GLOBAL AND LOCAL INSTABILITY OF THIN-WALLED COLUMNS BEYOND THE PROPORTIONAL LIMIT

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The problem of global and local stability in the inelastic range for thin-walled columns is examined on the basis of the deformation theory and the incremental theory of plasticity. Columns of closed and open cross-sections built from rectangular isotropic plates are subjected to the loading changing from uniform compression to pure bending. A solution of elastic buckling for a thin-walled orthotropic columns based on Koiter's asymptotic method is employed to investigate the elasto-plastic buckling mode of the column and to determine the critical load. The study is based on the numerical method of the transition matrix. The results of numerical calculations are presented in diagrams.

1. Introduction

Thin-walled structures consisting of plate elements have a number of buckling modes differing from one to another both in quantitative (by number of half waves) and in qualitative respects (by global and local buckling)

Isotropic materials (used in structural members) such as mild steel and aluminium alloys are characterised by typically linear elastic behaviour up to the proportional limit, and in the elasto-plastic range by rounded stress - strain curve.

The inelastic buckling of plates and columns can be investigated on the basis of the constitutive relations of the deformation theory and the incremental theory of plasticity. It has been well known that calculations based on the deformation theory which in fact does not represent physically admissible theory of plasticity gave reasonably good agreement with test results.

In recent years the inelastic buckling of plates and columns has been a point of interest in many works e.g. (3), (6).

In the present paper the problem of stability in the elasto-plastic range of thin-walled columns is examined using the method elaborated for the analysis of stability of thin-walled orthotropic beam-columns (5). The relationships between the stress and strain for a component elasto-plastic plate are derived on the basis of the deformation theory and the incremental theory of plasticity. On the other side the same relationships are written for an orthotropic elastic plate. Comparing the appropriate coefficients in both relations the instantaneous „conventional" parameters of orthotropy can be found out. So the problem of inelastic stability of isotropic columns can be investigated in the same way as the problem of stability of the elastic orthotropic columns. The elastic problem is solved using Koiter's asymptotic theory of stability (4). In the solution and in the prepared computer programme Byskov-Hutchinson asymptotic expansion (1) and the numerical transition matrices are employed. This approach enables to find all global and local buckling modes of the structures analysed (5). In the solution obtained the shear lag phenomenon and the effect of cross-sectional distortions are included.

The results of numerical calculations are plotted in diagrams showing the critical stress values calculated for a given column under assumed loading.

2. Constitutive relations in the inelastic range

In the studies concerning the stability of structures beyond the proportional limit it is essential to describe in the analytical way the uniaxial stress-strain curve of a material. In this work the Needleman-Tvergaard relation is used in modelling the behaviour of nonlinear characteristic of a material.

- Elastic behaviour:

$$\sigma = E*\varepsilon \tag{1}$$

- Inelastic behaviour (for $\sigma \geq \sigma_0$, where σ_0 is the proportional limit):

$$\sigma = \sigma_0 \left[n\left(\frac{\varepsilon * E}{\sigma_0} - 1 \right) + 1 \right]^{\frac{1}{n}} \tag{2}$$

According to the deformation theory the relations between the stresses and strains are:

$$\begin{aligned} &N_x = h*\left(A_{11}*\varepsilon_x + A_{12}*\varepsilon_y\right), \qquad N_y = h*\left(A_{12}*\varepsilon_x + A_{22}*\varepsilon_y\right), \\ &N_{xy} = h*A_{33}*\gamma_{xy}. \end{aligned} \tag{3}$$

where the coefficients:

$$A_{11} = E*\frac{3*\varphi_s + \varphi_t}{A_0}; \qquad A_{12} = E*\frac{2*\varphi_t - 2*(1-2*\nu)}{A_0}; \qquad A_{22} = E*\frac{4*\varphi_t}{A_0};$$

$$A_{33} = E*\frac{1}{3*\varphi_s - (1-2*\nu)}; \qquad A_0 = 3*\varphi_s*\varphi_t + 2*(1-2*\nu)*\varphi_t - (1-2*\nu)^2 \tag{4}$$

and $\varphi_t = E / E_t$; $\varphi_s = E / E_s$.

are obtained from the uniaxial stress-strain curve of the material of the column described by equation (2), where E_s -secant modulus and E_t -tangent modulus. The relations above correspond to the deformation theory. The relationships for the incremental theory are obtained simply by putting E_s=E.

3. Basic equations for orthotropic elastic columns

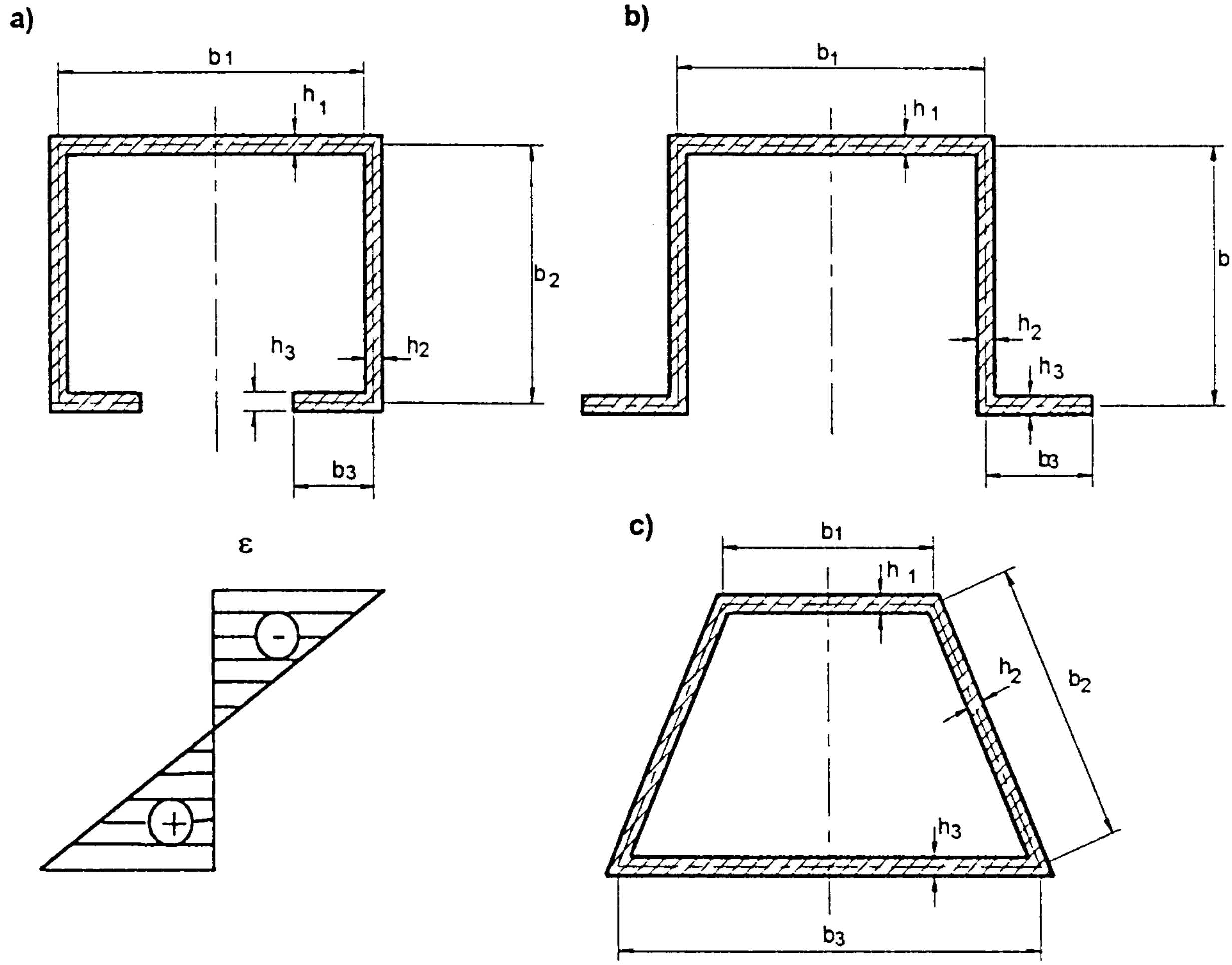

Fig 1. The considered cross-sections

The long thin-walled prismatic columns of length l and composed of plane, rectangular plate segments interconnected along longitudinal edges are considered. The columns of closed and open cross-sections, simply supported at the ends, are subjected to the axial strain ε varying linearly along the column height (Fig. 1).

A plate model is adopted for the columns. For a component plate exact geometrical relationships are taken into account in aim to consider both out-of-plane and in-plane bending of a plate (5):

$$\varepsilon_x = u_{,x} + 0.5(u_{,x}^2 + v_{,x}^2 + w_{,x}^2); \qquad \varepsilon_y = v_{,y} + 0.5(u_{,y}^2 + v_{,y}^2 + w_{,y}^2);$$
$$\gamma_{xy} = u_{,y} + v_{,x} + u_{,x}u_{,y} + v_{,x}v_{,y} + w_{,x}w_{,y}; \qquad (5)$$

while the bending strains are given by

$$\kappa_x = -w_{,xx}; \qquad \kappa_y = -w_{,yy}; \qquad \kappa_{xy} = -w_{,xy}. \qquad (6)$$

Physical relationships for a component plate treated as an orthotropic with principal axes of orthotropy parallel to its edges are formulated in the following way:

$$N_x = h*(K_{11}*\varepsilon_x + K_{12}*\varepsilon_y); \qquad N_y = h*(K_{12}*\varepsilon_x + K_{22}*\varepsilon_y);$$
$$N_{xy} = h*K_{33}*\gamma_{xy}. \tag{7}$$

where:

$$K_{11} = E_x/(1-\nu_{xy}*\nu_{yx}); \qquad K_{22} = E_y/(1-\nu_{xy}*\nu_{yx})$$
$$K_{12} = \nu_{yx}K_{11} = \nu_{xy}K_{22}; \qquad K_{33} = G_{xy}\gamma_{xy}. \tag{8}$$

The dependence in Young's moduli and Poisson's ratios in (7) and (8) is as follows:

$$E_x*\nu_{yx} = E_y*\nu_{xy}. \tag{9}$$

The elastic problem is solved by the asymptotic Koiter method (4). Displacement field $\overline{\mathbf{U}}$, and sectional force $\overline{\mathbf{N}}$, are expanded in power series in the buckling mode amplitude ζ , (ζ is the amplitude of buckling mode divided by the thickness of the first component plate):

$$\overline{\mathbf{U}} = \lambda*\overline{\mathbf{U}}^{(0)} + \zeta*\overline{\mathbf{U}}^{(1)} + \ldots \qquad \overline{\mathbf{N}} = \lambda*\overline{\mathbf{N}}^{(0)} + \zeta*\overline{\mathbf{N}}^{(1)} + \ldots \tag{10}$$

where the prebuckling fields are $\overline{\mathbf{U}}^{(0)}$, $\overline{\mathbf{N}}^{(0)}$ and the buckling modes fields are $\overline{\mathbf{U}}^{(1)}$, $\overline{\mathbf{N}}^{(1)}$.

By substituting the expansion (10) into equations of equilibrium, junction conditions and boundary conditions, the boundary value problems of zero and first order can be obtained. The zero approximation describes the prebuckling state while the first order approximation enables to determine the critical loads of global and local value and the buckling modes (5).

The described above method is applied to calculate the critical load values for isotropic columns in the inelastic range. The procedure is based on the fact that the relationships (3) and (7) have the identical form. So, the coefficients K_{11} -K_{33} can be replaced by coefficients A_{11} -A_{33} . In other words equating $K_{11}=A_{11}$, , etc. the „false" parameters of orthotropy E_x , E_y , G_{xy} , ν_{yx} can be found out as functions of E_s , E_t , E, ν calculated for each value of the axial strain ε from the relation (2) describing the inelastic behaviour of a material. It should be noted that when the loading of a column is described by strains varying linearly along the column height the coefficients A_{11} -A_{33} are constant for the upper and lower flange and they are variable for a web. Therefore the component plates such as flanges can be treated as plates of constant orthotropy parameters and the web has to be analysed as a plate of a variable orthotropy, modelled by strips of different othotropy parameters.
The problem is solved in a numerical way.

4. Results of numerical calculations

Some exemplary results are presented for columns of a cross-section shown in Fig.2b, subject to pure compression, of geometrical parameters as follows:
$b_1=40\ mm$, $b_2=20\ mm$, $b_3=4\ mm$, $h_1=h_2=h_3=1\ mm$, $l=140\ mm$;
The material properties correspond to the aluminium alloy: $E=0.7\times10^5\ MPa$, $\nu=0{,}32$; $\sigma_o=200\ MPa$, $n=5$.

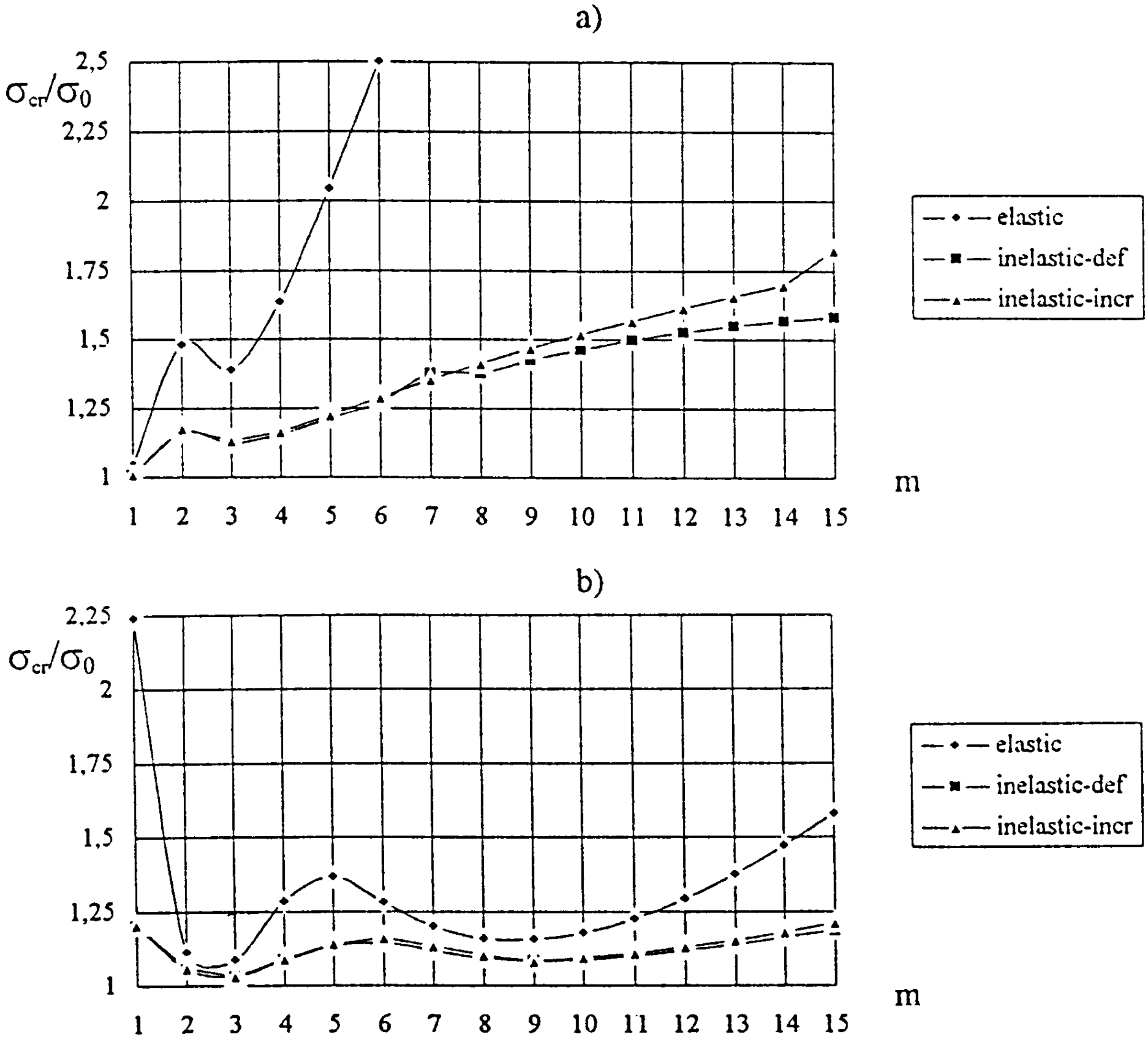

Fig. 2. Dimensionless critical stress σ_{cr}/σ_0 versus a number of axial half waves m

In Figs. 2a, 2b the dimensionless critical stress σ_{cr}/σ_0 as a function of axial half waves m is presented, assuming purely elastic and elastic-elasto-plastic behaviour of a material. In the inelastic range the equations of the deformation theory (*def*) and the incremental theory (*incr*) of plasticity have been applied. From Figs.2 follows that the value of global critical stress is lower when the antisymmetrical mode of buckling is assumed (Fig.2a) - it corresponds to the flexural-torsional mode both in the elastic and elasto-plastic range (Fig.3b). In the case of symmetrical buckling (Figs.2b and 3a) the change of buckling modes occurs- from a flexural-distortional mode when the elastic behaviour of a material is assumed to the flexural buckling (dash-dotted line) in the inelastic range .

It should be underlined that the adaptation of a plate model in the buckling analysis with the strain tensor expressed by eqs. (6) allows to find out other than classical modes - so called „mixed modes" (2) - and gives lower values of the critical stress than the values obtained when a beam model is assumed.

If local buckling is considered *m>1* for symmetry conditions the curves presenting the critical stress have two minima (Fig.2b), assuming antisymmetrical mode only one minimum can be found out (Fig.2a). For each minimum the local buckling modes are of different shape.

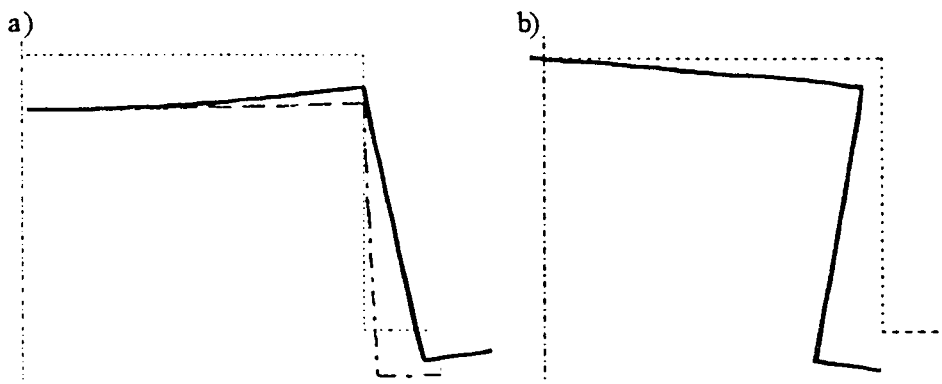

Fig. 3. Global buckling modes for columns with outside edge stiffeners

5. Final remarks

The presented method allows to find out all possible buckling modes (global and local, symmetrical and antisymmetrical) and also to investigate „mixed modes" occuring in columns of open cross-sections. The adaptation of a plate model in the buckling analysis with the strain tensor expressed by eqs. (6) gives lower values of the critical stress than the values obtained on the basis of the conventional theory of plates. The method elaborated for elastic orthotropic columns can be applied in the analysis of inelastic buckling of isotropic columns.

6. References

7. E.Byskov and J.W. Hutchinson, *AIAA J.*, 1977, pp. 941-948
8. D. Dubina, *Proc.of Sec.Int.Conf. on Coupled Instability in Metal Str.*, Imperial Press, 1996, pp.119-132
9. R.Grądzki and K.Kowal-Michalska, *Thin Walled Structures*,3, 1985, pp. 93-108
10. W.T. Koiter, , Proc. of the Symp. on Nonlinear Problems, Univ. of Wisconsin Press, Wisconsin, 1963
11. Z. Kołakowski and M. Królak, , *Engineering Transactions*, 43, 4, 1995, pp. 571-590, pp. 591-602
12. A. Needleman and V.Tvergaard, *Int.J.Mech.Sci.*,17,1975 pp.419-424