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Elementary Students' Strategies for Solving Visual Logic Tasks

Camille Schulman

Submitted in Partial Completion of the
Requirements for Departmental Honors in Elementary Education and Mathematics

Bridgewater State University

May 2014

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Introduction

Education is an important aspect of society and impacts the lives of countless individuals. That is why it is so important to understand and implement various standards deemed necessary for success. The Common Core State Standards (2010) are designed to do this on a national level in order to ensure that all students and teachers are provided with the information they need to be successful. The Common Core State Standards (2010) "are designed to be robust and relevant to the real world, reflecting the knowledge and skills that our young people need for success in college and careers." Currently the importance of science, technology, engineering and mathematics (STEM) are being emphasized considering the strong need for individuals possessing these skills.

In mathematics one of the standards for mathematical practice states that students must be able to "construct viable arguments and critique the reasoning of others." There are two major types of reasoning that appear in mathematics and allow different conclusions to be reached: inductive and deductive reasoning. Inductive reasoning refers to the generalization from many different patterns in order to make a prediction of future occurrences (Haylock, 2007). Conclusions resulting from inductive reasoning can be proven wrong with just one counterexample. On the other hand, deductive or logical reasoning requires very specific explanations to provide certainty of the outcome. "Deductive reasoning is the formulation of a valid, logical argument to explain, demonstrate, or convince others that a solution to a problem must be correct" (Haylock, 2007). Logic skills are used in a variety of problems ranging from simple problems to advanced mathematical proofs.

Deductive reasoning is essential to doing mathematics as well as to argumentation and proof in general. Deductive arguments are made of claims. In order to create a valid argument, there must be accurate and specific support for each claim being made. This is where the logic comes in; all claims must individually be backed by solid information, which cannot be argued against. This form of reasoning ranges in complexity level beginning with simple deductive arguments and progressing to much more complicated chains of deductions. The most basic form of logical reasoning is “one-step deductive reasoning, in which the reasoning is a single deduction from two or more premises” (Reid, 2002). Conclusions to these types of problems are in general, straightforward and easy to understand. Once multiple one-step deductions are linked together, it is called simple multi-step deductive reasoning. This occurs when reasoning requires a chain of deductions in order to reach a conclusion. Another more complicated form of deductive reasoning is hypothetical deductive reasoning, which involves a hypothesis (Reid, 2002). In this form of deductive reasoning, steps of a problem are completed based on unknown ideas in order to reach a conclusion. This type of reasoning is especially important in advanced mathematics where premises and concepts may not actually be known. The linking of all of these types of reasoning creates a basis for many mathematical procedures.

The ability to reason deductively plays a wide variety of roles in the field of mathematics and is especially important to prepare students for proofs. Proofs appear everywhere, not only in mathematics. Proofs, in different forms, appear in everyday situations, debates, science, mathematics, and social conflicts to name a few applications. Any time one must reach a conclusion, there must be sound evidence to support the conclusion being reached. Proofs are often used as a way to validate arguments and verify solutions. However, in order to ensure that these proofs are accurate, there must be a string of unarguable examples and explanations. Deductive reasoning provides these solid explanations that aid in reaching a valid conclusion. Proofs are especially important in mathematics education to encourage advanced competence.

In school, proofs are used to display mathematical understanding in a way that establishes and communicates what a person knows about mathematics (Stylianides, G., & Stylianides, A., 2008). This type of argumentation requires precise, step-by-step explanations of why certain mathematical solutions must be correct. Deductive reasoning provides a foundation for proofs because of the nature of reaching conclusions based on known premises. Linking logical steps together creates a valid proof of a mathematical statement. According to Reid (2002), mathematical proofs are simply that: long chains of deduction.

Deductive Reasoning Theories

Much research has been conducted on people's deductive reasoning, which provide explanations of the processes involved in completing deductive tasks; as well as an analysis of the cognitive processes used when completing such tasks. These theories emphasize different aspects of deductive reasoning. Some theories claim syntax, how words are arranged, are the most important; others rely heavily on semantic processes, the meaning of words. No matter the method used to explain deductive reasoning, each theory portrays the importance of reasoning skills throughout a person's lifetime.

One group of theories about deductive reasoning relies on the idea of the availability of information to determine the processes used to reason. The theories that fall under this category all depend on the idea that an individual's past and current mental processes help to create links between the premises in a problem. This contributes to their ability to accurately solve the problem. One such theory is Johnson-Laird's Mental Models Theory (1992), which is based upon creating mental models that parallel the given argument. These "models have a structure that corresponds directly to the structure of situations" (p. 419). The Mental Models Theory consists of three stages all of which work together in order to reach valid conclusions. The first stage is the comprehension stage during which verbal premises or observations take place. These are used to create a mental model of information based on "their meaning and any relevant general knowledge" (Johnson-Laird, P.N., 1992). Second is the description stage during which an attempt at a conclusion is made. The solver attempts to create a specific description of the model used and then reaches some new conclusion not stated in the premises (Stylianides, G., & Stylianides, A., 2008). This is where the main deductive reasoning occurs, to create a valid argument that backs-up the conclusion stated to be true according to the premises. The third and final stage is the validation stage. During this stage, the validity of the conclusion is determined by "ensuring that no model of the premises render it false" (Johnson-Laird, P.N., 1992). In other words, there is a search to see if there are any alternate mental models that would prove a conclusion to be false. If no other models can be found, then the conclusion is valid and the model has provided a positive result. However, if the third stage discovers an alternate model then the second stage must be revisited to see if the same conclusion is reached. This process continues until it has been determined that all models reach the same conclusion.

Another similar approach of deductive reasoning was presented by Cheng and colleagues (1985), the Pragmatic Reasoning Schemas Theory. This theory proposes that people use pragmatic reasoning schemas to reason through arguments. This approach uses "abstract knowledge structures induced from ordinary life experiences" (Cheng & Holyoak, 1985, p. 395). These structures are called pragmatic reasoning schemas and are more specifically defined as "generalized sets of rules defined in relation to classes of goals" (Cheng & Holyoak, 1985, p. 391). This theory states that people have difficulty reasoning with abstract ideas, but are able to reason when materials are concrete, realistic, or thematic. Thus, schemas, or diagrams of representation, are created in order to make sense of non-logical terms. Cheng and Holyoak

(1985) explain that permission, causal, and covariation schemas can be evoked helping to interpret the problem into a more relatable context. Permission schemas are the most widely used and are classified “as a type of regulation in which taking a particular action requires satisfaction of a certain precondition” (Cheng & Holyoak, 1985, p. 396). According to Cheng and Holyoak (1985), there are four rules that ultimately summarize the requirements of a permission schema:

> “Rule 1: If the action is to be taken, then the precondition must be satisfied.
> Rule 2: If the action is not to be taken, then the precondition need not be satisfied.
> Rule 3: If the precondition is satisfied, then the action may be taken.
> Rule 4: If the precondition is not satisfied, then the action must not be taken” (p. 397).

These rules are very similar to implications in formal logic: “if p, then q,” where p and q represent statements. However, not all schemas follow so closely to formal logic. In causal schemas, also referred to as obligations, “a certain condition requires performing a subsequent action” (Stylianides, G., & Stylianides, A., 2008, p. 120). The rules are similar to those of permission schemas. This type of schema is more likely to affirm the consequent, or reach the conclusion that “if the precondition is satisfied, then the action must have been taken.” Lastly, covariation schemas can be applied when two situations are expected to co-occur. Similar to the causal schema, the covariation schema is more likely to reach a conclusion that assumes the converse than the permission schema. Additionally, this schema is much more difficult for subjects to evoke due to the more complicated nature of understanding multiple situations simultaneously (Cheng & Holyoak, 1985).

The standard selection task is a classic problem. It is often used to study deductive reasoning and to test the pragmatic reasoning schemas theory. The selection task provides subjects with the following rule: if there is a vowel on one side of a card, then there is an even number on the other side. Thus, the distinct possibilities that the cards may be are as follows: 1) a vowel on one side, an even number on the other; 2) a vowel on one side, an odd number on the other; 3) a consonant on one side, an even number on the other; 4) a consonant on the other, an odd number on the other. Four cards are placed on the table and subjects are asked which cards need to be flipped over in order to determine if the originally stated rule is true. Researchers have used this basic outline to determine whether reasoning schemas provoke a higher rate of success. Cheng and Holyoak (1985) show that “people can solve selection problems if they are given cues that serve to trigger the permission schema” (Cheng, Holyoak, Nisbett & Oliver, 1986, p. 297). The use of these schemas will help the solver to logically reach a conclusion. If a rule evokes a pragmatic reasoning schema then the subject will implement the permission schema to solve a problem. However, when a schema is not readily available, that is when the subject will use syntactic clues and non-logical approaches (Cheng et al., 1986).

In contrast to the previous theories that build from necessary, relatable situations the Social Contract Theory (Cosmides, 1989), argues that human’s ability to reason has developed over time through the advancement of social exchange. This evolutionary theory claims that natural selection has played an important role in promoting “domain-specific cognitive mechanisms” that activate reasoning procedures based on the area being used (Cosmides, 1989, p. 190). This theory builds from the Social Exchange Theory, the cooperation between two humans in order to gain a mutual benefit, in order to show that humans have developed their own ability to reason through skills used in social exchange situations. There are two key concepts of social exchange that influence a person’s ability to reason through a task. First, humans need the ability to determine whether the benefit of a situation will be worth the cost; both parties must receive a benefit for engaging in the exchange. Second, humans must possess the ability to detect when their opponent is cheating or not being faithful to their obligations (Cosmides, 1989).

Cosmides, like Cheng and colleagues, uses the standard selection task to display how social exchange theory contributes to reasoning skills. The selection task parallels a social contract in which the solver must distinguish whether the cards given are "cheating" or following the given rule. Cosmides refers to this as the "look for cheaters" procedure where the solver uses the concepts of social exchange to determine the answer to the logic task.

Another approach to deductive reasoning defends the use of formal proof as the main means of reasoning through tasks. A Natural Deduction System (ANDS) Theory (Rips, 1983) uses natural deduction as the base for development of deductive reasoning. The main argument of this theory is that deductive reasoning occurs through the collection of multiple inference rules. This collection is then used to form a mental proof, based on the premises of the conclusion. In the words of Rips (1983) it "consists of propositional inference rules embodied in a set of computational routines." In order to create an organized view of the argument, Rips proposes the use of trees, assertion, and subgoal, which serve as a proof of the argument. Since ANDS uses working memory, the part of memory that remains actively involved in a task, to make inference rules pertaining to a given rule, these trees create an outline of assertions and subgoals made. The assertion tree contains the original premises and any propositions derived from them. Sometimes propositions are not necessarily true and are called suppositions and placed in a separate node of the tree. They are placed independently from other propositions so that their validity is not taken for granted. Despite their questionable truth, suppositions allow for exploration of different possible outcomes (Rips, 1983). Contrastingly, subgoal trees contain the conclusions and all propositions that prove it. The subgoal tree allows for "processing efficiency: subgoals keep the proof procedure aimed in the direction of the argument's conclusion rather than allowing it to produce implications at random from the premises" (Rips, 1983, p. 43).

Ultimately, trees are just an organizational list that allows each specific proposition to be kept separate. The combination of both the forward direction tree and the backward direction tree provides a clear picture of whether a rule is valid. If the inferences made in the assertion tree and the subgoals mesh together then the original argument will be labeled valid. However, if a proof runs out of rules to apply or the assertion tree and subgoal tree do not overlap, then the argument is invalid.

Each of these theories of deductive reasoning takes a different approach to the process of completing reasoning tasks. The research shows opposing views on the steps a person takes to reach a valid conclusion. However, they could all agree that deductive reasoning requires complex mental processes that are essential for solving certain problems.

Child Development of Deductive Reasoning Skills

Despite different theoretical approaches of deductive reasoning, researchers agree that deductive reasoning and proving are required to achieve proficiency in mathematics. The Common Core State Standards' current education goals are emphasizing Science, Technology, Engineering and Mathematics (STEM), thus the development of these skills are essential for young students. However, the development of deductive reasoning has been at the center of much debate. It is accepted that mastery of logical reasoning skills takes time to develop both in complexity and diversity throughout a child's lifetime. Much debate still exists in regards to the specific nature of this development. Some researchers, including Piaget and Markovits, believe that children do not possess the ability to reason deductively until their early teen years, while others, including Hawkins, Richards and Sanderson, state that children as young as preschool are able to understand simple forms of logical reasoning in familiar contexts.

Piaget was one of the first to explore and claim that children younger than seven years of age cannot reason logically. At seven to eight years, children become able to reason logically, but this ability "deals only with his personal conception of reality" (Piaget, 1966, p. 67). In other words, children of this young age are only able to use deduction when it has some necessity in their life. Thus, Piaget placed the development of formal deductive reasoning skills in his Formal Operational Stage, which begins at the age of eleven or twelve and continues into adulthood. Therefore, his theory of this development states that children are unable to deductively reason before the age of twelve. Formal reasoning skills require children to remember rules that are different from their own observations (Piaget, 1966). At this later age in childhood, children develop the ability to draw conclusions from a set of given premises regardless of their previous experience with the context. Piaget's theory of development of deductive reasoning created a basis upon which other researchers would build.

There have been many studies that support Piaget's theory of reasoning development. For example, Markovits, Schleiffer and Fortier (1989) outline the development of deductive reasoning in children. The study done by Markovits and colleagues tested 85 children ranging in age from kindergarten to fifth grade. Six pairs of syllogisms were used in the study half of the syllogisms reached logical conclusions, while the other half were tweaked in order to create an illogical conclusion with no link to the original two premises (see Table 1).

"Logical" Syllogisms	"Illogical" Syllogisms
1. Every Zobole is yellow. All yellow things have a nose. Do Zoboles have a nose?	7. Every Zobole is yellow. All red things have a nose. Do Zoboles have a nose?
2. Every Plouque is tall. No tall thing is green. Are Plouques green?	8. Every Plouque is tall. No crying thing is green. Are Plouques green?
3. Plitosores have legs. Nounou is a plitosore. Does Nounou have legs?	9. Plitosores have legs. Nounou is a Fiola. Does Nounou have legs?
4. Risomes cannot sing. Zapp is a Risome. Can Zapp sing?	10. Risomes cannot sing. Zapp is a Touki. Can Zapp sing?
5. Mibules cough when they are touched. All things that cough are smooth. Are Mibules smooth?	11. Mibules cough when they are touched. All things that cry are smooth. Are Mibules smooth?
6. Pliks sleep when they are cold. All sleeping things are blue. Are Pliks blue?	12. Pliks sleep when they are cold. All jumping things are blue. Are Pliks blue?

Table 1. *Syllogisms*. This table was taken from Markovits, Schleiffer and Fortier (1989).

The children were then asked about the validity of the final statement and how they knew that it was true. The analysis of the children's responses demonstrated a high level of correct responses to the logical problems (Markovits et al., 1989). However, the young children of six and eight years of age also had similar validation response rates to the illogical problems. As the children's age increased, their ability to identify illogical responses improved. According to the researchers, this result displays that the younger the children, the less reasoning skills they possess. Additionally, the young children's justifications were based on extraneous information, which was irrelevant to the original premises thus providing more evidence that they did not have the skills necessary to decipher the difference between logical and illogical conclusions (Markovits et al., 1989). Importantly, the order of the logical versus illogical syllogisms played a role in their response. Eight year old students presented with the illogical syllogisms first had increased performance on those and decreased performance on the logical syllogisms where as the six year olds made no link to the contrasting syllogisms. Children differentiated between the two syllogisms more frequently than when the logical syllogisms were presented first (Markovits et al., 1989). However, based on the discrepancy in definition of deductive reasoning, this was not a large enough factor to completely rule out the previous observations of how age plays a role in skill level.

Markovits, Schleiffer, and Fortier (1989) showed that although from a young age children are able to reach valid conclusions to reasoning problems they often do so through syntactic cues, or the format of the sentences, rather than logical arguments. Children are able to successfully complete logic tasks, but do not have "a full understanding of the relations of necessity that govern classes of propositions" (Markovits et al., 1989, p. 791). Often times strategies that rely "on the 'atmosphere effect,' in which the response is chosen as a function of the positive or negative mood of the premises" are used by young children (Markovits et al., 1989, p. 791). Although this strategy may reach a valid conclusion, it is not through the understanding of the problem rather through a gut feeling. Thus, despite the finding that young children can differentiate logical and illogical statements they are unable to describe how they reached their conclusions. These results illustrate that young children do not have the skills necessary to reason logically, but as they get older acquire the ability to do so.

A number of studies have challenged Piaget's theory of development of deductive reasoning, but did not reach conclusions to fully discredit Piaget. For example Moshman and Franks (1986) found that children go through multiple stages before fully comprehending deductive logic. The first stage in the preschool years is when a child acts with respect to logical norms without understanding the necessity of logic (Moshman, D. & Franks, B.A., 1986). In the second stage, developing around age six, a child is able to reach conclusions through logic, but only based on assumptions that are true. Finally, a child in the third stage is able to state the validity of a statement by backing up the conclusion with a series of arguments, which take a certain specified form. This last stage requires more research regarding the age at which children enter it (Moshman, D. & Franks, B.A., 1986). These stages are a good example of how the age of a child impacts his or her success at solving any logic problems. As a result of experiments, Moshman and Franks propose that this final stage of development begins at age 9-10 years old, but become even more fluent in years beyond that.

Each of three experiments' participants were in grades four, seven, and undergraduate students at a university. The purpose of these experiments was to determine at what age participants were able to complete the tasks by distinguishing the validity of arguments. The first experiment used seven "if, then" (Moshman, D. & Franks, B.A., 1986). The second experiment used arguments of the following five classes: transitive, a>b and b>c implies that a>c; class instantiation, using all as a category and placing something into the category based on an assumption; disjunction, using "or" as a means of distinction; conjunction, using "and" to connect two things with a similar property; and reverse conjunction, using “and” to categorize objects. Students were asked to determine the validity of a total of 40 arguments presented. Lastly, using the same arguments as experiment two, the third experiment provided half of the participants with an initial explanation about validity and instructed the other half to just sort the arguments in categories they thought best. Each of these experiments took a different approach to the presentation of arguments, but all analyzed the overall success participants of each age were able to assess validity of statements. Moshman and Franks found that no fourth graders chose to sort the arguments in relation to validity of the statements, rather they were more likely to sort them based on content and real life correspondence. The fourth graders showed no concept of understanding of validity, even in the third experiment when they were given explicit definitions of validity (Moshman, & Franks, 1986). Thus, Moshman and Franks study shows that a complete understanding of deductive reasoning does not begin until after age ten.

Despite their argument that children in fourth grade could not understand the concept of validity, Moshman and Franks’ (1986) showed that even in the preschool years, children possess a basic level of reasoning skills though they lack the ability to formulate full arguments until early adolescence. Although Moshman and Franks agree with Piaget that deductive reasoning skills take time to develop, they argue that skills are developed at an earlier age than Piaget predicted.

More recent research has claimed that Piaget’s theory of development of deductive reasoning may be incorrect, showing that under certain circumstances children as young as two and three are able to reason logically. Hawkins, Pea, Glick, and Scribner (1984) have researched how problem complexity, content and organization can all impact preschoolers’ ability to solve a syllogistic problem. Forty children age four and five years were presented with either fantasy, real-world, or opposing real-world statements and then asked a yes or no question related to the premises. They were also asked to justify the answer that they chose. Through the analysis of the results, Hawkins et al. found that children were able to reach valid conclusions through the use of deductive reasoning. However, this performance was contingent on multiple factors. Children were more likely to correctly justify their responses to fantasy based statements because there was no real-world application to confuse their reasoning. Thus, the context of the statements played a role in children's ability to reason. Additionally, the order that the syllogisms were presented impacted the results. If the real-world statements were presented first, children tended to base the rest of the questions of those premises (Hawkins et al., 1984). Nevertheless, Hawkins et al. concluded that children possess the initial stages of deductive reasoning even at the young age of four and five.

Richards and Sanderson (1999) further explored whether young children are able to reason deductively, supporting Hawkins et al. (1984). Richards and Sanderson argue that young children, through the use of their imagination to overcome belief bias, are able to correctly solve reasoning tasks. Their experiment tested ninety children age two, three, and four and their ability to correctly answer syllogistic logic tasks. The children were divided into subgroups where they received no make-believe cue, a word cue with low imagination, a word cue with high imagination, or visual imagery. They were then asked to listen to multiple statements and answer a question that followed. These statements involved "a cat barking, a fish in a tree, an elephant roller-skating, a dog driving, a bird swimming and a sheep riding a bicycle" (Richards & Sanderson, 1999, p. B3). It was found that the children cued to use their imagination were able to successfully answer the questions because they removed themselves from the reality of the statements. Children were even able to correctly justify their reasoning for answering a question in a specified manner. So, despite the premises contradicting the real world, children were able to use their imagination to reach logical conclusions even at the young age of two, three, and four.

Another study that opposes Piaget's theory even further is that of Galotti, Komastu and Voelz (1997). Galotti et al. examined students' reasoning skills in kindergarten, grades two, four, and six. Their study used fantasy syllogisms of deductive and inductive forms; children were asked to answer yes or no questions to verify their ability to understand the syllogisms. Not only were students able to correctly identify answers, students in grades two, four, and six were also able to justify their answers. Additionally, this study displays that students as young as second grade showed an understanding of the difference between inductive and deductive reasoning and were often more confident with the latter (Galotti, 1997). Galotti et al. proposed that task complexity could also contribute to children's ability to reason logically. Recently, two theories relating to complexity have been proposed.

The task of reasoning deductively is complex; children's ability to complete tasks depends on complexity level. Children starting as young as one year of age are able to solve lower complexity tasks, while older children can solve more complex tasks (Halford & Andrews, 2004). Complexity can be defined in two ways using the Cognitive Complexity and Control (CCC) theory and the Relational Complexity (RC) Theory. The first of these theories, Cognitive Complexity and Control theory suggests that the complexity of tasks relates to the number of distinct rules needed to complete the task (Zelazo & Frye, 1998). These rules are defined by categories by which objects can be classified. Children must have an understanding that objects can be categorized in multiple groups as well as how to differentiate between the groups. As the number of rules increase, it becomes more difficult to accurately complete a task. Zelazo and Frye (1998) state that two-year-old children are able to understand tasks with one rule, such as if characteristic A holds then it belongs to category A. Additionally, three-year-old children are able to understand tasks with two rules such as, if characteristic A holds then it belongs in category A and if characteristic B holds then it belongs in category B. Although as the number of rules increase, children have more difficulty completing the task, children as young as five are able to develop the skills to complete high-order ruled tasks (Zelazo & Frye, 1998).

The second theory that discusses the complexity of deductive reasoning is the Relational Complexity Theory. In this theory the amount of variables within a task adds to the complexity to solve it (Halford & Andrews, 2004). The number of variables relates to the arity of a relation, or the number of arguments within the task. Children are able to understand unary relations, tasks with one variable, at one year of age. Binary relations, tasks with two variables can be completed by children two years old; ternary relations, tasks with three variables, at five years old; quaternary relations, tasks with four variables at eleven years old (Halford & Andrews, 1998). As variables are added to the problem, the complexity rises; as children gain more experience with problems requiring them to reason deductively their ability levels also rise.

The CCC theory and RC theory differ in the approach they take to complexity level. However, in both of these theories, the complexity of a task directly relates to a child's ability to solve it. Each theory shows that as long as deductive reasoning tasks are age appropriate, young children are able to solve them (Halford et al., 2004).

Research has been focused on the development of these deductive reasoning skills, however different approaches to reasoning tasks play a role in the results has not been as thoroughly examined. Many studies have been done using syllogistic arguments, but not as many have been done using other techniques (English, 1998). The presentation of the task may impact the results; language-free visual representation of logic tasks may allow younger students to engage in and succeed at deductive reasoning tasks. Thus, this project will explore the success and strategies that elementary students use to complete visual logic tasks as opposed to syllogisms or relational problems.

Methodology

Participants

In this study data were collected from four elementary classes at two middle-class, suburban elementary schools in Southeastern Massachusetts. Two fourth grade classrooms were selected from a grade 4-5 school and two first grade classrooms were selected from a grade K-3 school. Written work was collected from 18 fourth grade students between the ages of 9-10 years and 22 first grade students between the ages of 6-7 years. Of the fourth grade students, fifty percent were girls and of the first grade students, forty-one percent were girls. Each teacher received a copy of MetaForms, a one player version of Logic Gym, for their classroom as a token of appreciation for participating in the study.

Materials

Visual logic tasks were selected from Logic Gym, a visual logic supplementary curriculum for elementary school students (Lyons, Lyons, and Sabinin in publication). Logic Gym requires that the player places nine objects (red square, blue square, yellow square, red circle, blue circle, yellow circle, red triangle, blue triangle, and yellow triangle) onto a 3x3 grid. Based on given clues these objects are arranged in a particular order on the board. Each clue contains two components, the object and the grid pattern. The object is the specific shape or color piece about which the clues make a statement. The grid pattern refers to the layout of the squares that shows the location of the object on the game board. Task difficulty increases with the addition of different types of clues and combination of clues. Positive, negative, implication, and attribute clues are the four main clues that must be deciphered to complete each task.

Figure 1 illustrates a positive clue. The shaded region in the grid pattern signifies the correct location of a shape. Thus, a shaded cell allows for a positive affirmation that guarantees the location of a specified object. For example, figure 1 means that there exists a red triangle in the right column, middle row.

Figure 2 displays a negative clue. The cells in the grid pattern with an X within them mean that a shape cannot go in that cell. Therefore an X-ed out box provides a negation or a certainty that a specific object cannot go in that location. For example, figure 2 means that there does not exist a blue square in the top middle, middle left, middle right, or bottom middle.

Figure 3 displays an implication clue. The clue shows one object in relation to another. And can be written as an if-then statement. For example, figure 3 means that if the red square is placed on the game board, then the blue circle goes diagonally below to the left.

Figures 4 and 5 display attribute clues. These clues contain either a certain colored pencil or a grey shape. The colored pencil means that the color of the object is specified, but the shape must be determined based on the other clues. Similarly, the grey shape means that the shape of the object is known, but the color must be determined based on the other clues. These clues provide one attribute of the object being placed, but the other is unknown until the other clues are used. For example, figure 4 means that there exists a yellow shape in the bottom middle and figure 5 means there exists a triangle in the bottom left.

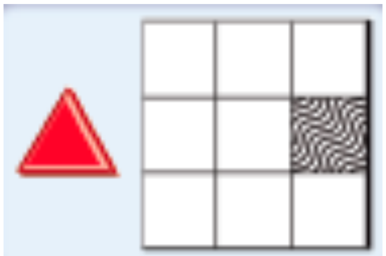


Figure 1. Positive Clue.

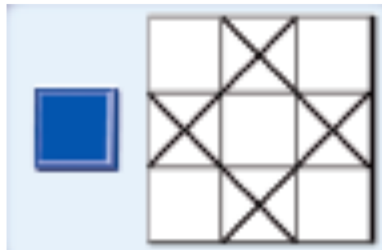


Figure 2. Negative Clue.

Figure 3. Implication Clue.

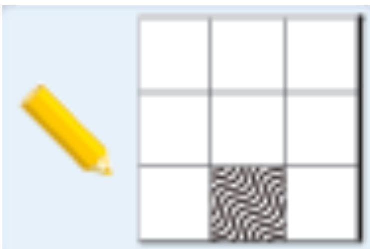


Figure 4. Attribute Clue.

Figure 5. Attribute Clue.

Data Sources

Three practice problems, twelve written problems, and six interview problems were used in this study. The three practice problems were tasks that contained six to nine clues sufficient to place all nine pieces on the board in a unique configuration (Appendix A). The problems became progressively more difficult, adding more clue types throughout. The first problem contained positive clues as well as attribute clues; the second problem added negative clues; the third problem contained all four types of clues. Interview problems were in the same form as the practice problems, but there were six tasks selected for participants to complete (Appendix B).

A written worksheet was developed based on the assessment pages of Logic Gym. The worksheet contained twelve problems for the participants to complete (Appendix C). Each problem required that only one shape be drawn onto an empty grid based on a given clue. An image of a partially filled board was shown as well as the remaining pieces to choose from. One clue was displayed for each numbered problem, this clue required participants to choose from the remaining objects and draw the correct one on their answer key. Once again, difficulty increased throughout the worksheet requiring different types of clues to be deciphered.

Procedure: The researcher introduced each classroom to the logic puzzles through the use of three practice problems. The students were given time to complete the three tasks, which contained positive clues, negative clues, implication clues, and attribute clues-all types of clues which appeared in the written work and interview tasks. Students were allowed to discuss the tasks with their classmates as they developed a better understanding of the problems. Throughout the introduction, participants were able to ask questions all of which were answered thoroughly and explained to the entire class. At the end of the introduction session, students were given the written worksheet to complete individually. Students were allowed to ask questions, however most questions were answered with encouragement to make their best guess based on the given clue. This written work was collected and coded as correct or incorrect. Each of the twelve problems was examined individually to determine whether students completed the task successfully or unsuccessfully. This examination of results was compared within the grades based on difficulty and complexity. A simple percentage of how many students found the correct solution for each problem was calculated. These percentages were used to determine if difficulty and complexity of a problem impact a students' ability to successfully solve no matter their grade. Additionally, the results were compared between grades to see whether there is a difference between first and fourth graders' success. A 2-sample T-test was performed based on the scores of both first and fourth grade students. This test calculated the mean score of the written work within in each grade. The results were compared between grades to determine if grade was a significant predictor of success. Also, chi-squared tests were performed on each question in order to determine if grade level was a significant predictor for each problem.

After the written work was completed, six students from each grade were selected based on teacher recommendation, observations, and random selection to participate in a 30-minute interview. The fourth grade students were selected the day of the written work and interviewed the following day. The first grade students were selected during the written work phase and interviewed later in the afternoon of the same day. Interviews were conducted and video-recorded in a private room to avoid distractions for the participants. Students were told that they would be asked to complete six puzzles similar to the practice problems from before. They were encouraged to "think out loud" as they completed the puzzles to better explain their thought process. Students who did not describe their thinking on their own were prompted to do so with questions such as: 1) Why did you place that piece there? 2) How did you decide to make that move? 3) Can you explain what that clue means in words? 4) How do you know that the solution to that puzzle is correct? 5) What clue will you use next?. Students were encouraged to complete as many problems as they could within the 30-minutes, but were allowed to stop if they no longer wanted to participate. These interviews were qualitatively analyzed for strategies students used in order to complete tasks as well as whether they were correct or incorrect strategies.

Data Analysis Process

All twelve interviews were transcribed including relevant gestures. For transcription, gestures were deemed relevant when they involved the puzzle clue being worked on such as pointing to a shape, pointing to a position on the game board, or mapping a specific grid pattern onto the game board.

Interviews were read multiple times and common strategies emerged which were coded and analyzed. Strategies were identified when the strategy was one other than rule recall or describing the clue in words. Rule recall often occurred when a student stated the definition of the different types of clues.

In most cases strategies were identifiable immediately before a student placed a piece on the game board. However some of these strategies occurred when the student first looked at a new clue and when he or she was checking the solution. Strategies that emerged included *clue mapping*, *mental representation*, *guess and check*, *listing of possibilities* and *process of elimination*.

The *clue mapping* strategy refers to the student process of physically drawing the grid pattern onto the game board with his or her finger. Students traced, or mapped, the image that appeared in the clue's grid pattern onto the game board to visualize the location given by the clue. The student used this strategy combined with the rules pertaining to each clue to decide where an object should be placed. Gestures were recorded in brackets and would say something like "[draws shape in P1C5]" or "[draws horizontal line]."

The *mental representation* strategy is when students verbalized that they were visualizing a specific representation of the grid pattern. Students used either direction words such as diagonal, left, right, horizontal, and vertical or they stated a shape that described the mental image they saw in the grid pattern of the clue. The student pictured the grid pattern on the game board and then used the rules of the clues to place the piece. A student using the mental representation strategy would say something like SB1, "the second one, I did that upside down T and then I put that right there. And then I did something like a Z there." Or like SA1, "it's kind of diagonal."

Guess and check strategy is when the student did not know where to place a shape and resorted to guessing. The student sometimes did not know why a shape went in a certain place or had a limited explanation of the decision. Sometimes the student would go back and check his or her decision by looking at the clues again. Other times the student simply assumed the answer was correct without checking. A student using the guessing strategy may say something similar to SE1, "cause, um, I didn't know so I just guessed where to put it."

Process of elimination strategy is where students made a decision based on the fact that they saw no other possibility. Process of elimination happened when the student was choosing which shape to use and there was only one shape of a specific color or one color of a specific shape. A student would say something like SD1, "cause the other, cause there's no more blues and it said to put any blue color." This strategy also occured when a student only had one open space on the game board thus automatically placing the final piece in the open box. Finally, this strategy sometimes occurred after a student listed all the possibilities for a piece, but some of the options were already taken, so those possibilities were eliminated leaving only one. Another typical example of a student using process of elimination would look like SE4, "well, here, if I did it there [points to GB6], there's already a shape there, so I had to move it up one and then put it there [points to GB3]."

The final strategy is the *listing of possibilities* strategy. The student either verbalized the different places that a shape could be placed, or physically pointed to the options on the game board. He or she then narrowed down the options using another strategy such as *clue mapping*, *mental representation*, *guess and check*, or *process of elimination*. In general, this strategy looked much like SC1's statement, "Well… it could go here [points to GB6] or here [points to GB4]."

Once all strategies were coded, a coding reliability check was conducted with three individuals to ensure that strategy definition was not subjective. There was an 83% agreement rate amongst each of the individuals and the researcher. To settle disagreements there was a discussion to determine where the difference was coming from. It was decided that some coded strategies could fall into multiple categories, however there was generally one strategy that fit the situation best. Next, these strategies were analyzed based on frequencies of strategy among all students and puzzles. Percentages were calculated to determine how many students overall used each strategy to show which strategies were used most frequently among all participants. Then strategies were analyzed within each grade to find if students in both first and fourth grade used the strategies at the same frequency. Finally, the frequency of strategies within each puzzle were analyzed to determine whether difficulty and complexity had an impact on the strategies students implemented both within the grades and between the grades.

Results

The first purpose of this study was to discover whether first and fourth grade students both possess the ability to successfully solve visual logic problems. After conducting student interviews and collecting written work, students in both first and fourth grade were able to successfully complete visual logic tasks. They were able to apply the different strategies while performing the tasks to reach the correct solution. Student written work was coded to detect if student scores are impacted by grade. Quantitative analysis shows that there is significant evidence that there is a difference in performance ability between first and fourth grade students.

A 2-sample T-test was performed with results indicating that grade is a significant predictor of mean scores on visual logic tasks. The mean score for the twelve-question written work was 53 percentage points higher in grade four than in grade one (Mean Grade 1= 2.81 (23%), Mean Grade 4= 9.11 (76%), $p \leq 0.001$). These results show that fourth grade students were able to independently solve written problems with more accuracy than first grade students.

Additionally, chi-squared tests were performed to determine whether grade was a significant predictor of student's success on each of the twelve problems. The results appear in the Table 2.

Question	Grade 1 Correct	Grade 4 Correct	P-value
1	100%	100%	N/A
2	45%	100%	<0.001
3	27%	78%	0.001
4	14%	72%	<0.001
5	23%	89%	<0.001
6*	0%	33%	0.003
7	17%	78%	<0.001
8	17%	61%	0.002
9	17%	83%	<0.001
10	23%	78%	<0.001
11	5%	72%	<0.001
12*	5%	50%	<0.001

Table 2. Results of Chi-Squared Tests on Written Work. *One or more of the predicted values was less than 5, decreasing the power of the Chi-squared test for these problems.

All problems show significant evidence that grade is a predictor of student success. The student results from each problem indicate that fourth grade students have a greater ability to solve visual logic tasks. Their overall performance on the written work as well as on individual questions shows that fourth grade students show more success completing visual logic tasks.

The second purpose of this study was to examine strategies used by students to solve visual logic tasks. Although results from the quantitative analysis of student written work display that fourth grade students possess a greater ability to solve visual logic tasks, results from the coded interviews show that students in first and fourth grade use similar strategies while completing such tasks despite their success level. Of the coded strategies: *clue mapping*, *mental representation*, *guess and check*, *listing of possibilities*, and *process of elimination*, all five strategies appeared in both first and fourth grade interviews. The overall frequencies of strategies show that *clue mapping* and *process of elimination* are the most used strategies amongst first and fourth grade students. The *process of elimination* strategy appeared in all twelve interviews and comprised 32% of the total coded strategies in both grades while the *clue mapping* strategy appeared in eleven interviews making up 29% of the total coded strategies. The other three strategies: *mental representation* (12%), *guess and check* (13%), and *listing of possibilities* (14%), although less prominent, were used by students in both grades. This shows that despite different success level, students are using common strategies in order to complete tasks.

Although all strategies appear in both grades, the frequency of which they appear slightly differs between the grades. The percent that each strategy appears within each grade is displayed in Table 3 below. Additionally, chi-squared tests were performed to determine whether grade was a significant predictor of strategy type used. The results appear in Table 3.

	Grade One	**Grade Four**	**P-value**
Clue Mapping	32%	26%	0.1609
Mental	14%	10%	0.2749
Guess and Check	17%	9%	0.0140
Listing of Possibilities	8%	20%	<0.001
Process of Elimination	29%	35%	0.2068

Table 3. Results of Chi-Squared Tests on Strategy Use.

The strategies that have the greatest percentage difference are the *guess and check* and *listing of possibilities* strategies. The *guess and check* strategy appeared in the first grade coded strategies much more frequently than in fourth grade. This may show that first grade students are more likely to use guessing as a means of solving a task than fourth grade students. The *listing of possibilities* strategy appears in the fourth grade coded strategies more frequently than first grade. This difference shows that fourth grade students are more likely to express the options that they have for a certain clue than first grade students.

The distribution of strategies throughout the progression of puzzles displays some interesting results. Figures 6, 7, 8, 9, 10, and 11 display the frequency that students employed each of the five strategies in each puzzle. Strategies are represented by the following: *Clue Mapping*, CM; *Mental Representation*, MR; *Process of Elimination*, POE; *Guess and Check*, GC; and *Listing of Possibilities*, PB.

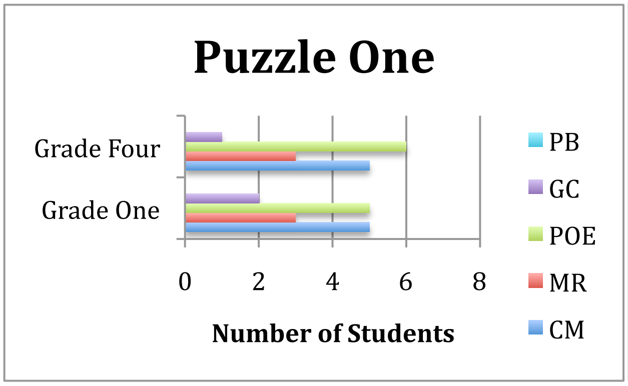


Figure 6. Puzzle One Strategy Frequency.

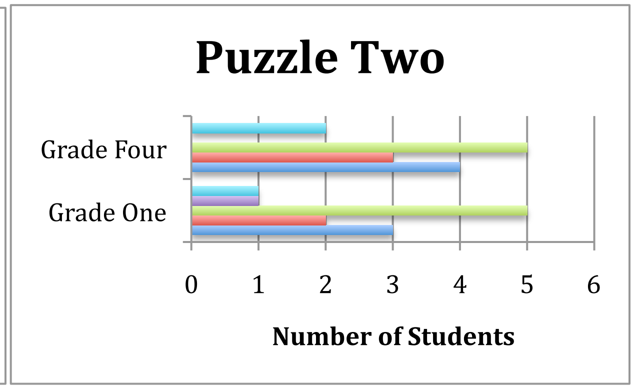


Figure 7. Puzzle Two Strategy Frequency.

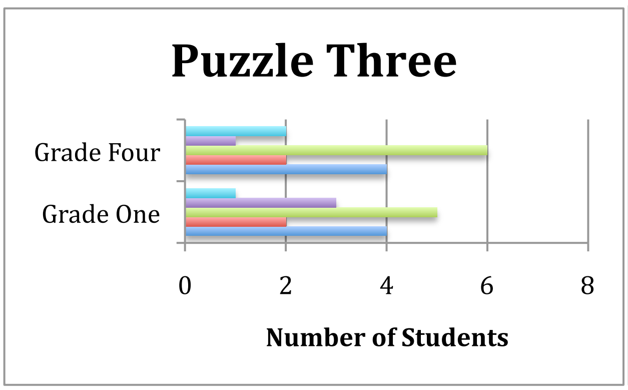


Figure 8. Puzzle Three Strategy Frequency.

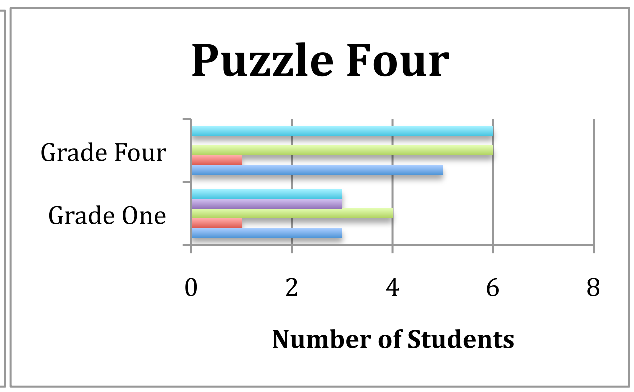


Figure 9. Puzzle Four Strategy Frequency.

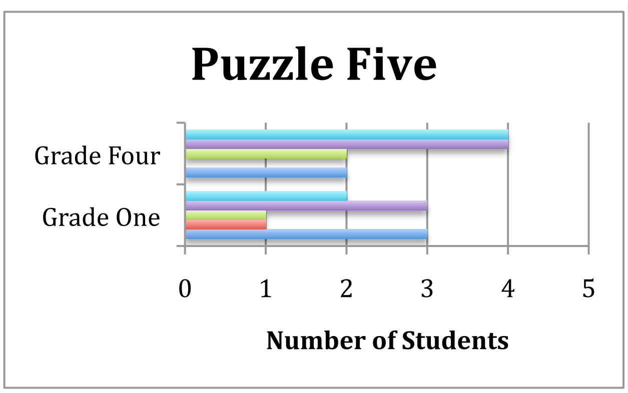


Figure 10. Puzzle Five Strategy Frequency.

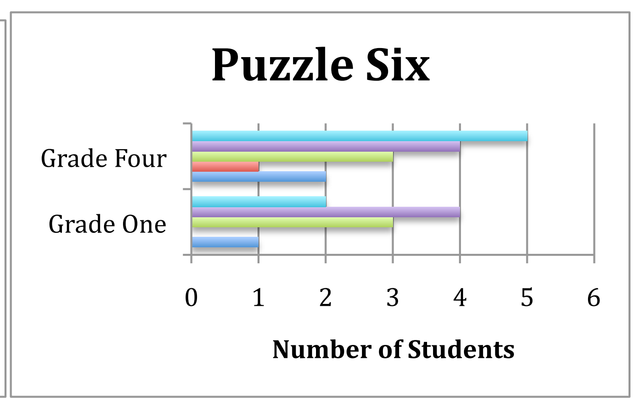


Figure 11. Puzzle Six Strategy Frequency.

When solving puzzles one and two, which consisted of only positive clues and attribute clues, students, in both grades, most frequently used the *clue mapping*, *mental representation* and *process of elimination* strategies. The *guess and check* strategy was used by only four students within these two puzzles and the *listing of possibilities* strategy was used by only three students in puzzle two. However, as the difficulty of the problems increased with the addition of negative clues in puzzles three and four, nine students employed the *listing of possibilities* strategy. Additionally, when implication clues were included in puzzles five and six, the number of students who used the *guess and check* strategy increased to nine.

The results of this study show a variety of important facts about students' ability and strategies for solving visual logic tasks. Although age was determined to be a significant predictor of success on the tasks, students in both first and fourth grade implemented similar strategies to solve them. In addition to the fourth grade students, first grade students could also verbalize logical steps to complete the problems.

Discussion

The results of this research show multiple findings. First, students in both first and fourth grade are able to solve visual logic tasks. Although at different success rates, participants in both of these age groups accurately completed visual logic tasks with a certain degree of difficulty and complexity with accuracy. Second, students in both of these grades used similar strategies when completing the tasks albeit at different frequencies.

Piaget (1958) is one of the best-known child development theorists whose ideas are important in the field of education. However, this research does not support his findings on children's ability to solve visual logic tasks. Students in first grade, ages six and seven, as well as students in fourth grade, ages nine and ten, were able to solve visual logic tasks successfully. This opposes Piaget's theory that logical reasoning does not develop until the formal operational stage when a child is eleven or twelve. This study showed that students who have not reached the formal operational stage can complete both simple and complex logic tasks with a variety of clues. This may be due to the visual representation of tasks. Young students who do not have strong language skills can use visual tasks instead of syllogisms to solve logic problems. The importance of language in syllogisms or word problems makes it more difficult for young students to successfully solve the problem. Introducing elementary students to visual logic tasks it allows them the opportunity to succeed without requiring decoding or comprehension skills.

After analyzing strategies used during interviews, patterns appeared that display differences in strategy choice between age groups. Supporting the research of Markovits and colleagues (1989), younger students were able to accurately solve tasks, but lacked a total understanding of how they went about doing so. This is displayed in the higher frequency of the *guess and check* strategy amongst first graders. Rather than having a deep understanding of their choices, students in first grade were more likely to take a guess and hope that they placed a piece in the correct spot. Between first and fourth grade, the frequency of the *guess and check* strategy changed; it decreased by eight percentage points. Students' understanding and ability to support their decisions with a logical reason increased with age. Like the atmosphere effect described by Markovits et al. (1989), students in first grade relied on a gut feeling to place pieces on the board whereas fourth grade students could create solid arguments for their solutions. This displays how the ability level to solve visual logic tasks differs between children of different ages. Although six and seven year olds were able to successfully complete a handful of tasks, their arguments were not valid as frequently as the nine and ten year olds. Thus, Markovits and colleagues may be correct to say that deductive reasoning skills are not inherent in children and do take time to development.

Moshman (1986) discusses the idea that young children are able to solve simple logic tasks. This research supported the conclusions that young children, age six, are able to reach conclusions based upon assumptions that are true. For example, first grade students had a high success rate on clue placement when using positive and negative clues. These simple clues provided a clear visual explanation of where a piece should be placed on the grid. However, as more difficult clues, implication and attribute clues, were introduced to a puzzle the success level decreased. This also contributed to the strategies that they used.

The difficulty of logic tasks impacts the strategy choice of students when completing problems. As the difficulty of tasks increased, student comprehension appeared to decrease. They began to employ strategies that they could not fully explain. The two most frequent strategies in the most difficult problems were *listing of possibilities* and *guess and check*. When students used the *listing of possibilities* strategy they decided to explicitly list all of the options because they were not initially sure of their answer. They needed to see each of the possibilities in order to make a decision. Also, students using the *guess and check* method did not have any clear reasoning to support their clue placement. They found no logical placement and resorted to making a guess. Ultimately as difficulty increased students became less likely to back up their decision-making processes with solid evidence and simply made a random guess. This is definitely true for first graders, however, fourth grade students were less likely to use *guess and check* as their first strategy choice. Fourth grade students were able to support their conclusions with valid arguments more frequently supporting Moshman's (1986) claim that students of this age are able to create support for their decisions. This increase in difficulty is directly related to the complexity of problems.

Throughout the interview process, the complexity of problems increased with the introduction of new clue types. The results showed a clear change in the strategy selection of students in both first and fourth grade. With the increase in complexity students were more likely to make a guess than with simpler tasks. Supporting the claims of Halford and Andrews (2004), this study displays that complexity of logic tasks plays a role in a child's ability to solve it. Tasks need to be age appropriate with the proper number of variables in order for a child to have success. Since the problems became increasingly more difficult throughout the study, students were being exposed to problems with more variables in the form of clue types. The increase in clue difficulty impacted the success rate at which students, in both grades, were able to complete tasks. The simpler problems that only used positive and negative clues had much higher success rates for students than when attribute and implication clues were added.

Elementary students, in both first and fourth grade, are able to successfully solve visual logic problems. However, results from the quantitative analysis display that there is significant evidence that fourth grade students' ability is greater than those in first grade. Despite this evidence students displayed the use of some of the same strategies when completing the tasks. Students' strategies showed patterns that reflect how the difficulty of tasks and age of children impacts ability and success level.

Some limitations may have had an impact on the results of this study. Due to the small sample size of students who were tested, results may not be generalized to the entire population of first and fourth grade students. Additionally, the study only took place in one school district that does not encompass a diverse population of students. Finally, during the interview process one first grade student only finished the first three interview problems thus data was not collected from problems four, five and six on this student.

Despite the limitations of this study, the results can still be used as an accurate way to examine the deductive reasoning skills of first and fourth grade students. These limitations did not have a direct impact on the performance of students. First and fourth grade students were still solving visual logic tasks independently and explaining their thought process.

However, future research could help address these limitations. Further research could be done using the same methodology, but with a larger amount of participants to see if similar results arise. This would allow the results to be generalized to a more diverse population of students. Also, including different aged participants, such as second and third grade students, generalizations could be made on a much wider spectrum encompassing multiple ages. Additionally, it would be beneficial to work with different school districts that may use different math instruction. Since some school districts' math programs may emphasize different standards, the results may appear different across districts. Giving students a longer amount of time to practice might be something that could be considered in future studies. Due to time constraints students only had one practice session before doing the written work and interviews. Additional time could improve the success rate of both grade levels.

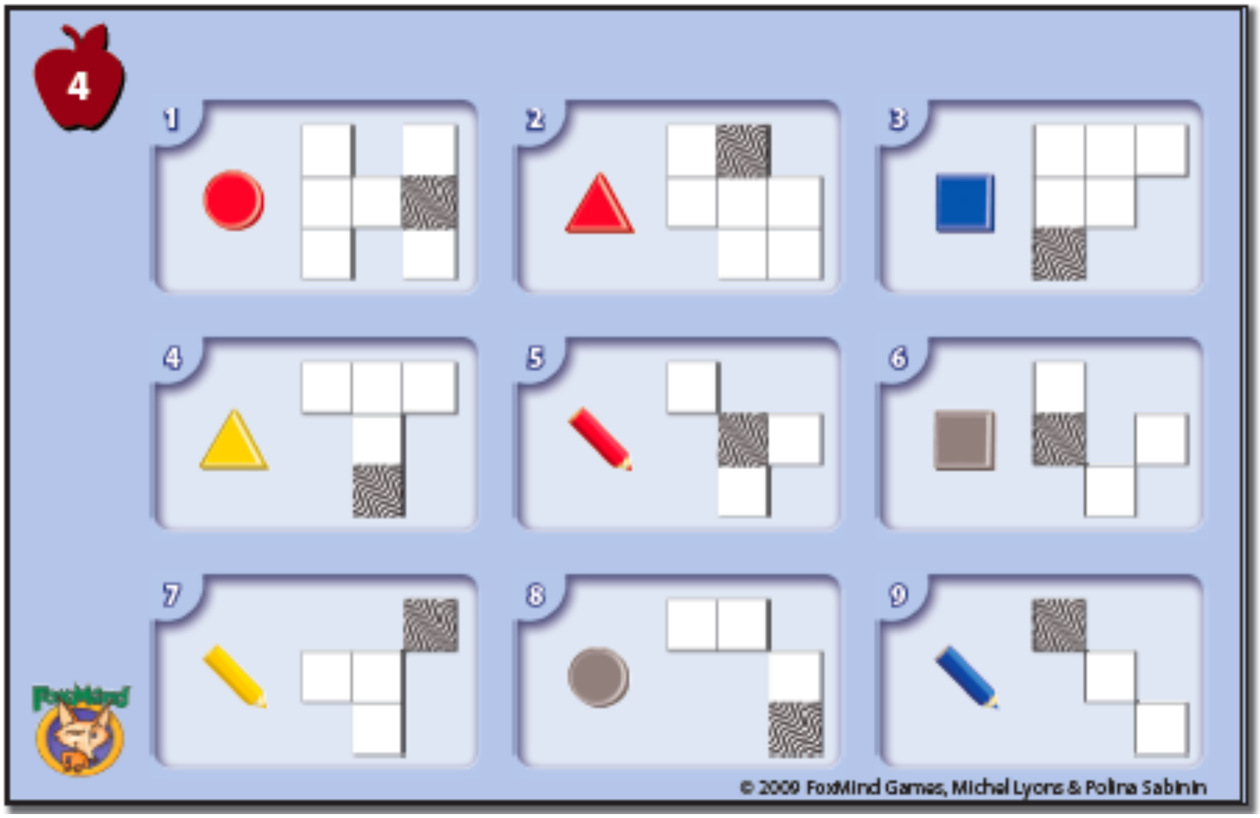
The findings from this research could also be used to further explore the success rate between students solving visual logic tasks versus another format. A study could be conducted that compares the same age students solving visual logic tasks and syllogisms. Another format to explore would be the use of digital visual logic tasks using an electronic display versus a task to be manipulated physically. Each of these approaches could be used to examine the deductive reasoning skills that elementary aged students possess.

The results of this research can be useful for educators to see what deductive reasoning skills their students possess. First and fourth grade students were able to solve visual logic problems, but at different success rates. This information will be helpful for educators when planning and implementing lessons; they must have an understanding of their students' ability levels in order to teach them most effectively. Using this study, teachers can think about ways to differentiate their instruction to best fit their students' needs while also emphasizing the importance of deductive reasoning skills. Based on the strategy results of first and fourth grade, teachers can determine what strategies need to be taught to students and what strategies are more inherent. Since fourth grade students were more likely to use the *listing of possibilities* strategy this one that teachers may need to focus on in the lower grades. By giving explicit instruction to students about this strategy, teachers can promote this important strategy at a younger age. Another important aspect to teach young students is to check their answers when they make a guess. Many times first graders made a guess without going back to check their answer, showing that this important skill needs to be taught to students more directly. Teachers can learn from this study what strategies are important to teach to students depending on their grade and ability.

Most importantly this study can be used to show teachers that first grade students do possess the ability to reason deductively. Many people may rule out the possibility of young students completing certain tasks deeming it developmentally inappropriate. However, as displayed in this study, students as young as six and seven are able to solve simple logic problems. With this knowledge teachers may give their students more deductive reasoning tasks, thus emphasizing and nurturing these important skills from a young age.

Deductive reasoning is an extremely important skill for a person to have, not only in mathematics, but in everyday life. This study was designed in order to determine whether first and fourth grade students have the ability to solve visual logic tasks and, if so, what strategies they use. Results show that both first and fourth grade students do possess the ability to reason deductively using a visual context, contrary to many theories that state young children do not have this ability. Fostering this ability throughout a child's education will ensure that they continue to build upon this pivotal life skill.

Appendix A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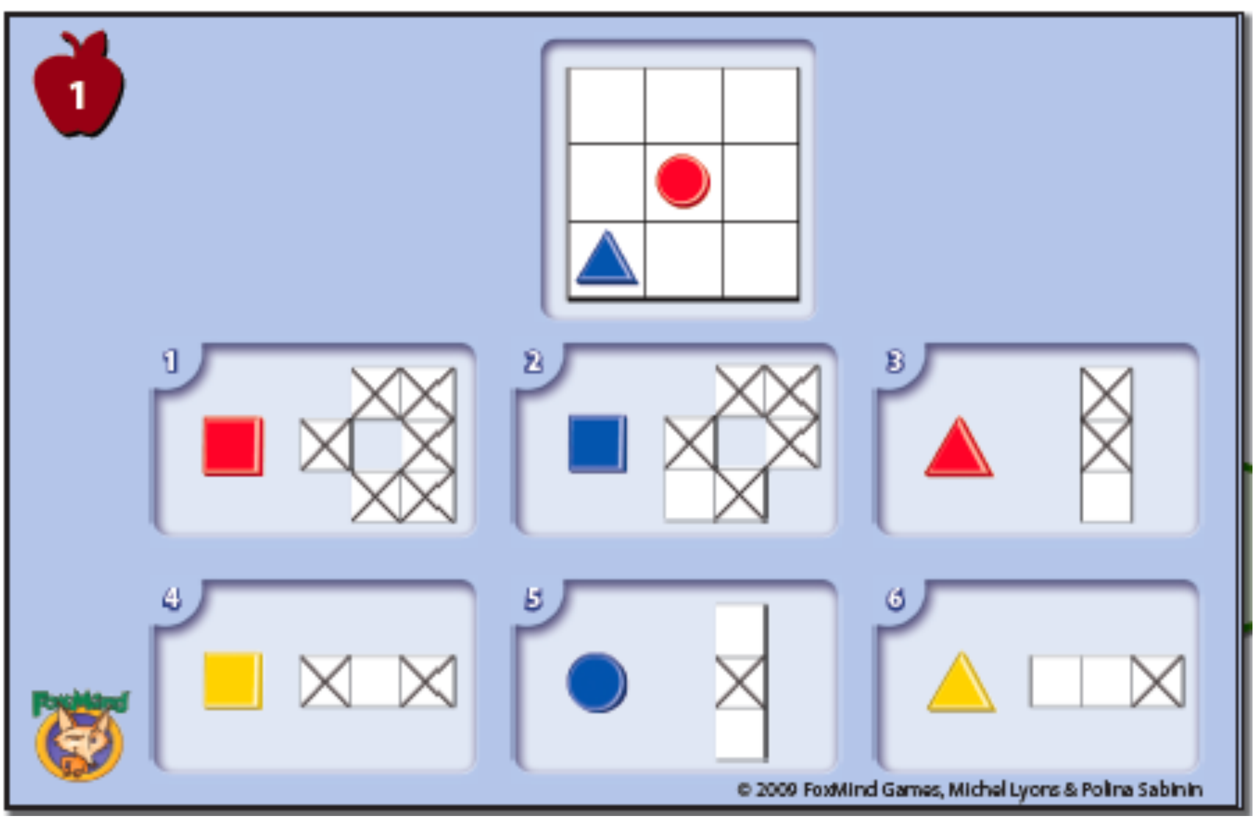
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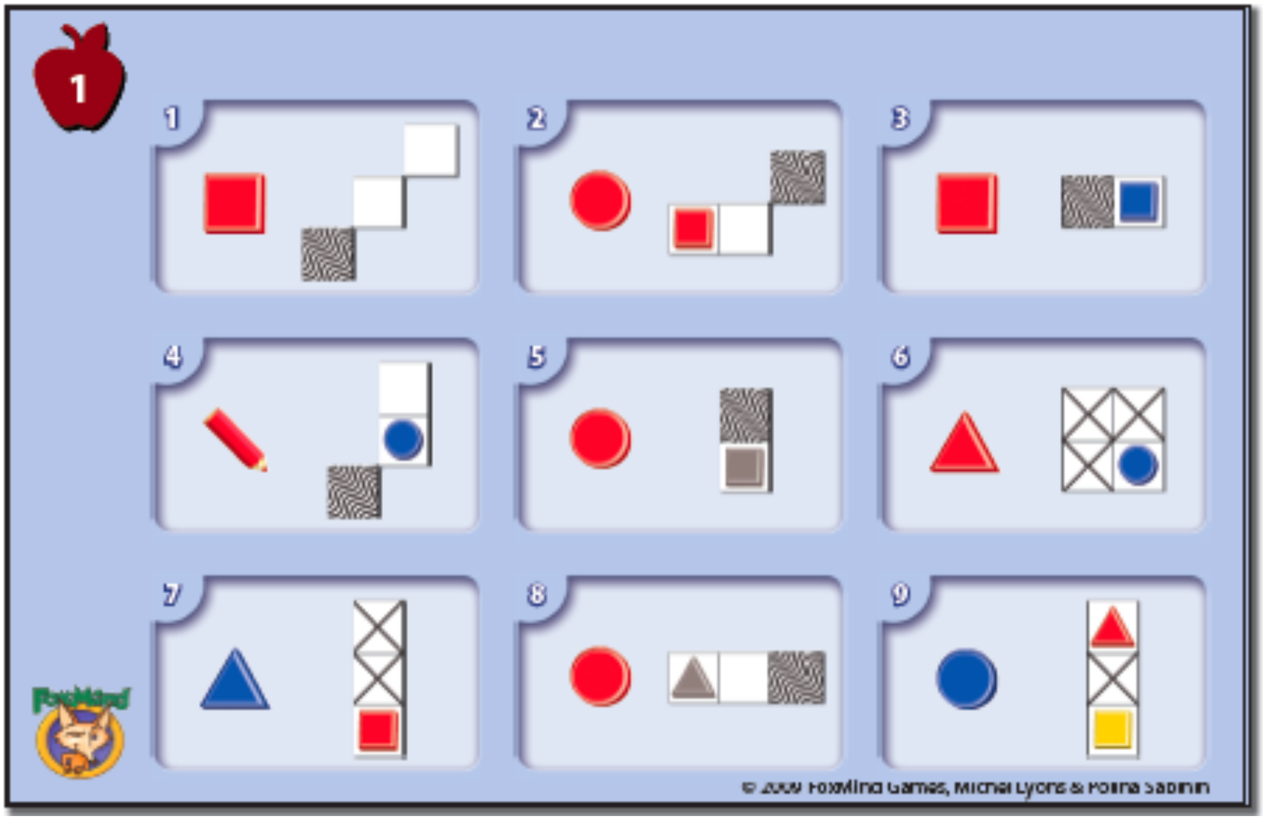
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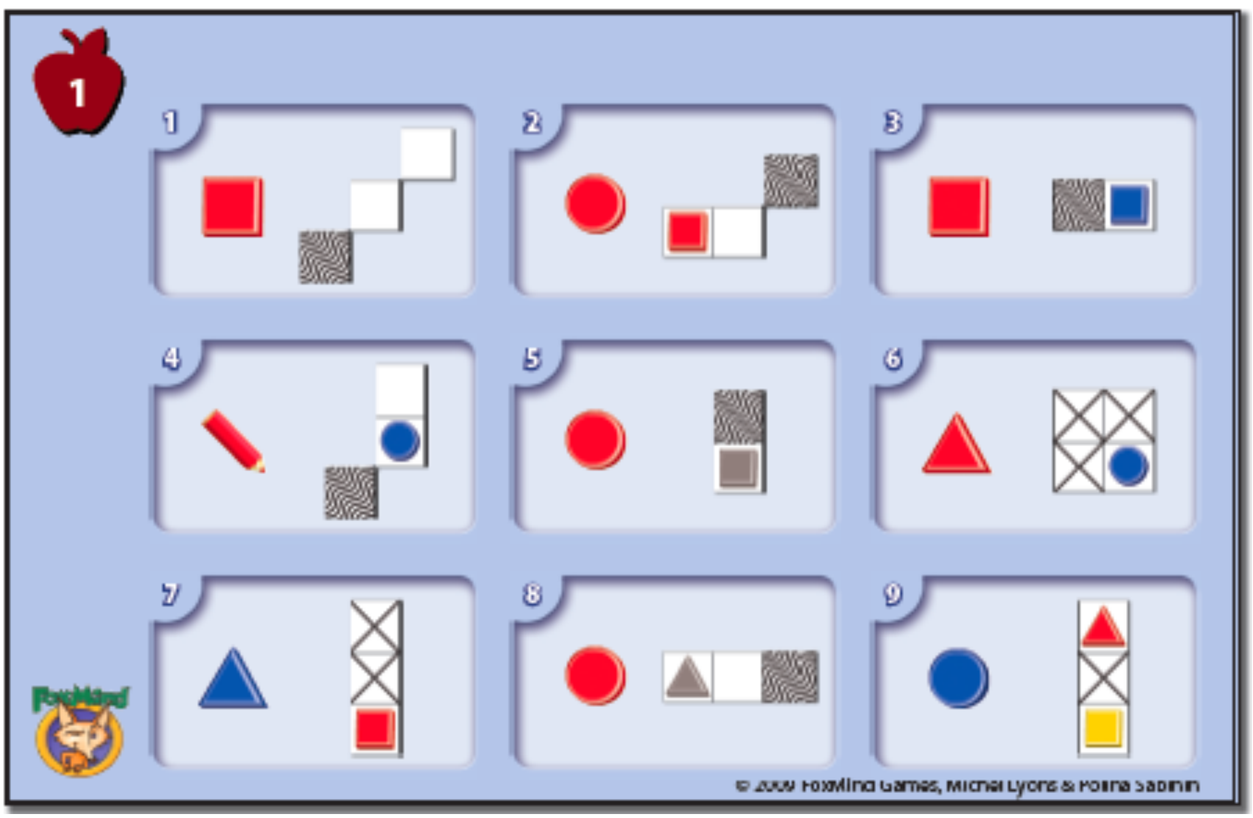
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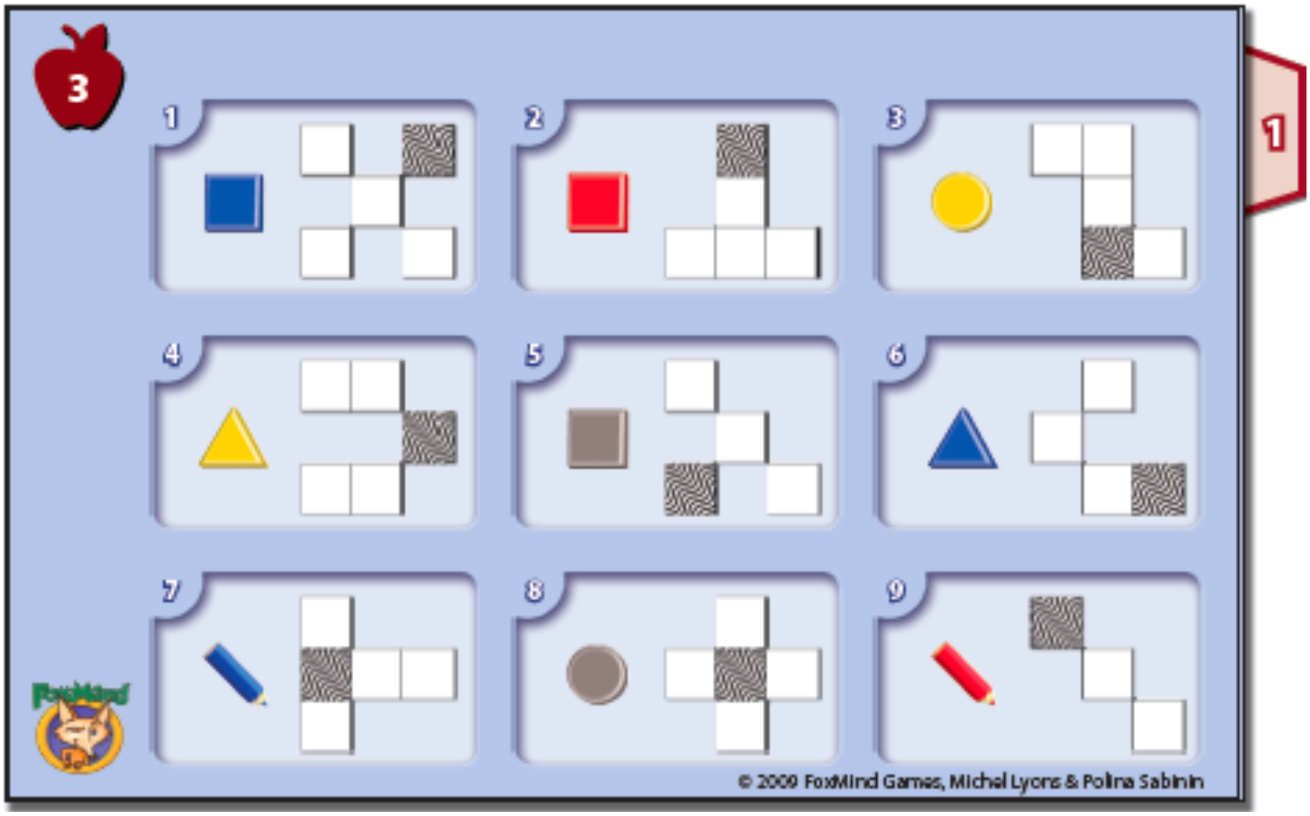
Appendix B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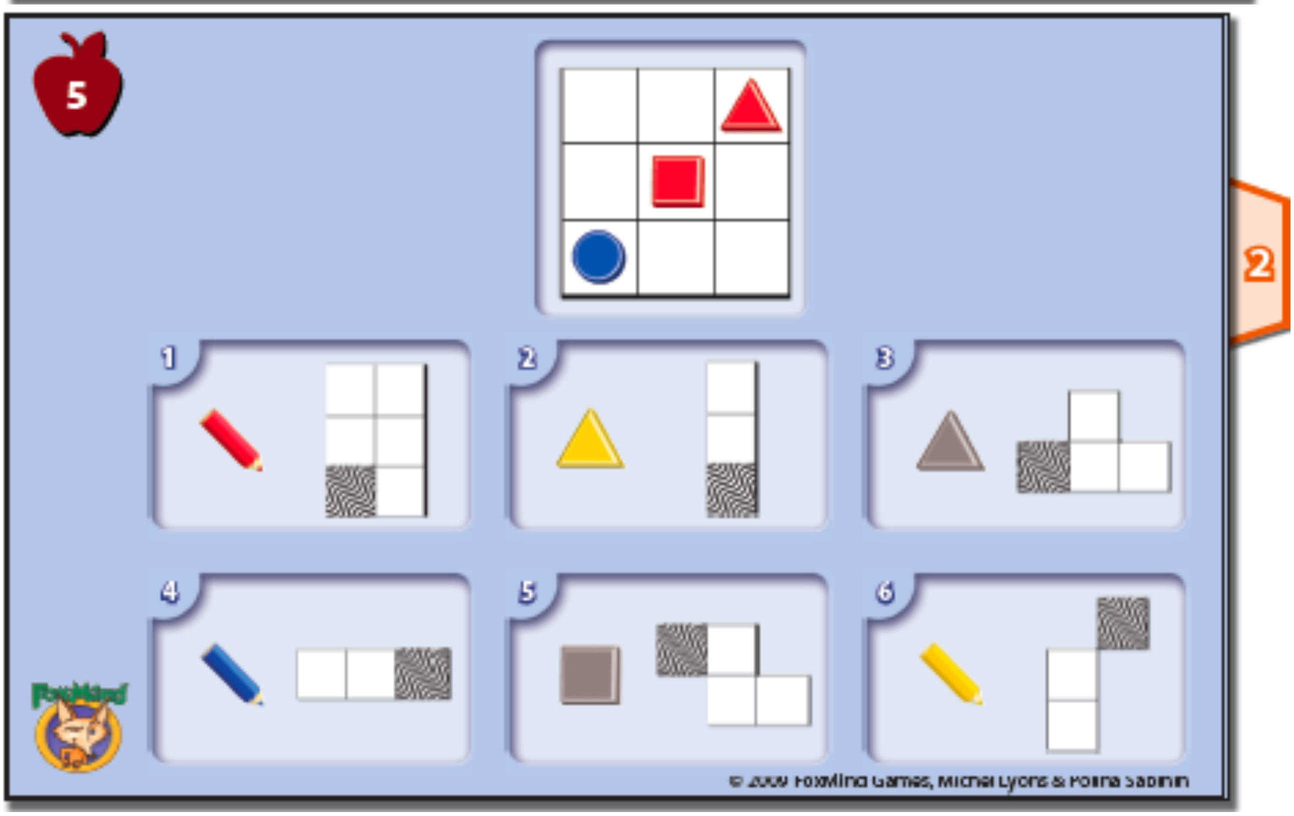
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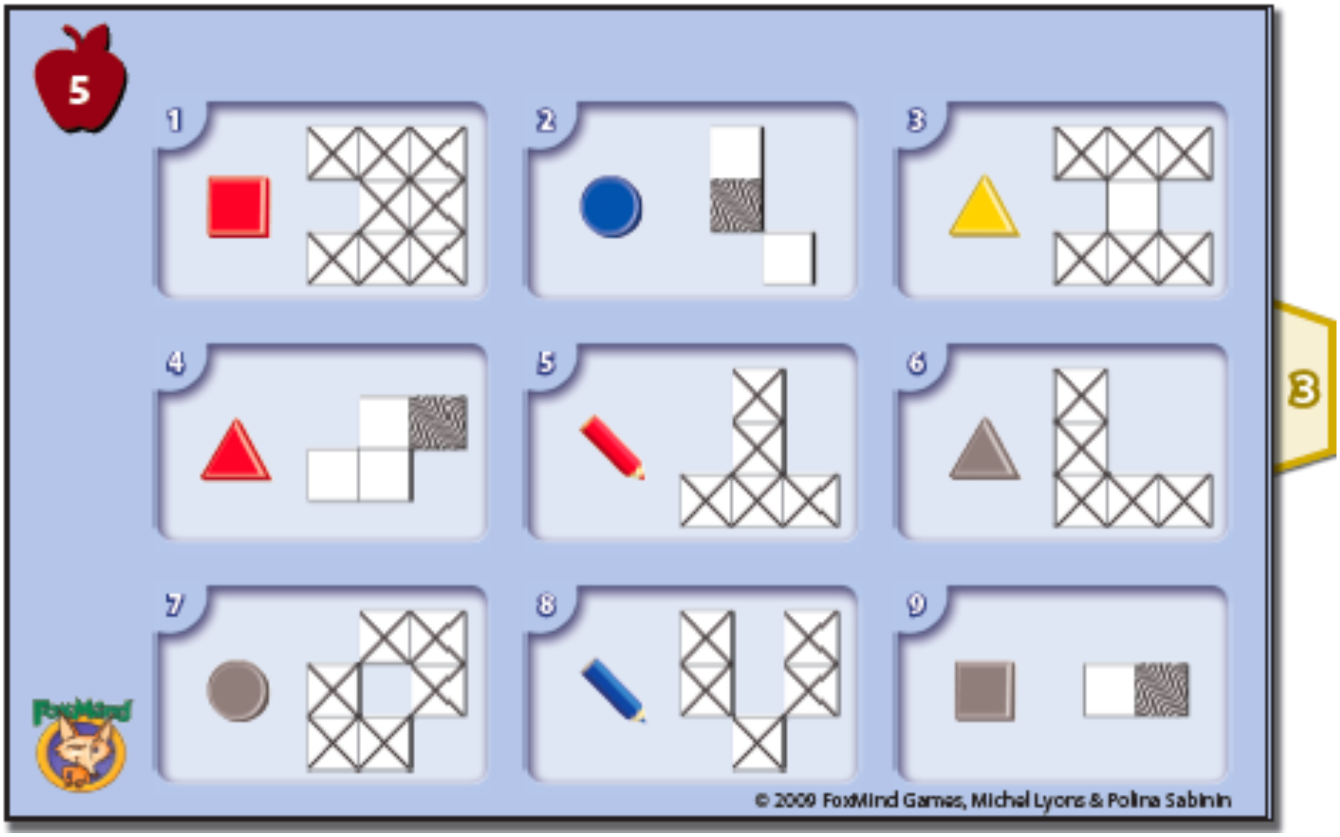
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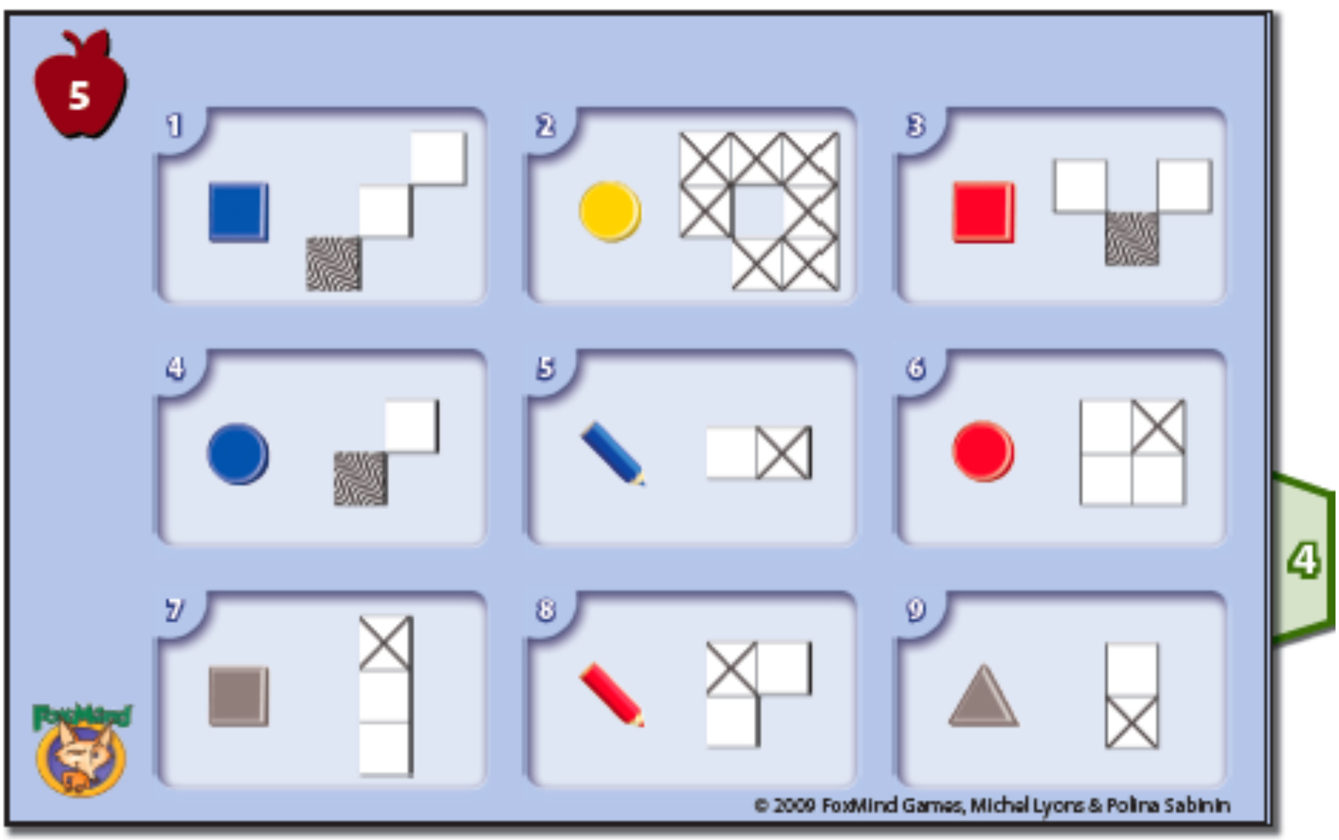
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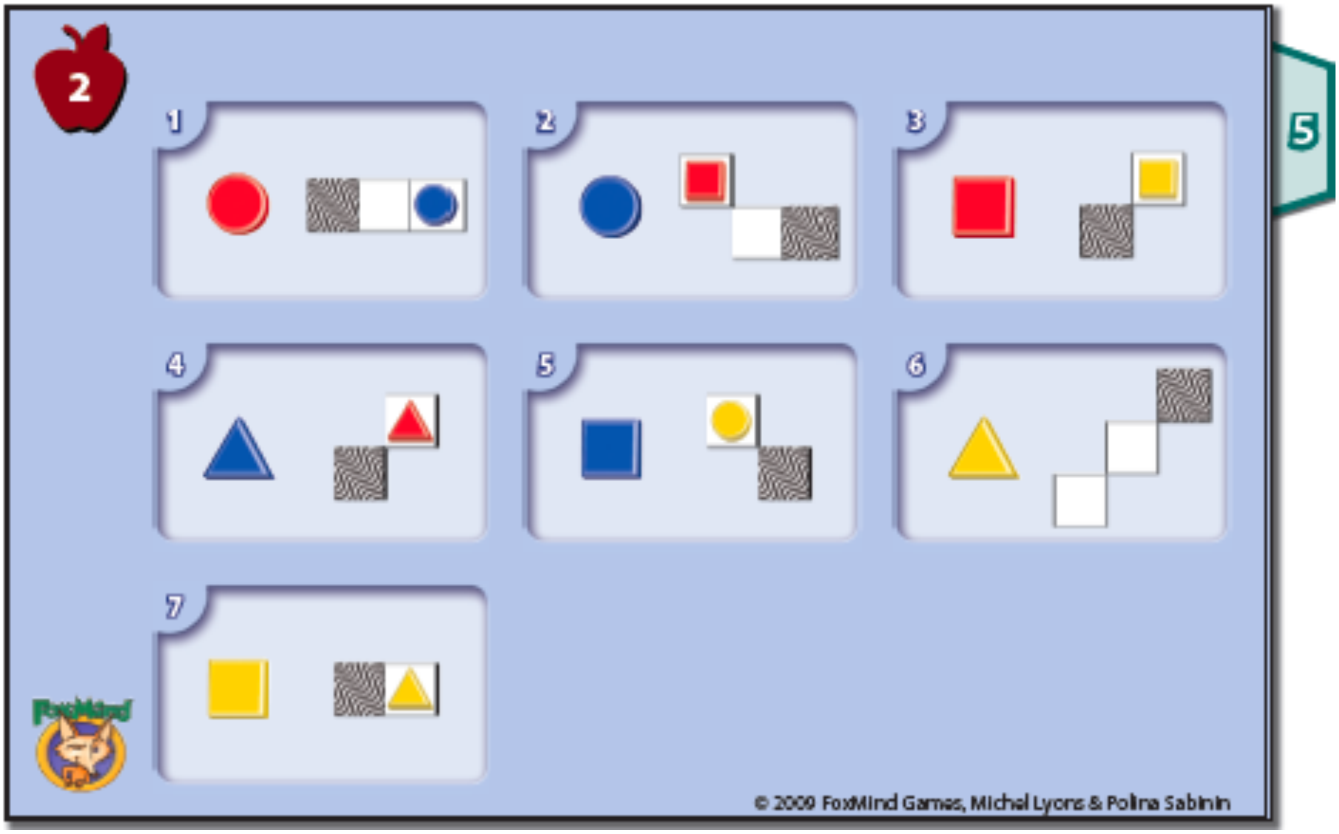
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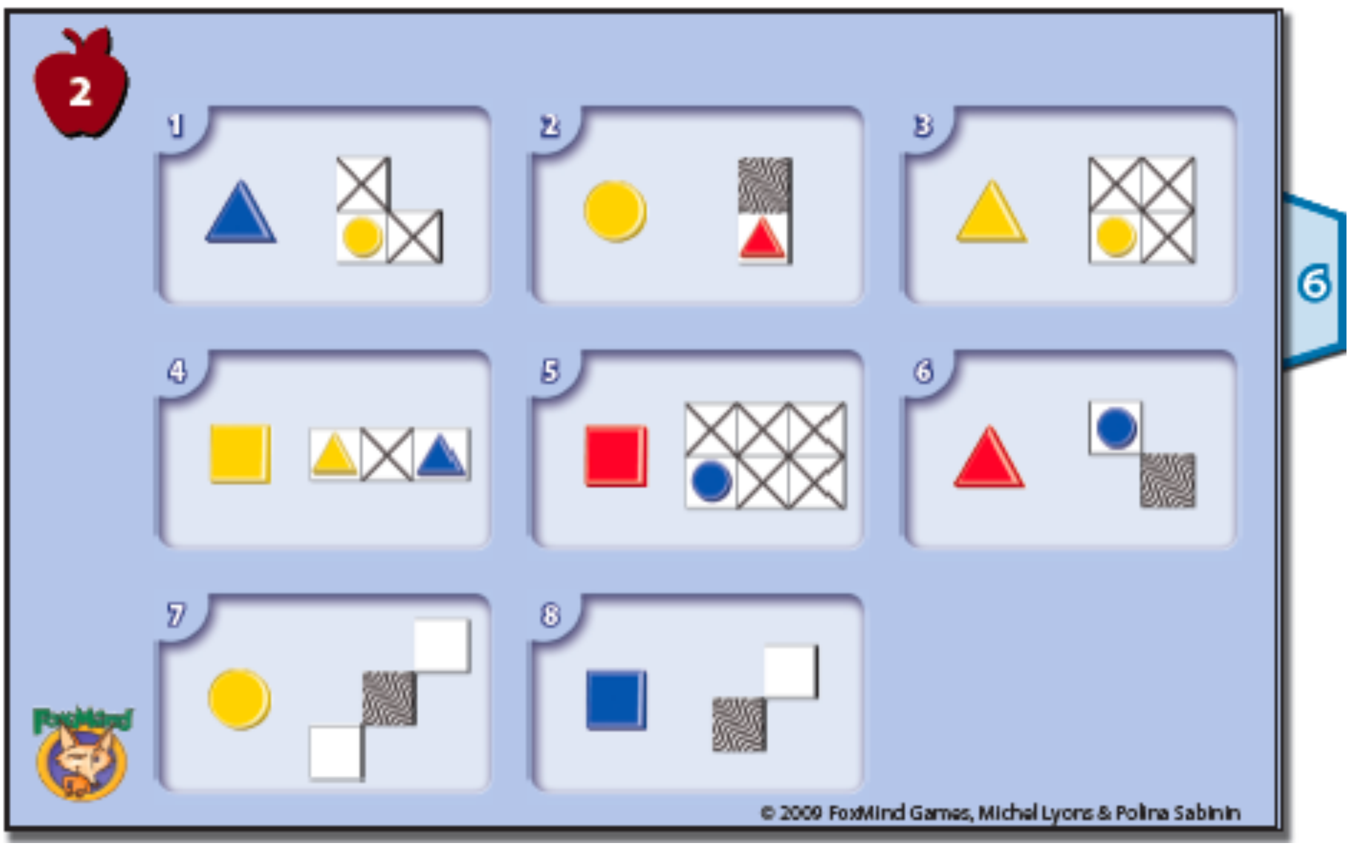
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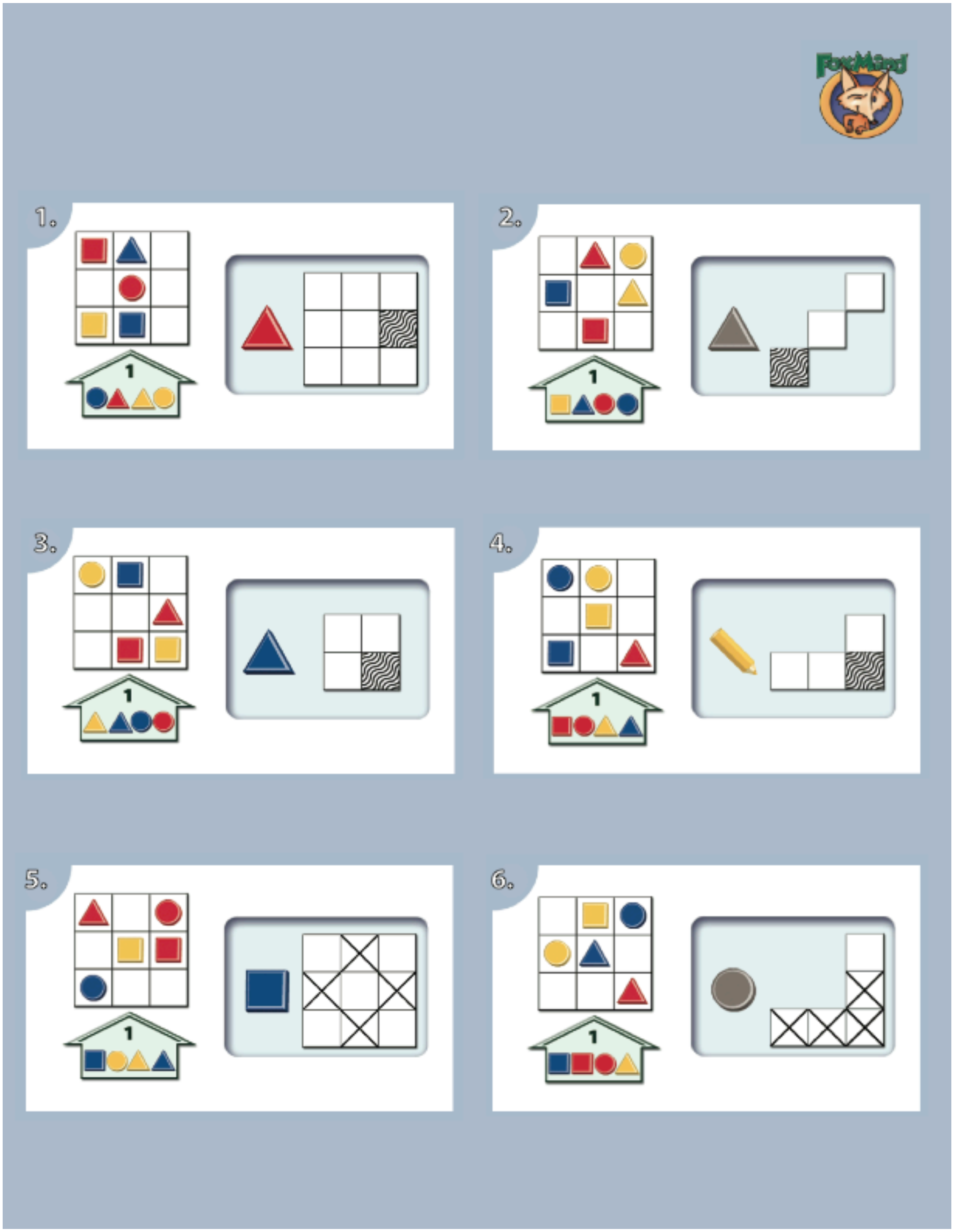

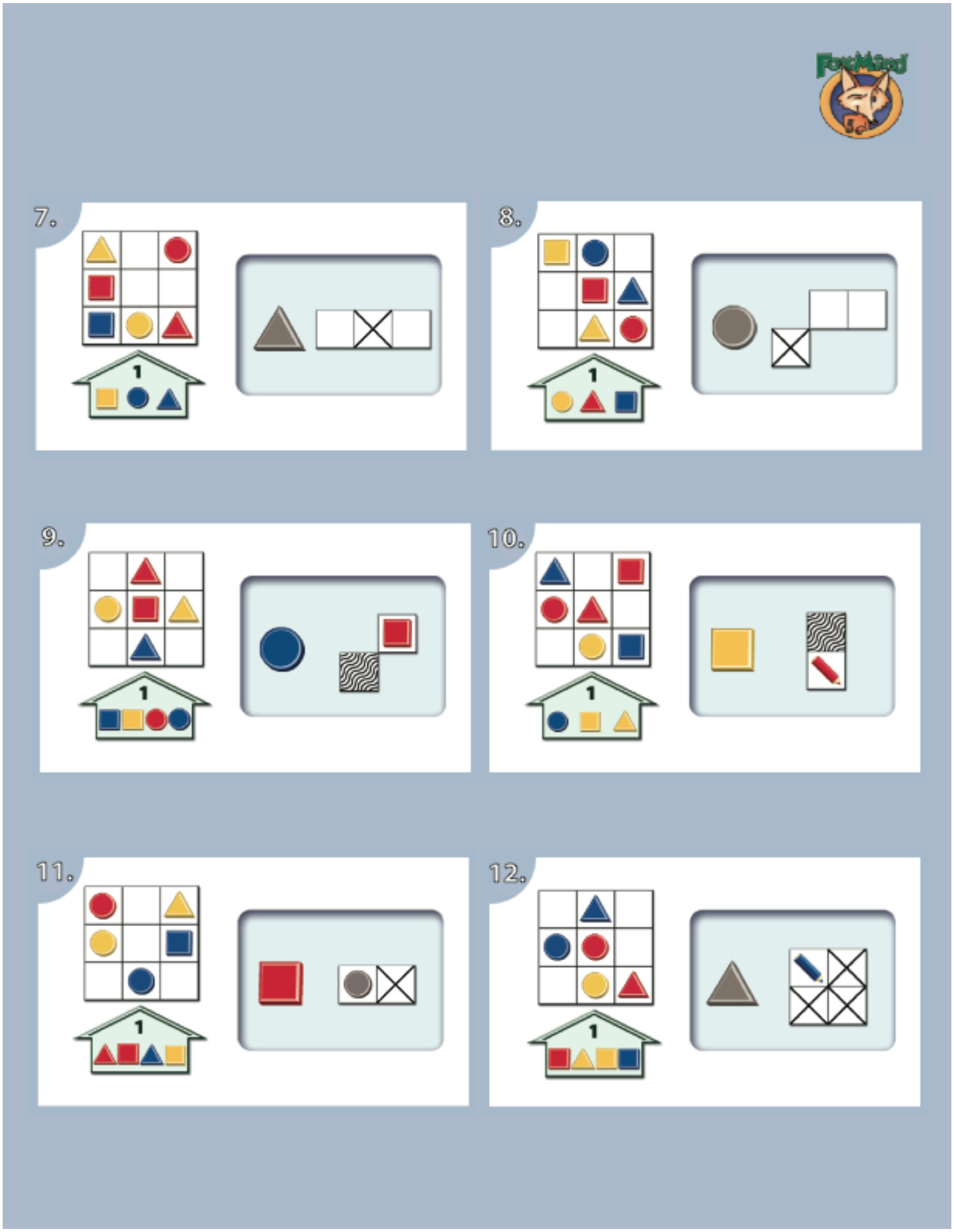
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