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Locality in the creation of electron-positron pairs

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We examine the mathematical solutions of the Dirac equation to predict the spontaneous electron-positron pair creation from the vacuum. The Dirac equation contains a position and time-dependent scalar potential to approximate the effect of an external force on the vacuum. We focus on forces that are localized in space as well as in time and find that the resulting creation process is also localized in time but delocalized in space. This illustrates that the Dirac equation can show nonlocal behavior as it predicts that particles can be created even in spatial regions where the force is zero. We also examine the spatial distribution of the created particles and show that for spatially extended force fields it is proportional to the square of the position dependence of the force. But when the force field is narrower than the Compton wavelength, the created electron density approaches a universal shape invariant form that is independent of the strength of the force for sufficiently weak field strength.

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I. INTRODUCTION

The Dirac equation provides a very fundamental description of atoms and their interaction with external electromagnetic fields [1]. Its prediction of the existence and properties of the electron's antiparticle, the positron, is viewed by many as one of the most striking examples for its validity and accuracy. At the same time, this equation, if interpreted as a single particle equation for a wave function (and not for a quantized Fermion field operator), can lead to interpretational difficulties that arise when states associated with the negative energy spectrum [2] are involved in the dynamics. The famous Klein paradox [3–5], the so-called Zitterbewegung [6], or the localization problem [6–9] are just a few examples.

Another significant prediction of the Dirac equation is the spontaneous creation of an electron-positron pair associated with the decay of the vacuum. The precise details of this breakdown mechanism in full spatial and temporal resolution are presently not well understood, despite the pioneering works of Schwinger [10]. Experimentally, this process was examined in the context of the collision of two charged ions [11]. For situations where the combined Coulomb potential of the ions becomes supercritical, the Dirac equation predicts the (spontaneous) creation of electron-positron pairs. Experiments at GSI in Germany [12] were interpreted to confirm such a process but a sequence of follow up experiments performed at Argonne National Laboratory [13] could not verify the data and suggested that the observed positrons were associated with complicated excitation processes of the involved nuclei and not with the supercriticality of the field. As a result it still remains an intriguing question whether the Dirac equation prediction of the destruction of the vacuum for a sufficiently strong force field is actually correct.

Several laboratories in Germany [14–18] and in France [19–22] are developing new laser systems that will have either sufficient intensity or frequency to reexamine the question about supercriticality in a more direct way without the unwanted complications of nuclear or other processes. We expect that these planned experiments will lead to a renewed interest in this important fundamental question. In order to prove or disprove the validity of the Dirac equation it is important that its predictions—whether unphysical or physical—are fully understood.

In this article, we will examine the predictions of this theory for subcritical but time-dependent model potentials. It is important to notice that it is not our intention to use this approach to make accurate predictions for real physical processes, but to examine extreme situations that can illustrate how the Dirac equation in its present mathematical formulation can lead to results that can violate locality. The principle of locality means that distant objects cannot have direct and instantaneous influences on each other.

There are two previous works that are most directly related to the present one. In 2006 it was shown how the total duration between a turned-on and turned-off external force field determines the final yield of created electron-positron pairs [23]. For a subcritical potential the number of particles can change only if the force field is time dependent. If those particles that are created during the temporal turn on have enough time to accelerate out of the spatial force region, the turn off can create another burst of particles. If, however, the turn off follows almost directly the turn on (corresponding to a plateau with very short duration) the created particles are annihilated, reducing the total yield. In 2007 we examined in more detail [24] the time dependence of the electron positron population during a linear turn on, $V(z,t)=V(z)t/T$. It was observed that during short times, the number of pairs can grow in an oscillatory fashion. We also tested the response of the system to purely periodic excitations, $V(z,t)=V(z)\sin(\omega t)$ and noted an enhanced response (largest growth of the number of pairs) for frequencies ω that are close to the energy difference between upper and lower energy continuum states, $\omega=2c^2$.

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In this article we focus on the precise location where the particles are created. In order to pinpoint this location of particle birth, we use a subcritical step potential that is abruptly turned on such that particles can be created only at a single instant in time. We also analyze the spatial density of the created electrons for force fields with various spatial extensions. It turns out that for a sufficiently wide force field, the position dependence of the spatial density of the created matter follows the square of the force. In the limit of a spatially narrow force field the shape of the distribution becomes universal and independent of the magnitude of the potential.

In Sec. II we describe our model system and in Sec. III we discuss for a spatially and temporally abruptly turned-on potential how the total number of created particles depends on the magnitude of the force field. In Sec. IV we discuss the more complicated situation where particles can be created at various moments in time. We finish with a brief summary and an extended discussion.

II. THE MODEL SYSTEM

In several works [11,25–27], the interaction of electrons or positrons with external forces along the z direction is modeled by the Dirac equation (in atomic units with $c=137.036$ a.u.),

$$i\partial\Psi(t)/\partial t = [c\alpha_z p_z + \beta c^2 + V(z,t)]\Psi(t), \tag{2.1}$$

where $\Psi(t)$ is the electron-positron field operator, α_z and β are the usual 4×4 Pauli matrices, and the scalar potential $V(z,t)$ approximates the interaction with an external force. In this approach the interaction between fermions is excluded. To examine this interaction would require a coupling of the fermions to a photon field that would need to be included as a fully dynamical variable.

The evolution of the corresponding electronic density matrix operator $\rho(t)$ can be obtained from quantum field theory [25,28] via $\rho(t) \equiv \langle\langle \Phi(t=0) \| \Psi^{(+)\dagger}(t)\Psi^{(+)}(t) \| \Phi(t=0)\rangle\rangle$. The symbol $\|\Phi(t=0)\rangle$ denotes the quantum field initial state which in this work will be the vacuum associated with the force-free Dirac Hamiltonian. The superscript $(+)$ denotes the electronic portion of the field operator $\Psi(t)$ that can be obtained by projecting [23,29] the operator Ψ onto the subspace covered by those instantaneous energy eigenvectors $|P\rangle$ of the generator in Eq. (2.1) that have electronic energies E_{P_t}, $[c\alpha_z p_z + \beta c^2 + V(z,t)]|P_t\rangle = E_{P_t}|P_t\rangle$, where for a positive potential $V(z,t)$ this corresponds to energies that are larger than $E_{P_t} > c^2$. The diagonal element of the electronic density operator in the spatial representation, defined as $P(z,t) \equiv \langle z|\rho(t)|z\rangle$, is interpreted here as the spatial probability density. The total number of created pairs $N(t)$ corresponds to the spatial integral of $P(z,t)$, leading to

$$N(t) = \sum_P \sum_n |\langle n(t)|P_t\rangle|^2, \tag{2.2}$$

where we denote with $|n(t)\rangle$ the time-evolved eigenstates of the force-free Dirac operator $c\alpha_z p_z + \beta c^2$ with the negative energies. For more computational details on numerical solutions of the Dirac equation, see Refs. [30–33].

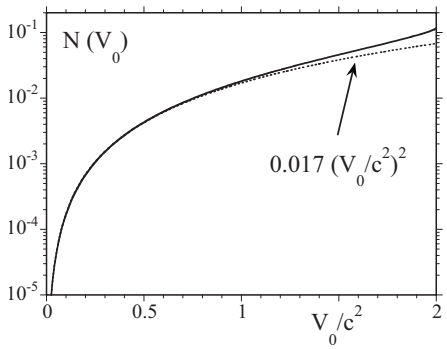

FIG. 1. The number of electrons $N(V_0)$ created during an electron-positron pair creation when a subcritical force field $V(z) = V_0\Theta(z)$ is turned on abruptly in time. For small values of V_0 it scales like $0.017V_0^2/c^4$ (dash curve). (Obtained by analytical integration as well as numerical integration of the time-dependent Dirac equation with $N_z=2048$ spatial grid points and a numerical box size of $L=0.3$ a.u. The momentum limits used were $k=-25\,600$ a.u. to $25\,600$ a.u.)

III. INSTANTANEOUSLY TURNED-ON FORCE FIELDS

As a model force field to examine the temporally induced pair creation process, we choose a one-dimensional potential [34] of the form $V(z,t) \equiv V_0\,[1+\tanh(z/W)]/2\;\Theta(t)$, where V_0 is the potential height and the width W is proportional to the region of nonzero force. In order to explore the importance of the force region, we have also run our simulations for the limiting case of a vanishing force zone, corresponding to $W=0$ and thus $V(z,t)=V_0\Theta(z)\;\Theta(t)$, where $\Theta(\ldots)$ denotes the Heaviside unit-step function, defined as $\Theta(z) \equiv (1+|z|/z)/2$.

This limiting case is of special interest. One could incorrectly expect that for any creation of particles to occur it is necessary for the spatial extension of the force to be nonzero. If this were true, then a force with a vanishing extension [associated with the potential $V(z) \sim \Theta(z)$] would not create pairs at all. We will illustrate below that this is not correct. It follows mathematically from the Dirac operator in Eq. (2.1) that particles can be created in spatial regions where there is no force, an apparent nonlocal effect.

To begin our discussion, we examine in Fig. 1 the final number of created electron-positron pairs associated with the potential $V(z,t)=V_0\Theta(z)\;\Theta(t)$ as a function of the potential height V_0 (solid curve). In this particular case all energy eigenstates $|P\rangle$ can be obtained analytically. In the Appendix we derive some expressions for the matrix elements $\langle n|P\rangle$ required to compute the final number of created particles.

As we see from Fig. 1, the created number increases monotonically from zero for $V_0=0$ to a value of 11.56% for $V_0=2c^2$. For smaller values of V_0 up to about $V_0 \approx 0.5c^2$ the curve follows $N(V_0) \sim 0.017\;(V_0/c^2)^2$ as indicated by the dashed line.

Such a quadratic dependence for small V_0 is expected from perturbation theory. In the case of an abrupt turn on, Eq. (2.2) reduces to $N=\Sigma_P\Sigma_n|\langle n|P\rangle|^2$. For small V_0 the state

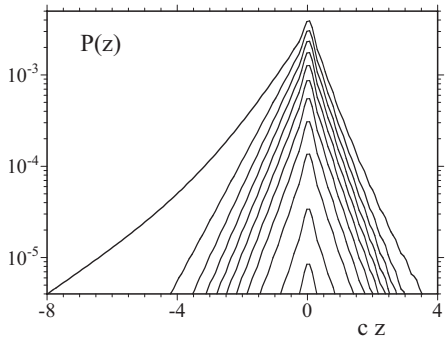

FIG. 2. The spatial electron probability density $P(z)$ created by a force field $V(z)=V_0\Theta(z)$ turned on abruptly in time, for $V_0=0.1c^2$, and then from $0.2c^2$ to $2.0c^2$ with an increment of $0.2c^2$. As V_0 increases the corresponding electron density grows monotonically. For fields $V_0 \leqslant 0.5c^2$ a shape-invariant distribution is formed which is localized in space with the shape $\Lambda(z)=e^{-2.3c|z|}$. ($N_z=512$, $L=0.3$ a.u.)

$|P\rangle$ can be expressed perturbatively [34] as $|P\rangle=|p\rangle+\Sigma_\alpha\langle\alpha|V|p\rangle/(e_p-e_\alpha)\ |\alpha\rangle$, where $|\alpha\rangle$ denotes all force-free energy eigen states with energy e_α except $|\alpha\rangle=|p\rangle$. Including only terms up to the second order in the potential, we obtain $N=\Sigma_p\Sigma_n|\langle n|V|p\rangle|^2/(e_p-e_n)^2$, suggesting the observed quadratic dependence, $N\sim V_0^2$. In addition to the fitting in Fig. 1, the proportionality constant $0.017/c^4$ was also confirmed numerically from the double integral $\Sigma_p\Sigma_n|\langle n|\Theta(z)|p\rangle|^2/(e_p-e_n)^2$ amounting to a value of $0.016\,88/c^4$.

In Fig. 2 we show the corresponding spatial density $P(z)$ of the instantaneously created particles for eleven strengths of the potential field V_0, ranging from $0.1c^2$ to $2c^2$. Note that the corresponding force, defined here as $-V'(z)$, is completely sharp and has zero spatial width. As indicated above, the particles are created outside the force zone, which is zero here. This finding seems to exhibit nonlocality. It is certainly interesting from a mathematical point of view to examine what is predicted by the Dirac equation in its present form of Eq. (2.1). The data also suggest that in the perturbative regime, $V_0<0.5c^2$, the shape of the spatial birth density $P(z)$ is independent of V_0 and falls off exponentially $P(z)\sim V_0^2\Lambda(z)$, where the universal function is given as

$$\Lambda(z) \equiv \exp(-2.3c|z|). \tag{3.1}$$

The area of each curve is the total number of created electrons shown in Fig. 1. The factor 2.3 was determined numerically from the slopes in Fig. 2 suggesting a localization length of $1/(2.3c)$. This length is related to the electron's Compton wavelength.

An almost identical localization length of $1/(2.3c)$ was already reported, although in a different context, in Refs. [6,35,36]. For a very wide force field, the joint probability to create a positron at $z=0$ and an electron at location z was computed from the corresponding two-particle wave function. This density also decayed exponentially on the same scale indicating that an electron cannot be created further away from the accompanying positron than the Compton wavelength.

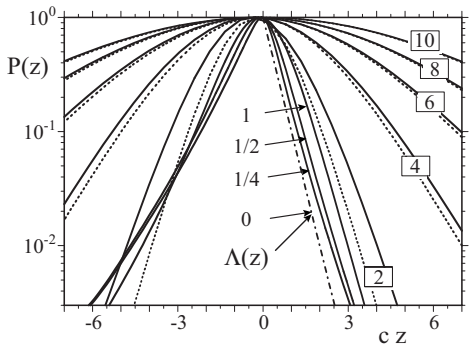

FIG. 3. The spatial electron probability density $P(z)$ created by a force field $V(z)=V_0 0.5\ [\tanh(z/W)+1]$ for $V_0=2.0c^2$ and various values of spatial widths as marked: $W=(10,8,6,4,2,1,0.5,0.25,0)/c$. The densities are compared with the corresponding $|V'(z)|^2$ for the first five W values. The deviation between $P(z)$ and $|V'(z)|^2$ grows as W decreases. For small values of W the densities approach the universal form of $\Lambda(z)=e^{-2.3c|z|}$ shown by the dotted-dashed line. ($N_z=256$, $L=1$ a.u.)

At first, this finding seems to contradict earlier claims that due to an entanglement-induced modification of the electron's localizability with the accompanying positron, an electron can be spatially arbitrarily narrow [6]. However, unlike previous calculations that computed the density after the field is instantaneously turned off, we compute here the density while the field is on. It turns out that if the field is turned off, our numerical data obtained by projecting the electron field operator on the force-free eigenstates suggest that there is no lower limit to the electron's localizability, meaning that after a suitable turn off the spatial density can be much narrower than the universal $\Lambda(z)$ function of Eq. (3.1). The distribution of created pairs after the turn off for a spatially narrow force field is interesting by itself and we will devote a separate publication to it. In previous works we examined the birth density associated with supercriticality and not necessarily solely due to a temporal change of the potential as discussed here.

For larger force fields $V_0>0.5c^2$, the localization length shown in Fig. 2 gradually widens from the weak field limit. Because the field produces a force pointing to the left the density is asymmetric about the center of the force and extended more to the left. In contrast to weaker forces, where the corresponding density of the created electrons is identical to those for the positrons, for $V_0\sim 2c^2$ the birth densities for the two different charges are different. This asymmetry between the electron's and positron's spatial density could lead to a joint probability that is not peaked at the zero distance between the two particles. But it does not necessarily contradict our previous finding [6] that the most likely distance between the two created particles is zero if one particle is created at zero.

Next we look at the case of force fields that have a nonzero spatial width W, returning to the potentials of the form $V(z,t)\equiv V_0\ [1+\tanh(z/W)]/2\ \Theta(t)$. We computed the spatial electron densities $P(z)$ in Fig. 3 for $V_0=2c^2$ for eight different widths, W ranging from $10/c$ to $0.25/c$. The five dashed lines (associated with $cW=10$, 8, 6, 4, and 2) represent the

square of the corresponding force $|F(z)|^2$, where $F(z)=-dV(z)/dz$. For better comparison, we have normalized each curve to be 1 at $z=0$. For a large force zone, we find excellent agreement between the electron density and $|F(z)|^2$. As we argued in a previous paper, this is the limit in which the force is not too strong and the created electrons will only acquire velocities much less than the speed of light.

A simple perturbation analysis can show that creation follows the force zone. In Eq. (A1) the density is given by $P(z)=\Sigma_P\Sigma_{P'}\Sigma_n\langle z|P_t\rangle\langle P_t|n(t)\rangle\langle n(t)|P'_t\rangle\langle P'_t|z\rangle$. During the abrupt turn on of the force we can neglect the time dependence and replace the state $|n(t)\rangle$ by $|n\rangle$ and $|P_t\rangle$ by $|P\rangle$. As we are only interested in the leading terms in V_0, we can use again the perturbative expansion as $|P\rangle\approx|p\rangle+\Sigma_\alpha\langle\alpha|V|p\rangle/(e_p-e_\alpha)|\alpha\rangle$ and obtain $P(z)=\Sigma_p\Sigma_{p'}\Sigma_n\langle z|p\rangle\times\langle p|V|n\rangle\langle n|V|p'\rangle\langle p'|z\rangle/[(e_{p'}-e_n)(e_p-e_n)]$. As a second approximation for a wide enough potential, only smaller values of the momentum are required and we can approximate the spinor components of the states discussed in the Appendix by $u_p\equiv p/[c+\sqrt{(c^2+p^2)}]\approx p/(2c)$ and $u_n\equiv n/[c+\sqrt{(c^2+n^2)}]\approx n/(2c)$. For nonrelativistic momenta, we can also approximate the energy differences in the denominator by the $2c^2$ energy gap, such that $[(e_{p'}-e_n)(e_p-e_n)]\approx(2c^2)\times(2c^2)$, leading to

$$P(z)=\sum_p\sum_{p'}\sum_n\langle z|p\rangle\langle p|V|n\rangle\langle n|V|p'\rangle\langle p'|z\rangle/(4c^4). \tag{3.2}$$

If we keep only the lowest order in $1/c$ in the spinor components, the factor $\langle z|p\rangle\langle p'|z\rangle$ becomes $\exp[i(p\text{-}p')z]$ leading to

$$P(z)\approx\sum_p\sum_{p'}\sum_n\langle p|V|n\rangle\langle n|V|p'\rangle\exp[i(p-p')z]/(4c^4) \tag{3.3}$$

In this approximation, the scalar product $\langle p|V|n\rangle$ amounts to $(p\text{-}n)\,\tilde{V}(p\text{-}n)/2c$, where $\tilde{V}$ is the Fourier transform of the real potential V. This reduces Eq. (3.3) to

$$\begin{aligned}P(z)&\approx\sum_p\sum_{p'}\sum_n\tilde{V}(p-n)\tilde{V}(n-p')(p-n)(p'-n)\\&\quad\times\exp[i(p-p')z]/(16c^6)\\&=\sum_n\sum_p\tilde{V}(p-n)(p-n)\exp[i(p-n)z]\\&\quad\times\left\{\sum_{p'}\tilde{V}(p'-n)(p'-n)\exp[i(p'-n)z]\right\}^*1/(16c^6)\\&=\sum_n\left|\sum_p\tilde{V}(p-n)(p-n)\exp[i(p-n)z]\right|^2 1/(16c^6)\\&=\sum_n\left|\sum_k\tilde{V}(k)k\exp[ikz]\right|^2 1/(16c^6)\\&=\sum_n|dV/dz|^2 1/(16c^6)\approx|dV/dz|^2.\end{aligned} \tag{3.4}$$

The densities for the smaller width ($cW=1$, $1/2$, and $1/4$) are much wider than the corresponding force field and approach the universal $\Lambda(z)$ function in the limit of small W. For comparison, we have indicated the function $\Lambda(z)$ by the dotted-dashed lines. Figure 3 illustrates how the birth density $P(z)$ changes from $|V'(z)|^2$ to $\Lambda(z)$ as the width of the force field shrinks while its norm grows to 11.56% as discussed above.

IV. LINEARLY TURNED-ON FORCE FIELDS

For a sudden turn-on of the force as discussed in Sec. III, all electrons are created simultaneously and there is a clear separation between the creation dynamics and the after acceleration stage when the particles leave the force field. If the temporal turn on of the force field is extended over a longer period, we can observe the nontrivial interaction of particles created at different moments in time involving the suppression of pair creation as well as annihilation. We note, however, that this "interaction" is not Coulombic in nature but a mere consequence of the fact that two electrons cannot simultaneously occupy the same single-particle quantum state. A linearly turned-on force field $V(z,t)=V(z)t/T$ introduces the duration of turn-on $T(\geqslant t)$ as a controllable parameter. The obvious question is whether there is an optimum duration T that can maximize the total yield of the created pairs. On the one hand we might expect that an instant turn on $(T=0)$ could create the largest number of particles as it has the largest temporal change of the potential. On the other hand, if the turn on is slow enough, the particles have sufficient time to escape from the creation zone and vacate the space for other particles to be created which could enhance the total yield. In principle, there could be an optimum T as a compromise between these two competing mechanisms. It is thus a nontrivial question if the final amount of pairs will be increased or reduced by changing the time T to reach the maximum potential strength.

In order to explore this question we have repeated the simulations described above with a linear turn on, $V(z,t)=V(z)t/T$, where T denotes the turn-on duration until the potential reaches is largest strength V_0, which we chose equal to $2c^2$ here. We presently do not know how to compute the electron and positron density in the force field under the supercritical case.

In Fig. 4 we plot the final total number of pairs after the potential has reached its maximum value $V_0=2c^2$. The graph decreases monotonically suggesting that the more rapidly the potential can reach its final height, the larger is the particle yield, in our case 11.56% as already discussed in Fig. 1. There also seem to be two different time regimes. If the turn on can be completed within times shorter than $2\pi/(2c^2)$ ($\approx 1.67\times10^{-4}$ a.u.) then the final yield is quite sensitive to the precise duration T as indicated by the rapid fall off of the graph. Once the time is long enough such that the spectrum of the time dependence of the force can contain a sufficient amplitude associated with the critical frequency $\omega_c=2c^2$, a further increase of T will reduce the total yield only insignificantly. We have commented on the significance of this particular frequency in a previous communication [24].

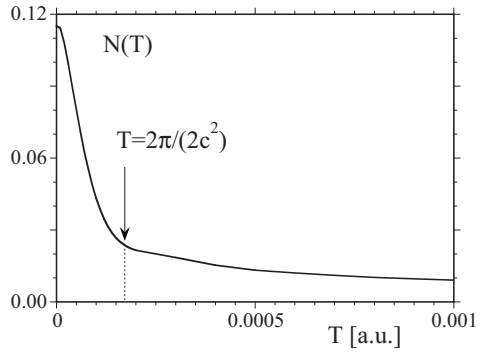

FIG. 4. The end-of-pulse electron-positron creation probability $N(T)$ due to a subcritical force field that changes linearly in time, $V(z,t)=2c^2\ t/T$. ($N_z=512$, $L=1.0$ a.u.)

In contrast to the end-of-period total electron production in Fig. 4 we analyze in Fig. 5 the dynamics *during* the turn on. First we study the zero force zone case, see Fig. 5(a), then we study a large force zone case of $W=6/c$ in Fig. 5(b). For each case we have selected three turn-on durations of fast ($T=5\times10^{-5}$ a.u.), intermediate ($T=1.5\times10^{-4}$ a.u.), and long ($T=10^{-3}$ a.u.) turn-on periods. It is clear that the more rapidly the force field is turned on, the more particles can be produced as we discussed above.

For the $W=0$ case of Fig. 5(a), the creation zone $[\Lambda(z)]$ is very narrow, and the particles can easily accelerate out of this zone, vacating the area such that additional particles can

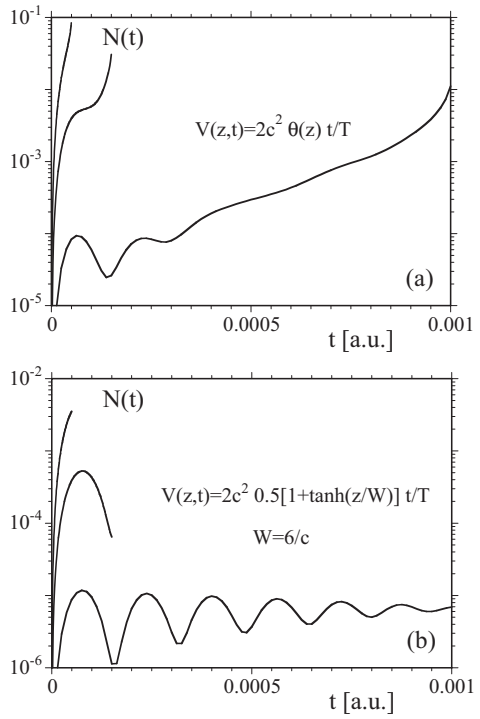

FIG. 5. The electron-positron pair creation probability $N(t)$ due to a subcritical force field that changes linearly in time, $V(z,t)=V(z)\ t/T$, for (a) $V(z)=2c^2\ \theta(z)$ as well as (b) $V(z)=2c^2\ 0.5[\tanh(z/W)+1]$. In these simulations parameter $W=6/c$ and turn-on times $T=5\times10^{-5}$, $T=1.5\times10^{-4}$, and $T=1\times10^{-3}$ a.u. ($N_z=512$, $L=1.0$ a.u.)

be created as the potential keeps growing. As a result we find a continued growth in the curve for the long turn on ($T=10^{-3}$ a.u.). The corresponding graph in Fig. 5(b) for the much wider creation zone $[|V'(z)|^2$ with $W=6/c]$, however, shows a very oscillatory growth reflecting the interaction between previously created particles and the birth process.

The oscillatory particle growth curve for wider potentials suggests that the yield after the potential reaches its maximum height can be maximized by an optimally chosen linear ramp-up time T. This is useful, as realistic force fields are typically wider than our mathematical limiting case of $W=0$. We note that the time scale of these oscillations is once again related to the energy difference between the upper and lower continuum states, $2\pi/(2c^2)=0.000\ 17$ a.u.

V. SUMMARY AND DISCUSSION

We have tested the predictions of the Dirac equation on the pair creation process from vacuum triggered by the time dependence of a subcritical potential. In this study we studied rapidly as well as linearly turned-on potentials. We examined how the spatial probability distribution of the created electron depends on the position dependence of the potential. It turns out that there are two distinct regimes. In case the potential is sufficiently wide, the electron's birth density is directly proportional to the square of the associated force field, given by the spatial derivative of the potential. This result for arbitrary position dependent potentials was derived analytically and confirmed by numerical simulations. In the opposite limit of a narrow potential, the resulting birth density becomes independent of the potential and takes the form of a universal exponential decay. If, for instance, the external force is nonzero only at a single position, the Dirac equation predicts that pairs can be created in a region around this point with radius proportional to $1/(2.3c)$, related to the electron's Compton wavelength. In other words, the electron can be created in the force-free region. It is interesting to note that the mathematical structure of the Dirac equation is capable of generating this nonlocal behavior. Even though the amount of electrons created in the force-free region is usually negligible, it points to a nonlocal and potentially unphysical behavior that could also lead to violations of causality. It therefore requires a few comments.

In quantum physics, nonlocal behavior is not so uncommon. For example, we are used to tunneling phenomena where we find nonvanishing wave functions in those spatial regions that are energetically forbidden from a classical mechanical point of view. But moving population into the spatial tunneling regions occurs with velocities less than c and does not violate causality, even though superluminal effects have been discussed in this context. [37–40]. Another famous physical situation is the well-known Aharonov-Bohm effect, which is nonlocal in the magnetic field, but local in the vector potential.

In our case, however, the prediction that particles can instantly appear when a force acts at a different location could also be viewed as a possible violation of causality that clearly requires an analysis about its mathematical cause. It could be related to the fact that we have included only a

time-dependent scalar potential $V(z,t)$ and omitted its associated vector potential $A(z,t)$. However, preliminary simulations that included $A(z,t)$ show that the birth density is still nonlocal and particles can be generated outside of both potentials. To be consistent we chose the four-potential as solutions to the Maxwell equations.

In order to have an unambiguous definition of the term locality, we examined only those four potentials that are exactly zero outside a certain spatial domain (the "force region"). As a result of this restriction, the potentials are nonanalytical and show an unavoidable discontinuity in their nth order derivative at the boundary with force-free region. We believe that this nonanalyticity could be the mathematical reason for the nonlocal behavior. In order to "shift" this singularity into higher derivatives we have used a sequence of potentials whose boundary scales similar to $z^n\Theta(z)$ (for $n=0,1,2$) where $\Theta(z)$ is the unit step function. The amount of the electrons in the force-free region decreases with n suggesting that the noncausality could vanish for analytical forces. We are not aware that the relationship between noncausality and nonanalyticity within a fully relativistic description has been discussed before.

Finally, we should also remark that the very concept of a force is an approximation by itself, and particles can affect each other only by an exchange of other particles, in our case, photons that are bound in speed by c. The semiclassical calculation discussed here could be improved if more sophisticated methods based on virtual photons are invoked. In a similar spirit, the Born-Infeld nonlinear electrodynamics [41] could be used to explore these processes on the smallest length scales.

The discovery of the universal Λ function [Eq. (3.1)] and its associated nonlocality [42] has also implications for the way previously obtained quantum field theoretical data were interpreted for situations in which the force field was present. We incorrectly assumed that the electronic density in the force-free region vanishes independent of whether there is a force in other spatial regions. As mentioned earlier, for subcritical forces we can compute the instantaneous energy eigenstates $|P\rangle$ as an unambiguous separability between electronic and positronic subspaces . In the case of supercritical potentials, however, we presently do not know how to obtain the electronic portion from the field operator that is necessary to compute the electronic spatial densities. Due to the fact that the Dirac equation is capable of violating locality, one has to be careful in interpreting data for supercritical fields in the close vicinity of the forces.

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APPENDIX

If the spatial and temporal dependence of the scalar potential is piecewise constant, one can find semianalytical solutions for the final number of created electron-positron pairs in such an interaction. We will derive the corresponding expressions here for the special case of an abruptly turned-on potential of the form $V(z,t)=V_0\Theta(z)\Theta(t)$, where $\Theta(\ldots)$ denotes the Heaviside unit step function, defined as $\Theta(z)\equiv 0.5(1+|z|/z)$ and V_0 is the positive barrier height. As we discussed in the main text, we have to compute first the electronic density operator defined as

$$\rho(t) = \langle\langle \Phi(t=0) \| \Psi^{(+)\dagger}(t)\Psi^{(+)}(t) \| \Phi(t=0)\rangle\rangle = \sum_P \sum_{P'} \sum_n \langle n(t)|P'_t\rangle\langle P_t|n(t)\rangle |P_t\rangle\langle P'_t|. \tag{A1}$$

Here and below we denote the eigenstates $[c\alpha_z p_z+\beta c^2+V_0\Theta(z)]|P\rangle=E_P|P\rangle$ with $E_P>c^2$ and the free states as $(c\alpha_z p_z+\beta c^2)|p\rangle=e_p|p\rangle$ with $e_p>c^2$ $(c\alpha_z p_z+\beta c^2)|n\rangle=e_n|n\rangle$ with $e_n<-c^2$. As the total field operator satisfies a differential equation, its evolution has to be continuous in time. In other words, $\Psi(t)$ cannot change during the infinitely small time interval when (at time $t=0$) the potential is turned on. However, the operator $\Psi^{(+)}(t)$ can jump due to the sudden change of the subspace of instantaneous energy eigenstates. As a result, we have $\Psi=\Sigma_p b_p|p\rangle+\Sigma_n d_n^\dagger|n\rangle$ and correspondingly $\Psi^{(+)}=\Sigma_P\ (\Sigma_p b_p\langle P|p\rangle+\Sigma_n d_n^\dagger\langle P|n\rangle)\,|P\rangle$ directly after the turn on, where b_p and $d_n^\dagger$ denote the fermionic annihilation and creation operators associated with the states $|p\rangle$ and $|n\rangle$. We then trace over all states $|P\rangle$ of the electronic density operator and obtain

$$N = \mathrm{tr}\,\rho(t) = \sum_{P''} \langle P''|\rho|P''\rangle = \sum_P \sum_n |\langle n|P\rangle|^2. \tag{A2}$$

In other words, the sum (integral) of all overlaps between the negative energy eigenstates of the force-free system with the positive energy states of the system with the force determines the final number of pairs.

Let us briefly summarize the analytical results. In order to simplify our notation we use for the Dirac matrices $\alpha_z=((0,1),(1,0))$ and $\beta=((1,0),(0,-1))$, which is also sometimes called the Weyl form [43] and neglects the spin. To include spin we simply have to double the final result. In the spatial representation, the field-free eigenstates $|p\rangle$ and $|n\rangle$ take the following form:

$$\langle z|p\rangle = N_p(1,u_p)\exp(ipz), \tag{A3a}$$

$$\langle z|n\rangle = N_n(-u_n,1)\exp(inz), \tag{A3b}$$

where $u_p\equiv p/[c+\sqrt{c^2+p^2}]$ and $u_n\equiv n/[c+\sqrt{c^2+n^2}]$ for $-\infty<p<\infty$ and $-\infty<n<\infty$, and the normalization constants are $N_p\equiv 1/\sqrt{2\pi}/\sqrt{1+u_p^2}$ and $N_n\equiv 1/\sqrt{2\pi}/\sqrt{(1+u_n^2)}$. They are associated with normalizations $\langle p|p'\rangle=\delta(p\text{-}p')$ and $\langle n|n'\rangle=\delta(n\text{-}n')$ and $\langle n|p\rangle=0$.

If the potential is subcritical $(V_0<2c^2)$ the electronic eigenvectors $|P\rangle$ for the system with the force field are

$$\langle z|P^1_{\mathrm{III}}\rangle = F_{p1}\{[(1,u_p)\exp(ipz) + f_p(1,-u_p)\exp(-ipz)]\Theta(-z) + [(1+f_p)(1,\tilde{u}_p)\exp(i\tilde{p}z)]\Theta(z)\}, \quad \text{(A4a)}$$

$$\langle z|P^2_{\mathrm{III}}\rangle = F_{p2}\{[(1-f_p)(1,-u_p)\exp(-ipz)]\Theta(-z) + [(1,-\tilde{u}_p)\exp(-i\tilde{p}z) - f_p(1,\tilde{u}_p)\exp(i\tilde{p}z)]\Theta(z)\}, \quad \text{(A4b)}$$

$$\langle z|P_{\mathrm{II}}\rangle = B_p\{[(1,u_p)\exp(ipz) + b_v(1,-u_p)\exp(-ipz)]\Theta(-z) + [(1+b_v)(1,iv_p)\exp(-\kappa z)]\Theta(z)\}, \quad \text{(A4c)}$$

$$\langle z|P_{\mathrm{I}}\rangle = B_p\{[(1,u_p)\exp(ipz) + b_w(1,-u_p)\exp(-ipz)]\Theta(-z) + [(1+b_w)(1,iw_p)\exp(-\kappa z)]\Theta(z)\}. \quad \text{(A4d)}$$

The normalization constants

$$F_{p1} \equiv 1/\sqrt{(\pi)}/\sqrt{[(1+b_p^2)(1+u_p^2) + (1+b_p)^2(1+\tilde{u}_p^2)\tilde{p}/p\sqrt{c^2+p^2}/\sqrt{c^2+\tilde{p}^2}]},$$

$$F_{p2} \equiv 1/\sqrt{(\pi)}/\sqrt{[(1+b_p^2)(1+\tilde{u}_p^2)\tilde{p}/p\sqrt{c^2+p^2}/\sqrt{c^2+\tilde{p}^2} + (1+b_p)^2(1+u_p^2)]}$$

and $B_p \equiv 1/\sqrt{(2\pi)}/\sqrt{(1+u_p^2)}$ follow from the normalization $\langle P|P'\rangle = \delta(P-P')$. Here we have used the abbreviations, $f_p \equiv (u_p - \tilde{u}_p)/(u_p + \tilde{u}_p)$, $\tilde{p} \equiv \sqrt{\{[\sqrt{(c^2+p^2)} - V_0/c]^2 - c^2\}}$, $\tilde{u}_p \equiv \tilde{p}/[c + \sqrt{c^2+\tilde{p}^2}]$, $\kappa \equiv \sqrt{\{c^2 - [\sqrt{(c^2+p^2)} - V_0/c]^2\}}$, $v_p \equiv \kappa/[c + \sqrt{c^2-\kappa^2}]$, $b_v \equiv (u_p - iv_p)/(u_p + iv_p)$, $w_p \equiv 1/v_p$, and $b_w \equiv (u_p - iw_p)/(u_p + iw_p)$. A discussion of these eigenstates for $V_0 > 2c^2$ can be found in Ref. [10].

The Roman subscripts I, II, and III denote the three distinct energy regions. Region III is characterized by energies that are above $V_0 + c^2$, leading to a twofold degeneracy. States in region II have an energy below V_0. The lowest energetic states associated with region I occur only if the potential height is sufficiently large such that V_0 exceeds c^2.

The inverse of the parameter κ [defined as $\sqrt{\{c^2 - [\sqrt{c^2+p^2} - V_0/c]^2\}}$] is proportional to the tunneling depth of the states with energy $E < V_0 + c^2$. As the energy approaches $E = c^2 + V_0$ from below, this tunneling depth increases to infinity. In the other limit, one could incorrectly expect that the smallest penetration depth under the barrier should be given by the lowest possible energy eigenstate with $E = c^2$, corresponding to zero momentum p. It turns out however, that in the case of $V_0 > c^2$, the smallest tunneling depth occurs for $E = V_0$. This purely relativistic effect is a consequence of the influence of the lower energy continuum states that are "pulled up" under the potential barrier.

A similar nonmonotonic behavior of a tunneling distance can observed for the spatial width of the ground-state wave function of the δ function potential, $V(z) = -V_0\delta(z)$. Here the spatial width of the ground state takes its smallest value if the ground-state energy E_g is at zero (exactly half way between the upper and lower energy continuum states). If the potential is sufficiently deep that the ground state can dive into the lower continuum, its spatial width grows to infinity.

Depending on the range of P and P' one may use expressions in Eqs. (A3) and (A4) to compute the required scalar products,

$$\langle n|P^1_{\mathrm{III}}\rangle = iF_{p1}N_n[-(u_p - u_n)/(p-n) - (u_p + u_n)f_p/(p+n) + (\tilde{u}_p - u_n)(1+f_p)/(\tilde{p}-n)], \quad \text{(A5a)}$$

$$\langle n|P^2_{\mathrm{III}}\rangle = iF_{p2}N_n[-(u_p + u_n)(1-f_p)/(p+n) + (\tilde{u}_p + u_n)/(\tilde{p}+n) - (\tilde{u}_p - u_n)f_p/(\tilde{p}-n)], \quad \text{(A5b)}$$

$$\langle n|P_{\mathrm{II}}\rangle = iB_pN_n[-(u_p - u_n)/(p-n) - (u_p + u_n)b_p/(p+n) + (iv_p - u_n)(1+b_p)/(i\kappa - n)], \quad \text{(A5c)}$$

$$\langle n|P_{\mathrm{I}}\rangle = iB_pN_n[-(u_p - u_n)/(p-n) - (u_p + u_n)b_p/(p+n) + (iw_p - u_n)(1+b_p)/(i\kappa - n)]. \quad \text{(A5d)}$$

Note that in obtaining expressions in Eq. (A5) only the principal values survive. In order to compute the final number of created particles, the remaining double integrals in Eq. (A2), $\Sigma_P\Sigma_n$, have to be done numerically. We used a simple Simpson method and obtained convergent results after the removal of several subtractive error cancellations associated with small denominators.

For example, for the largest potential height $V_0 = 2c^2$ we obtained that $2\Sigma_P\Sigma_n|\langle n|P\rangle|^2 = 0.1156$ which can be used to gauge the accuracy of our time-dependent numerical solution to the Dirac equation that was based on a finite space-time grid in a numerical box where all eigenstates are naturally discrete and normalized according to $\langle P|P'\rangle = \delta_{P,P'}$. For a numerical grid spacing of $\Delta z = 1.4 \times 10^{-4}$ a.u. and 2048 grid points we obtained $N = 0.115$ showing that even abruptly turned on potentials can be very well approximated on a finite grid if the grid spacing is sufficiently small.

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