STABILITY OF COMPOSITE PLATES UNDER VARYING AXIAL FORCES

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The aim of this contribution is to show how the known theories of elastic plates can be extended in order to describe property arbitrary inhomogeneous of the plates material for dynamics stability. Each layer being the composite components is taken to be inhomogeneous and anisotropic. The perfect contact between the components of the plate is assumed.

1. ANALYSIS

The analysis will be based on the assumption that the midplane of an undeformed plates coincides with the symmetry plane. Averaged equations of motion can be derived from the virtual work principle of three-dimensional elastodynamics. Applying the results of the papers [1] and [2], the governing equations take the following form:

$$\begin{gathered}(D_{\alpha\beta}-K_{\alpha\beta})(\varsigma_{\beta,\alpha}+w_{,\alpha\beta})+N_{\alpha\beta}(t)\,w_{,\alpha\beta}-m\ddot{w}=0\\ D_{\alpha\beta\gamma\delta}\,\varsigma_{\gamma,\delta\beta}-(D_{\alpha\beta}-K_{\alpha\beta})(\varsigma_{\beta}+w_{,\beta})-I\ddot{\varsigma}_{\alpha}=0\end{gathered}\tag{1}$$

where

$$\begin{gathered}D_{\alpha\beta\gamma\delta}\equiv\int_{-\frac{h}{2}}^{\frac{h}{2}}a_{\alpha\beta\gamma\delta}z^{2}dz\qquad D_{\alpha\beta}\equiv\int_{-\frac{h}{2}}^{\frac{h}{2}}A_{\alpha3\gamma3}\,dz\\ E^{a}_{\alpha\gamma}\equiv\int_{-\frac{h}{2}}^{\frac{h}{2}}A_{\alpha3\gamma3}\,l_{a,3}\,dz,\qquad F^{ab}_{\alpha\gamma}\equiv\int_{-\frac{h}{2}}^{\frac{h}{2}}A_{\alpha3\gamma3}\,l_{a,3}\,l_{b,3}\,dz\end{gathered}\tag{2}$$

The coefficients $F^{ab}_{\alpha\gamma}$ is non- singular, denoting by $G^{ab}_{\alpha\gamma}$ the inverse mapping, we can written $K_{\alpha\beta}$ as

$$K_{\alpha\beta}=E^{a}_{\alpha\gamma}\,G^{ab}_{\gamma\varepsilon}\,E^{b}_{\varepsilon\beta}\tag{3}$$

Now we restrict ourselves to the analysis dynamics stability of a rectangular plates which are simply supported on its edges $x_1=0,\ x_1=a,\ x_2=0,\ x_2=b.$

We assume that a composite plates consisting of n ortotropic inhomogeneous layers. Material properties of this plate are assumed to be known being determined by the components of the elasticity tensor field $a_{ijkl}(\cdot)$, which are piecewise constant functions, suffering jump discontinuities only across interfaces between material constituents. The uppers and the lowers layers have the same thickness and are made of the same material.

Let us seek the solution of the system of differential equations (1) in the form:

$$\begin{aligned} w &= W_{mn}(t)\sin(\lambda_m x_1)\sin(\mu_n x_2) \\ \vartheta_1 &= A_{mn}(t)\cos(\lambda_m x_1)\sin(\mu_n x_2) \\ \vartheta_2 &= B_{mn}(t)\sin(\lambda_m x_1)\cos(\mu_n x_2) \end{aligned} \tag{4}$$

where: $\lambda_m = \dfrac{m\pi}{a}$, $\mu_n = \dfrac{n\pi}{b}$

We see that solution (4) fulfils the boundary conditions, and is independent of the shape of the functions w_{mn}, ϑ_{1mn}, ϑ_{2mn} which are, so far, quite unknown. With the presuppositions (4) our problem is reduced to seeking the funktions w_{mn}, ϑ_{1mn}, ϑ_{2mn}, which are already only functions of the time *t*.

We now substitute the assumed solution (4) into the differential equations system (1). After rearrangement we obstain

$$\frac{d^2W_{mn}(t)}{dt^2} + \omega_{mn}^2\left(1 - \frac{P(t)}{N_{mn}^{11}}\right)W_{mn}(t) = 0 \tag{5}$$

where: ω_{mn}^2 is the angular natural frequency in *rad/sec* and N_{mn}^{11} is the critical load.

$$N_{mn}^{11} = -\frac{1}{\lambda_m^2}\left[D_{11}(1-\alpha_1)\lambda_m^2 + D_{22}(1-\alpha_2)\mu_n^2 + \frac{\alpha_{mn}}{\beta_{mn}}\right] \tag{6}$$

We now assume that $P(t)$ is cosinesoidal force

$$P(t) = S_a + S_b\cos\Theta t \tag{7}$$

This force consists of a stationary part S_a and a periodically varying part $S_b\cos\Theta t$, haveing amplitude S_b and radian frequency Θ. Substituting (7) into (5), we arrive at the following equation

$$\frac{d^2W_{mn}}{dt^2} + \Omega_{mn}(1 - 2\nu_{mn}\cos\Theta t)W_{mn}(t) = 0 \tag{8}$$

where we have denoted

$$
\Omega_{mn}^2 = \omega_{mn}^2\left(1-\frac{S_a}{N_{mn}^{11}}\right)
$$

$$
\nu_{mn} = \frac{S_b}{2\left(N_{mn}^{11}-S_a\right)} \tag{9}
$$

Equation (8) is known as the Mathieu equation.

In each particular case the quantities Ω_{mn} and ν_{mn} can be calculated by using Eq. (9). The character of the solution of Eq. (8) depends on the numerical values of Ω_{mn} and ν_{mn}. At certain of these quantities the solution gives a vibration which grows with time and thereby indicates an unstable condition. By introducing the following dimensionless values

$$
k_1 \equiv \frac{D_1}{D_2},\ k_2 \equiv \frac{D_1}{D_3},\ k_3 \equiv \frac{\widetilde{D}_{22}}{D_2},
$$

$$
k_4 \equiv \frac{\widetilde{D}_{11}}{D_1},\ c \equiv \frac{a}{b},\ h^* \equiv \frac{h}{a} \tag{10}
$$

where

$$
D_{11} = h\widetilde{D}_{11},\ D_{22} = h\widetilde{D}_{22},\ D_{1111} = D_1 h^3,\ D_{2222} = D_{22} h^3
$$

$$
D_{1122} = D_{2211} = D_2 \nu_1,\ D_{1212} = D_3 h^3 .
$$

Therefore the critical load can be written

$$
N_{mn}^{11} = {}^*N_{mn}^{11}(1+d) \tag{11}
$$

where

$$
d = -\frac{\alpha_1 k_1 (m\pi)^2 + c^2\alpha_2 (n\pi)^2}{k_1(m\pi)^2 + c^2(n\pi)^2} + \frac{1}{(h^*)^2}\cdot\frac{1}{k_1(m\pi)^2 + c^2(n\pi)^2}\cdot k_3\frac{a_{mn}}{b_{mn}},
$$

$$
\begin{aligned}
a_{mn} &= 2(1+\nu_1)(1-\alpha_1)(1-\alpha_2)c^2(m\pi)^2(n\pi)^2 + \\
&- k_1(1-\alpha_1)^2(m\pi)^2\left[\frac{k_1}{k_2}(m\pi)^2 + c^2(n\pi)^2 + k_3\left(\frac{1}{h^*}\right)^2(1-\alpha_2)\right] + \\
&- (1-\alpha_2)^2c^2(n\pi)^2\left[(m\pi)^2 + \frac{c^2}{k_2}(n\pi)^2 + k_3\left(\frac{1}{h^*}\right)^2(1-\alpha_1)\right]
\end{aligned}
$$

$$
\begin{aligned}
b_{mn} &= \left[(m\pi)^2 + \frac{c^2}{k_2}(n\pi)^2 + k_3\left(\frac{1}{h^*}\right)^2(1-\alpha_1)\right]\cdot\left[\frac{k_1}{k_2}(m\pi)^2 + \right. \\
&\left. + c^2(n\pi)^2 + k_3\left(\frac{1}{h^*}\right)^2(1-\alpha_2)\right] - \frac{1}{k_1}\left(\nu_1 + \frac{k_1}{k_2}\right)^2(m\pi)^2(n\pi)^2
\end{aligned} \tag{12}
$$

If $S_a = 0$ this the first two areas indicate regions of stability are given in the form.

$$\frac{\Theta_{mn}}{2\Omega_{mn}} = \sqrt{1 \pm \frac{S_b}{2 \cdot {}^*N_{mn}^{11}} \cdot \frac{1}{1+d}}, \quad \frac{S_b}{2 \cdot {}^*N_{mn}^{11}} \leq \frac{1+d}{2}$$

$$\frac{\Theta_{mn}}{2\Omega_{mn}} = \frac{1}{2}\sqrt{1 + \frac{1}{3}\left(\frac{S_b}{2 \cdot {}^*N_{mn}^{11}}\right)^2 \cdot \left(\frac{1}{1+d}\right)^2} \tag{13}$$

The influence of ratio α_1, α_2 upon areas indicate regions of stability are shown in Fig 1.

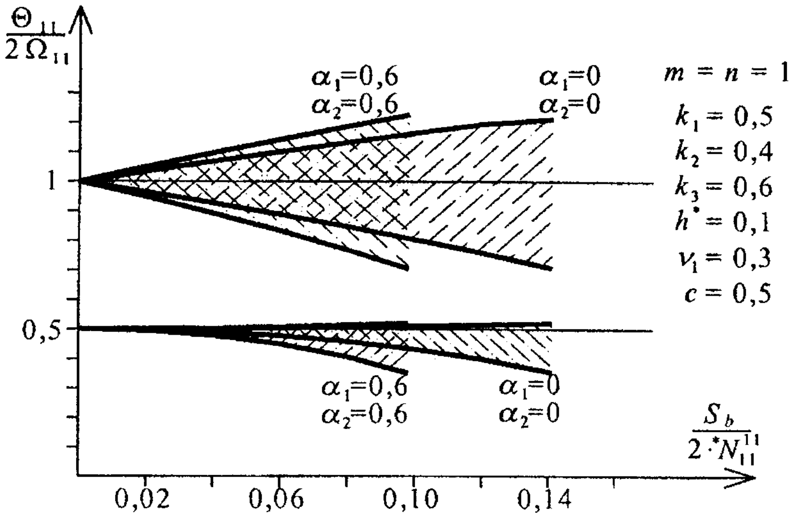

Fig. 1. The shaded areas indicate regions of stability.

Values of $\frac{\Theta_{11}}{2\Omega_{11}}$, $\frac{S_b}{2 \cdot {}^*N_{11}^{11}}$ given by coordinates of points in the unshaded areas represent an unstable condition.

2. FINAL CONCLUSIONS

In this paper, we have outlined a procedure leading from the equations of the linear elasticity for inhomogeneous plate-like bodies to the equations of the 2D-theories proposed for inhomogeneous plates. Every such procedure takes into account an arbitrary but fixed well-known plate kinematics constraints supplemented by an extra term which describe the effect of material inhomogeneity on plate stiffness. If the parameters α_1 and α_2 are different from zero then the areas that indicate regions of stability are greater than for $\alpha_1 = \alpha_2 = 0$.

REFERENCES

[1] Konieczny S., Langier Sz., Mes R., Dynamics and stability of multilayered composite plates. Studia Geotechnica et Mechanica, Vol. XXIII, No. 1 – 2, 2001, pp. 59 – 67.

[2] Konieczny S., Langier Sz., Mes R., Yevtushenko A., Dynamic analysis of inhomogeneous multilayered plates. Proceedings ICCE/9 San Diego, California 2002, pp. 759 – 760.