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Notation for Symmetric Images

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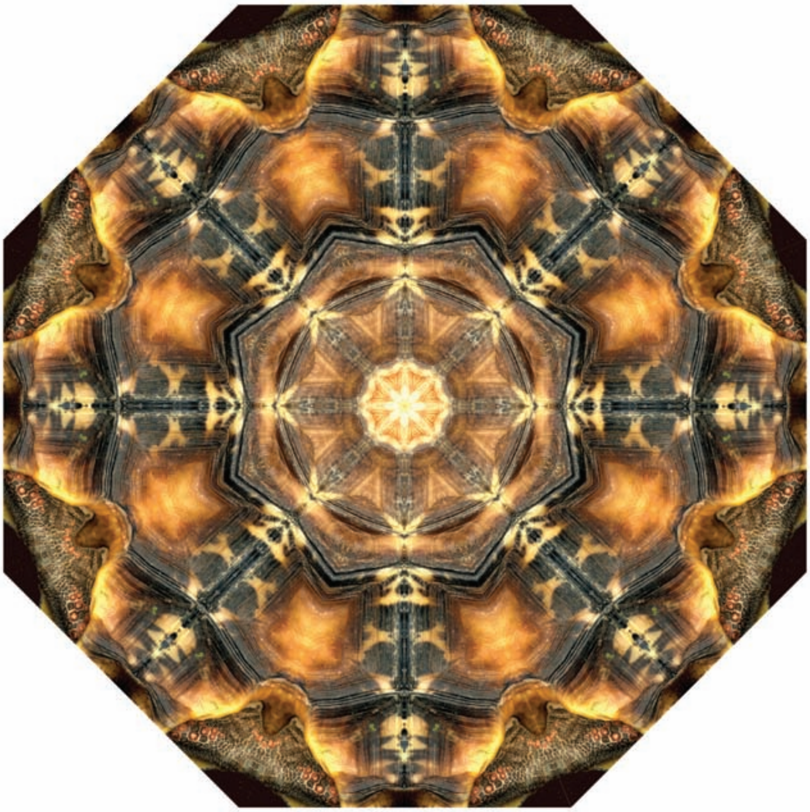
NOTATION FOR SYMMETRIC IMAGES

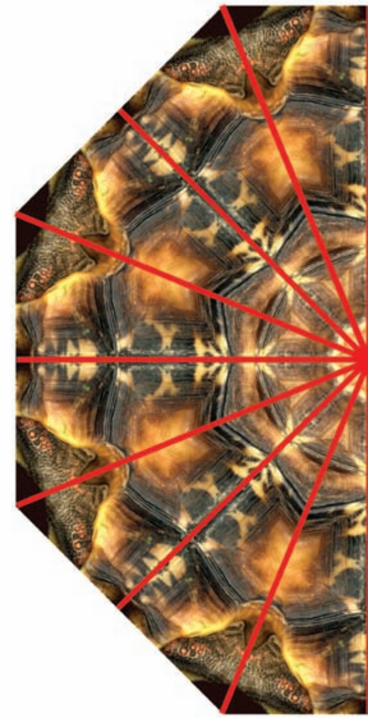
Jeffery Bowen and Heidi Burgiel



How do you describe the picture in Figure 1? As the shell and part of the leg of a red-footed tortoise? As a rosette or mandala? As modern art? The artist, Jeffery Bowen, describes it as a stack of copies of a 45-degree wedge taken from a reflected image of a tortoise. Heidi Burgiel describes it as a tortoise-shell motif reproduced according to the symmetry type *8.

FIGURE 1

FIGURE 2



Since 1993, Heidi Burgiel has been learning, teaching and using a new technique for classifying symmetric designs like the one in Figure 1. Symmetric patterns are integral to art and design; you see them every day on the floors and walls around you. William Thurston's "orbifold notation" provides an easy-to-learn way of describing these patterns mathematically, and also sheds light on the patterns' origins.

In the "signature" *8, the * indicates that the design has a mirror symmetry. Figure 2 shows the line of mirror symmetry Jeffery Bowen used to create the image that he later "stacked" to create the final design. The 8 in the signature refers to the fact that 8 lines of mirror symmetry cross at the center of the image, as shown in Figure 3. Although 45-degree rotations were used to create the image, these are not mentioned in the signature because those same rotations arise from combined reflections.

If you look closely at Figure 1, you will see that it's composed of 16 copies of the same pie-shaped wedge of tortoise shell and foot. (See Figure 4) This is the true origin of the signature—the * is in the signature because this "fundamental domain" has an edge (unlike a pattern produced by a cylindrical or conical roller) and the 8 refers to its single corner with angle 360/(2*8) = 22.5 degrees.

As another example, the signature of the square tiling commonly seen on kitchen floors is *442. The fundamental domain of the tiling is one-eighth of a square tile. It has a boundary (*), two 45-degree angles (44) and a 90-degree angle (2).

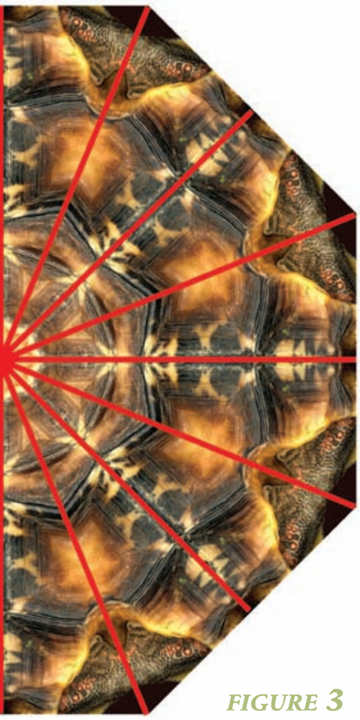


FIGURE 3

FIGURE 4

Figure 5 shows an image that was created by rotating the wedge shown in Figure 4. Once again, there are 16 copies of the wedge, but this time there are no mirror symmetries.

In Thurston's notation, this design's signature is 16. We identify the 16-fold center of rotational symmetry in the image by marking it with a blue dot. (See Figure 6) The fundamental domain of this image is the narrow cylindrical cone made by joining the long edges of the wedge shape in Figure 4.

In general, any pattern whose signature is a single number has a cone-shaped fundamental domain. The cone is an infinite, connected surface and is smooth except for the cone point. It has no edge and therefore there is no * in the signature. If we thought of the fundamental domain from Figure 5 as an inked stamp, rolling it on a blank sheet of paper would reproduce Figure 5—the cone point would stay at the center of a circle while the rest of the cone rolled around it.

Bill Thurston's revolutionary notation helps us understand the symmetries of patterns like those in Figures 1 and 5 by describing their fundamental domains. The notation extends to encompass translational and glide-reflective symmetries, patterns on the surface of a sphere and in three dimensions, and more.

—Heidi Burgiel is Associate Professor in the Department of Mathematics and Computer Science and Jeffery Bowen is Professor in the Department of Biological Sciences.

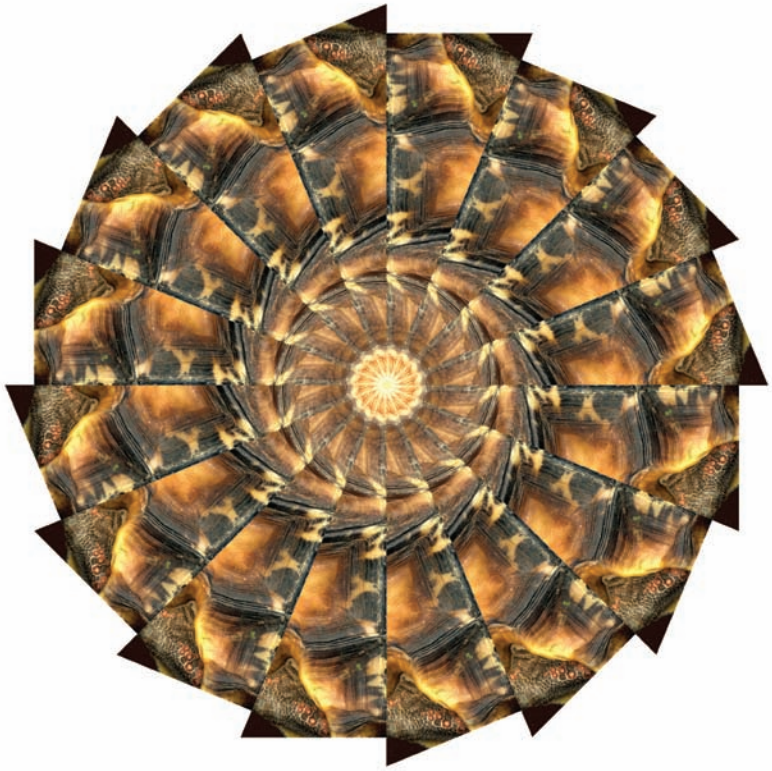


FIGURE 5 (ABOVE), FIGURE 6 (BELOW)