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Demonstrating the Principles of Aperture Synthesis with the Very Small Radio Telescope

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Abstract

We have developed a set of college-level, table-top labs for teaching the basics of radio interferometry and aperture synthesis. These labs are performed with the Very Small Radio Telescope (VSRT), an interferometer using satellite TV electronics as detectors and compact fluorescent light bulbs as microwave signal sources. The hands-on experience provided by the VSRT in these labs allows students to gain a conceptual understanding of radio interferometry and aperture synthesis without the rigorous mathematical background traditionally required.

1. MOTIVATION

The radio astronomical technique of aperture synthesis, in which high-resolution images are produced from interferometer arrays, has been a scientifically productive tool for astronomers for decades. From derivation of the theoretical concept in the 1950s to the creation of working instruments in the early 1960s, aperture synthesis was quickly recognized as a significant advancement for astronomy. Sir Martin Ryle shared the Nobel Prize for Physics in 1974 “for his observations and inventions, in particular of the aperture synthesis technique” (Lundqvist 1992). The best resolution attained with single radio antennas has historically been of order 20 arcseconds, while that achieved with aperture synthesis can be four orders of magnitude better. With the subsequent construction of the (now renamed) Jansky Very Large Array and continent-scale arrays for very long baseline interferometry, radio astronomers have been able to produce images with much higher angular resolution than has been possible at other wavelengths. This technique is so important to the advancement of astronomical knowledge that the National Radio Astronomy Observatory runs an aperture synthesis workshop every other summer for the training of, primarily, graduate students. Yet, because of the complexity of the math involved, aperture synthesis is often excluded from undergraduate curricula in physics and astronomy.

We introduce here a set of labs at the undergraduate level which provide first-hand experience with the basics of aperture synthesis observations and how the data reveal information about the size and structure of the observed sources. These labs are designed to give the students a general conceptual understanding of aperture synthesis without the need for the dense mathematical formalism that usually accompanies the topic in graduate-level classes.

2. AN OVERVIEW OF APERTURE SYNTHESIS

Successful completion of the labs does not require that the students, or instructors, know the mathematical formalism. As such, in this section, we give a brief discussion of what an aperture synthesis observation entails, touching on only those aspects necessary to comprehend the principles demonstrated in these labs. We give this overview to help the reader appreciate what these labs accomplish. The material in this section does not need to be presented to the students; the goal is that the students will *discover* the important principles of aperture synthesis observations from the perspective of a user at a national facility such as the the Jansky VLA. Further details about interferometry can be found in textbooks such as Wilson, Rohlfs, & Hüttemeister (2009), Burke & Graham-Smith (2010), Thompson, Moran, & Swenson (2001), or Kraus (1986).

In aperture synthesis a number of antennas, arranged in a particular pattern, or "array," receive radiation from an astronomical source simultaneously and the signals are combined pairwise. The method of signal combination in most modern interferometers is a cross-correlation, but the signals can also be added. The response of each pair of antennas contains an amplitude and a phase which, customarily, are represented as a complex number. (Note: the “amplitude” in a radio interferometer’s output is called the “modulus” of a complex number in standard mathematical usage.) For a single point of emission, the amplitude is proportional to the flux density of the source while the phase is related to the difference in path lengths to the two antennas, as depicted in Figure 1. For a general source of arbitrary structure, the detected amplitude and phase are the complex superposition of the responses due to each unresolved point of emission within the field of view.

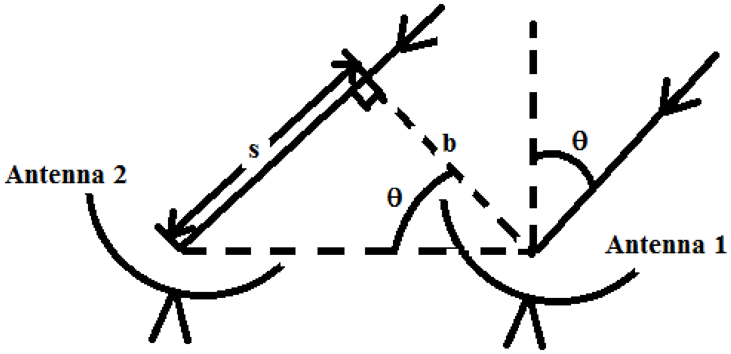

Figure 1: A pair of antennas with a projected separation b receive radiation from a celestial point source in a direction, θ, relative to the mid-plane, or transit, position. The path from the source to antenna 2 contains an extra distance, s, which causes a phase difference when the signals are combined. The projected baseline (the component perpendicular to the direction to the source) is shown as the length b.

In an aperture synthesis observation, the fully calibrated response of each pair of antennas, termed a "visibility," is a function of the separation of the antennas, generally referred to as the "baseline," $\vec{b}$. If the source is not located along the mid-plane of the baseline, $\vec{b}$ is the component of the baseline perpendicular to the direction of the source.

The visibility for any particular antenna pair is most sensitive to source structure on an angular scale proportional to $\lambda/|\vec{b}|$, where λ is the wavelength of the observation. This is a familiar concept from optics, in which the resolution of a telescope is proportional to λ/D, where D is the diameter of the objective lens or mirror. With an array of more than two antennas, the visibility for each pair is obtained and hence the visibility function can be probed over a wide range of baseline lengths and orientations.[1] In this way, information about the source structure on multiple angular scales enables one to produce an image of the source.

The expression "aperture synthesis" derives from the concept that each pair of antennas acts like one piece of a single large telescope, and by combining the responses of pairs on all baselines, the aperture of a telescope as large as the longest baseline is, in principle, synthesized. The mirror in a visible-wavelength telescope, similarly, can be considered a *filled aperture* since it contains the responses on all baseline separations up to the diameter.

In the labs described here, the presentation of the important principles is simplified in two ways. First, since the Very Small Radio Telescope (VSRT) measures only the amplitude of the visibilities and not the phase, complex numbers can be avoided. Second, the labs are performed with all sources and antennas in the horizontal plane and so the analysis is reduced to one-dimension. Students are introduced to the visibility function by measuring the interferometer response as a function of antenna separation, allowing them to discover the relationship between visibilities and simple source structure. By extrapolation, the students gain an appreciation of how information about source structure can be recovered from data obtained with many pairs of antennas.

2.1. Relation Between Visibility Amplitudes and Source Structure

The visibility as a function of baseline is analogous to the diffraction and interference patterns produced in the single and double slit experiments. In the case of an arbitrary pattern of brightness in a source, radiation from each point in the source is detected by each antenna and the different path lengths result in constructive or destructive interference depending on the point's position in the sky and the baseline geometry. The resulting visibility function is the sum of the interference patterns for all points of emission in the source, located at slightly different positions in the sky. When written as complex-values, the visibility function, with $\vec{b}/\lambda$ as the independent variable, is related to the image of the source, i.e. intensity as a function of angle $\vec{\theta}$ on the sky, via a Fourier transform. The radio astronomer, in general, obtains an image of a source with software which performs an inverse Fourier transform on the visibility data. Even

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[1] The visibilities for the baseline from antenna 1 to antenna 2, $\vec{b}_{12}$ and for that from 2 to 1, $\vec{b}_{21}$ are redundant because celestial sources have intensity distributions that have no imaginary part.

when there are not enough data to produce an image, simple source structures can be inferred by fitting source models directly to the visibilities.

We now discuss some simple examples of source structure to illustrate its effect on the visibility. We remind the reader that in general, there is a reciprocal relationship between the independent variables in a Fourier Transform, and therefore the source structure of angular size θ, most strongly affects visibilities on baselines of length b where $b/\lambda = 1/\theta$. Consider, for example, a source which contains just one unresolved point of emission at an angular position $\theta = 0.1$ radians west of the map center, as shown in Figure 2(a). Then, the real part of the visibility function, shown in Figure 2(b), is a cosine wave in the East-West direction with a "spatial period" L equal to the reciprocal of the angular position of the source relative to the map center, $L = \theta^{-1} = 10$ (in units of projected baseline divided by wavelength).[2] The imaginary part is a sine wave of the same frequency. The visibility amplitude, which equals the square root of the sum of the squares of the real and imaginary parts, is therefore independent of the baseline.

On the other hand, if the source is located at the map center, as in Figure 2(c), then the argument of the trig functions is zero and so the real part (the cosine term) is a constant, and the imaginary part (the sine term) vanishes, as shown in Figure 2(d).

If, now, the source contains two equally bright, unresolved points of emission, one at the map center and the other 0.1 radians to the West, as in Figure 2(e), then the real part of the visibility function in the East-West direction is the sum of the cosine function with period 10 and the constant term and so oscillates about a positive value and is never negative; see Figure 2(f). The imaginary part, though, oscillates about zero with period 10. The visibility amplitude, in this case, oscillates in the East-West direction with minima separated by 10 (in units of b/λ). If $\lambda = 2.5\,\text{cm}$, for example, then the minima in the visibility amplitude oscillation are separated by 25 cm.

More generally, with one point source at the map center and a second point source located $\Delta\theta$ radians to the East or West, the minima in the visibility amplitude in the East-West direction occur at baseline distances, b, given by

$$\frac{b}{\lambda} = \frac{N}{2\Delta\theta}, \tag{1}$$

where N is an odd integer.

The visibility amplitude of a single point source is constant, while for a pair of point sources the amplitude oscillates with a period inversely proportional to the separation angle of the point sources, as given in Eq. 1.

Consider, now, the observation of a single but resolved source. In this case, the visibility amplitude decreases with baseline length and the rate of decrease is inversely proportional to the angular size of the source. For a source with a Gaussian brightness profile, for example, the visibility amplitude follows a Gaussian profile as well, and the half-maximum width of the

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[2] We use the term "spatial period" to avoid confusion with the wavelength of observation or with a temporal period.

visibility function is inversely proportional to the half-maximum width of the source brightness distribution (Bracewell 1978).

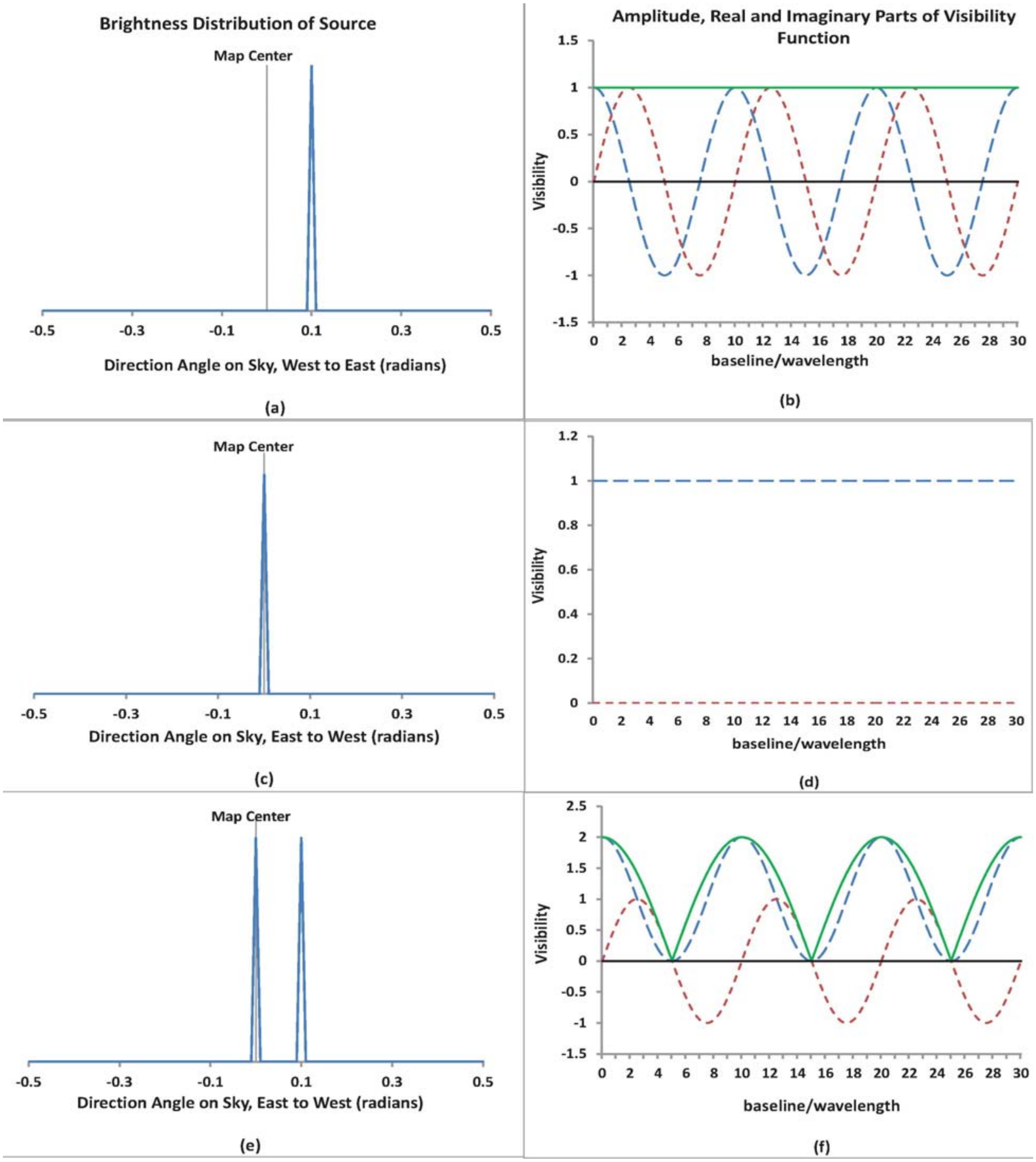

Figure 2: The left panels indicate the locations of point sources in the sky and the corresponding panels on the right display the visibility functions, which result from the Fourier transform of the functions on the left. The long-dashed, blue lines trace the real part and the short-dashed red lines represent the imaginary part. The solid green lines in (b) and (f) show the "magnitude" of the Fourier transform which is the same concept as the "amplitude" of the visibility function.

3. EQUIPMENT

The laboratory exercises described herein use the VSRT (Very Small Radio Telescope), a laboratory interferometer developed by MIT Haystack Observatory. Composed of commercially-available electronics and satellite TV equipment, the VSRT is a low-cost, easy to assemble instrument used to demonstrate the principles of radio interferometry in high school and college-level labs (Doherty, Fish, & Needles 2011). Full details of the system can be found at the VSRT project web page (http://www.haystack.mit.edu/edu/undergrad/VSRT/index.html).

The VSRT uses Ku-band satellite TV feeds to receive radiation near 12 GHz. The feeds can be installed in satellite dishes to increase the collecting area and directionality of the instrument, as is done for an experiment measuring the solar diameter (Fish et al. in preparation). Alternatively, the feeds can be used without the dishes to observe strong radio sources in the lab. The exercises in Section 4 use the instrument in this manner, with compact fluorescent light bulbs (CFLs) serving as radio sources (Figure 3).

Figure 3: Undergraduate students from Union College working with the VSRT. Two ``triple'' DirecTV feeds, taken from TV satellite dishes, used here as radio antennas, receive and detect the radio emission from CFLs. The CFLs can be moved to assorted separation distances. Only the bottom feed of each triple, as shown, is active. The output spectrum from the interferometer is displayed on the laptop screen, and the data files are recorded.

In addition to visible light, CFLs produce broadband radio emission from 100 MHz up to 100 GHz when the free electrons in the plasma collide with the glass walls of the bulb (Mumford & Schafersman 1995). The CFLs can be hidden by an optically-opaque material that is transparent to radio waves (e.g., a cardboard box) to demonstrate that the feeds are sensitive to the bulbs' radio emission, not their visible light.

4. LABORATORY EXERCISES

The labs described below are designed to help students develop an intuitive sense of how radio arrays can be used to infer aspects of the spatial structure of radio sources. To facilitate data analysis, we have produced a package of Java programs, known as the "VSRTI_Plotter", available at https://www1.union.edu/marrj/radioastro/labfiles.html or via the project web page.

4.1. Primary Beam

Astronomy students are usually familiar with the fact that the angular resolution of a single telescope, ignoring the atmosphere, is limited by diffraction and is approximately λ / D. With a radio telescope, in which a single detector is located at the focal plane, the central diffraction peak is known as the "primary beam." When an astronomical source is not at the center of the primary beam, the detected power is decreased. To map a source with a single-dish radio telescope, the observer must change the pointing direction of the antenna in steps comparable to the beam size, recording the detected power at each position.

With interferometers the angular resolution depends on the lengths of the baselines, which are much larger than the diameter of an individual telescope. The primary beam size of the component antennas of an array, though, places an effective upper limit on the maximum field of view in aperture synthesis.

In this first exercise, students measure the primary beam pattern of the VSRT feeds by placing the active feeds one above the other, making a baseline with zero horizontal length. They place a CFL two meters away at the mid-plane position and record data for about a minute. Keeping the feeds fixed, they subsequently record data with the CFL placed at various horizontal angles to the mid-plane. The students then drag and drop recorded data files into the VSRTI_Plotter "Plot Beam" program to graph their data and overlay a theoretical beam plot (Figure 4).

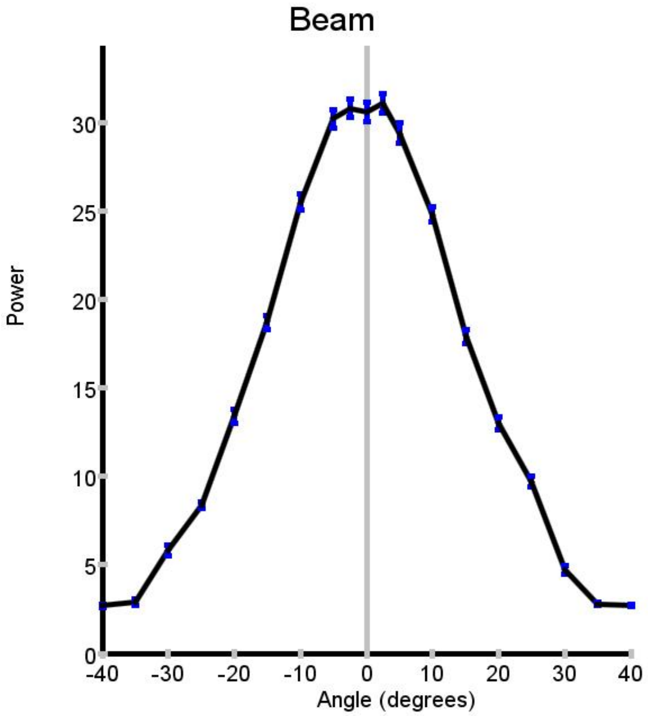

Figure 4: A plot of data obtained by undergraduate students showing the primary beam of a VSRT feed. The plot, produced with the VSRTI_Plotter, shows the detected power as a function of the angular position of a CFL.

4.2. One Resolved Source

The distance between the antennas (i.e. the baseline) determines the angular sizes of structure in the radio source that an interferometer is most sensitive to. In short, the baseline acts like a spatial filter. As the baseline length is increased, the interferometer response becomes more dependent on finer scale structure in the source, while its sensitivity to extended structure decreases. Flux distributed over larger angles is said to be "resolved out." If the baseline is too long, it will not detect the source at all. As discussed briefly in Section 2.1, the detected power of a resolved source decreases with increasing baseline length. One can use the rate of fall-off of detected power to determine the angular size of the source.

In this lab, students use a single CFL located at the mid-plane position between the two feeds and then vary the horizontal separation between the feeds. They produce a plot of the measured power versus the baseline length, finding that the power decreases as the baseline length increases. In this way students become acquainted with the visibility function, in which the independent variable is baseline length divided by wavelength.

Students are instructed to obscure the bottom half of the CFL using a metal plate. They find that the power decreases by about half and verify that this holds at a variety of baseline lengths. Next, they place two metal plates vertically obscuring the sides of the CFL in order to make a source whose apparent size is smaller. They find that the measured power at short baseline lengths is smaller than for an unobscured CFL, but the power at long baseline lengths is actually *greater*. With the VSRTI_Plotter program, the students can easily save the data to a file to be read into Excel for further analysis, such as overlaying multiple curves, as shown in Figure 5, which displays the results of this lab. The students discover that the rate of decrease of the measured power is inversely related to the angular size of the source.

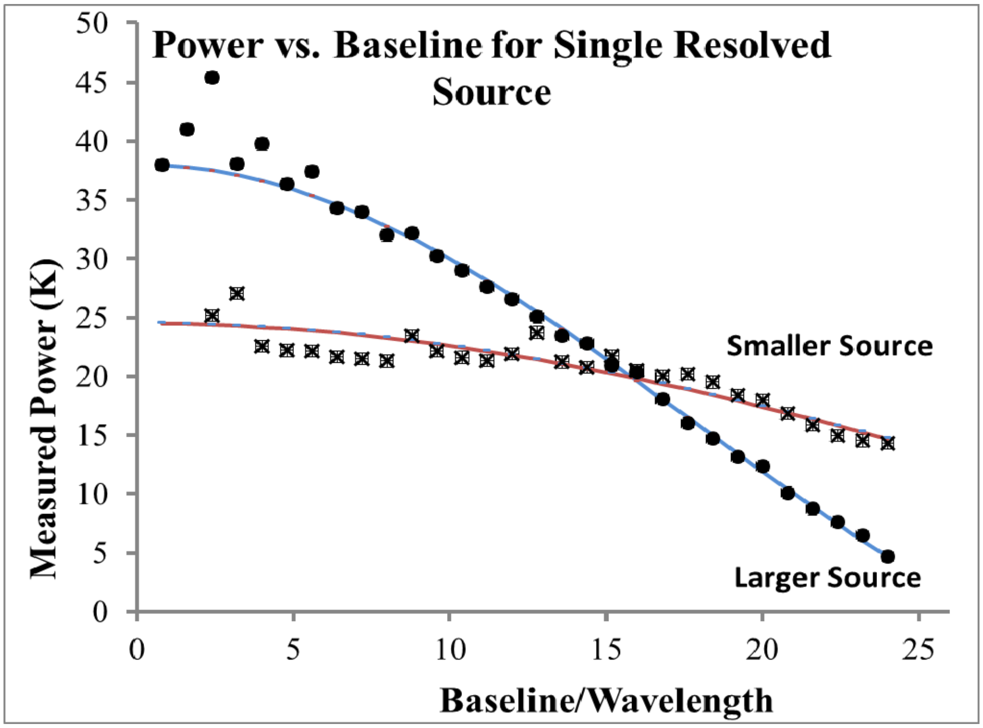

Figure 5: The power measured vs. baseline in observations of a single source of different angular sizes. Data from VSRT observations of single sources are plotted together to contrast the visibility functions of sources of different sizes. The curves show model fits. The detected power of the larger source (un-obscured CFL at distance of 2 m) falls off more quickly with baseline length than does that of the smaller source (4-cm width at distance of 2 m), demonstrating that the angular size of a source can be inferred from measurements at a variety of baseline lengths.

4.3. Two Sources

As discussed in Section 2.2, when the radio source is binary with two components separated by an angle resolved by the interferometer, the detected power varies with baseline in an oscillatory manner. In this exercise, students discover this by using two CFLs with a fixed separation between them. As the students move the VSRT feeds apart, they observe the oscillation of the measured power with baseline length. The students then move the two CFLs farther apart and repeat their measurements, discovering the reciprocal relationship between the angular separation of the two sources and the distance between minima in the visibility function (Figure 6).

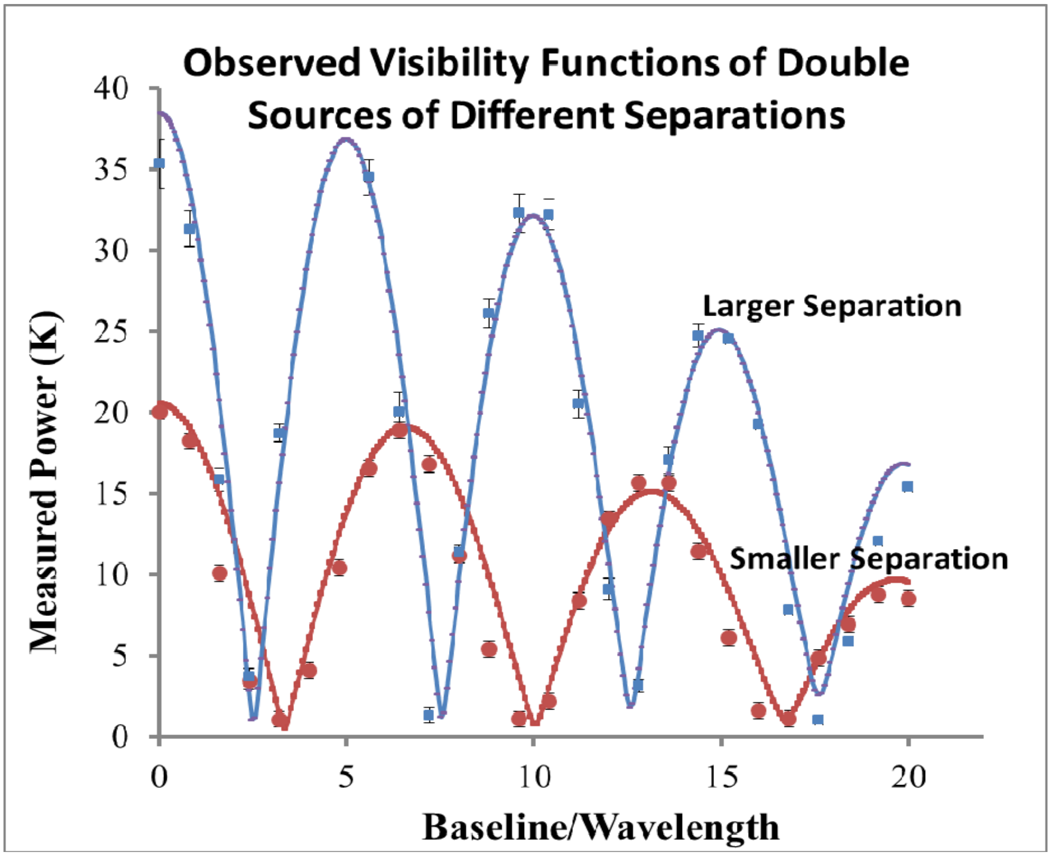

Figure 6: The detected power as a function of baseline length in observations of pairs of CFLs of different separation angles. The red points show the data with two CFLs separated by 30 cm at a distance of 2 meters, and blue points show the data when they are separated by 40 cm. The curves show model fits. The visibility function oscillates with baseline length, with the spacing between minima related inversely to the angular distance between the sources.

4.4. Fourier Transform Exercises

In actual aperture synthesis observations one obtains an image of the source by performing a Fourier transform on the complex visibilities. Since the VSRT data contain only amplitudes, and no phases, we have provided another java package which students can use to discover the relation between Fourier function pairs. They find that this relation is identical to that between the visibility function and source structure. Using the “Tool for Interactive Fourier Transforms” (or TIFT), also available at https://www1.union.edu/marrj/radioastro/labfiles.html, students use a simple click and drag operation to make functions representing the brightness distributions of the CFLs in the previous labs. In a companion plot, they discover that the amplitudes of the Fourier transforms are the same as the visibility amplitudes they measured with the VSRT (Figure 7).

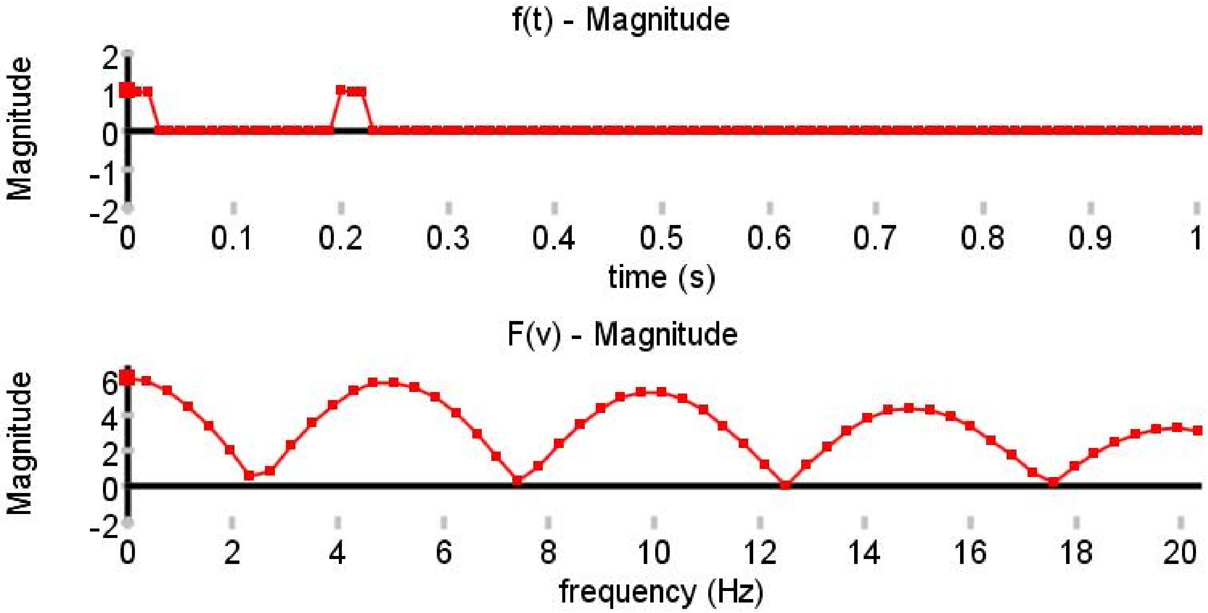

Figure 7: Plots displayed in TIFT showing the Fourier transform relation between functions representing the set-ups in the VSRT labs. The top function represents the brightness distribution with two resolved sources separated by 0.2 radians and the bottom function shows the magnitude of its Fourier transform function, which equates to the amplitude of the visibility function. The bottom function, which oscillates with a period equal to 5 and a decreasing envelope, can be seen to be identical, except for a vertical scale factor, to the blue curve in Figure 6, which is the visibility function when observing two CFLs separated by 0.2 radians.

4.5. Mystery Source

After completing the exercises discussed in 4.2 and 4.3, the students' new abilities to infer the angular separation of a pair of sources and the angular size of each source from the visibilities are tested by placing a cardboard box over a pair of CFLs. Students can obtain their data, infer the source structure and then check their answer by lifting the box.

A slight increase in complexity of source structure and a challenge to the students' understanding can then be demonstrated with an exercise using three CFLs hidden under a cardboard box. The instructor sets the CFLs with equal spacings, so that CFLs 1 and 2 and CFLs 2 and 3 create two pairs separated by angle θ and CFLs 1 and 3 make one pair with a separation of 2θ. The visibility data then show the sum of two sine waves -- one with a larger amplitude and period in $= 1/\theta$ and the other with a smaller amplitude and half the period (Figure 8). The students are not informed that there are three CFLs in the box, and are asked to infer, working as team, the source structure considering the principles they learned in the labs.

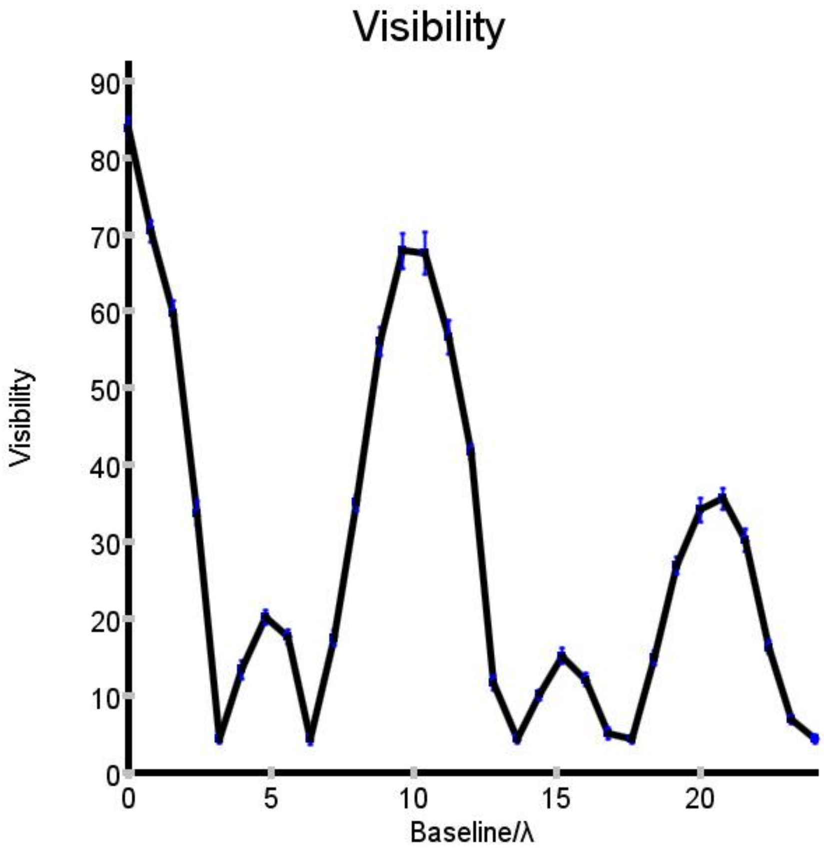

Figure 8: The detected power as a function of baseline length for a 'mystery source' hidden inside a box at a distance of 2 meters. Inside the box were three CFLs with separations of 20 cm between the middle CFL and each end CFL, yielding two pairs with angular separations of 0.1 radians and one pair with a separation of 0.2 radians.

4.6. Solar Diameter Measurement

As an exercise using the VSRT to make a measurement of an actual celestial source, the VSRT can also be used to determine the diameter of the Sun. This involves a more complicated set-up but also provides a practical culmination of the VSRT labs. This exercise will be discussed in forthcoming article (Fish et al. 2013).

5. EXPERIENCE OF THE FALL 2012 RADIO ASTRONOMY CLASS.

In Fall 2012, these labs were incorporated into a radio astronomy class at Union College. The class consisted of 8 students from Union College and 3 from Siena College. The class consisted of seniors, juniors and sophomores with majors in either physics or engineering.

As a test of the VSRT labs discussed above, the coverage of aperture synthesis in the course was accomplished using only the labs. No lectures or reading on aperture synthesis were provided.

During the first class meeting, the students were given a quiz on the basics of aperture synthesis, containing the following questions.

1. Explain conceptually how receiving signals with a number of radio telescopes, as with the VLA, contains information about the image of a radio source.
2. What is an 'array' of telescopes and what are the important criteria in designing the array?
3. What is the "Visibility function?"
4. Describe what a 1-dimensional visibility function looks like when observing:
    a) a single, unresolved source
    b) a single, resolved source. How does the shape of the visibility function change as the angular size of the source increases?
    c) a pair of unresolved sources. How does the shape of the the visibility function change as the separation of the two sources increases?

After completion of the VSRT labs (and with no lecture on the material), the students were given the same quiz. The average score on the quiz on the first day was 0.3 points out of 18 (or 1.5%), demonstrating that none of the students had any prior knowledge about aperture synthesis. The normalized gain (defined by the fraction of the material not known a priori that was learned by the time of the post-test) was 0.66.

The mystery source lab, in which three CFLs were hidden from view, was found to be challenging by the students. However, with all 11 students collaborating as a team to brainstorm, in the end, with extensive discussion, the class succeeded in inferring the actual source structure correctly.

6. CONCLUDING REMARKS

These labs were completed first by several undergraduate students undertaking an independent study at Union College in Fall 2010 and in a formal class at both Union College and Siena College in Fall 2012. The data for all the plots shown in the figures herein were obtained by these students. While single-dish radio observing methods are sometimes incorporated into undergraduate astronomy classes, aperture synthesis is often not emphasized due to the need to include high-level mathematics. Using these labs, these students were able to gain an intuitive understanding that data between pairs of antennas at different spacings leads to a function from which one can infer the size of a single source or the separation between sources in the sky.

These labs can be used to stimulate discussions about how the structural details of even a complicated source can be extracted from aperture synthesis data using standard observing setups and imaging algorithms. For instance, after learning that extended sources can be resolved out on long baselines, students can be asked about the importance of matching the angular resolution of an observing array to the scale of the expected structure in an astronomical source and asked to consider why the telescopes in the Jansky Very Large Array and the Atacama Large Millimeter Array are designed to be reconfigurable into arrays of different sizes. After the primary beam lab, students can be asked to speculate why small dishes are used in arrays designed for wide-field imaging, for example.

Acknowledgments

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