DESIGN AND MANAGEMENT OF MOBILITY-ON-DEMAND (MOD) TRANSPORTATION SYSTEMS CONSIDERING DEMAND UNCERTAINTY AND FLEXIBILITY

- A SIMULATION-BASED APPROACH

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NATIONAL UNIVERSITY OF SINGAPORE

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- A SIMULATION-BASED APPROACH

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A THESIS SUBMITTED

FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF INDUSTRIAL AND SYSTEMS ENGINEERING

NATIONAL UNIVERSITY OF SINGAPORE

2015

Declaration

I hereby declare that the thesis is my original work and it has been written by me in its entirety. I have duly acknowledged all the sources of information which have been used in the thesis.

This thesis has also not been submitted for any degree in any university previously.

Deng Yinghan

Deng Yinghan

29 July 2015

Acknowledgement

There are many people I am indebted to along this Ph.D. journey. Without them, I could never reach this goal.

First and foremost, I would like to express my deep gratitude to my supervisor, Dr. Michel-Alexandre Cardin, who gave me sincere support, consistent encouragement, and valuable suggestions throughout my research journey. Working with him in the past four years was an inspiring and rewarding experience. I feel so honored that I could work under his supervision.

I also would like to thank Prof. Amedeo R. Odoni. His patient guidance and immense knowledge enlightened many aspects of this research. I am very grateful to Prof. Daniel D. Frey for hosting me during my visit at MIT. It was the most illuminating and memorable period in my Ph.D. program. My gratitude also goes to my thesis committee members, Prof. Vladan Babovic and Prof. Chai Kah Hin, who helped me to think more deeply and comprehensively on the research topic.

I also want to thank all my friends and colleagues. They all contributed in making the journey less lonely. Particularly, thank you, Liao Lianjing, Wan Xingyun, Zhou Ke, and Dr. Zhang Si, for offering your company and help whenever I needed it. And thank you, Mr. Tian Zongxu. Most of this journey was shared with you. You were always there, standing beside me, listening and cheering me up. Although we went off different life courses in the very end of this journey, you will always have my best wishes and I sincerely believe that a great career is waiting for you to be realized.

Last but not the least, thanks to my dear family. Grown up in a big family full of warmth, I always considered myself such a lucky person. Thinking about having you around me, gave me so much courage and strength in many difficult times. Particularly, thank you, my dear mother. This thesis is for you. You set an example in every aspect of my life, teaching me to be honest, diligent, considerate of others, and to never give up. Your unconditional love contributes to every achievement in my life.

Thanks to everyone who offered me a hand during this four-year journey. This short acknowledgement is never enough to express my appreciation.

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Summary

Engineering systems today are exposed to various uncertainties. It has been demonstrated in many studies that designing flexible systems is an effective means to improve system performance in uncertain environment. A Mobility-on-Demand (MoD) system involves a fleet of vehicles strategically located at stations across the transportation network. Vehicle fleets can comprise bicycles, low emission cars, or electric vehicles. The stations mainly consist of parking areas for cars or bikes, and charging facilities if electric vehicles are used. Its performance is largely affected by both short-term demand fluctuations and long-term usage pattern changes. This thesis explores the “flexibility paradigm” to determine where and how to locate stations in an urban environment, i.e. number of parking spots, and allocate vehicles in one-way MoD systems making explicit considerations of uncertainty. A simulation-based approach is used for system modeling and solution computation. The systematic simulation-based approach and its advantages in terms of analyzing flexible systems are first introduced through a case study on an urban infrastructure. Then, the thesis applies the simulation-based approach to integrate the rebalancing operation – an operational-level flexibility – into the planning decisions of MoD systems, which deals with short-term demand fluctuations. A solution approach based on a discrete-event simulator (DES) and a computation algorithm combining Particle Swarm Optimization (PSO) and Optimal Computation Budget Allocation (OCBA) is devised to calculate the optimal planning decisions. The study then proceeds to analyze strategic flexibility – a capacity phasing or staging strategy – in MoD systems so as to target the uncertainty from long-term usage pattern changes. The same solution approach is adopted but modified to determine the optimal parameters of the flexible strategy. Also, inspired from the successful implementation of the PSO+OCBA algorithm, a computational framework combining

population-based search algorithms and the OCBA technique is proposed as another perspective to mitigate computational complexity in optimizing the flexible systems via simulation. This thesis provides distinct insights on the design and management of MoD systems as well as optimization of flexible transportation and engineering systems under uncertainty.

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List of Abbreviations

MoD	= Mobility-on-Demand,
VoF	= Value of Flexibility,
NPV	= Net Present Value,
ENPV	= Expected Net Present Value,
OFAT	= One-factor-at-a-time,
DES	= Discrete Event Simulator,
PSO	= Particle Swarm Optimization,
OCBA	= Optimal Computation Budget Allocation,
P(CS)	= Probability of Correct Selection,
MAS	= Maximum Budget Allowed for a Single Solution,
EA	= Equal Allocation,
SE	= Standard Error

Chapter 1 Introduction

1.1 Design and management of Mobility-on-Demand (MoD) systems

Urbanization is progressing at a high speed. According to the World Fact book, until 2010, 50.5% percent of the total population is living in urban areas, and the size of urban population is growing by 1.85% annually (De Lessio et al., 2013). The deepening urbanization leads to a rapid growth of demand for various urban recourses, like land usage and energy consumption, which requires urban system designers to think forward and provide efficient and sustainable solutions.

Among the challenges faced by urban system designers is the design and management of urban transportation systems. Due to the convenience of point-to-point transportation and increasingly lower prices, private automobiles have become a most common choice for urban mobility. It has been reported that in 2009, almost 90% of American workers choose private automobiles as the usual commute mode, contrasting with less than 10% who use public transit (Santos et al., June 2011). Even in Singapore where the charge of Certificate of Entitlement is severely high, the number of cars is still increasing ("Singapore Land Transport: Statistics In Brief 2013," 2013).

Consequently, scores of problems are emerging due to the popularity of private cars. The existing capacity of roads in a given city may no longer be able to carry increasing traffic flows, leading to frequent traffic congestion. In addition, more public space is occupied due to the mounting parking demand. Furthermore, more and more vehicle exhausts, such as CO_2, exacerbates the issue of air pollution.

Such situations call up more advocates for public transportation. However, traditional public transportation networks relying on buses and subways may not be able to satisfy the diverse needs on urban mobility, e.g. the last-mile and first-mile connectivity issue in daily commutes, which refers to the provision of travel service from a public transportation node to home or workplace(Wang & Odoni, 2014). Meanwhile, the growing size of the urban population often renders public transportation systems overloaded, especially during peak hours, further motivating people to use private cars in an ongoing vicious cycle. Admittedly, taxis can work as a supplement for public transportation and provide a more flexible form of mobility, but is a costly alternative and cannot be utilized by mass population as a regular transportation mode.

As a response to these challenges, Mobility-on-Demand (MoD) systems have drawn more and more attention in recent years. A MoD system, also referred as vehicle-sharing system (VSS) in this thesis, involves a fleet of vehicles strategically located at stations across the transportation network. Vehicle fleets can comprise bicycles, low emission cars, or electric vehicles (Nair & Miller-Hooks, 2011). Stations mainly consist of parking areas for cars or bikes, and charging facilities if electric vehicles are used. Figure 1.1 illustrates a the integration of existing transpiration network with a MoD system using electric vehicles.

MoD systems are perceived as a promising alternative for urban mobility due to the ease of use and potentially large overall societal benefits. For example, for car-sharing systems, users enjoy the convenience and comfort of private automobiles without the associated high costs, insurance requirements, need to refuel, service and repair demands, or parking problems ("Mobility on Demand: Future Transportation in Cities," 2008). Such benefits help to alleviate people's reliance on private cars, so that more parking areas can be freed up for other public uses, and consequently, alleviate road congestions.

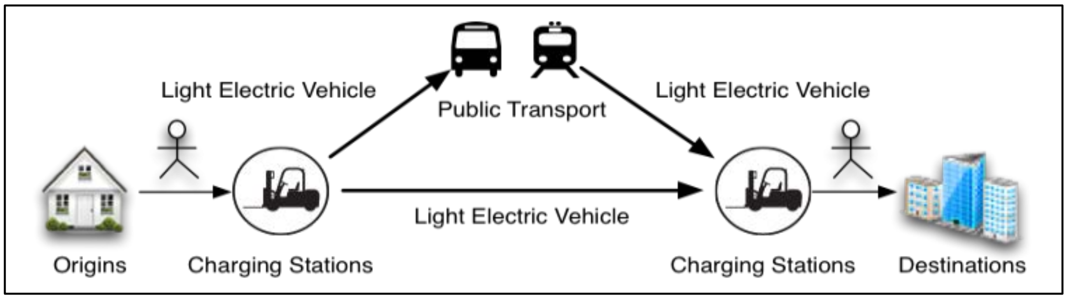

Figure 1.1 Illustration of a typical MoD system using electric vehicles

The design and operations of a MoD system involve multiple stakeholders. As illustrated in Figure 1.2, these major players can be roughly categorized into three groups. Resource providers are the group who possess and provide critical resources essential for the establishment and survival for a MoD system. Local government is one major resource provider who has the control over land, planning consent, and political support (as most MoD systems are subsidized). Meanwhile, vehicle providers can exist in various forms. They can be vehicle rental companies, e.g. Enterprise Rent-A-Car who provides corporate rental services, independent vehicle manufacturers, such as Honda who is one of the main partners with Zipcar (http://www.zipcar.com), and the mother company who owns the MoD system, which actually forms a more and more popular business model in auto industries with manufactures like Daimler and BMW starting up subsidiaries to provide MoD services. Financial support is another key resource for a MoD system, which comes from investors and / or creditors. Operating company is the one who determines the use of resources obtained from the aforementioned three parties. It makes planning decisions like how and where to set up stations and purchasing of vehicles, as well as operational decisions that respond to real-time demand realizations. While resource provides and operating company may encounter conflicts of interest somehow, they are both largely affected by customers who pay the price to the operating company and enjoy the MoD service. There are close interactions between the decisions and behaviors of the three groups of stakeholders and which, in return, have a collective effect on the performance of a MoD system. This study takes the perspective of an operating company of the MoD system. Therefore, the author mainly concerns with how to make better planning and operating decisions to obtain a sustainable growth of the system as well as the company.

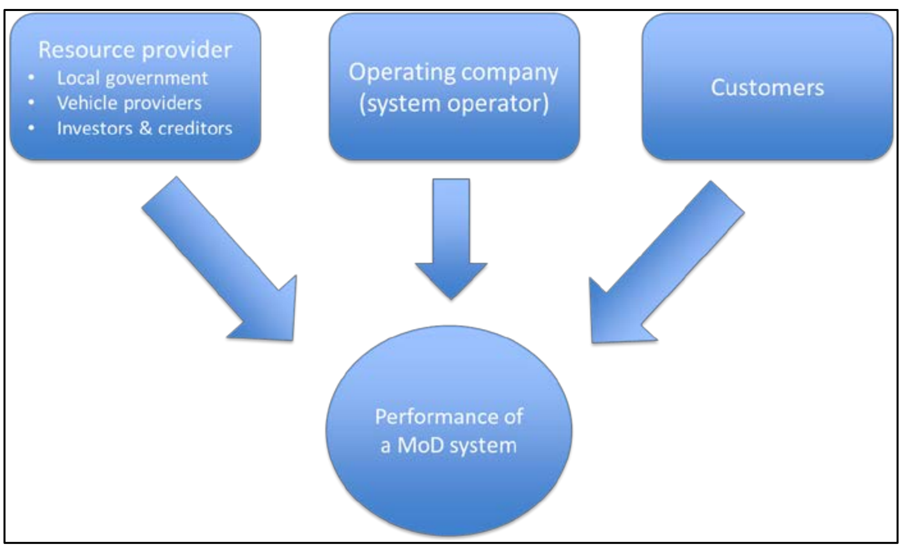

Figure 1.2 Major stakeholders in a MoD system

Existing MoD systems can be generally categorized into two types: one-way or two-way systems. For two-way sharing systems, users are required to return the used vehicle to the station where it was picked up. Car-sharing systems are mostly two-way. Companies like Zipcar and Hertz (https://www.hertz.com) currently operate such systems. Compared with the two-way sharing systems, one-way MoD systems are more flexible: users can walk to a nearby station to pick up a vehicle, and then they may drop it off at any station near their final destination.

Initially, most one-way MoD systems consisted of bike-sharing systems that gradually became a popular choice for urban mobility, such as Vélib in Paris (http://www.velib.paris.fr) and Hubway in Boston (http://www.thehubway.com). Recently, more and more one-way car-sharing systems have been implemented, with the success of Car2go (https://www.car2go.com) in North America indicating great potential for such systems. Because of the usage mode, one-way MoD systems are more likely to be utilized as a connection to existing public transport modes or as a substitute. As for the ownership of the system, currently a large proportion of car-sharing systems are operated by private companies, while for bike-sharing systems, they can be managed by local transportation authority, or public–private partnership, like Austin-B Cycle (http://austinbcycle.com), or even purely privately owned, like Citi Bike (http://www.citibikenyc.com) and Zagster (http://zagster.com/).

Admittedly there exists difference between car-sharing and bike-sharing systems, e.g. size of fleet and average travel distance, this study does not address these two types of one-way sharing systems seperatedly in terms of modelling and evaluation. As mentioned earlier, this study origins from the standpoint of a private company operating such systems, designing and managing either bike-sharing or car-sharing systems is motivated by the same objectives and faced with similar challenges. On the one hand, due to the large social benefit associated with MoD systems, as well as high cost of operating such systems in the urban area, particularly regarding the land cost as well as purchasing cost of electric vehicles if used, either car-sharing systems (Dudley, 2013) or bike-sharing systems (Tangel, 2014) are subsidized by the local government. Therefore, it is believed that at the inception stage of the system, no matter for bike-sharing systems or car-sharing systems, level of service (LoS) may be the first priority as it is critical to establish a large customer base to demonstrate the value of the subsidy and build a reputation. Nevertheless, for the long-term survival of the system that is owned privately, profitability may be still placed at a high priority. . On the other hand, Successfully deploying and operating one-way bike-sharing and car-sharing systems is faced with similar challenges. In addition to the demand being difficult to predict and highly fluctuating, which calls for dynamic decision-making and prompt actions, the inherent imbalance of urban traffic flows frequently leads to an inefficient use of the system: areas with higher rates of vehicle return may become overstocked with a large number of idle vehicles that could be better used if relocated to places where they are needed. Also, the performance of a MoD system is largely influenced by the intricate interactions between different levels of decisions, among which the trade-offs may not be very explicit. Because of such similarities in operating objectives and challenges, this study adopts the same practice as Nair and Miller-Hooks (2011) that does not distinguish between bike-sharing and car-sharing systems.

Given the aforementioned complexity involved in designing and operating one-way MoD systems, particularly with respect to managing the fluctuating and uncertain demand, this study adopts a different perspective of engineering design that addresses uncertainties by designing flexible systems.

1.2 Flexibility in engineering design

The only thing that never changes today is change itself. During the past several decades, we have witnessed tremendous changes in almost every aspect of human life, technology, economy, politics, etc. Take Internet as an example, it had been roughly estimated that in 2001 the number of global Internet users was only 495 million, and in just three years, the number doubled. In 2011, the number became 2,265 million (ITU). Meanwhile, various examples have shown that human beings are never good prophets for these changes. A. Wooldridge said "In the early 1980s, consultants at McKinsey and Company were hired by ATT to forecast the growth in the mobile market until the end of the millennium. They projected a world market of 900,000. Today [in 1999] 900,000 handsets are sold every three days" (de Neufville & Scholtes, 2011). These situations indicate that there is a need to develop an approach that can address the uncertainties inherent in a system, a process, or an organization. One of such approaches is designing flexible systems.

Flexibility – also referred in this thesis as a real option – is defined as the "right, but not the obligation to change a project or system in the face of uncertainty" (Trigeorgis, 1996). For example, a water catchment system can be designed with a flexibility to expand its original capacity in the future if necessary (Deng et al., 2013). Likewise, MoD system can also be designed with flexibilities such as a case where some vehicles or parking spots may be reserved for future expansion of the system.

There are generally two classes of flexibilities. Managerial flexibilities (also referred as real options "on" systems) involve high-level decisions, like real options to defer or stage an investment, abandon, switch, alter the operating scale, or grow a system, or find combinations of multiple real options (Trigeorgis, 1996). The other category is technical flexibilities (also referred as real options "in" systems) that are those inherited in the design configurations, which typically enable the managerial flexibility decisions (i.e. exercising the real options). Most of the past studies in this field focus on quantifying the economic value of flexibilities. Some commonly used analytical methods rely on a binomial lattice (John C Cox et al., 1979), decision analysis (Babajide et al., 2009) and Monte Carlo simulation (Deng et al., 2013).

Typically, systems are designed and evaluated under deterministic projections of the main uncertainties that affect their lifecycle performance (de Neufville & Scholtes, 2011). The most common practice consists of three phases: first, by collecting and analyzing relevant data, the scenario with the highest likelihood of occurrence is identified, which projects the major external drivers of the system, such as customer demand, market share, product price, etc.; then, according to those predictions, system designers generate design concepts and select design parameters that enable the system to perform optimally under the most likely or expected scenario; finally, it is the design evaluation, of which a standard methodology, like discounted cash flow (DCF) analysis, sensitivity analysis, scenario planning, etc., is applied. The result achieved through such practice is usually the “point optimal” design.

This kind of design practice based on deterministic forecasts and the assumption of fixed design parameters, however, may not help decision-makers identify and operate a system that performs well in the real world. In this case, the system may perform optimally only when the predicted scenario happens; while in other cases, the system stays passively and may not be able to perform well under unexpected and/or unfavorable conditions. Besides, standard design and evaluation analysis, which relies mostly on deterministic forecasts as inputs, can also mislead decision makers (de Neufville & Scholtes, 2011). This is because the response of engineering systems is typically not linear; and according to Jensen’s inequality (Jensen, 1906) as shown in Equation 1.1, the value of the function containing random variables does not equal to the value of the function under the expected value of those random variables. The benefits obtained from possible upside scenarios (e.g. higher product prices or demand) may not balance the losses incurred from potential downside scenarios (e.g. lower prices or demand).

$$\boldsymbol{f}[\mathbf{E}(\boldsymbol{x})] \neq \mathbf{E}[\boldsymbol{f}(\mathbf{x})] \qquad \mathbf{1.1}$$

Due to the reasoning above, a paradigm shift in systems design and evaluation is in need. The new methodology should enable systems to act pro-actively in the face of a variety of situations. Moreover, such adaptability needs to be considered as one of the metrics to assess the value of systems.

Designing flexible systems is one effective solution. This approach challenges the main assumptions of standard design and project evaluation approaches: the use of deterministic forecast and fixed design parameters. Under the "flexibility thinking", designers are required to consider a large number of possible scenarios and prepare for changes in these operating conditions, so that systems can better exploit the realized situations to capture extra profits or avoid excessive losses. Many recent studies have shown that flexibility can improve expected lifecycle performance (e.g. net present value, cost savings) by 10% to 30% as compared to the output from standard design and evaluation methods. This can be significant in the design and management of engineering systems, such as those considered in this thesis, since these typically represent large irreversible investments in infrastructures, in the order of \$ million and \$ billion. Flexibility typically improves such performance by reducing system exposure to downside risks (i.e. like an insurance policy), while providing contingencies to capitalize on upside opportunities. The net effect is typically to shift the entire distribution of possible performance outcomes towards better value, therefore improving the expected lifecycle performance of the system as a whole.

Finding the optimal flexible design, however, is not an easy task. For complex engineering systems, estimating their life-cycle performances can be very time-consuming, even without considering flexibility and optimization. For example, for a reservoir simulator, depending on the complexity of the reservoir and the resolution of each grid box, each simulation run over the field's lifecycle may take a few hours to a few days (Lin, 2008). Flexibility analysis further exacerbates the computational problem, since many scenarios and a variety of flexible designs are accounted for, which makes it even more difficult to assess the performance of candidate designs and find the optimal one. In this case, advanced computational techniques need to be developed.

1.3 Objectives and significance of the study

In sum, rapid urbanization results in unprecedented challenges to existing urban transportation network. As a response, MoD system emerges as a viable and promising solution. Due to unbalanced and highly fluctuating demand, however, effectively designing and managing MoD systems, particularly one-way systems, still remains a problem to be solved. This study aims to apply “flexibility thinking” to target the technical aspect of the aforementioned problem, as many past studies suggest that constructing a flexible system leads to better management of uncertainties in engineering systems. More specifically, the study intends to devise a methodology that provides systematic decision-support to the planning and operations of the one-way MoD systems that are exposed to both short-term (e.g. daily demand variations) and long-term (overall usage pattern changes) demand uncertainty. Definitions with more details of short-term and long-term uncertainty are provided in the next chapter. The specific research objectives are as follows:

- Develop a design procedure that provides high-level guidance on how to design and evaluate flexible engineering systems.
- Develop and solve a mathematical model that aims at finding the optimal planning decisions for a MoD system, where vehicle redistribution activities are considered as an operational level flexibility to address short-term demand fluctuations.
- Develop and solve a mathematical model that aims to determine the optimal flexible strategy for deploying a MoD system that copes with long-term demand uncertainty.

The contribution of this study stems from two aspects. On the one hand, with respect to MoD systems, it is the first study that integrates strategic planning and operational-level decisions into one decision-making framework. In addition to accounting for short-term stochastic demand fluctuations, this study is also the first to investigate flexible deployment strategies in MoD systems as a strategy to address long-term (and growing) demand uncertainty. On the other hand, regarding the methodological aspect, a computational framework based on an Optimal Computation Budget Allocation (OCBA) technique is first applied to the domain of optimization on flexibilities in engineering systems, which demonstrates another opportunity to solve the computational issues often encountered in optimizing flexible engineering systems.

1.4 Organization

This thesis contains 6 chapters, organized as follows. Chapter 2 reviews the background literature relevant to this study. Chapter 3 introduces a four-step simulation-based procedure that is used to analyze and evaluate flexible engineering systems design concepts. It also demonstrates that incorporating flexibilities can improve the performance of engineering systems and illustrates how simulations can be applied as an analytical framework to model and evaluate flexible systems design concepts. In chapter 4, this study explores the operational design and rebalancing dynamics in MoD systems that are used to deal with short-term demand uncertainty. In Chapter 5, a strategic level flexibility is incorporated into the deployment plan of the MoD system to accommodate another layer of uncertainty, the long-term usage pattern changes. Chapter 6 concludes the whole thesis, discusses results limitations and validity issues, and identifies opportunities for future research.

Chapter 2 Literature Review

This thesis is motivated by two bodies of research, i.e. studies investigating the planning and operations of MoD systems, and the ones exploring how to incorporate flexibilities into engineering systems. Past studies from these two communities are first presented. After that, this chapter concludes with the major research questions addressed by this study.

2.1 Research on MoD systems

The literature on the quantitative planning and detailed operation of MoD systems was virtually non-existent until a few years ago, but has been growing rapidly in recent years. Existing studies can be roughly categorized into three sub-streams: data analytics, strategic planning, and rebalancing operations. Some of these studies address bike-sharing systems, while others pertain to car-sharing systems.

Authors in the area of data analytics focus on understanding and characterizing the usage patterns of MoD systems. For example, Vogel et al. (2011) apply clustering analysis to the ride data from Vienna's bike-sharing system, "Citybike Wien," identifying five distinct such clusters based on pickup and return patterns over time. Borgnat et al. (2011) rely on non-stationary statistical modeling and data mining to describe the evolution of the dynamics of movements within the Vélib system in Paris. The spatial and temporary demand patterns are also described, and the social behavior of the users is explained. O'Brien et al. (2014) applied a similar approach to analyze the usage data of bike-sharing systems. However, their analysis is considered as the first to take a global view of bike-sharing characteristics, as data from 38 systems globally was used to identify the special and temporal patterns of different types of bike-sharing systems. Using demographics and travel survey data, Ciari et al. (2010) develop an agent-based simulation (ABS) model to estimate and predict demand for a car-sharing system during its planning phase.

The second category of papers presents models that deal with strategic planning issues. Lin et al. (2013) formulate a mixed integer programming model for optimizing the design of bike-sharing systems. Although the model encompasses a wide range of strategic decisions, their analysis is deterministic and does not capture dynamic behavior, such as the daily or even weekly fluctuations of demand that these systems experience. Rickenberg et al. (2013) devised a decision support system to optimize the location and sizing of car sharing systems. Although demand is assumed to be stochastic, their analysis lacks essential details, as only aggregated demand in a day is considered, and not the sequence of demand. However, a system design that targets aggregated demand may be installed with excessive capacity, as within a day, the traffic flows between stations somehow help to improve the utilization of the system, which, however, is not accounted in their analysis. For example, the travel demand from area A to area B can be partially resolved by the vehicles driven by customers from area B to area A earlier in the day, but taking an aggregated demand approach ignores such situation. Considering demand variations within a day, Raviv and Kolka (2013) explore the optimal number of vehicles to be made available at each station at the beginning of each day. They formulate a penalty cost function and further utilize it in a simulation-based optimization model to search for the best solution. Forma et al. (2015) takes a further step by considering the whole network of the system instead of a single station. They first define clusters of stations, and then try to find the optimal initial vehicle distribution among clusters by taking into account the penalty cost of unsatisfied customers, and the operating cost of transporting vehicles to achieve the ideal vehicle distribution plan. Shu et al. (2010) slightly simplify this same problem and adopt a linear approximation optimization model. Their analysis captures broader considerations by incorporating the day-to-day demand changes within a week. They also demonstrate the benefits of redistributing vehicles at the beginning of each day. Jorge et al. (2012) similarly apply a mixed integer programming model developed by Correia and Antunes (2012) to solve a similar problem under the deterministic environment, and further evaluate the solution via an ABS model. Romero et al. (2012) applied a simulation-optimization approach to find out the optimal locations of docking stations for a bicycle-sharing system. Bi-level programming was adopted where microscopic customer behaviors were accounted for. The study by Kumar and Bierlaire (2012) is oriented from a different perspective. Instead of applying operational research, their analysis is built upon a statistical model where they established a relationship between the “attractiveness” of stations and the socio-demographic-economic profile of the population residing nearby. In this way, new stations can be decided according to the ranking of attractiveness. García-Palomares et al. (2012) also solved the problem from another perspective by adopting a GIS approach to determine the optimal location of bike stations. The main contribution of their study is to apply GIS to extract demand information that serves as the input for the location-allocation model to calculate the final solution.

The above studies are helpful to determine an initial configuration of a MoD system. However, an essential part of daily operations, namely rebalancing of vehicle supplies, is missing from the formulations. In practice, imbalanced traffic flows frequently leave some stations empty of vehicles and others in need of additional parking space to accommodate an excess of vehicles. This necessitates redistributing vehicles and rebalancing of supplies as part of the daily operations of a MoD system.

Pavone et al. (2012) use a fluid-model approximation to develop quasi-optimal rebalancing policies. They first identify the conditions under which a MoD system reaches equilibrium, and then propose a simple optimization model to compute fixed rebalancing rates in such an equilibrium state. Schuijbroek, Hampshire, and Hoeve (2013) model vehicle flows at each station through a M/M/1/K queuing system which is then used to obtain an equation for computing bounds on the inventory of vehicles needed at each station. In addition, a cluster-first route-second heuristic model is proposed to obtain optimal routings of the rebalancing vehicles. Nair and Miller-Hooks (2011) and Nair et al. (2013) apply chance constraint programming to model the problem. An advanced algorithm is also developed to transform the stochastic problem to a set of mixed integer programming problems. Unlike the previous studies that analyze a particular period in a day, Vogel et al. (2014) formulated a mixed integer programming model to find the rebalancing operations for every time period of a day, and a hybrid metaheuristic algorithm was devised to find the solution. Kek et al. (2009) devise a decision support tool to determine the optimal rebalancing strategy. They apply the model to data from a Singapore-based car-sharing system and demonstrate the significant cost reduction that can be achieved through the introduction of vehicle relocation operations. Smith et al. (2013) takes a further step by exploring not only how to optimally rebalance vehicles, but also the best strategy for employing rebalancers.

2.2 Research on flexibilities in Engineering Design

There are three major research areas related to designing flexible engineering systems: concept generation, design space exploration, and process management.

Research on concept generation is concerned with devising effective methods and guidelines to organize such activities. Each flexible concept is comprised of (1) a strategy, and (2) enablers in design and management (Cardin, 2014). A strategy decides how the system will change when a certain situation happens; while enablers are the design features incorporated into the system initially that makes the strategy feasible in the future. Cardin et al. (2013) employ brainstorming, prompting, analogy, and explicit lecture to assist designers identify possible flexible strategies for an emergency system. They also evaluate the effectiveness of these ideation techniques in terms of generating flexible concepts. Fricke and Schulz (2005) extract general design principles that can be applied to enable changes in the system throughout its lifecycle. Similarly, Skiles et al. (2006) summarize principles and facilitators that enable products to acquire new or enhanced functionality in case of future requirements. Mikaelian et al. (2011) also propose generalized principles of designing enablers of flexible concepts. In addition, a number of modeling tools that analyze how flexibility initiates changes between system components are also proposed, such as the Change Propagation Analysis (Suh et al., 2007) Particularly, De Lessio et al. (2013) applies Design Structure Matrix (DSM) to identify the flexibility within a MoD system, Their analysis provides a high-level view of the structure of MoD system using DSM covering aspects like strategy, organization, infrastructure, stakeholders, operations and technology. The DSM is further utilized in the Change Propagation Anlaysis to discover the potential area to incorporate flexibility in the system. In sum, studies on concept generation aims to facilitate and inspire designers to generate “sketches” of the flexible concepts that will be analyzed in much more details later, namely the phase of design space exploration.

Design space exploration is associated with the quantitative evaluation and optimization of flexible designs. The value of flexibility (VoF) can be assessed from several perspectives. The classical works in pricing financial options (Black & Scholes, 1973; John C. Cox et al., 1979) are the origins of the real options valuation. Later, the binomial approach by John C. Cox et al. (1979) is applied to value options on real investments, hence real options (Trigeorgis, 1996). This approach has also been applied to assessing the value of real options in engineering systems (de Weck et al., 2004). Decision tree is another method to calculate the VoF, when the number of decisions is limited and the uncertainty can be modeled by discrete random variables. This method has been adopted to evaluate the VoF in oil deployment projects (Babajide et al., 2009). An important aspect of these two methodologies is that the evaluation process relies on dynamic programming where the decision rule essentially is decided by optimizing at each decision point based on expected value. This kind of approach, however, may not capture well the full realm of possible decision rules. Besides, as the number of decision-making periods and states increases, the computation may become intractable. Although the assumption of path-independency in the binomial approach allows a recombination structure to relief the computational burden, such assumption of path-independency may not hold for engineering systems. This is because different realizations of uncertainty may lead to distinct changes on system configurations, which consequently results in a disparity in realizations of the next time period. For instance, higher water price might trigger the upgrade on the efficiency of urban water systems, which may lead to lower water price in the next period. Another approach to assess the VoF is simulation that can be more generally applied. It has fewer restrictions on the number of time periods being considered as well as the distribution of uncertainties. Besides, this approach considers decision rules as explicit variables in the modeling framework, so that the model itself can be modified without too much endeavor so as to capture a wider range of design configurations.

Another community looks into efficient search mechanisms to explore the design space. Finding the optimal design for a flexible engineering system might be very daunting in some cases. First, a large number of design alternatives might be under consideration. For example, in a MoD system, flexibility can exist in multiple levels of decisions, e.g. changing the location and capacity of stations as a strategic-level flexibility or re-allocating vehicles in daily operations as an operational-level one. Besides, at each level of decisions exist many possible flexible designs or decision rules. Second, due to the complexity of the flexible system itself, there might be a lack of explicit mathematical formulation in terms of evaluating performance, which is commonly seen in the design of stochastic discrete-event systems, such as queuing networks (Shi, 2000).

Under this circumstance where analytic models cannot be formulated, simulation is gaining in popularity as the approach to model and analyze flexible designs. However, given the long planning horizon, which is very common in flexible design optimization, added by the intricate interactions within the system, obtaining a good statistical estimate of the performance of each design alternative is generally very time-consuming. Therefore, if requiring estimates with good quality and the design space is relatively large, the computation cost involved in searching the optimal flexible design can be prohibitively high. Past studies (Lin, 2009; Wang, 2005; Yang, 2009) have addressed this problem mainly by applying metamodels (or screening models), which intends to quickly estimate the performance of design alternatives via low-fidelity models and other statistical methods. The low-fidelity models are achieved by simplifying the physical relationship between performance and design variables, and/or representing the performance function by an approximate functional relationship. Once a metamodel is obtained, in principle, appropriate deterministic optimization can be applied to obtain an estimate of the optimal solution (Fu, 2002).

The last question that follows the calculation of the optimal flexible design goes into what is the favorable condition for its implementation and operation. Research on process management provides guidelines and techniques for such activities. Smit and Trigeorgis (2009) study the runaway capacity expansion option for two European airports, and show the importance of information-sharing and the timing of exercising flexibilities. Recently, another approach using serious gaming has been more and more applied to the design problems of engineering systems. This approach is defined as experience-focused, experimental, rule-based, interactive environments where participants learn by taking actions and by experiencing their effects through feedback mechanisms that are deliberately built into and around the game (Ligtvoet & Herder, 2012). Cardin et al. (2015) apply serious gaming to the design and management of flexible urban emergency systems. Their simulation platform suggests a way of experimentally studying and evaluating the effectiveness of training and other uncertainty management techniques.

2.3 Summary of research gaps

In the existing literature on MoD systems, it is common that the problems of planning/designing a MoD system and operating it are decomposed into several parts and then solved independently. Relocation/rebalancing decisions are made under an assumed system configuration and, conversely, the configuration of the system is determined without consideration of the impact of the relocation/rebalancing policies. In reality, however, ignoring the close interactions between these two sets of decisions may lead to significant problems and to poorly design and operating of MoD systems. For example, stations may be deployed with a larger capacity than would otherwise be necessary if rebalancing policies were taken into account. As demonstrated in a case study by Lin et al. (2013) on a bicycle-sharing system, bicycle stocks at stations can be overestimated by thousands if rebalancing policies are not considered. In fact, two studies have furthered the investigation on the interactions between planning and operational decision-making for MoD systems. Cepolina and Farina (2012) apply a simulation-based approach to determine the optimal distribution of vehicles for a MoD system consisting of Personal Intelligent City Accessible Vehicles (PICAVs) that solves the mobility problem in pedestrian area. Their analysis captures the variability in customers' travel patterns, and what's more, incorporates a relocation policy relying on a system supervisor to instruct customers to complete the relocation task. However, such rebalancing practice may not be easily executed in reality, especially for car-sharing systems where switching destination stations may lead to a relatively long walking distance, which consequently discourages customers to comply with such policy. In contrast, the paper by Boyacı et al. (2015) assumes that system operators takes the responsibility of rebalancing. Although their study provides interesting results on integrating the location and sizing issue of stations in MoD systems with operational decisions, the analysis is only deterministic since daily fluctuations in demand are not well captured. In fact, because of this deterministic approach, rebalancing decisions are made at the same time as planning decisions, even though rebalancing decisions are supposed to react in real-time to the realization of demand. In addition, the paper deals with a reservation-based system in which system operators are entitled with the power to choose among customers; however, for some VSS such as Car2go, reservations are not compulsory, let alone the fact that some VSS are purely on-demand, like the UCR IntelliShare system (Barth et al., 2000). Such systems allow greater flexibility for customers, but are generally more difficult in terms of system management. In sum, the existing literature does not provide an approach that helps to determine the initial set-up configuration of a MoD system considering the influence from operational decisions as well as the stochasticity of demand. Ignoring such influence may ultimately suggest a system design that suffers from inefficient use of resources and operating difficulties.

Moreover, not only are there very few studies capturing the short-term demand fluctuations, studies dealing with long-term uncertainty on overall usage patterns are almost nonexistent. However, as the total amount of urban population as well as its distribution constantly changes, accompanied by other factors that influence people’s behaviors on using MoD systems, e.g. reconfiguration of urban planning, assuming a constant overall usage pattern of such system seems unrealistic. The differing environment requires the configuration of a MoD system to evolve over time. Such situations suggest an opportunity of applying “flexibility thinking” to the design and management of MoD systems. In fact, one past study by Fassi et al. (2012) has already taken one further step by building a discrete-event simulation model to test growth strategies for two-way car-sharing systems where increasing demand is under consideration. Albeit their study demonstrates the importance of restructuring the system constantly, it can only be considered as a preliminary step to incorporate flexibilities into MoD systems. First, the study deals with two-way MoD systems where the design and especially the operational part of such systems is not so complicated as one-way systems, since the latter needs to address the unbalance traffic flows. Second, the study only considers too limited number of scenarios to be perceived as a stochastic analysis. Finally, although the simulation platform can be applied as a test bed for different combinations of flexible strategies, it does not provide any recommendation on how to find the optimal one. Given that the design space of flexible strategies can be very large, it may not be realistic to try every combination and then decide the best one, which, therefore, requires a proper optimization technique to be developed.

Above all, there exist two research gaps in the design and management of MoD systems, 1) an approach that integrates operational decisions into the strategic planning of the system as to better deal with the daily (short-term) demand fluctuations, and 2) a different deployment strategy that stems from a long-term forward view in system planning as to enable the system to adapt to changing usage patterns. .

Introducing flexibilities into engineering system, particularly regarding finding the optimal one to implement, however, can be computationally intractable. This study applies a simulation-based approach to the MoD system. In addition to the general benefits of the simulation approach, which is introduced earlier (i.e. ease of modelling a variety of flexible designs and uncertainties), there are other practical considerations. As demonstrated in past studies related to MoD systems, the operational-level decisions, namely rebalancing operations, play a very critical role in the overall performance of the system. The major advantage of applying a simulation-based approach is that the microscopic behaviors of frequent redistribution of vehicles within the system can be easily modeled; hence their influence on the final decisions is accounted for. Otherwise, if the analytic model is adopted, the computational complexity might be intractable. For example, in a system with eight stations, if hourly rebalancing is adopted and 17 hours are considered, 64 decision variables need to be created for just a one single rebalancing action, then it will be a number of 64*17=1,088 decision variables needed for just one day in one scenario. Such computational complexity can increase exponentially as the number scenarios and the number of stations increases.

Nevertheless, as introduced earlier, the computational issue may still remain a problem when the simulation-based approach is taken. Existing studies on this topic resorts to metamodel techniques, however, there may be some limitations with respect to using those surrogate models. First, it might be difficult to directly replicate the metamodelling techniques used by one specific problem to another. Different systems may prefer different functional models, which are the models with a generalized form does not necessarily apply to a particular kind of problem, with some systems appropriately represented by polynomial models or even linear models, while others requiring more advanced ones such as Gaussian process. Furthermore, in some cases, as indicated in Osorio and Chong (2014), functional models are not sufficient to capture the main system characteristics. Consequently, this calls for the development of physical models that represent the fundamental working principle of the system, but generally it requires significant efforts spent on the theoretical aspect of the systems. In the case of MoD systems, so far, there are no such physical models, either analytic equations or generalized laws that explain how different levels of decisions interact with each other as to influence the system performance. Second, the beauty of applying simulation to model and evaluate flexibilities is the ease of formulating different decision rules, which, however, may require significant effort to develop in a metamodel that suits each set of the decision rule. Such case results in difficulty in the comparison between the formulations of the decision rules. Last but not the least, the accuracy of metamodels is another concern, especially in the presence of uncertainties. On one hand, it has been found that the accuracy for evaluating the standard deviation of performance and the probability of constraint feasibility largely depends on the capability of a metamodel in capturing the nonlinearity and variations of a behavior (Jin et al., 2003). Therefore, for a system with significant randomness, using certain metamodels might not be sufficient to capture the variations of the system. A Kriging model performs relatively well in capturing various forms of functions, but the construction process can be very time-consuming and fitting problems due to singularities have been observed (Jin et al., 2003). On the other hand, when dealing with high dimensional stochastic problems, developing the metamodel itself may be challenging. For a single design point, a large number of runs may be required to obtain a good estimate of the performance, since the accuracy of the estimate cannot be improved faster than the rate $O(1/\sqrt{n})$ in the Monte Carlo simulation. Hence, when encountered with high dimensional problems, the overall computation cost can be very large. For example, in order to fit a reasonable metamodel via polynomial regression, the sample size should be at least two or three times the number of model coefficients (Jin et al., 2003). In sum, instead of using metamodelling techniques, it may worth exploring other techniques to resolve the computational complexity involved in optimizing flexible systems designs under explicit consideration of uncertainty.

Table 2.1 lists the major literatures that motivate this thesis. The contribution of this thesis poses on two aspects. In the domain of MoD systems, the thesis intends to propose a systematic methodology that addresses all the issues summarized in the table. Meanwhile, the thesis also would like to explore other opportunities to solve the computational complexity inherited in the optimization process of the flexible MoD systems, other than using metamodeling. This might provide further insights and instructions applicable to other flexible engineering systems.

In conclusion, this thesis aims to address the following research questions:

1) how to account for the influence of operational-level decisions and the stochasticity of demand when determining the configuration of a MoD system;
2) how to formulate flexibility into the deployment strategy of a MoD system as to address the uncertain usage pattern changes;
3) how to address the computational complexity in finding an optimal flexible MoD system

Table 2.1 Literatures motivating this thesis

Articles	Rebalancing operations	Strategic planning	Daily demand fluctuations	Long-term uncertainty
(Pavone et al., 2012; Schuijbroek, Hampshire, & van Hoeve, 2013)	✔		✔	
(Correia & Antunes, 2012; Lin et al., 2013)		✔		
(Barth et al., 2000; Jorge et al., 2014)		✔	✔	
(Boyacı et al., 2015)	✔	✔		
This thesis	✔	✔	✔	✔

The objectives and contributions of this thesis summarized in Section 1.3 aim to address the aforementioned research gaps and answer the research questions. The next chapter provides an overview of the simulation-based methodology that is at the heart of this thesis, and used to analyze MoD systems in Chapter 4-5. It also provides an example study showing how the method is used to analyze a complex engineered system considering uncertainty and flexibility explicitly.

Chapter 3 Methodological Approach: Simulation-Based Analysis

This chapter illustrates how simulation is incorporated into the design procedure to formulate and evaluate flexible design alternatives under uncertainty. A four-step procedure is introduced in this chapter that starts with the standard design practice and proceeds gradually towards the “flexibility thinking”, which gives designers a continuous education on “thinking out of box”. A case study on water management systems is presented as demonstration to offer a step-by-step explanation on the use of the simulation-based procedure for uncertainty and flexibility analysis. While the author fully acknowledges that water management systems is not the main application domain of this thesis, a similar analytical logic is used in Chapters 4-5 to analyze MoD systems under uncertainty and flexibility, although some steps, e.g. the deterministic analysis, may be skipped to focus on assessing the value of flexibility. Thus, it is deemed important to dedicate one chapter to introduce thoroughly the methodological approach. Also, this study has been published in Deng et al. (2013).

This chapter serves illustrates some of the drawbacks in standard design and project evaluation practice. It also highlights some the advantages of recognizing flexibility as a way to improve a system performance in the face of uncertainty. Also, the study illustrates the pros and cons of using a simulation-based approach to model and evaluate the performance of different design alternatives, including flexible ones, and explains why the thesis takes a simulation-based approach.

3.1 Procedure for designing and evaluating flexibilities via simulation

This study proposes a four-step procedure to design and evaluate flexible systems when simulation is applied to estimate system performance under uncertainty. The procedure is based on and modified from a design process proposed in a past study (Cardin et al., 2007). Similar to the original one, this proposed methodology is also a step-wise process for designing and evaluating flexibility in design of complex systems, starting with the baseline model and further stepping into the uncertainty analysis and the flexibility analysis. One additional step of sensitivity analysis is added to provide more reliable results.

3.1.1 Step 1: Baseline model

The starting point of the procedure is to build a baseline model. The objective here is to understand the main components of the system that influence its full life cycle performance. Costs and benefits involved in the system are calibrated by defining necessary design parameters and design variables. Additionally, assumptions are made on the working principle of the system. Following that, a preliminary deterministic cost-benefit analysis (Boardman et al., 2006) can be carried out. This step captures the standard practice in terms of design and project evaluation.

3.1.2 Step 2: Uncertainty analysis

In this step, designers need to model major uncertainty drivers, and investigate design alternatives under a range of possible scenarios. It is in this step that simulation model is built and begins to play a role in the analysis.

Historical data on the uncertainty drivers is first collected and calibrated into stochastic or probability models, like Geometric Brownian Motion (GBM) and normal distribution. Then, simulation is applied to assess the performance of design alternatives under these uncertainties. Through this analysis, designers capture a more comprehensive picture about the pros and cons of the design alternatives, compared with the deterministic analysis in Step 1. After these two steps, a simulation is obtained that is able to evaluate the system performance under uncertainty. The results can be displayed by a multi-metric table, where multifold indicators, e.g. expected value, value at risk, and value at gain, are shown for decision-makers who may have different risk profiles. Such tables will be presented in the analysis of this thesis. However, in terms of final decision-making, it assumes that decision-makers are risk-neutral.

3.1.3 Step 3: Flexibility analysis

In this step, designers first need to generate flexible design concepts. A complete flexible concept is defined by four elements: uncertainty source, flexible strategy, flexible enabler, and decision rule (Cardin et al., 2013). Flexible strategies are the actions designers can take when a particular path of uncertainties is realized (e.g. expand the capacity of the system if demand turns out to be higher than prediction), while flexible enablers are the design configurations that make the strategies feasible from a design and management standpoint. A decision rule is a triggering mechanism that is commonly represented by an “if” statement that specifies clearly when the flexible strategies will be exercised, based on some uncertainty realizations. For example, in the case of the HCSC building (Guma et al., 2009), the flexible concept can be "in order to deal with the uncertainty from working space demand, extra strength is added into the load bearing walls so that the building can be expanded if the working space is not enough". In this case, adding extra strength is the flexible enabler and capacity expansion is the flexible strategy, while the "if" statement about the working space is the decision rule.

After flexible concepts are formulated and the parameters (e.g. triggering point of exercising flexibilities) are determined, they are programmed into the simulation model and evaluated under the same condition as in step 2. The Value of Flexibility (VoF) is calculated by Equation 3.1.

$$VoF = ENPV_{flexible\ design} - ENPV_{baseline\ design} \tag{3.1}$$

This thesis does not explicitly formulate the cost of flexibility into the analytical process, which means VoF obtained by the above equation does not take account into the additional cost associated with enabling the system to possess such flexible features. Such assumptions results from the complexity inherited in the process of estimating such cost and may be highly subject to the profile of a decision-maker. For example, in the context of a MoD system, an expansion option may require the company to sign a contract with the local government who reserves some parking spaces for the MoD company to expand in the future. It is very difficult to estimate such contract cost. Besides, it is heavily case-by-case depended, which is largely affected by negotiation power of the company as well as the characteristics of geographic area. As such, formulating cost of flexibility is beyond the scope of this thesis. In contract, the thesis focuses on estimating the

VoF, which, in fact, provides an upper bound of the cost of flexibility. Decision-makers can compare the VoF and cost of flexibility later as to determine whether to implement the flexible design or not.

3.1.4 Step 4: Sensitivity analysis

Finally, sensitivity analysis is carried out in order to assess how the results obtained respond to changes in underlying assumptions. This step can be seen as a way to test the robustness of the design alternatives in response to the variations that may happen to the assumptions. There are several standard mathematical methods that can be applied in terms of doing sensitivity analysis. For example, one-factor-at-a-time method (OFAT) (Czitrom, 1999) is one of simplest and most common approaches.

3.2 Case study - valuing flexibilities in urban water management systems via simulation

This section presents a study where flexibility is incorporated into urban water management systems as intelligent decision-making mechanisms that enables the system to mitigate potential impact from downside risks and increase opportunities for upside gains over a range of possible futures. A simulation-based approach is applied here to estimate the performances of the system under different conditions, namely with and without flexibility.

The study further illustrates how the four-step procedure is implemented on a real-world urban infrastructure application. While not directly connected to the analysis of MoD systems, the case demonstrates the use of simulation for analyzing a complex system under uncertainty as done in Chapter 4, and estimating the VoF, which is crucial to Chapter 5. The numerical results obtained in this study also indicate whether a flexible system is able to outperform a more rigid, fixed design alternative, which in this case is achieved by reducing the initial investment and adapting to the changing environment.

3.2.1 Case introduction

As part of an effort to investigate possible solutions for next-generation water infrastructure systems, which aims to reduce damage caused by floods in rainy seasons and reuse of the run-offs, a new technology based on porous pavements and green roofs is being proposed. The technology allows rainwater to infiltrate into the sub-surface layer where it is temporarily stored. For porous pavements, the sub-surface layer is filled with porous materials, while for the green roofs, the vegetation cover and space underneath function as the storage facility. The stored rainwater is then either detained in the ground, or harvested by the pipe installed under pavements or the underneath space of green roofs. Later this harvested water can be recycled as “grey” water or be channeled to reservoirs. By implementing this technology, revenues (as cost savings of re-using rainwater) are generated. Besides, the porous pavements and green roofs reduce frequency and peak flow rate of rainwater that enters the drainage system. Consequently, less space is required for drainage, and the likelihood of flooding damage is also reduced (Zhang & Buurman, 2010).

A test site has been chosen for a preliminary analysis on the possibilities and limitations of this innovative solution. The site is located within the Kent Ridge campus of the National University of Singapore (NUS). The size of the catchment has an area of about 8.2 ha. The land use distribution of the catchment comprises the following: 41% of bushes, 35.5% of other green areas, mostly grass patches on mild and steep slopes, 16.8% of rooftop and 4.77% of road areas.

There are two considered design alternatives: a traditional expansion of the current drainage canal system (referred as design A) and alternative based on catchment measures of porous pavements and green roofs (referred as design B). Although design B has several aforementioned advantages compared with design A, since design B incurs a higher construction cost and maintenance cost, analysis is needed to better understand the costs and benefits. Also, one aims to assess whether there is potential to further improve the economic performance of those two design alternatives under uncertainties by applying the flexibility analysis.

3.2.2 Step 1: Baseline DCF model

The following is the list of notations used in the analysis.

$\mathrm{Area}_{\mathrm{total}}$	Total area of the test site under study (m^2)
$\mathrm{Area}_{\mathrm{p}}$	Area that can be deployed to porous pavements (m^2)
$\mathrm{Area}_{\mathrm{r}}$	Area that can be deployed to green roofs (m^2)
DC_{A}	Drainage capacity of canals in design A (m^3)
DC_{B}	Drainage capacity of canals in design B (m^3)
D_{c}	Depth of canals (m)
$\mathrm{Area}_{\mathrm{c}}$	Area of existing canals (m^2)

$AExp_c$	Area of expanded canals in design A (m^2)
SC_p	Storage capacity of porous pavements (m^3)
SC_r	Storage capacity of green roofs (m^3)
D_p	Depth of porous materials in porous pavements (m)
D_{rc}	Depth of vegetation covers in green roofs (m)
D_{rs}	Depth of underneath space in green roofs (m)
P_p	Porosity of porous materials in porous pavements
P_r	Porosity of vegetation covers in green roofs
Re_p	Recycle efficiency of porous pavements
Re_r	Recycle efficiency of green roofs
$Capex_A$	Initial investment of design A ($)
$Capex_B$	Initial investment of design B ($)
Uf	Unit flood damage cost ($\$/m^3$)
Um_A	Unit maintenance cost of design A ($\$/m^2$)
Um_p	Unit maintenance cost of porous pavements ($\$/m^2$)
Um_r	Unit maintenance cost of green roofs ($/m2)
Uc	Unit cost of water treatment ($\$/m^3$)

dr	Discounted rate
AC_k	Annual cost in kth year ($)
AR_k	Annual revenue in kth year ($)
Pr_k	Unit water price in kth year ($/m^3)
RQ_{ik}	Rainfall quantity of the ith rain of the kth year (m)
RN_k	Number of rain events of the kth year

The assumptions for the design parameters and input data needed in the case study are shown in Table 3.1. The cost information is based on personal communications with the design team members. Although it may not be perfectly accurate, it is based on experienced designers' inputs and reflects the essence of the system to some degree. The annual rainfall information is summarized from the online published data of National Environmental Agency ("Weather Statistics," 2013).

Table 3.1 Assumptions on parameters

Assumptions on parameters					
Catchment area (m^2)	82000	Pave area (m^2)	7500	Roofs area (m^2)	13000
Recycle efficiency (roofs)	0.45	Recycle efficiency (pavements)	0.65	Depth of canals (m)	0.5
Existing area of canals (m^2)	2600	Expanded area of canals (m^2)	5400	Porosity (pavements)	0.3
Depth (pavements) (m)	0.3	Porosity (roofs)	0.6	Depth (vegetation of roofs) (m)	0.15
Depth (space underneath of roofs) (m)	0.3	CAPEX (design A) ($)	150,000	CAPEX (design B) ($)	421,875
Maintenance cost (pavements) ($/$m^2$)	1.0	Maintenance cost (roofs) ($/$m^2$)	1.2	Maintenance cost (canals) ($/$m^2$)	0.85
Water price ($/$m^3$)	1.7	Water treatment cost ($/$m^3$)	0.3	Flood damage cost ($/$m^3$)	0.5
Average NO. of rain events (year)	178	Average rainfall in one rain (mm)	13.16	CAPEX (Flexible B) ($)	300,000
Expansion cost (pavements) ($/112.5m3)	70,000	Expansion cost (roofs) ($/650m3)	50,000	Expansion cost (canals) ($/740m3)	48,000

For design A, the following equations (Equation 3.2 - Equation 3.4) are developed. As there are no mechanisms of generating revenues, the analysis only needs to quantify the costs involved. There are two categories of costs under consideration. Flood damage cost is calculated based on the occurrence of the rain events where the rainfall quantity exceeds the drainage capacity. Maintenance cost is a variable cost that links with the drainage area. The maintenance cost is related to activities of physical cleansing, maintenance and minor structural repairs of drains and canals.

$$\mathrm{AC_k} = \sum_{\mathrm{i=1}}^{\mathrm{RN_k}} \max(0,\ \mathrm{RQ_{ik}} * \mathrm{Area_{total}} - \mathrm{DC_A}) * \mathrm{Uf} + (\mathrm{Area_c} + \mathrm{AExp_c}) * \mathrm{Um_A} \quad \mathbf{3.2}$$

$$\mathrm{DC_A} = (\mathrm{Area_c} + \mathrm{AExp_c}) * \mathrm{D_c} \quad \mathbf{3.3}$$

$$\mathrm{NPV_A} = -\mathrm{Capex_A} - \sum_{\mathrm{k=1}}^{50} \frac{\mathrm{AC_k}}{(1+\mathrm{dr})^{\mathrm{k}}} \quad \mathbf{3.4}$$

As to design B, more refined equations (Equation 3.5 – Equation 3.10) are developed, since not only costs need to be quantified but also the revenues generated as cost savings. In this case, the extra rainwater, that can neither be evacuated through drainage canals nor be captured by the new catchment measures, incurs flood damage cost. For the existing drainage canals, the maintenance cost is estimated by the same approach with design A. For porous pavements, the maintenance cost is mainly for required annual vacuum-sweeping activities, while for green roofs, it is used to carry out cleansing and vegetation maintenance. The area that installs this new catchment measure and the unit cost determines the total maintenance cost. The calculation of revenues is based on rainfalls, recycle efficiency and storage capacity, as indicated in Equation 3.9.

$$\mathrm{AC_k} = \sum_{\mathrm{i=1}}^{\mathrm{RN_k}} \max\big(0,\ \mathrm{RQ_{ik}} * \mathrm{Area_{total}} - \mathrm{DC_B} - \min\big(\mathrm{SC_p}, \mathrm{RQ_{ik}} * \mathrm{Area_p}\big) - \min(\mathrm{SC_r}, \mathrm{RQ_{ik}} * \mathrm{Area_r})\big) * \mathrm{Uf} + \mathrm{Uc} * \big(\min\big(\mathrm{SC_p}, \mathrm{RQ_{ik}} * \mathrm{Area_p}\big) * \mathrm{Re_p} + \min(\mathrm{SC_r}, \mathrm{RQ_{ik}} * \mathrm{Area_r}) * \mathrm{Re_r}\big) + \mathrm{Area_c} * \mathrm{Um_A} + \mathrm{Area_p} * \mathrm{Um_p} + \mathrm{Area_r} * \mathrm{Um_r} \qquad \textbf{3.5}$$

$$\mathrm{DC_B} = \mathrm{Area_c} * \mathrm{D_c} \qquad \textbf{3.6}$$

$$\mathrm{SC_p} = \mathrm{Area_p} * \mathrm{D_p} * \mathrm{P_p} \qquad \textbf{3.7}$$

$$\mathrm{SC_r} = \mathrm{Area_r} * (\mathrm{D_{rc}} * \mathrm{P_r} + \mathrm{D_{rs}}) \qquad \textbf{3.8}$$

$$\mathrm{AR_k} = \sum_{\mathrm{i=1}}^{\mathrm{RN_k}} \big(\min\big(\mathrm{SC_p}, \mathrm{RQ_{ik}} * \mathrm{Area_p}\big) * \mathrm{Re_p} + \min(\mathrm{SC_r}, \mathrm{RQ_{ik}} * \mathrm{Area_r}) * \mathrm{Re_r}\big) * \mathrm{Pr_k} \qquad \textbf{3.9}$$

$$\mathrm{NPV_B} = -\mathrm{Capex_B} + \sum_{\mathrm{k=1}}^{50} \frac{\mathrm{AR_k} - \mathrm{AC_k}}{(1 + \mathrm{dr})^{\mathrm{k}}} \qquad \textbf{3.10}$$

Based on the aforementioned assumptions and models, the deterministic analysis is carried out, where the two design alternatives are evaluated under deterministic values of unit water price, the number of rain events and the rainfall in a single rain event (Pr_k =1.7\$/m^3, RN_k =178, RQ_{ik} =13.16mm, $\forall i,k$). Table 3.2 summarizes the computation results. The deterministic DCF analysis shows that overall introducing porous pavements and green roofs may be more cost beneficial than the canal expansion alternative, as the former shows a less negative NPV compared with design A.

Table 3.2 Results of deterministic analysis

	Design A	Design B	Best Design
NPV ($)	-266,846	-252,274	Design B

3.2.3 Step 2: Uncertainty analysis

In this step, a simulation model is built as to assess the performance of the two design alternatives given that rainfalls and water prices subject to certain probabilistic models. The simulation model plays a core role in the rest of the analysis as the platform to test the performance of different design alternatives under the stochastic environment and varied assumptions. Results from this step provide a more comprehensive assessment on the design alternatives.

3.2.3.1 Assumptions

1) For the unit water price, the study relies on Geometric Brownian Motion (GBM) Process, captured by Equation 3.11 with drift assumed to be 1%, volatility 2% and Pr_0 1.7 $/m3.

$$dP_t = \mu P_t dt + \sigma P_t dW_t \qquad \textbf{3.11}$$

* μ-drift, σ-volatility, W_t-Wiener Process

2) Two major types of rain events are considered: normal rain events and storms. Normal rain events are simulated using only one scenario. Scenarios of storms are generated from IDF curves that are constructed by a company monitoring the rainfalls in Singapore.
3) Return period: 10 years. Based on Public Utilities Board (PUB) code of practice for surface water drainage ("Code of Practice-Drainage Design

and Considerations," 2011), since the area is less than 100 ha, a return period of 10 years is sufficient. The return period of each IDF curve also indicates the probability of the scenario. For example, for the scenario that has a return period of 1 month, the probability that it can happen in a specific day is 1/30. This assumption also indicates that only the ten IDF curves with return periods no more than 10 years are used to simulate storms.

4) Duration of a single rain event is normally distributed between 5 minutes and 420 minutes, with a mean of 60 and a variance of 100.
5) Only one rain event occurs in a single day.

3.2.3.2 Procedure of generating rainfalls in the simulation model

Under the assumptions above, the rainfall scenarios during the life cycle of the project have been simulated as follows (Figure 3.1).

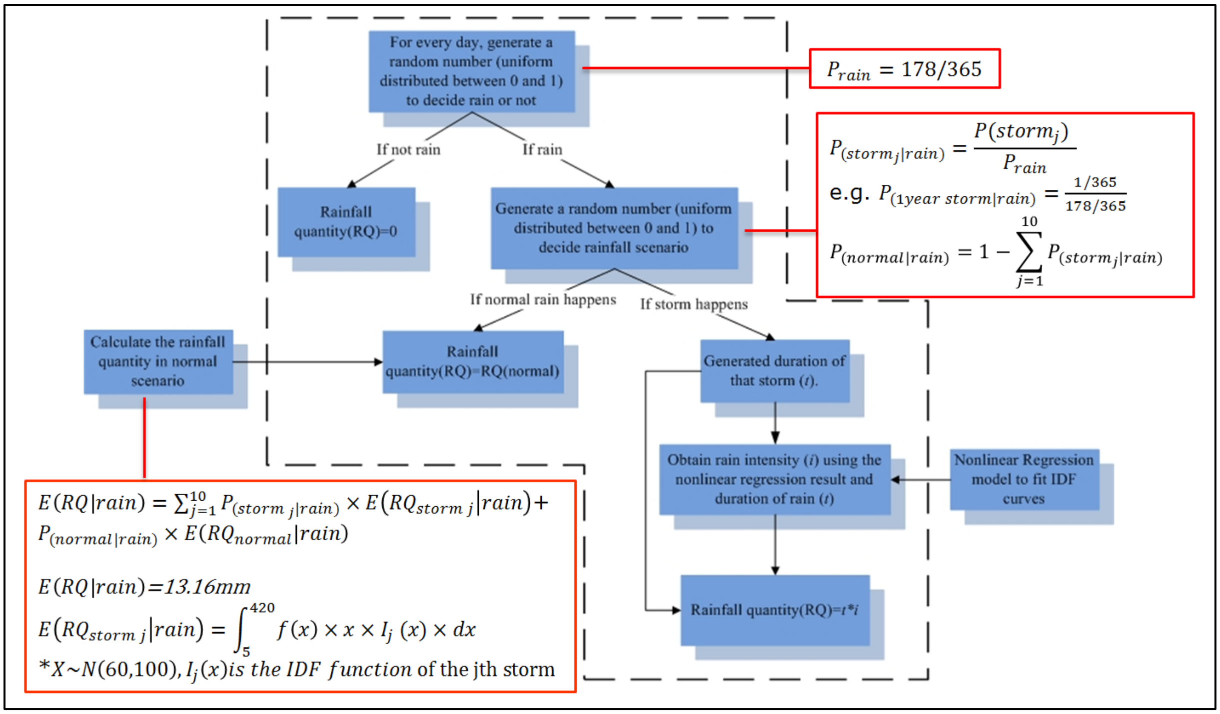

Figure 3.1 Procedure of generating rainfalls

1) Reverse engineering of the IDF curves by using nonlinear regression to calibrate the relationship between rain durations and intensities.
2) Calculate the rainfall of the normal rain event using the equations in Figure 3.1.
3) Apply the procedures described in the dashed box of Figure 3.1 to generate the rainfall scenario of a single day.
4) Repeating step 3) by 365 x 50 x *N* times, *N* scenarios of daily rainfalls in 50 years are obtained.

3.2.3.3 Evaluation results

2000 scenarios that contain the information of water prices and rainfalls are generated as to estimate the ENPV of the two design alternatives. Figure 3.2

shows the scenarios of yearly water prices, while Figure 3.3 illustrates the distribution of rainfalls.

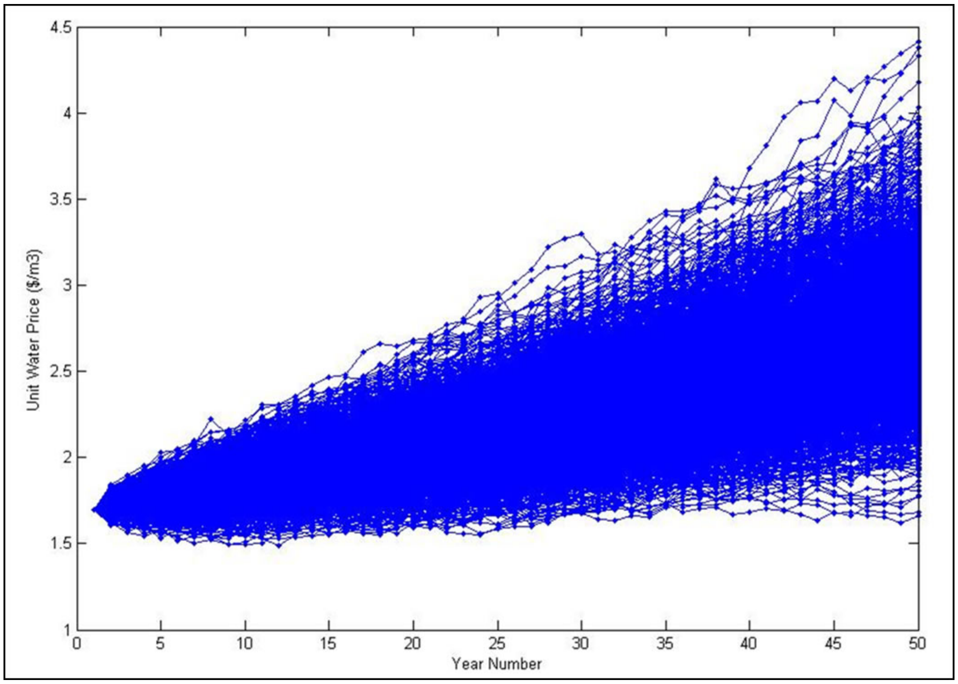

Figure 3.2 Yearly water price in the planning horizon

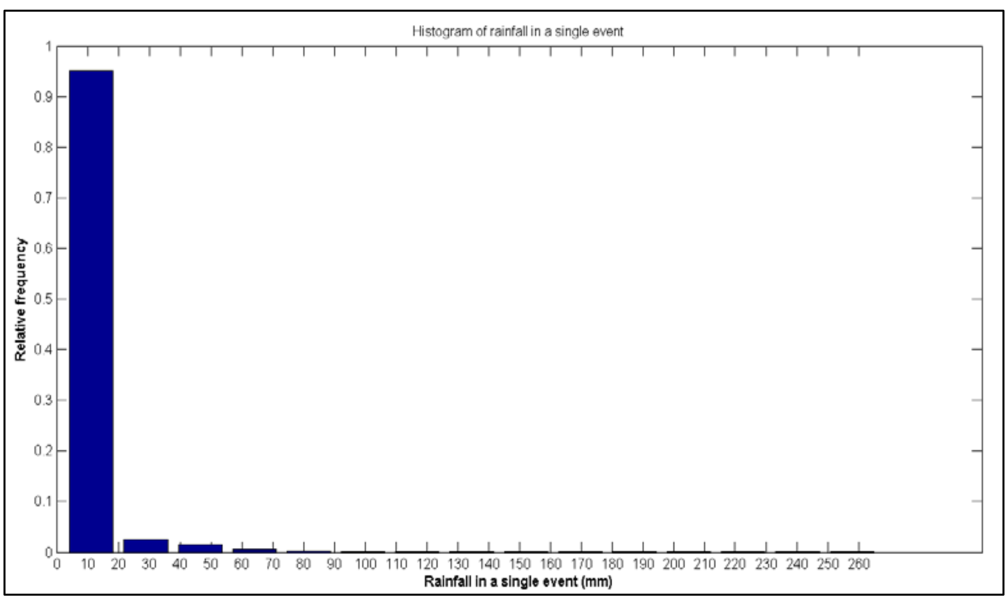

Figure 3.3 Histogram of rainfall in a single event

Combined with results from the deterministic analysis in the previous section and the uncertainty analysis, a probabilistic distribution (Figure 3.4) and a multi-criteria comparison table (Table 3.3) are constructed.

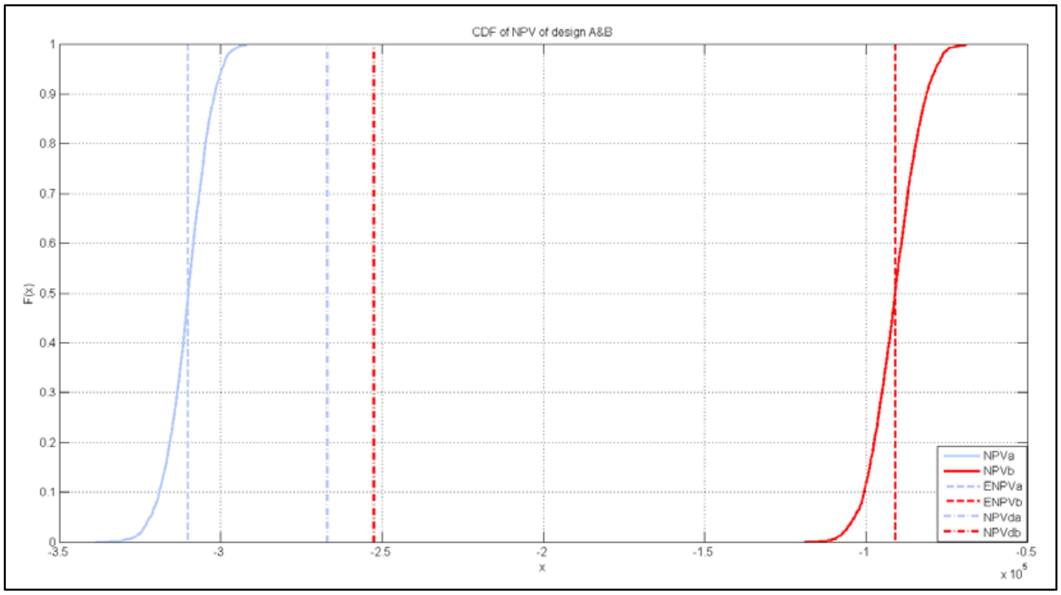

Figure 3.4 Distribution of NPV of Design A and Design B

Table 3.3 Multi-metrics table of Design A and Design B

	Deterministic NPV	ENPV	P5(VAR)	P95(VAG)	Standard error
Design A	-$266,846	-$310,207	-$321,648	-$299,584	$149
Design B	-$252,274	-$90,956	-$103,443	-$78,419	$169
Better Design	Design B	Design B	Design B	Design B	NA

As indicated from the above results, if only the deterministic analysis is referred as the basis of decision-making, although the ranking of design alternatives remains the same, the economic value of two design alternatives is either overestimated or underestimated. For design A, as shown in the cumulative probability curve, the likelihood that the realized NPV is smaller than the deterministic NPV is 1, which means the probability that such NPV can be obtained in the reality is negligible. This finding is supported by the Jensen's inequality (Jensen, 1906) shown in Equation 1.1. As we take the average of uncertainty drivers (unit price, rainfall quantity and number of rainfall events), the NPV in the upside scenarios cannot be averaged out by the downside scenarios. In fact, since here the flood cost is incurred when the rainfall is higher than the drainage capacity, as long as the assumed deterministic value of single rainfall is lower than the drainage capacity, there is no flood damage cost resulted in the whole life cycle of the system. In reality, however, the rainfall is subjected to high fluctuations, which leads to the presence of storms that lead to flood damage cost. The Flaw of Averages (Savage, 2000) is also observed in the result of design B but just turns out in an opposite direction. As shown in Figure 3.4, the deterministic value of NPV is even away from the lower tail of the CDF curve, which means the chance of obtaining such a low NPV in real world is very slim. As for the standard deviation, since design A is only subjected to the fluctuation of rainfall, while design B is influenced by rainfall and price of water, the variance of design A is relatively lower.

3.2.3.4 Further discussion

To further investigate how the two design alternatives perform under different scenarios, especially on the aspect of preventing flood damage, the flood damage costs under different rainfalls ranging from 0mm to 300mm are calculated. Figure 3.5 shows the computation results.

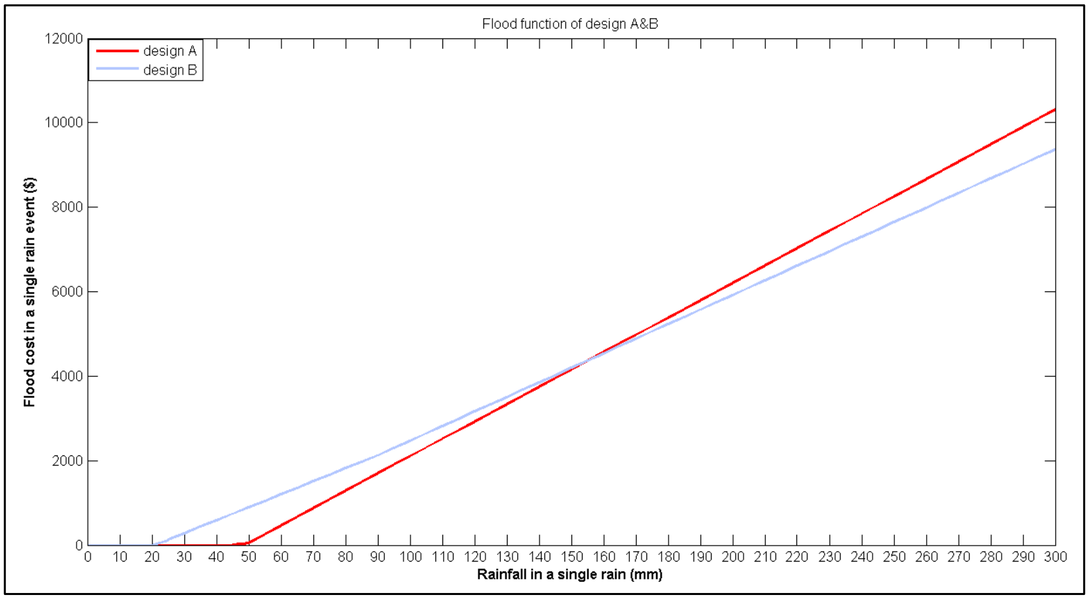

Figure 3.5 Flood functions of Design A and Design B

Based on Figure 3.5, when the rainfall is higher than 20mm, flood damage occurs to design B, while this threshold is almost doubled for design A. This indicates that there is a higher chance of flood damage in design B. Besides, since the rainfall in a single rain event is rarely higher than 160mm (shown in Figure 3.3), mostly higher flood damage costs happen to design B rather than design A.

This seemly counter-intuitive result may come from the fact that only a small proportion of the test area (40.02%) can be deployed to either green roofs or porous pavements. Therefore the rain dropping to other area that is not covered by the new technology can only be evacuated through existing drainage canals. Meanwhile, compared with design A that expands the capacity of canals, the drainage capacity in design B is much smaller. Because of the reasoning above, design B is more vulnerable to rainfall fluctuations in terms of flood damage.

3.2.4 Step 3: Flexibility analysis

In this step, flexibility is incorporated into both design A and design B. The conceptual designs are coded into the simulation model. The same number of scenarios (2000) is generated as to estimate their performances.

3.2.4.1 Flexible Designs

For design A, the flexible design is described as follows. The existing canals are not expanded at the beginning. If the number of floods happening within one year exceeds ten times, the drainage capacity will be expanded until it reaches the upper bound ($5000m^3$). This expansion option is further explained using the following expressions.

if $\sum_{i=1}^{RN_k} 1_{RQ_{ik}*Area_{total} \geq DC_A}(RQ_{ik}) \geq 10$ & $(DC_A + ExpSize) \leq maxA$

Then $\text{CapacityExpansion} == \text{true}$

* $1_{\mathrm{RQ_{ik}*Area_{total} \geq DC_A}}(RQ_{ik})$ is the indicator function of RQ_{ik}

As for design B, the same area is deployed for the new technology but only half of the depth is deployed for the pavements and the underneath space of roofs. If the number of floods happening within one year exceeds ten times, the storage capacity will be expanded by enlarging the depth until it reaches the upper bound. Details of this expansion option are shown below.

For green roofs,

if $\sum_{i=1}^{RN_k} 1_{RQ_{ik}*Area_r \geq SC_r}(RQ_{ik}) \geq 10$ & $(SC_r + ExpSize) \leq max_r$

$\text{Then CapacityExpansion} == \text{true}$

$*1_{\mathrm{RQ_{ik}*Area_r \geq SC_r}}(RQ_{ik})$ is the indicator function of RQ_{ik}

For porous pavements,

if $\sum_{i=1}^{RN_k} 1_{RQ_{ik}*Area_p \geq SC_p}(RQ_{ik}) \geq 10$ & $(SC_p + ExpSize) \leq max_p$

$\text{Then CapacityExpansion} == \text{true}$

$*\ 1_{\mathrm{RQ_{ik}*Area_p \geq SC_p}}(\mathrm{RQ_{ik}})$ is the indicator function of $\mathrm{RQ_{ik}}$

3.2.4.2 Evaluation Results

By summarizing results from the flexibility analysis and the uncertainty analysis, Figure 3.6 and Table 3.4are obtained. Figure 3.6 shows the distribution of the NPV of all alternatives, while Table 3.4 summarizes the information on the predefined metrics.

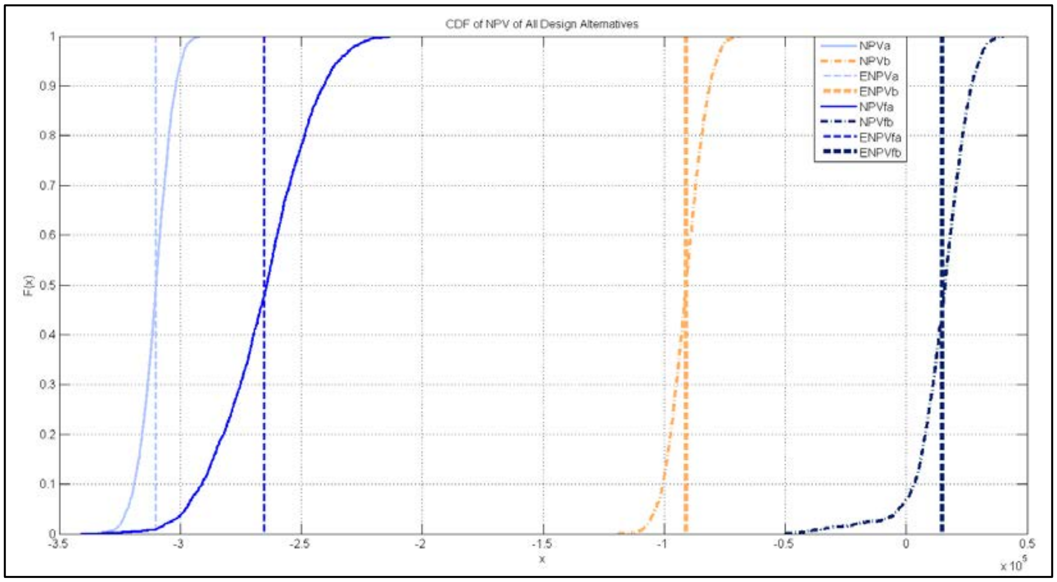

Figure 3.6 Distribution of NPV of all design alternatives

Table 3.4 Multi-metrics comparison table of all design alternatives

	ENPV	P5(VAR)	P95(VAG)	Standard error
Design A	-$310,207	-$321,648	-$299,584	$149
Design B	-$90,956	-$103,443	-$78,419	$169
Flexible A	-$265,367	-$297,644	-$234,641	$432
Flexible B	$14,843	-$3,121	$30,151	$257
Better Design	Flexible B	Flexible B	Flexible B	NA

For design B, based on Equation 3.12, the value of flexibility is $105,799. The results show that incorporating flexibility makes design B profitable as ENPV turns out to be positive. One interesting observation is that the value of flexibility closely corresponds to the difference in CAPEX between flexible design B and baseline design B ($121,875). This indicates that baseline design B may be designed with unnecessary storage capacity whereas flexible design B gains the advantage by reducing the redundant initial investment. This is further confirmed by the fact that among the 2000 times of simulation, porous pavements are expanded only in a small proportion of scenarios (197 out of 2000), and it has never reached the maximum capacity, while the expansion option is never exercised for green roofs.

$$VoF_B = ENPV_{fb} - ENPV_b \quad \textbf{3.12}$$

For design A, the economic performance is also improved by considering the flexibility of a staged capacity deployment approach. According to Figure 3.4, the extra value brought by this expansion option is \$44,840. The expansion decisions made through the simulation indicate that the improvement on design A is also achieved through reducing excessive capacity of the inflexible design. It is found that in less than 10% of the simulation does the drainage capacity of canals expands above 4000m^3, and mostly (over 70%) a capacity of 3520m^3 is considered sufficient based on the decision rule. The trade-off between the economy of scale and the time value of money may be another factor that leads to the better performance of flexible design A. The influence from the economy of scale suggests that a larger capacity deployed all at once is a more economic decision, while the time value of money favors that more investment should be placed later. In the case of design A, the latter factor seems to impose more impact on the final result.

The analysis does not make explicit assumptions on the cost of flexibility, as accurate information regarding cost of these flexible designs, e.g. opportunity cost of reserving the land for capacity expansion, cannot be obtained. However, the VoF calculated provides an upper bound of the associated cost the flexibility, which can be an important reference for making decisions on the implementation of these two flexible designs.

3.2.5 Step 4: Sensitivity analysis

After the flexibility analysis has been carried out, the best design alternative, flexible design B in this case study, is selected and subjected to the sensitivity analysis. Namely, the performance of flexible design B is reevaluated under the change of major assumptions made in Table 3.1. Recycle efficiency, maintenance cost, treatment cost, discounted rate expansion cost, and flood damage cost are assumed to be major influences on the performance of flexible design B. OFAT is applied in the sensitivity analysis. The values of the aforementioned factors are varied by ± 20% and 5% at a time, and then the ENPV of flexible design B is reevaluated under the new inputs.

According to Figure 3.7, the variation of expansion costs imposes almost no influence on the ENPV. This evidence supports the conclusion made on the flexibility analysis, that the expansion is rarely exercised. On the other hand, the recycle efficiency of green roofs is shown to affect the performance of flexible design B most, which is even stronger than that of the discounted rate. The observation here contrasts to the recycle efficiency of porous pavements that does not influence the result so much. This difference between porous pavements and green roofs is also observed on the maintenance cost where green roofs lead to a stronger degree of changes on the ENPV. The observation may be resulted from the fact, that a larger area is deployed for roofs so that the ENPV depends more on the change of roofs. Water treatment cost also influences the result to a certain degree that is close to that of the maintenance cost of pavements, but higher than that of the unit flood cost. The relatively weak effect of unit flood cost may be explained by the low frequency of flood. This result also indicates the robustness of choosing design B, as in the worst case, the ENPV of flexible design B is still far better than flexible design A. Even a higher unit flood cost cannot diminish the advantage of flexible design B.

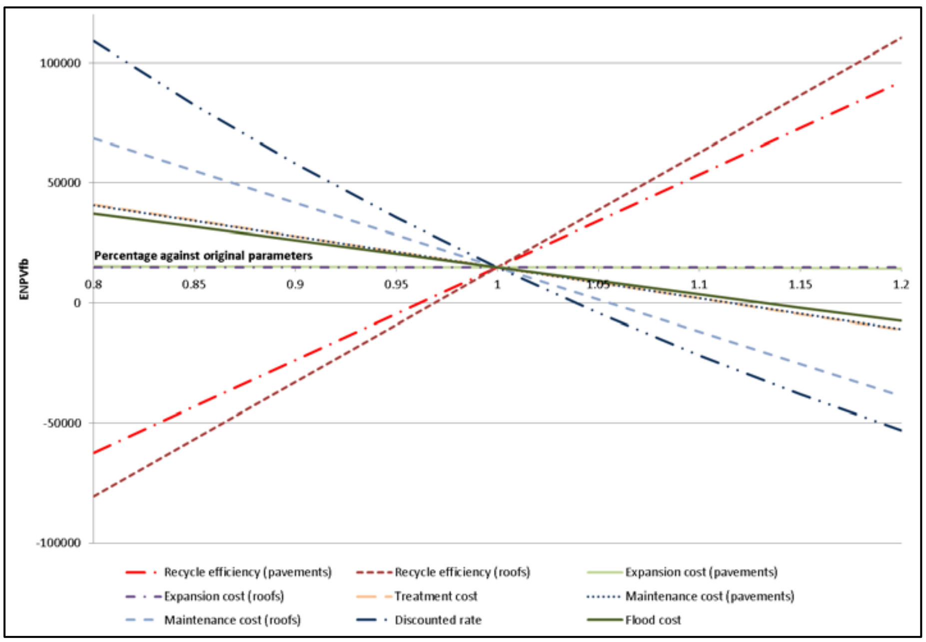

Figure 3.7 Sensitivity analysis of ENPVfb

By varying the same factors, this study also applies the OFAT to the VoFB. Results are shown in Figure 3.8. Due to the negligible influence of expansion cost and maintenance cost on the result, Figure 3.8 does not include these factors. According to Figure 3.8, discounted rate is the most critical factor on the VoF. The result on discounted rate also corresponds to the conclusion made in the flexibility analysis that a higher discounted rate contributes to a higher VoF. It is also interesting to note that increasing recycle efficiency of pavements leads to lower value of flexibility. One reason for this observation is that higher recycle efficiency may prefer developing a larger capacity at the beginning so as to generate more revenues by re-using more rainwater. On the contrary, the influence from the recycle efficiency of roofs is almost negligible. This is explained by the fact that the capacity of roofs is never expanded in the simulation. Treatment cost and flood damage cost only have a slight effect on the result.

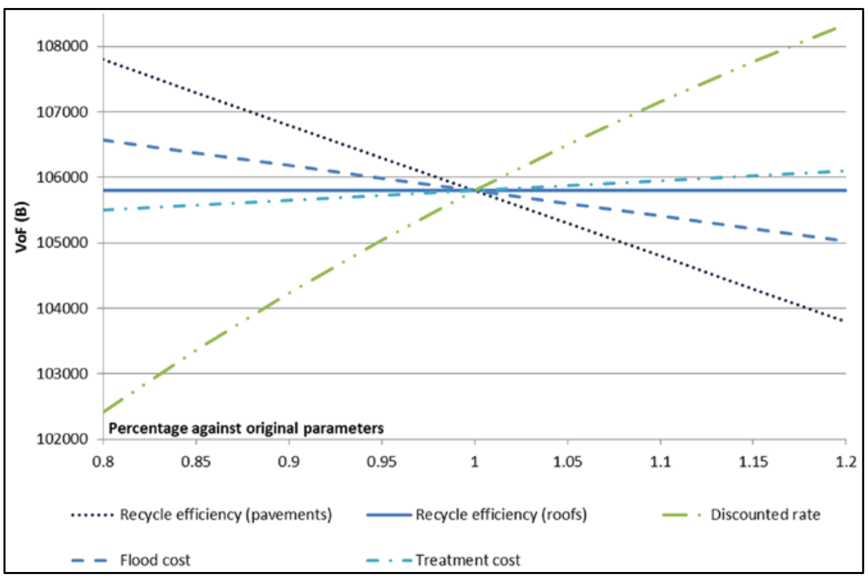

Figure 3.8 Sensitivity analysis of VoFB

3.2.6 Case study summary

This case study presents an application of the four-step procedure to the design and evaluation of an emerging water catchment technology. Several interesting findings are observed in this part of the analysis.

First, the comparison between the results from the uncertainty analysis and the deterministic analysis shows that the typical system design approach and evaluation might lead to suboptimal system performance and flaws in the evaluation results. It is found that deterministic analysis results in considerably inaccurate evaluation of design alternatives. The results here further confirm with the observation made by other studies (de Neufville & Scholtes, 2011).

Second, it is demonstrated in this case study again that designing flexible systems can be effective in improving the life cycle performance. For example, for flexible design B, the extra benefits are brought by reducing the initial excessive capacity, and by enabling an expansion option, so that the system is able to avoid unnecessary initial investment if downside scenarios happen (e.g. low cost savings by "grey water" that cannot balance the cost of the system). Meanwhile, the system is prepared to handle upside scenarios (e.g. high unit price of water which makes the system more profitable). This action is similar to buying insurance for the system by which the distribution of the system performance is shifted to the right side. This improvement on economic performance results from incorporating flexibility is also observed on design A.

However, flexible designs may not always result in improvement on system performance. As shown in the sensitivity analysis, there are many factors one would need to consider, such as the time value of money and opportunity cost. Although in the case study flexible design B is shown to be the best even under variations of assumptions, designers need to be careful about the trade-offs between those factors so that the system performance can be maximized when dealing with different systems.

Finally, the procedure in this study where the simulation model plays a central role can be generalized into the applications of other engineering systems. Since different systems are subjected to distinct costs and benefits, and faced with their respective source of uncertainties, details of modeling and computation may need to be adjusted to suit the particular system at hand. In addition, the way of combining historical data and IDF curves to simulate daily rainfall scenarios can be easily modified to another region with different IDF curves or requirements of return periods.

In sum, the case study is the first application of the “flexibility thinking” on the design and deployment of porous pavement and green roof technologies, as a way to recuperate and store grey water from natural rain events. It provides an example of how these ideas can be considered in urban water management systems and how this new catchment technology can be better deployed. The proposed methodology is general, and can be applied to the analysis of other engineering systems. This is demonstrated by using a similar analytical approach in Chapters 4-5 of this thesis, focusing on MoD transportation systems.

3.3 Summary

This chapter introduces the four-step procedure to analyze complex systems under uncertainty and flexibility, where the core analytical technique relies a simulation-based approach. The procedure is applied to value the flexibilities in design and management of urban water management systems.

Results from the case study support the argument that accounting for uncertainty and incorporating flexibility is an effective approach to enhance system performance. Equipped with the capability to change in the future, an engineering system is thus able to take the advantage of unexpected favorable condition as well as reducing exposure to downside scenarios.

On the other hand, the case study also shows that simulation plays an important role in estimating system performance more generally. The study demonstrates several advantages of simulation in the context of evaluating and optimizing systems design alternatives, both more rigid and flexible. First, a simulation model can be easily modified to cater to different formulations of flexible decision rules as well as varied assumptions on uncertainty modeling. Although in the case study, only one formulation of decision rule is considered for each design alternative, restructuring decision rule in the simulation model only requires a simple step, namely recoding the "if" statement. In fact, other approaches to modeling and evaluating flexibilities, such as mixed integer programming that relied on dynamic programming in the aspect of evaluation (Wang, 2005), may impose limitations on the decision rules being evaluated, and may be trapped in the curse of dimensionality when confronted with a multitude of decision periods and states. Besides, the ease of implementation of a simulation-based approach is also seen in performing a sensitivity analysis.

Second, the use of decision rule, which is the common practice of formulating flexibility in a simulation-based approach, is a direct extension of existing design and evaluation approaches, and is analyzed via a systematic step-wise process for more practical impact in engineering practice. For example in the case study, for the flexible drainage system, when the operators find that the number of flooding events exceeds a pre-specified level, they start to expand the capacity of the canals according to the decision rule. A similar strategy is used when analyzing MoD systems for flexibility in Chapter 5.

Finally, although only evaluation and comparison between a given set of standard and flexible designs are considered, there is more and more research related to simulation-based optimization techniques, which provides ample grounds to find optimal flexible design solutions. Comparing between optimal fixed (or rigid) designs and optimal flexible designs is an approach followed in the next chapters of this thesis.

However, there are also challenges when adopting a simulation-based approach to analyze complex systems under uncertainty and for flexibility. The main issue comes from the large computation cost, especially when optimization must be done. On the one hand, the decision space can be very large. As simulation model is able to adapt to different formulations of decision rules, and for each decision rule, there may be a considerable number of possible combinations for the decision variables. Therefore, identifying the optimal flexible design may not be an easy task. On the other hand, mostly, flexibility is planned for a relatively longer time horizon, which means the simulation model can grow to a very large computational scale. Consequently, estimating the performance of the candidate solutions may require a large amount of computation budget. Due to the reasoning above, it is critical to develop an approach to enhance the computational efficiency when simulation model is employed for analyzing complex systems under uncertainty and for flexibility.

The simulation-based analytical procedure introduced in this chapter provides a straightforward and readily-executable approach to design, analyze, and evaluate systems design alternatives. It offers designers a step-by-step framework on the "flexibility paradigm". Besides, the idea of using simulation further enhances wider applicability. The analytical logic that guides the case study in this chapter plays an essential role in the rest of this thesis. More advanced techniques relative to the simulation-based approach are considered to make a fair comparison between different systems design alternatives, some being more rigid, some being more flexible, and to overcome the computational challenges that such extensive analysis may create.

In sum, this chapter first indicates a direction to the research questions proposed in Section 2.3, suggesting that designing flexibility is an effective approach to deal with uncertainties and taking a simulation-based approach is advantageous in terms of modelling and evaluating flexibilities. Meanwhile, it also points out a challenge that may be encountered when a simulation-based approach is adopted, namely the computational cost.

Chapter 4 Integrating Operational Decisions into the Planning of MoD systems under Short-term Demand Fluctuation

This chapter targets the planning and operating issues of one-way MoD systems under short-term demand uncertainty. More specifically, it aims to assist stakeholders to determine where and how to set up stations and allocate vehicles when faced with stochastic and imbalanced demand. The use of "short-term" here indicates that although there are fluctuations in demand realization, the overall usage pattern remains the same. The main operational decisions, rebalancing activities, are incorporated as sub-problems, by which their influence on the higher-level decisions, and ultimately the overall performance of the system, is accounted for. The rebalancing operations can be perceived as an operational-level flexibility that helps to address the variations of daily demand.

The target problem can be illustrated in Figure 4.1 as an example. The geographic region in the figure contains two residential areas and one CBD. It is further divided into eight subareas with dots representing their centers. The subareas represent clusters of adjacent stations. Within each station cluster, there is no difference for customers to pick up or drop off vehicles at any specific station. Uncertain and imbalanced numbers of customers will travel between the subareas throughout the day. If customers cannot find any available vehicles within their origin subarea, the demand will be lost, while customers who cannot find available parking spots within their destination subarea will drop off the vehicle at a nearby location, but incur extra parking expenses to be paid by the operating company. Constrained by such user behaviors, this study devises a decision-support tool to help determine the following parameters for each subarea: 1) the number of parking spots to be rented and 2) the number of vehicles to be placed at the beginning of the day. The objective is to assist designers to render a system configuration that satisfies an adequate level of service (LoS) with a minimal overall cost.

This chapter provides a methodology that determines the initial configuration of a MoD system at its inception stage. It is assumed that at this stage, the priority resides on building a large customer base that can only be attained by providing an adequate LoS. As previously demonstrated in Singapore, poor service quality can be a main reason for a car-sharing system's demise ("End of the Road for Honda Car Sharing Scheme," 2008) Besides, as short-term demand uncertainty is the major concern in this chapter, more attention is placed on the operational-level decisions. Therefore, the chapter formulates the problem as a cost-minimization problem that is constrained by the LoS. Details of the optimization model are presented in the next section.

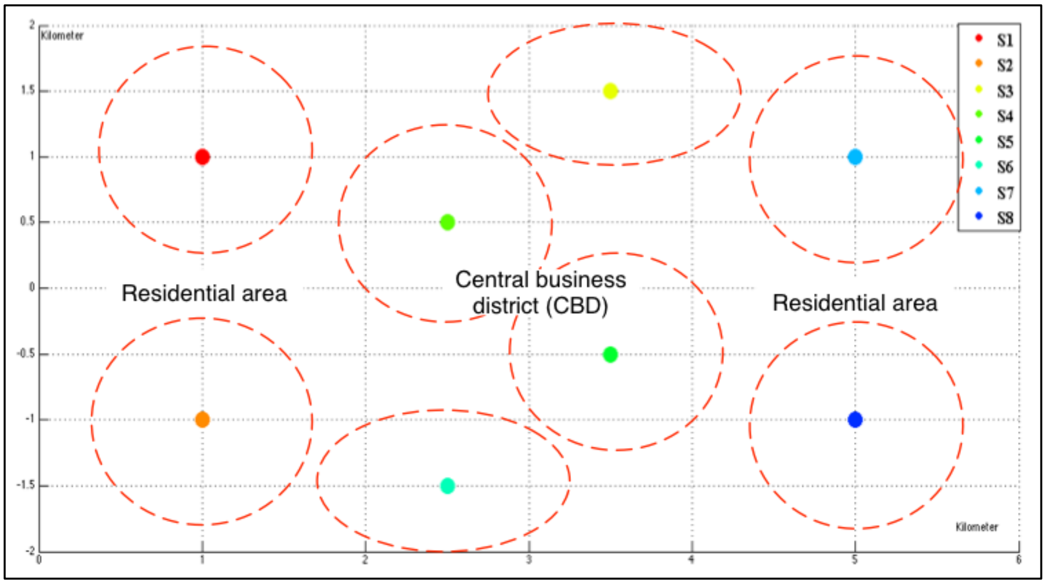

Figure 4.1 Illustration of the problem

4.1 A simulation-based methodology

In order to deal with the stochasticity of demand and model the rebalancing operations, this study relies on a simulation-based methodology. A constrained optimization model is first formulated and later solved using a simulation-based optimization approach. The solution approach consists of 1) a discrete event simulator (DES), which is applied to estimate the objective function of each candidate solution considering the stochastic demand and hourly rebalancing operations, and 2) a computational algorithm based on Particle Swarm Optimization (PSO) and Optimal Computation Budget Allocation (OCBA). The figure below illustrates the overall methodology.

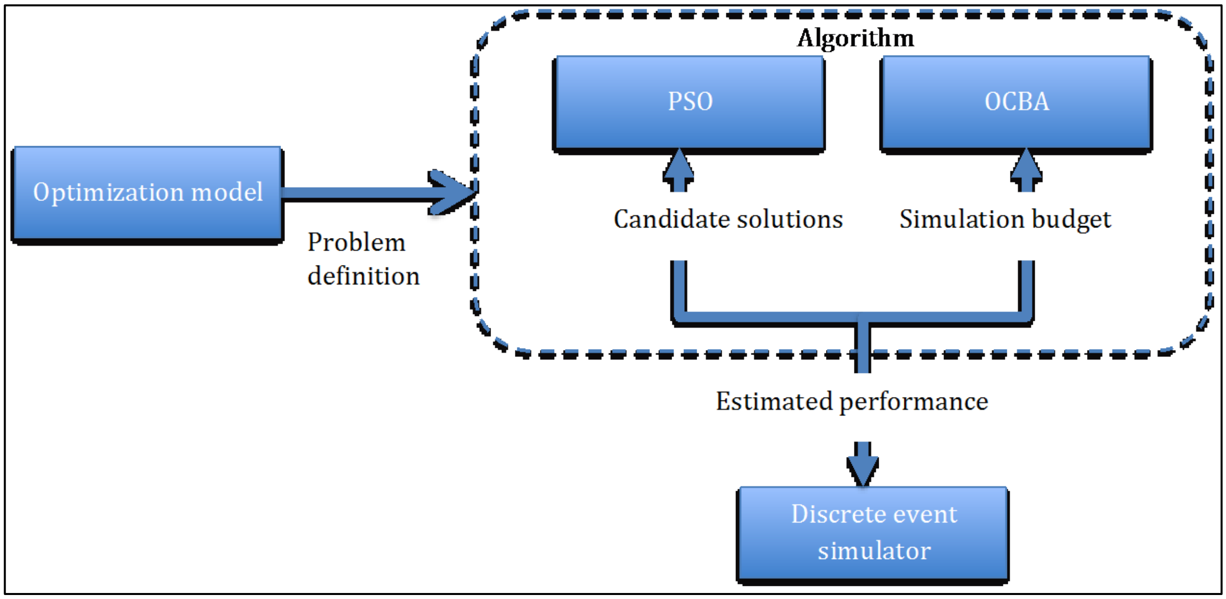

Figure 4.2 Overall methodology

4.1.1 Optimization model

$$min\ C_i \sum_{i \epsilon N} x_i + w * v + R(x, v) + P(x, v) \quad 4.1$$

$$g(x, v) \leq \alpha \quad 4.2$$

$$x_i \geq v_i \qquad \forall\ i \epsilon N \quad 4.3$$

$$v_i = v * \frac{\lambda_{i1}}{\sum_{i=1}^{N} \lambda_{i1}} \quad 4.4$$

$$\sum_{i=1}^{N} v_i = v \quad 4.5$$

$$v_i, x_i \geq 0, \in Integer \quad 4.6$$

In the optimization model, C_i is the rent cost of one parking spot at subarea i; x is a vector whose dimension is equal to the number of subareas and the value of its component is the number of parking spots at each subarea (also referred as capacity of each subarea) ; w is the depreciated daily cost of one vehicle; v is the total number of vehicles in the system; v_i is the number of vehicles placed at subarea i at the beginning of the day; $R(x, v)$ represents the rebalancing cost, which is a function of x and v; $P(x, v)$, which is also a function of x and v, represents the extra parking cost incurred when customers arrive at the destination subarea but cannot find any available parking spots, hence having to park somewhere nearby that is not rented in advance; $g(x, v)$ is the customer loss rate due to the lack of available vehicles upon arrival proportional to the total number of customers approaching the system; α is the requirement on the maximum customer loss rate; and λ_{i1} is the arrival rate of customers at subarea *i* during the first time segment of an operating day. Time segments are explained in the following section where the simulator is introduced.

As indicated in Equation 4.1, the objective of the optimization model is to minimize the overall daily cost, including rent cost for parking spots, depreciation cost of vehicles, and rebalancing cost. The practice of adopting daily cost has been seen in other studies, e.g., (Cepolina & Farina, 2012; Jorge et al., 2012). Equation 4.2 defines the main constraint of this optimization model, which sets an upper bound on the rate of lost customers. This constraint can be regarded as placing a requirement on the LoS that the system is able to provide. Equation 4.3-Equation 4.6 define the search space. In our optimization model, instead of directly optimizing each v_i, we optimize the total number of vehicles and apply the distribution rule specified by Equation 4.3-Equation 4.5. Basically, the heuristic rule allocates vehicles to each subarea at the beginning of the day in proportion to the customer arrival rates during the first time segment while being constrained by the capacity of the subarea. In the section of further analysis, this study will test the efficiency of the heuristic allocation rule by comparing the results obtained under such a rule to the results of an optimization analysis where the initial vehicle distribution is individually optimized.

For easier execution in computation, similar to the work by Cepolina and Farina (2012), the aforementioned model is transformed into the following relaxed model. In this transformed optimization model, violation of constrain 4.2 is incorporated into the objective function as a penalty cost. μ is considered as a large number since LoS is an important performance metric in this analysis.

$$min\, f(x,v) = C_i \sum_{i \epsilon N} x_i + w * v + R(x,v) + P(x,v) + \mu * (g(x,v) - \alpha)^2$$

$$x, v \,\epsilon\, D,\ D \text{ is the decision space} \qquad \textbf{4.8}$$

Since there is no analytical expression for $f(x,v)$, this study builds a discrete event simulator (DES) to estimate its value given x and v. Provided with the decision variables, parameters, and the number of simulations (for example, K), the simulator can generate the required number of sample points containing the rebalancing cost (r_k), the extra parking cost (p_k), and the rate of lost customers (g_k) under each realization of demand. Then, the value of the objective function is estimated using the following equation. The following section provides more details about the DES.

$$f(x,v) \sim C_i \sum_{i \epsilon N} x_i + w * v + \frac{\sum_{k=1}^{K} r_k + p_k + \mu * (g_k - \alpha)^2}{K} \qquad \textbf{4.9}$$

4.1.2 Discrete event simulator (DES)

The discrete event simulator (DES) was established using MATLAB. The randomness of the demand and a sub-optimization model to calculate rebalancing operations are coded into the simulator. The "passing of time" in the simulator is based on a fixed time increment.

4.1.2.1 Assumptions

This study makes several assumptions to define the behavior of customers and operators in the MoD system. These assumptions are coded into the simulator and applied to guide its progress.

1) 17 hours are simulated and analyzed (07:00-24:00) every day. Each time step in the simulation model represents 10 minutes in reality.
2) The travel demands between subareas are modeled as non-stationary Poisson processes. For each time step, there is an O-D matrix representing the arrival rates of the Poisson distributions that calibrate the travel demand between subareas.
3) Two concepts related to demand modeling are defined in this study: time segment and demand pattern. The time segment is defined as a period in a day in which the same O-D matrix applies for each time step. The demand pattern characterizes the demand during a whole day. More specifically, one particular demand pattern contains the O-D matrixes for all the time segments in a day.

4) Within each subarea, there may be one or more stations; it makes no difference at which station within a subarea customers pick up or return vehicles. However, if a subarea runs out of vehicles, customers who demand vehicles there will be lost. Similarly, if a subarea runs out of parking spots, customers will not travel to another subarea and will have to drop off the vehicle at a nearby location; while in this case, there will be temporary parking expenses incurred to the system operators Equation 4.10 accounts for the calculation of lost demand, whereas Equation 4.11 shows the calculation of the extra parking spaces needed. s_{it} is the number of vehicles at subarea *i* in time step *t*, while d_{ijt} is the simulated demand originating from subarea *i* and heading towards subarea *j* in time step *t*. In the simulation model, if the number of vehicles is not sufficient to satisfy all of the simulated demands, the satisfied demand between any two subareas will be proportioned according to the simulated demands, but the sum of all satisfied demands will be equal to the number of vehicles available.

$$l_{it} = \max(\sum_{j \in N, j \neq i}^{N} d_{ijt} - s_{it}, 0) \qquad \forall \mathrm{i}, \mathrm{j} \epsilon \mathrm{N}, \forall \mathrm{t} \epsilon \mathrm{T} \tag{4.10}$$

$$h_{it} = \max(\mathrm{s}_{it} - x_i, 0) \qquad \forall \mathrm{i}, \mathrm{j} \epsilon \mathrm{N}, \forall \mathrm{t} \epsilon \mathrm{T} \tag{4.11}$$

5) The DES incorporates an option to conduct rebalancing activities. In occasions when rebalancing is performed, at the beginning of every hour, an integer-programming model is applied to compute the rebalancing scheme. It is assumed that a number of part-time drivers are hired to perform the operations and how much they are paid depends on the number of vehicles

being transported and the distance between two subareas. The next section presents the formulation of the optimization model for calculating the rebalancing operations.

6) Rental time of vehicles is modeled as travel time between two subareas. Stopovers are not addressed separately in this study. Instead, this study assumes that a trip chain that involves multiple stops can be decomposed into several trips without stopovers. This assumption is the natural consequence resulting from how the customers are charged. Most MoD system charge customers by the usage time of the vehicle. In this case, even if customers have multiple stopovers, they will choose to end the current trip before their activities and restart a new trip after they finish the activity, which is perceived as a more economic choice. Therefore, the thesis does not make special consideration on trips with stopovers.

4.1.2.2 Integer-programming model for hourly rebalancing

$$min \sum_{j\epsilon N,\neq i,} \sum_{i\epsilon N} r_{ijt}\gamma_{ij} \tag{4.12}$$

$$L_{it} \le s_{it} - \sum_{j\epsilon N,\neq i} r_{ijt} + \sum_{j\epsilon N,\neq i} r_{jit} \le U_{it} \tag{4.13}$$

$$\textstyle\sum_{j\epsilon N,\neq i} r_{ijt} \le s_{it} \tag{4.14}$$

$$r_{ijt} \ge 0, \in \text{Integer} \tag{4.15}$$

r_{ijt} is the number of vehicles rebalanced out from subarea i to subarea j at the end of time t; γ_{ij} is rebalancing cost of transferring one vehicle from subarea i to subarea j at the end of time t; $\mathrm{U_{it}}$ and $\mathrm{L_{it}}$ are the upper bound and lower bound of vehicles that can be stocked at the subarea for the next hour.

As indicated in the model, the objective is to find the optimal rebalancing operation that minimizes cost and guarantees each subarea can meet a specified operational LoS in the next hour. Despite the LoS on satisfying arriving customers, a requirement is also set for the rate of successfully returning vehicles to the existing parking lots. Inspired by Schuijbroek, Hampshire, and Hoeve (2013), this model calculates the upper and lower bounds of vehicles that can be stocked at each subarea to meet the required LoS on vehicle returns and pickups, respectively. This formulation is indicted in Equation 4.13. Constraint 4.14 limits the rebalanced-out vehicles to no more than the total number of existing vehicles in the subarea, while Equation 4.15 constrains the solution to non-negative integers.

The following equations indicate how to calculate the bounds under different situations. To simplify the computation, instead of formulating vehicle returns and pick-ups as an M/M/1/K queue, as done by Schuijbroek, Hampshire, and Hoeve (2013), this paper identifies three situations and calibrates the arrival and departure of vehicles for each situation by a probability distribution. Here, λ_{it} and μ_{it} are the vehicle return rate and the customer arrival rate, respectively, for the next hour after time *t*.

1) $\lambda_{it} = 0,\ \mu_{it} \neq 0$

$$U_{it} = x_i \qquad \textbf{4.16}$$

$$L_{it} = arg\,min_k \sum_{x=0}^{k} \frac{(\mu_{it})^x e^{-\mu_{it}}}{x!} \geq 1-\alpha \qquad \mathbf{4.17}$$

2) $\mu_{it} = 0,\ \lambda_{it} \neq 0$

$$L_{it} = 0 \qquad \mathbf{4.18}$$

$$U_{it} = x_i - arg\,min_k \sum_{x=0}^{k} \frac{(\lambda_{it})^x e^{-\lambda_{it}}}{x!} \geq 1-\alpha \qquad \mathbf{4.19}$$

3) $\mu_{it} \neq 0,\ \lambda_{it} \neq 0$

$$L_{it} = arg\,min_k \sum_{x=-\infty}^{0} P(x, \mu_{it}, \lambda_{it}) + \sum_{x=0}^{k} P(x, \mu_{it}, \lambda_{it}) \geq 1-\alpha \qquad \mathbf{4.20}$$

$$U_{it} = x_i - arg\,min_k \sum_{x=-\infty}^{0} P(x, \lambda_{it}, \mu_{it}) + \sum_{x=0}^{k} P(x, \lambda_{it}, \mu_{it}) \geq 1-\alpha \qquad \mathbf{4.21}$$

* P is the Skellam distribution

Equation 4.16-Equation 4.17 show the calculation when there are only vehicle pickups in the next hour. In this case, the upper limit of stocked vehicles is simply the capacity of that subarea, as indicated in Equation 4.16, although there must also be a minimum number of vehicles so that a certain number of arriving customers can be served, which is the $(1-\alpha)$-th quantile of the Poisson distribution calculated by Equation 4.17. Similarly, Equation 4.18 and Equation 4.19 show the case when there are only vehicle returns. With no customers requiring vehicles, the lower bound of the vehicle stock is zero, while the upper bound is the total capacity minus the $(1-\alpha)$-th quantile of the Poisson distribution for vehicle returns since a sufficient number of parking spaces must be reserved for the coming vehicles in the next hour. When both returns and pickups exist, as shown in Equation 4.20 and Equation 4.21, the $(1-\alpha)$-th quantile of the Skellem distribution used to calibrate the difference between two Poisson distributions is applied to calculate the upper and lower bounds.

Infeasibility may occur when calculating solutions for the rebalancing optimization model, e.g., when the total number of vehicles is not sufficient to satisfy the required LoS or when extra parking spaces are needed to accommodate excessive vehicles. In the case of infeasibility, no rebalancing will be executed in the next hour.

4.1.2.3 Simulation procedure

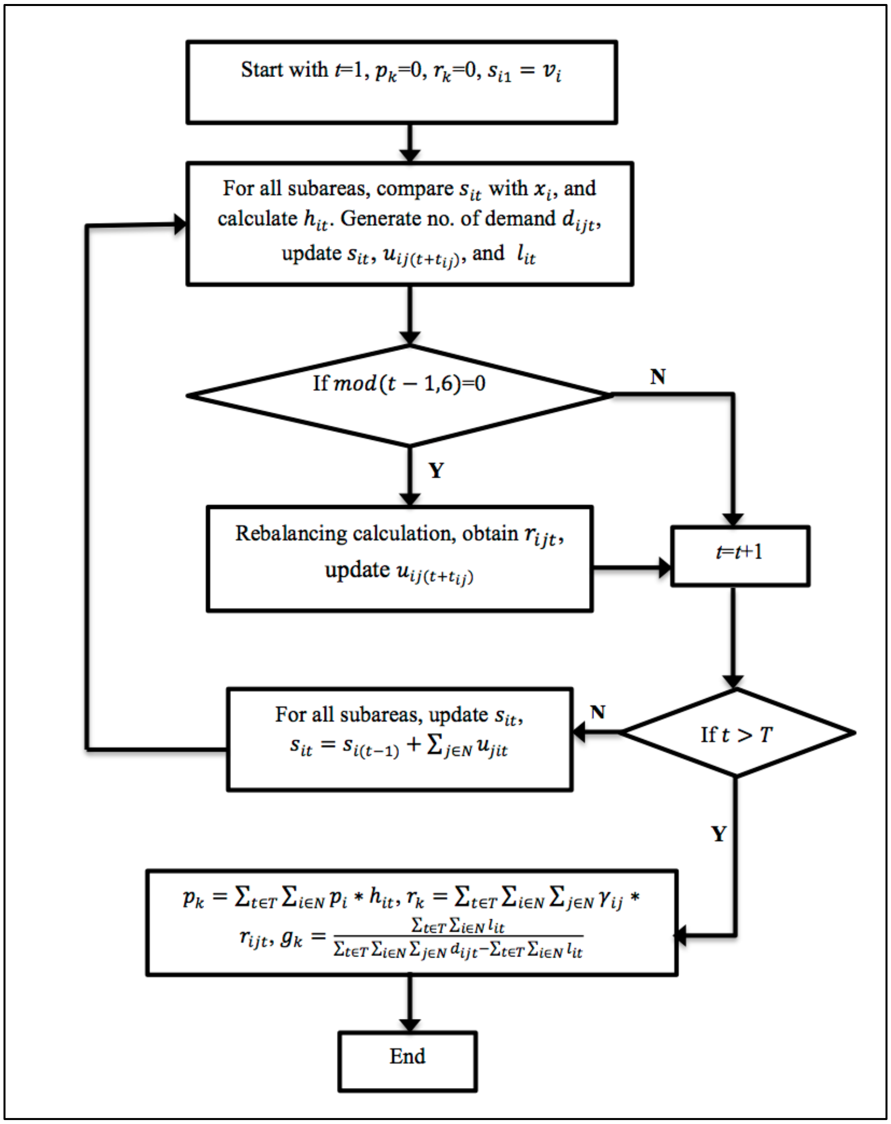

Figure 4.3 Simulation procedure

Figure 4.3 illustrates the process of the simulator starting with the first time step of the day. At the beginning of each time step, it calculates the number of temporary parking spots needed (h_{it}) by comparing the number of vehicles at each subarea to the total number of parking spots rented as part of the pre-determined planning decisions. Next, the simulator generates demands for each subarea (d_{ijt}), calculates the satisfied demand ($u_{ij(t+t_{ij})}$), lost demand (l_{it})

based on the travel time between subareas (t_{ij}). If this time step is the beginning of every hour, then the integer-programming model introduced earlier is applied to calculate the rebalancing operations. Following that, the number of arriving vehicles at each subarea in the subsequent time steps is further updated based on the optimization results of the integer-programming model. The simulator then moves to next time step and updates the number of vehicles at each subarea. This process is repeated until the simulation reaches the end of the day.

4.1.3 Computational procedure

This section addresses the computation issues arising in connection with the simulation-based optimization model. There are two main questions to be answered: 1) how do we identify potentially good solutions within the decision space and 2) how do we efficiently estimate their performance so that the computational effort is minimized. To answer these questions, this study modifies and implements a hybrid computation procedure combining PSO and OCBA. PSO aims to achieve an efficient search, while OCBA reduces the computational budget essential for comparing candidate solutions.

4.1.3.1 Particle swarm optimization (PSO)

Particle Swarm Optimization (PSO) is a population-based optimization technique developed by Eberhart and Kennedy (1995). The algorithm works similarly to the way individual members (or “particles”) in a group of birds or fish collaborate in search of the location of food (the optimal solution in the context of optimization). In the algorithm, the solutions are represented by the location of a swarm of particles that moves within the decision space to seek the best location. The swarm of particles represent the candidate solutions generated in each iteration. The goodness of each location is determined by the objective function value. Initially, the locations of all particles are randomly generated within the decision space. In later iterations, each particle moves towards its personal best location (*pbest*) and the global best location across the whole swarm (*gbest*). The search stops when a specified criterion is satisfied, e.g., after a maximum number of iterations or after only limited improvement can be achieved. Figure 4.4 shows how one particle in a swarm moves from one iteration to another.

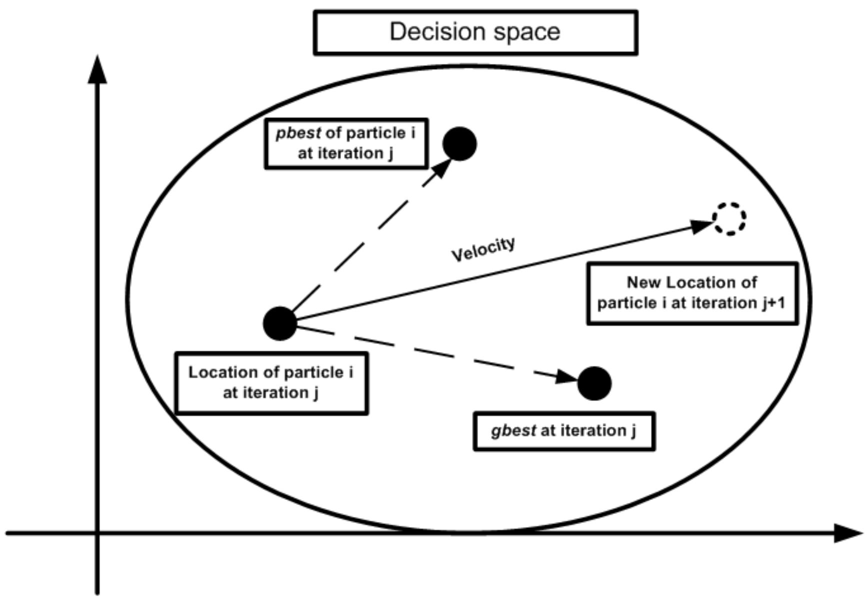

Figure 4.4 Illustration of PSO

Because PSO can be easily implemented for a vast array of problems, it has been applied to many fields including communication networks, control, robotics and even entertainment (Poli, 2008). In particular, its efficiency in terms of solving transportation problems is also demonstrated by Goksal et al. (2013). The convergence of this algorithm was theoretically studied and explained in Clerc and Kennedy (2002).

However, the efficiency of PSO may be affected when stochasticity is introduced into the search process, because information, namely the ranking of the goodness of locations, may be diluted by noise. In such cases, the particles may move towards locations where their performance is not better. Therefore, many studies have introduced optimal computing budget allocation (OCBA) into PSO, which aims to obtain a more accurate ranking of solutions using a limited simulation budget. Such practice leads to higher efficiency in terms of updating the solutions and, hence, better performance of the search algorithm (Pan et al., 2006; Zhang et al., 2011).

4.1.3.2 Optimal computing budget allocation (OCBA)

Optimal computing budget allocation (OCBA) derives from the concept of ordinal optimization that emphasizes the relative order of candidate solutions rather than their exact performance. The underlying principle is that, instead of spending great effort estimating the exact performance of every solution by running a large number of simulations, a ranking of solutions with sufficient accuracy can be obtained using a smaller computational budget. Studies on OCBA aim to generalize certain rules to determine the optimal allocation of the budget.

The accuracy of the ranking of solutions is measured by the probability of correct selection - or *P(CS)* - which is defined to give the probability that the selected best design is indeed the true best design (Dai, 1996). According to Shi (2000), *P(CS)* is approximated by Equation 4.22 and Equation 4.23. The derivation of the equations is not presented here and readers can refer to Shi (2000) if interested. Several budget allocation rules have been derived to maximize *P(CS)*. One such rule comes from Chen et al. (2000), whose numerical experiments supported that their allocation rule generally led to better results than others'. Therefore, Chen et al. (2000)'s allocation rule is adopted in this study. Equation 4.24-Equation 4.26 provide the details of this allocation rule. In brief, in each iteration, this rule picks the best solution and allocates a simulation budget according to its relative variance; for the rest of the solutions, the rule attempts to allocate more of the simulation budget to ones with a larger sample variance or a better sample mean. Readers can find more details about how this rule is deduced in Chen et al. (2000).

$$P_{cs(betwee\ i\ and\ j)} = P\left(\widetilde{X}_\iota < \ \widetilde{X}_J\right) = \phi\ \left(\frac{\overline{X_J} - \overline{X_\iota}}{\sqrt{\frac{s_j^2}{N_j} + \frac{s_i^2}{N_i}}}\right) \qquad \mathbf{4.22}$$

$$P_{cs(i\ is\ best\ design)} \approx \prod_{i \neq j}^{n} P_{cs(betwee\ i\ and\ j)} \qquad \mathbf{4.23}$$

$$\frac{N_i}{N_j} = \left(\frac{s_i/\delta_{b,i}}{s_j/\delta_{b,j}}\right),\ \ i, j \in N,\ i \neq j \neq b \qquad \mathbf{4.24}$$

$$N_b = s_b \sqrt{\sum_{i=1, i \neq b}^{k} \frac{N_i^2}{s_i^2}} \qquad \mathbf{4.25}$$

$$\delta_{b,i} = \overline{X}_\iota - \overline{X_b} \qquad \mathbf{4.26}$$

n is the number of solutions

ϕ is the standard normal distribution

$\widetilde{X_i}$ is the posterior distribution of the unknown mean X_i, $\widetilde{X_i} \sim N(\overline{X_i}, \frac{s_i^2}{N_i})$

$\overline{X_i}$ and s_i^2 are the sample mean and the sample variance of solution *i*

N_i is the number of simulations allocated to solution *i*

b is the best solution based on sample mean

4.1.3.3 Computation procedure in this study

This study developed a new version of PSO+OCBA to solve the optimization problem. The motivation is to guarantee a certain level of accuracy on the ranking of solutions in each iteration, namely to satisfy a pre-defined *P(CS)*. In such a case, PSO is expected to update the solutions more effectively. The target of achieving a certain level of *P(CS)* was not emphasized in past studies. The modified computation procedure is as follows.

1. Randomly generate the initial locations of all particles in the feasible design space.
2. Apply OCBA to estimate the performance of all particles at their current locations:
    i. Perform an equal number of simulations for all particles.
    ii. Calculate *P(CS)* using Equation 4.22 and Equation 4.23.
    iii. If *P(CS)* is satisfied or the maximum budget allowed for a single

solution (MAS) is reached, go to step 3; if not, increase the total value of the simulation budget, use Equation 4.24-Equation 4.26 to allocate the extra budget, run simulations accordingly, and go back to step ii.

3. Based on the results obtained from step 2, update *pbest* and *gbest*, and the velocity and location of each particle.

4. For each particle, use OCBA to determine its *pbest* between the new location and the existing *pbest*:

   i. Perform an initial number of simulations for the new location.

   ii. Use Equation 4.22 to estimate the *P(CS)* between the new location and the existing *pbest.*

   iii. If *P(CS)* is satisfied or MAS is reached, update *pbest* and go to step 5; if not, use Equation 4.24-Equation 4.26 to allocate the extra budget, run simulations accordingly, and go back to step ii.

5. For the current set of *pbests*, use OCBA to select the *gbest*:

   i. Use Equation 4.22-Equation 4.23 to estimate *P(CS).*

   ii. If *P(CS)* is satisfied or MAS is reached, update *gbest* and go to step 6; if not, use Equation 4.24-Equation 4.26 to allocate extra budget, run simulations accordingly, and go back to step i.

6. Check if either the maximum number of iterations is reached, or if *gbest* does not change for a number of consecutive iterations. If either is true, stop; if not, go to step 3.

4.2 Application

4.2.1 Case study of a prototype problem

This section presents a case study on a prototype problem. It illustrates the use of the proposed methodology and tests the efficiency of the methodology with respect to finding an optimal VSS design. A sensitivity analysis is also conducted to see how different factors may influence the final decision. In addition, to demonstrate the efficiency of the OCBA technique in terms of accelerating the computation process, the study performs additional numerical experiments where OCBA is removed from the optimization algorithm.

The analysis is implemented in MATLAB on a desktop computer (Intel 3.30 GHz Core i5 and 8 GB of memory) with a Microsoft Windows 7 operating system. At every six time steps, the IBM CPLEX optimizer for MATLAB is called to solve the hourly rebalancing optimization model.

4.2.1.1 Problem setting

The prototype problem is assumed to be a proposed VSS based in Singapore that includes three subareas. The whole geographic region is shown in Figure 4.5. The picture is provided by Map data ©2015, Google, Urban Redevelopment Authority. The three subareas are centered at Clement Mass Rapid Transit (MRT) (S1), Boon Lay MRT (S2), and Raffles Place MRT (S3) stations.

Only one demand pattern is considered in the prototype case. Table 4.1 displays the hourly arrival rates of the Poisson distributions used to model the travel demands. Assumptions of the daily costs are shown in Table 4.2. Except for the temporary parking cost is referred from the market rate in Singapore, all other costs are adopted from Jorge et al. (2014). For convenience of presentation, all costs here are displayed in Euros as Jorge et al. (2014) do in their study. The required LoS is set to 0.8, which means α is assumed to be 0.2.

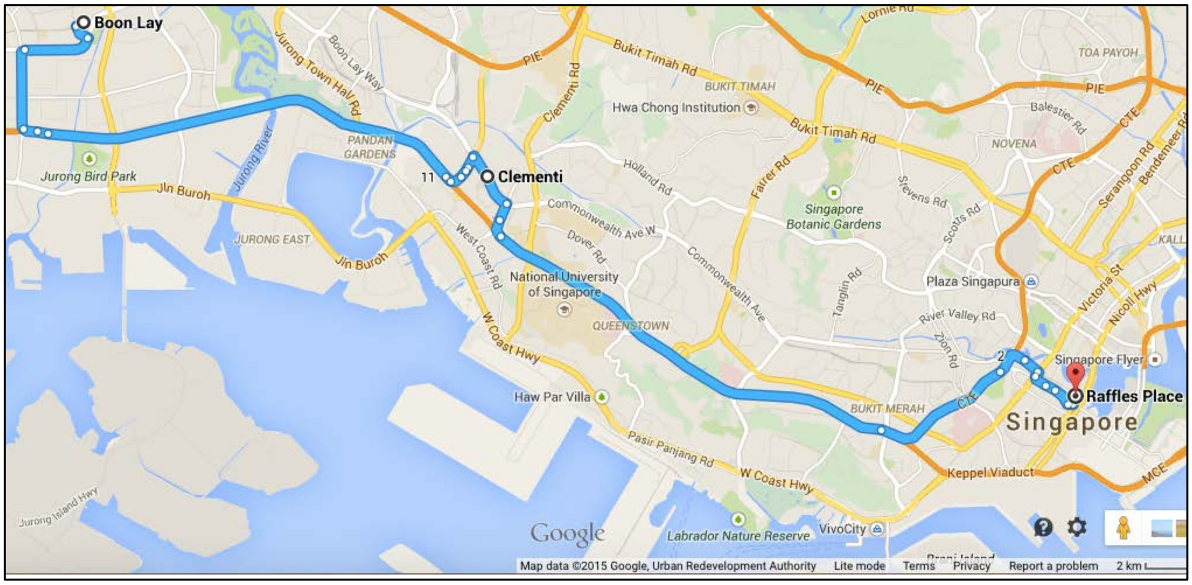

Figure 4.5 Geographic setting in the simplified problem

Table 4.1 Hourly arrival rates at different time segments in a day

	0700-0900			0900-1700			1700-1900			1900-2400		
	S1	S2	S3	S1	S2	S3	S1	S2	S3	S1	S2	S3
S1	0	0	1	0	0	1	0	0	0	0	0	0
S2	0	0	1	0	0	1	0	0	0	0	0	0
S3	0	0	0	1	1	0	1	1	0	1	1	0

Table 4.2 Cost parameters

Daily cost of unit capacity at each subarea	Daily cost of one vehicle	Rebalancing cost per vehicle per 10mins	Temporary parking cost per one space per 10mins
2 (for S1 and S2), 4 (for S3)	17	2	0.17 (for S1 and S2), 0.33 (for S3)

4.2.1.2 Optimization results

In this section, the proposed methodology is applied to identify the optimal system configuration for the aforementioned problem. The optimization analysis is conducted under two conditions: with and without hourly rebalancing activities. Under each condition, the optimization algorithm is run five times to allow fair comparisons between the two conditions, as well as to determine the robustness of the computation procedure. The parameters of PSO are set according to Eberhart and Shi (2000) with inertial weights of 0.729 and constriction factors of 1.49445. The number of particles in a swarm is set to 10, which is a relatively smaller number, to reduce the computation effort demanded by each iteration. The study sets MAS as 250, since under this condition the simulation is already well converged. The parameter μ is assumed to be 20000. This choice is tuned by running several rounds of optimization under different values, and this value of μ resulting in a relatively stable performance is selected.

Before conducting the optimization analysis, the study first validates the simulation model by examining the total number of vehicles at each time step. The vehicles on the road, driven either by customers or rebalancing staff, and those stocked at the stations and temporary parking spots are summed, and, at each time step, the total number of vehicles is equal to the number assumed before the simulation.

Table 4.3 and Table 4.4 display the optimal solutions obtained under the two conditions, i.e., with and without hourly rebalancing operations. For example, in Table 4.3 where optimal solutions under hourly rebalancing are displayed, the solution of the first run indicates 5 parking spots should be rented at subarea 1 and subarea 3, and 6 at subarea 2, where in total 5 vehicles should be purchased for the whole system. Although the solutions obtained from each run of the algorithm are not identical, the results within each group show that the overall performances are nearly the same. This demonstrates that the computation procedure is relatively stable.

On the other hand, between the two groups, rebalancing not only identifies solutions that cost less, but they also have a relatively higher LoS in terms of satisfying customers. This kind of win-win result suggests that rebalancing is an essential consideration when determining the optimal configuration of the system. The reallocation of vehicles not only reduces the number of parking spots rented, but also increases the utilization of vehicles by serving more customers and, hence, reducing the number of vehicles required. On the other hand, within each group, the trade-off between cost and LoS on meeting customers' requests can still be observed. For example, as shown in Table 4.4, Run 3 has a relatively larger cost but a higher LoS, while Run 5 shows the opposite situation.

This study also decomposes the total cost into individual cost items, including set-up cost (sum of rent of parking spots and depreciated cost of vehicles), temporary parking cost, and rebalancing cost (if considered). Figure 4.6 and Figure 4.7 illustrate the cost decomposition of the optimal solutions under the two conditions. It is interesting to note that, by conducting the rebalancing operations, the temporary parking cost is reduced almost to zero, which indicates it is rare that customers will need to park the vehicles outside the system. In both cases, with or without rebalancing, a trade-off between initial investment (set-up cost) and operating costs (sum of temporary parking and rebalancing cost) is observed.

Table 4.3 Optimal solutions assuming hourly rebalancing is conducted

Run No.	Solution				Estimated cost /standard error (SE)	Estimated customer loss rate /SE
	Capacity (S1)	Capacity (S2)	Capacity (S3)	No. of vehicles		
1	5	6	5	5	250/1.4	0.209/0.004
2	4	5	5	6	245/1.3	0.217/0.004
3	3	3	5	5	251/1.6	0.203/0.004
4	4	5	6	5	250/1.4	0.208/0.004
5	3	3	5	5	251/1.6	0.203/0.004

Table 4.4 Optimal solutions assuming no hourly rebalancing

No. of run	Solution				Estimated cost /SE	Estimated customer loss rate /SE
	Capacity (S1)	Capacity (S2)	Capacity (S3)	No. of vehicles		
1	4	4	10	14	333/0.9	0.224/0.006
2	4	4	11	14	333/1.0	0.224/0.006
3	4	4	13	14	338/1.0	0.205/0.006
4	4	5	8	14	332/1.0	0.217/0.006
5	4	5	11	13	310/0.9	0.236/0.006

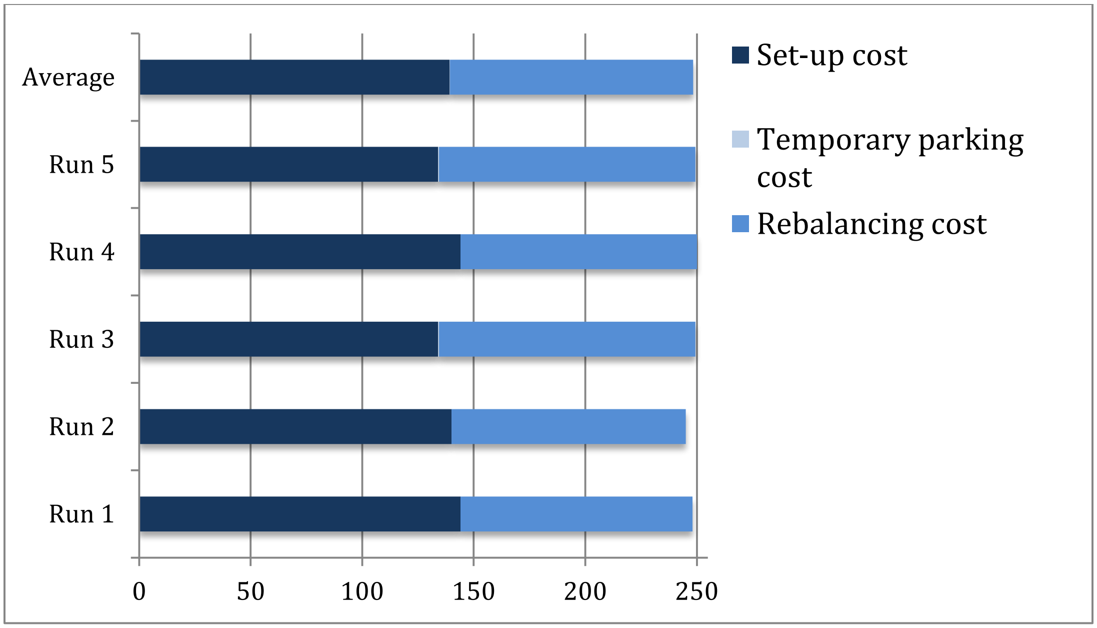

Figure 4.6 Cost structure of optimal solutions under rebalancing

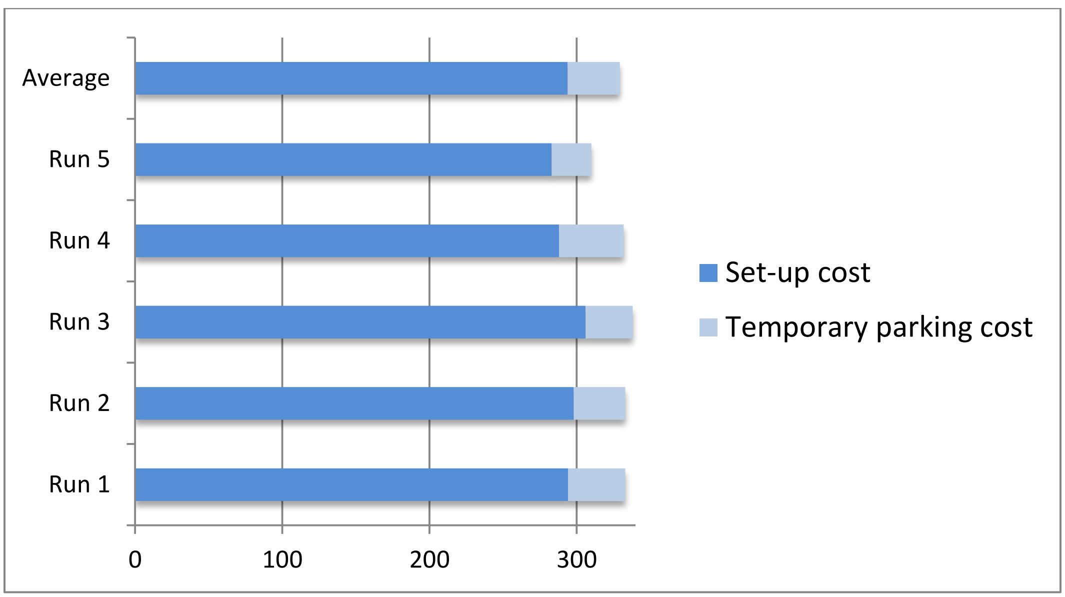

Figure 4.7 Cost structure of optimal solutions with no rebalancing

4.2.1.3 Further analysis

a) Test on the initial vehicle distribution rule

As mentioned earlier, a heuristic rule is applied to decide the distribution of vehicles at the beginning of the day after the total number of vehicles is determined. To investigate the efficiency of this rule, another round of optimization analysis is carried out where the number of vehicles at each subarea is optimized individually. Table 4.5 displays the solution and its estimated performance. Comparing these results with the ones in Table 4.3, there is no statistically significant difference between the solutions obtained using the more complex model here and the ones obtained using the heuristic rule. As such, the heuristic rule is applied in later analyses to reduce the dimensions of the problem.

Table 4.5 Optimization results by using a more complex model

Solution				Estimated cost /SE	Estimated customer loss rate /SE
Capacity (S1)	Capacity (S2)	Capacity (S3)	No. of vehicles		
5	3	5	6	249/1.3	0.21/0.004

b) Sensitivity analysis

In the sensitivity analysis, three parameters - namely the hourly arrival rates, required LoS, and frequency of rebalancing - are varied to see how the optimal solutions change in accordance with different assumptions. Under each varied set of assumptions, five rounds of optimization are conducted. The averaged results of each set of five solutions are presented here.

1) Increase in hourly arrival rates

The hourly arrival rates, namely the numbers in Table 4.1, are increased by 25% and 50%. For each increase, five runs of optimization analysis are conducted. Then, for all three sets of the optimal solutions, the study calculates the averaged performance, namely their averaged cost and LoS. The results are presented in Figure 4.8. An interesting observation can be made by looking at the relative change in set-up and rebalancing costs: When the demand grows by 25%, increasing the rebalancing operations, as opposed to immediately deploying a larger system, initially seems to be able to accommodate the additional demand, as the rebalancing cost seems to grow relatively faster than the set-up cost. However, when the demand increases by 50%, increasing the rebalancing operations is not as efficient as increasing the scale of the system. That explains why, in the later stage, the set-up cost increases relatively faster. On the other hand, in both cases, the customer loss rate and temporary parking cost remain quite similar.

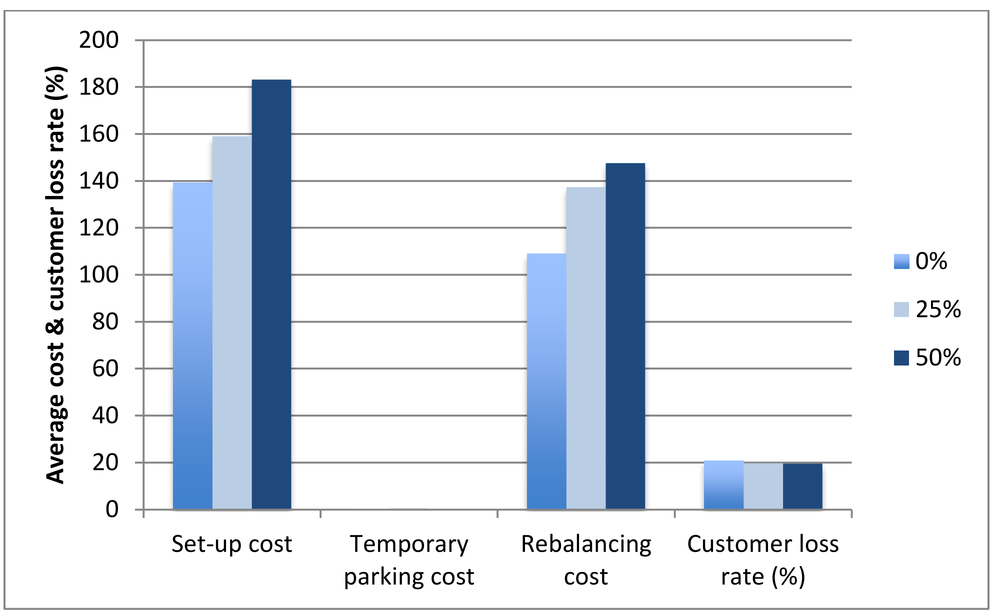

Figure 4.8 Sensitivity analysis on hourly arrival rates

2) Increase in LoS

There are three places in our model that need specified requirements for level of service (LoS). Strategically, the expected customer loss rate indicates the overall level of satisfied demand in a whole day. At the operational level, there exist two additional rates: the expected customer loss rate and the rate of temporary parking spots needed in the next hour. The two rates at operational level determine the upper and lower bounds of vehicle stocks at each subarea every hour. These three requirements are set at the same level and are presently at 0.2. In this part of sensitivity analysis, the levels are changed to 0.15 and 0.1. Using the same logic as in the previous section, the results are obtained and summarized in Figure 4.9. Unlike the previous case, variation in LoS seems to have little influence on the rebalancing cost, as satisfying the increased LoS is mainly achieved by scaling up the system. This may be due to the fact that small variation in LoS has little effect on the upper and lower bounds of vehicle stocks each hour, especially when LoS changes from 0.2 to 0.15, which consequently leads to similar rebalancing schemes. Hence, as rebalancing operations remain the same, a larger system is needed to cope with the higher requirement for LoS. On the other hand, since the requirement for LoS is raised, the realized customer loss rate decreases accordingly.

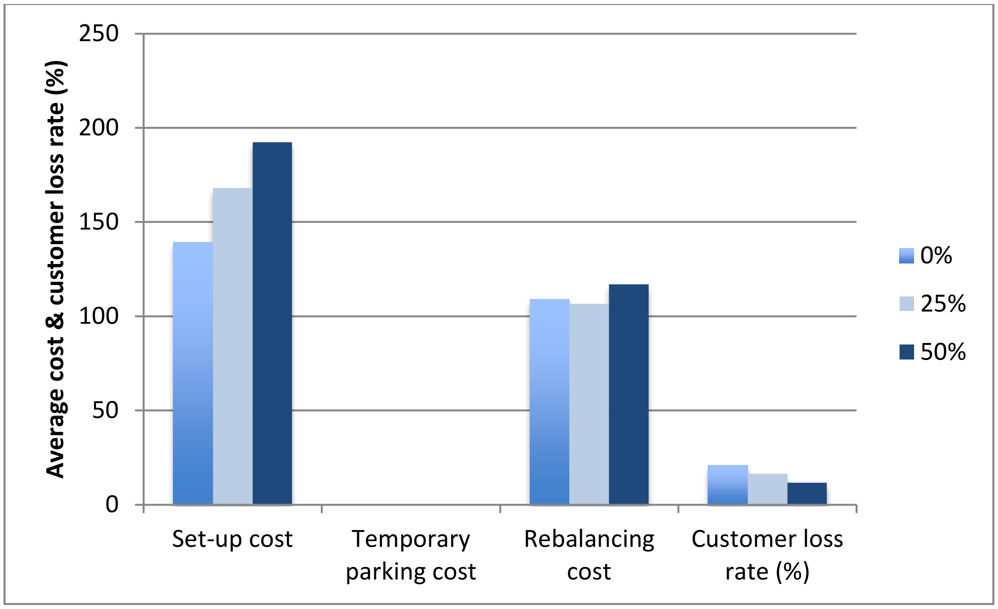

Figure 4.9 Sensitivity analysis on LoS

3) Decrease in rebalancing frequency

In the previous analysis, vehicles were redistributed in the system every hour. However, the benefit of such a high frequency of rebalancing activities is questionable. Therefore, this section considers a situation where rebalancing occurs every two hours. Figure 4.10 displays the results. It seems counter-intuitive that reducing the rebalancing frequency would not significantly reduce the rebalancing cost. In fact, as the purpose of rebalancing is to keep the vehicle stocks in each subarea within the proper bounds for the next period (here, 2h), enlarging the interval between two rebalancing activities, which does reduce the number of actions (8 instead of 17), means more vehicles are transported during one single action. The other negative effect is the stricter upper and lower bounds, which require a larger scale system. Considering a case where the rebalancing frequency is reduced in an extreme way, that is the is situation where no rebalancing is conducted throughout the day, as shown in the previous analysis, even more parking spots and more vehicles are needed. Although the temporary parking cost remains the same for the 1h and 2h rebalancing intervals, there is a slight increase in the customer loss rate under the lower rebalancing frequency. As less rebalancing operations are conducted, satisfying the required customer loss rate requires greater initial investment to configure a larger system. It may be that the required level of investment exceeds the penalty cost of slightly violating the LoS requirement. This is also consistent with the result in the case without rebalancing, as a larger deviation of constraint occurs.

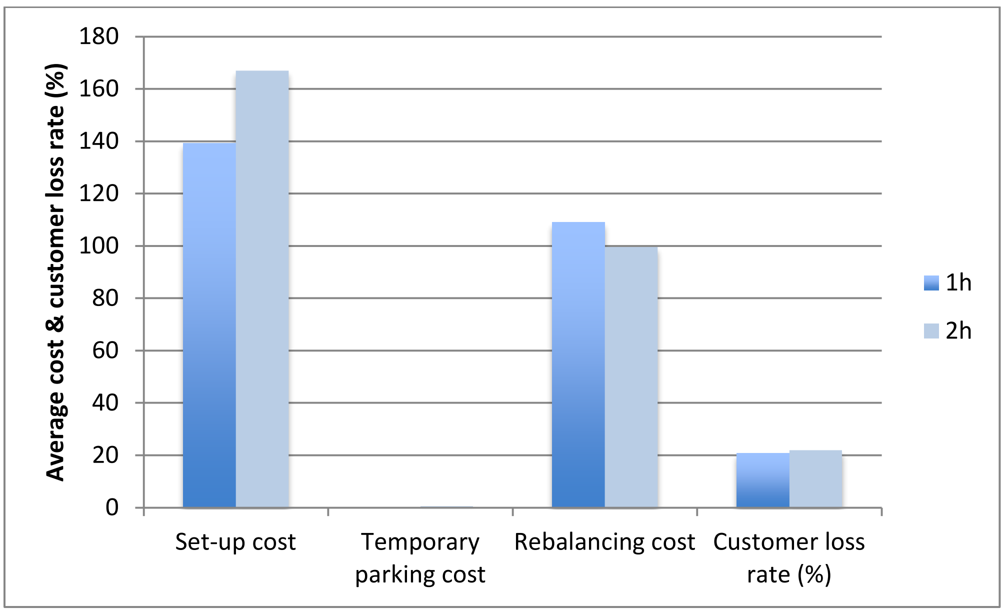

Figure 4.10 Sensitivity analysis on rebalancing frequency

c) Further analysis on the solution approach

1) Results validation

As the algorithm developed in this study is based on a heuristic search method, the optimality of the final solution cannot be justified theoretically. In this case, the approach by Shu et al. (2010) is taken to calculate the optimal number of vehicles for the condition without rebalancing operations. By comparing results obtained by their approach with the algorithm in this study (namely results in Table 4.4), it is believed that the quality of the results can be assessed to some degree.

Similar to our study, Shu et al. (2010) assumes demand to be a non-stationary Poisson process. Besides, their analysis is also based on no waiting time for the customers. Because of such similarities in problem setting, their approach is taken as a benchmark. However, differences do exist between their approach and this study. First, they do not consider rebalancing operations. Second, in their optimization model, the objective is to maximize total satisfied demand, which is approximated under the assumption that the system reaches equilibrium state. Finally, there is no constrains on the capacity of stations.

Despite the different formulation of the optimization model the solution approach, a final solution with 16 vehicles to purchase is obtained, which is very close to the ones obtained in Table 4.4. This observation, therefore, somehow validates the solution approach proposed in this study.

2) Comparison between PSO+OCBA and PSO+EA

This study adopts OCBA technique to reduce the computation effort, while at the same guarantee a certain level *P(CS)* in each iteration. To further illustrate how this computation budget allocation rule helps accelerate the computation process, this study performs another five runs of the optimization analysis under the condition where an equal simulation budget is allocated to each candidate solution instead of using the OCBA rule, which is referred to as PSO+EA. The solutions obtained under this case were not identical to those obtained under PSO+OCBA, but no statistically significant difference could be observed. More importantly, the average computation time of these five runs is 2495 seconds with a standard deviation of 556 seconds, which is obviously not as efficient as PSO+OCBA whose average computation time is 1632 seconds with a standard deviation of 397 seconds. Figure 4.11 and Figure 4.12 illustrate the convergence speed of PSO+OCBA and PSO+EA, respectively. One thing that needs attention here is that the objective value mentioned in these two figures includes not only set-up cost and rebalancing cost but also the penalty cost when required LoS is violated. According to the results, compared with PSO+EA, PSO+OCBA requires fewer function evaluations to converge to the optimal solution. This is achieved by reducing the simulation budget that is spent on the obviously inferior solutions generated by the searching algorithm.

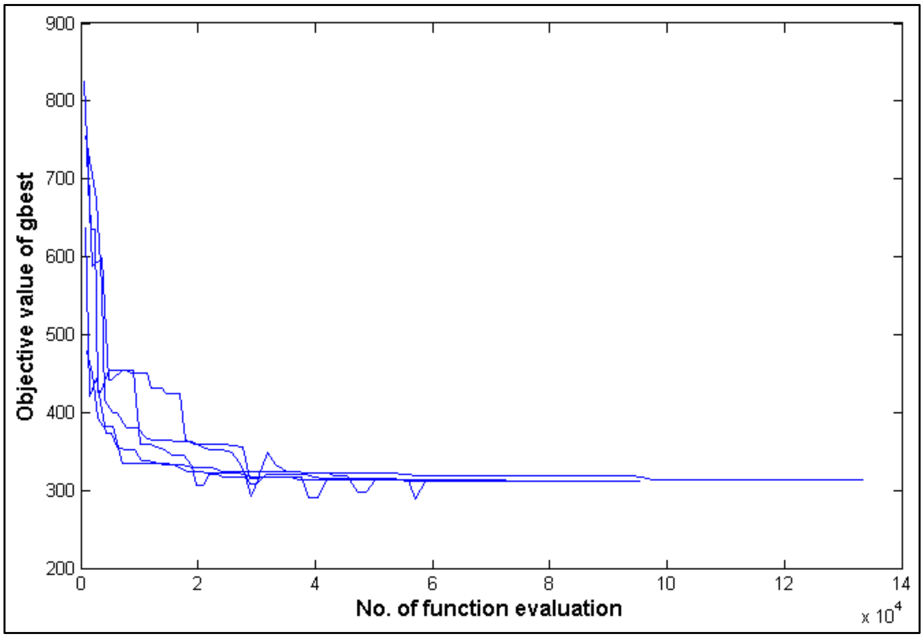

Figure 4.11 Convergence speed of PSO+OCBA

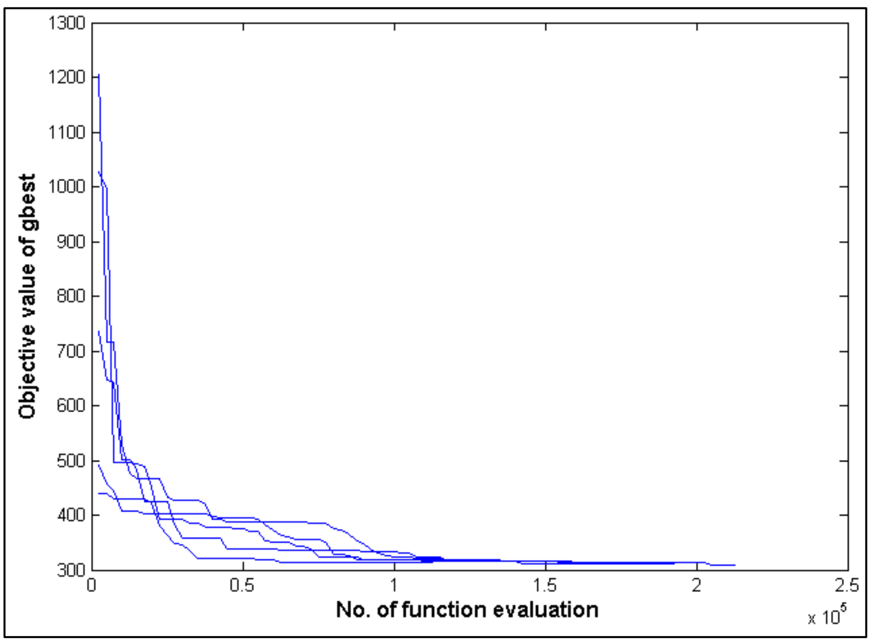

Figure 4.12 Convergence speed of PSO+EA

4.2.2 Case study of a more complex problem

4.2.2.1 Problem setting

In the second case study, a larger-scale problem with eight subareas - Clementi MRT (S1), Boon Lay MRT (S2), One-North MRT (S3), Vivo City MRT (S4), Raffles Place MRT (S5), Orchard MRT (S6), Bishan MRT (S7), and Ang Mo Kio MRT (S8) stations - and two demand patterns (weekdays and weekends) is studied. The purpose of this study is to test the efficiency of the proposed methodology when handling a more complex problem. The geographic information is presented in Figure 4.13. Since more subareas are under consideration, additional assumptions are made regarding the rent cost of parking spots as well as the temporary parking cost, as shown in Table 4.6. The other assumptions remain the same.

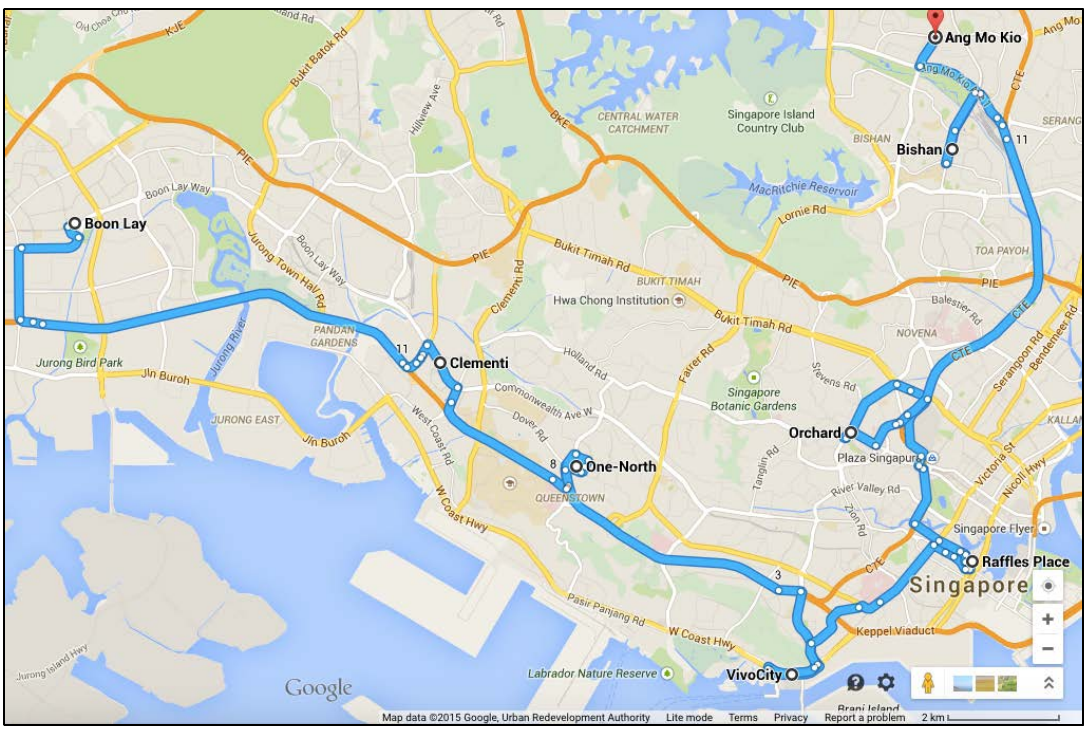

Figure 4.13 Area of study in the second case study

Table 4.6 Additional cost parameters

Daily cost of unit capacity at each subarea	Temporary parking cost per one space per 10mins
2 (S1, S2, S7 and S8), 3 (S3), 4 (S4, S5, and S6)	0.17 (S1, S2, S7, and S8), 0.25 (S3), 0.33 (S4, S5, and S6)

The demand profiles on weekdays and weekends are illustrated in the following figures. Assuming all trip requests are satisfied, the figures are obtained by averaging the sample points from 250 runs of the simulation. Figure 4.14 and Figure 4.15 illustrate the vehicle return and pickup demand during weekdays, while Figure 4.16 and Figure 4.17 show the situation on weekends. The probability transition matrix for both weekend and weekdays are also presented in the Appendix.

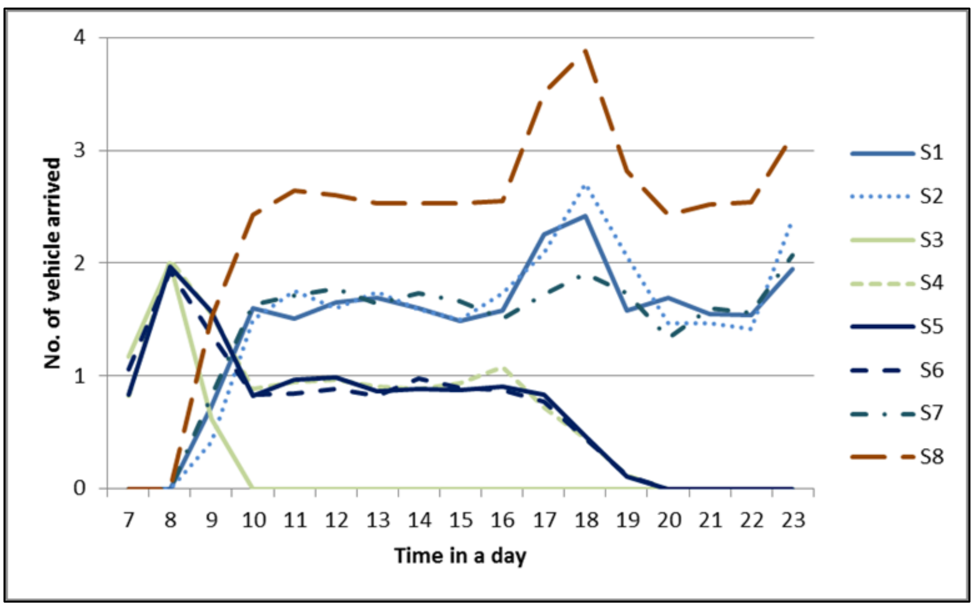

Figure 4.14 Averaged vehicle arrivals at each subarea during a day in weekdays

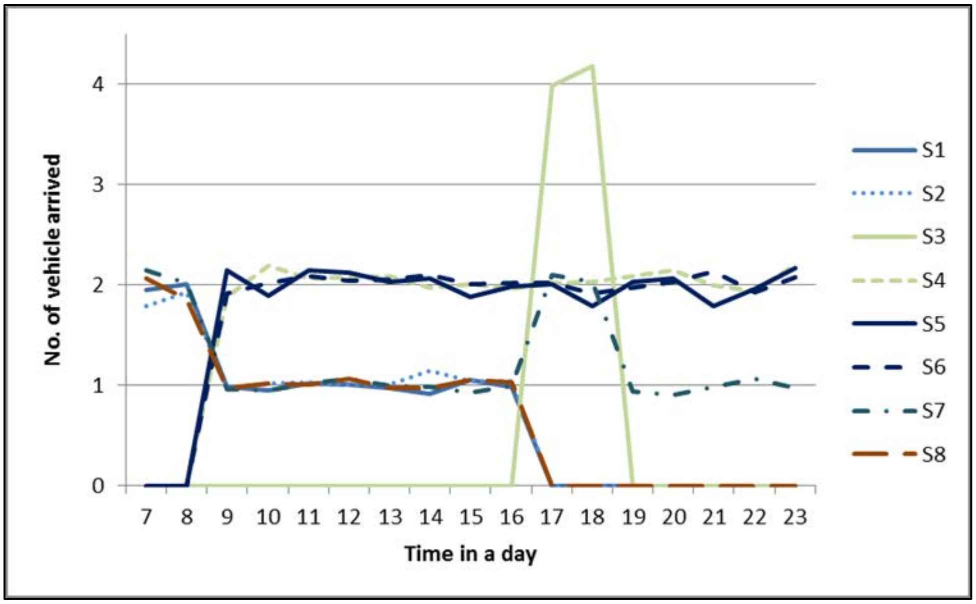

Figure 4.15 Averaged customer arrivals at each subarea during a day in weekdays

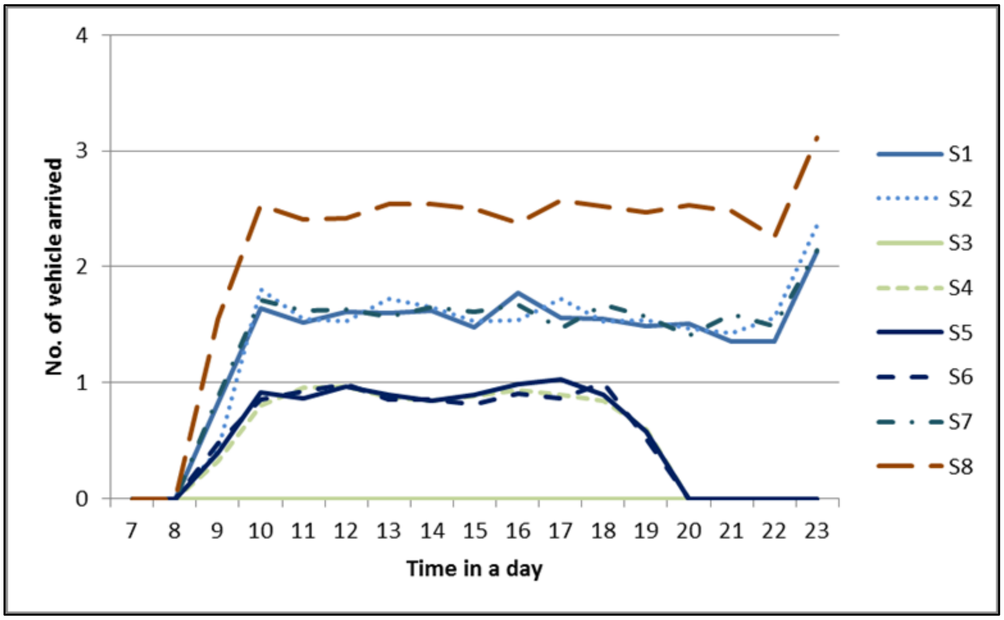

Figure 4.16 Averaged vehicle arrivals at each subarea during a day in weekends

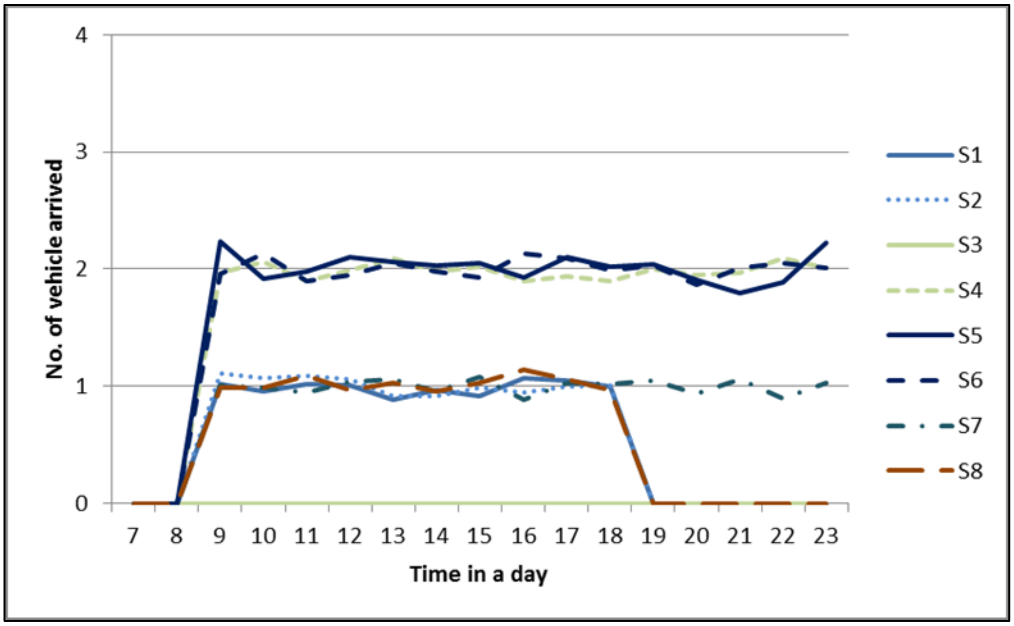

Figure 4.17 Averaged customer arrivals at each subarea during a day in weekends

4.2.2.2 Optimization results

Since the more complicated problem assumes two demand patterns, the optimization model is slightly modified to cater to this change. Equation 4.27 shows the altered objective function. The weighted operating cost is such that the one-day operating cost on weekdays accounts for 5/7 of the total cost, while that on weekends accounts for 2/7. The requirement for the customer loss rate is the same for both weekdays and weekends. With the exception that the size of the swarm changes to 15 to cope with the higher dimensions of the problem, the other parameters in the computation procedure remain the same. This part of the analysis only assesses the situation where hourly rebalancing is adopted.

$$\min f(x,v) = C_i \sum_{i\epsilon N} x_i + w * v + \frac{5}{7} * \left(R_{wd}(x,v) + P_{wd}(x,v)\right) + \frac{2}{7} * \left(R_{wn}(x,v) + P_{wn}(x,v)\right) + \mu * (g_{wd}(x,v) - \alpha)^2 + \mu * (g_{wn}(x,v) - \alpha)^2 \quad \mathbf{4.27}$$

Similar to the previous analysis, the complex case is solved by five rounds of the computation procedure. The results are summarized in Table 4.7. By examining the results, no statistically significant difference is found between the performances of the solutions. The average computation time is about three and half hours.

The results are consistent with the previous case where the total number of vehicles does not fluctuate so much across solutions compared with decisions on the number of parking spots at each subarea. One explanation may be that the unit daily cost of one vehicle is assumed to be much higher than that of a parking spot, which makes the results less sensitive to changes in the number of parking spots. On the other hand, the results indicate that a larger capacity must be installed at S1, S2, S3, S7, and S8. Although the hourly customer arrival rates are generally higher at S4, S5, and S6 than at S1, S2, S7, and S8, the vehicle arrival rates seem to play a more important role here in deciding the essential capacity. According to Figure 4.14 and Figure 4.16, the latter group expects more vehicles to be returned every hour. Since S3 has an obvious customer arrival peak, it requires a relatively larger capacity to accommodate sufficient vehicles. The results also show that the customer loss rate is lower on weekends than weekdays under the optimal solutions. This is because lower hourly arrival rates of demand are assumed for weekends. Consequently, the design with a scale adequate for weekdays is capable of achieving a higher LoS on weekends.

Table 4.7 Optimization results

No. of runs	Solution									Estimated Cost / SE	Estimated customer loss rate (Weekdays) /SE	Estimated customer loss rate (Weekends) /SE
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	v			
1	7	8	6	3	5	3	6	6	18	727/1.5	0.206/0.002	0.177/0.003
2	6	6	6	4	3	3	5	8	18	725/1.5	0.216/0.003	0.179/0.002
3	8	7	6	3	3	4	5	5	18	726/1.7	0.210/0.003	0.179/0.002
4	5	7	6	5	3	3	7	7	18	727/1.6	0.215/0.003	0.184/0.002
5	5	6	7	5	3	3	6	6	18	729/1.7	0.207/0.003	0.174/0.003

4.3 Summary

This part of the study has developed a decision-support tool to assist with determining the optimal configuration of a MoD system accounting for stochastic demand and the effect of conducting vehicle redistribution as part of daily operations. An optimization model is formulated that aims to identify the solution with the minimal cost but an adequate LoS. A simulation-based approach is devised to solve the model. The computation approach consists of a DES that assesses the performance of different system configurations, as well as a hybrid algorithm combining PSO and OCBA to efficiently search the decision space. The chapter presents two case studies to illustrate the application of the methodology, leading to several interesting observations.

First, rebalancing is demonstrated as an essential consideration when determining the configuration of a MoD system. Without considering such operations, decision makers may design a system with an excessive number of parking spots and vehicles. As reported in the simplified case study, the optimal solutions obtained under the condition of no rebalancing end up with significantly more parking spots to be rented and more vehicles to be purchased. In addition, an inefficient system design may make it difficult for operators to conduct rebalancing operations later on. As shown in this study, the hourly rebalancing optimization model may be infeasible due to the overall lack of vehicles or parking spaces. When rebalancing is incorporated, not only can a system design with a lower overall cost be identified, but a higher LoS in terms of satisfying more customers can also be realized.

Second, this study identifies a heuristic rule to allocate vehicles among subareas at the beginning of the day. This rule distributes vehicles proportionally to the customer arrival rates during the first time segment of an operating day. Numerical experiments show that statistically no difference exists between the results obtained using such a heuristic rule and those obtained by optimizing the number of vehicles at each subarea individually. This finding suggests an opportunity to simplify the decision-making process and reduce computation effort with respect to solving the problem.

Third, by conducting optimization under varied assumptions in the simple case study, the trade-offs between set-up cost, rebalancing cost, and LoS are explicitly illustrated. In particular, the results show that by increasing rebalancing activities, less additional investment is required to satisfy higher demands or stricter requirements of LoS, while reducing rebalancing activities has the opposite effect.

Finally, as demonstrated by the two case studies and several runs of the numerical experiments, the computation procedure is shown to have a stable performance. In particular, as the optimal number of vehicles obtained for the case without rebalancing is very close to the solution calculated by the approach from another paper, it somehow validates the solution approach proposed in this study. On the other hand, the numerical results also indicate that incorporating OCBA into the procedure accelerates the computation process and leads to a higher convergence speed.

In sum, the study in this chapter makes a contribution by integrating the operational-level decisions, namely rebalancing operations, into the planning of MoD systems considering daily demand fluctuations. The mathematical model and solution approach address the first and third research questions proposed in Section 2.3, respectively. The rebalancing operations, which redistribute vehicles among subareas frequently according to the realization of demand, can be perceived as an operational-level flexibility as to address fluctuations in a short time period. The overall usage pattern, which is captured by the hourly arrival rates of the Poisson processes, is assumed known and unchanged. However, as indicated in the sensitivity analysis for the prototype case, changes on such usage pattern have a clear effect on the final solutions.

The alternative solutions found in this study can be deemed as rigid from a strategic standpoint, as they follow a structured and well established plan over time. Further work is needed to account for the ability to adapt to changing demographic conditions, as the system is operated and deployed over a longer time scale (e.g. months). Such situation calls for the implementation of strategic-level flexibility in the system, which is analyzed in the next chapter.

There are some insights regarding designing and operating MoD systems generalized from the analysis in this chapter. These insights provide general guidelines as decision-making reference, but given that they are based on the particular assumptions made in this study, it may need further consideration to determine how they can be applied to other systems.

1) There is a trade-off between rebalancing cost, set-up cost, and LoS. Increasing LoS leads to increase in overall cost, but not necessarily both rebalancing cost and set-up cost.

2) Rebalancing is an essential consideration when making planning decisions. It creates a distinct difference on both the final planning decisions and their final performance.

3) Reducing rebalancing operations may not lead to significant reduction in rebalancing cost but is very likely to require a system with a larger capacity.

Chapter 5 Incorporating Strategic Flexibilities into MoD systems to Address Long-term Demand Uncertainty

The previous analysis assumes that demand patterns remain the same throughout the planning horizon. However, such assumption may not hold in reality, especially when a longer planning horizon is under consideration, e.g. a quarter or half a year. Demographics change, level of satisfaction with the system, construction of new buildings, and other relevant factors may lead to usage pattern changes in MoD systems. As explained earlier, existing studies do not provide an effective approach to cope with this higher level of uncertainty. This part of the analysis adds another layer of flexibility for the analysis of MoD systems, namely strategic flexibility, to address such long-term demand uncertainty.

As this part of the analysis focuses on long-term shift in usage trends, instead of prioritizing LoS that is more critical to the initial survival issue, the operating objective switches to profit maximization that weighs more in continuous development of the system. Therefore, the analysis in this chapter places more emphasis on profit instead of only cost.

A phasing strategy is formulated into the deployment plan so that the system can be expanded when certain conditions are met. Different from the analysis in the previous chapter, the optimization model here is constructed to find the optimal parameters for the phasing strategy that maximize the overall profit in the planning horizon. Similar to the previous chapter, a simulation-based approach is applied to solve the optimization problem. The DES is extended and modified to account for the formulation of the phasing strategy and a longer planning horizon. The PSO+OCBA algorithm developed in the last chapter is further applied in this analysis. In addition, a computational framework inspired by the successful implementation of this algorithm is generalized and proposed to address the optimization problem of other flexible systems.

5.1 A simulation-based methodology

5.1.1 Notations

Before introducing the methodology, the notations used throughout this chapter are first summarized as follow.

i, j Subareas in the geographic region

k Scenarios

m Months in a planning horizon;

w Weeks in a month;

d Days in a week;

t Time steps in a day;

N Set of the subareas;

T No. of time steps in a day;

K No. of scenarios;

w Depreciated daily cost of one vehicle;

C_i Rent cost of one parking spot at subarea *i*;

γ_{ij} Rebalancing cost between subarea *i* and subarea *j*;

t_{ij} Travel time between subarea *i* and subarea *j*;

q_i Temporary parking cost per hour in subareas *i*

p_{ij} Profit obtained of serving one customer from subareas *i* to subarea *j*;

d_{ij}^{tdwm} Satisfied demand at time step *t* of day *d* in week *w* of month *m*;

g_i^{tdwm} Temporary parking spots needed at time step *t* of day *d* in week *w* of month *m*;

r_{ij}^{tdwm} No. of vehicles rebalanced from subarea *i* to subarea *j* at time step *t* of day *d* in week *w* of month *m*;

s_i^{tdwm} No. of vehicles at subarea *i* at time step *t* of day *d* in week *w* of month *m*;

μ_i^{tdwm} Expected No. of vehicle pickup demand at subarea *i* at time step *t* of day *d* in week *w* of month *m*;

vs_i^{tdwm} Vehicle surplus at subarea *i* at time step *t* of day *d* in week *w* of month *m*;

h_i^{tdwm} Utilization rate of parking spots in subarea *i* at time step *t* in day *d* of week *w* of month *m*;

l^m No. of customer loss in month *m*;

z_m No. of realized demand in month *m;*

$\overline{h_i^m}$ Average hourly utilization rate of parking spots of subarea *i* in month *m*;

v_i^{dm} No. of vehicles placed at subarea *i* at the beginning of day *d* of every week in month *m*;

v^m No. of vehicle in the system in month *m*;

x_i^m No. of parking spots at subarea *i* in month *m*;

$f1$ No. of vehicles added into the system;

$f2$ No. of parking spots added into the system;

$\alpha1$ Minimum monthly customer loss rate that triggers adding vehicles;

$\alpha2$ Minimum vehicle-to-capacity ratio that triggers adding parking spots;

L_k Profit gained in scenario *k*

5.1.2 Phasing deployment strategy

As mentioned earlier, this chapter assumes that the overall usage pattern is subjected to uncertain change. In other word, the customer arrival rates, which are assumed deterministic in the previous analysis, now change randomly over time, reflecting possible (longer term) changes in factors like demographics, customer behaviors, etc. that may influence how the system is utilized. To deal with this situation, a phasing strategy is adopted. Defined by this strategy, the system starts with a relatively smaller capacity scale, and more parking spots and vehicles are added into the system if certain conditions are fulfilled. More specifically, it is assumed that the status of the system, namely the number of parking spots in each subarea and the total number of vehicles in the system, is updated monthly.

The initial system configuration is optimized for the first month using the methodology from the previous chapter but the objective function is changed to maximizing profit and the rebalancing schemes is calculated using the hubristic rule that will be introduced later. Meanwhile, at the end of every month, a set of decisions rules will be applied to reconfigure the system. Basically, there are decisions on two aspects in the decision rules. If loss of customers is too high, extra vehicles are added into the system. The vehicle distribution is then decided by the heuristic rule introduced in the previous chapter (Equation 4.4 and 4.5). On the other hand, if the overall vehicle-to-capacity ratio (defined as the total number of vehicles divided by total number of parking spots) exceeds a certain threshold, more parking spots are rented at the subarea having the highest average hourly rate of parking spots utilization. The mathematical expression of the decision rules are presented in the next section along with the optimization model.

5.1.3 Optimization model

Equation 5.1-5.6 present the optimization model. As defined in Equation 5.1 below, the objective of the optimization model in this chapter is to maximize the expected profit in the planning horizon by finding the proper set of parameters $(\alpha 1, \alpha 2, f1, f2)$ for the decision rule. The objective function consists of a revenue function $P(\alpha 1, \alpha 2, f1, f2)$, a vehicle cost function $V(\alpha 1, \alpha 2, f1, f2)$, a fixed parking cost function $S(\alpha 1, \alpha 2, f1, f2)$, a temporary parking cost function $Q(\alpha 1, \alpha 2, f1, f2)$, and a rebalancing cost function $R(\alpha 1, \alpha 2, f1, f2)$. The mathematical formulation of the decision rule is displayed in Equation 5.2-5.3. Equation 5.4-5.6 characterize the decision space.

$$\max\ L(\alpha 1, \alpha 2, f1, f2) = E[P(\alpha 1, \alpha 2, f1, f2) - S(\alpha 1, \alpha 2, f1, f2) - Q(\alpha 1, \alpha 2, f1, f2) - R(\alpha 1, \alpha 2, f1, f2)] \quad \textbf{5.1}$$

$$If\ \ \frac{l^m}{z^m} > \alpha 1, v^{m+1} = v^m + f1,\ \forall\, m \geq 2 \quad \textbf{5.2}$$

$$If\ \frac{v^{m+1}}{\sum_{i=1}^{n} x_i^m} > \alpha 2,\ x_s^{m+1} = x_s^m + f2,\ {*s} = arg\, max_{i \in N}(\overline{h_i^m}),\ \forall\, m \geq 2 \quad \textbf{5.3}$$

$$0 \leq \alpha 1 \leq 1 \quad \textbf{5.4}$$

$$0 \leq \alpha 2 \leq 1 \quad \textbf{5.5}$$

$$f1, f2 \geq 0, \in Integer \quad \textbf{5.6}$$

The following equations provide more details about the random functions aforementioned. Since the system configuration in the first month is obtained by maximizing the profit in that month, which is independent of the optimization of the phasing strategy, $\boldsymbol{d_{ij}^{tdw1}}$, $\boldsymbol{g_i^{tdw1}}$, and $\boldsymbol{r_{ij}^{tdw1}}$ are random variables that depend on the realization of demand. On the other hand, for the other months, as the decision rule is applied to update the system configuration, the value of these random functions depends on the parameters of the decision rule$(\alpha 1, \alpha 2, f1, f2)$. Here bold characters are used to highlight that the symbols are either random variables or random functions.

$$P(\alpha 1, \alpha 2, f1, f2) = \sum_{i,j,t,d,w} p_{ij} \boldsymbol{d}_{\boldsymbol{ij}}^{\boldsymbol{tdw1}} + \sum_{i,j,t,d,w,m\geq 2} p_{ij} \boldsymbol{d}_{\boldsymbol{ij}}^{\boldsymbol{tdwm}}(\alpha 1, \alpha 2, f1, f2) \quad \mathbf{5.7}$$

$$S(\alpha 1, \alpha 2, f1, f2) = \sum_{i,} C_i * x_i^1 + w * v^m + \sum_{i,m\geq 2} C_i * \boldsymbol{x}_{\boldsymbol{i}}^{\boldsymbol{m}}(\alpha 1, \alpha 2, f1, f2) + \sum_{m\geq 2} w * \boldsymbol{v}^{\boldsymbol{m}}(\alpha 1, \alpha 2, f1, f2) \quad \mathbf{5.8}$$

$$Q(\alpha 1, \alpha 2, f1, f2) = \sum_{i,t,d,w} q_i \boldsymbol{g}_{\boldsymbol{i}}^{\boldsymbol{tdw1}} + \sum_{i,t,d,w,m\geq 2} q_i \boldsymbol{g}_{\boldsymbol{i}}^{\boldsymbol{tdwm}}(\alpha 1, \alpha 2, f1, f2) \quad \mathbf{5.9}$$

$$R(\alpha 1, \alpha 2, f1, f2) = \sum_{i,j,t,d,w} \gamma_{ij} \boldsymbol{r}_{\boldsymbol{ij}}^{\boldsymbol{tdw1}} + \sum_{i,j,t,d,w,m\geq 2} \gamma_{ij} \boldsymbol{r}_{\boldsymbol{ij}}^{\boldsymbol{tdwm}}(\alpha 1, \alpha 2, f1, f2) \quad \mathbf{5.10}$$

Since there is no analytical expression for $L(\alpha 1, \alpha 2, f1, f2)$, this study builds a discrete event simulator (DES) to estimate its value given $\alpha 1, \alpha 2, f1$ and $f2$. Provided with the decision variables, parameters, and the number of simulations (for example, *K*), the simulator can generate the required number of sample points of L_k. Then, the value of the objective function is estimated using Equation 5.11. The following section provides more details about the DES.

$$L(\alpha 1, \alpha 2, f1, f2) \sim \frac{\sum_{k=1}^{K} L_k}{K} \quad \mathbf{5.11}$$

5.1.4 Discrete event simulator (DES)

Similar to the previous chapter, the fundamental component of the simulation-based methodology is the discrete event simulator (DES). The simulator intends to estimate the profit obtained by any given phasing strategy, namely a combination of $\alpha 1, \alpha 2, f1$, and $f2$, over a relatively longer planning horizon (assumed to be half a year in the analysis). Compared with the simulator developed in the previous chapter, there are some major changes in this DES.

First, this model considers a change in the overall usage pattern that was assumed to remain the same in previous studies. Second, the formulation of the phasing strategy is coded into the simulator that may constantly alter the configuration of the system. Finally, although hourly rebalancing is still considered in this part of the analysis, this simulator adopts a heuristic rule to guide the rebalancing operations, which aims at easy implementation and computational efficiency. The details of these modifications on the DES are as follows. Other assumptions are the same with the ones in Chapter 4.

5.1.4.1 Model assumptions

The simulation environment is also developed using MATLAB. Several assumptions are made on the simulation model to characterize the system and code its behavior. If users are interested in other settings, those assumptions can be easily modified.

1) Planning horizon and time scale

Half-year performance is evaluated. Each month contains four weeks. 17 hours are analyzed for each day, and each time step represents one hour in reality.

2) Demand generation

The hourly arrivals of passengers at each subarea throughout a day are modeled as non-stationary Poisson processes. Assumptions are also made about what proportion of the demands originating in one subarea will end up in other subareas. Just like the previous chapter, a dataset containing arrival rates of the

Poisson processes for all subareas throughout a day is defined as a demand pattern.

For each week, realized demand is the same for each demand pattern. For example, if only one demand pattern exists for a week, then a one-day scenario is generated and repeatedly used for the seven days of that week; while if two demand patterns are considered, e.g. weekdays and weekends, two one-day scenarios are generated, with one representing a typical weekday and the other representing a typical day on weekends.

The demand patterns, or the sets of arrival rates of hourly demand in a day, are assumed to change every month. As indicated in Equation 5.12, monthly growth rates of these arrival rates are formulated as a geometric browning motion (GBM) process.

$$d\delta_m = \mu\delta_m dt + \sigma\delta_m dW_m \quad \textbf{5.12}$$

* δ_m-vector of arrival rates of vehicle pickup demand at month m, μ-drift, σ-volatility, W_m-Wiener Process

3) Rebalancing operation

Rebalancing operations are to be carried out every hour in the simulator, i.e. at the end of each time step. Instead of using the integer-programming model to calculate the rebalancing decisions as in the previous chapter, the following heuristic rule is applied. In brief, the rule tries to send vehicles from subareas with extra vehicles to the nearby subareas that require more vehicles.

I. For each $i \in N$, calculate the vehicle surplus based on Equation 5.13. The vehicle surplus indicates the extra vehicles for a particular subarea between the number of vehicles and the expected number of vehicle pickup demand at the next time step. Then, obtain the ranking of subareas ($O_{(i)}$) based on the descend order of the vehicle surplus. Set *i*=1.

$$vs_i^{tdwm} = s_i^{tdwm} + \sum_{j=1,\neq i}^{n} d_{ji}^{(t+1-t_{ij})dwm} + r_{ji}^{(t+1-t_{ij})dwm} - \mu_i^{(t+1)dwm} \quad \textbf{5.13}$$

II. Obtain the ranking of other subareas ($O_{(i)}^{j}$) based on the ascend order of the distance between the selected subareas. Set *j*=1.

III. Calculate the number of vehicles being transported from $O_{(i)}$ to $O_{(i)}^{j}$ ($r_{O_{(i)}O_{(i)}^{j}}^{tdwm}$) using Equation 5.14, and update *j* to *j*+1

$$r_{O_{(i)}O_{(j)}^{i}}^{tdwm} = \min\left[s_{O_{(i)}}^{tdwm}, \quad vs_{O_{(i)}}^{tdwm}, \quad -\min\left(0, \quad vs_{O_{(i)}^{j}}^{tdwm}\right)\right] \quad \textbf{5.14}$$

IV. Check if *j*=*n*-1 or Equation 5.15 is satisfied or not. If not, go to step III. If so, update *i* to *i*+1.

$$s_{O_{(i)}}^{tdwm} - \sum_{l=1,O_{(i)}^{l}\neq O_{(i)}}^{j} r_{O_{(i)}O_{(i)}^{l}}^{tdwm} + \sum_{j=1,\neq O_{(i)}}^{n} d_{jO_{(i)}}^{(t+1-t_{O_{(i)}j})dwm} + r_{jO_{(i)}}^{(t+1-t_{O_{(i)}j})dwm} \leq \mu_{O_{(i)}}^{(t+1)dwm} \quad \textbf{5.15}$$

V. Check if *i*=*n*. If not, go to step II. If so, this round of rebalancing is finished.

5.1.4.2 Activity mapping

Before running the simulation, there are a set of inputs to be specified, namely the number of parking spots at each subarea and the total number of vehicles at the first month. These two decisions are obtained by optimizing the profit in the first month beforehand. Besides, users need to determine the parameters of the flexible strategies ($\alpha 1$, $\alpha 2$, $f1$, and $f2$) when flexibility is under consideration.

Figure 5.1 shows the steps in one run of the simulation. In each run of the simulation, for each month, the simulation begins with generating the demand growth rate for that month, which is further used to calculate the arrival rates of the Poisson processes. Then, starting with Week 1 and Day 1 of that week, at the beginning of every time step and for every subarea, the simulation first calculates utilization rate (h_i^{tdwm}), and generates travel demand (z). For each realized demand, if vehicles are available, the destination is simulated, and the relevant parameters on system status, e.g. the number of vehicles at each site (s_i^{tdwm}), revenues obtained (d_{ij}^{tdwm}), are updated accordingly. At the end of each time step, the heuristic rebalancing rule introduced earlier is applied to determine the vehicle redistribution decisions, and the relevant system parameters are recalculated based on the results. After 17 hours are simulated, the simulation process will proceed to another day, then another week, until the end of that month. Next, the trigger points of decision rules ($\alpha 1$ and $\alpha 2$) are checked, and the system is reconfigured if the rules are satisfied. This run of simulation is ended when all six months are simulated. For a particular run of simulation, the main output, profit obtained (L_k), is calculated by .

$$L_k = \sum_{i,j,t,d,w,m} p_{ij} d_{ij}^{tdwm} - \sum_{i,m} C_i * x_i^m + \sum_m w * v^m - \sum_{i,t,d,w,m} q_i g_i^{tdwm} - \sum_{i,j,t,d,w,m} \gamma_{ij} r_{ij}^{tdwm} \quad \textbf{5.16}$$

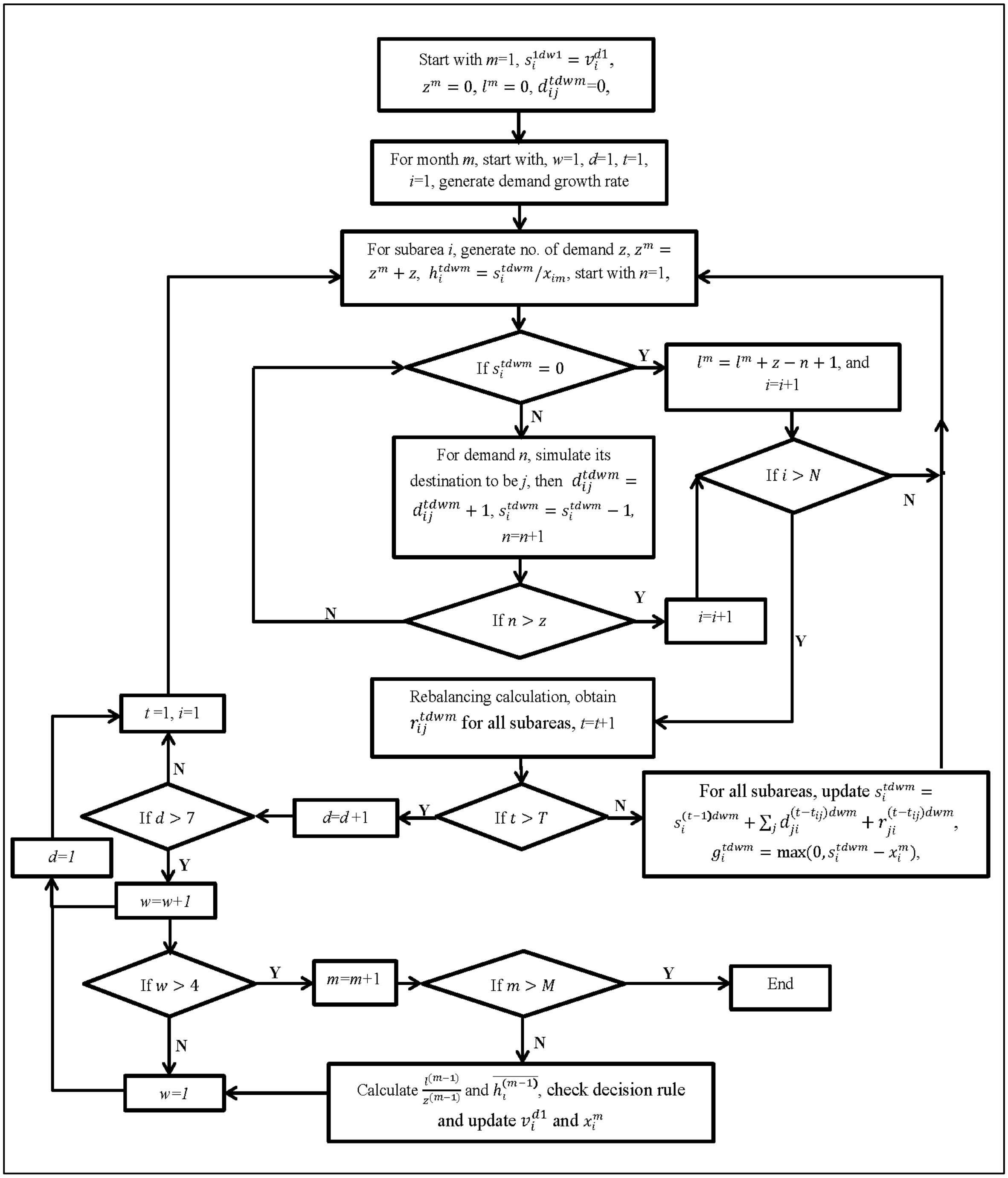

Figure 5.1 Activity map of the simulation model

5.2 Application

5.2.1 Case study of a prototype problem

5.2.1.1 Problem setting

Similar to the analysis in Chapter 4, the proposed methodology is applied to a prototype case first where only three subareas are under consideration. The same geographic area as for section 4.2.1 is under study, and the cost parameters also remain the same. Additional parameters on modelling demand pattern changes and unit revenue that are not under consideration in the previous chapter are displayed in Table 5.1. The drift and volatility of the demand growth rate is chosen to give a relatively high expected growth rate and its variance, while the revenue parameter is derived from Jorge et al. (2012). Those assumptions will be further tested in the sensitivity analysis.

Table 5.1 Additional parameters

Drift of the demand growth rate	Volatility of the demand growth rate	Revenues of servicing one customer per hour
0.25	0.20	17.58

5.2.1.2 Simulation results

To illustrate how the simulator works, a combination of parameters, as displayed in Table 5.2, is tested in an example simulation run. As explained earlier, the initial system configuration (x_{11}, x_{21}, x_{31}, and v_1) is obtained by optimizing the profit in the first month, while the parameters for the phasing strategy (α1, α2, f1, and f2) are randomly chosen. Here, α1 stands for the threshold of the customer loss rate that triggers adding f1 more vehicles into the system; α2 represents the threshold of the ratio of vehicle-to-capacity that triggers renting f2 more parking spots in the system; x_i^1 is the No. of parking spots rented at subarea *i* at the beginning of the first month; v^1 is the total No. of vehicles in the system at the beginning of the first month.

Table 5.2 The decision in the illustration example

α1	α2	f1	f2	x_1^1	x_2^1	x_3^1	v^1
0.2	0.8	5	5	5	6	5	9

Figure 5.2 shows the total demand every month in the 500 runs of the simulation. As indicated in the figure, although variations exist between scenarios, an overall trend of gradual increase every month can be observed. To further illustrate how the system changes over time under the test decisions, results from one run of the simulation are examined in more details. Several indicators are summarized and presented in Table 5.3.

In this particular scenario (whose demand realization is highlighted in Figure 5.2), demand increases relatively fast especially for the last month. Customer loss rate first increases, then decreases, and increases again in the last two months. As in the first two months, demand increases, but the scale of the system remains the same because the customer loss rate is not high enough (namely less than the α1 = 0.2 specified in Table 5.2) to trigger the expansion decision. However, in the third month, the customer loss rate exceeds the triggering point, so f1=5 vehicles are added into the system at the beginning of the fourth month. Meanwhile, since the ratio of vehicle-to-capacity is 0.88 that is higher than the α2=0.8 specified in Table

5.2, f2=5 more parking spots are rented. As in the third month, the subarea 3 has the highest average utilization rate of parking spots (0.86), the extra parking spots are installed there. This expansion of the system at the beginning of the fourth month reduces the customer loss rate from 0.28 to 0.16, although demand increased. As the time approaches to the fifth month, the existing scale of the system is not sufficient to satisfy the required LoS again, as the customer loss rate exceeds $\alpha 1 = 0.2$. Such situation triggers the execution of the decision rule again, and f1=5 more vehicles are added into the system in the sixth month. However, since the ratio of vehicle-to-capacity is below f2=0.8, no more parking spots are rented. It is also interesting to notice that, although the system is expanded at the beginning of the last month, the customer loss rate does not go down, which may be due to the demand increasing so fast in this month that the number of vehicles added into the system is not sufficient. The final profit obtained under this particular scenario is €135,722.

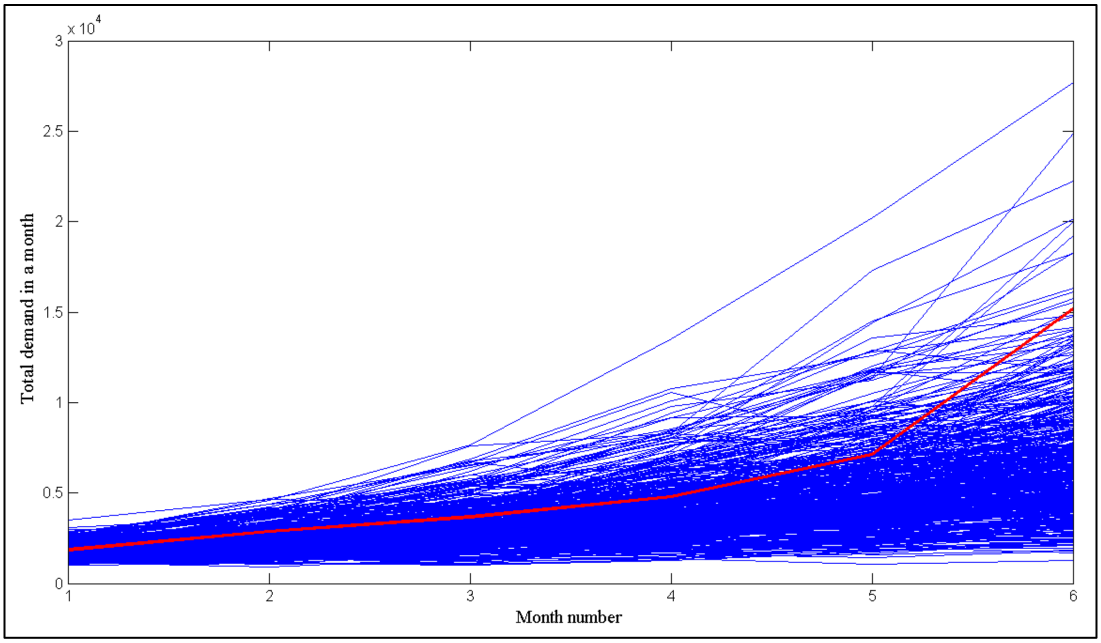

Figure 5.2 500 scenarios of monthly demand

Table 5.3 Illustration of changes in the system under the test decision over time

	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
Total demand	1,855	2,863	3,661	4,795	7,147	15,162
Customer loss rate	0.16	0.19	0.28	0.16	0.36	0.56
No. of vehicles/Total capacity (Before adding capacity)	0.56	0.56	0.56	0.88	0.67	0.90
Utilization_s1	0.55	0.55	0.46	0.73	0.83	0.50
Utilization_s2	0.35	0.43	0.39	0.61	0.51	0.75
Utilization_s3	0.83	0.74	0.86	0.67	0.68	0.95
Capacity_s1	5	5	5	5	5	10
Capacity_s2	6	6	6	6	6	6
Capacity_s3	5	5	5	10	10	10
Total No. of vehicles	9	9	9	14	14	19

Based on the sample points obtained in these 500 scenarios, the convergence of the estimation on the objective function value, namely the half-year profit, is also examined. Figure 5.3 displays how this estimation varies with the number of scenarios considered. It indicates that the results start to become quite stable after 400 runs of the simulation, which also suggests that 500 scenarios are sufficient to provide an accurate estimation of half-year profit for a given decision.

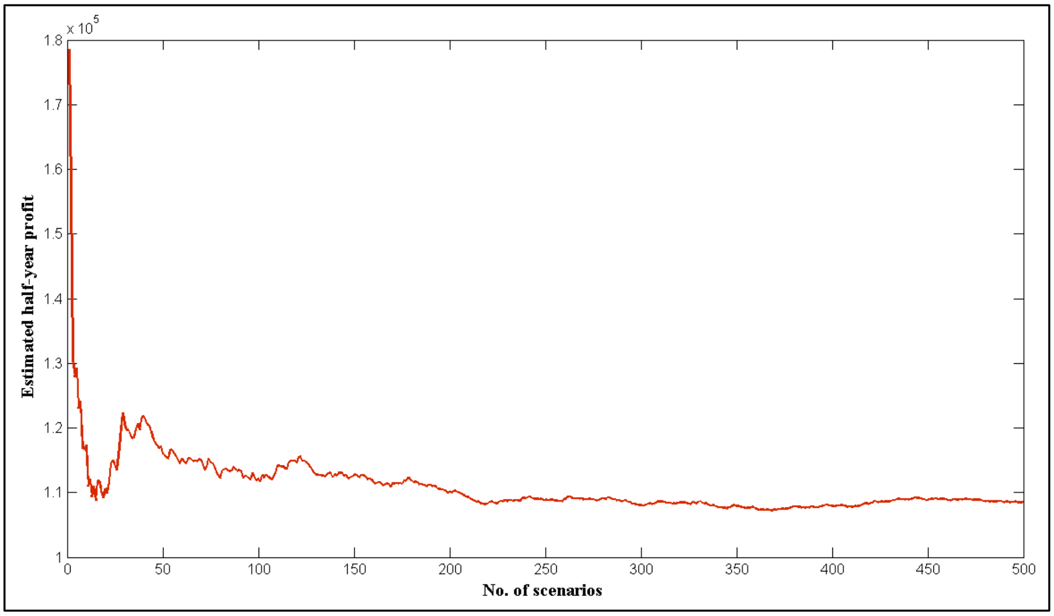

Figure 5.3 Convergence on the estimation of half-year profit

5.2.1.3 Optimization results

This part of the study adopts the same solution approach as the previous chapter, namely PSO and OCBA. Furthermore, to assess the value of the flexible strategies, optimization is conducted on the fixed design, where the system is deployed once at the beginning and remains the same throughout the planning horizon (i.e. a stochastically robust solution). For both fixed and flexible design, five runs of optimization are conducted. The average computation time of optimization on the flexible designs is 3857 seconds with a standard deviation of 657 seconds, while the average time for optimizing the fixed designs is 5635 seconds with a standard deviation of 1275 seconds. Table 5.4 and Table 5.5 display the optimal fixed and flexible solutions respectively. As indicted in Table 5.5, the initial configuration of the system is the same for the five runs of the simulation.

Table 5.4 Optimal fixed solutions

	x_1	x_2	x_3	v
fixed-1	10	12	11	19
fixed-2	12	12	10	19
fixed-3	8	10	12	20
fixed-4	11	11	11	19
fixed-5	9	10	13	17

Table 5.5 Optimal flexible solutions

	α1	α2	f1	f2	x_1^1	x_2^1	x_3^1	v^1
flex-1	0.075	0.275	6	7				
flex-2	0.050	0.475	6	10				
flex-3	0.050	0.450	5	7	5	6	5	9
flex-4	0.100	0.200	6	8				
flex-5	0.050	0.200	5	9				

Figure 5.4 shows the CDF curves of all solutions (Dashed lines represent the optimal flexible solutions, while solid lines are the optimal fixed solutions), and Table 5.6 presents the performances of all solutions under different criteria.

Combined with the results in Table 5.7 that show a statistically significant difference between the mean values of the flexible designs and the fixed ones, these results indicate that the flexible designs are clearly better than the fixed solutions. Not only do the flexible designs have a better average performance, but they have a better ability to avoid downside losses when demand turns out to be lower than expected (indicated by the higher P5 value) as well as to capture upside gains when market turns out to be better (as flexible designs have higher P95 value). This is because flexible designs enable waiting and changing the system configuration at the right time. When lower demand happens, since flexible designs start with smaller capacity scale, decision-makers can choose to delay the expansion of system to avoid losses caused by an excessive (and often times unused) capacity, while in such case the fixed designs are not able to adapt to the situation, and may suffer from wasted initial investments in renting parking spots and purchasing vehicles. In fact, as illustrated in Figure 5.4, there are some situations where fixed designs end up with negative profits. On the other, when demand is higher than expected, the flexible designs are able to add more vehicles and parking spots into the system as to serve more customers, hence, leading to a higher profit.

Meanwhile, results from Table 5.7 also show the robustness of the computational procedure. Under the two optimization conditions, namely flexible design optimization and fixed design optimization, although the final solutions are not exactly the same, there is no statistical difference between them. There is, as expected, a statistically significant difference between the fixed and flexible solutions.

Furthermore, Equation 5.17 is applied to calculate the value of flexibility (VoF). Here, $flex_i$ is the estimated profit of flexible design i, and $fixed_i$ is the estimated profit of fixed design i. By taking the average of the five fixed and flexible solutions, respectively, and calculate the difference between the two averaged values, the VoF is estimated to be €11,777, which is roughly 11% of the averaged value of the fixed solutions.

$$VoF = \frac{\sum_{i=1}^{5} flex_i}{5} - \frac{\sum_{i=1}^{5} fixed_i}{5} \qquad \textbf{5.17}$$

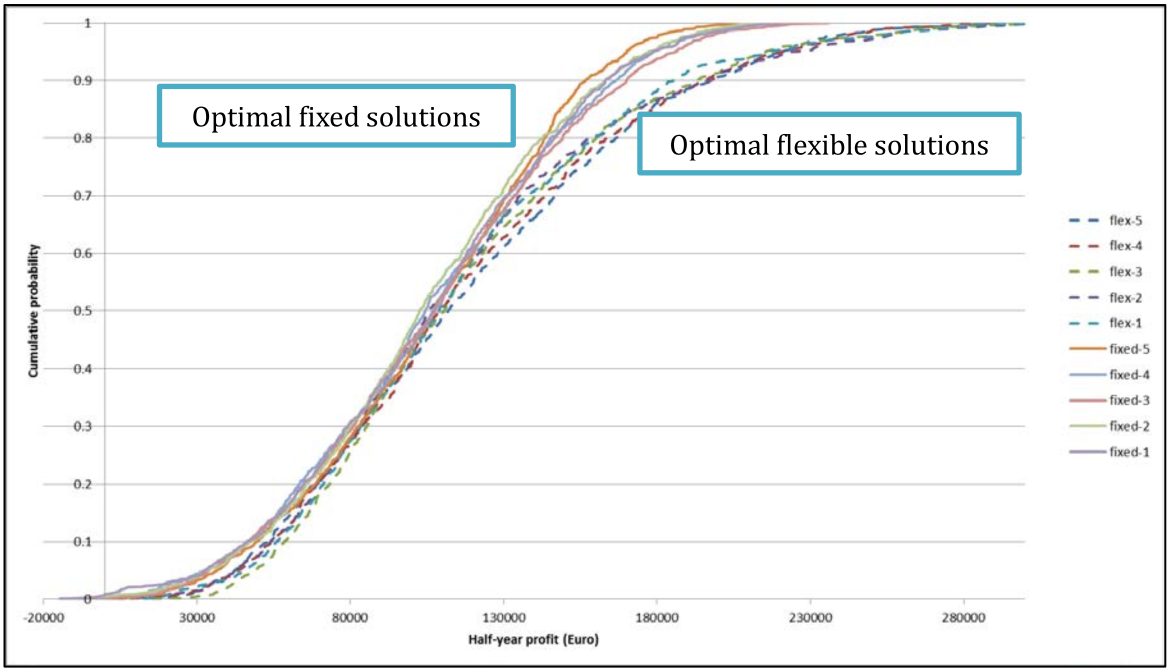

Figure 5.4 CDF curves of half-year profit for all solutions

Table 5.6 Performance metrics of all solutions

	fixed-1	fixed-2	fixed-3	fixed-4	fixed-5	flex-1	flex-2	flex-3	flex-4	flex-5
Mean	105,772	104,662	107,702	106,084	105,876	116,522	116,528	118,439	118,334	119,160
SE	1,425	1,368	1,450	1,426	1,295	1,674	1,748	1,654	1,708	1,733
P5	32,575	32,332	33,874	33,056	36,519	45,475	42,550	48,046	43,592	42,413
P95	178,715	177,167	185,886	178,853	170,112	217,192	223,804	215,387	220,726	220,543

Table 5.7 P-value of pairwise t-test between solutions

	fixed-1	fixed-2	fixed-3	fixed-4	fixed-5	flex-1	flex-2	flex-3	flex-4	flex-5
fixed-1	N.A.	0.57	0.34	0.88	0.96	0.00	0.00	0.00	0.00	0.00
fixed-2		N.A.	0.13	0.47	0.52	0.00	0.00	0.00	0.00	0.00
fixed-3			N.A.	0.43	0.35	0.00	0.00	0.00	0.00	0.00
fixed-4				N.A.	0.91	0.00	0.00	0.00	0.00	0.00
fixed-5					N.A.	0.00	0.00	0.00	0.00	0.00
flex-1						N.A.	1.00	0.42	0.45	0.27
flex-2							N.A.	0.43	0.46	0.29
flex-3								N.A.	0.96	0.76
flex-4									N.A.	0.73
flex-5										N.A.

5.2.1.4 Sensitivity analysis

To see the robustness of the results, OFTA analysis is applied to conduct a sensitivity analysis on the cost parameters and the volatility of demand growth rate. The values of these parameters are varied by ± 50% and 25% at a time, and the VoF is recalculated each time using Equation 5.17.

Figure 5.5 shows the result of the sensitivity analysis. Among all the factors, unit revenue seems to influence the VoF most. As the unit revenue goes higher, the profit brought by one additional customer increases accordingly. Due to the ability of capturing more demand by gradual expansion, the flexible design is able to make higher profit than the fixed one. Demand volatility also demonstrates a strong positive relation with the VoF. As the demand becomes more volatile, flexible designs are more capable of adapting to the changing environment, with higher realized demand leading to more frequent exercising of the expansion option, while in the opposite situation delaying such decision. However, the larger variation of demand only makes the fixed design worse, as the initial investment may become excessive if demand is lower than expected while unexpected higher demand, on the other hand, cannot be satisfied. Furthermore, car cost is also observed to increase along with VoF, although its influence is not as strong as the previous two factors. As the flexible design starts with a smaller scale and expands only if necessary, it saves more cost than the fixed one by having fewer vehicles, and a higher car cost enhances such advantage. On the other hand, it is interesting to see that another cost parameter linking to the initial investment, the parking cost, seems to have no effect on VoF. This may be explained by the fact that the daily cost of a parking spot is much smaller compared with the depreciated daily car cost. Temporary parking cost also has very limited influence on VoF, which occurs because under both fixed and flexible designs, there are very few occasions when temporary parking is needed. The only factor that seems to negatively relate to VoF is the rebalancing cost. The result here indicates that rebalancing cost seems to have more impact on the flexible design than the fixed one. Contrary to the flexible design, initially, fixed design has more vehicles and parking spots, resulting in less rebalancing operations to be conducted, which is why a higher rebalancing cost affects less the fixed design.

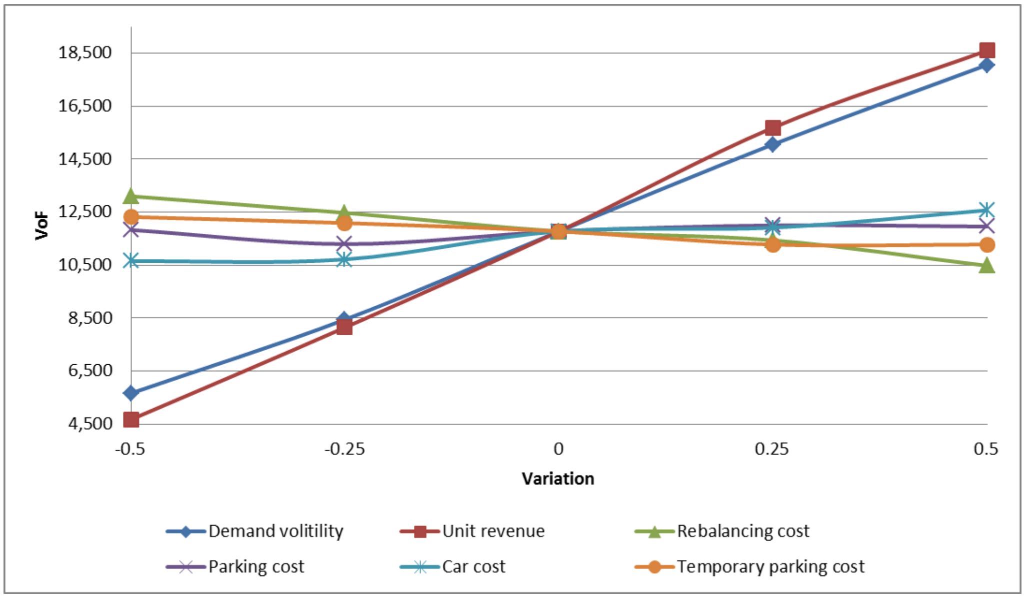

Figure 5.5 Sensitivity analysis on VoF against changes (percentage) on main parameters

5.2.2 Case study of a more complex problem

Similar to the analysis in the previous chapter, the methodology is applied to a more complex problem to study the scalability of the proposed approach. The problem setting is the same as section 4.2.2. Parameters regarding demand modeling and unit revenue remain the same with the prototype case (shown in Table 5.1).

5.2.2.1 Optimization results

The same solution approach is applied to the second case study. For both fixed and flexible design, this study runs five times the computational procedure. The average computation time of optimization for the flexible designs is 21,321 seconds with a standard deviation of 4,706 seconds, while the average time for optimizing the fixed designs is 38,204 seconds with a standard deviation of 6,391. Optimizing the fixed designs takes longer because there are more decision variables to determine than for the flexible systems (9 for the fixed designs, 4 for the flexible systems).

Table 5.8 and Table 5.9 display the optimal solutions obtained. For example, in Table 5.8, the fixed-1 solution indicates 6 parking spots should be rented at subarea 1 and 5, 7 at subarea 2 and 4, 8 at subarea 6,10 at subarea 8, 12 at subarea 7, while no parking spots are needed for subarea 3. The solution also recommends purchasing 28. On the other hand, in Table 5.9, the first four columns specify the flexible strategy. For example, in the solution of flex-1, when customer loss rates exceeds $\alpha 1 = 0.2$, f1=9 vehicles are added into the system, while when the ratio of vehicle-to-capacity is higher than $\alpha 2 = 0.625$, f2=8 more parking spots are rented. The system configuration at the beginning, namely the beginning of the first month, is presented in the last nine columns, where x_i^1-x_i^8 indicate the parking spots at each subarea, while v^1 indicates the total number of vehicles. Similar to the prototype case, the same initial system configuration is adopted for the five runs of the optimization for the flexible systems. The performances of the solutions obtained are summarized into Table 5.10 and Figure 5.6. For each solution, the performance is estimated by averaging over 500 scenarios.

Table 5.8 Optimal fixed design for the complex problem

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	v
fixed-1	6	7	0	7	6	8	12	10	28
fixed-2	7	10	0	8	6	8	8	8	28
fixed-3	5	7	0	6	6	6	5	8	28
fixed-4	5	6	0	10	6	9	7	8	27
fixed-5	8	8	0	7	7	7	7	8	29

Table 5.9 Optimal flexible designs for the complex problem

	α1	α2	f1	f2	x_1^1	x_2^1	x_3^1	x_4^1	x_5^1	x_6^1	x_7^1	x_8^1	v_1
flex-1	0.200	0.625	9	8									
flex-2	0.150	0.650	7	8									
flex-3	0.200	0.450	10	9	2	4	0	3	3	6	3	6	15
flex-4	0.200	0.575	15	9									
flex-5	0.175	0.700	14	10									

Table 5.10 Performance metrics for all the solutions

	fixed-1	fixed-2	fixed-3	fixed-4	fixed-5	flex-1	flex-2	flex-3	flex-4	flex-5
Mean	116,883	117,508	120,255	120,276	118,656	140,968	141,788	138,585	136,715	136,063
SE	1,821	2,029	1,811	1,756	2,090	2,331	2,337	2,403	2,363	2,558
P5	43,767	37,374	44,632	46,389	40,093	54,415	54,004	55,974	59,351	48,304
P95	181,047	186,734	181,499	180,483	190,083	224,584	225,798	234,650	236,140	235,707

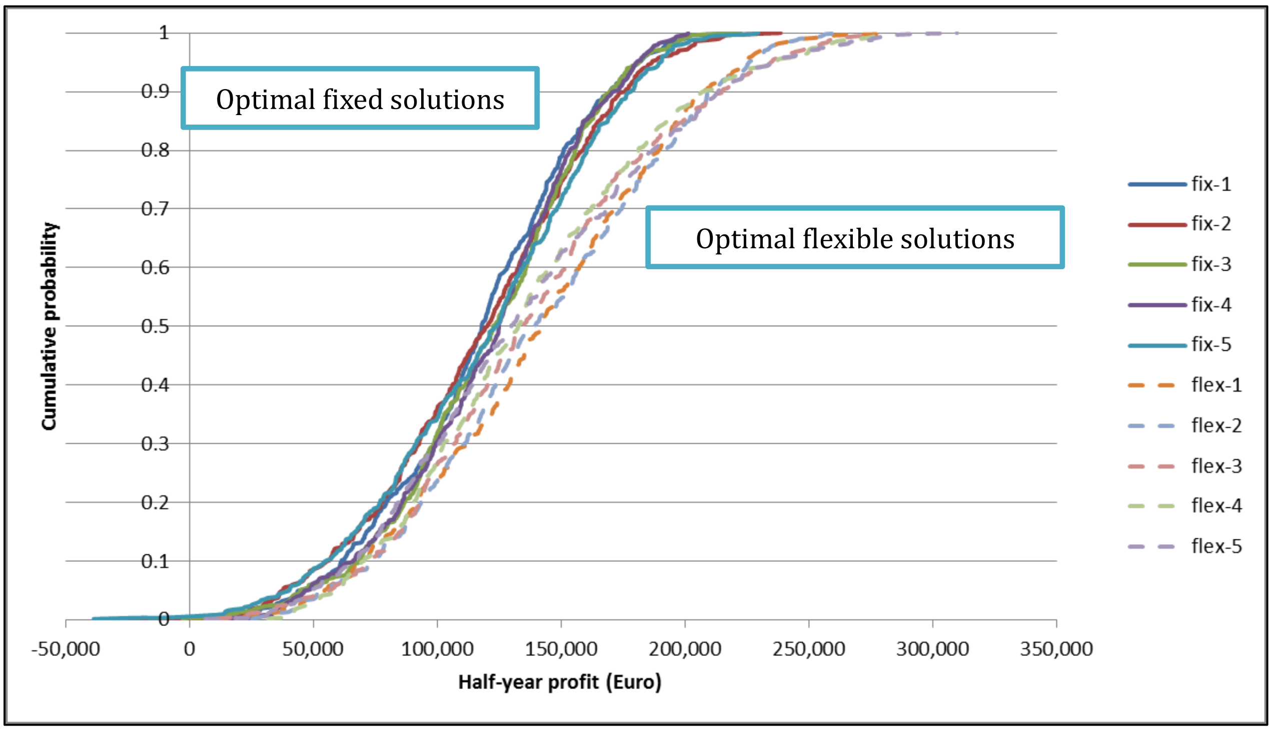

Figure 5.6 CDF of half-year profit for all solutions

Although according to Table 5.8 and Table 5.9, a slightly different solution is obtained in each run of optimization, within each group, either the fixed design or the flexible design, there is no statistically difference as indicated in Table 5.10. However, between the optimal fixed and flexible solutions, clear difference can be observed. No matter on mean value, P5 value, or P95 value, all of the flexible designs are better than fixed ones. In fact, as shown in Figure 5.6, fixed designs are almost all dominated by flexible ones. By applying Equation 5.11 again, this study obtains a VoF of €20,108 for the complex problem, which accounts for nearly 17% improvement on the fixed designs. This percentage of improvement is higher than that achieved for the prototype case.

5.2.2.2 Further analysis

To further investigate how the flexible and fixed designs perform along the planning horizon, for each month and each solution, the study calculates the average revenue, set-up cost (sum of depreciated cost of vehicles and rent cost of parking spots), rebalancing cost, cost for temporary parking, and profit. Then, the average performances of the two groups of designs are summarized into Figure 5.7.

As shown in Figure 5.7, with an assumed increase in demand every month, both fixed designs and flexible designs reap more and more revenues. Fixed designs start with higher revenue, as initially they are built with larger capacity and more vehicles, which makes them able to capture more demand at the beginning. However, since there is a phasing option incorporated into the flexible designs, they are expanded gradually, which is indicated by their increasing set-up costs. Consequently, more and more demand is satisfied due to the gradual expansion on the flexible designs. Finally, beginning in Month 3, flexible designs start to harvest more revenues than the fixed ones.

Regarding set-up cost, as the scale of fixed designs remain the same throughout the planning horizon, this cost item does not change accordingly. On the other hand, the flexible designs keep scaling up along with increasing demand, leading to an increase in set-up cost. Another interesting observation can be found in Month 3. In this month, the set-up cost of flexible designs is slightly lower than the fixed ones, which indicates that they still remain in a smaller scale than fixed ones, whereas they manage to satisfy more demand than the fixed ones. This may be achieved by conducting more rebalancing operations, and allowing more customers to use temporary parking. As shown in Figure 5.7, both rebalancing cost and extra parking cost is higher for flexible designs.

With respect to operating cost, on the one hand, extra parking cost remains very small for both flexible and fixed designs during the whole planning horizon, although slight fluctuations can be observed. Initially, this cost item is higher in fixed designs than flexible designs. However, because the flexible design keeps expanding, and according to Table 5.9, it seems to purchase additional vehicles faster than rent more parking spots, since in Month 2, more temporary parking is needed for flexible designs. On the other hand, rebalancing cost, which is the other operating cost item, of flexible designs gradually increases during the planning horizon, as increase in demand leads to more rebalancing operations. However, for the fixed designs, this cost first increases from Month 1 to Month 2 and then decreases until the end of the planning horizon. This observation seems somehow counter-intuitive. For the first two months, increase in demand indeed requests more rebalancing operations, leading to the increase in rebalancing cost.

At this early stage, although demand is increasing but it is still relatively low compared to the whole planning horizon, there is a higher chance that there are surplus vehicles in some subareas during the day. As such, rebalancing operations are triggered to place the idle vehicles to the subareas where they are in need. However, as demand goes up gradually in the later stage, the system may be faced with an overall lack of vehicles, which consequently leads to less and less idle vehicles to be rebalanced, and hence, a decrease in rebalancing cost. This finding demonstrates the close link between the planning decisions (where and how to install stations and allocate vehicles) and rebalancing operations. On the one hand, rebalancing operations increase the utilization of vehicles and parking spots, and on the other hand, the higher-level decisions also influence the efficiency of rebalancing operations in terms of satisfying more demand.

Finally, the monthly profits for both flexible and fixed designs grow gradually with increasing demand. Initially, due to the redundant parking spots and vehicles, profit for the fixed designs is much lower than that for the flexible designs. As the demand increases, a system with a larger scale becomes more essential, which explains why the profit gap between flexible designs and fixed ones shrinks month after month. Until Month 3, the profits of these two groups are basically the same. However, along with the continuous growth of the demand, starting from Month 5, the number of parking spots and vehicles in the fixed designs is no longer sufficient. Meanwhile, as the flexible designs are allowed for further expansion, more vehicles and parking spots are added into the system, which makes the flexible designs able to reap more profit. The advantage of such phasing strategy becomes even clearer in Month 6 with even higher demand.

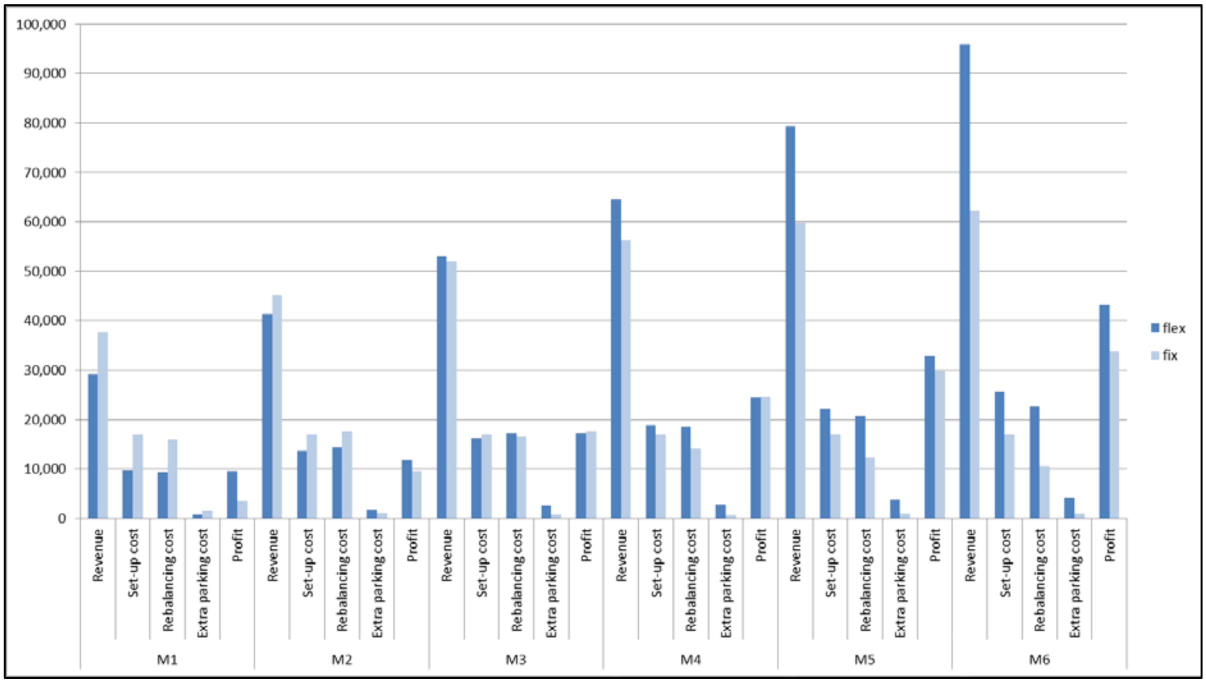

Figure 5.7 Average monthly performance of fixed and flexible designs

5.3 Further discussion

As explained earlier in Chapter 2, the computational complexity involved in optimizing flexible systems via simulation may need to be addressed from another perspective rather than just simplify the original simulator. In this chapter, the computational procedure developed in Chapter 4, namely PSO+OCBA, is applied again to determine the optimal parameters for the strategic-level flexibility. The analysis indicates that this solution approach is able to identify the optimal solution within an acceptable amount of time and a stable performance. It is believed that the acceleration of the computation process is achieved by the way how the search algorithm collaborates with the OCBA technique. Therefore, this study proposes a generalized computational framework combining population-based search algorithms and OCBA techniques.

For discrete event models, such as used in this study, a heuristic search method may be more applicable than a gradient-based approach. This is because a heuristics-based approach does not require gradient information that is usually not available for discrete event models. Furthermore, there are two categories of heuristic search methods depending on sampling strategies: population-based approaches (such as genetic algorithm, nested partition, and particle swarm optimization) and single-point-based approaches (such as simulated annealing and Tabu search). Although in a particular iteration, single-point-based search methods require less computation time (as only one solution needs to be evaluated), population-based approaches are able to search the design space more thoroughly. More importantly, a population-based approach integrates well with an OCBA approach, so that in each iteration, the simulation budget is optimized to maximize the probability of correct selection of the best individual in that population. Using OCBA enhances the efficiency of the algorithm in terms of updating the population in the subsequent iterations, as well as saving the computational effort in a current iteration.

The figure below shows the structure of the optimization framework. The computation framework relies on collaboration between the OCBA rule, the simulator, and the population-based search algorithm. The initial set of solutions is randomly generated in the decision space or based on certain rules that depend on the specific algorithm being used, and then a small amount of simulation budget is allocated to each solution whose performance is further estimated by the simulator. After that, the OCBA rule plays its role to allocate extra simulation budget to solutions until a specified accuracy is achieved or the total budget for a single iteration consumes up. Next, the estimated performances of solutions are imported into the search algorithm where such information is processed to generate a new set of population. The computation ends if certain stopping criteria are satisfied, e.g. limited improvement on the best individual in two consecutive iterations.

As commonly there is no strict confinement that the OCBA rule or population-based search algorithms can only be applied to a particular kind of problem or simulator, it is believed that this computational framework is arguably very generalizable. It is expected that this computational framework provides another angle to cope with the large computational cost when optimizing flexible systems via simulation.

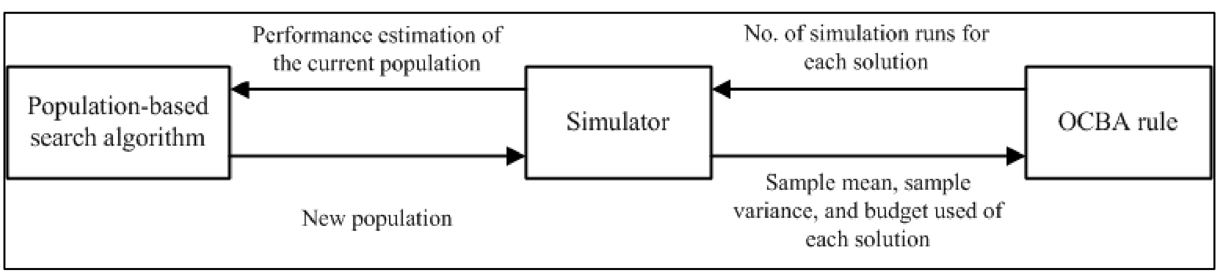

Figure 5.8 Computational framework

5.4 Summary

This chapter incorporates strategic flexibilities into design and management of MoD systems to address long-term demand uncertainty. It addresses the third research question proposed in Section 2.3. A phasing strategy is formulated into the deployment plan that enables the system to expand every month if necessary, which aims to deal with uncertain demand growth. Similar to the previous chapter, a simulation-based methodology is adopted to determine the optimal parameters in this phasing strategy. An optimization problem is first defined. Different from the previous chapter, the optimization model here tries to find the optimal solution that maximizes profit instead of minimizing cost. Besides, a longer planning horizon is considered as this chapter focuses on changes in the overall demand pattern. The DES, which is used to estimate the objective function value of a given solution, is also modified from the one developed in the previous chapter, namely to account for the formulation of the phasing strategy, a longer planning horizon, and a heuristic rule to guide the rebalancing operations. The computational procedure, namely PSO+OCBA, which is shown to be effective in the previous chapter, is further applied to calculate the optimal solution.

The two case studies in this chapter both demonstrate that designing a flexible MoD system is an effective method to increase the profit of the system. A VoF that accounts for 11% of the fixed design is obtained in the first case study. In the second case study with a more realistic setting, this improvement increases further to 17%. As the overall demand pattern is subjected to uncertain change in a longer planning horizon, when demand is lower than expectations, the flexible system is able to avoid excessive loss by staying in a relatively smaller scale compared with the fixed one. Meanwhile, if demand turns out higher than expected, the flexible system can be expanded to a larger scale to service more customers, thus a higher profit. Such advantage of the flexible system is further illustrated by the decomposition of monthly performance in the second case study.

For the prototype study, sensitivity analysis is also conducted. The results are tested under variations of major assumptions, namely cost and revenue parameters, as well as the volatility of demand growth rate. It is shown that demand volatility and unit revenue play the most important role. Besides, it is interesting to notice that, in any case, a positive VoF can be obtained in the context of this case study, which demonstrates the value of the proposed flexibility concept.

Meanwhile, as explained in Chapter 3, the design procedure proposed in this thesis does not account for cost of flexibility, as in general, it needs many efforts to obtain such value and requires additional assumptions that may restrain the applicability of the methodology proposed in this thesis. Decision-makers, however, should be aware the existence of such cost, such as signing a contract with local government to access more parking spaces or making agreements with vehicle rental company or manufacturers. Although formulating such cost is beyond the scope of the thesis, the VoF obtained using the methodology proposed in this thesis provides a valuable reference for decision-makers to compare with their specific cost of implementing flexibility.

Finally, a generalized computational framework is proposed as the combination of PSO and OCBA is demonstrated to be an effective computational procedure in this chapter and Chapter 4. This computational framework provides guidelines from a different perspective other than metamodels on how to efficiently find optimal flexible designs.

In sum, this chapter provides guidelines and a methodology on how to design a flexible MoD system considering strategic level flexibility to address the long-term demand uncertainty. Also, it shows how different factors may influence the additional value that can be brought by adopting such flexible deployment strategy.

Similar to Chapter 4, generalizable knowledge informative to the design and management of MoD systems is also derived from the analysis in this chapter, but as the analysis is also based on some explicit assumptions, applicability to other systems require a further examination on similarities and differences between the target systems and the one assumed in this study.

1) With the demand growing in an uncertain trend, a phasing deployment strategy is effective to improve the profit of a MoD system compared with a rigid deployment strategy.
2) The more volatile the demand is, the more additional value results from taking a phasing strategy.
3) The higher the charge of ride is, the more additional value is resulted from taking a phasing strategy.
4) Decision-maker should be aware of the trade-off between revenue, expansion cost, and operating cost (the sum of rebalancing cost and temporary parking cost) in a flexible MoD system.

Chapter 6 Conclusion

6.1 Summary

This thesis explores the design and management issue of MoD systems under both short-term and long-term demand uncertainty. On the one hand, rebalancing operations are incorporated as an operational-level flexibility to address daily demand fluctuations. On the other hand, a phasing deployment strategy is formulated as a strategic-level flexibility to deal with the overall demand pattern changes. A simulation-based approach is adopted throughout the thesis as an effective approach in terms of systems modeling and solution calculation. In retrospect, three research objectives were proposed in Chapter 1. Each was addressed by a distinct chapter of the thesis, respectively.

Objective 1: Develop a design procedure based on simulation that provides high level of instructions on how to design and evaluate flexible engineering systems.

In Chapter 3, a four-step procedure for designing and evaluating flexibilities via simulation is introduced. The analytical logic that derives from the four-step procedure runs through the whole thesis and is repeatedly applied to guide the analysis in this thesis, although the part concerned with deterministic analysis is omitted mostly, since it is typical for illustrative purposes and model development. To illustrate how this procedure works and why the thesis takes a simulation-based approach, as well as to demonstrate that incorporating flexibility is effective to improve system performance, a case study on a water management system is introduced, based on the work recently published by Deng et al. (2013). The case study provides a demonstration on how to apply the four-step procedure step by step in an urban infrastructure system, an example of a complex engineering system, and it also shows the convenience of applying simulation to model and evaluate flexible systems. The case study also serves as another example that incorporating flexibility leads to improvement in system lifecycle performance.

Objective 2: Develop and solve a mathematical model that aims at finding out the optimal planning decisions for a MoD system where vehicle redistribution activities are considered as an operational level flexibility to address short-term demand fluctuations.

Chapter 4 applies the simulation-based approach described in Chapter 3 to optimize the planning decisions of the MoD systems considering stochastic demand and operational-level decisions. Although demand is modeled by stochastic processes in this chapter, the overall usage pattern represented by the parameters in the stochastic model remain unchanged throughout the planning horizon. The rebalancing operations, which can be regarded as a type of operational-level flexibility, are integrated into the planning decisions in order to cope with the day-to-day variations in realized demand and the imbalanced traffic flows in the system. In this part of the study, the optimization problem is defined so as to find the configuration of a MoD system that incurs minimal cost to satisfy a predefined LoS. A DES that includes a sub-optimization model to calculate hourly rebalancing schemes, is built to estimate the performance of a given configuration. Furthermore, the thesis devises an algorithm that combines PSO and OCBA techniques to efficiently search the design and decision space.

Numerical results on two case studies demonstrate the necessity of considering the effect of the rebalancing operations when making planning decisions. By taking this approach, the system configuration identified not only results in a lower cost but also a smaller chance of violating the requirement on LoS. Besides, the results also suggest that the proposed solution approach has a stable performance. In particular, the results indicate that the OCBA technique plays a critical role in accelerating the computation process. The analysis and results from Chapter 4 are available in another journal paper recently submitted for review (see Deng and Cardin (2015)).

Objective 3: Develop and solve a mathematical model that aims to determine the optimal flexible strategy for deploying a MoD system that copes with long-term demand uncertainty.

Chapter 5 investigates strategic decision-making for MoD systems operating under uncertainty, assuming that overall demand patterns evolve over time. A vehicle capacity phasing strategy – an example of strategic-level flexibility – is formulated into the deployment plan to address longer-term uncertain demand growth. Different from Chapter 4, this chapter aims to identify the optimal solution that maximizes the profit rather than minimizes the cost. The DES developed previously is extended and modified to account for a longer planning horizon, changing demand patterns over time, and the formulation of the phasing flexibility strategy. The PSO+OCBA algorithm is applied again to find the optimal solution. The proposed methodology is implemented on the same two case studies as in the previous chapter. In both cases, compared with the fixed designs that deploys the system once at the beginning and remain the same throughout the planning horizon (i.e. a robust design solution), the phasing strategy provides better performance, namely higher expected profit. Such improvement is achieved by staying at a relatively smaller scale when the demand is low, but adding more parking spots and vehicles when the demand reaches an adequate level. In fact, the results show that the fixed design solutions are only able to reap slightly more profit than the flexible ones during the middle of the planning horizon. This is because until this time point, demand is relatively moderate compared with the beginning or the end of the planning horizon. On the other hand, the numerical results in this chapter further confirm that the proposed solution approach is helpful in terms of determining the optimal flexible strategy. Inspired by the successful implementation of the PSO+OCBA approach, this part of the study proposes the computational framework based on a population-search algorithm and the OCBA technique as a general approach to resolve the computational complexity involved in optimizing flexible systems. A third journal paper focusing on the longer-term flexibility analysis is currently under preparation.

6.2 Results validity and study limitations

This thesis investigates both operational and strategic planning of MoD transportation systems by incorporating flexibility into the system to address both short-term and long-term demand uncertainty. There are two major discoveries from this study. First, results indicate that considering explicitly uncertainty and flexibility into a MoD system contributes to better performance, either through dynamic rebalancing operations, or by strategically phasing system capacity over time. Second, the simulation-based approach is effective to help determine the deployment strategy of a MoD system.

This section examines the quality of the results from three perspectives, internal validity, external validity, and reliability. Internal validity considers any bias or errors that may compromise the causal relationships established in this study; external validity is the extent to which the results of this study can be generalized to other situations and contexts; reliability concerns with the consistency and replicability of the results.

6.2.1 Internal validity

In order to demonstrate that designing flexibility is effective in terms of improving the performance of MoD systems, this thesis makes a comparison between the optimal solutions obtained under three conditions: without flexibility (benchmark), with operational-level flexibility (i.e. incorporating rebalancing operations), and with both operational-level and strategic-level flexibility (i.e. the phasing strategy). By comparing the optimal solutions that are calculated using the same solution approach, the biases that can be caused by wrongly choosing the design variables for a particular condition are avoided. Results, which are demonstrated to be statistically significant, show that each time when one more layer of flexibility is added, systems designs with better performance are generated.

Meanwhile, the solution approach mentioned earlier, namely the simulation-based methodology proposed in this thesis that consists of the DES and the PSO+OCBA algorithm, is also carefully devised to minimize any bias or error.

The DES is established to evaluate the performance of one-way MoD systems adopting a first-come-first-to-service policy. Admittedly, it is most straightforward to validate a simulator by comparing the simulated results with the data collected in reality. As real-world data is not available in this study, however, other methods have been exploited to validate the simulator. On the one hand, conceptually, the assumptions made in the simulator regarding the system behavior can be supported by past studies, such as the one by Jorge et al. (2012), as well as other real-world examples, e.g. car2go (https://www.car2go.com/). On the other hand, the computerized model is validated by checking the consistency of the simulated results with the input information. For example, the analysis makes sure that the total number of vehicles at each time step equals the decision made before running the simulation, and that the sample mean of the simulated demand is close to the assumed parameters of the demand.

Regarding validation of the PSO+OCBA algorithm, the main issue stems from the quality of the solutions obtained. Due to the large scale of the problem, exhaustive search is not a realistic approach to examine whether the solution identified by PSO+OCBA is the true optimal solution or not. In this case, in Chapter 5, the embedded solver from MATLAB is employed to compute the optimal solutions. Results show that there is no statistically difference between the solutions obtained by the proposed algorithm and the MATLAB solver, which demonstrates the optimality of the solutions identified by the PSO+OCBA approach to a certain degree. Besides, the PSO+OCBA algorithm takes much less time to identify the optimal solution than the MATLAB solver. Furthermore, the performance of PSO+EA and PSO+OCBA is examined in Chapter 4, which further illustrates the importance of incorporating the OCBA technique into the algorithm. These observations suggest that the proposed computational procedure is an effective decision-support tool to find optimal flexible MoD systems.

6.2.2 External validity

External validity of the results is concerned with the questions of how the results of this study are applicable to 1) the same type of MoD system under different settings, 2) other types of MoD systems, and 3) other engineering systems.

For the recommendation regarding designing flexible MoD systems, the first question is partially addressed through the sensitivity analysis. In Chapter 5, major parameters are varied to see how the VoF responds to these changes. Although there is a clear variation in terms of VoF when some of the parameters are altered, the VoF remains positive, which supports the recommendation of applying a phasing strategy to deploy a MoD system that follows the decision rules assumed in this thesis. It is still possible, however, that the VoF may become negative when some parameters vary in an extreme way, namely out of the ranges assumed in the sensitivity analysis. In fact, the recommendations generated in the numerical studies, particularly the very specific ones, e.g. optimal decisions, are based on the assumptions and parameters made for the case that is based in Singapore. The recommendations may change if the study is done in other contexts or cities, where these parameters may change. For example, if the system is built on another city other than Singapore, the temporary parking cost will vary and the travel distance will also be different. In this case, a different set of recommendations may be derived from the numerical analysis. Furthermore, it also remains a question whether other flexible strategies, e.g. the abandonment or the deferring, are effective to address uncertainties in the assumed MoD systems or not. Moreover, the questions whether incorporating flexibility is beneficial to other types of MoD systems or other engineering systems or not, are beyond the scope of thesis, and cannot be answered here. More work is needed to demonstrate that flexibility may generate value in other MoD systems (e.g. two-way MoD systems or reservation-based MoD systems).

Regarding the general applicability of the methodology, for the same type of MoD systems, the DES will not be influenced by changes in the parameters or the formulation of the decision rule, as the simulator is devised to easily accommodate such changes. If different types of MoD systems, however, such as two-way MoD systems, or another category of engineering systems are the objects under study, the discrete event simulation methodology may still be applicable, but significant changes may be required to redevelop the simulator. On the other hand, the use of the PSO+OCBA algorithm is not restricted to a particular MoD system or even a particular category of engineering systems. Its computational efficiency, however, may be influenced by the dimension of the problem and the degree of fluctuations in the uncertainty factors and random variables. For example, for the MoD system studied here, if the problem goes larger (i.e. more subareas under consideration) or higher demand volatility is considered, it may take longer to find the optimal solution.

6.2.3 Reliability

Examining the reliability of the results is to see whether the conclusion can be replicated under the similar conditions, or not. As shown in both Chapter 4 and Chapter 5, multiple runs of the numerical experiments are conducted to investigate the reliability of the results, and gain more statistical significance for the results. Although slightly different solutions are obtained for each replicated experiment due to stochasticity inherent to both the simulation and optimization processes, statistical tests indicate that there is no significant difference between the performances of the solutions. On the other hand, the consistency of the results is also implied by the fact that when experimental conditions are changed, e.g. flexibility is added, statistically significant differences exist between solutions obtained under different conditions.

6.3 Future work

Opportunities for future research can be pursued from the following suggestions. On the one hand, in terms of application, as discussed earlier, this thesis focuses on the type of MoD systems that adopts a first-come-first-to-service policy, as this is the most flexible form of the non-floating one-way MoD systems. There are other types of MoD systems being operated, however. As such, it remains an opportunity to see how flexibility and real options can be incorporated into those systems and how much benefit can be brought in. Even for the particular type of MoD systems addressed in this thesis, the analysis can be further extended to consider other or multiple uncertainty sources, as well as other types of flexible strategies. For example, the analysis can be extended to account for the uncertainty that exists in rental time of customers. This study assumes that rental time is only influenced by the distance between two stations, while a more complex analysis can introduce random noise into this parameter. Regulations changes regarding CO2 emissions can also be a factor modelled in future studies. In fact, as one of the main advantages of adopting a simulation-based approach is the ease of modeling various uncertainties and flexible strategies, the solution package proposed in this study can be easily modified to cart to the aforementioned needs. Besides, it also remains an opportunity to investigate other rebalancing policies and their interactions with the planning decisions. For example, compared with relying on the system operator to redistribute vehicles, how will the system configuration change if using price incentives to motivate customers to do the rebalancing? Furthermore, future work can be carried out using historical demand data in a real-world system. In such case, not only can the simulator be further validated but also helpful suggestions can be obtained to better design and operate that system where the data is collected. Besides, as introduced earlier, the analysis in this study takes the perspective of a private company, who operates the one-way MoD system, although two mathematical models are developed for different objectives at different stage of a company, it still remains an interesting opportunity to see how changes on this analytical set-up will affect the final result. For example, what kind of designs of a MoD system will be preferred by a local government?Furthermore, it also remains an opportunity to see how the flexible MoD systems are implemented in reality. Will the practitioners follow the optimal decision rule to make gradual adjustment to the system? Is there any difficulty in terms of the documentation and implementation of the flexible design? Such research questions may be addressed by having discussion with practitioners , using the simulation-gaming technique introduced in Section 2.2, or building active safeguards as proposed by (Gil, 2007).. In addition, as illustrated in Figure 1.2, there are multiple stakeholder who may somehow have conflicts of interest existing in a MoD system, between which the interactions may hinder the implementation or exercising the flexible designs. For example, intensified scarcity of urban land may prevent local government to provide more subsidized parking spots to the operating company. It may worth further exploration on this social aspect of the system. Research by (Gil, 2015) provides a direction to address such issue who suggest building a polycentric commons to steer the development process of a flexible system.

On the other hand, methodologically speaking, this thesis generalizes a computational framework that combines population-based search algorithms and OCBA techniques, and proposes this framework as a different perspective for reducing the computational burden when optimizing flexible systems via simulation. This computational framework takes advantage of the population-based search algorithms where the decision space is explored more thoroughly through a single iteration. At the same time, adopting the OCBA technique helps alleviate the computational burden in every iteration. This thesis only considers, however, a combination of PSO and OCBA rules from Chen et al.

(2000). Further studies can explore other possible combinations. In addition, it may also be worthwhile to apply the proposed computational framework to optimize other flexible systems, as to further demonstrate and validate its efficiency more generally for engineering systems design and management.

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Appendix

Table A.1 Probability transition matrix for weekends for case study II

	0900-1700								1700-1900								1900-2400			
	S1	S2	S3	S4	S5	S6	S7	S8	S1	S2	S3	S4	S5	S6	S7	S8	S1	S2	S7	S8
S1	0	0	0.25	0.25	0.25	0.25	0	0	0	0	0.25	0.25	0.25	0.25	0	0	0	0	0	0
S2	0	0	0.25	0.25	0.25	0.25	0	0	0	0	0.25	0.25	0.25	0.25	0	0	0	0	0	0
S3	0.2	0.2	0	0	0	0.2	0.2	0.2	0.2	0.2	0	0	0	0.2	0.2	0.2	0.25	0.25	0.25	0.25
S4	0.25	0.25	0	0	0	0	0.25	0.25	0.25	0.25	0	0	0	0	0.25	0.25	0.25	0.25	0.25	0.25
S5	0.25	0.25	0	0	0	0	0.25	0.25	0.25	0.25	0	0	0	0	0.25	0.25	0.25	0.25	0.25	0.25
S6	0.2	0.2	0.2	0	0	0	0.2	0.2	0.2	0.2	0.2	0	0	0	0.2	0.2	0.25	0.25	0.25	0.25
S7	0	0	0.25	0.25	0.25	0.25	0	0	0	0	0.25	0.25	0.25	0.25	0	0	0	0	0	0
S8	0	0	0.25	0.25	0.25	0.25	0	0	0	0	0.25	0.25	0.25	0.25	0	0	0	0	0	0

Table A.2 Probability transition matrix for weekdays

	0700-0900								0900-1700							
	S1	S2	S3	S4	S5	S6	S7	S8	S1	S2	S3	S4	S5	S6	S7	S8
S1	0	0	0.25	0.25	0.25	0.25	0	0	0	0	0.25	0.25	0.25	0.25	0	0
S2	0	0	0.25	0.25	0.25	0.25	0	0	0	0	0.25	0.25	0.25	0.25	0	0
S3	0	0	0	0	0	0	0	0	0.2	0.2	0	0	0	0.2	0.20	0.20
S4	0	0	0	0	0	0	0	0	0.25	0.25	0	0	0	0	0.25	0.25
S5	0	0	0	0	0	0	0	0	0.25	0.25	0	0	0	0	0.25	0.25
S6	0	0	0	0	0	0	0	0	0.2	0.2	0.2	0	0	0	0.20	0.20
S7	0	0	0.25	0.25	0.25	0.25	0	0	0	0	0.25	0.25	0.25	0.25	0	0
S8	0	0	0.25	0.25	0.25	0.25	0	0	0	0	0.25	0.25	0.25	0.25	0	0

	1700-1900								1900-2400							
	S1	S2	S3	S4	S5	S6	S7	S8	S1	S2	S3	S4	S5	S6	S7	S8
S1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S3	0.2	0.2	0	0	0	0.2	0.2	0.2	0.25	0.25	0	0	0	0	0.25	0.25
S4	0.25	0.25	0	0	0	0	0.25	0.25	0.25	0.25	0	0	0	0	0.25	0.25
S5	0.25	0.25	0	0	0	0	0.25	0.25	0.25	0.25	0	0	0	0	0.25	0.25
S6	0.2	0.2	0.2	0	0	0	0.2	0.2	0.25	0.25	0	0	0	0	0.25	0.25
S7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0