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Disenfractioned: How a Blind Spot for Fractions May Be a Hindrance to Democracy

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If Old MacDonald has seven cows and three horses on his farm, which is greater, his cows, or his animals?

"He has more cows," the casual reader would be forgiven for saying. A familiar mathematics problem, he or she would think, inviting a quick computation and immediate answer, and perhaps best expressed with arcane symbols such as "7 > 3" as though to encode a truth so pure it surpasses written language.

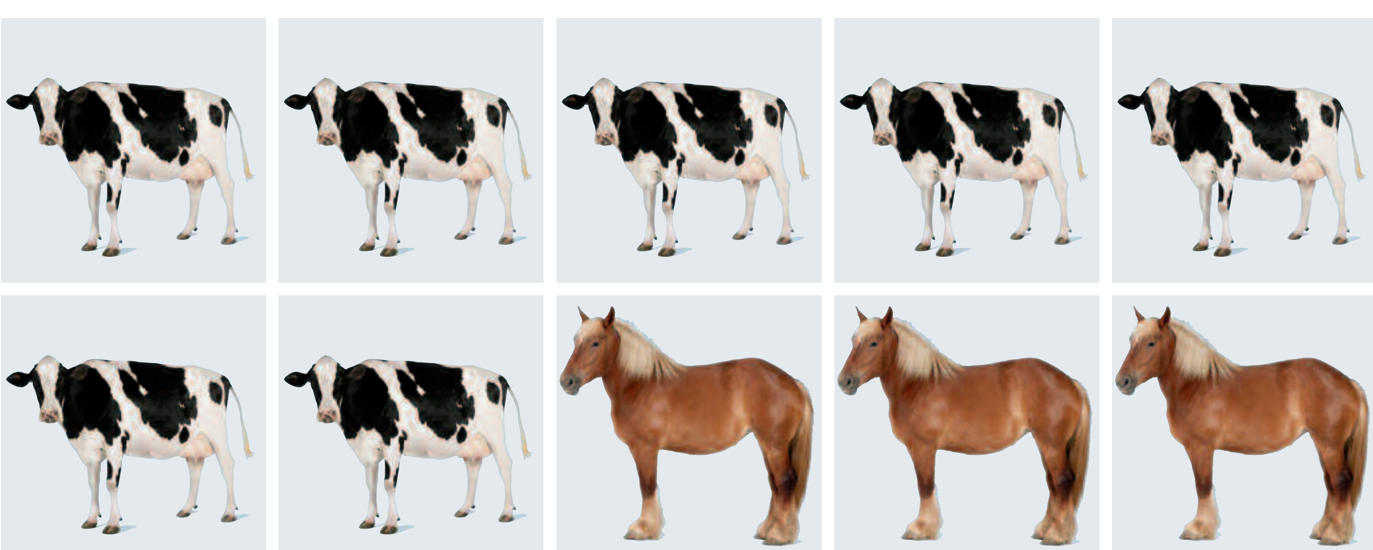
But the casual reader would be invited to re-read the question carefully. MacDonald has seven cows, but ten animals in total. Ten being greater than seven, he has more animals than he has cows. "A trick question," decides the reader, a phrase that invariably means "a question whose precise wording and immediate solution I haven't yet been shown."

If a trick lies in this question, it may be more in the mind of the reader than in the words of the examiner. As part of his 1960's study of how childrens' logical minds develop, the education theorist Jean Piaget posed this cows-and-animals question to schoolchildren, with the predictable result that majorities of students up to the age of 10 make the inappropriate comparison of cows to horses. He writes that the students' failure to attend to detail in this question is due to their developing "part-whole reasoning." At this stage, he suggests, students have been trained in the habit of comparing sets that do not overlap (the cows with the horses), and not sets that do (the cows with the animals).

For these children, it is not long before math educators typically introduce them to the object whose purpose is to solve all their part-whole reasoning problems: the fraction. This is also the earliest stage to which many adults, even highly educated ones, trace the roots of their persistent disengagement and anxiety with numbers. This disengagement may have far-reaching implications for how these adults conduct their personal, professional, and public lives.

Keeping it Real

Critical thinking about numbers is increasingly vital to both productivity and public discourse. Anthony Carnevale, then Vice President at Educational Testing Service, wrote in 2003 that higher-order cognitive skills such as mathematical reasoning are the gatekeepers of economic opportunity, "especially in the United States, where poorly educated individuals, not employers or governments, pay the price of educational inequality." Citing the National Adult Literacy Survey, he contends that 40 percent of the U.S. labor force possesses minimal or basic quantitative skills, placing them in the most sluggish and lowest-salaried segments of the economy. Meanwhile, he writes, "those who get the best jobs have taken the most mathematics."



However, Carnevale's analysis also suggested that the key to upward mobility is not necessarily advanced mathematics, but advanced reasoning. The mathematical skill required of those workers in the most highly-paid jobs "does not, on average, rise much above independent application of basic mathematical operations in complex situations." What our workforce seems to need, and reward, most is the ability to join relatively simple math with sophisticated reasoning; to interrogate our world with numbers, and to interrogate the numbers in our world.

Yet bad experiences with simple math derail many learners developing this ability. Fractions are the first and most fundamental instance of what author Sheila Tobias calls "dropped stitches" in a person's developing number sense — the implication being that, once dropped, none of the stitches that follow will line up, no matter how expertly they are sewn. The resulting cognitive deficit, if not patched, may pose a serious obstacle to critical thinking about numerical quantities.

So what gain is it for an adult to know fractions? For that matter, what does it even mean to "know" fractions? For those that have dropped the stitch, fractions can seem a hopeless morass of techniques and terminology: "cross multiplication," "common denominators," "reciprocals." This is no surprise: traditional mathematics instruction at this level tends to be skill-focused. As math educator Barbara J. Rose puts it, "The standardized curriculum expects students to *do* mathematics, not to *think about* its nature or raise questions about its existence." Yet "doing" fractions without understanding them only develops abstract skills with little relevance or power to interrogate the world.

To re-empower our number skills, Rose's and Carnevale's perspectives together point to a need for more circumspection and application in mathematics. Rather than treating fractions as a necessary annoyance of arithmetic on our way to algebra, geometry, and calculus, they might say, slow down and explore how fractions and fractional reasoning help us make sense of real-world problems. Rather than reaching up toward higher levels of abstraction, reach out toward more diverse contexts of application. Contemplate before you calculate.

Viewed through this lens, our understanding of a mathematical concept (fractions) derives from our understanding of our world. Thus can mathematical fluency be acquired – not taught – much in the same way as language and literacy develops: within a context. The skills acquired in this mode are then more easily accessed in new contexts later in an education, later in a career, and later in life. This skill set is known variously as "numeracy" or "quantitative literacy," which unlike "mathematics" is necessarily acquired in a broad spectrum of contexts. Educationally, this means numeracy is not only a goal for a mathematics course, but for coursework across the curriculum.

As an example of how understanding of fractions is acquired through context, let us take a moment to contemplate one of their most confounding aspects: the denominator.

Uncommon Denominators

That "most" quantities are represented by a single whole number can make understanding a fraction – represented by a pair – challenging. In the language of part-whole comparisons, the "top" number of the pair in a proper fraction (the numerator) represents the size of the part while the bottom number (the denominator) represents the size of the whole. Because these two numbers serve opposing roles in the comparison, however, they effect opposite changes on the value of the fraction, interfering with our inherent desire to estimate its size.

The part-whole comparison of cows to animals in the Old MacDonald problem can be encapsulated in the fraction 7/10. If one of his horses is replaced by a cow, the numerator of this fraction increases, and the new fraction 8/10 is larger than the old. We might say that cows make up a larger proportion of the farm animals than they did before. In this sense, the numerator of a fraction has a "straightforward" effect on its size: when the numerator increases, so does the fraction.

If, on the other hand, we were instead to add one more horse to the farm, then the proportion of cows to animals then goes from 7/10 to 7/11. The addition of an extra animal to the denominator, *but not* the numerator (since our new animal is not a cow), results in a *smaller* fraction since the cows are now making up a smaller proportion of the farm animals. So the denominator of a fraction has a "backward" effect on its size: when the denominator increases, the fraction *decreases*.

This tension between the opposing effects of changes in numerators and denominators is crucial to the understanding of fractions' meaning. Indeed, its implications in mathematics are far-reaching. It is no stretch to say that many of the concepts and computations central to calculus — such as the measurement of rates of change on an ever-shrinking scale — rely upon striking a balance between these two opposing forces of arithmetic.

Resolving this cognitive dissonance is not a matter of learning rules or processes, but merely of understanding the nature of part-whole comparisons. Yet, as Barbara Rose noted, students at age 10 are typically not given the opportunity to be cognitively dissonant in mathematics (let alone in their other subjects), and this teachable moment slips away. Students respond by discarding the dissonance instead of confronting it and integrating it into a more sophisticated understanding. In short, they develop a habit of being attentive only to numerators of fractions and not denominators.

Cornell University psychologist Valerie Reyna contends that neglect of denominators can be a matter of life or death. A consultant to the National Cancer Institute, Reyna makes the case that "low numeracy translates to poorer medical treatment and poorer health outcomes," since both patients and health professionals rely upon statistics and risk analyses to make informed treatment decisions. "In some experimental situations," she writes, "people prefer 10-in-13 odds over 9-in-11 odds, even though the latter are more favorable," because fraction confusion makes the larger numerator more enticing even though the still-larger denominator more than washes out its effect. Reyna promotes better education, or at minimum better writing, about health statistics and risks to help doctors and patients make more informed decisions.

Neglect of denominators is a persistent issue in other public spheres as well. Broadly speaking, every percentage statistic — a percentage being merely a fraction whose denominator ("whole") is one hundred — invites the critique: "Percentage *of what*?"

In 2009, Massachusetts legislators voted to raise the state sales tax rate from 5% to 6.25%. In April of that year, a Boston Globe article reported this to be a "1.25 percent sales tax increase." Later, in July, another Globe article termed it a "25 percent hike in the state sales tax." Aside from the difference in rhetorical force in which 1.25 is an "increase" while 25 is a "hike", this reporting highlights two different answers to the percentage-of-what question. In April the change in sales tax was reported as a percentage of the tax *base* (1.25 out of 100), and in July as a percentage of the tax *rate* (1.25 out of 5). This subtle difference in denominators has a dramatic effect on the size of the fractions used to characterize the *same* phenomenon.

In fact, stealth denominators are a perennial source of fiscal misinformation, particularly surrounding issues of economic justice. At the time of this writing, claims such as "the top 1% in America control 42% of our financial wealth" and "a 9% federal sales tax will help create a fair tax code" are animating politicians and demonstrators on both sides of the political spectrum. Each, however, is critically missing a denominator: Top 1 percent of whom? ("People?") Tax 9 percent of what? ("Sales?") It is impossible to assess such claims without their denominators. A cynic might suggest that, despite their pretense of precision, these political statements are not to be taken literally, but emotionally.

On that point, Valerie Reyna would likely agree. Her "fuzzy-trace theory" contends that people by default prefer quick, vague, instinctual judgments to more precise analyses requiring attention to detail. When assessing which of two fractions is larger, for instance, we are inordinately swayed by a gut instinct to prefer larger numbers. So we might erroneously conclude 10/13 is larger than 9/11, because at a glance it "looks bigger," and not pause even for a brief computation that shows our instinct to be mistaken.

Finding Perspective Through Fractions

So how do we keep our gut instincts in check long enough to develop our sense of fractions? If the goal is to develop a better sense of fractions' magnitudes and their meaning within the contexts of our world (numeracy), arithmetic drills are likely to be insufficient. Child psychologist Steven Hecht, in a 2003 study, found that conceptual understanding, more than arithmetic skill, determines whether a learner will succeed in computing, estimating, and using fractions in context. In other words, we may find fractions difficult more because we lack understanding of what they *are*, not merely what they *do*. Improving this understanding calls for more mindfulness, not merely more practice.

In the spirit of numerate citizenship, then, let us advance a more mindful definition of fractions that makes it easy to locate and understand fractions within their many contexts: *A fraction is a relationship between two numbers which is unchanged when those numbers are scaled upward or downward equally.* This definition has the advantage of highlighting what is important about fractions (the relationship between the part and the whole) over what is not (the scale, or absolute sizes, of the part and whole). In the commonly-abused mental picture, a fraction does not describe the size of a slice of pizza, nor the size of the whole pizza itself, but rather the connection between the sizes of slice and whole, which for a given fraction is the same whether the pizza is a personal size or whether it is large enough to feed an army. The scale can then be adjusted to fit the circumstance, leaving the fraction itself unchanged.

In this definition we can locate fractions as tax rates: 6.25 cents out of every 100 scales up to $625 on a $10,000 car purchase. We can locate fractions as cancer treatment odds: 10 out of 13 patients responding positively to a treatment scales up to roughly 77 out of 100. Fractions are dosages: prescribing 5 milligrams of a drug per 1 kilogram of a patient's weight scales up to 250 milligrams taken by a patient weighing 50 kilograms. Fractions are concentrations: carbon dioxide present at 389 parts per million in the atmosphere scales to an eight-foot cube of the greenhouse gas within an air space roughly the size of a football stadium.

This conceptual understanding, and intuitive estimation, of fractions provides perspective to balance our gut instincts. At an objective level, 6.25 sounds like a small tax, and 389 sounds like a large amount of carbon dioxide. With their denominators in place, however, these numbers may tell a different story. After all, any comparison of a part to a whole is in essence an establishment of perspective: how big is the part really, when compared to the whole?

In 1991, Valerie Reyna re-examined Piaget's Old MacDonald problem. Her analysis showed that the confusion over part and whole, cows and animals, was indeed related to a conceptual disconnect, an inability to access and implement logical knowledge. She also showed that this disconnect was present independently of misleading wording in the question. So no, the Old MacDonald question is not a trick — it's just the simplest of examples illustrating that our numerate minds are not always in the habit of comparing parts with wholes. Is it any surprise, then, that the fractions that embody those comparisons remain frustratingly opaque for so many otherwise-capable adults?

For many who struggle with fractions, a pause to consider fractions' meaning within a context that matters to them may be enough to pick up their "dropped stitch," alleviate some of their long-standing math anxiety, and permit them to more fully engage with the numbers in their world. It need not be the end of the conversation, particularly for scientists, engineers, and mathematicians who will require a much deeper understanding of how to operate using fractions, but it can be a common ground on which we all can begin. Our data-drenched democracy would be better for it.

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