function, several possibilities are analyzed in this paper, and the different obtained results are compared. At first, it is considered no optimization. Then two cost functions and their combination are considered: the constant and the oscillatory component of the active power. The constant component is related to the power consumption and will be denominated consumed active power, while the oscillatory component is related to the DC link voltage ripple. The expression of the power is always a quadratic function of the current, thus the cost function can be expressed with a quadratic form like in equation (8). However, the expressions of the matrix H and the vector f must be computed.

1) The consumed active power: The expression of the constant component of the active power is given by the dot product of the currents and the voltages of the same harmonic frequency, thus:

$$\begin{cases} \bar{P}_k = \begin{pmatrix} V_k^{5^{th}} & V_k^{7^{th}} \end{pmatrix} \begin{pmatrix} I_k^{5^{th}} \\ I_k^{7^{th}} \end{pmatrix}, k = 1, 2, 3, 4 \\ \bar{P} = \sum_{k=1}^{4} \bar{P}_k \end{cases} \tag{10}$$

In order to express the cost function in a form like in (8), the voltage must be expressed as a function of the vector x. From the knowledge of the line impedance, $\bar{H}$ and $\bar{f}$ are defined as

$$\begin{cases} \bar{H} = \begin{pmatrix} Z_{l1} & Z_{l1} & Z_{l1} & Z_{l1} \\ Z_{l1} & Z_{l1}+Z_{l2} & Z_{l1}+Z_{l2} & Z_{l1}+Z_{l2} \\ Z_{l1} & Z_{l1}+Z_{l2} & Z_{l1}+Z_{l2}+Z_{l3} & Z_{l1}+Z_{l2}+Z_{l3} \\ Z_{l1} & Z_{l1}+Z_{l2} & Z_{l1}+Z_{l2}+Z_{l3} & Z_{l1}+Z_{l2}+Z_{l3}+Z_{l4} \end{pmatrix} \\ \bar{f} = \begin{pmatrix} V_{pcc} & V_{pcc} & V_{pcc} & V_{pcc} \end{pmatrix}^T \end{cases} \tag{11}$$

where V_{pcc} represents the vector of the voltage harmonics in the PCC, defined as $V_{pcc} = \begin{pmatrix} V_{pcc}^{5^{th}} & V_{pcc}^{7^{th}} \end{pmatrix}$. From (11) the vector of the output voltages of the PV converters is defined as:

$$\bar{V} = \bar{H}x + \bar{f} \tag{12}$$

Since the term $\bar{f}$ gives a relatively low contribution in the cost function, it can be neglected. With this assumption, the constant active power can be expressed by combining equation (10), (11) and (12) as

$$\bar{P} = x^T \bar{H} x \tag{13}$$

which is coherent with the standard formulation of a quadratic programming problem in equation (8).

2) The oscillatory power: the other cost function is related to the amplitude of the oscillatory component of the active power, expressed in equation (5). By minimizing it, the DC link ripple in all the converters will be consequently minimized. In order to formulate the optimization problem, the norm of $\dot{\tilde{p}}$ in (5) must be expressed as a quadratic function of the vector x defined in equation (9). For the *k-th* converter:

$$\left\| \dot{\tilde{p}}_k \right\|^2 = \left(\|V_{1st}\| \left\| \begin{pmatrix} I_d^{5^{th}} + I_d^{7^{th}} \\ I_q^{5^{th}} - I_q^{7^{th}} \end{pmatrix}_k \right\| \right)^2 \tag{14}$$

The total oscillatory power term to minimize is chosen as $\sum_{k=1}^{4} \left\| \dot{\tilde{p}}_k \right\|^2$, and can be expressed with the quadratic form

$$\tilde{P} = x^T \tilde{H} x \tag{15}$$

with

$$\begin{cases} \tilde{I} = \|V_{1st}\| \left(\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{array} \right) \\ \tilde{H}_{sqrt} = diag\left(\tilde{I}, \tilde{I}, \tilde{I}, \tilde{I}\right) \\ \tilde{H} = \tilde{H}_{sqrt}^T \tilde{H}_{sqrt} \end{cases} \tag{16}$$

The cost function given by the combination of (13) and (15) can be defined in the form of (8) as $\frac{1}{2} x^T H x$ with

$$H = 2\left(\bar{\rho}\bar{H} + \tilde{\rho}\tilde{H}\right) \tag{17}$$

The coefficients $\bar{\rho}$ and $\tilde{\rho}$ are weighting factors for the two different optimization objectives. The vector f in (8) is null.

C. The constraints

The correct definition of the constraints is fundamental to obtain a meaningful result from the optimization algorithm. In the case of the PV park, the constraints to model are two: the rated current of the converter for each harmonic and the full compensation of the voltage distortion in the PCC. The expression of the two constraints will be computed in the next paragraphs.

1) The rated current constraint: The maximum harmonic current constraint for a generic converter can be defined as follow:

$$|\dot{I}^{j^{th}}| \leq i_r^{j^{th}} \quad j = 5, 7 \tag{18}$$

where $i_r^{j^{th}}$ is the rated current for the *j-th* harmonic. In this paper it is assumed $i_r^{5^{th}} = i_r^{7^{th}}$, and the simplified notation i_r is used for the rated current. The p.u. values of the currents in (9) are referred to i_r. The constraint (18) expressed in term of optimization variables as defined in (9) corresponds to a circle with unitary radius: its parametric equation in Cartesian coordinates is nonlinear, and can not be expressed with a linear inequality like the one in (8). Thus, in order to reconduct the formulation to a QP problem, the constraint is reduced to its biggest linear subset, that is a square of side $\frac{2}{\sqrt{2}}$. By defining

$$\begin{cases} I_p = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad I_n = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \\ b_l = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \end{cases} \tag{19}$$

$$\begin{cases} A_1 = diag\left(I_p, I_p, I_p, I_p\right) & A_3 = -A_1 \\ A_2 = diag\left(I_n, I_n, I_n, I_n\right) & A_4 = -A_2 \end{cases} \tag{20}$$

$$\begin{cases} A_{lim} = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix} \quad b_{lim} = \begin{pmatrix} b_l \\ b_l \\ b_l \\ b_l \end{pmatrix} \end{cases} \tag{21}$$

the constraint for the maximum current can be expressed by the linear inequality

$$A_{lim} x \leq b_{lim} \tag{22}$$

TABLE I: Voltage active filtering parameters.

P_r	Power rating of each converter	105 kW
V_{DC}	Voltage in the DC link	700 V
C_{DC}	DC link capacitor	500 μF
$\dot{V}_{pcc}^{1^{st}}$	Fundamental voltage amplitude (peak)	315V
$\dot{V}_{pcc}^{5^{th}}$	5^{th} voltage harmonic (peak)	18.9V $\angle 0°$ (6%)
$\dot{V}_{pcc}^{7^{th}}$	7^{th} voltage harmonic (peak)	17.325V $\angle 0°$ (5.5%)
$\dot{Z}_{pcc}^{5^{th}}$	Grid impedance at 250 Hz	$0.18\Omega\angle 52°$
$\dot{Z}_{pcc}^{7^{th}}$	Grid impedance at 350 Hz	$0.26\Omega\angle 56°$
$\dot{Z}_{l}^{5^{th}}$	Line impedance at 250 Hz	$25m\Omega\angle 0°$
$\dot{Z}_{l}^{7^{th}}$	Line impedance at 350 Hz	$25m\Omega\angle 0°$
$i_r^{5^{th}}$	Rated current for the 5^{th} harmonic	$17A$
$i_r^{7^{th}}$	Rated current for the 7^{th} harmonic	$17A$
$\dot{I}_{pcc}^{5^{th}}$	Compensation current for the 5^{th} harmonic	$35A\angle -52°$
$\dot{I}_{pcc}^{7^{th}}$	Compensation current for the 7^{th} harmonic	$18.2A\angle -56°$

2) The PCC harmonic compensation constraint: According to (2), the current I_{pcc} is equal to the sum of all the currents injected by each PV converter. Considering the compensation current computed in (1), the equality $I_{pcc} = I_{comp}$ represents the PCC harmonic compensation constraint. Although the formulation of a QP problem in (8) includes only inequality constraints, the latter equality can be expressed through two inequalities in this way:

$$\begin{cases} \dot{I}_1 + \dot{I}_2 + \dot{I}_3 + \dot{I}_4 \leq \dot{I}_{comp} \\ -\dot{I}_1 - \dot{I}_2 - \dot{I}_3 - \dot{I}_4 \leq \dot{I}_{comp} \end{cases} \tag{23}$$

and, in term of matrices and with p.u. currents

$$A_{pcc} x \leq b_{pcc} \tag{24}$$

with:

$$\begin{cases} A_{comp} = \begin{pmatrix} I_{4\times4} & I_{4\times4} & I_{4\times4} & I_{4\times4} \\ -I_{4\times4} & -I_{4\times4} & -I_{4\times4} & -I_{4\times4} \end{pmatrix} \\ b_{comp} = \begin{pmatrix} \dfrac{I_{comp}}{i_r} \\ -\dfrac{I_{comp}}{i_r} \end{pmatrix} \end{cases} \tag{25}$$

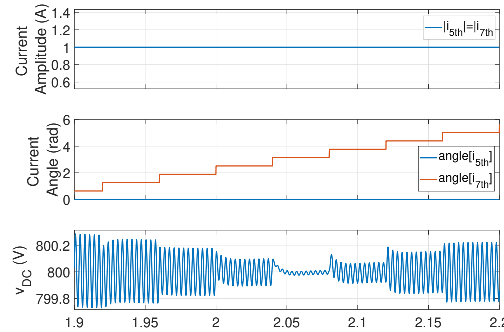
where $I_{2\times2}$ is the identity matrix of dimension 2.

The two constraints, namely the one expressing the rated current of each converter in (22) and the one for the full PCC harmonic compensation in (24) can be expressed together as $Ax \leq b$ where

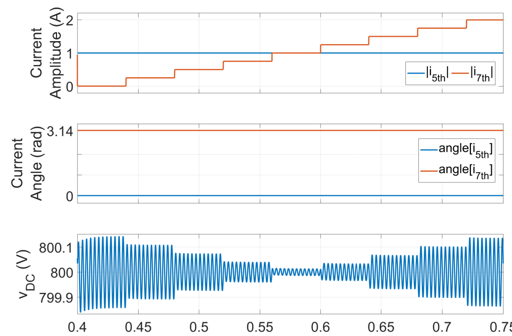
$$\begin{cases} A = \begin{pmatrix} A_{lim} \\ A_{comp} \end{pmatrix} \quad b = \begin{pmatrix} b_{lim} \\ b_{comp} \end{pmatrix} \end{cases} \tag{26}$$

V. Simulation Results

In Section III, the relation between the harmonic current injection and the DC link ripple is analytically derived. This relation is used by the optimization algorithm to minimize the DC link ripple. The validation of this analysis is performed in the next subsection. Then, the optimization algorithm is simulated in a PV park like the one in Fig. 1b under different weightings of the cost function.

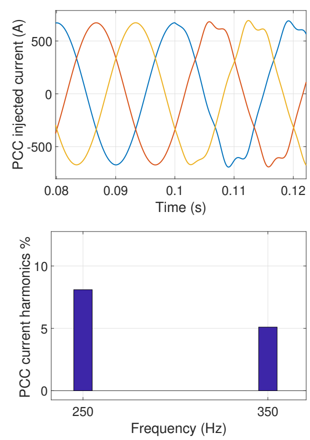


(a)

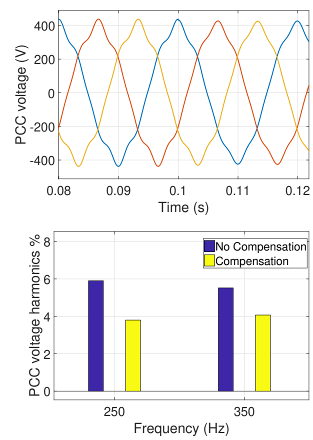


(b)

Fig. 4: (a) The DC link ripple variation depending on the phase displacement between the 5^{th} and the 7^{th} harmonic current. (b) The DC link ripple variation depending on the amplitude of the 5^{th} and the 7^{th} harmonic current.

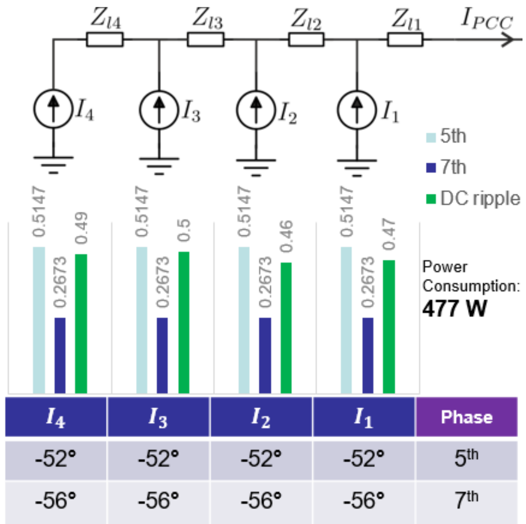


(a)

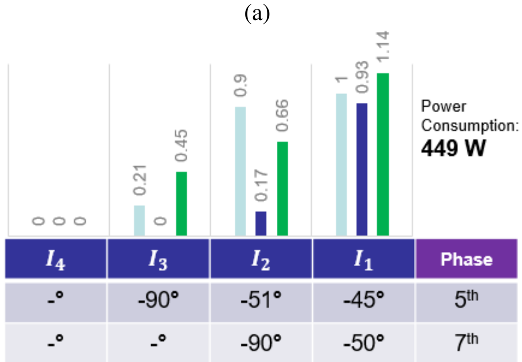


(b)

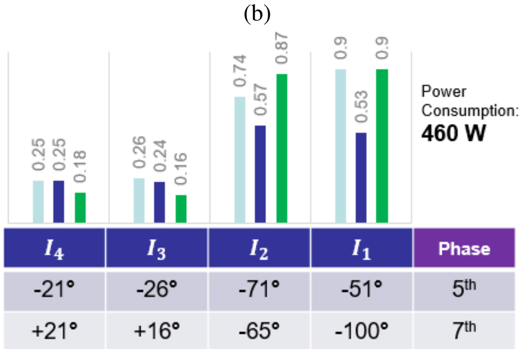
Fig. 5: Simulation of the active filtering enabled at the instant 0.1 s: (a) The PCC injected current waveform (b) The voltage at the PCC.



(a)



(b)



(c)

Fig. 6: The harmonic current injection distribution among the different converters of the PV farm for three optimization cost functions: (a) no optimization, (b) constant active power consumption, (c) balanced optimization.

A. DC Link Ripple Analysis Validation

The main result of Section III is the relation (5) which is used for the optimization algorithm. Fig. 3 realizes equation (5) with a three-dimensional. In this section, the aforementioned relation is validated through a simulation on an average model as in Fig. 2 and the results are shown in Fig. 4. The 5^{th} harmonic current is set to:

$$\begin{cases} |\dot{I}_{5th}| = 1A \\ \underline{/\dot{I}_{5th}} = 0rad \end{cases} \tag{27}$$

and the 7^{th} is varied at first in its phase and then in its amplitude. In Fig. 4(a) it is set $|\dot{I}_{7th}| = 1$ and the phase is varied with intervals of $\frac{\pi}{5}$. The minimum DC link ripple is obtained for an angle difference of π between the 5^{th} and the 7^{th} harmonic current. In Fig. 4(b) the phase of the 7^{th} harmonic is fixed $\underline{/\dot{I}_{7th}} = \pi$ and the amplitude is varied with intervals of 0.25A. The minimum DC link ripple is obtained when the amplitude of the two injected currents are equal.

B. PV Park Optimization

In section IV, the matrices and the vectors which formulate the PV farm optimization problem as described in (8) have been computed. The latter form is very suitable for numerical solvers, which compute the optimal solution from the input parameters H, f, A and b. A model of the PV park like in Fig. 1b has been implemented in simulation; the converters have been modeled with an average model as in Fig. 2. The parameters used for the simulation are described in Table I, and the compensation current I_{comp} computed through equation (1) is declared in the bottom of the table. The simulation results in Fig. 5 shows how the injection of the compensation current is able to reduce until the 4% the amplitude of the each PCC harmonic, in order to comply with the standards [3].

The simulation of the optimization algorithm has been carried out with three different weightings of the cost function defined in (17). The PCC injected current shown in Fig. 5 is divided in different ways among the converters according to the selected cost function. The distribution graphics are shown in Fig. 6. For a more meaningful analysis, the currents obtained in the dq frame from the optimization solver are expressed in p.u. magnitude and phase; the DC link ripple is expressed in p.u. respect to a ripple of the 3% of the nominal DC link voltage.
The first result in Fig. 6a shows the case with no optimization ($\bar{\rho} = 0, \tilde{\rho} = 0$); this weighting gives the same result of the case in which only the DC link ripple is optimized ($\bar{\rho} = 0, \tilde{\rho} = 1$). In this case the total current needed in the PCC is equally divided among the four converters. The DC link ripple index is almost the same for each of the four converters and the active power consumption for the filtering is 477 W.

In the second case, only the constant active power is optimized ($\bar{\rho} = 1, \tilde{\rho} = 0$). As it can be seen in Fig. 6b, the converters that are closer to the PCC inject as much current as possible, and the remaining current is injected by the next converters. Indeed, due to the line impedances the voltage in the AC bus gets higher as the distance from the PCC increases and the converters that are far away from the PCC consume more power to inject the same amount of current. The consumed active power in this case is 449 W, the 6% less than the case without optimization. However, the strong injection of 5^{th} and 7^{th} harmonics by the closest converters determines huge DC link ripples in those converter, that reduce their reliability.

The balanced optimization approach $(\bar{\rho} = 1, \tilde{\rho} = 1)$ shown in Fig. 6c realizes the concept of using the closest converters more than the others to save power, but in the same time takes into account the DC link ripple problem. In Fig. 6c it can be seen as, also in this case, the closest converters inject a higher amount of current respect to the others. However, the converter 1 injects the 7^{th} harmonic with a phase of $-100°$, much different from the value requested in the PCC, that is $-56°$. Since the oscillatory power is dependent on the phase displacement of the 5^{th} and 7^{th} harmonic currents according to equation (5), the optimization algorithm plans a high current injection by the first converter with a phase displacement between the two harmonics as close as possible to the condition (7), whereby the injection doesn't produce fluctuations on the DC link voltage. However, the phase of the total current injected the PCC must be the one computed through equation (1), and for this reason the further converters inject smaller currents such as to correct the phase of the current injected by the first converter. The consumed power in this case is 460 W, the 4% less than the case without optimization. Nevertheless, the DC link ripple in the closest converter is much lower respect to the case of Fig. 6b, and in all the converter is kept smaller than 3%.

VI. Conclusion

In this paper, a PCC voltage harmonic filtering functionality has been embedded in a PV farm. A QP optimization algorithm has been used to minimize the active power consumption and the DC link ripple in the PV converters, due to the filtering currents injection. From the results it emerges that an equal sharing of the compensation current between the PV converters is an effective strategy for the DC link ripple reduction, but still presents poor performances in term of power consumption. The optimization of the power consumption realizes a different sharing of the currents which brings a significant reduction on the consumed active power, equal to the 6% less than the previous case. However, in this case a critical DC link ripple affects the converters that are closer to the PCC. A balanced optimization which realizes a compromise between power consumption and DC link ripple reduction is the proposed solution. The consumed power is the 4% less than the equal sharing case, and the DC link ripple in the first converter is the 21% less than in the case of optimized power consumption.

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