ROBUST LOCATION PLANNING OPTIMIZATION MODEL FOR ENERGY AND ENVIRONMENTAL SUSTAINABILITY

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DECLARATION

I hereby declare that this thesis is my original work and it has been written by me in its entirety. I have duly acknowledged all the sources of information which have been used in the thesis.

This thesis has also not been submitted for any degree in any university previously.

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Contents

Abstract

We consider an expansion planning problem for Waste-to-Energy (WtE) systems facing uncertainty in future waste supplies. The WtE expansion plans are regarded as strategic, long term decisions, while the waste distribution and treatment are medium to short term operational decisions which can adapt to the actual waste collected. We propose a prediction set uncertainty model which integrates a set of waste generation forecasts and is constructed based on user-specified levels of forecasting errors. Next, we use the prediction sets for WtE expansion scenario analysis. More specifically, for a given WtE expansion plan, the guaranteed net present value (NPV) is evaluated by computing an extreme value forecast trajectory of future waste generation from the prediction set that minimizes the maximum NPV of the WtE project. This problem is essentially a multiple stage min-max dynamic optimization problem. By exploiting the structure of the WtE problem, we show this is equivalent to a simpler min-max optimization problem, which can be further transformed into a single instance of mixed integer linear program. Furthermore, we extend the model to optimize the guaranteed NPV by searching over the set of all feasible expansion scenarios, and show that this can be solved by an exact cutting plane approach. We also propose a heuristic based on a constant proportion distribution rule for the WtE expansion optimization model, which reduces the problem into a moderate size mixed integer program. Finally, our computational studies demonstrate that our proposed expansion model solutions are very stable and competitive in performance compared to scenario tree approaches.

Keywords: Waste-to-Energy (WtE) systems; expansion planning; guaranteed NPV; extreme value forecast; adaptive min-max optimization; mixed integer program

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Table 1: Notation of model parameters and decision variables

	Indices
i	index for residential zones, $i \in \boldsymbol{\mathcal{I}} := \{1, 2, \cdots, \lvert\boldsymbol{\mathcal{I}}\rvert\}$
j	index for potential WtE sites, $j \in \boldsymbol{\mathcal{J}} := \{1, 2, \cdots, \lvert\boldsymbol{\mathcal{J}}\rvert\}$
t	index for stages (time periods), $t \in \boldsymbol{\mathcal{T}} := \{1, 2, \cdots, \lvert\boldsymbol{\mathcal{T}}\rvert\}$
	Parameters
h	Transportation cost per unit distance travelled for per unit amount of waste
ξ_{ti}	The waste collected from zone i at time t (uncertainty)
c_j	The annualized fixed cost to open potential site j
v	Unit operating cost of WtE facility
θ_i	The purity ratio of the waste collected from zone i
q_E	Unit energy price
μ	Conversion ratio from waste to energy
ρ_j	residue ratio of the WtE facility j
c_D	Unit disposal cost
d_{ij}	Distance between the resident zone i and WtE facility site j
s_j	Total capacity of WtE facility j
	Decision variables
$x_{t-1,j}$	Binary decision variable of opening or not WtE facility site j at period $t \in \boldsymbol{\mathcal{T}}$ (1 is to open)
y_{tij}	The amount of waste transported from zone i to WtE site j at period t
z_{ti}	The amount of excess waste generated from zone i at period t that will be disposed

Chapter 1

Introduction

The recent years have seen an increased interest in the area of sustainable waste management, especially in modern societies where the factors of rapid population growth, increasing consumption and subsequent waste generation, and diminishing landfill space continue to pose a large problem to the government.

Trends show clearly today that we are headed into an era of unprecedented waste generation. In just one decade, the municipal solid waste (MSW) collected in industrialised countries France and Luxembourg has increased by 11% and 14% respectively [13], and China, a developing country, has similarly seen an 11% increase [11]. The world bank estimates that by 2025, waste generation will be double of that in 2012, a 100% increase in just thirteen years [39]. At the same time, we are rapidly using up our available landfill space. In less than 25 years, the United States of America has seen its disposal facilities drop by 75% [37]. Similarly, Singapore estimates that the island will be running out of landfill space by around 2035 [28].

Faced with this problem, most societies choose to use a two pronged approach. On one hand, recognising that waste is being generated faster than it can be treated, societies actively encourage the population to generate less waste. On the other hand, societies are also attempting to find better ways to treat the waste that are being generated. To be truly successful at tackling this problem, this approach requires participation from all levels of society, from the citizens being more environmentally aware to the industries being more willing to invest in more technologically advance systems and methods used to treat waste.

From the side of societal awareness, programs have been implemented in many countries for a good number of decades, and we see that results are encouraging. Most countries adopt the 3Rs slogan: Reduce, Reuse and Recycle. This encourages people to generate less waste by reducing their consumption habits to cut off what is not needed (buying less clothes), to reuse what can be reused instead of throwing them away (using reusable bags when shopping) and to recycle and support recycled products. In Singapore alone, the commitment to this program has seen a steady increase in recycling points around residential homes as well as frequent collection drives where residents are able to donate their unwanted items. As a result of this effort, Singapore has seen our recycling rate jump from 40% in 2000 to 60% in 2014, a 50% increase in fourteen years.

However, even with the stark increase of recycling efforts, there is still a significant increase in the waste disposed to landfills [49]. Such trends are common for most countries, both developed and developing, and highlights the need for waste treatment technologies to play a bigger part in our waste management strategy.

Technology has been advancing at a rapid pace in the past decade and this has allowed industries to come up with increasing green technology used to treat waste. (consuming less energy, producing less carbon, etc...) Among them, Waste-to-Energy (WtE) technologies are of particular interest and have been seen as a key waste management solution that has shown significant advances and merits since their introduction. WtE is defined by the United States Environmental Protection Agency (U.S. EPA, see [47]) as the energy recovery from waste, or equivalently the conversion of non-recyclable waste materials into usable heat, electricity, or fuel. The state of the art of WtE now encompasses a broad range of alternatives with environmentally friendly processes, including anaerobic digestion, gasification, pyrolysis, and highly enhanced incineration. These are widely acknowledged to be significantly cleaner and more energy efficient than conventional combustion. Thermal technologies such as gasification have the ability to produce combustible gas, including hydrogens and synthetic fuels, while non-thermal technologies such as anaerobic digestion is able to produce biogas that are rich in methane.

Landfilling is on the other hand becoming a less viable and less acceptable option. With these advances of green technology (see [22, 40]), more states and countries have turned to WtE for sustainable environmental and energy solutions. From 2003 to 2012, the 27 members of the European Union (EU-27) reported a 35% increase in the amount of WtE treated waste (see [13, 46]), and the U.S. has similarly witnessed an increase of 22.4% in WtE implementation from the years 2000 to 2010 (see [47]). Motivated by the heightened interests and investments in the WtE industry, it is the objective of this paper to identify and address one of the key challenges arising from the strategic implementation and expansion planning for WtE systems.

In our paper, we would like to consider a medium-to-long term expansion planning problem of WtE units (systems). The WtE units are deployed in different locations and treat waste collected from different populated regions. For example, we can consider the case of Singapore, in which the country is divided into different 29 different constituencies, each managed by a town council who takes care of the population within its constituency. These 29 points can be regarded as the different residential zones from which waste originates. At the same time, the waste treatment facilities are situated at other places and treat the waste delivered by these residential zones. We consider the planning problem of WtE units as a primary motivation and interest in this problem arise from the increasing commercial available of compact (small scale), clean, modular WtE units, which makes them easier to install and socially more acceptable in even highly populated locations (see [27]).

On the other hand, we recognise that waste generation, which is the input feedstock to the WtE, can be highly uncertain in reality, and can have a significant impact on the economic sustainability of the WtE system. Since WtE systems make money by selling the energy converted from waste, it is clear that lower-than-projected waste supply directly impacts revenue margins of the WtE operations as insufficient waste would lead to a low amount of energy produced for sale. At the same time, excessive waste can create capacity strains and inflate the disposal costs. Since WtE companies are usually charged with taking care of the waste management of the entire residential zone, a high amount of waste would mean that the company has to pay out of its own pockets for the waste to be landfilled if this waste cannot be accommodated by the waste treatment facilities due to capacity constraints.

In practice, despite the best of the intentions, WtE operators can incur major financial deficits and even be forced to shut down due to the unrealistic estimations of the capacity utilization. This observation has been made based on cases of actual WtE operators, for instance, IUT Global Ltd [20, 38], one of the largest food waste recyclers in Singapore, ceased operations after incurring three successive years of losses since its inception. The company had estimated that it would be able to collect and process 800 tonnes of food waste daily, but only ended up collecting an average of about 120 – 130 tonnes daily. This was because the company only realised after operations had started that most of the food waste was contaminated by other inorganic materials (for example, food disposed of still in a plastic bag) and hence had to be incinerated and landfilled, causing a huge raise in operating costs.

In this paper, we propose a WtE expansion planning model under uncertain future waste supply. WtE expansion plans are usually regarded as strategic, long term decisions, while the waste distribution and treatment are medium to short term operational decisions which can adapt to the actual waste collected. Sound strategic decisions hence clearly depend on the quality of the waste generation forecasts available. In reality however, these forecasts can be highly inaccurate and unreliable due to two main reasons. The first is that while waste generation shows a clear increasing trend in the extremely long run, it is hard to predict the period-on-period fluctuations which can often have a cascading effect as small errors in the waste generation forecast add up. Waste generation itself is dependent on a lot of other factors, some of which are measurable such as the population size and household income, and others which are hard to quantify such as the level of environmental awareness. These factors are by themselves unpredictable and add another level of complexity to the waste generation forecast. The second is that data in previous years is difficult and extremely time-consuming to obtain. Without sufficient and reliable waste generation data for each region, it is impossible to come up with a forecast model that can provide sufficient and reliable forecast data. In addition, different countries, states, local governments and industries can also differ widely in data availability and data collection efforts (see [10, 18]), and hence consolidating all the data to come up with a single consistent model can be an arduous task.

While it is impossible to reduce these forecast errors to zero, it is possible to account for them in mathematical models. It is hence imperative for practitioners to recognize the fact that such nominal point-value forecasts are no longer sufficient, and it is important to account for the impact of forecast errors during the WtE expansion planning and analysis. The level of forecasting error allowed should be appropriate given the amount of data currently available. If only a small amount of forecasting error is allowed, the model would not be useful as it would be unable to reflect the reality of the situation, while if too high an amount is taken into account, then the solution obtained would be largely sub-optimal.

In view of this, we propose a *prediction set* uncertainty model which consists of a set of waste generation forecasts, instead of only a single forecast value. The prediction sets are constructed based on a specified level of forecasting errors, termed as *forecast error budgets*, that the user would like to consider based on their preference and available data. These forecast error budgets also capture the period-to-period, multi-location characteristics of the system.

Next, we use the prediction sets for WtE expansion scenario evaluation and analysis. More specifically, for a given WtE expansion plan and waste generation forecast error budget level, we assume that the short-term decisions of waste distribution and treatment are chosen to optimize the net present value of the WtE project. A min-max adaptive optimization model is then solved to find an *extreme value forecast* of the future waste generation within the prediction set that minimizes the maximum net present value of the WtE project, or what is termed as the *guaranteed net present value*. Evaluating the guaranteed net present value makes sense when the WtE project owners are uncertainty-averse and is more concerned with avoiding economic losses due to bad forecasts. This also provides the decision-maker a simple-to-use performance criteria when making comparisons of different WtE expansion scenarios.

Finally, we extend the model to optimize the guaranteed net present value by searching over the set of all feasible expansion scenarios.

By using this approach, we are able to get a solution that would be robust to the uncertainty set, that is, our solution is able to guarantee a certain level of performance as long as the actual waste generation in the future does not stray too far from our waste generation forecast. Robust models have long been used for decision making under uncertain circumstances [5], and the key to obtaining solutions close to the actual optimal is to use robust optimization models which are adaptable to the information set. A model is said to be adaptable if each decision the model makes is conditioned upon the full set of information available at that time in actuality. Unfortunately, this is difficult to implement both theoretically and computationally. There is hence a tradeoff between the computational efficiency and the solution performance when employing different types of robust approaches. In this paper, we provide two different models with different computational efficiencies, an exact solution approach using a cutting plane algorithm as well as a heuristic using a constant proportion waste distribution rule.

The key technical contributions of our work are as follows. First, in this work, we make novel connections between well-known time series forecasting models with robust optimization constructs. This allows users who are familiar with standard forecasting techniques to assimilate and integrate the two methodologies in a more seamless manner. Next, based on the problem structure, and the constructed prediction sets, we show that the resulting problem of evaluating guaranteed net present values of the WtE project can be reduced from a multiple stage min-max dynamic optimization problem into a format of min-max adaptive optimization. We then show that this can be re-formulated into a single instance of a mixed integer linear problem of moderate dimensions. Finally, to optimize the guaranteed net present value, we propose a heuristic based on a *constant proportion distribution rule*, which also reduces the optimization model to a moderate size mixed integer program. Our computational studies demonstrate that the proposed heuristic performs very competitively compared to exact methods using cutting plane algorithm and scenario tree stochastic models in terms of solution quality, and requires much lower computational effort.

The rest of the paper is outlined as follows. In the next chapter, a review of relevant literature is provided. Chapter 3 introduces the basic setting, notation and the deterministic version of the WtE planning problem. This is also the problem typically solved by using a single point-value forecast of the future waste uncertainty. A short analysis is then done to show why this is insufficient in the face of uncertain waste generation. In Chapter 4, we first develop the prediction sets motivated by forecasting errors of the future waste growth. Next, we present the guaranteed net present value evaluation model for performing WtE expansion scenario analysis. We show the technical results to achieve a tractable formulation for the evaluation problem. In Chapter 5, we extend the model to consider the optimization over the set of feasible expansion scenarios to maximize the guaranteed net present value. We describe an exact solution approach to the problem using a cutting plane algorithm. To reduce the computational requirements, the constant proportions distribution heuristic is then developed, and we show its reformulation as a linear mixed integer program. Chapters 4 and 5 are largely contributed by Shuming Wang et al. [43]. However, while Shuming Wang et al. chooses to place a larger emphasis on the method presented and the analysis of results, we are more concerned with the applications in real life and how the work presented can go towards solving the problems highlighted in the WtE industry. Results of computational studies are reported in Chapter 6, where extreme value forecasts analysis and robust designs analysis are performed with different forecast error budgets. Our proposed model solutions are also compared to those generated by using a single point forecast model and a scenario tree stochastic approach. Finally, Chapter 7 provides some concluding remarks and future research directions of our work.

Chapter 2

Literature Review

In this chapter, we aim to give a complete overview of the current research being done in the Sustainable Waste Management Planning industry. We will show that the subset of facility location planning to be an important one in this area, and review the different methods currently studied in this domain. This is further split into two parts, deterministic waste treatment facility location models as well as waste management planning under uncertainty. In addition, we will also consider what has been done in the area of adaptive robust optimization models, which we will be using to aid us in the resolution of our problem.

2.1 General Methods used in Sustainable Waste Management Planning

Primarily, sustainable waste management planning is based on a series of decisions. Decision making tools were hence popular from the start. Due to the complexity of this matter, multi-criteria decision making methods were largely applied, and case studies showed that they had good potential.

In one study, Vego et al. [41] adopted two multi-criteria decision-making methods, GAIA and PROMETHEE, to assist the analysis and evaluation for the waste management system in the coastal part of Croatia, the analysis consists of two levels: the first level is to study the potential number of waste management centers and the second level was to investigate the relative preference of siting of the waste management centers. Aragonés-Beltrán et al. [2] analyzed the problem of selecting the best location for the construction of a waste management facility in Valencia, Spain. They approached the problem using the Analytic Network Process by considering 21 varied criteria in areas such as economical, environmental, legal and social. They concluded that their approach was useful in helping decision makers make traceable and reliable decisions, and also help them to reflect more comprehensively on the problem by getting them to consider the various factors involved.

While such methods were able to provide much insight into the decision making process, they lacked the rigour of mathematical models and were easily swayed by expert opinion. As such, there was a need to consider models which were more mathematically sound. These models would be able to provide solutions closer to the true optimal, and in our case, would even be able to guarantee a certain amount of profit that would be made under a fix set of scenarios. However, they might be unable to take into account the large number of factors multi-criteria decision making models typically consider. An important fact to take into account when considering which method to use is hence the trade off between mathematical rigour and the complexity of the setting.

2.2 Waste treatment facility location: deterministic and stochastic models

The application of mixed integer programming models to site resource recovery facilities for solid waste management in a deterministic setting was proposed in an early work by Jenkins [23]. The author studied and proposed various heuristic rules and computational procedures for the parametric analysis and solution of the model. Antunes et al. [1], working on a plan to help deal with Portugal's rising waste generation in the 1980s, proposed a two-level discrete facility location model to site waste treatment plants. They then conducted a case study in central Portugal which showed that their model was a credible and efficient one. Erkuta et al. [12] presented a multi-criteria mixed-integer linear programming model to solve the location-allocation problem for municipal solid waste management systems at the regional level where the lexicographic mini-max approach was used to obtain a “fair” non-dominated solution. This ”fair” non-dominated solution is such that all normalized objective values are as equal to each other as possible. The authors considered five objectives in their case study, including minimizing the green house effect and operational costs, as well as maximizing the total energy and material recovery. Fabbricino [14] considered an integrated planning problem for municipal solid waste management which involves facility location selection, waste collection approaches, as well as waste treatment choices, and the developed model aimed to evaluate the economic advantages pertaining to different municipal solid waste collection and treatment options. The advantage of their network model based on the waste management cycle is that is it easy to apply and provides results that can help to facilitate management decisions. Khadivi and Fatemi Ghomi [24] proposed a location-planning tool that uses analytical network process and data envelopment analysis as a leverage and can effectively take managerial preferences and subjective data in waste management into consideration, along with quantitative factors.

All the above cited works assume that the problem environment is deterministic, which is never the case in reality. We hence see in the past ten years a surge of works dealing with the same waste management problems but which also allow for uncertainty to occur. The uncertainty modelling in such cases is especially important. Somplaka et al. [31] considered an incineration facility planning problem under uncertainty. The problem is to determine the optimal waste treatment capacity and the steam turbine units, when parameters such as heat demand and energy prices are uncertain. The problem is formulated as a stochastic programming model and the uncertainties are assumed to be discrete random variables with known probability mass functions. Wang et al. [42] developed a waste management system planning model using interval-valued triangular fuzzy sets to describe the cost coefficients, and discrete random variables to characterize the waste generation quantities. The solution of the model relies on a discretization method that generates a number of intervals from the interval-valued triangular membership function, so as to transfer the fuzzy constraints of the problem to deterministic forms. Yeomans [45] employed an evolutionary simulation-optimization approach to assist in the solid waste management planning. The objective of the proposed model is to minimize the impact of negative outcomes due to uncertainty, and the concept of outcome minimization through the use of penalty functions is combined with grey programming into an evolutionary simulation-optimization procedure to solve solid waste management problems containing significant sources of uncertainty. More related studies can be found in Sun et al. [34], Tan et al. [35], and Xu et al. [44].

2.3 General methodologies for facility location under uncertainty

Most applications of facility location under uncertainty are based on general approaches such as two-stage stochastic models, probabilistic models (max-probability, chance-constrained models), mean-variance models, robust models and regret models. Readers may refer to Baron and Milner [3], Gulpilar et al. [19], Lian et al. [25], and Tian and Yue [36] for the details, and Snyder [33] for an excellent review. Most of these works assume that the probability distributions for the uncertainty are known, and very few studies consider multiple period cases. To the best of our knowledge, the most relevant studies to our work are Baron and Milner [3], and Gulpilar et al. [19]. In Baron and Milner [3], the authors developed robust robust facility location models with multiple periods using a box-uncertainty set and an ellipsoid-uncertainty set, respectively. However, their models assume the following restrictions: (i) the multiple period uncertainty set is formed in a time-independent setting; (ii) the operations decisions are forced to be made before the uncertainty being realized, which are not adaptable to the uncertainty realizations. This can limit the flexibility and performance of the solution. In Gulpilar et al. [19], the authors considered a robust stochastic facility location problem assuming a normal distribution with uncertainty parameter (mean, variance) or ambiguous distributions for the uncertain demand. However, their model also does not allow the operations decision to be adaptable to the uncertainty.

In this work, our proposed WtE expansion models (both the guaranteed NPV evaluation and optimization models) can be regarded as special applications of *adaptive robust optimization problem* (see [4]), in which the operations decisions are adaptable in the fashion of a dynamic min-max game. Unfortunately, adaptive robust problems are in general computationally intractable (see [4, 15]) due to the large problem size as well as the sheer number of possibilities that will have to be considered. Considering multiple decisions also increases the problem size exponentially. Popular solution approaches include approximations using affine decision-rule models (see [6, 7, 32]), and the exact solution approach (see [8, 26, 48]) for two-stage adaptive problems which uses cutting plane techniques. Affine decision-rule models for two-stage and multi-stage adaptive problems which solve a more restrictive version of the problem, and while the model might be simpler to solve, the solution obtained is only optimal under a very strict set of conditions. Finding the exact solution using a cutting plane approach yields the optimal solution, but the model itself is harder and more time-consuming to resolve.

In our work, given a WtE 'expansion scenario', we solve a multi-stage adaptive robust (min-max) problem to compute the extreme value forecast of the waste generation, and the corresponding optimal adaptive operational decisions. This model is useful for evaluating a guaranteed performance level for the given expansion scenario. As we will subsequently show, this multi-stage min-max dynamic evaluation problem is equivalent to a much simpler formulation of a min-max optimization problem, which can be further transformed into a single mixed integer linear program. As a consequence, our proposed WtE expansion optimization problem to maximize the guaranteed NPV can also be cast directly in the format of a two-stage adaptive max-min-max optimization problem, and can therefore be handled exactly by cutting plane approaches.

Based on the literature review, the important research gaps identified and contributions of our work can be summarized as follows:

- The facility location planning models in the above literature of waste management either studied deterministic problems which cannot handle the uncertainty, or stochastic problems that heavily rely on the known distribution assumptions for the uncertain parameters. Our proposed WtE models on the other hand, account for uncertainty using waste forecast prediction sets that can be built directly using data observations, without necessarily imposing probability distribution assumptions. While our proposed prediction sets are based on general constructs in robust optimization, we are able to make novel connections with the area of time-series forecasting models. This will improve the acceptability of our approach in practice and makes integration more seamless.

- In contrast to the robust location planning models in Baron and Milner [3] and Gulpilar et al. [19], our proposed prediction set models allow users to control the period-to-period waste generation uncertainty propagation and dependence, through the use of forecast error budgets. We also allow the operations decisions (waste distribution and treatment in our case) to adapt to the uncertainty revealed at each stage. This is more realistic in practice, and can also improve the solution performance.

- Although it is not the aim of our work to contribute to the solution methodology in adaptive robust optimization, we identify some interesting properties for our models in the WtE planning context that can simplify the problem complexity. More specifically, we show that the WtE expansion scenario evaluation problem can be reduced from a multi-period, dynamic min-max problem into a simpler min-max optimization (Proposition 1), and this can be further reduced to a single instance of linear mixed integer model (Proposition 2), which can be solved conveniently by off-the-shelf software. An interesting consequence of this is: it also allows the convenient articulation of a specific extreme value forecast for the given WtE expansion scenario (Proposition 3). Furthermore, this directly results in a two-stage adaptive max-min-max problem format for the WtE expansion optimization problem, hence permitting the application of existing solution algorithms such as cutting plane methods. Finally, for the cutting plane implementation, we show that the cuts in our problem can be described efficiently in a closed form. This can help improve cut enumeration procedures if required. Such results are novel and not available in the reviewed literature to the best of our knowledge.

Chapter 3

WtE expansion planning using point-value forecasts

3.1 Problem Setting and Notation

We start with the WtE expansion problem given a point value forecast of the future waste generation. Some basic assumptions are: (i) the capacity of the WtE facilities is assumed to be expanded in fixed modular units. This simplifies the presentation to focus on the key features of the problem. (ii) The residues after the WtE treatment, and any untreated waste, incur additional disposal costs, e.g. for landfilling.

Let i, j and t be the indices for residential zones, potential WtE sites, and time periods of planning, with $i \in \boldsymbol{\mathcal{I}} := \{1, 2, \cdots, |\boldsymbol{\mathcal{I}}|\}$, $j \in \boldsymbol{\mathcal{J}} := \{1, 2, \cdots, |\boldsymbol{\mathcal{J}}|\}$ and $t \in \boldsymbol{\mathcal{T}} := \{1, 2, \cdots, |\boldsymbol{\mathcal{T}}|\}$, respectively. For WtE expansion problems, each planning bucket t is typically in the range of one to three years (see [21]). Denote by ξ_{it} the waste collected from residential zone i at period t, d_{ij} the distance between the zone i and WtE facility site j, and h the transportation cost per unit distance travelled for per unit amount of waste. Let c_j be the annualized fixed cost to open potential site j, v be the unit variable operating cost of the WtE facility, c_D be the unit disposal cost, that is, the cost to landfill one unit of waste, and q_E the unit energy price for selling the recovered energy. We also let μ be the conversion ratio from waste to energy, that is, how much energy can be obtained per unit of waste processed.

Furthermore, recognising that not all waste can be processed (for example, metal and non-biodegradable plastics cannot be used in most waste-to-energy processes), a 'purity ratio' θ_i is introduced, representing the percentage of waste collected from zone i which can be processed.

We set ρ_j to be the residue ratio of the WtE facility j, and s_j is the modular capacity of WtE facility j. Define $x_{j,t}$ as a binary decision variable, so that $x_{jt} = 1$ if WtE facility j is installed and opened at the beginning of the period $t \in \boldsymbol{\mathcal{T}}$, $x_{jt} = 0$ otherwise. Finally, y_{ijt} and z_{it} are the waste distribution flow decisions, where y_{ijt} is the amount of waste from zone i distributed to WtE site j in period t, and z_{it} the amount of excess waste generated from zone i at period t that is disposed without treatment at any WtE site.

In practice, the decision variables x_{jt} are usually made at time zero, without any knowledge of what the future waste generation will be. This is because the planning is done and fixed in advance, bound by a contract before the process can start. Once these x_{jt} are fixed, negotiation will start for the sale of the land and the construction of the facilities. For waste management planning carried out over a longer term, there may be a review of the decisions taken when required, though this might prove costly. On the other hand, it is clear that the operational decision variables y_{ijt} and z_{it} are made of a period-on-period basis, and we can assume that this decisions are made with full knowledge of the waste generated at the current time t.

3.2 Problem Description and Formulation

Suppose the waste supply forecast are the actual values $\boldsymbol{\xi} = [\xi_{it}]_{|\mathcal{I}|\times|\mathcal{T}|}$, the expansion plan $\boldsymbol{x} = \left[\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{|\mathcal{T}|}\right]$ and distribution flows $\boldsymbol{y} = [y_{ijt}]_{|\mathcal{I}|\times|\mathcal{J}|\times|\mathcal{T}|}$ and $\boldsymbol{z} = [z_{it}]_{|\mathcal{I}|\times|\mathcal{T}|}$ are determined jointly to maximize the *net present value* (NPV) of the project over the entire planning horizon. In the following, we first describe the items accounted for in the NPV function. In each $t = 1, 2, \cdots, |\mathcal{T}|$, the amount of waste processed at facility j from waste sent by residential zone i is given by the decision variable y_{ijt}. Out of this only $y_{ijt}\theta_i$ can be processed, and this amount produces a total of $y_{ijt}\theta_i\mu$ units of energy. Sold at a price of q_E per unit of energy, the revenue accrued from sales of recovered energy for each facility j from processing the waste sent from residential zone i is given by $y_{ijt}\theta_i\mu q_E$. Summing this up for all residential zones and all facilities, the total revenue made per time period t is given by:

$$\left[\sum_{j\in\mathcal{J}}\sum_{i\in\mathcal{I}} y_{ijt}\theta_i\mu\right] q_E.$$

In a similar manner, we obtain that the total transportation cost of shipping collected waste from resident zones to the WtE sites at time t is:

$$\sum_{j\in\mathcal{J}}\sum_{i\in\mathcal{I}} y_{ijt}d_{ij}h.$$

The total variable cost of operating the WtE is given by:

$$\sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{I}} y_{ijt} v,$$

and the final disposal cost in each $t \in \mathcal{T}$ is:

$$\sum_{i \in \mathcal{I}} c_D \left[z_{it} + \sum_{j \in \mathcal{J}} (1 - \theta_i) y_{ijt} + y_{ijt} \rho_j \right],$$

where the components in the above formula refer respectively to the disposal costs for collected but untreated waste, the untreatable portions of the waste, and finally the treatment residues. To simplify the presentation of the model, we denote by

$$r_{ij} := \theta_i \mu q_E - d_{ij} h - v - c_D (1 - \theta_i + \rho_j) \tag{3.1}$$

the WtE revenue coefficient for the waste collected from zone i and processed at facility j. The net WtE revenue in period $t \in \mathcal{T}$ before capacity repayments is then revenue from sales of energy recovered, less transportation cost, operational cost and disposal cost, i.e.,

$$\sum_{i \in \mathcal{I}} \left[\sum_{j \in \mathcal{J}} r_{ij} y_{ijt} - c_D z_{it} \right].$$

In addition, the WtE installation $\boldsymbol{x}_t, t \in \mathcal{T}$ incurs a fixed cost $\boldsymbol{c}' \boldsymbol{x}_t = \sum_{j \in \mathcal{J}} c_j x_{jt}$. Consolidating the above items, the NPV contribution in period t, with $\beta \in (0, 1)$ being the stage-wise discount ratio used when considering the time value of money, is given by:

$$\phi_t \left(\boldsymbol{x}_{t-1}, \boldsymbol{y}_t, \boldsymbol{z}_t \right) := \beta^t \sum_{i \in \mathcal{I}} \left[\sum_{j \in \mathcal{J}} r_{ij} y_{ijt} - c_D z_{it} \right] - \beta^{t-1} \boldsymbol{c}^\top \boldsymbol{x}_t, \tag{3.2}$$

Denote also $\phi(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) = \sum_{t \in \boldsymbol{\mathcal{T}}} \phi_t(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{z}_t)$ as the total NPV function. Hence, given the waste generation forecast $\boldsymbol{\xi}$, the WtE expansion problem can be formulated as the following mixed integer linear program, which solves for the expansion plan $\boldsymbol{x}$ and waste distribution $(\boldsymbol{y}, \boldsymbol{z})$ to maximize the total NPV:

$$\max_{\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}} \quad \phi(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) := \sum_{t \in \boldsymbol{\mathcal{T}}} \phi_t(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{z}_t) \tag{3.3}$$

$$\text{s.t.} \quad \sum_{j \in \boldsymbol{\mathcal{J}}} y_{ijt} + z_{it} = \widehat{\xi}_{it}, \; i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}} \tag{3.4}$$

$$\sum_{i \in \boldsymbol{\mathcal{I}}} y_{ijt} \le \sum_{\tau=1}^{t} x_{j\tau} s_j, \; j \in \boldsymbol{\mathcal{J}}, t \in \boldsymbol{\mathcal{T}} \tag{3.5}$$

$$x_{jt} \in \{0, 1\}, y_{ijt}, z_{it} \in \Re_+, \; i \in \boldsymbol{\mathcal{I}}, j \in \boldsymbol{\mathcal{J}}, t \in \boldsymbol{\mathcal{T}} \tag{3.6}$$

$$\boldsymbol{x} \in \mathcal{X}_0. \tag{3.7}$$

In the above model, constraint (3.4) ensures that the waste treatment requirement from each zone must be satisfied by either the WtE processing or disposal. Constraint (3.5) guarantees that the waste supply at time period t can be distributed to all the WtE facilities opened up to period t, and the maximum quantity treated is limited by the available capacity. Constraint (3.6) covers the binary requirements for location decisions $\boldsymbol{x}$ and the non-negative requirement for the waste allocation decision $(\boldsymbol{y}, \boldsymbol{z})$.

Finally, $\mathcal{X}_o$ in (3.7) is used to capture additional requirements on the expansion variables $\boldsymbol{x}$. We assume such requirements can be modelled so that $\mathcal{X}_o$ is polyhedral. For instance, if the decision maker allows each potential location j to be sited at most once for WtE system installation, then $\mathcal{X}_o$ can be defined as:

$$\mathcal{X}_o := \left\{ \boldsymbol{x} : \sum_{t \in \boldsymbol{\mathcal{T}}} x_{jt} \leq 1, j \in \boldsymbol{\mathcal{J}} \right\}.$$

3.3 Uncertainty Impact Illustration

As an initial analysis, let us illustrate the impact of the variation in waste supply $\boldsymbol{\xi}$ to the location designs $\boldsymbol{x}$ as well as its NPVs for the WtE system. Here we employ 15 different scenarios for $\boldsymbol{\xi} = [\xi_{it}]_{I \times T}$, and $\boldsymbol{\xi}^k$ is the k-th scenario of the waste supply. Also, we set $I = 10, J = 20$ and a planning horizon of 2 stages ($T = 2$). More specifically, each ξ_{i1}^k is the k-th waste supply at stage-1 which can only been observed after the initial location planning $\boldsymbol{x}_0$ (at stage-0), and the possible values of ξ_{i1}^k can be regarded as a variation or perturbation based on the current waste supply at zone i; while each ξ_{i2}^k is the k-th waste supply at stage-2, which is a perturbation based on the realized waste supply ξ_{i1}^k realized at stage-1. For a concise presentation, here we set a feedstock quantity benchmark of $10,000$ tons, and generate 15 scenarios of $\boldsymbol{\xi}^k, k = 1, 2, \cdots, 15$ for each feedstock supply ξ_{it}^k at zone i at stage-t by increasing this benchmark value by from 0% to 15%. All other basic parameter setting are similar to that in the computational study (Chapter 6).

Table 3.1 shows that under two different feedstock supply scenarios (Scenarios-I and II are selected from the 15 generated ones), the location decisions for WtE systems are influenced significantly. Furthermore, we plot the values of NPV for all the different scenarios in Figure 3.1, from which I see the NPV value is also largely impacted by the feedstock supply uncertainty.

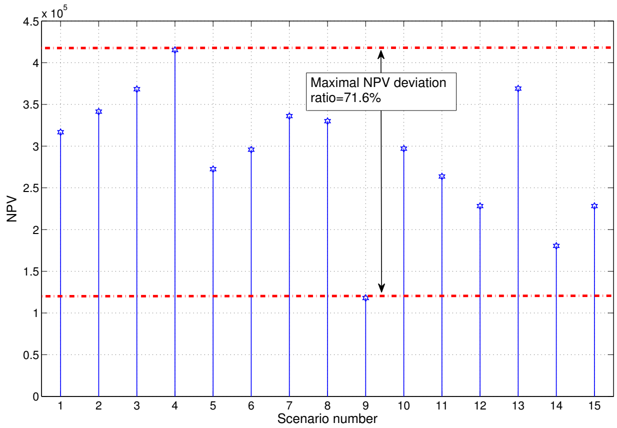


Figure 3.1: The NPV plots of WtE location design cross all the generated scenarios, where the maximal NPV deviation ratio:=$((\text{NPV}_{\text{max}} - \text{NPV}_{\text{min}})/\text{NPV}_{\text{max}}) \times 100\%$.

Table 3.1: Different WtE systems location decisions under different scenarios of feedstock supply, where the total feedstock quantity increases (compared with the benchmark quantity of 10, 000 tons) by 2% at Stage-1 and by 15% at Stage-2 in Scenario-I, while increases by 10% at Stage-1 and by 5% at Stage-2 in Scenario-II.

Planning period	Sites of WtE systems opened in scenario I	Sites of WtE systems opened in scenario II
Stage 0	No.2-8, No.15, No.17-18	No.1-8, No.10, No.13, No.15, No.17-18
Stage 1	No.1, No.9-11, No.13,	No.9, No.11

Chapter 4

WtE Scenario Analysis using Forecast Prediction Sets

In reality, location-expansion plan $\boldsymbol{x}$ generated by (3.3)–(3.7) using point-valued forecasts can be arbitrarily bad, unless we have perfect information of the future waste generation $\boldsymbol{\xi}$. The practical issue of interest is then on achieving expansion plans that are relatively stable despite the uncertainty over the planning horizon. This is the major focus of our work in the rest of the paper. In this chapter, we first propose a general model of prediction sets for the uncertain future waste supply. We then show how these prediction sets can be used to compute a guaranteed NPV forecast (denoted $\mathcal{Z}_{\text{NPV}}(\boldsymbol{x})$) for a given *expansion scenario* $\boldsymbol{x}$, where the WtE expansion scenario here refers to a specified instance of expansion decisions $\boldsymbol{x}$.

4.1 Modeling waste generation prediction sets

We consider the following growth model for the waste supply:

$$\xi_{it} = \xi_{it-1} + \eta_{it}, \;\; i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}, \tag{4.1}$$

where ξ_{it} is the waste generated in zone i and period t, which is the sum of the generation in the previous period, and η_{it} the growth (which could possibly be negative) in the waste in zone i over the t-th period. $\xi_{i0} = \widehat{\xi}_{i0}$ is defined as the observed initial waste supply in zone i at the beginning of the planning horizon, which we denote as period $t = 0$. Also we denote by $\widehat{\boldsymbol{\eta}} = [\widehat{\eta}_{i,-k}]_{|\boldsymbol{\mathcal{I}}|\times K}$ the observed historical waste growth (K years back) of each zone $i \in \boldsymbol{\mathcal{I}}$ with $k \in [0\!:\!K]$.

Let $\mathcal{F}_{it}$ be the t-period ahead forecast of the average waste growth level η_{it} in zone i and period t, made at the period $t = 0$. $\mathcal{F}_{it}$ can be based on any forecasting model of choice by the user. Popular time series models (see Gilchrist [17]) include for example Exponentially Weighted Moving Average (EWMA) model:

$$\text{(EWMA)}: \quad F_{it} := \sum_{k=0}^{K} \left[\frac{\alpha^k}{\sum_{k=0}^{K} \alpha^k} \right] \widehat{\eta}_{i,-k}, \ \forall t \in \boldsymbol{\mathcal{T}},$$

where $\widehat{\eta}_{i,-k}$ denotes the observed waste growth in zone i and period $-k = 0, \cdots, -K_0 (K_0 < K)$, that is, before the beginning of the planning horizon at $t = 0$, and $\alpha \in (0, 1]$ is the discounting factor to be estimated. Another example is the Autoregressive Moving Average of order (p, q) (ARMA(p, q)) model (see Gilchrist [17]):

$$\begin{aligned}\text{(ARMA}(p,q)): \quad F_{it} :=& \chi_{\{t\geq 2\}} \left[\sum_{\ell=1}^{t-1} \alpha_\ell F_{i\ell} \right] + \sum_{r=t}^{p} \alpha_r \widehat{\eta}_{i,-(r-t)} \\ &+ \sum_{\varsigma=t}^{q} \beta_\varsigma \left[\widehat{\eta}_{i,-(\varsigma-t)} - F_{i,-(\varsigma-t)}^{-\varsigma} \right], \ \forall t \in \boldsymbol{\mathcal{T}}\end{aligned}$$

where $\alpha_\ell, \ell \in [1:p]$ and $\beta_\varsigma, \varsigma \in [1:q]$ are the constant coefficients to be estimated, $F_{i,-(\varsigma-t)}^{-\varsigma}$ is the t-period ahead forecast of $\widehat{\eta}_{i,-(\varsigma-t)}$, and $\chi_{\{t\geq 2\}}$ is an indicator function taking on the value of one when $t \geq 2$ and zero otherwise.

We assume that the data for the t-period ahead forecast errors, $\eta_{it} - F_{it}$, can be correspondingly collected. Based on historical observations of these forecast errors, various statistics S_{it} such as mean absolute error, sample variations, can be computed. In the following, a basic construct of interest is to replace the waste growth point-value forecast F_{it} with the prediction interval $[F_{it} - S_{it}, F_{it} + S_{it}]$. The motivation of the prediction intervals is to take into account the presence of forecast errors, through the use of the statistics S_{it}. For example, the t-step ahead mean absolute error MAE_{it} for zone i can be estimated as:

$$\text{MAE}_{it} := \frac{1}{K_0 + 1} \sum_{k=0}^{K_0} \left| F_{i,-k}^{(-k-t)} - \widehat{\eta}_{i,-k} \right| \tag{4.2}$$

where $F_{i,-k}^{(-k-t)}$ denotes the t-step ahead forecast of $\eta_{i,-k}$, made in period $-t-k$. The above prediction interval with $S_{it} := \text{MAE}_{it}$ can then be defined as $[F_{it} - \text{MAE}_{it}, F_{it} + \text{MAE}_{it}]$.

Another example is as follows. Assume that the forecasting error follows a zero-mean normal distribution. We then have (Bowerman et al. [9] and Geisser [16]):

$$\frac{\eta_{it} - F_{it}}{\hat{\text{v}}_{it}^{K}} \sim \text{t}_{K-1},$$

where $\hat{\text{v}}_{it}^{K}$ is the sample variance of the t-period ahead forecast error $\eta_{it} - F_{it}$ computed using a sample of size K, and t_{K-1} is the standard t-distribution with $K-1$ degrees of freedom. We can then construct the $100 \times (1-\alpha)\%$ prediction interval on the future waste growth η_{it}, by defining $S_{it} := \hat{\text{v}}_{it}^{K} \text{t}_{K-1,\frac{\alpha}{2}}$, where $\text{t}_{K-1,\frac{\alpha}{2}}$ is the $100 \times (1 - \frac{\alpha}{2})$-percentile of the t-distribution with $K-1$ degrees of freedom.

We next define the following reformulation of the prediction interval of η_{it}:

$$\eta_{it} \in [F_{it} - S_{it}, F_{it} + S_{it}] \Longleftrightarrow \eta_{it} = F_{it} + \varrho_{it} S_{it},\ \ \varrho_{it} \in [-1, 1],$$

where the variables $\varrho_{it} \in [-1, 1]$ are used to indicate the direction and level of forecast errors accounted for in the prediction intervals. For instance, ϱ_{it} with a magnitude of one indicates that a forecast error of value S_{it} is accounted for in the prediction of the growth η_{it}. A positive (negative) value of ϱ_{it} indicates that the adjustment from the point-value forecast F_{it} is in the upward (downward) direction. When ϱ_{it} takes a value of zero, no forecast errors are considered in the prediction of η_{it}.

Based on the above, we propose the *prediction set* of the future waste generation $\boldsymbol{\xi}$ as follows:

$$\mathcal{U}^{\Gamma} := \left\{ \boldsymbol{\xi} \in \Re^{|\boldsymbol{\mathcal{I}}| \times |\boldsymbol{\mathcal{T}}|} : \begin{array}{l} \xi_{it} = \widehat{\xi}_{i0} + \sum_{\tau=1}^{t} F_{i\tau} + \left[\sum_{\tau=1}^{t} \varrho_{i\tau} S_{i\tau} \right],\ i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}} \\ \sum_{i \in \boldsymbol{\mathcal{I}}} |\varrho_{it}| \leq \Gamma^{\mathrm{Z}}_{t}, t \in \boldsymbol{\mathcal{T}} \\ \sum_{t \in \boldsymbol{\mathcal{T}}} |\varrho_{it}| \leq \Gamma^{\mathrm{T}}_{i}, i \in \boldsymbol{\mathcal{I}} \\ \varrho_{it} \in [-1, 1], i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}} \end{array} \right\}. \tag{4.3}$$

We now explain the components in (4.3). First, note that for each $i \in \boldsymbol{\mathcal{I}}$, $t \in \boldsymbol{\mathcal{T}}$, the item

$$\xi_{it} = \widehat{\xi}_{i0} + \sum_{\tau=1}^{t} F_{i\tau} + \left[\sum_{\tau=1}^{t} \varrho_{i\tau} S_{i\tau} \right]$$

computes a forecast of the waste generation ξ_{it} by applying the growth model (4.1) and the above mentioned prediction intervals $[F_{i\tau} - S_{i\tau}$, $F_{i\tau} + S_{i\tau}]$, repeatedly, for $\tau \in [1 : t]$. Note that by the above construction, each ξ_{it} has up to one additional unit of forecast error S_{it}, than in ξ_{it-1}. This reflects the uncertainty propagation effect in future waste supply over time, which is also consistent with practical observations.

Next, the items:

$$\sum_{t \in \boldsymbol{\mathcal{T}}} |\varrho_{it}| \leq \Gamma_i^{\mathrm{T}}, \ \forall\, i \in \boldsymbol{\mathcal{I}}, \quad \sum_{i \in \boldsymbol{\mathcal{I}}} |\varrho_{it}| \leq \Gamma_t^{\mathrm{Z}}, \ \ \forall\, t \in \boldsymbol{\mathcal{T}}$$

constrain the number of forecast errors that are included in the prediction set, where $\Gamma := \left[\Gamma_t^{\mathrm{Z}}, \Gamma_i^{\mathrm{T}}\right]_{|\boldsymbol{\mathcal{T}}|+|\boldsymbol{\mathcal{I}}|}$ are regarded as *forecast error budgets*, which can be chosen based on user attitudes towards the uncertainty. In particular, Γ_i^{T} controls the total forecast errors in waste growth of each zone i over the entire planning horizon $|\boldsymbol{\mathcal{T}}|$, and Γ_t^{Z} controls the total forecast errors in waste growth at each period t, across all zones. Typically, users will choose some value of $\Gamma_i^{\mathrm{T}} \in [0, |\boldsymbol{\mathcal{T}}|]$, and $\Gamma_t^{\mathrm{Z}} \in [0, |\boldsymbol{\mathcal{I}}|]$. When the upper bounds (full budgets) are chosen, the user adopts an extremely conservative position, and believes that a full forecast error will occur in every waste growth prediction F_{it}. When the budgets are 0, the user adopts the other extreme, and is very optimistic about his point-value forecasts F_{it}. Here, it is assumed that the budget choices are integer-valued. This represents little to no loss in the problem consideration as the budget choice, a measure of conservatism, only requires a finite scale to be well represented. In addition, we will later show that this will help in simplifying the definition of the prediction set in (4.5).

4.2 Forecasting guaranteed NPV levels

A typical decision process in the WtE expansion project is as follows. An expansion scenario $\boldsymbol{x} := (\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{|\mathcal{T}|})$ is proposed, and the project owners would like to evaluate its feasibility by obtaining a forecast of the resulting net present value (NPV) of the project. To account for the future waste generation uncertainty, we assume that an appropriate prediction set $\mathcal{U}^\Gamma$ in (4.3) for some forecast error budget setting Γ has been articulated. Because WtE projects are extremely expensive and involve huge, long-term and irreversible investments in capital infrastructure, stakeholders are likely to adopt a rather cautious stance in the presence of uncertainty. For this reason, what is often required is a *guarantee* on the NPV level, i.e. some lower bound on the achievable NPV upon actual implementation. If the stakeholders deem the guaranteed level acceptable, then the proposed expansion plan $\boldsymbol{x}$ is also acceptable.

In reality, the uncertain waste generations $\boldsymbol{\xi} := \left(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \cdots, \boldsymbol{\xi}_{|\mathcal{T}|}\right)$ are revealed in stages, they are assumed to take on values in the prediction set $\mathcal{U}^\Gamma$. Similarly, the operational decisions $\boldsymbol{y}_1, \cdots, \boldsymbol{y}_{|\mathcal{T}|}$ depend on the actual waste generated in stages. Intuitively, we can regard the process of determining the guaranteed NPV as a leader-follower game over $|\mathcal{T}|$ stages. In this context the 'leader' is nature itself, who selects waste generation $\boldsymbol{\xi}_t$ at the beginning of each period t. The 'follower' is the WtE owner, who chooses waste distribution action $\boldsymbol{y}_t$ in response, so as to maximize the NPV contribution. Hence, the objective of the 'leader' is to choose a forecast $\boldsymbol{\xi} \in \mathcal{U}^\Gamma$ that strategically minimizes the realized NPV, subject to its forecast error budget Γ. The outcome of this process is the desired guaranteed NPV level.

To formalize the above descriptions mathematically, we first denote $\boldsymbol{x}_{[t]} := (\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_t)$ as the expansion up to t. Correspondingly, $\boldsymbol{x}_{[0]}$ refers to the initial state before any decision has been made. In each time period $t \in \boldsymbol{\mathcal{T}}$, given the revealed waste generation $\boldsymbol{\xi}_t$, define:

$$\mathcal{Y}_t\left(\boldsymbol{x}_{[t]}, \boldsymbol{\xi}_t\right) := \left\{(\boldsymbol{y}_t, \boldsymbol{z}_t) \in \Re_+^{|\boldsymbol{\mathcal{I}}| \times |\boldsymbol{\mathcal{J}}|} \times \Re_+^{|\boldsymbol{\mathcal{I}}|} \mid (3.4) - (3.5)\right\}$$

as the set of feasible waste distribution decisions $(\boldsymbol{y}_t, \boldsymbol{z}_t)$ at time period t given $\boldsymbol{x}_{[t]}$ and $\boldsymbol{\xi}_t$. The decisions $\boldsymbol{y}_t, \boldsymbol{z}_t$ are selected to maximize the NPV at the period t, which, by the problem structure in (3.3)-(3.7), are only influenced by $\boldsymbol{x}_{[t]}$ and the waste supply $\boldsymbol{\xi}_t$ at that period. This is stated as:

$$\max_{(\boldsymbol{y}_t, \boldsymbol{z}_t) \in \mathcal{Y}_t\left(\boldsymbol{x}_{[t]}, \boldsymbol{\xi}_t\right)} \phi_t\left(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{z}_t\right)$$

where $\phi_t\left(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{z}_t\right)$ is the NPV contribution in the period t as defined in (3.2). For convenience, we also denote by

$$\Upsilon_t(\boldsymbol{x}_{[t]}, \boldsymbol{\xi}_t) := \max_{(\boldsymbol{y}_t, \boldsymbol{z}_t) \in \mathcal{Y}_t\left(\boldsymbol{x}_{[t]}, \boldsymbol{\xi}_t\right)} \phi_t\left(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{z}_t\right) \tag{4.4}$$

the optimal NPV in period t given $\boldsymbol{x}_{[t]}$ and $\boldsymbol{\xi}_t$.

Next, denote $\boldsymbol{\xi}_{[t-1]} = (\widehat{\boldsymbol{\xi}}_0, \boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \cdots, \boldsymbol{\xi}_{t-1})$ as the revealed waste generation up to $t-1$, with $\boldsymbol{\xi}_{[0]} := (\widehat{\boldsymbol{\xi}}_0)$. Recall that $\boldsymbol{\xi}_{[t-1]}$ is assumed to take on values from the prediction set $\mathcal{U}^{\Gamma}$. In the context of the leader-follower game, this corresponds to that up to period $t-1$, the leader has expended some portion of his forecast error budget Γ through his selection of the forecast error allocation variables $\varrho_{i\tau}$, $\forall i \in \boldsymbol{\mathcal{I}}, \tau \in [1 : t-1]$, that determined $\xi_{i\tau}, i \in \boldsymbol{\mathcal{I}}, \tau \in [1 : t-1]$. The set of realizable values for $\boldsymbol{\xi}_t$ then depends on the 'budget-to-go' at the beginning of period t. More specifically, we define this prediction set for $\boldsymbol{\xi}_t$, conditional on $\boldsymbol{\xi}_{[t-1]}$ as:

$$\mathcal{U}_t^{\Gamma}(\boldsymbol{\xi}_{[t-1]}) := \left\{ \boldsymbol{\xi}_t \in \Re^{|\boldsymbol{\mathcal{I}}|} : \begin{array}{l} \xi_{it} = \xi_{it-1} + F_{it} + \varrho_{it} S_{it}, i \in \boldsymbol{\mathcal{I}} \\ \sum_{i \in \boldsymbol{\mathcal{I}}} |\varrho_{it}| \leq \Gamma_t^{\mathrm{Z}} \\ |\varrho_{it}| \leq \Gamma_i^{\mathrm{T}} - \chi_{\{t \geq 2\}} \left[\sum_{\tau=1}^{t-1} |\varrho_{i\tau}| \right], i \in \boldsymbol{\mathcal{I}} \\ \varrho_{it} \in [-1, 1], i \in \boldsymbol{\mathcal{I}} \end{array} \right\}, \quad (4.5)$$

where $\chi_{\{t \geq 2\}}$ is an indicator function, such that $\chi_{\{t \geq 2\}} = 1$ if $t \geq 2$, and $\chi_{\{t \geq 2\}} = 0$ otherwise. Note that the constraints:

$$|\varrho_{it}| \leq \Gamma_i^{\mathrm{T}} - \chi_{\{t \geq 2\}} \left[\sum_{\tau=1}^{t-1} |\varrho_{i\tau}| \right], \varrho_{it} \in [-1, 1], i \in \boldsymbol{\mathcal{I}}$$

in (4.5) jointly ensure that for each $i \in \boldsymbol{\mathcal{I}}$, if the 'expended' forecast error budget $\sum_{\tau=1}^{t-1} |\varrho_{i\tau}|$ has already reached the total budget level Γ_i^{T}, then $|\varrho_{it}| = 0 \Leftrightarrow \varrho_{it} = 0$; otherwise $|\varrho_{it}| \leq 1$ (noting that Γ_i^{T} is integer-valued).

At the beginning of the final period $T = |\boldsymbol{\mathcal{T}}|$ in the planning horizon, nature (the 'leader') solves the following problem to minimize the NPV contribution achieved when the waste generation is revealed:

$$\begin{aligned} &\min_{\boldsymbol{\xi}_T \in \mathcal{U}_T^{\Gamma}(\boldsymbol{\xi}_{[T-1]})} \Upsilon_T(\boldsymbol{x}_{[T-1]}, \boldsymbol{\xi}_{T-1}) = \\ &\min_{\boldsymbol{\xi}_T \in \mathcal{U}_T^{\Gamma}(\boldsymbol{\xi}_{[T-1]})} \max_{(\boldsymbol{y}_T, \boldsymbol{z}_T) \in \mathcal{Y}_T(\boldsymbol{x}_{[T-1]}, \boldsymbol{\xi}_{T-1})} \phi_T\left(\boldsymbol{x}_{[T-1]}, \boldsymbol{y}_T, \boldsymbol{z}_T\right). \end{aligned} \quad (4.6)$$

Given that the total NPV obtained is the sum of the NPV each year, and that the decisions made at time t only depend on what has happened in the past in time $s, s < t$, we apply (4.6) recursively backwards in time $t \in \boldsymbol{\mathcal{T}}$ to formulate the evaluation of the guaranteed NPV, denoted $\mathcal{Z}_{\mathrm{NPV}}(\boldsymbol{x})$, as the following optimization problem:

$$\mathcal{Z}_{\text{NPV}}(\boldsymbol{x}) = \min_{\boldsymbol{\xi}_1 \in \mathcal{U}_1^{\Gamma}(\boldsymbol{\xi}_{[0]})} \Bigg[\Upsilon_1(\boldsymbol{x}_{[0]}, \boldsymbol{\xi}_1) + \\ \min_{\boldsymbol{\xi}_2 \in \mathcal{U}_2^{\Gamma}(\boldsymbol{\xi}_{[1]})} \Bigg[\Upsilon_2(\boldsymbol{x}_{[1]}, \boldsymbol{\xi}_2) + \cdots + \min_{\boldsymbol{\xi}_T \in \mathcal{U}_T^{\Gamma}(\boldsymbol{\xi}_{[T-1]})} \Big[\Upsilon_T(\boldsymbol{x}_{[T-1]}, \boldsymbol{\xi}_T) \Big] \cdots \Bigg] \Bigg], \tag{4.7}$$

where $\mathcal{U}_t^{\Gamma}(\boldsymbol{\xi}_{[t-1]}), t \in [1 : |\boldsymbol{\mathcal{T}}|]$ are the state-dependent prediction sets given by (4.5).

Note that (4.7) is in the format of a dynamic optimization problem under uncertainty, which is generally difficult to solve. Fortunately, due to structure of the WtE problem, we can show that the (4.7) can be reduced to a much simpler format of a two-stage optimization problem. This can be further shown to be transformable into a mixed integer linear programming formulation that can be solved directly using commercial applications.

To simplify the notation, in the following we denote the feasible set of the waste distribution decisions $(\boldsymbol{y}, \boldsymbol{z})$, given $\boldsymbol{x}$ and $\boldsymbol{\xi}$, as:

$$\mathcal{Y}(\boldsymbol{x}, \boldsymbol{\xi}) := \Big\{ (\boldsymbol{y}, \boldsymbol{z}) \in \Re_+^{|\boldsymbol{\mathcal{T}}| \times |\boldsymbol{\mathcal{I}}| \times |\boldsymbol{\mathcal{J}}|} \times \Re_+^{|\boldsymbol{\mathcal{T}}| \times |\boldsymbol{\mathcal{I}}|} \mid (3.4) - (3.5) \Big\}. \tag{4.8}$$

We then have the following result.

Proposition 1. *Given WtE expansion scenario* $\boldsymbol{x}$, *we have*

$$\mathcal{Z}_{\text{NPV}}(\boldsymbol{x}) = \min_{\boldsymbol{\xi} \in \mathcal{U}^{\Gamma}} \max_{(\boldsymbol{y}, \boldsymbol{z}) \in \mathcal{Y}(\boldsymbol{x}, \boldsymbol{\xi})} \phi(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}), \tag{4.9}$$

where $\phi(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ *is the total NPV objective defined in (3.3).*

Proof. The technical proof details are provided in Appendix A.1. □

The implication of Proposition 1 is the following. Given expansion scenario $\boldsymbol{x}$, there exist a corresponding waste generation forecast $\boldsymbol{\xi}^{\dagger} = [\xi^{\dagger}_{it}]_{|\boldsymbol{\mathcal{I}}|\times|\boldsymbol{\mathcal{T}}|}$ in $\mathcal{U}^{\Gamma}$, which if realized, will result in the guaranteed NPV level $\mathcal{Z}_{\text{NPV}}(\boldsymbol{x})$. We refer to $\boldsymbol{\xi}^{\dagger}$ as the *extreme value forecast* of the waste supply. The guaranteed NPV can be evaluated by determining the NPV of the waste distribution $(\boldsymbol{y}, \boldsymbol{z}) \in \mathcal{Y}(\boldsymbol{x}, \boldsymbol{\xi})$ that is optimal under the extreme value forecast.

Essentially, the result in (4.9) is a consequence of the fact that the optimal total NPV function in the WtE operations problem is decomposable in each period t:

$$\max_{(\boldsymbol{y},\boldsymbol{z})\in\mathcal{Y}(\boldsymbol{x},\boldsymbol{\xi})} \phi(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) = \sum_{t\in\boldsymbol{\mathcal{T}}} \Upsilon_t(\boldsymbol{x}_{[t]}, \boldsymbol{\xi}_t).$$

The above is possible because of the assumption that all collected waste has to be either treated or disposed in each period, and no storage of waste across periods is allowed. This is due to the characteristics of the WtE problem, since the planning bucket is in the unit of years, and untreated municipal waste cannot be stockpiled at this scale due to public health restrictions.

Although the resulting min-max optimization problem in (4.9) is simpler than the dynamic optimization formulation (4.7), the former problem in general is still intractable (see [4]). Fortunately, it can be shown that under our problem setting with the structure of the budget prediction set $\mathcal{U}^{\Gamma}$, it can be solved as a single instance of mixed integer linear program of moderate size which is more computationally tractive. This result is formally presented in the following proposition.

Proposition 2. *Given the expansion scenario* $\boldsymbol{x}$ *and uncertainty budget parameters* Γ*, the guaranteed NPV evaluation problem (4.9) is equivalent to the following mixed integer programming model:*

$$\mathcal{Z}_{\text{NPV}}(\boldsymbol{x}) = \min_{\boldsymbol{q},\boldsymbol{g},\boldsymbol{\pi},\boldsymbol{h},\boldsymbol{\gamma}} \gamma_1 + \gamma_2 - \gamma_3 \tag{4.10}$$

$$\text{s.t.}\quad \gamma_1 \geq \sum_{j\in\boldsymbol{\mathcal{J}}} \left[\sum_{t\in\boldsymbol{\mathcal{T}}} \sum_{\tau=0}^{t-1} x_{j\tau} s_j g_{jt} - \beta^{t-1} \boldsymbol{c}^\top \boldsymbol{x}_t\right] \tag{4.11}$$

$$\gamma_2 \geq \sum_{i\in\boldsymbol{\mathcal{I}}} \sum_{t\in\boldsymbol{\mathcal{T}}} q_{it} \left[\widehat{\xi}_{i0} + \sum_{\tau=1}^{t} F_\tau(\widehat{\boldsymbol{\eta}}_i)\right] + \left[\sum_{\tau=1}^{t} \left(h^+_{it\tau} - h^-_{it\tau}\right) S_{i\tau}\right] \tag{4.12}$$

$$\gamma_3 \leq \sum_{i\in\boldsymbol{\mathcal{I}}} \sum_{t\in\boldsymbol{\mathcal{T}}} \left\{\widehat{\xi}_{i0} + \sum_{\tau=1}^{t} F_\tau(\widehat{\boldsymbol{\eta}}_i) + \left[\sum_{\tau=1}^{t} \left(\pi^+_{i\tau} - \pi^-_{i\tau}\right) S_{i\tau}\right]\right\} \beta^t c_D \tag{4.13}$$

$$q_{it} + g_{jt} \geq \beta^t r_{ij} + \beta^t c_D,\ i \in \boldsymbol{\mathcal{I}}, j \in \boldsymbol{\mathcal{J}}, t \in \boldsymbol{\mathcal{T}} \tag{4.14}$$

$$\sum_{i\in\boldsymbol{\mathcal{I}}} (\pi^+_{it} + \pi^-_{it}) \leq \Gamma^{\text{Z}}_t, t \in \boldsymbol{\mathcal{T}} \tag{4.15}$$

$$\sum_{t\in\boldsymbol{\mathcal{T}}} (\pi^+_{it} + \pi^-_{it}) \leq \Gamma^{\text{T}}_i, i \in \boldsymbol{\mathcal{I}} \tag{4.16}$$

$$h^+_{it\tau} \leq q_{it},\ \forall\ i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}, \tau \in [1:t] \tag{4.17}$$

$$h^+_{it\tau} \geq q_{it} + (\pi^+_{i\tau} - 1)M,\ \forall\ i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}, \tau \in [1:t] \tag{4.18}$$

$$h^+_{it\tau} \leq \pi^+_{i\tau} M,\ \forall\ i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}, \tau \in [1:t] \tag{4.19}$$

$$h^-_{it\tau} \leq q_{it},\ \forall\ i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}, \tau \in [1:t] \tag{4.20}$$

$$h^-_{it\tau} \geq q_{it} + (\pi^-_{i\tau} - 1)M,\ \forall\ i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}, \tau \in [1:t] \tag{4.21}$$

$$h^-_{it\tau} \leq \pi^-_{i\tau} M,\ \forall\ i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}, \tau \in [1:t] \tag{4.22}$$

$$\boldsymbol{q} \in \Re_+^{|\boldsymbol{\mathcal{I}}|\times|\boldsymbol{\mathcal{T}}|}, \boldsymbol{g} \in \Re_+^{|\boldsymbol{\mathcal{J}}|\times|\boldsymbol{\mathcal{T}}|},$$

$$\boldsymbol{\pi}^+, \boldsymbol{\pi}^- \in \{0,1\}^{|\boldsymbol{\mathcal{I}}|\times|\boldsymbol{\mathcal{T}}|}, \boldsymbol{h}^+, \boldsymbol{h}^- \in \Re_+^{|\boldsymbol{\mathcal{I}}|\times|\boldsymbol{\mathcal{T}}|^2}, \tag{4.23}$$

where M is a positive finite number which can be determined explicitly by model inputs, and $F_\tau(\widehat{\boldsymbol{\eta}}_i)$ is the τ-period ahead forecast as defined in Section 4.1.

Proof. The technical proof details are provided in Appendix A.1.

□

Along with the guaranteed NPV $\mathcal{Z}_{\text{NPV}}(\boldsymbol{x})$, the extreme value forecast of the waste generation, $\boldsymbol{\xi}^\dagger = [\xi^\dagger_{it}]_{|\boldsymbol{\mathcal{I}}|\times|\boldsymbol{\mathcal{T}}|}$, can also be recovered by using the auxiliary variables in (4.10)–(4.23). Specifically, we can use the following formula to extract the $\boldsymbol{\xi}^\dagger$.

Proposition 3. *Given an expansion scenario $\boldsymbol{x}$, the corresponding extreme value forecast $\xi^\dagger_{it}$ of waste supply for zone $i \in \boldsymbol{\mathcal{I}}$ and time period $t \in \boldsymbol{\mathcal{T}}$ under forecast error budget Γ can be obtained by the following formula:*

$$\xi^\dagger_{it} = \widehat{\xi}_{i0} + \sum_{\tau=1}^{t} F_{i\tau} + \left[\sum_{\tau=1}^{t} \left(\bar{\pi}^+_{i\tau} - \bar{\pi}^-_{i\tau}\right) S_{i\tau}\right], \tag{4.24}$$

where $\bar{\pi}^+_{i\tau}, \bar{\pi}^-_{i\tau}, i \in \boldsymbol{\mathcal{I}}, \tau \in [1:t]$ are auxiliary binary decision variables solved from (4.10)-(4.23).

Proof. The result can be obtained directly from the proof of Proposition 2, we therefore omit the proof details. □

The extreme value forecast $\xi^\dagger_{it}, i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}$ is useful when the practitioners would like to study the characteristics of the most adverse future waste generations trajectory corresponding to expansion scenario $\boldsymbol{x}$ under the given forecast error budget level. We note that the obtained $\boldsymbol{\xi}^\dagger_1, \boldsymbol{\xi}^\dagger_2, \ldots, \boldsymbol{\xi}^\dagger_T$ are precisely the extreme value forecasts for the multi-stage guaranteed NPV evaluation problem in (4.7) rather than an approximation.

It is hence of practical value, since users and decision-makers can often assimilate and interpret the solution values better than if presented with a dynamic policy. By adjusting the budget parameters $\Gamma = \left[\Gamma_t^{\mathrm{Z}}, \Gamma_i^{\mathrm{T}}\right]_{|\boldsymbol{\mathcal{T}}|+|\boldsymbol{\mathcal{I}}|}$, a series of extreme value forecasts can also be generated efficiently. We show examples of using the extreme value forecast information in the computational studies in Section 6.1.

Chapter 5

Expansion Optimization with Guaranteed NPV

In this chapter, we focus on the optimization of the WtE expansion problem using the waste generation prediction set model and maximizing the guaranteed NPV level developed in the previous chapter. To simplify notation, we denote by $\mathcal{X}$ the following expansion decision constraints:

$$\mathcal{X} := \left\{ \boldsymbol{x} \in \Re^{|\boldsymbol{\mathcal{T}}| \times |\boldsymbol{\mathcal{J}}|} : \boldsymbol{x} \in \mathcal{X}_o, x_{tj} \in \{0,1\},\ t \in \boldsymbol{\mathcal{T}}, j \in \boldsymbol{\mathcal{J}} \right\}. \tag{5.1}$$

By Proposition 1, the WtE expansion optimization problem of interest:

$$\max_{\boldsymbol{x} \in \mathcal{X}} \mathcal{Z}_{\text{NPV}}(\boldsymbol{x})$$

can be stated compactly as a two-stage max-min-max optimization problem. That is

$$\max_{\boldsymbol{x} \in \mathcal{X}} \mathcal{Z}_{\text{NPV}}(\boldsymbol{x}) = \max_{\boldsymbol{x}} \left\{ \min_{\boldsymbol{\xi} \in \mathcal{U}^{\Gamma}} \max_{(\boldsymbol{y},\boldsymbol{z}) \in \mathcal{Y}(\boldsymbol{x},\boldsymbol{\xi})} \phi(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) \ \mid\ \boldsymbol{x} \in \mathcal{X} \right\}, \tag{5.2}$$

where $\phi(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$ and $\mathcal{Y}(\boldsymbol{x},\boldsymbol{\xi})$ are as defined in (3.3) and (4.8) respectively.

In summary, for a given $\boldsymbol{x}$, the problem allows the waste distribution $(\boldsymbol{y}, \boldsymbol{z})$ to adapt to the waste generation outcomes $\boldsymbol{\xi}$ as well as possible. The guaranteed NPV is evaluated by minimizing over all possible outcomes $\boldsymbol{\xi}$ in the prediction set $\mathcal{U}^{\Gamma}$. Finally, the maximum guaranteed NPV is achieved by optimizing over $\boldsymbol{x} \in \mathcal{X}$. Structurally, (5.2) belongs to the class of *two-stage adaptive robust optimization problems* which are known to be computationally intractable in general (see [4, 15]). Nevertheless, by exploiting the structure in the WtE problem (Proposition 2), we propose an iterative cutting plane algorithm with a closed-form of cuts that solves (5.2) via a sequence of linear mixed integer programs. The advantage of such algorithms is that it can be implemented easily using available integer optimization solvers. This is presented in Section 5.1. Next, in Section 5.2, we also propose a heuristic approach to obtain reasonably good (lower bound) solutions to (5.2) by considering a class of restrictions on the waste distribution actions $\boldsymbol{y}$. In particular, this class of restrictions require the distribution actions to follow a constant proportions rule, which is subsequently optimized. We show that under such assumptions, the problem can be formulated as a single instance of a mixed integer linear program of moderate dimensions. In addition, we also identify that if the waste supply constraints can be relaxed, then a tractable model can be developed for the WtE expansion optimization problem. To keep the content consistent, we leave such results in Appendix A.2.

5.1 Exact solution approach using cutting plane algorithm

In this section we develop a cutting plane approach for solving (5.2). Essentially, by extending the results in Proposition 2, we are able to construct a closed-form for the cuts. To describe the algorithm, we first formally present the following equivalent waste generation prediction set $\mathcal{V}(\Gamma)$ as the following:

$$\mathcal{V}(\Gamma) := \left\{ \boldsymbol{\xi} \in \Re^{|\boldsymbol{\mathcal{I}}|\times|\boldsymbol{\mathcal{T}}|} : \begin{array}{l} \xi_{it} = \widehat{\xi}_{i0} + \sum_{\tau=1}^{t} F_{i\tau} \\ \qquad + \left[\sum_{\tau=1}^{t} \left(\pi_{i\tau}^{+} - \pi_{i\tau}^{-}\right) S_{i\tau} \right], i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}} \\ \sum_{i \in \boldsymbol{\mathcal{I}}} \left(\pi_{it}^{+} + \pi_{it}^{-}\right) \le \Gamma_{t}^{\mathrm{Z}}, t \in \boldsymbol{\mathcal{T}} \\ \sum_{t \in \boldsymbol{\mathcal{T}}} \left(\pi_{it}^{+} + \pi_{it}^{-}\right) \le \Gamma_{i}^{\mathrm{T}}, i \in \boldsymbol{\mathcal{I}} \\ \pi_{it}^{+}, \pi_{it}^{-} \in \{0,1\}, i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}} \end{array} \right\}. \tag{5.3}$$

Note that $\mathcal{V}(\Gamma)$ is a *finite* set, and has been used in Proposition 2 to replace $\mathcal{U}^{\Gamma}$ (in dual form of problem (4.9)), so as to achieve the equivalent mixed integer linear formulation (4.10)-(4.23). Here $\mathcal{V}(\Gamma)$ plays a central role in establishing the cutting plane formulation for solving (5.2).

Furthermore, to represent the cuts in a compact manner in the formulation, we define in the following the set $\mathcal{K}(\boldsymbol{x}, \boldsymbol{\xi})$, given $\boldsymbol{x}$ and $\boldsymbol{\xi}$, as:

$$\mathcal{K}(\boldsymbol{x}, \boldsymbol{\xi}) := \left\{ \gamma \mid \exists(\boldsymbol{y}, \boldsymbol{z}) \text{ s.t. } \begin{array}{l} \gamma \leq \sum\limits_{t \in \boldsymbol{\mathcal{T}}} \left[\beta^t \sum\limits_{i \in \boldsymbol{\mathcal{I}}} \sum\limits_{j \in \boldsymbol{\mathcal{J}}} r_{ij} y_{ijt} - \sum\limits_{i \in \boldsymbol{\mathcal{I}}} c_D z_{it} \right] \\ \sum\limits_{j \in \boldsymbol{\mathcal{J}}} y_{ijt} + z_{it} = \xi_{it}, \ i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}} \\ \sum\limits_{i \in \boldsymbol{\mathcal{I}}} y_{ijt} \leq \sum\limits_{\tau=1}^{t} x_{j\tau} s_j, \ j \in \boldsymbol{\mathcal{J}}, t \in \boldsymbol{\mathcal{T}} \\ y_{ijt}, z_{it} \in \Re_+, \ i \in \boldsymbol{\mathcal{I}}, j \in \boldsymbol{\mathcal{J}}, t \in \boldsymbol{\mathcal{T}} \end{array} \right\} \quad (5.4)$$

With the above constructs, we can reformulate (5.2) as a large scale mixed integer linear program, by noting the finiteness of the set $\mathcal{V}(\Gamma)$. This is formally presented in the following result.

Proposition 4. *The WtE expansion optimization problem (5.2) is equivalent to the following mixed integer program:*

$$\max_{\boldsymbol{x}} \quad \sum_{t \in \boldsymbol{\mathcal{T}}} \left[-\beta^{t-1} \boldsymbol{c}^\top \boldsymbol{x}_t \right] + \gamma \quad (5.5)$$

$$\text{s.t.} \quad \gamma \in \mathcal{K}(\boldsymbol{x}, \boldsymbol{\xi}), \ \boldsymbol{\xi} \in \mathcal{V}(\Gamma) \quad (5.6)$$

$$x \in \mathcal{X} \quad (5.7)$$

where $\mathcal{K}(\boldsymbol{x}, \boldsymbol{\xi})$ is the set defined by (5.4).

Proof. The technical proof details can be found in Appendix A.1. □

The proposed reformulation (5.5)-(5.7) is termed as the *complete master problem*, which is generally of high dimensions, since it is based on the enumeration of $\mathcal{V}(\Gamma)$. On the other hand, because the elements of $\mathcal{V}(\Gamma)$ can be efficiently described in our problem, the explicit formulation in (5.5)-(5.7) can be easily obtained. This is useful if computational resources permit the direct solution of (5.5)-(5.7). Note that in general, this may not be possible when there is no efficient description of $\mathcal{V}(\Gamma)$.

Denote ω as the index for the elements of $\mathcal{V}(\Gamma)$, that is $\omega = 1, 2, \cdots, |\mathcal{V}(\Gamma)|$. By relaxing the set of constraint $\gamma \in \mathcal{K}(\boldsymbol{x}, \boldsymbol{\xi})$, $\boldsymbol{\xi} \in \mathcal{V}(\Gamma)$ into the case with $\omega = 1, \cdots, k, k \leq |\mathcal{V}(\Gamma)|$, we arrive at the following *relaxed master problem* formulation:

$$\begin{aligned} \mathrm{UB}(k) := \max_{\boldsymbol{x}} \quad & \sum_{t \in \boldsymbol{\mathcal{T}}} \left[-\beta^{t-1} \boldsymbol{c}^\top \boldsymbol{x}_t \right] + \gamma && (5.8) \\ \text{s.t.} \quad & \gamma \in \mathcal{K}(\boldsymbol{x}, \boldsymbol{\xi}^\omega), \ \omega = 1, \cdots, k && (5.9) \\ & x \in \mathcal{X} && (5.10) \end{aligned}$$

where $k \leq |\mathcal{V}(\Gamma)|$. Clearly, the relaxation (5.8)-(5.10) can be significantly smaller in size compared to (5.5)-(5.7), and provides an upper bound of the problem (5.2). Also, the upper bound improves as more cuts $\mathcal{K}(\boldsymbol{x}, \boldsymbol{\xi}^\omega)$ are added into the model. On the other hand, using the solution $\hat{\boldsymbol{x}}$ from (5.8)-(5.10), we can solve the following problem to obtain a lower-bound to (5.2):

$$\mathrm{LB}(\hat{\boldsymbol{x}}) := \mathcal{Z}_{\mathrm{NPV}}(\hat{\boldsymbol{x}}) = \max_{(\boldsymbol{y}, \boldsymbol{z}) \in \mathcal{Y}(\hat{\boldsymbol{x}}, \boldsymbol{\xi})} \phi_{\mathrm{NPV}}(\hat{\boldsymbol{x}}, \boldsymbol{y}, \boldsymbol{z}) \qquad (5.11)$$

where $\boldsymbol{\xi} \in \mathcal{V}(\Gamma)$. By Proposition 2 this can be solved as the mixed integer linear program (4.10)-(4.23). The optimal solution is achieved when $\mathrm{LB}(\hat{\boldsymbol{x}}) = \mathrm{UB}(k)$. Note that because $\mathcal{V}(\Gamma)$ is finite, the algorithm is guaranteed to converge in a finite number of iterations.

In practice, the choice of the cut $\mathcal{K}(\boldsymbol{x}, \boldsymbol{\xi}^\omega)$ added can play an important role in improving the computational efficiency. An observation is that when we input the $\boldsymbol{x}$ obtained from (5.8)-(5.10) in (5.11), by Proposition 3, we can also recover an instance of $\boldsymbol{\xi}^{\dagger} \in \mathcal{V}(\Gamma)$ by using the solution $(\bar{\boldsymbol{\pi}}^{+}, \bar{\boldsymbol{\pi}}^{-})$:

$$\xi^{\dagger}_{it}(\bar{\boldsymbol{\pi}}^{+}, \bar{\boldsymbol{\pi}}^{-}) := \widehat{\xi}_{i0} + \sum_{\tau=1}^{t} F_{i\tau} + \left[\sum_{\tau=1}^{t} \left(\bar{\pi}^{+}_{i\tau} - \bar{\pi}^{-}_{i\tau} \right) S_{i\tau} \right], \; i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}. \quad (5.12)$$

Since the obtained $\boldsymbol{\xi}^{\dagger}(\bar{\boldsymbol{\pi}}^{+}, \bar{\boldsymbol{\pi}}^{-})$ is the extreme value forecast yielding the guaranteed NPV corresponding to the most recent expansion solution $\boldsymbol{x}$ in the cutting plane iterations, it is at least heuristically better (more effective) than a randomly selected $\boldsymbol{\xi}^{\omega} \in \mathcal{V}(\Gamma)$. Hence, $\boldsymbol{\xi}^{\dagger}(\bar{\boldsymbol{\pi}}^{+}, \bar{\boldsymbol{\pi}}^{-})$ can be used to construct the new cut $\mathcal{K}(\boldsymbol{x}, \boldsymbol{\xi}^{\dagger}(\bar{\boldsymbol{\pi}}^{+}, \bar{\boldsymbol{\pi}}^{-}))$. The following algorithm outlines the cutting plane procedure:

Algorithm 1 Cutting plane algorithm for solving WtE expansion optimization problem (5.2)

Set ϵ as the error tolerance, Take some $\boldsymbol{\xi}^{\omega} \in \mathcal{V}(\Gamma)$, $\mathrm{UB} \leftarrow \infty, \mathrm{LB} \leftarrow -\infty$

while $\frac{\mathrm{UB}-\mathrm{LB}}{\mathrm{LB}} \times 100\% \geq \epsilon$ **do**

Solve (5.8)-(5.10) and obtain optimal objective value UB_N and solution $\boldsymbol{x}_N$, where N is the iteration number.

$\mathrm{UB} \leftarrow \mathrm{UB}_N$

Set $\boldsymbol{x}_{LB} = \boldsymbol{x}_N$, solve (4.10)-(4.23), and obtain optimal objective value LB_N and solution $(\bar{\boldsymbol{\pi}}^{+}, \bar{\boldsymbol{\pi}}^{-})$. Recover $\boldsymbol{\xi}^{\dagger}(\bar{\boldsymbol{\pi}}^{+}, \bar{\boldsymbol{\pi}}^{-})$ via (5.12), and form a new cut $\mathcal{K}(\boldsymbol{x}, \boldsymbol{\xi}^{\dagger}(\bar{\boldsymbol{\pi}}^{+}, \bar{\boldsymbol{\pi}}^{-}))$.

$\mathrm{LB} \leftarrow \max\{\mathrm{LB}, \mathrm{LB}_N\}$

end while

return $\boldsymbol{x}_{LB}$

5.2 A heuristic using constant proportion waste distribution rule

Although the cutting plane approach in the previous section can solve expansion problem (5.2) to optimality, in practice the computational effort required may still be formidable for large problem sizes. In this section we propose a heuristic procedure to obtain reasonably good (though not necessarily optimal) solutions that can be more computationally attractive. The concept of the approach is simply to restrict the set of feasible operational actions $\boldsymbol{y}$ and $\boldsymbol{z}$ in (5.2) to a smaller and more manageable subset.

In the following, we define p_{ijt} as the proportion of waste generated from zone i, period t distributed to WtE site j, and q_{it} is the proportion of excess waste from zone i in period t. Denote also $(\boldsymbol{p}, \boldsymbol{q})$ as the vector of all these proportion parameters. Clearly we have $\sum_{j\in\boldsymbol{\mathcal{J}}} p_{ijt} + q_{it} = 1,\ i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}$. For a given $(\boldsymbol{p}, \boldsymbol{q})$, and a given waste generation level $\boldsymbol{\xi}$, the waste distribution variables $\boldsymbol{y}$ and $\boldsymbol{z}$ are then evaluated as:

$$\boldsymbol{\Lambda}(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{\xi}) := \Big\{(\boldsymbol{y}, \boldsymbol{z}) \mid y_{ijt} = p_{ijt}\xi_{it},\ z_{it} = q_{it}\xi_{it}, t \in \boldsymbol{\mathcal{T}}, i \in \boldsymbol{\mathcal{I}}, j \in \boldsymbol{\mathcal{J}}\Big\}. \tag{5.13}$$

In the proposed heuristic, $(\boldsymbol{p}, \boldsymbol{q})$ are treated as control parameters, whose values are optimized together with the expansion solution $\boldsymbol{x}$, and waste distribution is always restricted to $\boldsymbol{\Lambda}(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{\xi})$. Under the above assumption, problem (5.2) can be formulated as the following:

$$\max_{\boldsymbol{x},\boldsymbol{p},\boldsymbol{q}} \quad \min_{\boldsymbol{\xi}\in\mathcal{U}^{\Gamma}} \max_{(\boldsymbol{y},\boldsymbol{z})\in\mathcal{Y}(\boldsymbol{x},\boldsymbol{\xi})\cap\boldsymbol{\Lambda}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{\xi})} \phi(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \tag{5.14}$$

$$\text{s.t.} \quad \sum_{j\in\boldsymbol{\mathcal{J}}} p_{ijt} + q_{it} = 1,\ i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}} \tag{5.15}$$

$$p_{ijt}, q_{it} \geq 0,\ i \in \boldsymbol{\mathcal{I}}, j \in \boldsymbol{\mathcal{J}}, t \in \boldsymbol{\mathcal{T}} \tag{5.16}$$

$$\boldsymbol{x} \in \mathcal{X}, \tag{5.17}$$

where $\mathcal{Y}(\boldsymbol{x}, \boldsymbol{\xi})$ given in (4.8) is the feasible set of $(\boldsymbol{y}, \boldsymbol{z})$ given $\boldsymbol{\xi}$.

Although restricting $(\boldsymbol{y}, \boldsymbol{z})$ using the constant proportion distribution rule may not produce the optimal solution to (5.2) in general, a key advantage is that the model can be solved efficiently. In particular we will show that the problem can be re-formulated as a linear mixed integer program of moderate dimensions, which can be handled directly by off-the-shelf solvers. This result is stated in the following proposition.

Proposition 5. *The expansion optimization model with proportional waste distribution rules (5.14)-(5.17) is equivalent to the following mixed integer linear program:*

$$\max_{\boldsymbol{x},\boldsymbol{p},\boldsymbol{q}} \quad \sum_{i\in\boldsymbol{\mathcal{I}}}\sum_{t\in\boldsymbol{\mathcal{T}}}\left[\widehat{\xi}_{i0}+\sum_{\tau=1}^{t}F_{i\tau}\right]d_{it}^{o}+\sum_{t\in\boldsymbol{\mathcal{T}}}\left[\Gamma_t^{\mathrm{Z}}b_t^o-\beta^{t-1}\boldsymbol{c}^\top\boldsymbol{x}_t\right]$$

$$+\sum_{i\in\boldsymbol{\mathcal{I}}}\Gamma_i^{\mathrm{T}}\varphi_i^o+\sum_{i\in\boldsymbol{\mathcal{I}}}\sum_{t\in\boldsymbol{\mathcal{T}}}\left[\psi_{it}^o-\nu_{it}^o\right] \tag{5.18}$$

$$\text{s.t.} \quad \beta^t\left[\sum_{j\in\boldsymbol{\mathcal{J}}}r_{ij}p_{ijt}-c_Dq_{it}\right]-d_{it}^o=0,\ \ i\in\boldsymbol{\mathcal{I}},t\in\boldsymbol{\mathcal{T}} \tag{5.19}$$

$$b_t^o+\varphi_i^o+\gamma_{it}^o+\varpi_{it}^o=0,\ \ i\in\boldsymbol{\mathcal{I}},t\in\boldsymbol{\mathcal{T}} \tag{5.20}$$

$$\psi_{it}^o+\nu_{it}^o+\varpi_{it}^o-\gamma_{it}^o-\sum_{\tau=t}^{|\boldsymbol{\mathcal{T}}|}d_{i\tau}^oS_{i\tau}=0,i\in\boldsymbol{\mathcal{I}},t\in\boldsymbol{\mathcal{T}} \tag{5.21}$$

$$\sum_{i\in\boldsymbol{\mathcal{I}}}\left[\widehat{\xi}_{i0}+\sum_{\tau=1}^{t}F_{i\tau}\right]p_{tij}+\sum_{\varsigma\in\boldsymbol{\mathcal{T}}}\Gamma_\varsigma^{\mathrm{Z}}b_\varsigma^{tj}+\sum_{i\in\boldsymbol{\mathcal{I}}}\Gamma_i^{\mathrm{T}}\varphi_i^{tj}$$

$$+\sum_{\varsigma\in\boldsymbol{\mathcal{T}}}\sum_{i\in\boldsymbol{\mathcal{I}}}\left[\psi_{i\varsigma}^{tj}-\nu_{i\varsigma}^{tj}\right]\le\sum_{\tau=0}^{t-1}x_{j\tau}s_j,\ \ j\in\boldsymbol{\mathcal{J}},t\in\boldsymbol{\mathcal{T}} \tag{5.22}$$

$$b_\varsigma^{tj}+\varphi_i^{tj}+\gamma_{i\varsigma}^{tj}+\varpi_{i\varsigma}^{tj}=0,\ \ i\in\boldsymbol{\mathcal{I}},\varsigma\in\boldsymbol{\mathcal{T}},j\in\boldsymbol{\mathcal{J}},t\in\boldsymbol{\mathcal{T}} \tag{5.23}$$

$$\psi_{i\varsigma}^{tj}+\nu_{i\varsigma}^{tj}+\varpi_{i\varsigma}^{tj}-\gamma_{i\varsigma}^{tj}=0,$$

$$i\in\boldsymbol{\mathcal{I}},j\in\boldsymbol{\mathcal{J}},t\in\boldsymbol{\mathcal{T}},\varsigma\in[t+1:|\boldsymbol{\mathcal{T}}|] \tag{5.24}$$

$$\psi_{i\varsigma}^{tj}+\nu_{i\varsigma}^{tj}+\varpi_{i\varsigma}^{tj}-\gamma_{i\varsigma}^{tj}-p_{tij}S_t(\widehat{\boldsymbol{\eta}}_i)=0,$$

$$i\in\boldsymbol{\mathcal{I}},j\in\boldsymbol{\mathcal{J}},t\in\boldsymbol{\mathcal{T}},\varsigma\in[1:t] \tag{5.25}$$

$$(\boldsymbol{x},\boldsymbol{p},\boldsymbol{q})\in\{(\boldsymbol{x},\boldsymbol{p},\boldsymbol{q})\mid(5.15)-(5.17)\} \tag{5.26}$$

$$\boldsymbol{d}^o\in\Re^{|\boldsymbol{\mathcal{I}}|\times|\boldsymbol{\mathcal{T}}|},\boldsymbol{b}^o\in\Re_-^{|\boldsymbol{\mathcal{T}}|},\boldsymbol{\varphi}^o\in\Re_-^{|\boldsymbol{\mathcal{I}}|},$$

$$\boldsymbol{\gamma}^o, \boldsymbol{\varpi}^o, \boldsymbol{\nu}^o \in \Re_+^{|\boldsymbol{\mathcal{I}}|\times|\boldsymbol{\mathcal{T}}|}, \boldsymbol{\psi}^o \in \Re_-^{|\boldsymbol{\mathcal{I}}|\times|\boldsymbol{\mathcal{T}}|} \tag{5.27}$$

$$\boldsymbol{b}^{tj} \in \Re_+^{|\boldsymbol{\mathcal{T}}|}, \boldsymbol{\varphi}^{tj} \in \Re_+^{|\boldsymbol{\mathcal{I}}|},$$

$$\boldsymbol{\gamma}^{tj}, \boldsymbol{\varpi}^{tj}, \boldsymbol{\nu}^{tj} \in \Re_-^{|\boldsymbol{\mathcal{I}}|\times|\boldsymbol{\mathcal{T}}|}, \boldsymbol{\psi}^{tj} \in \Re_+^{|\boldsymbol{\mathcal{I}}|\times|\boldsymbol{\mathcal{T}}|}, j \in \boldsymbol{\mathcal{J}}, t \in \boldsymbol{\mathcal{T}}, \tag{5.28}$$

where variables in (5.27) and (5.28) are auxiliary decision variables and $S_t(\widehat{\boldsymbol{\eta}}_i)$ *is the forecast error as defined in Section 4.1.*

Proof. The technical proof details can be found in Appendix A.1. □

We remark that the optimal objective function value (5.18) always provides a safe lower bound for that in problem (5.2). Hence, the solution in (5.14)-(5.17) can be used as a good starting point for the cutting plane algorithm described in the previous section. Also, because it is not necessary to actually implement the constant proportion rule $(\boldsymbol{p}, \boldsymbol{q})$ in practice, the lower bound can be further improved by using the expansion solution $\boldsymbol{x}$ achieved in (5.14)-(5.17) to evaluate the guaranteed NPV by solving the derived mixed integer model (4.10)-(4.23).

Chapter 6

Computational Study

In this chapter, we perform computational studies of the WtE expansion evaluation and optimization models. We assume a planning horizon of 8 time periods, where each period consists of a time bucket of two years. These assumptions are consistent with guidelines from the International Solid Waste Association [21] for the long term waste management planning, allowing for a reasonable time frame for planning changes. In our case study, we consider 10 residential zones and 20 potential WtE system sites, and the waste generation forecasting models are based on the 10 years' local waste of the city state Singapore.

We apply an exponential weighted moving average forecast model with mean absolute errors calibrated from the available data to construct the prediction set model $\mathcal{U}^{\Gamma}$ (see Chapter 4.1). The computational study consists of three parts: (i) An analysis of the extreme value forecasts for the given WtE expansion scenario (Section 6.1). (ii) Analysis of expansion solutions based on optimizing guaranteed NPVs (Section 6.2). (iii) An out-of-sample performance analysis of expansion solutions and comparison with stochastic programming approaches (Section 6.3). All the models presented below are coded in MATLAB 2015a and solved by the MOSEK solver.

6.1 Expansion scenario analysis with extreme value forecasts

We assume a WtE expansion scenario $\boldsymbol{x}$ given in Table 6.1, and use the guaranteed NPV evaluation model (4.10)-(4.23) to identify a series of extreme value forecasts over periods for the future waste generation for different residential zones under different forecast error budget assumptions. These forecasts are useful in providing the following information: (i) the conditions of future waste growth that aversely impact WtE expansion scenario $\boldsymbol{x}$, and (ii) the effect of forecast error budget on these conditions.

Table 6.1: WtE expansion scenario assumed for waste forecast analysis

Period	Expansion scenario $\boldsymbol{x}$
t=1 (Years 1-2)	Sites 3, 5, 11, 14, 17–18
t=2 (Years 3-4)	Site 2
t=3 (Years 5-6)	–
t=4 (Years 7-8)	Site 10
t=5 (Years 9-10)	Site 1
t=6 (Years 11-12)	Site 4
t=7 (Years 13-14)	Site 20
t=8 (Years 15-16)	–

In the following, the forecast error budget parameters are set as $\Gamma_i^{\mathrm{T}} \equiv$

$\Gamma_\text{T}, \forall\ i \in \boldsymbol{\mathcal{I}}$ and $\Gamma_t^\text{Z} \equiv \Gamma_\text{Z}, \forall\ t \in \boldsymbol{\mathcal{T}}$, with $\Gamma_\text{Z} \leq 10, \Gamma_\text{T} \leq 8$. This simplifies our discussion, and yet captures the major features of our prediction set models. We also change the budget parameters $(\Gamma_\text{Z}, \Gamma_\text{T})$ from $(5, 4)$ to $(10, 8)$ to analyze the obtained extreme value forecasts. First, the guaranteed NPV evaluation model (4.10)-(4.23) is solved for the binary indicator variables π_{it}^+ and π_{it}^-. Recall that these indicators imply adjustments of the nominal waste growth forecast η_{it} in the positive and negative directions respectively, by the designated forecast error level. The extreme value forecasts $\xi_{it}^\dagger$ are then recovered by applying (4.24) in Proposition 3. For the purpose of illustration, we consider two residential zones, Zone 4 and Zone 8, that have low and high annual waste generations respectively. We present the guaranteed NPVs and the associated extreme value forecasts qualitatively using $(\boldsymbol{\pi}^+, \boldsymbol{\pi}^-)$ of two residential zones (Zone 4 and Zone 8) in Table 6.2.

An observation of the results is the following. First, when the forecast error budget is relatively low, the extreme value forecasts indicate that a lower-than-nominal forecast of the waste generation will impact the expansion scenario most negatively. This can be seen from the results in Table 6.2, for the cases when budget parameter varies from $(5, 4)$ to $(7, 5)$. That is, all the components of the $\boldsymbol{\pi}^+$ are zero, while selected components of $\boldsymbol{\pi}^-$ take on the value of one. Interestingly, when the error budget changes, the periods in which these lower-than-nominal forecasts occurs does not necessarily always remain the same. Generally, these extreme value forecasts are a consequence of the interaction between the expansion scenario and waste generation uncertainty, and typically cannot be obtained by simple inspection. When the forecast error budget increases above $(8, 6)$, the extreme value forecasts switch directions, and higher-than-nominal waste

Table 6.2: Guaranteed NPVs and directions $\boldsymbol{\pi}_i^+$ and $\boldsymbol{\pi}_i^-$ of the extreme value forecasts $\boldsymbol{\xi}_i^\dagger$ for Zones 4 and 8 under budget parameters (Γ_Z, Γ_T). Note that $\boldsymbol{\pi}_i^+ = (\pi_{i1}^+, \pi_{i2}^+, \pi_{i3}^+, \pi_{i4}^+, \pi_{i5}^+, \pi_{i6}^+, \pi_{i7}^+, \pi_{i8}^+)$ and $\boldsymbol{\pi}_i^- = (\pi_{i1}^-, \pi_{i2}^-, \pi_{i3}^-, \pi_{i4}^-, \pi_{i5}^-, \pi_{i6}^-, \pi_{i7}^-, \pi_{i8}^-)$ indicates the positive and negative forecast error adjustments in the growth components $\boldsymbol{\eta}_i = (\eta_{i1}, \eta_{i2}, \eta_{i3}, \eta_{i4}, \eta_{i5}, \eta_{i6}, \eta_{i7}, \eta_{i8})$, respectively. The Guaranteed NPVs are given for the all the residential zones together.

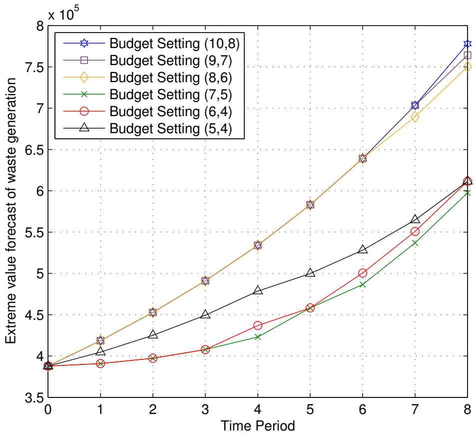
(Γ_Z, Γ_T)	Guaranteed NPV (Overall)	Forecast $\boldsymbol{\xi}_4^\dagger$ (Zone 4)	
		Indicator $\boldsymbol{\pi}_4^+$	Indicator $\boldsymbol{\pi}_4^-$
(10, 8)	1.6562×10^8	(1, 1, 1, 1, 1, 1, 1, 1)	(0, 0, 0, 0, 0, 0, 0, 0)
(9, 7)	1.6899×10^8	(1, 1, 1, 1, 1, 1, 1, 0)	(0, 0, 0, 0, 0, 0, 0, 0)
(8, 6)	1.7012×10^8	(1, 1, 1, 1, 1, 1, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0)
(7, 5)	1.7118×10^8	(0, 0, 0, 0, 0, 0, 0, 0)	(1, 1, 1, 1, 0, 1, 0, 0)
(6, 4)	1.7346×10^8	(0, 0, 0, 0, 0, 0, 0, 0)	(1, 1, 1, 0, 1, 0, 0, 0)
(5, 4)	1.7444×10^8	(0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 1, 1, 1, 1)
(Γ_Z, Γ_T)	Guaranteed NPV (Overall)	Forecast $\boldsymbol{\xi}_8^\dagger$ (Zone 8)	
		Indicator $\boldsymbol{\pi}_8^+$	Indicator $\boldsymbol{\pi}_8^-$
(10, 8)	1.6562×10^8	(1, 1, 1, 1, 1, 1, 1, 1)	(0, 0, 0, 0, 0, 0, 0, 0)
(9, 7)	1.6899×10^8	(0, 0, 0, 0, 0, 0, 1, 1)	(0, 0, 0, 0, 0, 0, 0, 0)
(8, 6)	1.7012×10^8	(0, 0, 0, 0, 0, 0, 1, 1)	(0, 0, 0, 0, 0, 0, 0, 0)
(7, 5)	1.7118×10^8	(0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 1, 1, 1, 1)
(6, 4)	1.7346×10^8	(0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 1, 1, 1, 1)
(5, 4)	1.7444×10^8	(0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 1, 1, 1)

generation will now be most critical. Under this condition, the components of the $\boldsymbol{\pi}^+$ take on values of one, while the components of $\boldsymbol{\pi}^-$ take on zero values. Operationally, this means that the disposal costs associated with the excess waste will most negatively impact the NPV of the project.

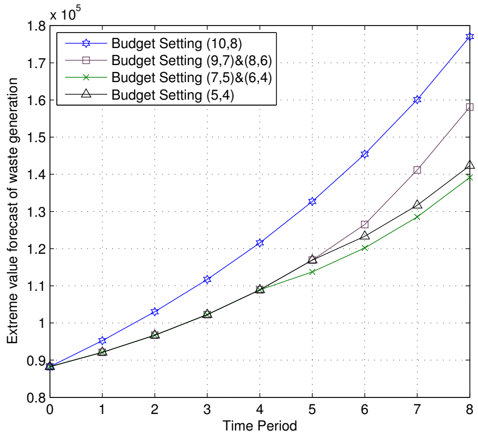
The extreme value forecasts of these two zones are also plotted in Figure 6.1-(a) and Figure 6.1-(b), respectively. The average of the extreme value forecasts across all different residential zones are plotted in Figure 6.1-(c). These plots essentially provide the users a simple visualization of a 'band' of extreme forecast levels that most negatively impact the expansion scenario under the various forecast error budget settings. When the budget is between $(5, 4)$ to $(7, 5)$, the lower-than-nominal forecast regime dominates, and the extreme value waste forecasts of waste generation decreases as the forecast error budget increases. Further increases in the forecast error budget bring the extreme value forecast to the higher-than-nominal regime. When the error budget continues to increase above $(8, 6)$, the extreme value waste forecast increases. These observations are consistent across all zones. Intuitively, over the range of forecast error budgets, waste generation trajectories that are within the band of extreme forecasts are desirable and can improve the actual NPV levels. This can provide useful information in the situation when the decision-maker can exert influence on the future waste generation, e.g. through waste management and disposal policies.

6.2 Analysis of WtE expansion solutions

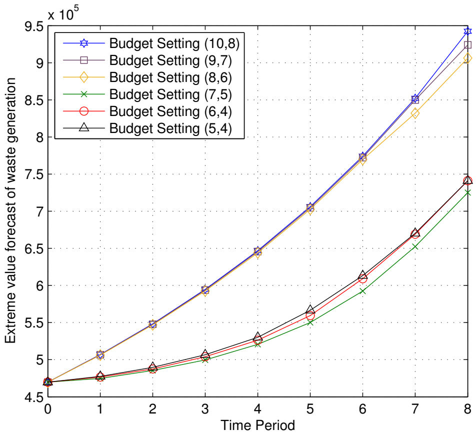
In this section we study the WtE expansion plans obtained from the optimal solution of (5.2) using the cutting plane algorithm in Section 5.1, and an approximate solution derived from the constant proportions waste distribution heuristic, based on (5.14)-(5.17) in Section 5.2. For convenience, in the rest of the computational studies we refer to this as the $(\boldsymbol{p}, \boldsymbol{q})$-heuristic, where $(\boldsymbol{p}, \boldsymbol{q})$ refer the waste distribution proportion control parameters introduced in (5.14)-(5.17). Table 6.3 reports the siting and corresponding



(a)



(b)



(c)

Figure 6.1: Extreme value forecasts of waste generation under different error budget settings: (a) Extreme value forecast for Zone 4 (low annual waste generation). (b) Extreme value forecast for Zone 8 (high annual waste generation). (c) The average of the extreme value forecasts across different residential zones.

capacity levels by the two planning solutions under different budget settings (Γ_Z, Γ_T). The accumulated capacity expansions are also plotted in Figure 6.2.

Some discussions are as follows. Generally, it can be observed from Table 6.3 and Figure 6.2, that as the forecast error budget decreases from (10,8) to (5,4), both WtE planning solutions decrease in the total capacity expansion and possibly also the number of sites opened. This reduction in

Table 6.3: WtE expansion solutions using exact method (*Cut. plane*) and constant proportions heuristic ((*$\boldsymbol{p}, \boldsymbol{q}$*)-*heuristic*) under different forecast error budgets (Γ_Z, Γ_T). Underlined values refer to capacity expansion levels in the respective periods.

WtE expansion solution (opened sites) with associated capacity volume

Solution Approach 1: *Cutting Plane Algorithm*

(Γ_Z, Γ_T)	(10,8)	(8,6)	(5,4)
$t = 1$	Sites 2, 3, 6, 11, 14, 17–18 5408830	Sites 2, 3, 6, 11, 14, 17–18 5408830	Sites 2, 3, 6, 11, 14, 17–18 5408830
$t = 2$	–	–	–
$t = 3$	Site 5 669650	Site 5 669650	Site 5 669650
$t = 4$	–	–	–
$t = 5$	Site 20 1511400	Site 20 1511400	Site 20 1511400
$t = 6$	–	–	–
$t = 7$	Site 8 928870	Site 1 720260	–
$t = 8$	–	–	Site 1 720260

Solution Approach 2: $(\boldsymbol{p}, \boldsymbol{q})$-*heuristic*

(Γ_Z, Γ_T)	(10,8)	(8,6)	(5,4)
$t = 1$	Sites 2, 3, 5, 11, 14, 17–18 5658200	Sites 2, 3, 5, 11, 14, 17–18 5658200	Sites 2, 3, 5, 11, 14, 17–18 5658200
$t = 2$	–	–	–
$t = 3$	Site 8 928870	Site 6 420280	Site 8 928870
$t = 4$	–	Site 1 720260	–
$t = 5$	Site 1 720260	Site 8 928870	Site 1 720260
$t = 6$	Site 20 1511400	Site 20 1511400	Site 20 1511400
$t = 7$	–	–	–
$t = 8$	Site 13 933860	–	–

capacity investments translates into direct quantifiable cost-savings. Such information is useful for decision-makers who may be able to exert influence on controlling the future waste generation variability. Next, it can be observed that the solution from the cutting plane method installs slightly less capacity in the initial period, compared to the heuristic solution, and may choose to execute larger expansions in subsequent periods. The total installed capacity is also lower than that achieved in the $(\boldsymbol{p}, \boldsymbol{q})$ heuristic. In most instances, the cutting plane solutions either utilize less periods for WtE expansion than the heuristic solution (e.g., four periods v.s. five periods under budget settings of (8,6) and (10,8)), or expand the capacity in latter stages (e.g., periods 1-3-5-7 v.s. periods 1-3-4-5-6 under budget (8,6) and periods 1-3-5-8 v.s. 1-3-5-6 under budget (5,4)). All these observations demonstrate the effectiveness of the exact solutions in reducing the capacity investment and fixed costs while adapting to the waste generation uncertainties.

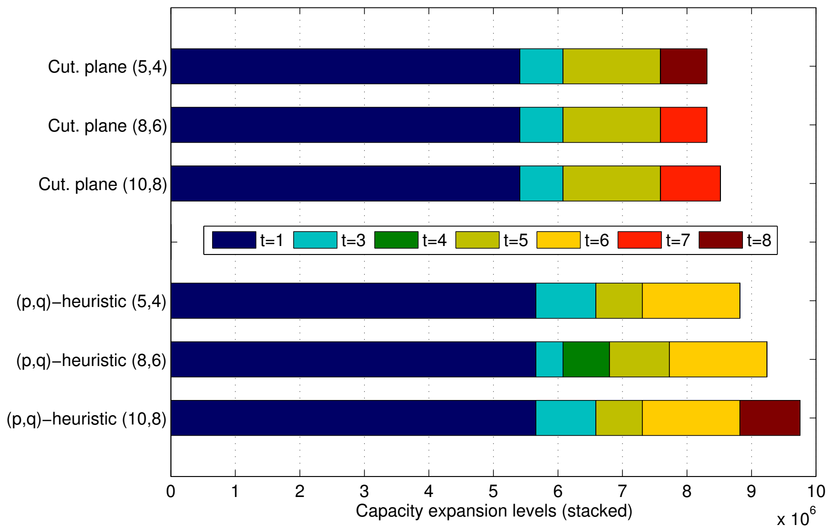


Figure 6.2: Capacity expansion profiles of the exact cutting plane solutions (top three plots) and $(\boldsymbol{p}, \boldsymbol{q})$-heuristic (bottom three plots) under different forecast error budget settings.

Finally, by using the $(\boldsymbol{p}, \boldsymbol{q})$-heuristic solutions as expansion scenarios in the NPV evaluation model (4.10)-(4.23), the guaranteed NPV levels of these solutions can be computed. These are tabulated together with the guaranteed NPV levels of the exact solutions in Table 6.4. It can be seen that the achieved NPV from the heuristic solutions are actually very close to the optimal guaranteed NPV levels, despite the fact that the heuristic solutions appear more conservative from the earlier observations. This gives reasonable justification and confidence for using the heuristic as an alternative to solving for the exact optimal expansions.

Table 6.4: Guaranteed NPVs of exact (*Cut. plane*) and constant proportions waste distribution heuristic solutions ($(\boldsymbol{p}, \boldsymbol{q})$-heuristic) under different error budget settings, where the 'Ratio of increased guaranteed NPV' is defined as the percentage of increased guaranteed NPV of exact solution to the heuristic solution out of the guaranteed NPV of former.

(Γ_Z, Γ_T)	*Cut. plane*	$(\boldsymbol{p},\boldsymbol{q})$*-heuristic*	Ratio of increase in guaranteed NPV
(10,8)	16.89×10^7	16.79×10^7	0.59%
(8,6)	17.21×10^7	16.97×10^7	1.39%
(5,4)	17.57×10^7	17.51×10^7	0.34%

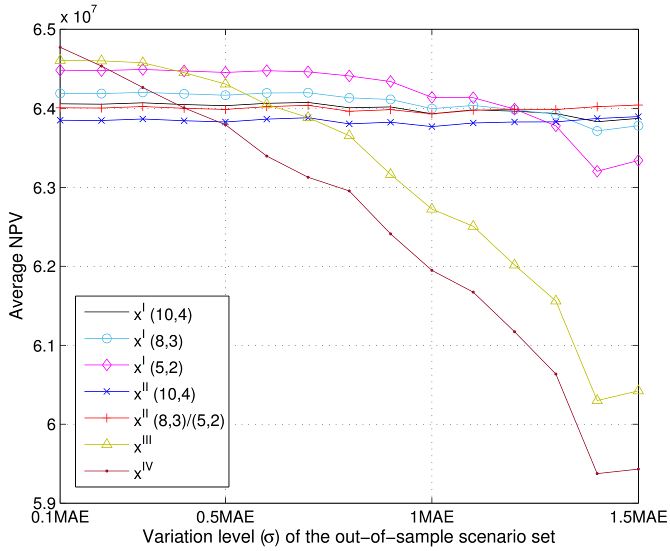
6.3 Out-of-sample performance analysis

We now study the out-of-sample performance of the WtE expansion solutions of our proposed guaranteed NPV optimization model solved using cutting plane algorithm and $(\boldsymbol{p}, \boldsymbol{q})$-heuristic. We compare the performance of our proposed model solutions to two other approaches: the expansion solution based on single point-value forecast, and a multi-stage stochastic programming model. The former is obtained by solving the deterministic model (3.3)–(3.7) with the nominal waste growth forecasts. The stochastic programming solution is an optimal policy obtained by using a scenario-tree approach (see Appendix A.3 for construction details) generated using randomly sampled outcomes from the same forecast prediction set.

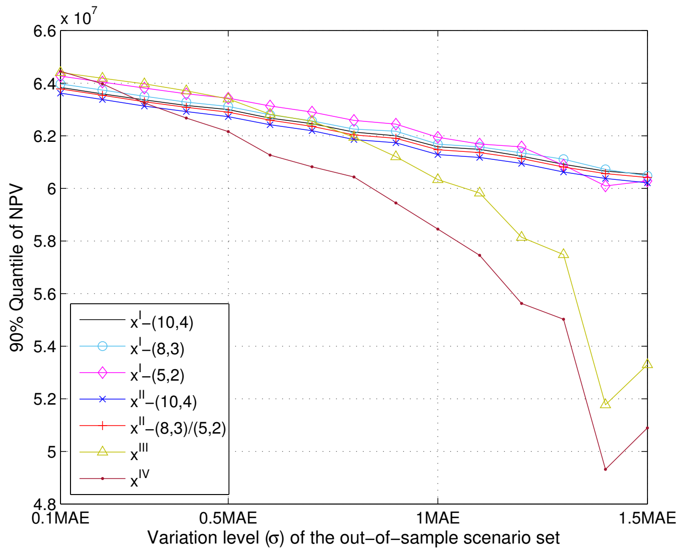
In order to solve the multi-stage stochastic programming model within a reasonable timeframe, we assume all problem instances of dimensions ($|\boldsymbol{\mathcal{I}}| = 10, |\boldsymbol{\mathcal{J}}| = 20, |\boldsymbol{\mathcal{T}}| = 4$). Under this setting, the stochastic programming model instances are solved in approximately five hours on our workstation (CPU i7-4510u 2GHZ, 8GB RAM). Beyond $\mathcal{T} = 4$ unfortunately the scenario tree model could not be solved even after twelve hours of runtime. For our proposed guaranteed NPV maximization model (solved using the cutting plane algorithm), the computations are completed within five iterations in all cases. The CPU time varies from one and up to two hours for all instances. Lastly, the CPU time for the $(\boldsymbol{p}, \boldsymbol{q})$-heuristic varies from ten to a maximum of forty minutes.

We designed a collection of waste generation test sets, each with random waste growths $\boldsymbol{\eta}$ with different levels of variation. Specifically, the out-of-sample random waste growth η_{it} for each zone i and time period t is assumed to be normally distributed $\eta_{it} \sim \mathbb{N}(F_{it}, \sigma^2)$, and we increase the variation level across the out-of-sample scenario sets by setting from $\sigma = 0.1\text{MAE}$ to $\sigma = 1.5\text{MAE}$, where MAE is the mean absolute error levels assumed in the forecast prediction sets. Each test set consists of 1000 randomly simulated waste generation sample paths. We remark that, the settings with small σ correspond to the situations when the forecasting model can accurately predict the future waste generation (only small variations around the nominal forecasts F_{it}), and the fluctuation becomes higher as σ increases. When $\sigma = 1.5\text{MAE}$, the generated waste growths fall into the ±MAE range with a probability around 50%, while about 50% of the outcomes fall outside the assumed prediction set. In the following results, we denote for convenience $\boldsymbol{x}^{\mathrm{I}}(\Gamma_Z, \Gamma_T)$ and $\boldsymbol{x}^{\mathrm{II}}(\Gamma_Z, \Gamma_T)$ as solutions of our proposed guaranteed NPV optimization model solved using cutting plane algorithm and $(\boldsymbol{p}, \boldsymbol{q})$-heuristic respectively, under the forecast error budgets (Γ_Z, Γ_T). Denote also $\boldsymbol{x}^{\mathrm{III}}$ as the stochastic programming solution (policy corresponding to the scenario tree), and $\boldsymbol{x}^{\mathrm{IV}}$ as the solution based on nominal value forecast.

Figure 6.3 plots the sample average and 90%-quantile NPV achieved by the different expansion solutions across increasing levels of out-of-sample variance. In general, the performance of all the solutions degrade as the out-of-sample variance increase. First consider the regime when the future waste generation can be well predicted, e.g. in the case when $\sigma = 0.1$MAE (here around 68% out-of-sample scenarios fall into ±0.1MAE region while 95% scenarios fall into ±0.2MAE region). Here it can be observed that the WtE expansion solution based on single-point value forecast ($\boldsymbol{x}^{\mathrm{IV}}$) performs the best in both average and 90% quantile NPV. This is not surprising since it turns out that the actual waste growths are very close to the nominal forecast. When $\sigma \leq 0.3$MAE (68% out-of-sample scenarios fall into ±0.3MAE region while 95% scenarios fall into ±0.6MAE region), the expansion policy from the stochastic programming model ($\boldsymbol{x}^{\mathrm{III}}$) achieves slightly better NPV on average compared to our proposed model solutions based on extreme value forecasts. However, as the waste generation continue to deviate from the forecasts, the expansion plan based on the point-value forecast quickly deteriorates in performance, and is the worst in both the average and 90% quantile NPV (i.e. when $\sigma \geq 0.5$MAE and $\sigma \geq 0.4$MAE, respectively). Similarly, as the variance increases, the stochastic programming solution



(a)



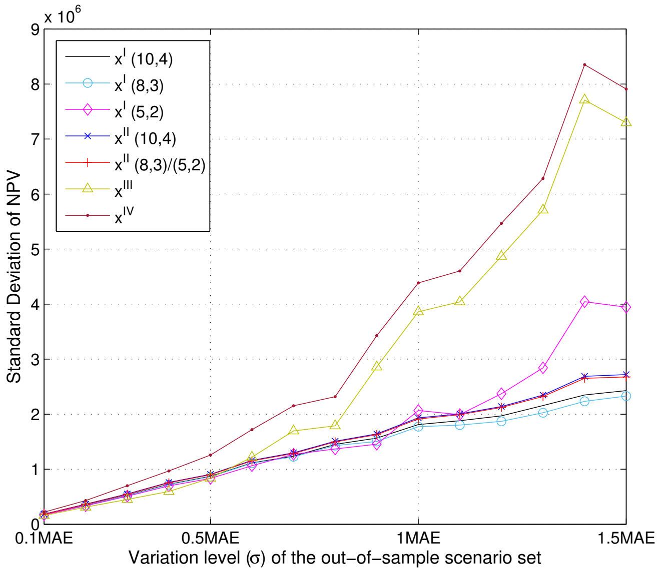
(b)

Figure 6.3: Expansion solution performance across increasing out-of-sample variation levels (σ) (a) Average NPV, (b) 90%-quantile NPV, which refers to the NPV level that is achieved by at least 90% of 1000 sample paths in each test set.

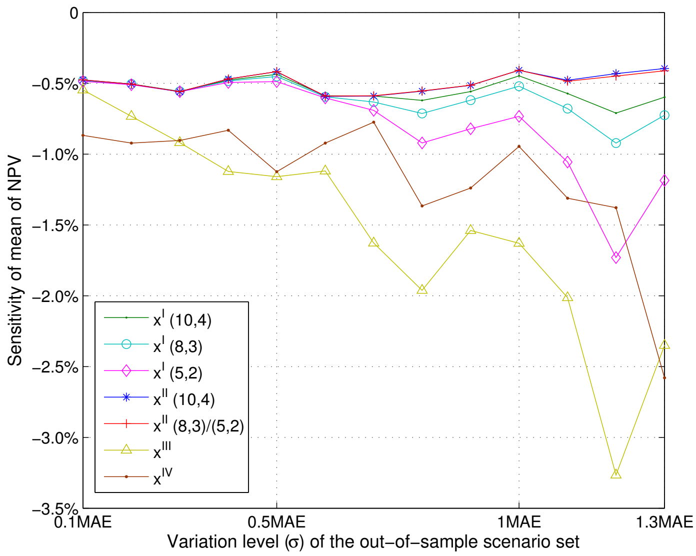
deteriorates in performance compared to our proposed models. For the regime with $\sigma \geq 0.8$MAE, our proposed model solutions ($\boldsymbol{x}^{\mathrm{I}}$ and $\boldsymbol{x}^{\mathrm{II}}$) dominate in performance for the average NPV level, and when $\sigma \geq 0.9$MAE, they also dominate in terms of the 90% NPV quantile. Now as for our proposed expansion model based on extreme value forecasts, the exact solutions $\boldsymbol{x}^{\mathrm{I}}$ outperform the solutions from the $(\boldsymbol{p}, \boldsymbol{q})$ heuristic $\boldsymbol{x}^{\mathrm{II}}$ (across all the forecast error budget settings) in average NPV when $\sigma \leq 1.1\mathrm{MAE}$ and in 90% NPV quantile when $\sigma \leq 1.3\mathrm{MAE}$. Interestingly, beyond this level of out-of-sample variation, i.e. when $\sigma \geq 1.4\mathrm{MAE}$, the $(\boldsymbol{p}, \boldsymbol{q})$-heuristic expansion solution outperforms all the other solutions.

Some managerial insights in above NPV performance comparison are as follows. First, the point value forecast model for expansion planning can only work well when the future waste generation is very close to the nominal forecast made, while the stochastic programming model is good when there are variations of waste generation accounted for in the assumed prediction set. However, when the actual variations fall outside the forecast prediction set, the stochastic programming solutions are highly unstable, and it is here where our proposed models based on extreme value forecasts dominate in performance. It can be observed from Figure 6.3 that this trade-off is indeed attractive in both the average and 90%-NPV quantile, since the loss in performance in the low variance regime is extremely small compared to the improvements achieved in the high variance regime. Finally, the performance of the proposed guaranteed NPV optimization models indicate that users need not be overly-conservative when calibrating the forecast prediction sets. It can be seen from the results that even when the assumed MAE is much lower than the out-of-sample variance, the NPV performance is remarkably stable, compared to the rest of the solutions. This implies that in reality, even accounting for a small part of the forecast errors in the expansion planning can lead to significant improvements.

Next, Figure 6.4 depicts the performance of the expansion solutions in



(a)



(b)

Figure 6.4: Comparison of stability in NPV performance: (a) Standard deviation of NPV, (b) Sensitivity of Average NPV (ANPV) per unit change (0.2 MAE) in the out-of-sample variation level, i.e. vertical ordinate plots values Sensitivity := $\frac{\text{ANPV}(\sigma+0.2\text{MAE})-\text{ANPV}(\sigma)}{\text{ANPV}(\sigma)} \times 100\%$ across increasing out-of-sample variance.

terms of the out-of-sample NPV standard deviation, and percentage change in average NPV to the increases of the waste growth variation level. Both performance measures describe the stability and sensitivity of the expansion solutions to the future waste growth uncertainties. The general conclusions are similar to that in the previous analysis. That is, when the out-of-sample variance is low, there is no significant difference across the various expansion solutions. When the out-of-sample variance increases, we see that both the point forecast model solution $\boldsymbol{x}^{\mathrm{IV}}$ and the stochastic programming model solution $\boldsymbol{x}^{\mathrm{III}}$ deteriorate significantly, and have both high NPV variances and sensitivities to changes in the variance assumptions. On the other hand, it can be seen that these performance measures remain very stable for our proposed models, even when the forecast error budget changes. Noting that statistics such as standard deviation are often related to measurements of economic risks, these results provide indication that our proposed models are suitable and consistent with the preference of risk-averse decision-makers.

Finally, in Figure 6.5, we look at the *'non-guaranteed NPV premiums'* characteristic of the various expansion solutions across increasing out-of-sample variance. As a surrogate for this characteristic, we consider the value of $\frac{(\mu_{\text{sample}} - \text{OBJ}^*)}{\text{OBJ}^*} \times 100\%$, where μ_{sample} refers to mean of the out-of-sample NPV and OBJ^* is the optimal objective function value obtained from the corresponding model. That is, the non-guaranteed NPV premium evaluates the extent that the out-of-sample objective value actually turns out better (if non-negative) or worse (if negative) than the model's optimal objective function value. Intuitively, large values on either side are not desirable, since it reflects over-pessimism or over-optimism of the model objective function value with respect to the true performance.

From the results plotted in Figure 6.5, all our proposed model solutions $\boldsymbol{x}^{\mathrm{I}}$ and $\boldsymbol{x}^{\mathrm{II}}$ score positive values on the non-guaranteed NPV premiums. On the other hand, the point value forecast solution $\boldsymbol{x}^{\mathrm{IV}}$ and stochastic

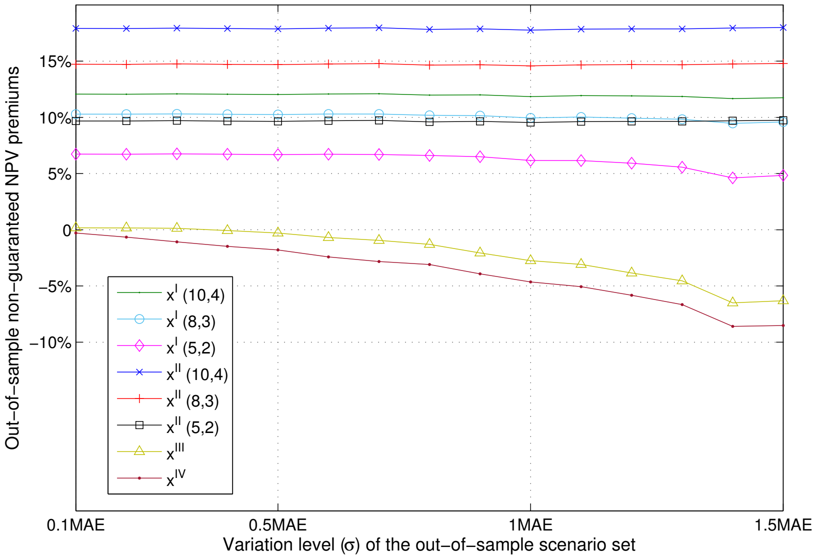


Figure 6.5: Comparison of non-guaranteed NPV premiums with increasing out-of-sample variation levels.

programming model solution $\boldsymbol{x}^{\text{III}}$ report non-positive values of the non-guaranteed premiums, implying that these models generally produce over optimistic performance projections. The key point to note, however, is that this 'optimism' tends to grow rapidly as the out-of-sample variance increases, which is undesirable since this implies that the optimal NPV projections made by these models could be very unreliable in practice. On the other hand, it can be seen that non-guaranteed NPV premiums accrued by our proposed model solutions are extremely stable across changing waste generation variances.

Chapter 7

Conclusion

This study investigates a WtE expansion planning problem with uncertain future waste generations. A prediction set model is proposed to deal with the waste generation uncertainty, which incorporates a set of time series forecasts with waste generation data information and forecasting error budget settings. Based on the prediction set, a WtE expansion scenario evaluation model is developed which computes a guaranteed NPV level and an extreme value forecast trajectory for the given WtE expansion scenario under a user-specified forecast error budget level. The evaluation model can be transformed and solved as a single instance of mixed integer linear program. Based on the evaluation model, a WtE expansion optimization model is developed which maximizes the guaranteed NPV by searching over the set of all feasible expansion scenarios, and can be handled exactly by solving a finite number of linear mixed integer programs with cutting plane iterations. Also, a constant proportion distribution rule heuristic model is proposed which can generate a safe approximation (lower bound) for the optimal guaranteed NPV and is more computationally attractive (a moderate size mixed integer program).

Our computational study demonstrates: (i) the extreme value forecasts recovered by the evaluation model can provide useful information which helps the decision maker better understand the adverse conditions of the future waste generation to the given WtE expansion plan under different predetermined forecast error levels. In particular, the waste generation 'band' constructed by extreme value forecast trajectories with different error levels can help the decision maker to utilize more effectively the waste management tools (e.g., waste collection contracts) or polices so as to improve the achievable NPV level of the WtE project. (ii) Our proposed WtE expansion plans obtained under different forecast error levels provide more options (hence higher flexibility) for the decision maker to make the trade-offs between the NPV performance and the uncertainty aversion, which could be more acceptable by practitioners with different preferences. (iii) The WtE expansion solutions produced by our developed models exhibit good stability compared to other approaches such as stochastic programming, and the NPV performances are very attractive, especially when the variation level of future waste generation is high.

It is clear that in the area of robust optimisation, there is still much to be done. In our paper, we have done a case-study of a specific scenario and have shown that our proposed method is able to obtain good results. It is expected that in a real life scenario, the results might not be as optimal, but should still show a marked improvement over the current methods used. An advantage of our paper is that the prediction set defined is generalised in nature and can thus be representative of a large portion of the constraints in reality. We are hence confident that the results captured by our case study will remain valid in reality.

However, we do acknowledge that our approach would be more suited for high-level planning and does not include all the uncertainty that comes from the operation of the WtE plant. In particular, we have taken both the operating cost and the profit to be constants in our model, which is not representative of the reality, as both of these will fluctuate with a number of other factors, including manpower cost and the cost of electricity. In addition, several other factors such as the environmental awareness of the society and the green policies put forth by the government are also capable of largely impacting the profitability of a WtE business. Unfortunately, incorporating such variables would over-complicate the model and reduce its utility. For that reason, we see our model being used more in the initial stages of planning where there is a need to evaluate of the business sustainability of carrying out a WtE business.

More interestingly, we foresee that one is able to easily extend our model to systems outside of the WtE industry. For example, many industries rely on the same supply and demand mechanism, whereby both supply and demand could be uncertain and decisions must be made based on their predictions. While the equations may differ depending on the scenario, using the same approach proposed by this paper, we should be able to arrive at a problem that is capable of providing the optimal solution to the problem at hand. However, the complexity of this would depend on the constraints considered in the system. Where appropriate, assumptions can be put into place to simplify the problem so as to get an approximately optimal solution.

For future work, more research and development can be devoted to improve the solution efficiency of the guaranteed NPV optimization problem. In the current work, a basic cutting plane algorithm is implemented, primarily for the purpose of using its solutions as benchmarks. More sophisticated extensions of the constant proportions waste distribution heuristic can also be explored. Another future research challenge is to incorporate uncertainty in energy prices, which can impact the cash inflows of the WtE project significantly through the sales of its recovered energy products. Given the historical observations of these energy prices, our proposed forecast prediction set models can be applied and calibrated.

Appendix A

Appendix

A.1 Technical Proofs

Proof of Proposition 1.

Proof. First of all, given expansion $\boldsymbol{x}$ and waste supply forecast $\boldsymbol{\xi} \in \mathcal{U}^{\Gamma}$, by the structure of our WtE operations problem in (3.3)-(3.5), we can have

$$\max_{(\boldsymbol{y},\boldsymbol{z})\in\mathcal{Y}(\boldsymbol{x},\boldsymbol{\xi})} \phi(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) = \sum_{t\in\boldsymbol{\mathcal{T}}} \Upsilon_t(\boldsymbol{x}_{[t]},\boldsymbol{\xi}_t).$$

We now claim that

$$\begin{aligned}\mathcal{Z}_{\mathrm{NPV}}(\boldsymbol{x}) = \min_{\boldsymbol{\xi}_1\in\mathcal{U}_1^{\Gamma}(\widehat{\boldsymbol{\xi}}_0)} \min_{\boldsymbol{\xi}_2\in\mathcal{U}_2^{\Gamma}(\boldsymbol{\xi}_{[1]})} \cdots \min_{\boldsymbol{\xi}_T\in\mathcal{U}_T^{\Gamma}(\boldsymbol{\xi}_{[T-1]})} \Big[\Upsilon_1(\boldsymbol{x}_{[1]},\boldsymbol{\xi}_1)+ \\ \Upsilon_2(\boldsymbol{x}_{[2]},\boldsymbol{\xi}_2)+\cdots+\Upsilon_T(\boldsymbol{x}_{[T]},\boldsymbol{\xi}_T)\Big].\end{aligned} \tag{A.1}$$

Note that given $\boldsymbol{x}$ and any $\boldsymbol{\xi} = (\boldsymbol{\xi}_1,\boldsymbol{\xi}_2,\cdots,\boldsymbol{\xi}_{|\boldsymbol{\mathcal{T}}|})$,

$$\sum_{t\in\boldsymbol{\mathcal{T}}} \Upsilon_t(\boldsymbol{x}_{[t]},\boldsymbol{\xi}_t)$$

solves $|\boldsymbol{\mathcal{T}}|$ independent sub-problems. Therefore, for the given path $\boldsymbol{\xi}_{[T-1]}$,

$$
\begin{aligned}
& \min_{\boldsymbol{\xi}_T \in \mathcal{U}_T^\Gamma(\boldsymbol{\xi}_{[T-1]})} \Big[\Upsilon_1(\boldsymbol{x}_{[1]}, \boldsymbol{\xi}_1) + \Upsilon_2(\boldsymbol{x}_{[2]}, \boldsymbol{\xi}_2) + \cdots + \Upsilon_T(\boldsymbol{x}_{[T]}, \boldsymbol{\xi}_T) \Big] \\
= \; & \Upsilon_1(\boldsymbol{x}_{[1]}, \boldsymbol{\xi}_1) + \Upsilon_2(\boldsymbol{x}_{[2]}, \boldsymbol{\xi}_2) + \cdots + \min_{\boldsymbol{\xi}_T \in \mathcal{U}_T^\Gamma(\boldsymbol{\xi}_{[T-1]})} \Upsilon_T(\boldsymbol{x}_{[T]}, \boldsymbol{\xi}_T).
\end{aligned}
$$

Furthermore, given path $\boldsymbol{\xi}_{[T-2]}$, we can have

$$
\begin{aligned}
& \min_{\boldsymbol{\xi}_{T-1} \in \mathcal{U}_{T-1}^\Gamma(\boldsymbol{\xi}_{[T-2]})} \min_{\boldsymbol{\xi}_T \in \mathcal{U}_T^\Gamma(\boldsymbol{\xi}_{[T-1]})} \Big[\Upsilon_1(\boldsymbol{x}_{[1]}, \boldsymbol{\xi}_1) + \\
& \Upsilon_2(\boldsymbol{x}_{[2]}, \boldsymbol{\xi}_2) + \cdots + \Upsilon_T(\boldsymbol{x}_{[T]}, \boldsymbol{\xi}_T) \Big] \\
= \; & \min_{\boldsymbol{\xi}_{T-1} \in \mathcal{U}_{T-1}^\Gamma(\boldsymbol{\xi}_{[T-2]})} \Bigg[\Upsilon_1(\boldsymbol{x}_{[0]}, \boldsymbol{\xi}_1) + \Upsilon_2(\boldsymbol{x}_{[1]}, \boldsymbol{\xi}_2) + \cdots \\
& + \min_{\boldsymbol{\xi}_T \in \mathcal{U}_T^\Gamma(\boldsymbol{\xi}_{[T-1]})} \Upsilon_T(\boldsymbol{x}_{[T-1]}, \boldsymbol{\xi}_T) \Bigg] \\
= \; & \Upsilon_1(\boldsymbol{x}_{[1]}, \boldsymbol{\xi}_1) + \cdots + \min_{\boldsymbol{\xi}_{T-1} \in \mathcal{U}_{T-1}^\Gamma(\boldsymbol{\xi}_{[T-2]})} \Bigg[\Upsilon_{T-1}(\boldsymbol{x}_{[T-1]}, \boldsymbol{\xi}_{T-1}) + \\
& \Bigg[\min_{\boldsymbol{\xi}_T \in \mathcal{U}_T^\Gamma(\boldsymbol{\xi}_{[T-1]})} \Upsilon_T(\boldsymbol{x}_{[T]}, \boldsymbol{\xi}_T) \Bigg] \Bigg].
\end{aligned}
$$

If we repeat the above process recursively, the equivalence of (A.1) can be achieved.

The remaining of the proof is to show the equivalence between $\mathcal{U}^\Gamma$ and joint the path-dependent nested prediction set

$$
\Big\{ \boldsymbol{\xi} = \Big(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \cdots, \boldsymbol{\xi}_{|\boldsymbol{\mathcal{T}}|} \Big) : \boldsymbol{\xi}_1 \in \mathcal{U}_1^\Gamma(\widehat{\boldsymbol{\xi}}_0), \boldsymbol{\xi}_2 \in \mathcal{U}_2^\Gamma(\boldsymbol{\xi}_{[1]}), \cdots, \boldsymbol{\xi}_T \in \mathcal{U}_T^\Gamma(\boldsymbol{\xi}_{[T-1]}) \Big\}.
$$

On the one hand, for each $\boldsymbol{\xi} = (\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \cdots, \boldsymbol{\xi}_{|\boldsymbol{\mathcal{T}}|})$ with $\boldsymbol{\xi}_t \in \mathcal{U}_t^\Gamma(\boldsymbol{\xi}_{[t-1]}), t \in \boldsymbol{\mathcal{T}}$, we have

$$
\xi_{i1} = \widehat{\xi}_{i0} + F_{i1} + \varrho_{i1} S_{i1}, \tag{A.2}
$$

$$\xi_{it} = \quad \xi_{it-1} + F_{it} + \varrho_{it} S_{it} = \widehat{\xi}_{i0} + \sum_{\tau=1}^{t} F_{i\tau} + \left[\sum_{\tau=1}^{t} \varrho_{i\tau} S_{i\tau}\right], i \in \boldsymbol{\mathcal{I}}, t \in [2 : |\boldsymbol{\mathcal{T}}|], \tag{A.3}$$

with some

$$\boldsymbol{\varrho} \in \left\{ \boldsymbol{\varrho} \in \Re^{|\boldsymbol{\mathcal{I}}| \times |\boldsymbol{\mathcal{T}}|} : \begin{array}{l} \sum_{i \in \boldsymbol{\mathcal{I}}} |\varrho_{it}| \leq \Gamma_t^{\mathrm{Z}}, t \in \boldsymbol{\mathcal{T}} \\ |\varrho_{it}| \leq \Gamma_i^{\mathrm{T}} - \chi_{\{t \geq 2\}} \left[\sum_{\tau=1}^{t-1} |\varrho_{i\tau}|\right], i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}} \\ \varrho_{it} \in [-1, 1], i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}} \end{array} \right\}. \tag{A.4}$$

Note that,

$$|\varrho_{it}| \leq \Gamma_i^{\mathrm{T}} - \chi_{\{t \geq 2\}} \left[\sum_{\tau=1}^{t-1} |\varrho_{i\tau}|\right], t \in \boldsymbol{\mathcal{T}} \Rightarrow \sum_{\tau=1}^{|\boldsymbol{\mathcal{T}}|} |\varrho_{i\tau}| \leq \Gamma_i^{\mathrm{T}}$$

for each $i \in \boldsymbol{\mathcal{I}}$. Therefore, $\boldsymbol{\xi} \in \mathcal{U}^{\Gamma}$.

Conversely, for each $\boldsymbol{\xi} = (\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \cdots, \boldsymbol{\xi}_{|\boldsymbol{\mathcal{T}}|}) \in \mathcal{U}^{\Gamma}$, we must have

$$\sum_{t=1}^{|\boldsymbol{\mathcal{T}}|} |\varrho_{it}| \leq \Gamma_i^{\mathrm{T}}, i \in \boldsymbol{\mathcal{I}},$$

which by the fact that each $|\varrho_{it}| \geq 0$ implies

$$|\varrho_{it}| \leq \Gamma_i^{\mathrm{T}} - \chi_{\{t \geq 2\}} \left[\sum_{\tau=1}^{t-1} |\varrho_{i\tau}|\right], t \in \boldsymbol{\mathcal{T}}, i \in \boldsymbol{\mathcal{I}}.$$

Hence, the condition in (A.2)-(A.4) holds, which implies that $\boldsymbol{\xi}_t \in \mathcal{U}_t^{\Gamma}(\boldsymbol{\xi}_{t-1}), t \in \boldsymbol{\mathcal{T}}$. The equivalence (4.9) has been verified. □

Proof of Proposition 2.

Proof. By Proposition 1, the formulation for $\mathcal{Z}_{\text{NPV}}(\boldsymbol{x})$ spells out as follows:

$$\min_{\boldsymbol{\xi}\in\mathcal{U}^{\Gamma}} \max_{\boldsymbol{y},\boldsymbol{z}} \quad \sum_{t\in\boldsymbol{\mathcal{T}}} \left[\beta^t \sum_{i\in\boldsymbol{\mathcal{I}}} \left(\sum_{j\in\boldsymbol{\mathcal{J}}} r_{ij} y_{ijt} - c_D z_{it} \right) - \beta^{t-1} \boldsymbol{c}^{\top} \boldsymbol{x}_t \right] \tag{A.5}$$

$$\text{s.t.} \quad \sum_{j\in\boldsymbol{\mathcal{J}}} y_{ijt} + z_{it} = \xi_{it}, \ i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}} \tag{A.6}$$

$$\sum_{i\in\boldsymbol{\mathcal{I}}} y_{ijt} \le \sum_{\tau=1}^{t} x_{j\tau} s_j, \ j \in \boldsymbol{\mathcal{J}}, t \in \boldsymbol{\mathcal{T}} \tag{A.7}$$

$$\boldsymbol{y}_t \in \Re_+^{|\boldsymbol{\mathcal{I}}|\times|\boldsymbol{\mathcal{J}}|}, \boldsymbol{z}_t \in \Re_+^{\times|\boldsymbol{\mathcal{I}}|}, t \in \boldsymbol{\mathcal{T}}. \tag{A.8}$$

Applying the strong duality of linear program theory to the inner maximization problem, we can arrive at the following dual form for problem (A.5)–(A.8):

$$\min_{\boldsymbol{\xi}\in\mathcal{U}^{\Gamma}} \min_{\boldsymbol{f},\boldsymbol{g}} \quad \sum_{t\in\boldsymbol{\mathcal{T}}} \left[\sum_{i\in\boldsymbol{\mathcal{I}}} \xi_{it} f_{it} + \sum_{j\in\boldsymbol{\mathcal{J}}} \sum_{\tau=1}^{t} x_{j\tau} s_j g_{jt} - \beta^{t-1} \boldsymbol{c}^{\top} \boldsymbol{x}_t \right] \tag{A.9}$$

$$\text{s.t.} \quad f_{it} + g_{jt} \ge \beta^t r_{ij}, \ i \in \boldsymbol{\mathcal{I}}, j \in \boldsymbol{\mathcal{J}}, t \in \boldsymbol{\mathcal{T}} \tag{A.10}$$

$$f_{it} \ge -\beta^t c_D, \ i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}} \tag{A.11}$$

$$\boldsymbol{f} \in \Re^{|\boldsymbol{\mathcal{I}}|\times|\boldsymbol{\mathcal{T}}|}, \quad \boldsymbol{g} \in \Re_+^{|\boldsymbol{\mathcal{J}}|\times|\boldsymbol{\mathcal{T}}|} \tag{A.12}$$

with $\boldsymbol{f}, \boldsymbol{g}$ being dual variables.

In order to derive the mixed integer program formulation for the $\mathcal{Z}_{\text{NPV}}(\boldsymbol{x})$, we have the following 3 steps to go:

STEP1. We first claim that the dual variables $f_{it}, i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}$ are bounded by a finite number M, that is

$$f_{it} \le M_1, \ \forall \ i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}. \tag{A.13}$$

To ease the presentation, let us denote by OBJ the optimal objective value of (A.9)–(A.12). Furthermore, we denote

$$M_0 := \sum_{t\in\mathcal{T}}\left[-\sum_{i\in\mathcal{I}}\left(\xi_{it}^{+}\beta^t c_D + \sum_{j\in\mathcal{J}}\sum_{\tau=1}^{t} x_{j\tau}s_j\beta^t(r_{ij}+c_D)\right) - \beta^{t-1}\boldsymbol{c}^\top\boldsymbol{x}_t + \Delta\right] > 0,$$

where Δ can be any suitable positive constant and ξ_{it}^{+} is a finite upper bound of ξ_{it}.

It is then easy to see that

$$\text{OBJ} < M_0.$$

On the other hand, we let

$$M_1 := \frac{M_0 + \sum_{t\in\mathcal{T}}\beta^{t-1}\boldsymbol{c}^\top\boldsymbol{x}_t + \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\xi_{it}^{+}\beta^t c_D}{\min_{i\in\mathcal{I},t\in\mathcal{T}}\xi_{it}^{-}}, \tag{A.14}$$

where $\xi_{it}^{-} > 0$ is a finite lower bound of ξ_{it}. It is easy to see that, given an $\boldsymbol{f} \in \Re^{|\mathcal{I}|\times|\mathcal{T}|}$, if there is any $(i^*, t^*) \in \mathcal{I}\times\mathcal{T}$ such that $f_{i^*t^*} > M_1$, we must have for any $\boldsymbol{\xi} \in \mathcal{U}^\Gamma$, the objective value

$$\begin{aligned}
&\sum_{t\in\mathcal{T}}\left[\sum_{i\in\mathcal{I}}\xi_{it}f_{it} + \sum_{j\in\mathcal{J}}\sum_{\tau=0}^{t-1}x_{j\tau}s_j g_{jt} - \beta^{t-1}\boldsymbol{c}^\top\boldsymbol{x}_t\right]\\
\geq\ & \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}}\xi_{it}f_{it} - \sum_{t\in\mathcal{T}}\beta^{t-1}\boldsymbol{c}^\top\boldsymbol{x}_t\\
\geq\ & M_0 + \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\xi_{it}^{+}\beta^t c_D + \sum_{t\in\mathcal{T}\setminus\{t^*\}}\sum_{i\in\mathcal{I}\setminus\{i^*\}}\xi_{it}f_{it}\\
\geq\ & M_0 + \sum_{i\in\mathcal{I}\setminus\{i^*\}}\sum_{t\in\mathcal{T}\setminus\{t^*\}}\xi_{it}\left(\beta^t c_D + f_{it}\right)\\
\geq\ & M_0,
\end{aligned}$$

by noting that $\xi_{it} > 0, g_{jt} \geq 0, f_{it} \geq -\beta^t c_D,\ i \in \mathcal{I}, t \in \mathcal{T}$. Therefore, such $\boldsymbol{f}$ cannot be an optimal solution. In other words, we can add the redundant constraints (A.13) into the problem (A.9)–(A.12) without affecting the optimality.

STEP2. Furthermore, we claim that the prediction set $\mathcal{U}^{\Gamma}$ of (4.3) is equivalent to the following form:

$$\mathcal{U}^{\Gamma} = \left\{ \boldsymbol{\xi} \in \Re^{|\boldsymbol{\mathcal{I}}| \times |\boldsymbol{\mathcal{T}}|} : \begin{array}{l} \xi_{it} = \widehat{\xi}_{i0} + \sum_{\tau=1}^{t} F_{i\tau} + \left[\sum_{\tau=1}^{t} \left(\varrho_{i\tau}^{+} - \varrho_{i\tau}^{-} \right) S_{i\tau} \right], \\ i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}} \\ \sum_{i \in \boldsymbol{\mathcal{I}}} (\varrho_{it}^{+} + \varrho_{it}^{-}) \leq \Gamma_{t}^{\mathrm{Z}}, t \in \boldsymbol{\mathcal{T}} \\ \sum_{t \in \boldsymbol{\mathcal{T}}} (\varrho_{it}^{+} + \varrho_{it}^{-}) \leq \Gamma_{i}^{\mathrm{T}}, i \in \boldsymbol{\mathcal{I}} \\ \varrho_{it}^{+}, \varrho_{it}^{-} \in [0,1], i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}} \end{array} \right\}.$$

To achieve this, we just need to show the equivalence between the following two sets:

$$\mathcal{P}_1 := \left\{ \boldsymbol{\varrho} \in \Re^{|\boldsymbol{\mathcal{I}}| \times |\boldsymbol{\mathcal{T}}|} : \begin{array}{l} \sum_{i \in \boldsymbol{\mathcal{I}}} |\varrho_{it}| \leq \Gamma_{t}^{\mathrm{Z}}, t \in \boldsymbol{\mathcal{T}} \\ \sum_{t \in \boldsymbol{\mathcal{T}}} |\varrho_{it}| \leq \Gamma_{i}^{\mathrm{T}}, i \in \boldsymbol{\mathcal{I}} \\ \varrho_{it} \in [-1,1], i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}} \end{array} \right\},$$

and

$$\mathcal{P}_2 := \left\{ (\boldsymbol{\varrho}^{+} - \boldsymbol{\varrho}^{-}) \in \Re^{|\boldsymbol{\mathcal{I}}| \times |\boldsymbol{\mathcal{T}}|} : \begin{array}{l} \sum_{i \in \boldsymbol{\mathcal{I}}} (\varrho_{it}^{+} + \varrho_{it}^{-}) \leq \Gamma_{t}^{\mathrm{Z}}, t \in \boldsymbol{\mathcal{T}} \\ \sum_{t \in \boldsymbol{\mathcal{T}}} (\varrho_{it}^{+} + \varrho_{it}^{-}) \leq \Gamma_{i}^{\mathrm{T}}, i \in \boldsymbol{\mathcal{I}} \\ \varrho_{it}^{+}, \varrho_{it}^{-} \in [0,1], i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}} \end{array} \right\}.$$

On the one hand, for any $\boldsymbol{\varrho} \in \mathcal{P}_1$, we can always find $\varrho_{it}^{+}, \varrho_{it}^{-} \in [0,1], i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}$ such that $\varrho_{it} = \varrho_{it}^{+} - \varrho_{it}^{-}, i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}$. Furthermore, if $|\varrho_{it}| = 0$, we then must have $\varrho_{it} = 0 = \varrho_{it}^{+} = \varrho_{it}^{-}$, which implies $\varrho_{it}^{+} + \varrho_{it}^{-} = 0$. Otherwise, for any $|\varrho_{it}| > 0$, we have $\varrho_{it} = \varrho_{it}^{+} - \varrho_{it}^{-} \neq 0$, and if both $\varrho_{it}^{+} > 0, \varrho_{it}^{-} > 0$, we then can always scale down ϱ_{it}^+ and ϱ_{it}^- by the same amount until one of the them is zero, while keeping $\varrho_{it}^+ - \varrho_{it}^-$ unchanged. This essentially results in ϱ_{it}^+ and ϱ_{it}^- being the positive and negative parts of ϱ_{it}, respectively, and $\varrho_{it}^+ + \varrho_{it}^- = |\varrho_{it}|$. Therefore,

$$\sum_{i\in\boldsymbol{\mathcal{I}}}(\varrho_{it}^+ + \varrho_{it}^-) = \sum_{i\in\boldsymbol{\mathcal{I}}}|\varrho_{it}| \leq \Gamma_t^{\mathrm{Z}},\ t \in \boldsymbol{\mathcal{T}},$$

and

$$\sum_{t\in\boldsymbol{\mathcal{T}}}(\varrho_{it}^+ + \varrho_{it}^-) = \sum_{t\in\boldsymbol{\mathcal{T}}}|\varrho_{it}| \leq \Gamma_i^{\mathrm{T}},\ i \in \boldsymbol{\mathcal{I}}.$$

In sum of above, we can always find $\boldsymbol{\varrho} = \boldsymbol{\varrho}^+ - \boldsymbol{\varrho}^- \in \mathcal{P}_2$.

On the other hand, for any $\boldsymbol{\varrho}^+ - \boldsymbol{\varrho}^- \in \mathcal{P}_2$, we have $\varrho_{it} := \varrho_{it}^+ - \varrho_{it}^- \in [-1, 1]$, and then $|\varrho_{it}| \leq \varrho_{it}^+ + \varrho_{it}^-$, for any $i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}$, which implies

$$\sum_{i\in\boldsymbol{\mathcal{I}}}|\varrho_{it}| \leq \sum_{i\in\boldsymbol{\mathcal{I}}}(\varrho_{it}^+ + \varrho_{it}^-) \leq \Gamma_t^{\mathrm{Z}},\ t \in \boldsymbol{\mathcal{T}},$$

and

$$\sum_{t\in\boldsymbol{\mathcal{T}}}|\varrho_{it}| \leq \sum_{t\in\boldsymbol{\mathcal{T}}}(\varrho_{it}^+ + \varrho_{it}^-) \leq \Gamma_i^{\mathrm{T}},\ i \in \boldsymbol{\mathcal{I}}.$$

Therefore, $\boldsymbol{\varrho}^+ - \boldsymbol{\varrho}^- = \boldsymbol{\varrho} \in \mathcal{P}_1$.

STEP3. We derive the mixed integer program formulation for problem (A.9)–(A.12). Given any feasible $\boldsymbol{f}$, we look at the following sub-problem of (A.9)-(A.12):

$$\min_{\boldsymbol{\xi}\in\mathcal{U}^{\Gamma}} \sum_{t\in\boldsymbol{\mathcal{T}}}\sum_{i\in\boldsymbol{\mathcal{I}}} \xi_{it} f_{it}$$

which by the discussion in STEP2 is equivalent to the following formulation:

$$\min_{\boldsymbol{\varrho}^+,\boldsymbol{\varrho}^-} \quad \sum_{t\in\boldsymbol{\mathcal{T}}}\sum_{i\in\boldsymbol{\mathcal{I}}} f_{it}\left[\widehat{\xi}_{i0} + \sum_{\tau=1}^{t} F_{i\tau}\right] + \left[\sum_{\tau=1}^{t}\left(\varrho_{i\tau}^+ f_{it} - \varrho_{i\tau}^- f_{it}\right) S_{i\tau}\right] \tag{A.15}$$

$$\text{s.t.} \quad \sum_{i\in\boldsymbol{\mathcal{I}}}(\varrho_{it}^+ + \varrho_{it}^-) \leq \Gamma_t^{\mathrm{Z}}, t \in \boldsymbol{\mathcal{T}} \tag{A.16}$$

$$\sum_{t\in\boldsymbol{\mathcal{T}}}(\varrho_{it}^{+}+\varrho_{it}^{-})\leq\Gamma_{i}^{\mathrm{T}},i\in\boldsymbol{\mathcal{I}} \tag{A.17}$$

$$\varrho_{it}^{+},\varrho_{it}^{-}\in[0,1],i\in\boldsymbol{\mathcal{I}},t\in\boldsymbol{\mathcal{T}}. \tag{A.18}$$

It is not difficult to see that the constraint system (A.16)-(A.18) can be abstracted into the following matrix form:

$$\left\{\begin{pmatrix}\boldsymbol{\varrho}^{+}\\ \boldsymbol{\varrho}^{-}\end{pmatrix}\in\Re^{2\times|\boldsymbol{\mathcal{I}}|\times|\boldsymbol{\mathcal{T}}|}:\begin{bmatrix}\boldsymbol{A}\\ \boldsymbol{I}\end{bmatrix}\begin{pmatrix}\boldsymbol{\varrho}^{+}\\ \boldsymbol{\varrho}^{-}\end{pmatrix}\leq\begin{pmatrix}\boldsymbol{\Omega}^{T}\\ \boldsymbol{\Omega}^{Z}\\ \mathbf{1}\end{pmatrix},\begin{pmatrix}\boldsymbol{\varrho}^{+}\\ \boldsymbol{\varrho}^{-}\end{pmatrix}\geq\mathbf{0}\right\}.$$

Note that $\boldsymbol{A}$ satisfies the following property: (i) each column of $\boldsymbol{A}$ contains two non-zero elements with the same sign; (ii) the rows of $\boldsymbol{A}$ can be partitioned into two subsets, and the two non-zero elements in each column are in different subsets of rows. Therefore, $\boldsymbol{A}$ is Total Unimodular (TU, see Schrijver [30]), then $[\boldsymbol{A}',\boldsymbol{I}]'$ is also TU. This together with the fact that $\Gamma_t^T,\Gamma_i^Z,i\in\boldsymbol{\mathcal{I}},t\in\boldsymbol{\mathcal{T}}$ are integers implies that the extreme points of the linear system (A.16)-(A.18) are also integers. Hence, the sub-problem (A.15)-(A.18) is further equivalent to the following mixed integer program form:

$$\min_{\boldsymbol{\pi}^{+},\boldsymbol{\pi}^{-}}\quad\sum_{t\in\boldsymbol{\mathcal{T}}}\sum_{i\in\boldsymbol{\mathcal{I}}}f_{it}\left[\widehat{\xi}_{i0}+\sum_{\tau=1}^{t}F_{i\tau}\right]+\left[\sum_{\tau=1}^{t}\left(\pi_{i\tau}^{+}f_{it}-\pi_{i\tau}^{-}f_{it}\right)S_{i\tau}\right] \tag{A.19}$$

$$\text{s.t.}\quad\sum_{i\in\boldsymbol{\mathcal{I}}}(\pi_{it}^{+}+\pi_{it}^{-})\leq\Gamma_{t}^{\mathrm{Z}},t\in\boldsymbol{\mathcal{T}} \tag{A.20}$$

$$\sum_{t\in\boldsymbol{\mathcal{T}}}(\pi_{it}^{+}+\pi_{it}^{-})\leq\Gamma_{i}^{\mathrm{T}},i\in\boldsymbol{\mathcal{I}} \tag{A.21}$$

$$\pi_{it}^{+},\pi_{it}^{-}\in\{0,1\},i\in\boldsymbol{\mathcal{I}},t\in\boldsymbol{\mathcal{T}}. \tag{A.22}$$

Finally, recall in STEP1 that we have established the truth $f_{it}\leq M_1$, $\forall\, i\in\boldsymbol{\mathcal{I}},t\in\boldsymbol{\mathcal{T}}$, change the variables by applying $q_{it}=f_{it}+\beta^t c_D$, we can have

$\pi^+_{i\tau} f_{it} = \pi^+_{i\tau} q_{it} - \pi^+_{i\tau}\beta^t c_D$, and $\pi^-_{i\tau} f_{it} = \pi^-_{i\tau} q_{it} - \pi^-_{i\tau}\beta^t c_D$, and

$$0 \le q_{it} \le M := M_1 + \beta^t c_D,\ \forall\, i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}.$$

By the standard linearization techniques, we have

$$h^+_{it\tau} = \pi^+_{i\tau} q_{it},\ \forall\, i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}, \tau \in [1:t]$$

is equivalent to the following linear system

$$\left\{\begin{array}{l} h^+_{it\tau} \ge 0,\ \forall\, i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}, \tau \in [1:t] \\ h^+_{it\tau} \le q_{it},\ \forall\, i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}, \tau \in [1:t] \\ h^+_{it\tau} \ge q_{it} + (\pi^+_{i\tau} - 1)M,\ \forall\, i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}, \tau \in [1:t] \\ h^+_{it\tau} \le \pi^+_{i\tau} M,\ \forall\, i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}, \tau \in [1:t] \end{array}\right\}.$$

By a similar linearization treatment, we can replace $\pi^-_{i\tau} q_{it}$ with $h^-_{it\tau}$, for $i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}, \tau \in [1:t]$.

Packaging all the above transformations, we can arrive at the linear mixed integer program formulation (4.10)–(4.23). □

Proof of Proposition 4.

Proof. We first deal with the inner min-max sub-problem of (5.2):

$$\min_{\boldsymbol{\xi} \in \mathcal{U}^\Gamma} \max_{(\boldsymbol{y},\boldsymbol{z}) \in \mathcal{Y}(\boldsymbol{x},\boldsymbol{\xi})} \phi(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}). \tag{A.23}$$

As we have done in Proposition 2, we take the dual form of the inner maximization problem in (A.23), we can arrive at the dual equivalent form of (A.23) as follows:

$$\min_{\boldsymbol{\xi}\in\mathcal{U}^{\Gamma}} \min_{\boldsymbol{f},\boldsymbol{g}} \quad \sum_{t\in\boldsymbol{\mathcal{T}}}\left[\sum_{i\in\boldsymbol{\mathcal{I}}}\xi_{it}f_{it}+\sum_{j\in\boldsymbol{\mathcal{J}}}\sum_{\tau=1}^{t}x_{j\tau}s_j g_{jt}-\beta^{t-1}\boldsymbol{c}^{\top}\boldsymbol{x}_t\right] \tag{A.24}$$

$$\text{s.t.} \quad f_{it}+g_{jt}\geq\beta^t r_{ij},\ i\in\boldsymbol{\mathcal{I}},j\in\boldsymbol{\mathcal{J}},t\in\boldsymbol{\mathcal{T}} \tag{A.25}$$

$$f_{it}\geq-\beta^t c_D,\ i\in\boldsymbol{\mathcal{I}},t\in\boldsymbol{\mathcal{T}} \tag{A.26}$$

$$\boldsymbol{f}\in\Re^{|\boldsymbol{\mathcal{I}}|\times|\boldsymbol{\mathcal{T}}|},\ \ \boldsymbol{g}\in\Re_+^{|\boldsymbol{\mathcal{J}}|\times|\boldsymbol{\mathcal{T}}|}. \tag{A.27}$$

where $\boldsymbol{f}\in\Re^{|\boldsymbol{\mathcal{T}}|\times|\boldsymbol{\mathcal{I}}|}$ and $\boldsymbol{g}\in\Re_+^{|\boldsymbol{\mathcal{T}}|\times|\boldsymbol{\mathcal{J}}|}$ are the dual variables.

From the proof of Proposition 2, the optimal objective of above problem stays unchanged when prediction set $\mathcal{U}^{\Gamma}$ is replaced with the discrete set $\mathcal{V}(\Gamma)$. That is

$$\min_{\boldsymbol{\xi}\in\mathcal{V}(\Gamma)} \min_{\boldsymbol{f},\boldsymbol{g}} \left\{\sum_{t\in\boldsymbol{\mathcal{T}}}\left[\sum_{i\in\boldsymbol{\mathcal{I}}}\xi_{it}f_{it}+\sum_{j\in\boldsymbol{\mathcal{J}}}\sum_{\tau=1}^{t}x_{j\tau}s_j g_{jt}-\beta^{t-1}\boldsymbol{c}^{\top}\boldsymbol{x}_t\right] : (A.25)-(A.27)\right\}.$$

Furthermore, we dualize back the inner minimization problem for each $\boldsymbol{\xi}\in\mathcal{V}(\Gamma)$ and then obtain the following equivalent form of problem (A.24)-(A.27) (or the primal min-max sub-problem (A.23)):

$$\min_{\boldsymbol{\xi}\in\mathcal{V}(\Gamma)} \max_{(\boldsymbol{y},\boldsymbol{z})\in\mathcal{Y}(\boldsymbol{x},\boldsymbol{\xi})} \phi(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}). \tag{A.28}$$

We express the above (A.28) in an epigraph form:

$$\begin{aligned} \max \quad & \gamma \\ \text{s.t.} \quad & \gamma\leq \max_{(\boldsymbol{y},\boldsymbol{z})\in\mathcal{Y}(\boldsymbol{x},\boldsymbol{\xi})}\phi(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}),\ \boldsymbol{\xi}\in\mathcal{V}(\Gamma), \end{aligned}$$

which can be rewritten as following LP formulation (by noting that $\mathcal{V}(\Gamma)$

is a discrete set):

$$\begin{array}{ll} \max & \gamma \\ \text{s.t.} & \left[\begin{array}{c} \gamma \leq \phi(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \\ (\boldsymbol{y}, \boldsymbol{z}) \in \mathcal{Y}(\boldsymbol{x}, \boldsymbol{\xi}) \end{array} \right], \ \boldsymbol{\xi} \in \mathcal{V}(\Gamma), \end{array}$$

or, by the definition of set $\mathcal{K}(\boldsymbol{x}, \boldsymbol{\xi})$ given by (5.4), equivalently,

$$\max \quad \gamma \tag{A.29}$$

$$\text{s.t.} \quad \gamma \in \mathcal{K}(\boldsymbol{x}, \boldsymbol{\xi}), \ \boldsymbol{\xi} \in \mathcal{V}(\Gamma). \tag{A.30}$$

Now putting back the fixed cost term into (A.29)-(A.30), we can arrive at the following equivalent linear mixed integer program formulation for the overall max-min-max problem (5.2):

$$\begin{array}{ll} \max\limits_{\boldsymbol{x}} & \sum\limits_{t \in \boldsymbol{\mathcal{T}}} \left[-\beta^{t-1} \boldsymbol{c}^\top \boldsymbol{x}_t \right] + \gamma \\ \text{s.t.} & \gamma \in \mathcal{K}(\boldsymbol{x}, \boldsymbol{\xi}), \ \boldsymbol{\xi} \in \mathcal{V}(\Gamma) \\ & \boldsymbol{x} \in \mathcal{X} \end{array}$$

as claimed. We are done. □

Proof of Proposition 5.

Proof. Given any $(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{q}) \in \{(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{q}) \mid (5.15) - (5.17)\}$ and $\boldsymbol{\xi} \in \mathcal{U}^\Gamma$, the inner maximization item

$$\begin{aligned} & \max_{(\boldsymbol{y}, \boldsymbol{z}) \in \mathcal{Y}(\boldsymbol{x}, \boldsymbol{\xi}) \cap \boldsymbol{\Lambda}(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{\xi})} \phi(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \\ = & \begin{cases} \phi\left(\boldsymbol{x}, \left[\boldsymbol{p}_j^\top \boldsymbol{\xi}\right]_{|\boldsymbol{\mathcal{J}}|}, \boldsymbol{q}^\top \boldsymbol{\xi}\right), & \text{if } \mathcal{Y}(\boldsymbol{x}, \boldsymbol{\xi}) \cap \boldsymbol{\Lambda}(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{\xi}) \neq \emptyset \\ -\infty, & \text{otherwise.} \end{cases} \end{aligned}$$

Therefore, for each $(\boldsymbol{x},\boldsymbol{p},\boldsymbol{q}) \in \{(\boldsymbol{x},\boldsymbol{p},\boldsymbol{q}) \mid (5.15)-(5.17)\}$, we can further rewrite the inner min-max sub-problem in the overall problem (5.14)-(5.17) in the following form:

$$
\begin{aligned}
&\min_{\boldsymbol{\xi}\in\mathcal{U}^\Gamma}\left[\max_{(\boldsymbol{y},\boldsymbol{z})\in\mathcal{Y}(\boldsymbol{x},\boldsymbol{\xi})\cap\boldsymbol{\Lambda}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{\xi})}\phi(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})\right]\\
=&\begin{cases}-\infty, & \text{if } \exists\ \boldsymbol{\xi}\in\mathcal{U}^\Gamma,\ \text{s.t. } \mathcal{Y}(\boldsymbol{x},\boldsymbol{\xi})\cap\boldsymbol{\Lambda}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{\xi})=\emptyset\\ \min_{\boldsymbol{\xi}\in\mathcal{U}^\Gamma}\phi\left(\boldsymbol{x},\left[\boldsymbol{p}_j^\top\boldsymbol{\xi}\right]_{|\boldsymbol{\mathcal{J}}|},\boldsymbol{q}^\top\boldsymbol{\xi}\right), & \text{otherwise.}\end{cases}
\end{aligned}
$$

By the fact that

$$
\bigcap_{\boldsymbol{\xi}\in\mathcal{U}^\Gamma}\mathcal{Y}(\boldsymbol{x},\boldsymbol{\xi})\cap\boldsymbol{\Lambda}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{\xi})\neq\emptyset
$$

is equivalent to

$$
\boldsymbol{p}_{tj}^\top\boldsymbol{\xi}_t\le\sum_{\tau=1}^{t}x_{j\tau}s_j,\ \forall\ \boldsymbol{\xi}\in\mathcal{U}^\Gamma, t\in\boldsymbol{\mathcal{T}}, j\in\boldsymbol{\mathcal{J}}
$$

for each $(\boldsymbol{x},\boldsymbol{p},\boldsymbol{q}) \in \{(\boldsymbol{x},\boldsymbol{p},\boldsymbol{q}) \mid (5.15)-(5.17)\}$, the overall problem (5.14)-(5.17) can then be rewritten in the following form:

$$
\begin{aligned}
\max_{\boldsymbol{x},\boldsymbol{p},\boldsymbol{q}}\quad & \min_{\boldsymbol{\xi}\in\mathcal{U}^\Gamma}\sum_{t\in\boldsymbol{\mathcal{T}}}\left[\beta^t\sum_{i\in\boldsymbol{\mathcal{I}}}\left(\sum_{j\in\boldsymbol{\mathcal{J}}}r_{ij}p_{ijt}-c_Dq_{it}\right)\xi_{it}-\beta^{t-1}\boldsymbol{c}^\top\boldsymbol{x}_t\right] && \text{(A.31)}\\
\text{s.t.}\quad & \sum_{i\in\boldsymbol{\mathcal{I}}}p_{ijt}\xi_{it}\le\sum_{\tau=1}^{t}x_{j\tau}s_j,\ \forall\ \boldsymbol{\xi}\in\mathcal{U}^\Gamma, j\in\boldsymbol{\mathcal{J}}, t\in\boldsymbol{\mathcal{T}} && \text{(A.32)}\\
& (\boldsymbol{x},\boldsymbol{p},\boldsymbol{q})\in\{(\boldsymbol{x},\boldsymbol{p},\boldsymbol{q})\mid(5.15)-(5.17)\}. && \text{(A.33)}
\end{aligned}
$$

The inner minimization problem in the objective (A.31):

$$
\min_{\boldsymbol{\xi}\in\mathcal{U}^\Gamma}\sum_{t\in\boldsymbol{\mathcal{T}}}\sum_{i\in\boldsymbol{\mathcal{I}}}\beta^t\left[\sum_{j\in\boldsymbol{\mathcal{J}}}r_{ij}p_{ijt}-c_Dq_{it}\right]\xi_{it},
$$

can be expressed as the following linear programming problem:

$$\min_{\boldsymbol{\xi},\boldsymbol{\lambda},\boldsymbol{\varrho}} \quad \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}}\beta^t\left[\sum_{j\in\mathcal{J}} r_{ij}p_{ijt} - c_D q_{it}\right]\xi_{it} \tag{A.34}$$

$$\text{s.t.} \quad \xi_{it} - \sum_{\tau=1}^{t}\varrho_{i\tau}S_{i\tau} = \widehat{\xi}_{i0} + \sum_{\tau=1}^{t}F_{i\tau},\ i\in\mathcal{I}, t\in\mathcal{T} \tag{A.35}$$

$$\sum_{i\in\mathcal{I}}\lambda_{it} \le \Gamma_t^{\mathrm{Z}}, t\in\mathcal{T} \tag{A.36}$$

$$\sum_{t\in\mathcal{T}}\lambda_{it} \le \Gamma_i^{\mathrm{T}}, i\in\mathcal{I} \tag{A.37}$$

$$\lambda_{it} - \varrho_{it} \ge 0, i\in\mathcal{I}, t\in\mathcal{T} \tag{A.38}$$

$$\lambda_{it} + \varrho_{it} \ge 0, i\in\mathcal{I}, t\in\mathcal{T} \tag{A.39}$$

$$\varrho_{it} \ge -1, i\in\mathcal{I}, t\in\mathcal{T} \tag{A.40}$$

$$\varrho_{it} \le 1, i\in\mathcal{I}, t\in\mathcal{T} \tag{A.41}$$

$$\boldsymbol{\xi}, \boldsymbol{\lambda}, \boldsymbol{\varrho} \in \Re^{|\mathcal{I}|\times|\mathcal{T}|}. \tag{A.42}$$

Applying the strong duality of linear programming, we have its equivalent dual form:

$$\begin{aligned}
\max \quad & \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}}\left[\widehat{\xi}_{i0} + \sum_{\tau=1}^{t}F_{i\tau}\right]d_{it}^o + \sum_{t\in\mathcal{T}}\Gamma_t^{\mathrm{Z}}b_t + \sum_{i\in\mathcal{I}}\Gamma_i^{\mathrm{T}}\varphi_i + \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}}\left[\psi_{it} - \nu_{it}\right] \\
\text{s.t.} \quad & \beta^t\left[\sum_{j\in\mathcal{J}} r_{ij}p_{ijt} - c_D q_{it}\right] - d_{it}^o = 0,\ i\in\mathcal{I}, t\in\mathcal{T} \\
& b_t + \varphi_i^o + \gamma_{it}^o + \varpi_{it}^o = 0,\ i\in\mathcal{I}, t\in\mathcal{T} \\
& \psi_{it}^o + \nu_{it}^o + \varpi_{it}^o - \gamma_{it}^o - \sum_{\tau=t}^{|\mathcal{T}|} d_{i\tau}^o S_{i\tau} = 0, i\in\mathcal{I}, t\in\mathcal{T} \\
& \boldsymbol{d}^o \in \Re^{|\mathcal{I}|\times|\mathcal{T}|}, \boldsymbol{b}^o \in \Re_-^{|\mathcal{T}|}, \boldsymbol{\varphi}^o \in \Re_-^{|\mathcal{I}|}, \\
& \boldsymbol{\gamma}^o, \boldsymbol{\varpi}^o, \boldsymbol{\nu}^o \in \Re_+^{|\mathcal{I}|\times|\mathcal{T}|}, \boldsymbol{\psi}^o \in \Re_-^{|\mathcal{I}|\times|\mathcal{T}|}.
\end{aligned}$$

By an analogous treatment to the robust constraint (A.32) for each

$j \in \boldsymbol{\mathcal{J}}, t \in \boldsymbol{\mathcal{T}}$, and package all the transformations, we can arrive the claimed formulation (5.18)-(5.28). $\square$

A.2 A Tractable Design Model with Waste Supply Constraint Relaxation

Suppose we simplify the problem setting by allowing the waste treatment requirement not to be fully satisfied, which refers to the situation when the WtE operator is not obligated (e.g. by contract with the local township council) to fully collect the waste from the residential zones. In such condition, the equality constraint (3.4) is relaxed into an inequality:

$$\left\{ \sum_{j \in \boldsymbol{\mathcal{J}}} y_{ijt} + z_{it} = \xi_{it},\ i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}} \right\} \Rightarrow \left\{ \sum_{j \in \boldsymbol{\mathcal{J}}} y_{ijt} \leq \xi_{it}, i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}} \right\}, \tag{A.43}$$

by noting that the excessive waste quantity $\boldsymbol{z}$ is now redundant in the formulation. We then denote by $\mathcal{Y}_{\mathrm{S}}(\boldsymbol{x}, \boldsymbol{\xi})$ the new feasible set for $\boldsymbol{y}$ with the constraint replacement (A.43).

On the other hand, we utilize a new error budget parameter $\Omega := [\Omega_{it}]_{|\boldsymbol{\mathcal{T}}| \times |\boldsymbol{\mathcal{I}}|}$ and each $\Omega_{it} \in [0, 1], i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}$ to mitigate the conservativeness of the prediction set. Specifically, we set $\varrho_{it} \in [-\Omega_{it}, \Omega_{it}], i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}$. In other words, instead of controlling the variation in $\boldsymbol{\xi}$ collectively over periods and across zones as the original budget parameter $\Gamma = \left[\Gamma_t^{\mathrm{Z}}, \Gamma_i^{\mathrm{T}}\right]_{|\boldsymbol{\mathcal{T}}|+|\boldsymbol{\mathcal{I}}|}$ functions, the parameter Ω controls the forecast error level of waste supply ξ_{it} for each $i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}$ in a separate manner. As the following Proposition 6 shows, the new error budget parameters defined in this way ensure the tractability of the relaxed problem. The resulting new prediction set, denoted by $\mathcal{U}^{\Omega}$, can be formed as follows:

$$\mathcal{U}^{\Omega} := \left\{ \boldsymbol{\xi} \in \Re^{|\boldsymbol{\mathcal{I}}|\times|\boldsymbol{\mathcal{T}}|} : \begin{array}{l} \xi_{it} = \widehat{\xi}_{i0} + \sum_{\tau=1}^{t} F_{i\tau} + \left[\sum_{\tau=1}^{t} \varrho_{i\tau} S_{i\tau}\right], \; i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}} \\ \varrho_{it} \in [-\Omega_{it}, \Omega_{it}], i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}} \end{array} \right\}. \tag{A.44}$$

With the new feasible set $\mathcal{Y}_{\mathrm{S}}(\boldsymbol{x}, \boldsymbol{\xi})$ and prediction set $\mathcal{U}^{\Omega}$, the expansion optimization problem (5.2) can then be formulated in the following form:

$$\max_{\boldsymbol{x}} \quad \min_{\boldsymbol{\xi}\in\mathcal{U}^{\Omega}} \max_{\boldsymbol{y}\in\mathcal{Y}_{\mathrm{S}}(\boldsymbol{x},\boldsymbol{\xi})} \phi(\boldsymbol{x}, \boldsymbol{y}) \tag{A.45}$$

$$\text{s.t.} \quad \boldsymbol{x} \in \mathcal{X}, \tag{A.46}$$

where $\phi(\boldsymbol{x}, \boldsymbol{y})$ is the new overall NPV function (without variable $\boldsymbol{z}$). We now show that the relaxed WtE expansion optimization problem solves exactly a single mixed integer program.

Proposition 6. *The expansion optimization problem (A.45)-(A.46) under forecast error budget setting Ω is equivalent to the following mixed integer program:*

$$\max_{\boldsymbol{x},\boldsymbol{y}} \sum_{t\in\boldsymbol{\mathcal{T}}} \left[\beta^t \sum_{i\in\boldsymbol{\mathcal{I}}} \sum_{j\in\boldsymbol{\mathcal{J}}} r_{ij} y_{ijt} - \beta^{t-1} \boldsymbol{c}^{\top} \boldsymbol{x}_t\right] \tag{A.47}$$

$$\text{s.t.} \sum_{j\in\boldsymbol{\mathcal{J}}} y_{ijt} - \widehat{\xi}_{i0} - \sum_{\tau=1}^{t} F_{i\tau} + \left[\sum_{\tau=1}^{t} \Omega_{i\tau} S_{i\tau}\right] \leq 0, \; i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}} \tag{A.48}$$

$$\sum_{i\in\boldsymbol{\mathcal{I}}} y_{ijt} \leq \sum_{\tau=1}^{t} x_{j\tau} s_j, \; j \in \boldsymbol{\mathcal{J}}, t \in \boldsymbol{\mathcal{T}} \tag{A.49}$$

$$\boldsymbol{x} \in \mathcal{X}, \boldsymbol{y} \in \Re_{+}^{|\boldsymbol{\mathcal{T}}|\times|\boldsymbol{\mathcal{I}}|\times|\boldsymbol{\mathcal{J}}|}. \tag{A.50}$$

Proof. Under the new prediction set $\mathcal{U}^{\Omega}$, it follows from the setting $\varrho_{it} \in$

$[-\Omega_{it}, \Omega_{it}]$, $i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}$ that for any $\boldsymbol{\xi} \in \mathcal{U}^{\Omega}$ we have

$$
\begin{aligned}
\xi_{it} \quad &= \widehat{\xi}_{i0} + \sum_{\tau=1}^{t} F_{i\tau} + \left[\sum_{\tau=1}^{t} \varrho_{i\tau} S_{i\tau} \right] \\
&\leq \widehat{\xi}_{i0} + \sum_{\tau=1}^{t} F_{i\tau} - \left[\sum_{\tau=1, S_{i\tau} \geq 0}^{t} \Omega_{i\tau} S_{i\tau} \right] + \left[\sum_{\tau=1, S_{i\tau} < 0}^{t} \Omega_{i\tau} S_{i\tau} \right] =: \underline{\xi}_{it}, \ i \in \boldsymbol{\mathcal{I}}, t \in \boldsymbol{\mathcal{T}}.
\end{aligned}
$$

Therefore, if we denote by $\underline{\boldsymbol{\xi}} := \left[\underline{\xi}_{it} \right]_{|\boldsymbol{\mathcal{I}}| \times |\boldsymbol{\mathcal{T}}|}$, it is clearly that given any $\boldsymbol{x} \in \mathcal{X}$,

$$
\mathcal{Y}_{\mathrm{S}}(\boldsymbol{x}, \underline{\boldsymbol{\xi}}) \subseteq \mathcal{Y}_{\mathrm{S}}(\boldsymbol{x}, \boldsymbol{\xi}), \ \forall \ \boldsymbol{\xi} \in \mathcal{U}^{\Omega},
$$

which implies

$$
\max_{\boldsymbol{y} \in \mathcal{Y}_{\mathrm{S}}(\boldsymbol{x}, \underline{\boldsymbol{\xi}})} \phi(\boldsymbol{x}, \boldsymbol{y}) \leq \max_{\boldsymbol{y} \in \mathcal{Y}_{\mathrm{S}}(\boldsymbol{x}, \boldsymbol{\xi})} \phi(\boldsymbol{x}, \boldsymbol{y}), \ \forall \ \boldsymbol{\xi} \in \mathcal{U}^{\Omega}.
$$

Since $\underline{\boldsymbol{\xi}} \in \mathcal{U}^{\Omega}$, we then have

$$
\min_{\boldsymbol{\xi} \in \mathcal{U}^{\Omega}} \max_{\boldsymbol{y} \in \mathcal{Y}_{\mathrm{S}}(\boldsymbol{x}, \boldsymbol{\xi})} \phi(\boldsymbol{x}, \boldsymbol{y}) = \max_{\boldsymbol{y} \in \mathcal{Y}_{\mathrm{S}}(\boldsymbol{x}, \underline{\boldsymbol{\xi}})} \phi(\boldsymbol{x}, \boldsymbol{y}).
$$

This leads to the equivalent formulation (A.47)–(A.50) of the expansion optimization problem (A.45)–(A.46). □

An intuitive insight of Proposition 6 is that with the new forecast error budget parameters, when the full waste treatment requirement is relaxed, the extreme value forecast of waste generation is always its lower-bound, i.e., $\boldsymbol{\xi}^{\dagger} = \underline{\boldsymbol{\xi}}$. Therefore, any change in the upper bound of future waste supply (forecasts) will have no effect onto the guaranteed NPV, and it is the variation in waste supply lower bounds that concerns the (uncertainty-averse) decision-makers.

A.3 Multi-Stage Stochastic Programming Model with Scenario Tree

A common way to present the multi-stage stochastic program is the scenario formulation in which the scenarios of the uncertainty (waste supply in our problem) as data are inputs to the model. However, one of the key difficulties in multi-stage stochastic program is modelling the "*non-anticipativity*" constraints, which requires the rationality that the decisions made at any stage of the process do not depend on future realizations of random parameters or on future decisions (see [29]). In the scenario form of a multi-stage stochastic program model, this means that if two uncertainty scenario paths before stage t are identical, then the corresponding decisions that depend on these two scenario paths must be the same. We employ the commonly used scenario tree approach to model the waste supply uncertainty in our multi-stage WtE expansion optimization problem, which groups automatically the identical scenarios along the paths based on the tree structure and therefore can guarantee the non-anticipativity.

To present the scenario tree model in a concise manner, we assume w.l.o.g. for each stage t an identical scenario number for the future waste supply ξ_{it} for different zones $i \in \mathcal{I}$, and denote $\mathcal{N}(t)$ as the node (scenario) set for the waste supply scenarios at stage t with the maximum node number $N_t = |\mathcal{N}(t)|$, for $t \in \{0\} \cup \mathcal{T}$, where $\mathcal{N}(0) := \{1\}$ which is used exclusively for $\boldsymbol{x}_0$ to keep a consistent presentation. Also, we denote $\mathcal{P}(n)$ as the set of all the parent nodes of non-root node n. With the defined scenario index set, we can move the index t from ξ_{it} into $\mathcal{N}(t)$ and represent the waste supply scenarios of zone i in a cleaner way as $\xi_i^n, n \in \mathcal{N}(t), t \in \mathcal{T}$, which can be recovered without any loss as $\xi_{ti}^{k_t}$ standing for some k_tth node (scenario) of waste supply at stage t of zone i. Similarly, we can represent the expansion and waste allocation decisions as y_{ij}^n, z_i^n and x_j^n for $n \in \boldsymbol{\mathcal{N}}(t), t \in \boldsymbol{\mathcal{T}}$.

Applying the waste supply scenario tree structure, the multi-stage stochastic WtE expansion optimization model can be formulated as follows:

$$
\begin{aligned}
\min \quad & \sum_{t \in \boldsymbol{\mathcal{T}}} \left[\frac{\beta^{t-1}}{N_{t-1}} \sum_{n \in \boldsymbol{\mathcal{N}}(t-1)} \sum_{j \in \boldsymbol{\mathcal{J}}} c_j x_j^n - \frac{\beta^t}{N_t} \sum_{n \in \boldsymbol{\mathcal{N}}(t)} \sum_{j \in \boldsymbol{\mathcal{J}}} \sum_{i \in \boldsymbol{\mathcal{I}}} \left(r_{ij} y_{ij}^n - c_D z_{ij}^n \right) \right] \\
\text{s.t.} \quad & \sum_{j \in \boldsymbol{\mathcal{J}}} y_{ij}^n + z_i^n = \xi_i^n, \ n \in \boldsymbol{\mathcal{N}}(t), t \in \boldsymbol{\mathcal{T}}, i \in \boldsymbol{\mathcal{I}} \\
& \sum_{i \in \boldsymbol{\mathcal{I}}} y_{ij}^n \leq \sum_{m \in \boldsymbol{\mathcal{P}}(n)} x_j^m s_j, \ n \in \boldsymbol{\mathcal{N}}(t), t \in \boldsymbol{\mathcal{T}}, j \in \boldsymbol{\mathcal{J}} \\
& \sum_{m \in \boldsymbol{\mathcal{P}}(n)} x_j^m \leq 1, \ n \in \boldsymbol{\mathcal{N}}(|\boldsymbol{\mathcal{T}}|), j \in \boldsymbol{\mathcal{J}} \\
& x_j^n \in \{0, 1\}, \ n \in \boldsymbol{\mathcal{N}}(t-1), t \in \boldsymbol{\mathcal{T}}, j \in \boldsymbol{\mathcal{J}} \\
& y_{ij}^n, z_i^n \in \Re_+, \ n \in \boldsymbol{\mathcal{N}}(t), t \in \boldsymbol{\mathcal{T}}, i \in \boldsymbol{\mathcal{I}}, j \in \boldsymbol{\mathcal{J}} \\
& t \in \boldsymbol{\mathcal{T}}.
\end{aligned}
$$

It can be seen that, although the above model is a mixed integer linear programming, the total number of scenarios and the number of decision variables grow exponentially with increase of the number of stages. Therefore, it is in general time-consuming to solve this multi-stage stochastic problem.

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