ESSAYS ON GROWTH, INEQUALITY AND FISCAL POLICIES

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Declaration

I hereby declare that this thesis is my original work and it has been written by me in its entirety. I have duly acknowledged all the sources of information which have been used in the thesis.

This thesis has also not been submitted for any degree in any university previously.

Signed:

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Contents

Summary

This thesis consists of three independent chapters related to endogenous growth models. Chapter 1 and Chapter 3 examine the effects of taxation; Chapter 2 discusses the patent length policy in an endogenous growth model with two types of innovation.

The first chapter examines the effects arising from the imposition of various taxes in a Schumpeterian economy. The growth rate and the level of inequality are jointly determined in equilibrium. The effects of fiscal policies may depend on the time path of the economy. Therefore, I analyze both the transitional dynamics and the steady state effects of the taxes. I find that there is an optimal capital income tax that minimizes wealth inequality; before tax income inequality is increasing in capital income tax, while both after tax income inequality and welfare inequality are decreasing in capital income tax. Then I examine the distributional effects of a taxation reform from lump-sum tax to capital income tax or labor income tax. I show that capital income tax financing leads to larger dynamic adjustment of inequality and also results in higher wealth inequality and before tax income inequality. Furthermore, I do several robustness checks by modifying model parameters and I show that the patterns of the taxation effects are unchanged.

In Chapter 2, I analyze the patent length in an endogenous growth model in which growth is driven by variety expansion. In this model, there are two types of R&D activities: Innovation and Standardization. Successful innovation introduces a new variety and the new product is produced with high cost; Standardization targets on the existing high cost variety and lowers the production cost if successful. The patent protection for innovation and standardization then leads to resource reallocation between the two types of R&D investments and also between high cost production and low cost production by changing the returns to innovation and standardization. I show that the steady state equilibrium can be classified by three different regimes. First, if patent length for innovation is too long, there will be only innovation. Second, if patent length for standardization is too long, the economy may fall into a poverty trap that eliminates innovation. Finally, both innovation and standardization can exist in equilibrium. When both innovation and standardization present, the growth rate is increasing in the patent length for standardization, but is unchanged by patent length for innovation. Given patent length for innovation, there is a finite patent length for standardization that maximizes welfare. Welfare is increasing in patent length for innovation.

Chapter 3 examines the effects of taxation on the long run distance to frontier of the economy as well as the welfare and growth rate. The technological progress of an economy is assumed to be obtained from both innovation and imitation. In this chapter, both innovation and imitation improve technology level of the economy. Innovation targets on local technology frontier, while imitation targets on global technology frontier. I show that higher capital income tax results in longer steady state distance to frontier while it increases steady state welfare. By analyzing the transitional dynamics, I find that higher capital income tax will lead to lower current growth rate. The effect of capital income tax on total welfare is inverse-U shaped.

List of Figures

List of Tables

Chapter 1

The Distributional Role of Fiscal Policies in an Endogenous Growth Model

1.1 Introduction

From 1970s, there is a rapid rise in the worldwide income inequality such as in US and UK, reviving the interest in the issue of the trade-off between growth and inequality. The distributional effects of fiscal policies in the neoclassical models and AK growth models have been extensively studied in the literature. However, it received less attention in the Schumpeterian growth models. In this chapter, we construct a Schumpeterian growth model to examine the effect of fiscal policies on the trade-off between growth and inequality.

The growth rate and inequality are simultaneously determined at equilibrium. Economic growth comes from endogenously determined innovation. In addition, there is monopoly power in the intermediate goods sector. Monopoly power leads to less than optimal output on the one hand. On the other hand, it provides more incentive to innovate and thus stimulates economic growth. In the Schumpeterian model, fiscal policies redistribute income but they can also affect the profits and thus the incentives to innovate. Therefore, it is necessary to analyze the effects of fiscal policies on the growth rate and inequality within the Schumpeterian growth model.

The joint determination of growth rate and distributions seems to be supported by the disagreement of empirical economists on the relationship between growth and inequality. Many researchers find that there is a negative correlation between economic growth and inequality (e.g. Alesina and Rodrik (1994), Person and Tabellini (1994) and Perotti (1996)). However, later studies argue that inequality has positive or no effect on economic growth (e.g. Forbes (2000) and Barro (2000)). It is not surprising that economists find controversial evidence, because both the inequality level and growth rate interact with each other and are jointly determined at equilibrium. Policies that affect the return to different factors will affect the distributions of income and wealth. Likewise, policies targeted on distributions are also likely to influence the aggregate economy. Consequently, if both reducing inequality and enhancing growth are the economic objectives, the governments may face a trade-off between economic growth and inequality.

To understand the trade-off between growth and inequality, it is necessary to incorporate economic growth and distributional dynamics into one model. Garcia-Penalosa and Turnovsky (2007) examine the roles of different fiscal policies on the relationship between growth and inequality using an AK model. However, one notable drawback of their model is that there is no transitional dynamics in their model and thus it is not possible to study the wealth inequality as well as income dynamics. Garcia-Penalosa and Turnovsky (2011) investigate the distributional dynamics with an Ramsey model, however economic growth is missing. In the present Schumpeterian model, the wealth and income dynamics are present, and economic growth is driven by purposeful R&D investments.

We first study the revenue-neutral taxation effects on inequality measures. In line with Aghion, Akcigit, and Fernandez-Villaverde (2013), a higher capital income tax leads to a lower growth rate. As for distributions, the results show that a higher capital income tax leads to lower after-tax income inequality and welfare inequality, which is in line with Garcia-Penalosa and Turnovsky (2007). Thus, the conventional trade-off between growth and inequality largely remains in the quality ladder model. A novel finding is that there is a capital income tax that minimizes wealth inequality. Therefore, it is possible to enhance economic growth and reduce wealth inequality by reducing the capital income tax when the capital income tax is initially high enough.

We then examine the distributional effects of a taxation reform from lump-sum tax financing to capital income tax financing or labor income tax financing. We find that capital income tax financing leads to larger dynamic adjustment of inequality and results in higher wealth inequality and before-tax income inequality. The after-tax income inequality and welfare inequality will be lower than that under labor income tax financing. The results are largely consistent with Garcia-Penalosa and Turnovsky (2011). We also analyze the effect on growth. Capital income tax financing results in lower growth rate.

This chapter belongs to the large literature of the relationship between growth and inequality. The classical view of the relationship generally supports a positive correlation between inequality and growth. Since marginal propensity to save increases in personal wealth, higher inequality is beneficial to capital accumulation and therefore leads to a higher growth rate. More recent examinations of the relationship between growth and inequality seem to depart from this view via two different channels. One strand is the credit market imperfection channel. Galor and Zeira (1993) show that there is a negative relationship between inequality and growth based on the assumptions of imperfect credit market and fixed cost associated with human capital investment. Researchers such as Aghion and Bolton (1997) and Benabou (1996) argue that the redistribution to the poor may enhance growth based on the assumption of credit market imperfection. Another strand is the political economy channel. Due to the importance of the median voter, the poorer the median voter, the higher demand for redistribution, thus the lower the growth rate (e.g. Bertola (1993), Persson and Tabellini (1994), Alesina and Rodrik (1994) and Benabou (2000)).

This chapter also contributes to the Schumpeterian literature on inequality. Foellmi and Zweimuller (2006) and Zweimuller (2000) examine the relationship between growth and inequality from the demand side. Income distribution determines the demand for innovative goods and thus the incentive to innovate. Chol-Won Li (1998) argues that income distribution determines the market structure. When inequality is relatively small, the market is monopolistic and inequality leads to lower growth. However, when inequality is relatively high, the market is in duopoly and inequality enhances growth. Garcia-Penalosa and Wen (2008) argue that redistribution provides insurance to unsuccessful entrepreneurs and thus enhances growth. Several papers explained the widening inequality in US with Schumpeterian framework. Dinopoulos and Segerstrom (1999) and Sener (2001) demonstrate that trade liberalization is a source of widening inequality. Acemoglu (1998, 2000) shows that skill-biased technology lead to widening inequality. Aghion, Howitt and Violante (2002) explain the within educational group inequality based on the assumption of transferability of knowledge and general purpose technological progress. Although in this chapter we do not deal with the causal relationship between growth and inequality, we consider the distributional effects of government policies which received less attention in this strand of literature. In order to simplify the calculation and to obtain analytical results, we consider only proportional taxes, instead of progressive taxes.

The rest of the chapter is organized as follows. Section 2 sets up the model. Section 3 discusses the steady state equilibrium. Section 4 examines the dynamics and steady states of the distributions of wealth, income and welfare. Section 5 presents the numerical results of the revenue-neutral tax change and the taxation reform. Section 6 concludes.

1.2 The model

The basic framework with discrete time in this paper builds on the model of Aghion, Akcigit, and Fernandez-Villaverde (2013), in which economic growth is driven by technological progress resulting from intentional investment in R&D that improves the quality of the intermediate inputs. Their model discusses the optimal capital income tax and labor income tax, but not the effects on the wealth and income distributions. By extending their model with the assumption of heterogeneous agents with different initial capital endowments, we discuss the distributional effects of the two taxes.

1.2.1 Households

Consider an economy populated with a mass of 1 infinitely lived households. The individual household is indexed by $i \in [0, 1]$. Households are heterogeneous in the sense that they have different initial capital endowment K_{i0}. The relative position of household i's capital endowment is $k_{i0} \equiv K_{i0}/K_0$, where K_0 is the average capital endowment at time $t = 0$. Generally, the relative capital stock at any instant $k_{it} \equiv K_{it}/K_t$ follows a distribution of $G(k_{it})$ with the mean $\int_0^1 k_{it}di = 1$, and the variance σ_{kt}^2. Here, aggregate capital is $K_t = \int_0^1 K_{it}di$.

The labor supply of the households is elastic. Each household has 1 unit of time which can be allocated to leisure or to work. Denote l_{it} as the leisure consumed by the household. Then the labor supplied by the household is $L_{it} = 1 - l_{it}$. The discounted life-time utility is:

$$\Sigma_{t=0}^{\infty} \beta^t \frac{1}{\gamma} (C_{it} l_{it}^{\eta})^{\gamma} \tag{1.1}$$

where β is the rate of time preference, C_{it} is the consumption, l_{it} is the leisure time, η is the elasticity of leisure in utility, and γ is the coefficient of relative risk aversion. Households own physical capital and the firms, and they also supply labor to earn labor income. Thus, the household maximizes the discounted life-time utility subject to the following capital accumulation function:

$$K_{it+1} = (1-\tau_k)r_t K_{it} + (1-\delta)K_{it} + (1-\tau_l)w_t(1-l_{it}) + (1-\tau_d)D_{it} - C_{it} \quad (1.2)$$

where τ_k, τ_l, and τ_d stand for tax rates on capital income, labor income, and firm profit, respectively. r_t, w_t and δ denote the rental rate of capital, wage rate and depreciation rate, respectively. D_{it} stands for the firm profit household i gets. We will see that the net firm profit is always equal to 0 from the innovator's problem below.

The first order conditions of the household are as follows:

$$\beta^t C_{it}^{\gamma-1} l_{it}^{\eta\gamma} = \mu_{it} \quad (1.3)$$

$$\beta^t \eta C_{it}^{\gamma} l_{it}^{\eta\gamma-1} = \mu_{it} w_t (1-\tau_l) \quad (1.4)$$

$$\frac{\mu_{it+1}}{\mu_{it}} = \frac{1}{(1-\tau_k)r_{t+1} + 1 - \delta} \quad (1.5)$$

where μ_{it} is the shadow price of capital for agent i. The corresponding transversality condition is:

$$\lim_{t\to\infty} \beta^t U^{'}(C_{it})K_{it} = 0 \quad (1.6)$$

From (1.3) and (1.4), we can get:

$$C_{it} = \frac{1-\tau_l}{\eta} w_t l_{it} \quad (1.7)$$

Iterating (1.3) and (1.7) forward by one period, we can have:

$$\beta\Big(\frac{C_{it+1}}{C_{it}}\Big)^{\gamma-1}\Big(\frac{l_{it+1}}{l_{it}}\Big)^{\eta\gamma} = \frac{\mu_{it+1}}{\mu_{it}} \quad (1.8)$$

$$\frac{C_{it+1}}{C_{it}} = \frac{w_{t+1}}{w_t}\frac{l_{it+1}}{l_{it}} \tag{1.9}$$

From (1.5), the growth rate of μ_{it} is the same for all individual, i.e. $\frac{\mu_{it+1}}{\mu_{it}} = \frac{\mu_{t+1}}{\mu_t}$. Thus, we can also see that $\frac{C_{it+1}}{C_{it}} = \frac{C_{t+1}}{C_t}$ and $\frac{l_{it+1}}{l_{it}} = \frac{l_{t+1}}{l_t}$. Then the growth rates of consumption and leisure time are determined by:

$$\beta\Big(\frac{C_{t+1}}{C_t}\Big)^{\gamma-1}\Big(\frac{l_{t+1}}{l_t}\Big)^{\eta\gamma} = \frac{\mu_{t+1}}{\mu_t} = \frac{1}{(1-\tau_k)r_{t+1}+1-\delta} \tag{1.10}$$

$$\frac{C_{t+1}}{C_t} = \frac{w_{t+1}}{w_t}\frac{l_{t+1}}{l_t} \tag{1.11}$$

1.2.2 Firms

In the economy, there is only one final good and a continuum of intermediate goods indexed by $j \in [0, 1]$. The final firm produces under perfect competition using physical capital and the continuum of intermediate goods. The intermediate goods are produced using labor by the producers who possess the most advanced technologies. The progress of technologies is driven by innovations.

Final Good Producers

Final good can be used interchangeably for consumption, capital good or innovation investment. Thus it serves as the numeraire. The production function of a final good producer is

$$Y_t = K_t^\alpha Z_t^{1-\alpha} \tag{1.12}$$

where K_t is the capital stock and Z_t is the intermediate goods basket aggregated according to

$$\ln Z_t = \int_0^1 \ln z_{jt} dj \tag{1.13}$$

where z_{jt} is the demand for intermediate good j to produce the intermediate goods basket. Both Y_t and Z_t are produced under perfect competition.

Because final good production is competitive, the prices of the inputs should be equal to their marginal products. Thus,

$$r_t = \alpha K_t^{\alpha-1} Z_t^{1-\alpha} \tag{1.14}$$

$$p_{jt} = (1-\alpha)\frac{Y_t}{z_{jt}} \tag{1.15}$$

where p_{jt} denotes the price of the good j. Equation (1.13) implies the producer of Z_t will spend the same amount on each intermediate input j.

Intermediate Good Producers

Each intermediate input j is produced by a one-period-lived monopolist. That is, the intermediate producer lives only for one period and it will be replaced by another producer who successfully improved the productivity of the corresponding product. This firm holds the patent to the most advanced technology. Thus the production function of j is

$$z_{jt} = q_{jt} L_{jt} \tag{1.16}$$

where L_{jt} is the labor demand by producer j and q_{jt} is the labor productivity. The marginal cost of producing j is

$$MC_{jt} = \frac{w_t}{q_{jt}} \tag{1.17}$$

The intermediate good producer j faces a competitive fringe of imitators who can produce the same product using the previous technology. The imitators are unable to produce with the leading edge technology, thus the productivity of the imitators are q_{jt-1}. The production function of the imitators is

$$z_{jt} = q_{jt-1}L_{jt} = \frac{q_{jt}}{\lambda}L_{jt} \tag{1.18}$$

where λ is the step of the innovation, which indicates the size of the productivity improvement. We can see that $q_{jt} = \lambda q_{jt-1}$. Thus, by Bertrand competition, the producer of j can not set the price p_{jt} higher than the marginal cost of the imitators which is $w_t\lambda/q_{jt}$. That is, the price of good j should be equal to the marginal cost of the imitators. Thus,

$$p_{jt} = \frac{\lambda w_t}{q_{jt}} \tag{1.19}$$

The equilibrium profit of the producer j is thus

$$\pi_{jt} = (p_{jt} - MC_{jt})z_{jt} = \Big(1 - \frac{MC_{jt}}{p_{jt}}\Big)(1-\alpha)Y_t = \frac{\lambda - 1}{\lambda}(1-\alpha)Y_t \tag{1.20}$$

Thus the aggregate profit of all intermediate firms is:

$$\Pi_t = \int_0^1 \pi_{jt}dj = \frac{\lambda - 1}{\lambda}\hat{Z}_t = \frac{\lambda - 1}{\lambda}(1-\alpha)Y_t \tag{1.21}$$

From (1.16) and (1.19), we have

$$L_{jt} = \frac{(1-\alpha)Y_t}{\lambda w_t} \tag{1.22}$$

Then the aggregate labor demand is $L_t = \int_0^1 L_{jt} dj$, so

$$L_t = \frac{(1-\alpha)Y_t}{\lambda w_t} \tag{1.23}$$

Substitute (1.22) into (1.13), we can have

$$w_t = \frac{1-\alpha}{\lambda}\frac{Q_t Y_t}{Z_t} \tag{1.24}$$

where $\ln Q_t \equiv \int_0^1 \ln q_{jt} dj$ is an aggregate productivity index. On the balanced growth path where $\frac{K_t}{Z_t}$ is constant, the wage rate will grow at the same rate as the aggregate productivity.

Innovation

Innovations result from R&D investments. Successful innovators get monopolistic power to produce with the previous technologies. A potential innovator in sector j need to invest $x_{jt}\psi Q_t$ to get a innovation rate of $\frac{x_{jt}}{\bar{x}_t}$. $\bar{x}_t$ is the innovation intensity of the potential innovators. Q_t appears here to show that the difficulty of doing innovation and the required R&D investment is increasing along with the productivity growth. That is, as technology progresses, the difficulty of innovation increases. Thus the maximization problem of the innovator is:

$$\max_{x_{jt}} \left\{ \frac{x_{jt}}{\bar{x}_t}\pi_{jt} - x_{jt}\psi Q_t \right\} \tag{1.25}$$

In a symmetric equilibrium, we can have

$$x_{jt} = \bar{x}_t = \frac{\lambda - 1}{\lambda} \frac{1 - \alpha}{\psi} \frac{Y_t}{Q_t} \tag{1.26}$$

The equilibrium growth rate is then

$$\frac{Q_{t+1}}{Q_t} = 1 + (\lambda - 1)\bar{x}_t = 1 + \frac{(\lambda - 1)^2}{\lambda} \frac{1 - \alpha}{\psi} \frac{Y_t}{Q_t} \tag{1.27}$$

The total innovation expenditure is

$$X_t = \bar{x}_t \psi Q_t = \frac{\lambda - 1}{\lambda}(1 - \alpha)Y_t \tag{1.28}$$

Total innovation expenditure equals total profit of the intermediate producers. The net return to the household D_{it} is then zero.

1.2.3 Government

Government collects tax revenue from capital income and labor income to finance the government expenditure which is a constant proportion of final output. Thus, the balanced budget constraint of the government is

$$G_t = G_0 Y_t = \tau_k r_t K_t + \tau_l w_t L_t \tag{1.29}$$

The aggregate resource constraint of the economy is then

$$K_{t+1} = Y_t + (1 - \delta)K_t - C_t - X_t - G_t \tag{1.30}$$

1.3 Equilibrium

The macroeconomic equilibrium can be defined as following:

Definition 1. *The equilibrium is an allocation* $\{C_{it}, K_{it}, l_{it}, L_t, Y_t, Q_t, D_t, \Pi_t, X_t\}_{t=0}^{\infty}$ *and the market prices* $\{r_t, w_t\}_{t=0}^{\infty}$ *such that:*

(1) given prices and the initial capital distribution $G(K_{i0})$*, the household maximizes the expected lifetime utility;* $\{$*(1.2)-(1.6)*$\}$

(2) given r_t *and* w_t*, firms maximize profit by choosing price and production;* $\{$*(1.14), (1.24), and (1.27)*$\}$

(3) the government policies satisfy the balanced budget; $\{$*(1.29)*$\}$

(4) markets clear and aggregate resource constraint satisfied. $\{$*(1.30) and* $\int_0^1 l_{it} di = 1 - L_t$, $\int_0^1 D_{it} di = \Pi_t - X_t\}$

The balanced growth path can be defined as following:

Definition 2. *The balanced growth path is a dynamic equilibrium where* $\{r_t, L_t\}$ *are constants and the aggregate variables* $\{C_t, Y_t, K_t, Q_t\}$ *grows at the same constant rate* g*.*

Solving, we can get the following system of equations in aggregate terms:

$$C_t = \frac{1}{\eta}(1 - \tau_l) w_t l_t \tag{1.31}$$

$$\beta\Big(\frac{C_{t+1}}{C_t}\Big)^{\gamma-1}\Big(\frac{l_{t+1}}{l_t}\Big)^{\eta\gamma} = \frac{1}{(1-\tau_k)r_{t+1} + 1 - \delta} \tag{1.32}$$

$$r_t = \alpha K_t^{\alpha-1} Z_t^{1-\alpha} \tag{1.33}$$

$$w_t = \frac{1-\alpha}{\lambda} Q_t \Big(\frac{K_t}{Z_t}\Big)^{\alpha} \tag{1.34}$$

$$\frac{Q_{t+1}}{Q_t} = 1 + \frac{(\lambda-1)^2}{\lambda} \frac{1-\alpha}{\psi} \frac{Y_t}{Q_t} \tag{1.35}$$

$$Y_t = K_t^{\alpha} Z_t^{1-\alpha} \tag{1.36}$$

$$Y_t = K_t^{\alpha} (Q_t L_t)^{1-\alpha} \tag{1.37}$$

$$G_t = \tau_k r_t K_t + \tau_l w_t L_t + \tau_d D_t \tag{1.38}$$

$$K_{t+1} = Y_t + (1-\delta) K_t - C_t - X_t - G_t \tag{1.39}$$

$$X_t = \frac{\lambda - 1}{\lambda} (1-\alpha) Y_t \tag{1.40}$$

To facilitate the computational exercise, we can convert the above equilibrium conditions to a system of technology-adjusted variables. Assume J_t is an arbitrary variable. Then denote the technology-adjusted variable by $\tilde{J}_t \equiv J_t / Q_t$, except that $\tilde{Q}_{t+1} \equiv Q_{t+1}/Q_t$. The stable system we obtain is the following:

$$\tilde{C}_t = \frac{1}{\eta} (1-\tau_l) \tilde{w}_t (1 - L_t) \tag{1.41}$$

$$\beta \Big(\frac{\tilde{C}_{t+1}}{\tilde{C}_t} \tilde{Q}_{t+1}\Big)^{\gamma-1} \Big(\frac{1-L_{t+1}}{1-L_t}\Big)^{\eta\gamma} = \frac{1}{(1-\tau_k) r_{t+1} + 1 - \delta} \tag{1.42}$$

$$\tilde{Q}_{t+1} = 1 + \frac{(\lambda-1)^2}{\lambda} \frac{1-\alpha}{\psi} \tilde{K}_t^{\alpha} L_t^{1-\alpha} \tag{1.43}$$

$$\frac{\tilde{K}_{t+1}}{\tilde{K}_t} = \Big[1 - G_0 - \frac{\lambda-1}{\lambda}(1-\alpha)\Big] \frac{\tilde{K}_t^{\alpha-1} L_t^{1-\alpha}}{\tilde{Q}_{t+1}} + \frac{1-\delta}{\tilde{Q}_{t+1}} - \frac{\tilde{C}_t}{\tilde{Q}_{t+1} \tilde{K}_t} \tag{1.44}$$

$$\tilde{w}_t = \frac{1-\alpha}{\lambda}\tilde{K}_t^{\alpha} L_t^{-\alpha} \tag{1.45}$$

$$r_t = \alpha \tilde{K}_t^{\alpha-1} L_t^{1-\alpha} \tag{1.46}$$

$$\tilde{Y}_t = \tilde{K}_t^{\alpha} L_t^{1-\alpha} \tag{1.47}$$

On a BGP, the labor and leisure are both constant. Suppose $r_t = r^*$, $L_t = L^*$ on the balanced growth path. We have

$$g = \frac{C_{t+1}}{C_t} = \Big(\frac{1}{\beta}\frac{1}{(1-\tau_k)r^* + 1 - \delta}\Big)^{1/(\gamma-1)} \tag{1.48}$$

$$g = \frac{K_{t+1}}{K_t} = 1 - \delta + \frac{r^*}{\alpha}\Big[1 - G_0 - \frac{\lambda-1}{\lambda}(1-\alpha) - \frac{1-\tau_l}{\eta}\frac{1-\alpha}{\lambda}\frac{1-L^*}{L^*}\Big] \tag{1.49}$$

$$g = \frac{Q_{t+1}}{Q_t} = 1 + \frac{(\lambda-1)^2}{\lambda}\frac{1-\alpha}{\psi}\Big(\frac{r^*}{\alpha}\Big)^{\frac{\alpha}{\alpha-1}} L^* \tag{1.50}$$

Solve for g, r^*, and L^* from above,

$$L^* = \frac{(g-1)\lambda\psi}{(\lambda-1)^2(1-\alpha)}\Big[\frac{(\delta-1)\beta + g^{1-\gamma}}{\beta(1-\tau_k)}\Big]^{\frac{\alpha}{1-\alpha}} \alpha^{\frac{\alpha}{\alpha-1}} \tag{1.51}$$

and

$$L^* = \frac{\phi_1(g)}{\phi_2(g)} \tag{1.52}$$

where $\phi_1(g) \equiv (1-\alpha)(1-\tau_l)[(\delta-1)\beta + g^{1-\gamma}]$ and $\phi_2(g) \equiv [(\delta-1)\beta + g^{1-\gamma}][(1-\tau_l)(1-\alpha) + \lambda\eta(1 - G_0 - \frac{\lambda-1}{\lambda}(1-\alpha))] - \lambda\eta\alpha\beta(1-\tau_k)(g+\delta-1)$.

The equilibrium of the economy is the same as that in Garcia-Penalosa and Turnovsky (2007) in the sense that the aggregate behavior is independent of wealth and income distributions. However, the growth rate in our model is determined endogenously by productivity growth that results from intentional R&D activity as in Aghion, Akcigit and Fernandez-Villaverde (2012).

1.4 Distributions

1.4.1 Relative Capital Distribution

In the above section, we already denoted the relative capital position by $k_{it} \equiv \frac{K_{it}}{K_t}$. Summing over the individual capital accumulation function (1.2), we can get the aggregate capital accumulation function:

$$K_{t+1} = (1-\tau_k)r_t K_t + (1-\delta)K_t + (1-\tau_l)w_t L_t - C_t \tag{1.53}$$

Together with (1.31), we have

$$\frac{K_{t+1}}{K_t} = (1-\tau_k)r_t + 1 - \delta + \frac{1-\tau_l}{\eta}\frac{w_t}{K_t}\Big[(\eta+1)L_t - 1\Big] \tag{1.54}$$

Similarly, by combining (1.2) and (1.7), we get the individual capital accumulation function

$$\frac{K_{it+1}}{K_{it}} = (1-\tau_k)r_t + 1 - \delta + \frac{1-\tau_l}{\eta}\frac{w_t}{K_{it}}\Big[(\eta+1)L_{it} - 1\Big] \tag{1.55}$$

From the definition of k_{it}, we know $\frac{k_{it+1}}{k_{it}} = \frac{K_{it+1}}{K_{t+1}} / \frac{K_{it}}{K_t}$. Thus, the evolution of household i's relative capital position can be described by

$$\frac{k_{it+1}}{k_{it}} = \frac{(1-\tau_k)r_t + 1 - \delta + \frac{1-\tau_l}{\eta}\frac{w_t}{K_{it}}[(\eta+1)L_{it} - 1]}{(1-\tau_k)r_t + 1 - \delta + \frac{1-\tau_l}{\eta}\frac{w_t}{K_t}[(\eta+1)L_t - 1]} \tag{1.56}$$

On the balanced growth path, the relative capital position of household i is constant, i.e. $k_{it+1} = k_{it} = k_i^*$. Thus, we can find the relationship between the relative capital position and individual labor supply on the balanced growth path,

$$k_i^* = \left[1 - (1+\eta)L_i^*\right] / \left[1 - (1+\eta)L^*\right] \tag{1.57}$$

The household transversality condition implies the following aggregate transversality condition must be satisfied

$$\lim_{t\to\infty} \beta^t U^{'}(C_t)K_t = 0 \tag{1.58}$$

It is then obvious that $\frac{\beta^{t+1}C_{t+1}^{\gamma-1}K_{t+1}}{\beta^t C_t^{\gamma-1}K_t} < 1$. Thus, $\frac{K_{t+1}}{K_t} < (1-\tau_k)r_t + 1 - \delta$. From (1.54), we can see the aggregate transversality condition is equivalent to

$$(1+\eta)L^* < 1 \tag{1.59}$$

It also holds for each individual i, $(1+\eta)L_i^* < 1$. Then from equation (1.57), there is a positive correlation between relative capital position and leisure time on the balanced growth path.

Equation (1.55) shows the evolution of the capital stock of household i. From the dynamics of the aggregate variables, it is straightforward to derive the evolution of the relative capital positions. Suppose the initial capital of household i is K_{i0}. The capital stock at time t can be solved by iterative substitution using the stationary form of (1.55):

$$\frac{\tilde{K}_{it+1}}{\tilde{K}_{it}} = \frac{(1-\tau_k)r_t + 1 - \delta}{\tilde{Q}_{t+1}} + \frac{1-\tau_l}{\eta}\frac{\tilde{w}_t}{\tilde{K}_{it}\tilde{Q}_{t+1}}\Big[(\eta+1)L_{it} - 1\Big] \tag{1.60}$$

In the household's problem, we have known that $\frac{l_{it+1}}{l_{it}} = \frac{l_{t+1}}{l_t}$. It implies that $l_{it} = v_i l_t$ and v_i is constant over time. In other words, household i's relative leisure time is constant over time. According to (1.57),

$$v_i = \frac{\eta}{(1+\eta)(1-L^*)} + \frac{1-(1+\eta)L^*}{(1+\eta)(1-L^*)\tilde{K}^*}\tilde{K}_i^* \tag{1.61}$$

Then we rewrite (1.60) as

$$\tilde{K}_{it} = \Big(\Pi_{m=0}^{t-1} S_m\Big)\Big\{\tilde{K}_{i0} + \Sigma_{m=0}^{t-1}\frac{\Lambda_m}{\Pi_{n=0}^{m} S_n}\Big\} \tag{1.62}$$

where $S_m \equiv \frac{(1-\tau_k)r_m+1-\delta}{\tilde{Q}_{m+1}}$, $\Lambda_m \equiv \frac{1-\tau_l}{\eta}\frac{\tilde{w}_m}{\tilde{Q}_{m+1}}\big[(\eta+1)(1-v_i(1-L_m)) - 1\big]$. On the balanced growth path, individual capital stock should be constant. Let $t \to \infty$, then $\tilde{K}_{it} \to \tilde{K}_i^*$. Thus, we can solve the steady state $\tilde{K}_{it}$ from (1.62), resulting in

$$\tilde{K}_i^* = \Big\{\Pi_{m=0}^{\infty} S_m\Big\}\Big\{\tilde{K}_{i0} + \Sigma_{m=0}^{\infty}\frac{\Lambda_m}{\Pi_{n=0}^{m} S_n}\Big\} \tag{1.63}$$

Since we have already got the dynamics of the aggregate variables, it is straightforward to get the dynamics of individual capital stock from equation (1.62) and (1.63).

Substitute (1.61) into (1.63), we get

$$\tilde{K}_{it} = \Gamma_t\Big\{\tilde{K}_{i0} - \Psi_t\tilde{K}_i^* + \Delta_t\Big\} \tag{1.64}$$

where $\Gamma_t \equiv \Pi_{m=0}^{t-1} S_m$, $\Delta_t \equiv \Sigma_{m=0}^{t-1}\left[\frac{(1-\tau_l)\tilde{w}_m}{\eta \tilde{Q}_{m+1}\Pi_{n=0}^{m}S_n}(\eta - \frac{\eta(1-L_m)}{1-L^*})\right]$, and $\Psi_t \equiv \Sigma_{m=0}^{t-1}\left[\frac{(1-\tau_l)\tilde{w}_m}{\eta \tilde{Q}_{m+1}\Pi_{n=0}^{m}S_n}\frac{(1-(1+\eta)L^*)(1-L_m)}{(1-L^*)\tilde{K}^*}\right]$. In the long run,

$$\tilde{K}_i^* = \tilde{K}_{i0}\frac{\Gamma_\infty}{1+\Gamma_\infty \cdot \Psi_\infty} + \frac{\Gamma_\infty \cdot \Delta_\infty}{1+\Gamma_\infty \cdot \Psi_\infty} \tag{1.65}$$

Thus (1.64) can be written as

$$\tilde{K}_{it} = \Gamma_t\left\{\tilde{K}_{i0}\left[1 - \Psi_t \cdot \frac{\Gamma_\infty}{1+\Gamma_\infty \cdot \Psi_\infty}\right] - \Psi_t \cdot \frac{\Gamma_\infty \cdot \Delta_\infty}{1+\Gamma_\infty \cdot \Psi_\infty} + \Delta_t\right\} \tag{1.66}$$

The law of motion of the relative capital stock $\tilde{K}_{it}$ can also be described by (1.60) since (1.61) gives us v_i. This is an equivalent way to describe the path of $\tilde{K}_{it}$. Moreover, we can see from the above equation that the relative capital ranking of the households will be unchanged across time. Household with a higher initial relative capital position will always have a higher relative capital position.

According to (1.65), the standard deviation of relative capital position in the steady state is

$$\sigma_{k_i^*} = \frac{\tilde{K}_0}{\tilde{K}^*}\frac{\Gamma_\infty}{1+\Gamma_\infty \cdot \Psi_\infty}\sigma_{k_{i0}} \tag{1.67}$$

We can also derive the standard deviation of relative capital position at any period,

$$\sigma_{k_{it}} = \Theta_t \sigma_{k_{i0}} \tag{1.68}$$

where $\Theta_t \equiv \frac{\tilde{K}_0}{\tilde{K}_t}\Gamma_t \cdot \{1 - \frac{\Gamma_\infty}{1+\Gamma_\infty \cdot \Psi_\infty} \cdot \Psi_t\}$.

In the neoclassical or AK model, there is no transitional dynamics. Therefore, the steady state relative capital position is the same as the initial value. In contrast, our model has incorporated the transitional dynamics of the relative capital position.

1.4.2 Relative Income Distribution

Distribution of Before-Tax Relative Income

The household i with capital stock K_{it} obtains before-tax income $F_{it} = r_t K_{it} + w_t L_{it}$. The aggregate before-tax income of all households is $F_t = r_t K_t + w_t L_t$. The relative income is denoted by $f_{it} \equiv F_{it}/F_t$. On the balanced growth path, from (1.57), we can express L_i by k_i. Since $r_t = \alpha \frac{\tilde{Y}_t}{\tilde{K}_t}$ and $\tilde{w}_t = \frac{1-\alpha}{\lambda} \frac{\tilde{Y}_t}{L_t}$, the relative income can be written as

$$f_i^* = \frac{F_{it}}{F_t} = \frac{1}{\tilde{F}^*} \Big[r^* \tilde{K}_i^* + \tilde{w}^* \frac{k_i^* [(1+\eta)L^* - 1] + 1}{1+\eta} \Big] \tag{1.69}$$

and further as

$$f_i^* = \frac{1}{\tilde{F}^*} \Big[\alpha \frac{\tilde{Y}^*}{\tilde{K}^*} \tilde{K}_i^* + \frac{1-\alpha}{\lambda} \frac{\tilde{Y}^*}{L^*} \frac{k_i^* [(1+\eta)L^* - 1] + 1}{1+\eta} \Big] \tag{1.70}$$

Then we have

$$f_i^* = \frac{1}{\lambda\alpha + 1 - \alpha} \Big[(\lambda\alpha + \frac{1-\alpha}{(1+\eta)L^*} ((1+\eta)L^* - 1)) k_i^* + \frac{1-\alpha}{(1+\eta)L^*} \Big] \tag{1.71}$$

Thus,

$$f_i^* - 1 = \rho(L^*)(k_i^* - 1) \tag{1.72}$$

where $\rho(L^*) \equiv 1 - \frac{1-\alpha}{(\lambda\alpha+1-\alpha)(1+\eta)L^*}$.

Claim 1. *In the laissez-faire economy,* $0 < \rho(L^*) < 1$.

Proof *It is obvious that* $\rho(L^*) < 1$. *Next, we show that* $\rho(L^*) > 0$. *Suppose the growth rate is strictly positive on the balanced growth path, i.e.* $g > 1$. *From aggregate capital accumulation function (1.53),*

$$g - 1 = r^* - \delta + \tilde{w}^* \frac{L^*}{\tilde{K}^*} - \frac{\tilde{C}^*}{\tilde{K}^*}$$

Thus,

$$g - 1 = r^* - \delta + \frac{1}{\eta}\frac{1-\alpha}{\lambda\alpha L^*}\Big[(1+\eta)L^* - 1\Big]r^* > 0$$

Thus,

$$(\lambda\alpha + 1 - \alpha)(1+\eta)L^* - (1-\alpha) > 0$$

Therefore, $0 < \rho(L^*) < 1$.

According to (1.72), household with higher wealth also obtains higher relative income. Also, we have the standard deviation of relative income $\sigma_{f_i^*} < \sigma_{k_i^*}$. On the balanced growth path, the income inequality is smaller than the wealth inequality.

The dynamics of income inequality can be obtained similarly to that of the wealth inequality. The relative income of individual i is

$$f_{it} = \frac{r_t K_{it} + w_t L_{it}}{r_t K_t + w_t L_t} = \frac{r_t \tilde{K}_{it} + \tilde{w}_t L_{it}}{r_t \tilde{K}_t + \tilde{w}_t L_t} \tag{1.73}$$

Thus,

$$f_{it} = \frac{r_t \tilde{K}_{it} + \tilde{w}_t - \tilde{w}_t(1-L_t)v_i}{(\alpha + (1-\alpha)/\lambda)\tilde{Y}_t} \tag{1.74}$$

Substitute (1.61) (1.65) (1.66) to the above equation, we can obtain the standard deviation of the relative income $\sigma_{f_{it}}$:

$$\begin{aligned} \sigma_{f_{it}} &= \left\{ \frac{r_t \tilde{K}_t \Theta_t}{(\alpha + (1-\alpha)/\lambda)\tilde{Y}_t} - \frac{\tilde{w}_t(1-L_t)(1-(\eta+1)L^*)}{(\alpha + (1-\alpha)/\lambda)\tilde{Y}_t(1+\eta)(1-L^*)\tilde{K}^*} \right. \\ & \left. \frac{\Gamma_\infty \cdot \tilde{K}_0}{1 + \Gamma_\infty \cdot \Psi_\infty} \right\} \sigma_{k_{i0}} \end{aligned} \tag{1.75}$$

Especially, the standard deviation of the relative income in the long run is

$$\sigma_{f_i^*} = \rho(L^*)\Theta_\infty \sigma_{k_{i0}} \tag{1.76}$$

The transitional dynamics affect the income inequality measure through its effect on the steady state relative capital position.

Distribution of Relative After-tax Income

After-tax individual income can be defined as $\hat{F}_{it} = (1-\tau_k)r_t K_{it} + (1-\tau_l)w_t L_{it}$. The aggregate after-tax income is $\hat{F}_t = (1-\tau_k)r_t K_t + (1-\tau_l)w_t L_t$. Denote $\hat{f}_{it} = \hat{F}_{it}/\hat{F}_t$. On the balanced growth path, we can get similar results as above.

$$\begin{aligned} \hat{f}_i^* &= \frac{1}{(1-\tau_k)\lambda\alpha + (1-\tau_l)(1-\alpha)} \Big[(\lambda\alpha(1-\tau_k) + \frac{(1-\tau_l)(1-\alpha)}{(1+\eta)L^*} \\ & ((1+\eta)L^* - 1))k_i^* + \frac{(1-\tau_l)(1-\alpha)}{(1+\eta)L^*} \Big] \end{aligned} \tag{1.77}$$

Similarly,

$$\hat{f}_i^* - 1 = \hat{\rho}(L^*)(k_i^* - 1) \tag{1.78}$$

where $\hat{\rho}(L^*) \equiv 1 - \frac{(1-\tau_l)(1-\alpha)}{((1-\tau_k)\lambda\alpha+(1-\tau_l)(1-\alpha))(1+\eta)L^*}$.

Claim 2. $0 < \hat{\rho}(L^*) < 1$

Proof *It is obvious that* $\hat{\rho}(L^*) < 1$. *Next, we show that* $\hat{\rho}(L^*) > 0$. *Suppose the growth rate is strictly positive on the balanced growth path,* $g > 1$. *From aggregate capital accumulation function (1.53),*

$$g - 1 = (1 - \tau_k)r^* - \delta + (1 - \tau_l)\tilde{w}^* \frac{L^*}{\tilde{K}^*} - (1 + \tau_c)\frac{\tilde{C}^*}{\tilde{K}^*}$$

Thus,

$$g - 1 = (1 - \tau_k)r^* - \delta + \frac{1}{\eta}(1 - \tau_l)\frac{1 - \alpha}{\lambda\alpha L^*}[(1 + \eta)L^* - 1]r^* > 0$$

Thus,

$$[\lambda\alpha(1 - \tau_k) + (1 - \tau_l)(1 - \alpha)](1 + \eta)L^* - (1 - \tau_l)(1 - \alpha) > 0$$

Therefore, $0 < \hat{\rho}(L^*) < 1$.

We can see that after-tax relative income is higher for wealthier household. Also, we have the standard deviation of relative income $\sigma_{\hat{f}_i^*} < \sigma_{k_i^*}$. The after-tax relative income inequality is also smaller than the wealth inequality on the balanced growth path.

According to the above results, both before and after tax relative income are increasing in relative capital position. It implies that although the richer individuals supply less labor and obtain lower labor income, the effect is not strong enough to offset the higher capital income. More intuitively, it is straightforward to see that $\sigma_{L_i^*} = (L^* - \frac{1}{1+\eta})\sigma_{k_i^*}$ from (1.57). According to (1.59), the variability of labor supply is smaller than the variability of relative capital position.

From the expression of $\hat{\rho}(L^*)$, we can have

$$\hat{\rho}(L^*) = \rho(L^*) + (1 - \rho(L^*)) \frac{\lambda\alpha(\tau_l - \tau_k)}{(1-\tau_k)\lambda\alpha + (1-\tau_l)(1-\alpha)} \tag{1.79}$$

Thus, for the same equilibrium capital variation, the variation of pre-tax income is higher than post-tax income variation if and only if $\tau_k > \tau_l$. The effects of taxes on after-tax income can be measured by the first derivatives:

$$\frac{\partial\hat{\rho}(L^*)}{\partial\tau_k} = \frac{\partial\rho(L^*)}{\partial L^*}\frac{\partial L^*}{\partial\tau_k}\frac{1-\hat{\rho}(L^*)}{1-\rho(L^*)} + (1-\rho(L^*))\frac{-\lambda\alpha(1-\tau_l)(\lambda\alpha+1-\alpha)}{[(1-\tau_k)\lambda\alpha + (1-\tau_l)(1-\alpha)]^2} \tag{1.80}$$

$$\frac{\partial\hat{\rho}(L^*)}{\partial\tau_l} = \frac{\partial\rho(L^*)}{\partial L^*}\frac{\partial L^*}{\partial\tau_l}\frac{1-\hat{\rho}(L^*)}{1-\rho(L^*)} + (1-\rho(L^*))\frac{\lambda\alpha(1-\tau_k)(\lambda\alpha+1-\alpha)}{[(1-\tau_k)\lambda\alpha + (1-\tau_l)(1-\alpha)]^2} \tag{1.81}$$

Both income taxes τ_l and τ_k have direct and indirect effects on the after-tax income distribution. Firstly, taxes can directly affect the after-tax income variations which is implied by the second terms of the right hand side of the above two derivatives. From the derivatives, it is easy to see that capital income tax has a negative direct effect on after-tax income variation, while the effect of labor income tax is positive. Secondly, taxes can affect the after-tax income through their effects on the equilibrium labor supply and further the before-tax income. This is implied by the first term of the right hand side of the derivatives.

The dynamics of after-tax income inequality can be obtained similarly to that of the wealth inequality. The relative income of individual i is

$$\hat{f}_{it} = \frac{(1-\tau_k)r_t K_{it} + (1-\tau_l)w_t L_{it}}{(1-\tau_k)r_t K_t + (1-\tau_l)w_t L_t} \tag{1.82}$$

We can obtain the standard deviation of the after-tax relative income $\sigma_{\hat{f}_{it}}$:

$$\begin{aligned} \sigma_{\hat{f}_{it}} = & \frac{1}{(1-\tau_k)\alpha + (1-\tau_l)(1-\alpha)/\lambda} \Bigg\{ (1-\tau_k)\frac{r_t \tilde{K}_t \Theta_t}{\tilde{Y}_t} \\ & -(1-\tau_l)\frac{\tilde{w}_t(1-L_t)(1-(\eta+1)L^*)}{\tilde{Y}_t(1+\eta)(1-L^*)\tilde{K}^*}\frac{\Gamma_\infty \cdot \tilde{K}_0}{1+\Gamma_\infty \cdot \Psi_\infty}) \Bigg\} \sigma_{k_{i0}} \end{aligned} \tag{1.83}$$

In the long run, the standard deviation of the relative after-tax income is

$$\sigma_{\hat{f}_i^*} = \hat{\rho}(L^*)\Theta_\infty \sigma_{k_{i0}} \tag{1.84}$$

1.4.3 Distribution of Welfare

The individual welfare equals the individual utility function (1.1) evaluated along the equilibrium path. Thus,

$$\Omega_i = \Sigma_{t=0}^{\infty}\beta^t \frac{1}{\gamma}(C_{it} l_{it}^{\eta})^{\gamma} = \Sigma_{t=0}^{\infty}\beta^t \frac{1}{\gamma}\Big[\frac{(1-\tau_l)w_t l_{it}^{1+\eta}}{\eta(1+\tau_c)}\Big]^{\gamma} \tag{1.85}$$

The welfare of individual i relative to the individual with average wealth is then

$$\varpi_i = \frac{\Sigma_{t=0}^{\infty}\beta^t \frac{1}{\gamma}\left(\frac{(1-\tau_l)w_t l_{it}^{1+\eta}}{\eta(1+\tau_c)}\right)^{\gamma}}{\Sigma_{t=0}^{\infty}\beta^t \frac{1}{\gamma}\left(\frac{(1-\tau_l)w_t l_{t}^{1+\eta}}{\eta(1+\tau_c)}\right)^{\gamma}} = v_i^{(1+\eta)\gamma} \tag{1.86}$$

The second equality comes from the fact that $l_{it} = v_i l_t$. Note that v_i is time-invariant. We can express a monotonic transformation of the relative welfare of individual i in terms of the relative capital position of individual i,

$$\varpi_i^{1/(1+\eta)\gamma} = v_i = \frac{\eta}{(1+\eta)(1-L^*)} + \frac{1-(1+\eta)L^*}{(1+\eta)(1-L^*)}k_i^* \tag{1.87}$$

A natural measure of welfare inequality is then the standard deviation of the relative utility

$$\sigma_{\varpi_i} = \frac{1-(1+\eta)L^*}{(1+\eta)(1-L^*)}\sigma_{k_i^*} = \frac{1-(1+\eta)L^*}{(1+\eta)(1-L^*)}\frac{\tilde{K}_0}{\tilde{K}^*}\frac{\Gamma_\infty}{1+\Gamma_\infty\cdot\Psi_\infty}\sigma_{k_{i0}} \tag{1.88}$$

Comparing with the income inequality, we can show that $\sigma_{\varpi_i} < \sigma_{f_i^*}$ and $\sigma_{\varpi_i} < \sigma_{\hat{f}_i^*}$. The welfare inequality is smaller than both the before-tax and after-tax income inequality, thus also smaller than wealth inequality on the balanced growth path.

We summarize some of the results in the following proposition:

Proposition 1. *On the balanced growth path,*

1. both before-tax and after-tax income inequalities are smaller than the wealth inequality;

2. the before-tax income inequality is higher than the after-tax income inequality if and only if the capital income tax rate is larger than the labor income tax rate;

3. welfare inequality is smaller than before-tax and after-tax income inequalities, thus also smaller than wealth inequality.

1.5 Numerical Analysis

To study the dynamics of inequality measures and the effects of fiscal policy, we apply the numerical analysis in this section. The system of equations (1.41) -(1.47) forms a typical two-point boundary value problem in discrete time. We adopt the tool provided by Trimborn et al (2008) to solve this problem. The tool in Trimborn et al (2008) is only applicable to the continuous models, so we modified the codes and extended it to discrete models.

Our choice of $\gamma = -0.8$ indicates that the inter-temporal elasticity of substitution is 0.5556. It is well in the range used in existing literature. The elasticity of leisure in the utility η is 2.1, which implies that agents spent 70 percent of their discretionary time on leisure. β equals to 0.998 delivers an annual after-tax interest rate of 4.9%. The depreciation rate δ is 0.05. $\alpha = 0.28$ and $\lambda = 1.08$ implies that the labor share of output is about two thirds. $\psi = 0.067$ gives an equilibrium growth rate of about 2.6 percent.

Parameter	γ	η	β	δ	α	λ	ψ	G_0
Value	-0.8	2.1	0.998	0.05	0.28	1.08	0.067	0.2367

Table 1.1: Baseline parameter values

variable	g^*	L^*	$\tilde{K}^*$	$\tilde{C}^*$	$\tilde{Y}^*$	$\tilde{w}^*$	r^*
steady-state	1.0258	0.3024	0.8592	0.2225	0.4051	0.8931	0.1320

Table 1.2: Steady-state values of variables

In our benchmark model, we assume a base income tax rate of 0.25. Thus, $\tau_k = \tau_l = 0.25$ in the benchmark case. In order to keep the government budget balanced at every period, the following equation must hold:

$$G_0 = \alpha \tau_k + \frac{1-\alpha}{\lambda} \tau_l \tag{1.89}$$

It then implies a government spending share of $G_0 = 0.2367$. Table (1.1) summarizes the benchmark parameter values.

Table (1.2) shows the steady state values of the model.

r^* represents the rental rate, and $r^* = 0.1320$ implies 4.9% of after-tax interest rate.

In the following sections, we will examine the effects of taxation on growth and several inequality measures by beginning with negative capital income tax and gradually increasing capital income tax to 0.7. Negative capital income tax should be regarded as subsidy to capital. Therefore, the labor income tax will decrease from a high level accordingly.

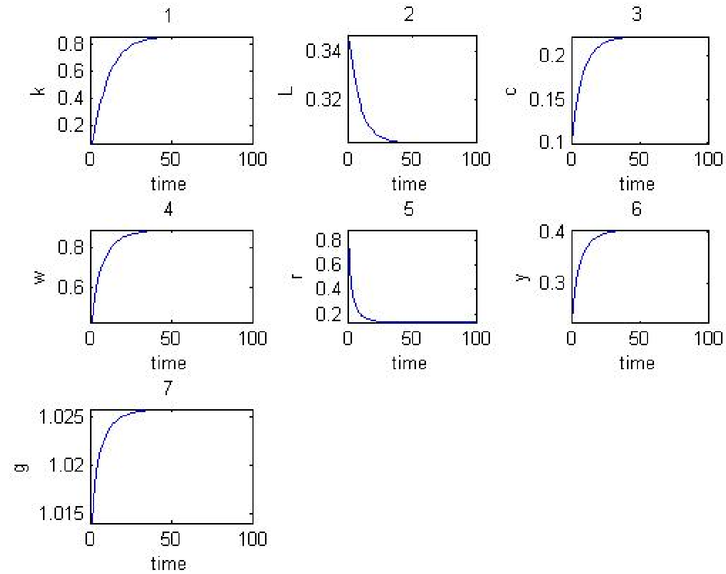

Figure 1.1: Dynamics of aggregate variables

1.5.1 Aggregate and Distributional Dynamics

Suppose the initial aggregate capital endowment is 0.07, which is lower than all of the steady state capital levels under the tax rates we are going to study below. We have the aggregate dynamics as shown in Figure (1.1).

Starting from a level lower than the steady state, the capital stock will increase over time. Meanwhile, the equilibrium labor supply will decrease. The price of capital r_t and the price of labor W_t will evolve towards the opposite direction to that of capital stock and labor supply, respectively. Output, consumption and growth rate will all increase until they reach the steady state values.

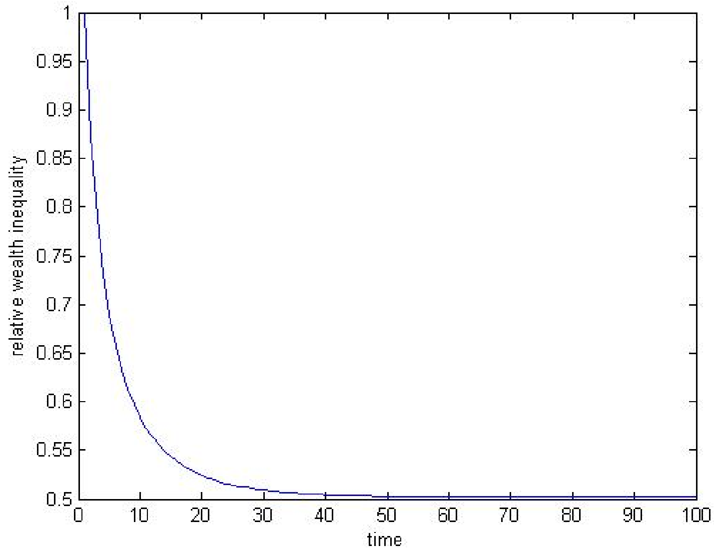

Figure 1.2: Dynamics of wealth inequality

The distribution of the economy can be described by Figure (1.2) and (1.3).

Along with the increase of capital stock, the average level of labor supply is decreasing. The labor supply of individuals with initial capital both lower than (the poor) and higher than (the rich) the average level will decrease. From the first order condition of the households, the individuals will keep the relative leisure level constant over time. The relative labor supply of the rich will decrease when the average labor supply decreases. On the contrary, the relative labor supply of the poor will increase. ($\frac{L_{it}}{L_t} = \frac{1-v_i}{L_t} + v_i$.) In another word, the labor supply of the rich decrease more than the poor. Moreover, since $\frac{C_{it+1}}{C_{it}} = \frac{C_{t+1}}{C_t}$, the relative consumptions of all individuals are constant over time. The relative saving of the poor will increase and that of the rich

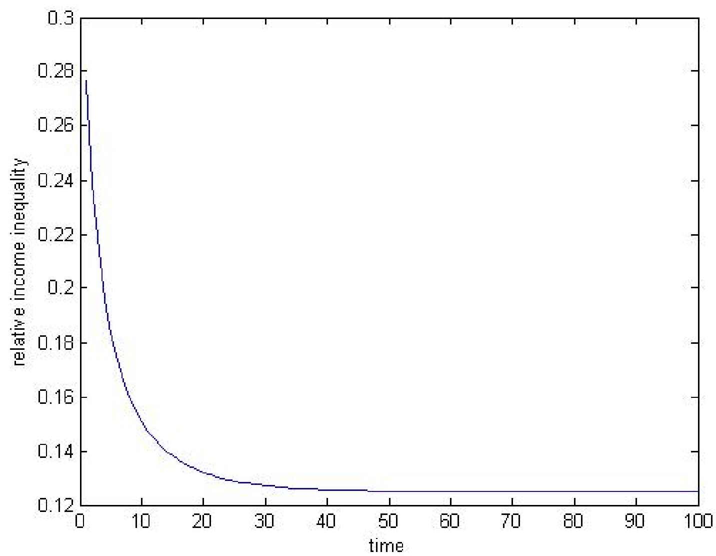

Figure 1.3: Dynamics of income inequality

will decrease. The relative capital position of the rich will decrease while the relative capital position of the poor will increase. Therefore, the standard deviation of the relative capital position (wealth inequality) is decreasing over time.

The income of the household includes two parts: capital income and labor income. The relative capital income of individuals with lower initial capital will increase and that of the rich will decrease, because the relative capital of the poor increases. The relative labor income of the poor also will increase as the relative labor supply of the poor is increasing.

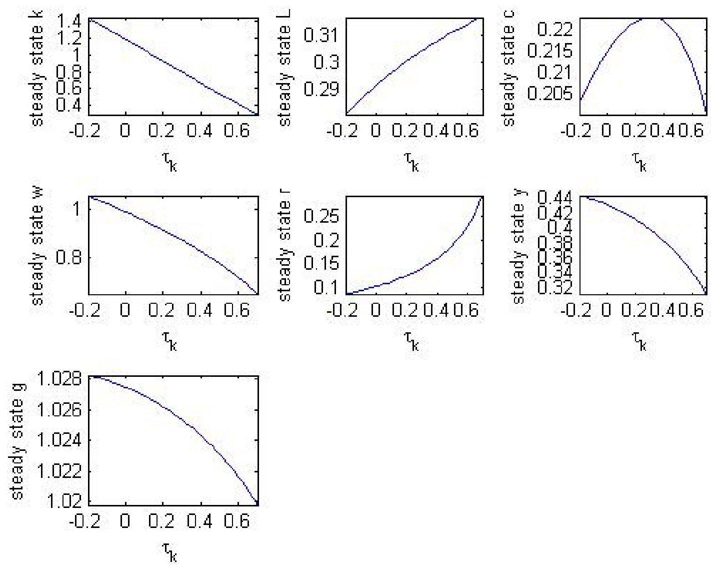

Figure 1.4: Capital income tax and key variables

1.5.2 Revenue Neutral Taxation Effects

The following Figure (1.4) shows the steady state growth rates as well as other main variables under different tax rates.

Figure (1.4) shows the effect of taxation on the aggregate economy. We can see that a higher capital income tax leads to a lower steady state growth rate. This is consistent with the conventional wisdom that capital income tax reduces capital accumulation and investment (e.g. Chamley (1986) and Lucas (1990)). In the Schumpeterian model, it will also hurt the demand for intermediate goods and thus the incentive to innovate, which is the source of economic growth in this model. A higher capital income tax (a lower labor income tax) will lead to a lower steady state capital level and a higher labor supply level. This is to say that the direct effect from lower incentive to accumulate outweighs the effect from higher return to capital. The effect on consumption is more complicated. When capital income tax is low, increasing capital income tax can increase consumption, because marginal utility gain from consumption is greater than that of accumulation. However, it will reduce consumption to increase capital income tax, if the capital income tax is already at a high level. That is to say there is a "golden rule" capital income tax that maximizes consumption.

We normalize the standard deviation of the initial relative capital position to 1, i.e. $\sigma_{k_{i0}} = 1$. Under different tax rates, the following Figure (1.5) and Figure (1.6) show the effects of taxation on different inequality measures.

Figure (1.7) shows the effects of taxation on different components of the before and after tax incomes of the household, i.e. capital income r^*K and labor income w^*L.

The figure shows that higher capital income tax will lead to higher before-tax income inequality but lower after-tax income inequality and welfare inequality. As to the wealth inequality, when initial capital income tax is small, increasing capital income tax will decrease wealth inequality. However, if initial capital income tax is already high enough, increasing capital income tax

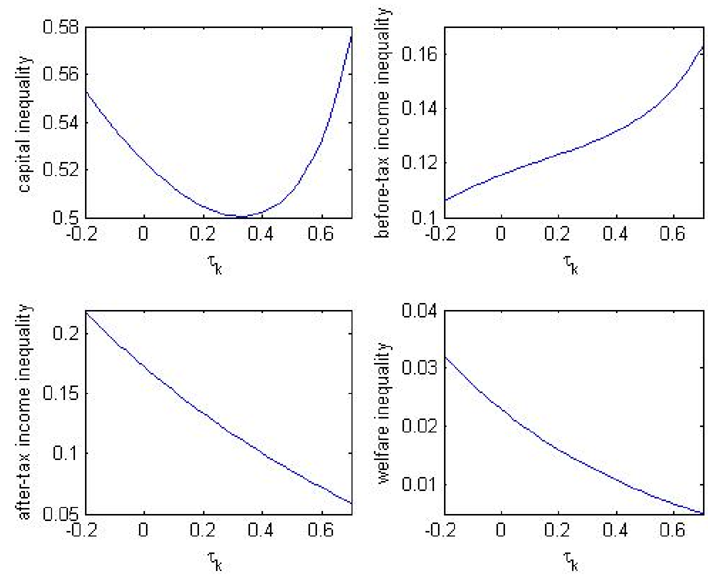

Figure 1.5: Capital income tax and inequality measures

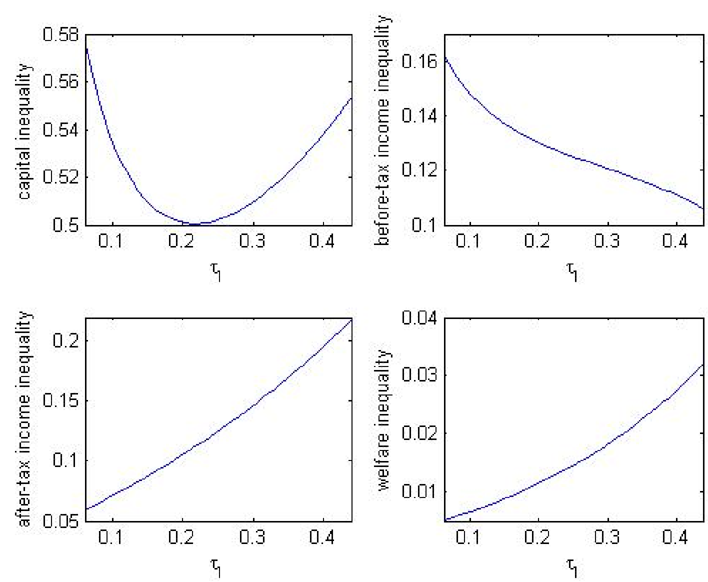

Figure 1.6: Labor income tax and inequality measures

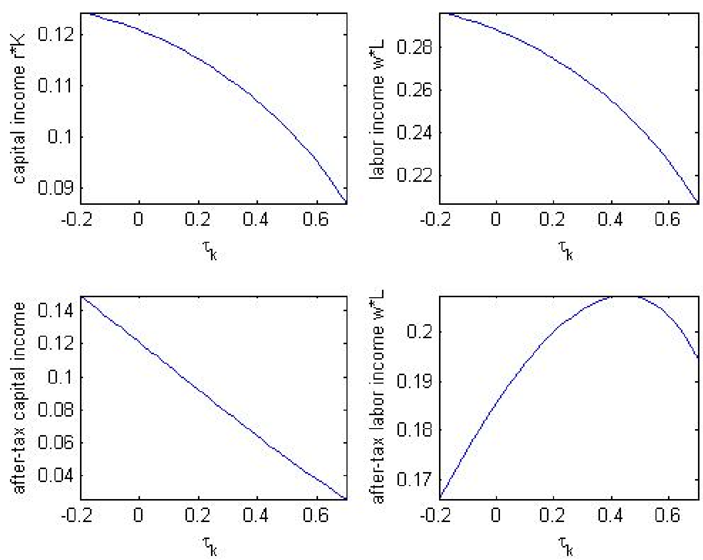

Figure 1.7: Capital income tax and income shares

will lead to higher wealth inequality. We then have the following results:

Result 1. *On the balanced growth path,*

1. *Consumption is an inverse-U shape function of the capital income tax.*

2. *Growth rate is decreasing in the capital income tax.*

3. *There is a capital income tax that minimizes wealth inequality.*

4. *After-tax income inequality and welfare inequality are decreasing in capital income tax, while before-tax income inequality is increasing.*

The result regarding the growth rate is consistent with Aghion et al (2013). Consistent with Garcia-Penalosa and Turnovsky (2007), after tax income inequality and welfare inequality are decreasing in capital income tax. However, in their paper, the effects on wealth inequality can not be analyzed, because the economy is always on the balanced growth path and there is no transitional dynamics.

When the capital income tax is low, increasing capital income tax will reduce wealth inequality, after-tax income inequality and welfare inequality. At the same time, growth rate will be reduced by increasing capital income tax. Government faces the conventional trade-off between growth and inequality. When the capital income tax is high, this trade-off between growth and after-tax income inequality and welfare inequality remains. However, in this case, it is possible for the government to reduce wealth inequality and enhance economic growth at the same time by reducing the capital income tax.

1.5.3 Taxation Reform

In this section, we will consider the following cases when the economy transit from lump-sum tax financed government spending to labor income tax or capital income tax financed government spending. We will analyze the impacts of this tax transformation on the inequality measures. Here, however, we assume the share of government expenditure is 0.1 to avoid an unrealistically high capital income tax or labor income tax. To avoid the direct distribution effects of lump-sum tax, we assume that $T_{it}/T_t = K_{it}/K_t$, where T_{it} is the lump sum tax of agent i, $T_t \equiv \int_0^1 T_{it} di$ is the aggregate lump sum tax. We also assume

variable	g^*	L^*	$\tilde{K}^*$	$\tilde{C}^*$	$\tilde{Y}^*$	$\tilde{w}^*$	r^*
steady-state	1.0307	0.3324	1.2485	0.3070	0.4815	0.9657	0.1080

Table 1.3: Initial steady-state values of variables

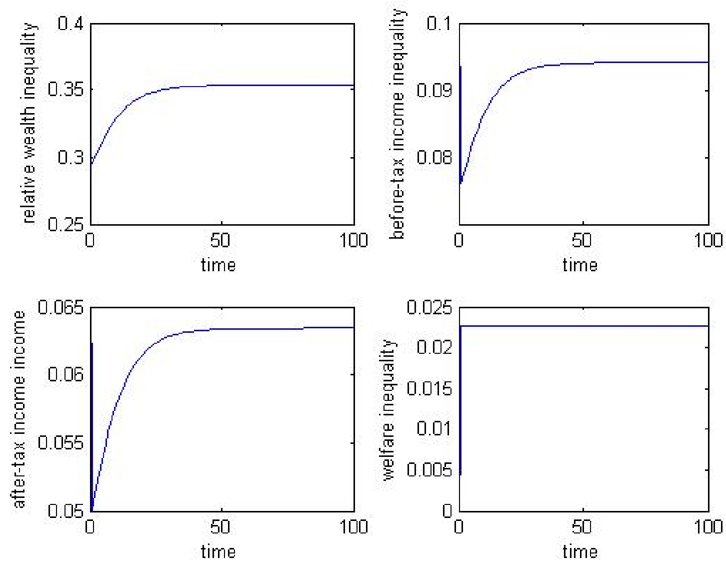

Figure 1.8: Transiting from lump sum financing to capital income tax financing

initially the economy is in the steady state, which is reported in Table (1.3).

Firstly, we consider the case when lump sum tax financing switching to capital income tax financing. The capital income tax increases from 0 to 0.3571. From the following Figure (1.8), the increase of capital income tax leads to a rise in the wealth inequality. The before-tax and after-tax income inequality both have an initial downward adjustment followed by a gradual increase which finally lead to a bit higher inequality than that of the lump sum tax financing. Welfare inequality also increases.

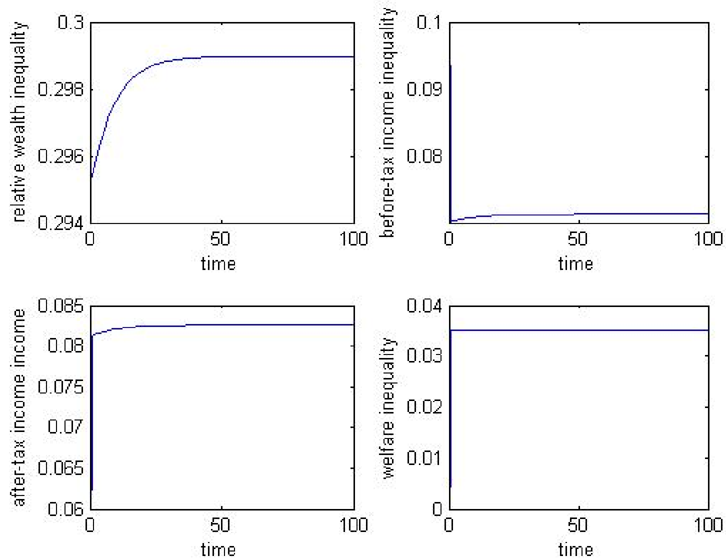

Figure 1.9: Transiting from lump sum financing to labor income tax financing

When switching from lump sum tax financing to labor income tax financing, wealth inequality will increase gradually, similar to the above case. Before-tax income inequality and after-tax income inequality, however, have different pattern of changes. Before-tax income inequality experiences a downward initial adjustment while after-tax income inequality experiences an upward initial adjustment. After the initial adjustments, they both increase towards the new steady state, but the increases are relatively small compare to the initial adjustments. Welfare inequality also increases.

We can summarize the results as follows:

Result 2. *On the balanced growth path,*

1. *Capital income tax financing results in a larger dynamic adjustment of*

inequality.

2. Compare to labor income tax financing, capital income tax financing leads to higher wealth inequality and before-tax income inequality, but lower after-tax income inequality and welfare inequality.

3. Capital income tax financing leads to a lower growth rate and steady state capital stock.

The above results are largely consistent with Garcia-Penalosa and Turnovsky (2011). The result that capital income tax financing results in lower growth rate is consistent with Garcia-Penalosa and Turnovsky (2007) and Aghion et al (2013). It is in line with Garcia-Penalosa and Turnovsky (2007) that capital income tax financing leads to lower welfare inequality, comparing to labor income tax financing.

1.5.4 Sensitivity Checks

Neoclassical Case

If we assume $\lambda = 1$, the model becomes a neoclassical model with zero growth rate. The effects of taxes on the several inequality measures are reported in Figure (1.10):

The effects of taxes on inequality measures are almost the same as the endogenous growth case with slightly different magnitude.

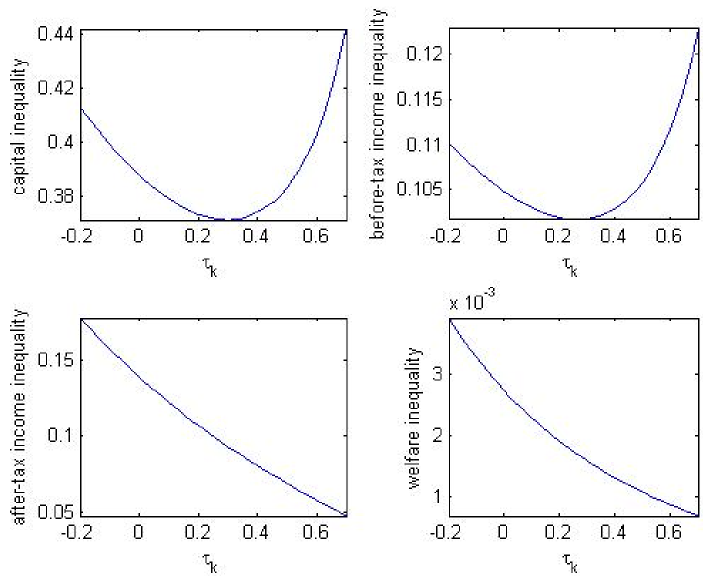

Figure 1.10: Capital income tax and inequality measures in neoclassical case

Sensitivity check of parameters

To check whether our results are dependent on the chosen set of parameters, we do several sensitivity checks for all the main parameters of the model. The following figures show that the patterns of the effects of taxation on inequality measures are robust to different parameter values.

We show the changes of inequality measures when γ changes from -0.2 to -1.4. From Figure (1.14), we can see that all the four inequalities are decreasing in γ. Increase of γ indicates an increase of inter-temporal elasticity of substitution. The result is contrary to Garcia-Penalosa and Turnovsky (2006), in which increasing inter temporal elasticity of substitution leads to

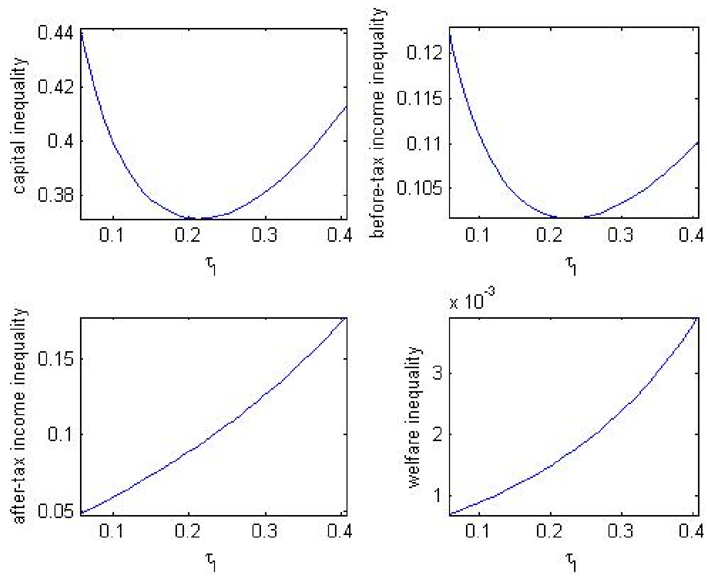

Figure 1.11: Labor income tax and inequality measures in neoclassical case

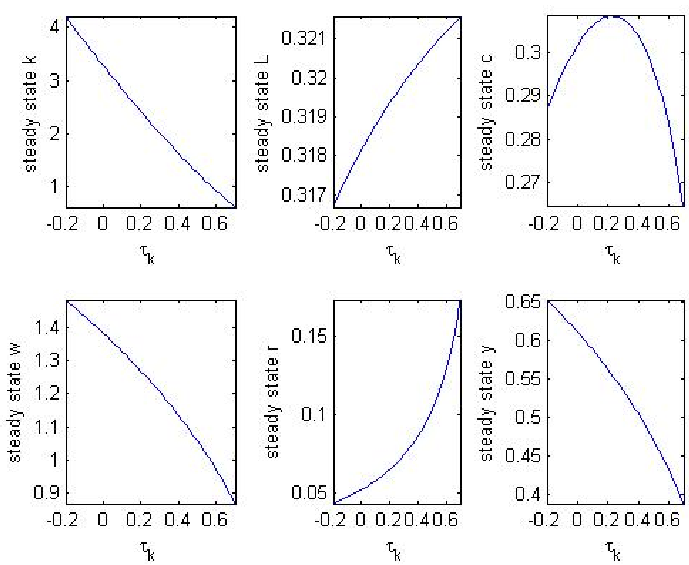

Figure 1.12: Capital income tax and key macro variables in the neoclassical case

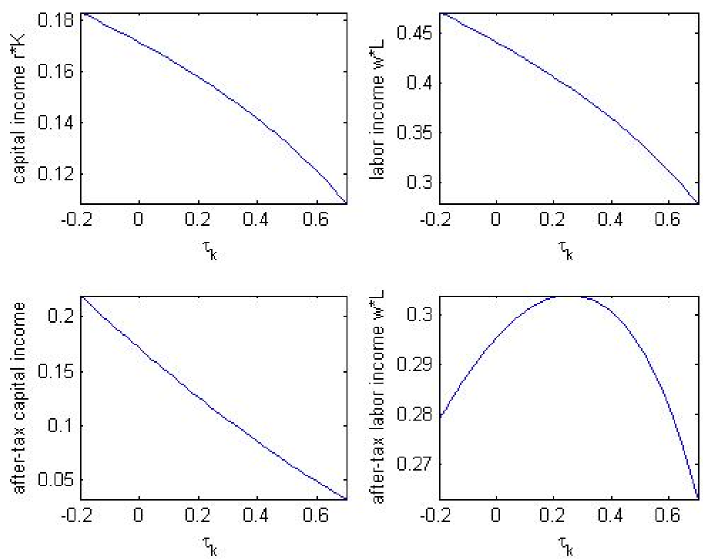

Figure 1.13: Capital income tax and income components in the neoclassical case

higher income inequality, due to the presence of transitional dynamics.

We show in Figure 1.15 the change of η from 1.5 to 2.7. The above graph shows that capital and before-tax income inequality are increasing in η while welfare inequality is decreasing in η. However, all the changes in the inequality measures are relatively small.

We allow β to change from 0.95 to 0.998. Capital inequality and welfare inequality are decreasing in β. After-tax income inequality is increasing in β. Before-tax income inequality is increasing in β when τ_k is small, but it is decreasing in β when τ_k is large enough.

When δ increases from 0.02 to 0.08, capital inequality, before-tax and after-

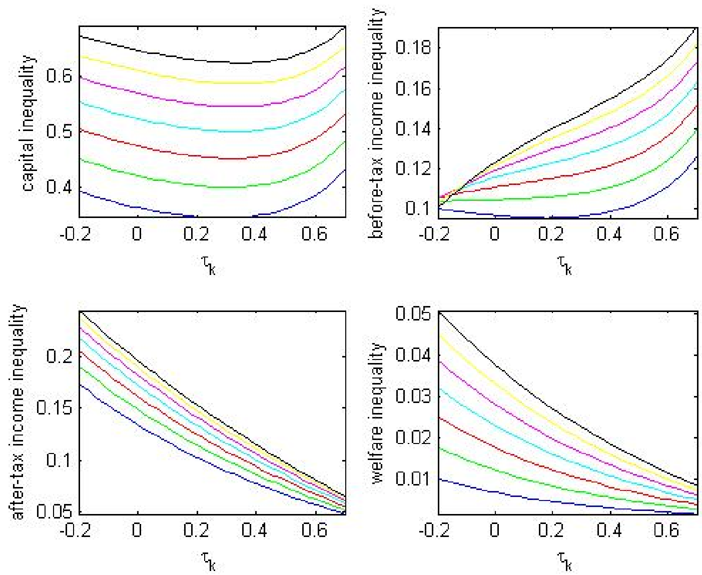

Figure 1.14: Variations in γ and inequality measures

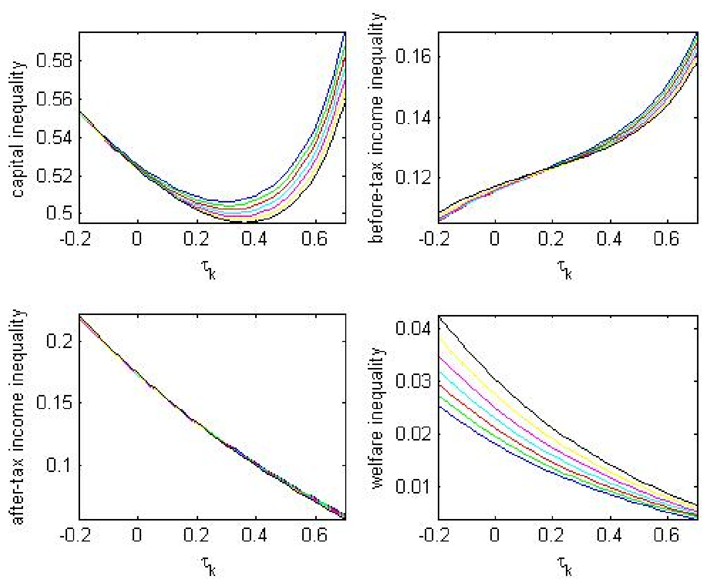

Figure 1.15: Variations in η and inequality measures

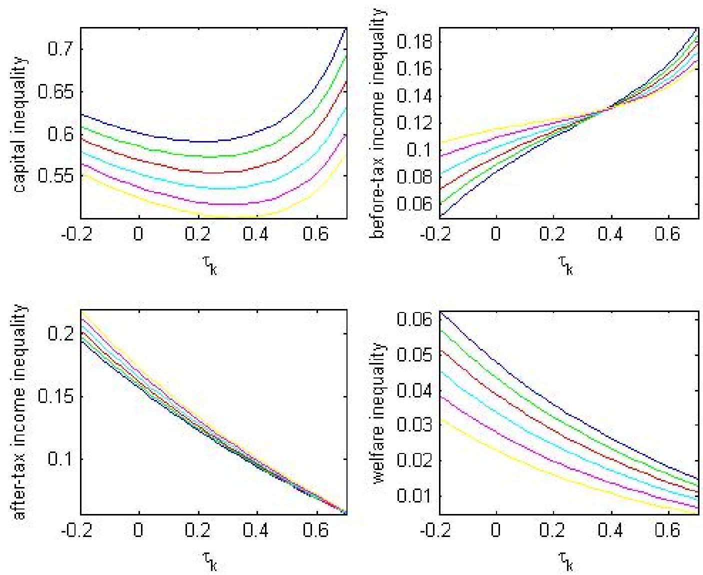

Figure 1.16: Variations in β and inequality measures

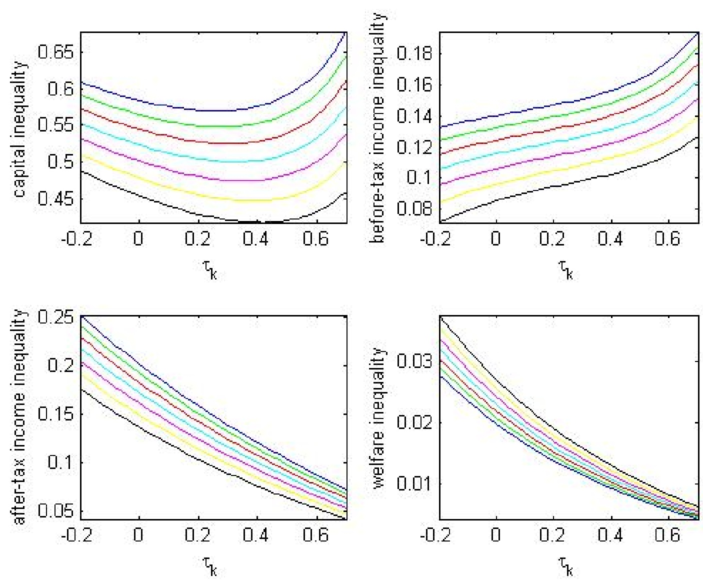

Figure 1.17: Variations in δ and inequality measures

tax inequality all increase. Only welfare inequality decreases when δ decreases. See Figure 1.17.

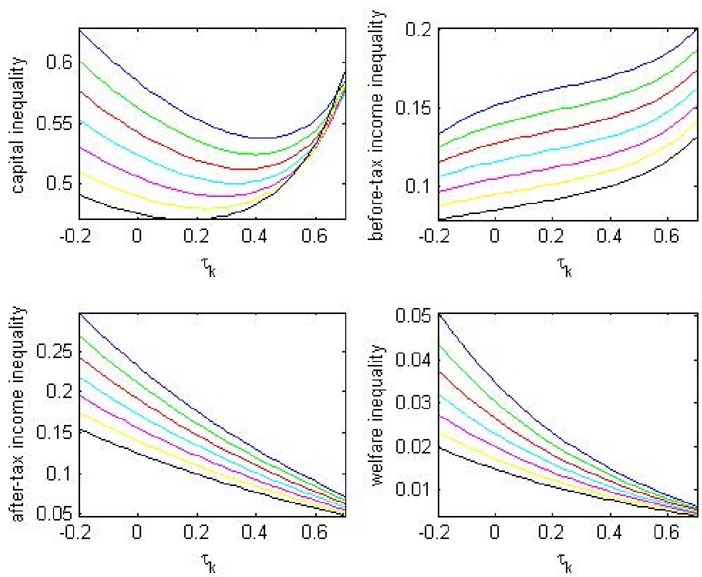

Figure 1.18: Variations in α and inequality measures

Figure 1.18 shows that when α increases from 0.22 to 0.34, the four inequality measures all increase.

We also show in Figure 1.19 that when λ increases from 1.02 to 1.14, all the four inequality measures increase.

When ϕ increases from 0.037 to 0.097, all the inequalities decreases. See Figure 1.20.

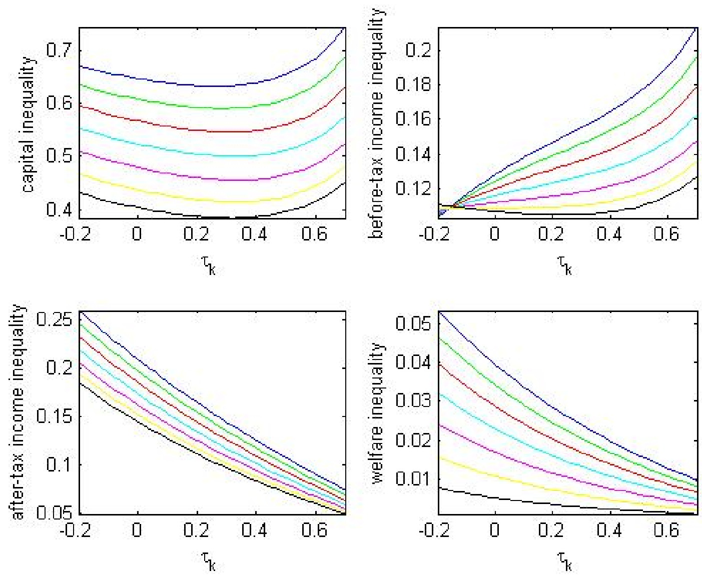

Figure 1.19: Variations in λ and inequality measures

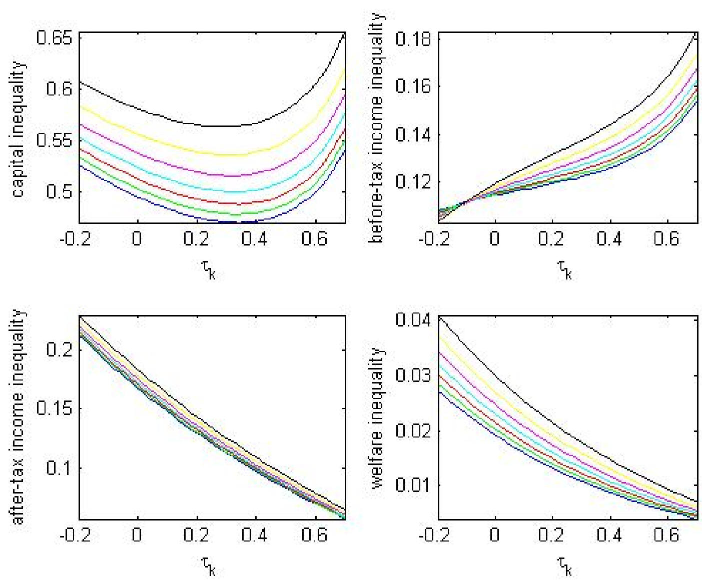

Figure 1.20: Variations in ϕ and inequality measures

1.6 Conclusion

This chapter investigates the distributional roles of different taxations. We construct a Schumpeterian growth model with heterogeneous households. The only heterogeneity comes from different initial capital endowments. We show that the steady state inequality depends on the transitional dynamics of the economy as well as the initial inequality. By analyzing both the transitional dynamics and the steady state effects of the taxes, we conclude that there is a capital income tax that minimizes wealth inequality. Thus, it is possible for the government to reduce wealth inequality and enhance growth at the same time by reducing the capital income tax from a relatively high level. Before-tax income inequality is increasing in the capital income tax, while both after-tax income inequality and welfare inequality are decreasing in the capital income tax.

We also examine the reform from a lump-sum tax to a capital income tax or a labor income tax. We show that capital income tax financing results in a larger dynamic adjustment of inequality. Compare to labor income tax financing, capital income tax financing leads to higher wealth inequality and before-tax income inequality, but lower after-tax income inequality and welfare inequality. Moreover, capital income tax financing leads to a lower growth rate and a lower level of steady state capital stock.

Chapter 2

Patent Length in an Endogenous Growth Model with Innovation and Standardization

2.1 Introduction

When a new product is first introduced, the production technology of that product is usually complex and thus the production cost is high. With the widespread adoption and acceptance of the technology and the product, respectively, a secondary innovation targeting on the high-cost products will emerge. Successful secondary innovations can reduce the complexity of the technology and the production cost is then lowered. We call this secondary innovation as "Standardization". On the one hand, the standardization process destroys the monopoly profits of the high-cost goods producers and thus lowers the innovation incentive. On the other hand, it expands the output by lowering production cost. Thus, standardization is both an engine of economic growth and a barrier to it.

In this chapter, we examine the optimal combination of patent length for innovation and standardization. Patent protection policy is one commonly used instrument for the government to balance the dynamic gain and the static loss from monopoly power of the innovators in production. Without patent protection, innovation can not be compensated and there will be no incentive to do innovation. However, with permanent patent protection, monopolistic price above marginal cost discourages aggregate demand for goods and services. With two types of innovations, monopolistic pricing also hinders the speed of technology diffusion from high cost production to low cost production. With both innovation and standardization in the economy, patent length for innovation and standardization may have different effects on economic growth as well as social welfare.

Although the distinctions between innovation and standardization are not always clear enough for the policy makers to apply different patent length protections, Acemoglu, Garcia and Zilibotti (2012) listed many examples on the differences between innovation and standardization. Among the examples cited is the production of transistors. Bell Laboratories initiated the introduction of this new product in 1947. However, this process required skill manual work and the production technology was complex. In 1959, a newly founded company Fairchild Semiconductor introduced the planar transistor which became the first industry standard. This secondary innovation replaced the irregular surface of the transistors with flat surface, which significantly reduced the production cost and complexity of production. The differences between innovation and standardization are also quite similar to that between innovation and imitation in the literature. The difference is that standardization requires R&D inputs and the production technology is changed. Basically, the R&D activities that introduces newly emerged varieties or industries can be categorized as innovation; the R&D activities that lower the technology complexity and production cost of the existing product should be regarded as standardization.

There are numerous studies on the optimal patent length to balance the conflicting interests between innovation incentive and market efficiency. Earlier studies adopted models with no economic growth or exogenous growth rate. For example, Nordhaus (1969), Sherer (1972), Gilbert and Shapiro (1990), Judd (1985). Gilbert and Shapiro (1990) consider both patent length and patent breadth. They show that the optimal patent length can be infinite with patent breadth adjustments. On his seminal paper, Judd (1985) shows that infinite patent length is optimal with an exogenous growth model. More recent papers discuss the patent policy under endogenous growth models. Horowitz and Lai (1996) consider the issue with a quality ladder model and they show that the optimal patent length that maximizes welfare is finite and lower than patent that maximizes rate of innovation. Kwan and Lai (2003), Futagami and Iwaisako (2007) also find that the patent length that maximizes welfare is finite based on variety expansion models. Zeng, Zhang and Fung (2014) study the combination of price regulation and patent length instruments in a variety expansion model too. They also find that welfare maximizing patent length is finite. The literature only considers one type of innovation in the model. However, the implications of patent policy will be different when there are two types of innovation.

In essence, patent length affects growth rate and social welfare through its impact on resource allocation among different economic activities: high cost and low cost monopoly production, competitive production, R&D input to innovation and R&D input to standardization. Patent length for innovation affects the return to innovation and the pace of technology diffusion to low cost production. Patent length for standardization affects the return to standardization. We show that given patent lengths, the equilibrium may fall in three regimes: one with both innovation and standardization; one with only innovation; and one with no innovation. Firstly, when patent for innovation is too long, there will be only innovation at equilibrium, because standardization is always less profitable than innovation. Secondly, when patent for standardization is too long, there might be no innovation, because return to innovation might be always less than standardization. Thirdly, we find that the optimal combination of patent lengths that maximizes welfare should fall in regime with both innovation and standardization. Optimal patent for innovation is finite. The reason is that there will be no standardization at equilibrium if patent length for innovation is too long. Therefore, the economy is unable to reap the benefit of an increase in output resulting from secondary innovation that lowers production cost. The optimal patent for standardization should also be finite not only because of the possible poverty trap, but also because of the static loss from lowering consumption.

Furthermore, when there are both innovation and standardization at equilibrium, only patent length for standardization can affect growth rate. That is also a result of free entry that equalizes return to innovation and standardization. Patent length for innovation will not affect growth rate, because arrival rate of standardization will adjust to make return to innovation equals to the return to standardization which only depends on patent length for standardization.

Different from Acemoglu, Garcia and Zilibotti (2012), in which standardization shifts production input from skilled labor to unskilled labor, our setup of standardization here assumes that standardization reduces production cost. This assumption brings our standardization process closer to the cost reduction innovation. Therefore, our model combines the variety expansion innovation with cost reduction innovation. Acemoglu (2009) shows that cost reduction innovation and quality improving innovation are essentially equivalent in the context of typical growth models. [1]

Extending our model to an open economy setup by assuming the North does innovation and the South does standardization, our paper is also related to another literature on intellectual property rights (IPR) policy in North-South trade and worldwide technology diffusion models. Helpman (1993) find that tighter IPR policy in the South would hurt the South in terms of trade and with endogenous innovation rate, stronger IPR policy will generate an initial increase and following decrease in innovation rate. Glass and Saggi (2002), however, show that stronger IPR makes multinationals more secure from imitation but reduces FDI and innovation. On the contrary, Lai (1998) and Dinopoulos and Segerstrom (2010) argue that stronger IPR protection in the South causes an increase in FDI and innovation. By studying the trade off between quality improvement innovation and variety expansion innovation, Glass and Wu (2007) find that stronger IPR protection may shift innovation away

[1] For the survey of quality improving innovation literature, you can refer to Aghion and Howitt (2009).

from quality improving innovation to variety expansion innovation. In this chapter, stronger patent protection for standardization will definitely lead to an increase in innovation rate, because the returns to innovation and standardization are equalized. However, our model is a closed economy model where there is one central government which can choose a combination of patent protection for innovation and for standardization, while it is not possible in the North-South trade models as there are separate governments.

This chapter is organized as follows. In Section 2, we construct the basic model with two types of innovation. Section 3 analyzes the steady state equilibrium of the model and the effects of patent length on welfare and economic growth. Section 4 shows that numerical analysis based on the model in Section 2. Section 5 concludes.

2.2 Model Setup

We consider an closed economy in continuous time and admits a representative household. The endogenous growth of the economy results from the expansion of product varieties. Similar to Acemoglu, Garcia and Zilibotti (2012), the innovation is followed by a costly process of standardization.

2.2.1 Households

The continuum of households are infinitely lived with a mass of L. Each household has one unit of time which is supplied inelastically to work. They maximize the present discounted lifetime utility by choosing their consumption of commodities, such that:

$$U = \int_0^\infty e^{-\rho t} \frac{C_t^{1-\gamma} - 1}{1 - \gamma} dt \tag{2.1}$$

whereC_t is consumption expenditure, γ represents the elasticity of marginal utility.

The budget constraint of the representative household is:

$$\dot{a}_t = w_t L + r_t a_t - C_t \tag{2.2}$$

where w_t is the wage rate and r_t is the interest rate. A dot above a variable represents the change rate of that variable with respect to time.

Solving the household problem gives the standard Euler equation:

$$\frac{\dot{C}_t}{C_t} = \frac{r_t - \rho}{\gamma} \tag{2.3}$$

According to the above equation, the growth rate then depends on r_t which also indicates the returns to the R&D investment.

2.2.2 Firms

In the economy, there is only one final good which is produced with labor and a continuum of intermediate products under perfect competition. There are three types of intermediate producers. One produces monopolistic high cost goods. Low tech goods are produced by two types of intermediate producers: one is a monopoly, and the other is competitive. Same as Romer (1990), economic growth comes from the increase of the total variety of intermediate goods which is the result of intentional R&D activities.

Final firm

The final good is produced with labor and intermediate goods. Final good can be used interchangeably for consumption, innovation and standardization investment. Thus it serves as the numeraire. The production function of the final firm is

$$Y = AL^{\alpha} \int_{0}^{N} x_i^{1-\alpha} di \tag{2.4}$$

where L is the labor input to final good production, A is a constant scale factor, x_i is the demand for intermediate good i, and N is the number of all intermediate products, a proxy for the technology level of the economy. As in Romer (1990), N grows endogenously through purposeful innovation by intermediate firms.

Upon introduction, each intermediate product is a high cost good created by innovation and produced from final good with an one to one technology. We denote this type of intermediate product by x_{Hi}. The innovator then enjoys a period of patent protection during which the innovator can produce and obtain profit. After the innovation patent expires, the newly introduced product may face with a costly standardization process. Once standardized, the product can be produced by a less costly technology. The innovator of the product then loses monopoly power and exits the market. We denote the standardized good by x_{Mi}. The standardizer also enjoys a period of patent protection. Immediately after the expiration of the standardization patent, this good becomes competitive which we denote by x_{Ci}.

In sum, there are three types of intermediate goods that are used to produce the final good. x_{Hi} is high cost good, while x_{Mi} and x_{Ci} are both low cost goods. Thus, we can rewrite the production function of the final good as:

$$Y = AL^{\alpha}\Big[\int_0^{N_H} x_{Hi}^{1-\alpha} di + \int_0^{N_M} x_{Mi}^{1-\alpha} di + \int_0^{N_C} x_{Ci}^{1-\alpha} di\Big] \tag{2.5}$$

where N_H is the number of high-tech products, N_M is the number of monopolistic low-tech goods and N_C is the number of competitive low cost goods. Suppose N_L is the total number of low cost goods. Then $N_M + N_C = N_L$. So $N_H + N_L = N$. Because of perfect competition in final good production, the price of each intermediate input will be equal to its marginal product:

$$P_{ji} = (1-\alpha)AL^{\alpha}x_{ji}^{-\alpha} \tag{2.6}$$

$$w = \alpha Y/L \tag{2.7}$$

where w is the wage rate, P_{ji} is the price of the intermediate good i and $j \in H, M, C$.

Intermediate firms

The high cost intermediate goods are produced with an one to one production technology using final good. The intermediate goods' markets are monopolistic competitive. Thus, intermediate producers of good i can obtain positive profits. Hence the profit for a high cost intermediate good producer is:

$$\pi_{Hi} = P_{Hi}x_{Hi} - x_{Hi} \tag{2.8}$$

According to (2.6), we can have

$$x_{Hi} = (1-\alpha)^{2/\alpha}A^{1/\alpha}L \tag{2.9}$$

and

$$\pi_{Hi} = \frac{\alpha}{1-\alpha}(1-\alpha)^{2/\alpha}A^{1/\alpha}L \tag{2.10}$$

The low-cost intermediate goods are produced with a lower cost in terms of final good. One unit of intermediate good is produced with ξ unit of final good where $0 < \xi < 1$. Thus, the profit that the monopolistic low cost intermediate goods producers can obtain is:

$$\pi_{Mi} = P_{Mi}x_{Mi} - \xi x_{Mi} \tag{2.11}$$

Then we can have

$$x_{Mi} = \xi^{-1/\alpha}(1-\alpha)^{2/\alpha}A^{1/\alpha}L \tag{2.12}$$

and

$$\pi_{Mi} = \xi^{(\alpha-1)/\alpha}\frac{\alpha}{1-\alpha}(1-\alpha)^{2/\alpha}A^{1/\alpha}L \tag{2.13}$$

Here, we can see that the output of low cost production x_M is greater than that of the high cost production x_H. The low cost production also offers the monopolist a larger profit.

The competitive low cost goods producers make zero profit. Therefore, the price of the competitive low cost goods is equal to its marginal cost:

$$P_{Ci} = \xi \tag{2.14}$$

Together with (2.6), we have:

$$x_{Ci} = \xi^{-1/\alpha}(1-\alpha)^{1/\alpha}A^{1/\alpha}L \tag{2.15}$$

Comparing the expression for x_{Hi}, x_{Mi} and x_{Ci}, we can see that $x_{Mi} > x_{Hi}$. The input and output of monopolistic low cost goods production are larger than that of high cost goods production. This results from the lower marginal cost of low cost goods production. In addition, we can have $x_{Ci} > x_{Mi}$. It means the input and output of monopolistic production are less than that of competitive production. This results from the presence of monopoly power.

2.2.3 Innovation and Standardization

We assume that both innovation and standardization are costly activities. Upon introduction, the intermediate good producer will have a period of patent protection with length T_H, when producing that product. Within the patented period, the product is not allowed to be standardized. When the patent expires, the product is available for a costly secondary innovation which we call standardization. After standardization, the standardizer of that product will enjoy a patent length of T_L to produce it.

One possible conjecture is that unlike the above assumption that the new product is not allowed to be standardized by anyone within the patented period, the producer of a certain product can implement the cost reduction innovation upon their own product at any time. However, when the profit from standardized product is not large enough to cover both the standardization cost and the loss from the destruction of the patent for innovation, the producer will have no incentive to do standardization within the patented period. We will derive the condition on the low cost monopolistic profit that exclude this possibility in next section.

After the patent for innovation expires, it is also not beneficial for a producer to standardize its own product due to the usual "Arrow effect". The value of a standardization to the incumbent is strictly less than the value to an outsider, because the incumbent has to bear the loss from current profit flow of high cost production.

Define V_H and V_L as the present discounted value of an innovation and standardization, respectively. The value of an innovation consists of two components: the value before the patent expiring date and the value after the patent expiring date, as shown below:

$$\begin{aligned} V_H &= \int_t^{t+T_H} e^{-\int_t^\tau r ds} \pi_H d\tau + e^{-\int_t^{t+T_H} r d\kappa} \Big\{ \\ & \int_{t+T_H}^{\infty} \Big(\int_{t+T_H}^{\tau} e^{-\int_{t+T_H}^{s} r d\kappa} \pi_H ds \Big) m e^{-m(\tau - t - T_H)} d\tau \Big\} \qquad (2.16) \end{aligned}$$

The first term on the right-hand side of equation (2.16) is the expression for the discounted value of an innovation before the patent expires. There is no uncertainty regarding the profit flow that the monopoly firm can obtain in this period. The second expression on the right-hand side of equation (2.16) indicates the value of the innovation after the patent expires. That is, the discounted profit flow weighted by the probability that the product has not been standardized. m is the standardization rate which will be discussed in next section. The value in the second period can also be determined from the following condition which is similar to the standard no arbitrage condition from Aghion and Howitt (1992):

$$rV_A = \pi_H + \dot{V}_A - mV_A \tag{2.17}$$

where V_A is the value of innovation after the expiration of the patent. The instantaneous profit plus capital gain minus the expected loss of monopoly power from standardization should be equal to the risk free return from lending the value of innovation to the capital market.

Furthermore, we assume the cost of introducing a new product is μ_H. The free entry to innovation drives the net profit of innovation to zero:

$$V_H \leq \mu_H \tag{2.18}$$

The value of a standardization is:

$$V_L = \int_t^{t+T_L} e^{-\int_t^\tau r ds} \pi_M d\tau \tag{2.19}$$

Thus, in steady state, we have:

$$V_L = (1 - e^{-rT_L})\pi_M / r \tag{2.20}$$

The free entry in standardization also drives the net gain from standardization down to zero:

$$V_L \leq \mu_L \tag{2.21}$$

The aggregate resource constraint of the economy is then:

$$\begin{aligned} Y &= C + \dot{N}\mu_H + \dot{N}_L\mu_L + N_H(1-\alpha)^{2/\alpha}A^{1/\alpha}L + \\ & N_M(1-\alpha)^{2/\alpha}\xi^{(\alpha-1)/\alpha}A^{1/\alpha}L + N_C(1-\alpha)^{1/\alpha}\xi^{(\alpha-1)/\alpha}A^{1/\alpha}L \end{aligned} \quad (2.22)$$

The second and the third terms on the right are the investments on innovation and standardization, respectively. The last three terms on the right are the production inputs of final good to the high cost intermediate goods and the low cost intermediate goods.

2.3 Steady State Equilibrium

In the steady state equilibrium, consumption, final output and the number of variety grow at the same rate g, which is also the growth rate of the number of high cost goods and the growth rate of the low cost goods. Thus, $\frac{\dot{N}}{N} = \frac{\dot{N}_H}{N_H} = \frac{\dot{N}_L}{N_L}$. It implies that the share of high cost goods n is a constant. It is straightforward that the share of monopolized low cost goods $\frac{N_M}{N}$ and the share of competitive low cost goods $\frac{N_C}{N}$ are also constant in steady state. Given n, we can determine the steady state $\frac{N_M}{N}$ and $\frac{N_C}{N}$. Next we explain how to determine them. Suppose the number of total low cost goods is N_0 at time 0. At time t, it becomes N_L. Since growth rate is g, we have $N_0e^{gt} = N_L$. Thus, $N_0 = N_Le^{-gt}$. The number of competitive low cost goods is $N_0e^{g(t-T_L)}$ and the number of monopolized low cost goods is $N_L - N_0e^{g(t-T_L)}$. Therefore,

$N_C = N_L e^{-gT_L}$ and $N_M = N_L - N_L e^{-gT_L}$. The total share of low cost goods is $1-n$. Then $\frac{N_M}{N} = (1-n)(1-e^{-gT_L})$ and $\frac{N_C}{N} = (1-n)e^{-gT_L}$.

In the steady state equilibrium, the free entry conditions ensure that $V_H = \mu_H$ and $V_L = \mu_L$. Since μ_H and μ_L are constant, V_H and V_L are also constant. It then implies that $\dot{V}_H = 0$ and $\dot{V}_L = 0$. Thus, the value of an innovation is

$$V_H = \frac{(1-e^{-rT_H})\pi_H}{r} + \frac{e^{-rT_H}\pi_H}{r+m} \tag{2.23}$$

The standardization rate is defined as $m = \frac{\dot{N}_L}{Ne^{-gT_H}-N_L}$, where $Ne^{-gT_H} - N_L$ is the number of high cost goods whose patents have expired. Since $\frac{\dot{N}}{N} = \frac{\dot{N}_H}{N_H}\frac{\dot{N}_H}{N} + \frac{\dot{N}_L}{Ne^{-gT_H}-N_L}\frac{Ne^{-gT_H}-N_L}{N}$, together with the definition of m, we can solve for n in terms of g and m

$$n = 1 - \frac{m}{m+g}e^{-gT_H} \tag{2.24}$$

The free entry condition of standardization determines the interest rate

$$\frac{1-e^{-rT_L}}{r}\pi_M = \mu_L \tag{2.25}$$

To make sure the existence of a solution, we just need a sufficient condition on parameters:

$$T_L > -\frac{1}{\rho}ln(1-\frac{\mu_L}{\pi_M}\rho) \tag{2.26}$$

The above condition ensures that with positive growth rate, standardization is profitable, thus there is positive standardization in the equilibrium.

Then from the free entry condition of innovation, we can solve for the standardization rate m:

$$(\frac{1-e^{-rT_H}}{r}+\frac{e^{-rT_H}}{r+m})\pi_H=\mu_H \tag{2.27}$$

The standardization rate is then

$$m=\frac{\pi_H-r\mu_H}{\mu_H-(1-e^{-rT_H})\pi_H/r} \tag{2.28}$$

When $\frac{(1-e^{-rT_H})}{r}\pi_H<\mu_H<\frac{\pi_H}{r}$, we can have $m>0$. That is, there are both positive innovation and standardization activities at equilibrium. However, this condition may not always be satisfied at equilibrium. Different equilibria may arise due to the violation of this condition as a result of different policy choices (change of T_H and T_L). The following Figure (2.1) depicts the possible equilibria. In essence, there could be three regimes, which will be discussed in detail.

Proposition 1: *The equilibria can be classified into three regimes and the economy may fall into any of the three different regimes at steady state:*

Regime 1: When $\frac{(1-e^{-rT_H})}{r}\pi_H<\mu_H<\frac{\pi_H}{r}$*, both innovation and standardization exist.*

Regime 2: When $\mu_H\leq\frac{1-e^{-rT_H}}{r}\pi_H$*, there will be no standardization at equilibrium.*

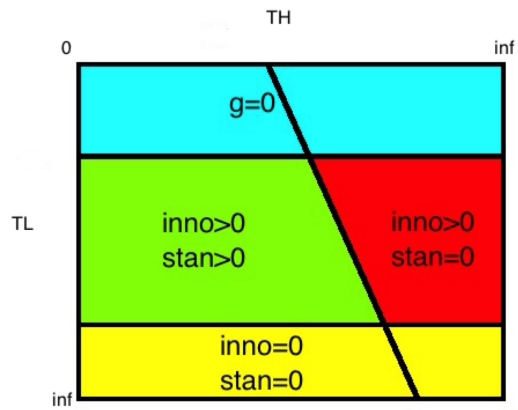

Figure 2.1: Possible Equilibria

Regime 3: When $\mu_H \geq \frac{\pi_H}{r}$*, there will be no innovation and economic growth. The economy stagnates.*

Proof. *From the expression of* m*, the standardization rate is:*

$$m = \frac{\pi_H - r\mu_H}{\mu_H - (1 - e^{-rT_H})\pi_H / r}$$

That means when $\frac{(1-e^{-rT_H})}{r}\pi_H < \mu_H < \frac{\pi_H}{r}$*, we can have* $m > 0$*. Thus, for any given combination of* T_H *and* T_L*, there is an equalized return to R&D investment to both innovation and standardization.*

If $\mu_H \leq \frac{1-e^{-rT_H}}{r}\pi_H$*, the first part of the inequality is violated.* $v \equiv \frac{1-e^{-rT_H}}{r}\pi_H$ *is the value of innovation while* $m \to \infty$*. When* v *is always larger than* μ_H*, the innovation value is always higher than its cost. All the R&D investment will be reallocated to innovation. Thus, no one will do standardization at equilibrium.*

If $\mu_H \geq \frac{\pi_H}{r}$, the value of an innovation $v \equiv \frac{\pi_H}{r}$ is always lower than its cost μ_H. No resource will be allocated to innovation. In equilibrium, there will be no innovation and nothing available for standardization. Q.E.D.

The three conditions above can be rewritten as the boundary conditions for T_H and T_L. We will discuss these conditions in the following subsections. Basically, the condition for Regime 2 requires T_H to be large enough or T_L to be small enough. The condition for Regime 3 requires T_L to be large enough. Regime 3 may not always exist. When $\pi_M/\mu_L < \pi_H/\mu_H$, there will be both innovation and standardization even if $T_L \to \infty$. We will elaborate it in the following subsections.

One concern we want to discuss here is the possibility that the innovator himself has the incentive to do the secondary innovation targeting on its own product. By doing this, the innovator can obtain higher profit ($\pi_M > \pi_H$) even before the patent for the innovation expires. If so, there is no chance for other standardizers to do standardization. The model then degenerates to the conventional growth model with one type of innovation. If the innovator does standardization during the patent length T_H, he would do it immediately after the innovation of the new product to maximize profit. The value of innovation with immediate standardization is now

$$V_H^N = \int_t^{t+T_H+T_L} e^{-\int_t^\tau r ds} \pi_M d\tau \tag{2.29}$$

In the steady state, $V_H^N = \frac{\pi_M}{r}[1 - e^{-r(T_H+T_L)}]$. If the value of innovation with standardization is smaller than the summation of the cost of innovation and the cost of standardization, the innovators have no incentive to do standardization. That requires the following condition:

$$\frac{\pi_M}{r}[1 - e^{-r(T_H+T_L)}] < \mu_H + \mu_L \tag{2.30}$$

From the next section, we will see that with positive m, there is an upper bound for T_H in terms of T_L: $T_H < T_H^* = -\frac{1}{r(T_L)} ln\big(1 - \frac{\mu_H}{\pi_H} r(T_L)\big)$. Substitute the expression of π_M into the above inequality (2.30). Together with the fact that $r > \rho$, the condition becomes

$$T_L > \frac{1}{\rho} ln\Big(\frac{\mu_H}{\mu_H + \mu_L} \frac{\pi_M}{\pi_H}\Big) \tag{2.31}$$

Thus, to exclude the uninteresting case when innovators do standardization, it is sufficient to expose the following condition:

$$\xi^{(\alpha-1)/\alpha} < \frac{\mu_H + \mu_L}{\mu_H} \tag{2.32}$$

This condition requires that the cost reduction should not be too large and thus the profit of low cost production is not too large compare with that of high cost production. This is a very weak condition that can be easily met. With this condition, the innovators have no incentive to do standardization targeting on their own product for all positive patent length for standardization.

To transform the system into a stationary one, we define $c \equiv \frac{C}{N}$, $y \equiv \frac{Y}{N}$, $\omega \equiv \frac{w}{N}$. The stationary final output can be rewritten as:

$$y = (1-\alpha)^{2(1-\alpha)/\alpha} A^{1/\alpha} L \Big\{ n + (1-n)(\frac{1}{\xi})^{(1-\alpha)/\alpha} \Big[(1-e^{-gT_L}) + e^{-gT_L}(1-\alpha)^{(\alpha-1)/\alpha} \Big] \Big\} \tag{2.33}$$

According to the aggregate resource constraint (2.22), we can write c as:

$$\begin{aligned} c &= y - \mu_H g - \mu_L(1-n)g - n(1-\alpha)^{2/\alpha} A^{1/\alpha} L - (1-n) \\ &\quad \xi^{(\alpha-1)/\alpha}(1-\alpha)^{1/\alpha} A^{1/\alpha} L \Big[(1-e^{-gT_L})(1-\alpha)^{1/\alpha} + e^{-gT_L} \Big] \end{aligned} \tag{2.34}$$

Define the social welfare Γ as the lifetime utility of the representative household. We assume the economy starts from the steady state and the social welfare function[2] will be

$$\Gamma = max \int_0^\infty e^{-\rho t} \frac{C_t^{1-\gamma} - 1}{1-\gamma} dt = \frac{c^{1-\gamma}}{1-\gamma} \frac{1}{\rho - g(1-\gamma)} - \frac{1}{\rho(1-\gamma)} \tag{2.35}$$

Next, we discuss the welfare and growth effects of patent protection in the three regimes respectively.

2.3.1 Equilibrium With Both Innovation And Standardization

When $\frac{(1-e^{-rT_H})}{r}\pi_H < \mu_H < \frac{\pi_H}{r}$, the standardization rate is positive. This condition says that the innovation cost is lower than the value of innovation

[2]When $\gamma = 1$, the utility function becomes log utility. Therefore, the social welfare becomes: $\Gamma = \frac{1}{\rho} ln(c) + \frac{g}{\rho^2}$.

with no standardization, while the innovation cost is larger than the value of innovation with infinite standardization rate. Therefore, there is always a positive standardization rate that equalizes innovation cost and innovation benefit.

For a given patent length for standardization, there is an upper bound for patent length for innovation T_H^*, beyond which the model degenerates to the standard variety expansion model with only innovation. T_H^* is determined as follows:

$$\frac{1 - e^{-r(T_L)T_H^*}}{r(T_L)} = \frac{\mu_H}{\pi_H} \tag{2.36}$$

where $r(T_L)$ is the solution to equation (2.25) given T_L. Thus,

$$T_H^* = -\frac{1}{r(T_L)} ln\Big(1 - \frac{\mu_H}{\pi_H} r(T_L)\Big) \tag{2.37}$$

From this equation, we can see that the upper bound of T_H is a function of T_L. When $T_H = T_H^*$, the standardization rate goes to infinity. A high cost good will be standardized immediately after its patent expires.

In summary, the conditions for the regime that both innovation and standardization to be present at equilibrium are:

$$-\frac{1}{\rho} ln(1 - \frac{\mu_L}{\pi_M}\rho) < T_L < -\frac{\mu_H}{\pi_H} ln\Big(1 - \frac{\mu_L \pi_H}{\mu_H \pi_M}\Big) \tag{2.38}$$

$$T_H < -\frac{1}{r(T_L)} ln\Big(1 - \frac{\mu_H}{\pi_H} r(T_L)\Big) \tag{2.39}$$

Next, we analyze the growth effects of the patent durations. According to equation (2.25), longer patent length for standardization enhances the return on standardization. Thus r is increasing in T_L. The Euler equation (2.3) from household maximization shows that g is increasing in r. Thus, within this regime, longer T_L increases growth rate g. Furthermore, according to the Euler equation, g is independent of T_H. It is interesting that the return on standardization is the sole determinant of the growth rate of the economy. The intuition is that the return on investments across activities should be equal at steady state. Thus, if the return on innovation and the return on standardization are different, there will be a reallocation of resources across R&D activities. For example, if the return on standardization is larger than the return on innovation, R&D investment will be reallocated to standardization. The return on innovation will then increase as there will be less competition in innovation.

We summarize the above results in the following proposition:

Proposition 2: *Within the steady state regime with both innovation and standardization, growth rate is increasing in the patent length for standardization, while independent of the patent length for innovation.*

From the expression of welfare (equation (3.46)), we can see that the social welfare is determined by two factors: growth rate and the steady state consumption. The welfare effect of T_H depends solely on its effect on steady state consumption.

Proposition 3: *For a given patent length for standardization, social welfare is increasing in patent length for innovation if* $\gamma \geq 1$ *and* $\xi^{(\alpha-1)/\alpha} > 2 - \alpha$.

Proof. *With patent length for standardization* T_L *fixed, the interest rate and the growth rate of the economy will be fixed. Thus, the variation of* T_H *can only affect welfare through its effect on consumption. Then* $\frac{dc}{dT_H} = \frac{dc}{dn}\frac{dn}{dT_H}$. $\frac{dn}{dT_H} = \frac{\partial n}{\partial m}\frac{\partial m}{\partial T_H} + \frac{\partial n}{\partial T_H}$. *According to equation (2.44),* $\frac{\partial m}{\partial T_H} = m(r+m)$. *And according to equation (2.24),* $\frac{\partial n}{\partial m} = \frac{-ge^{-gT_H}}{(m+g)^2}$ *and* $\frac{\partial n}{\partial T_H} = \frac{mg}{m+g}e^{-gT_H}$. *Thus, we can have* $\frac{dn}{dT_H} = \frac{mg}{m+g}e^{-gT_H} - \frac{mg(m+r)e^{-gT_H}}{(m+g)^2} = \frac{mge^{-gT_H}}{(m+g)^2}(g-r) < 0$.

From (2.22), we can see that c *is a linear function of* n. *Thus,* c *is monotonic in* T_H. *We can see that:*

$$\begin{aligned}\frac{dc}{dn} &= (1-\alpha)^{1/\alpha}\xi^{(\alpha-1)/\alpha}A^{1/\alpha}L\Big[(1-e^{-gT_L})(1-\alpha)^{1/\alpha}+e^{-gT_L}\Big]\\ &\quad -(1-\alpha)^{2/\alpha}A^{1/\alpha}L+(1-\alpha)^{2(1-\alpha)/\alpha}A^{1/\alpha}L\\ &\quad \Big\{1-(\frac{1}{\xi})^{(1-\alpha)/\alpha}\Big[1-e^{-gT_L}+e^{-gT_L}(1-\alpha)^{(\alpha-1)/\alpha}\Big]\Big\}+\mu_L g\end{aligned}$$

Then we can rewrite it as:

$$\begin{aligned}\frac{dc}{dn} &= \mu_L g+(1-\alpha)^{2/\alpha}A^{1/\alpha}L\Big[\frac{1}{(1-\alpha)^2}-1\Big]+(1-\alpha)^{2/\alpha}A^{1/\alpha}L\xi^{(\alpha-1)/\alpha}\\ &\quad [1-\frac{1}{(1-\alpha)^2}](1-e^{-gT_L})+(1-\alpha)^{1/\alpha}A^{1/\alpha}L\xi^{(\alpha-1)/\alpha}\Big[1-\frac{1}{1-\alpha}\Big]e^{-gT_L}\end{aligned}$$

The summation of the last two terms lies between $(1-\alpha)^{1/\alpha}A^{1/\alpha}L\xi^{(\alpha-1)/\alpha}[1-1/(1-\alpha)]$ *and* $(1-\alpha)^{2/\alpha}A^{1/\alpha}L\xi^{(\alpha-1)/\alpha}[1-1/(1-\alpha)^2]$*. Since* $(2-\alpha)(1-\alpha)^{(1-\alpha)/\alpha} < 1$*, the summation of the last two terms is smaller than* $(1-\alpha)^{2/\alpha}A^{1/\alpha}L\xi^{(\alpha-1)/\alpha}\big[1-1/(1-\alpha)^2\big]$*. Thus,*

$$
\begin{aligned}
\frac{dc}{dn} &< \mu_L g + (1-\alpha)^{2/\alpha}A^{1/\alpha}L\Big[\frac{1}{(1-\alpha)^2}-1\Big] \\
&\quad + (1-\alpha)^{2/\alpha}A^{1/\alpha}L\xi^{(\alpha-1)/\alpha}\Big[1-\frac{1}{(1-\alpha)^2}\Big] \\
&= \mu_L g + (1-\alpha)^{2/\alpha}A^{1/\alpha}L(1-\xi^{(\alpha-1)/\alpha})\Big[\frac{1}{(1-\alpha)^2}-1\Big]
\end{aligned}
$$

Since $\mu_L = \frac{1-e^{-rT_L}}{r}\pi_M$, $\mu_L g < \pi_M$ *if* $\gamma \geq 1$*. Thus,* $\frac{dc}{dn} < \pi_M + (1-\alpha)^{2/\alpha}A^{1/\alpha}L\big(1-\xi^{(\alpha-1)/\alpha}\big)\big[\frac{1}{(1-\alpha)^2}-1\big]$*. Plugging in the expression of* π_M*, we find that* $\frac{dc}{dn} < \frac{2-\alpha-\xi^{(\alpha-1)/\alpha}}{1-\alpha}\pi_H$*. Therefore, if the given condition is satisfied,* $\frac{dc}{dn} < 0$*. The welfare is then increasing in* T_H*. Q.E.D.*

The condition $\xi^{(\alpha-1)/\alpha} > 2-\alpha$ confines the marginal cost of low cost production with an upper bound. Thus, this condition requires the profit for low cost production to be large enough.

With a longer patent protection for innovation, more high cost products are under patent protection. Longer patent protection will increase the return to innovation. However, because of free entry to R&D activities, the resource reallocation will equalize the returns to the two types of R&D activities. Thus, the increase of standardization rate m and the decrease in the share of high cost products n will exactly offset the positive effect of longer patent on the incentive to innovate. Therefore, higher T_H always leads to lower n.

A lower fraction of high cost products n has two countervailing effects on consumption. On the one hand, since low cost products are produced with lower cost, more final output y will be produced with lower n. Moreover, lower n means lower R&D investment on innovation and lower production input in high cost goods. They all have positive effects on consumption. On the other hand, lower n means higher R&D investment and production input to low cost productions, which have negative effects on consumption. Proposition 3 shows that the positive effect of lowering n on consumption always dominates as long as the profit for high cost goods production is large enough. That is because a lower n can release more production ability to final good production when the profit gap between high cost production and low cost production is large enough.

Proposition 4: *With longer patent length for standardization, the optimal patent length for innovation that maximizes social welfare is also longer.*

Proof. *According to Proposition 2, the optimal patent length for innovation is the largest possible T_H that leads to both innovation and standardization at equilibrium. Denote it by T_H^*. T_H^* is the solution to the equation (2.44) while $m = \infty$. That is $T_H^* = -\frac{1}{r}ln\left(1 - \frac{\mu_H}{\pi_H}r\right)$. Here, r is determined by the free* *entry condition in standardization. Thus r is an increasing function of T_L. Since T_H^* is increasing in r, it is straightforward that T_H^* is increasing in T_L. Q.E.D.*

When $T_H > T_H^*$, there will be a regime switch in the economy. To find the optimal T_H in the whole range, we also need to consider the welfare across different regimes. We will analyze the optimal T_H across regimes in the numerical section.

The effect of patent protection for standardization on welfare works through both directions. On the one hand, longer patent protection for standardization leads to higher growth rate. The reason is that the return on standardization increases in T_L, while the return on standardization and the return on innovation will be equalized at steady state. Higher return on R&D attracts more investment to R&D, which leads to higher growth rate. On the other hand, however, more investment to R&D sector takes up the final good that was originally allocated to consumption. Moreover, longer protection for standardization leads to higher fraction of high-tech goods. This leads to lower aggregate production of final good and thus decreases equilibrium consumption. The net effect of higher T_L on welfare depends on the relative magnitude of the two forces.

2.3.2 Equilibrium With Only Innovation

As discussed in Proposition 1, there will be no standardization when $\mu_H \leq \frac{1-e^{-rT_H}}{r}\pi_H$. This condition says that the value of an innovation is greater than innovation cost even if the standardization rate is infinite. Thus, the return from doing innovation is always higher than doing standardization. Therefore, there is only innovation at equilibrium.

From the above section, we can see that when T_H increases such that $T_H > T_H^*$, the economy encounters a regime switch which eliminates standardization. This condition can also be rewritten as $T_L < T_L^*$ where T_L^* is a function of T_H.

This regime, however, is different from the model we setup previously. Because there is no standardization, it is impossible to find a consistent setup for both Regime 1 and Regime 2 after the patent for innovation expires. In this paper, we adopt two relatively consistent assumptions for Regime 2. (1) When patent for innovation expires, the monopolistic firm continue to produce with high cost forever and no one is able to imitate; (2) When patent for innovation expires, the corresponding product is immediately imitated and been produced competitively. Generally speaking, the first assumption is a special case of the second assumption when patent for innovation goes to infinity.

With the first assumption, the equilibrium is independent of patent policies. Firstly, there is no standardization. Thus T_L does not influence any variable of interest. Secondly, since the monopolistic firm can produce forever, T_H is equivalent to ∞.

With the second assumption, the growth rate is now determined by T_H, while independent of T_L. The final output is then:

$$Y = AL^{\alpha}\Big[\int_0^{N_H} x_{Hi}^{1-\alpha} di + \int_0^{N_C} x_{Ci}^{1-\alpha} di\Big] \tag{2.40}$$

Different from previous setup, x_{Ci} is competitively produced with high cost here. Thus,

$$y = (1-\alpha)^{(1-\alpha)/\alpha} A^{1/\alpha} L\Big[(1-e^{-gT_H})(1-\alpha)^{(1-\alpha)/\alpha} + e^{-gT_H}\Big] \tag{2.41}$$

The aggregate resource constraint is also different:

$$Y = C + \dot{N}\mu_H + N_H(1-\alpha)^{2/\alpha} A^{1/\alpha} L + N_C(1-\alpha)^{1/\alpha} A^{1/\alpha} L \tag{2.42}$$

Thus, the expression for stationary consumption becomes:

$$c = y - \mu_H g - (1-e^{-gT_H})(1-\alpha)^{2/\alpha} A^{1/\alpha} L - e^{-gT_H}(1-\alpha)^{1/\alpha} A^{1/\alpha} L \tag{2.43}$$

The interest rate and therefore the growth rate is determined by:

$$\frac{1-e^{-rT_H}}{r}\pi_H = \mu_H \tag{2.44}$$

From the above equation, we can see that the growth rate is increasing in T_H. This is consistent with the literature such as Zeng, Zhang and Fung (2014).

Stronger patent protection enhances incentive to innovate and thus drives up economic growth. This is the dynamic gain from stronger patent protection.

The effect of patent length T_H on welfare is still determined by both growth rate and consumption. From the expression for c, we can see that c is decreasing in T_H. This is the static loss from stronger patent protection. The effect of T_H on social welfare then depends on the relative magnitude of the dynamic gain and static loss. Thus, according to Zeng, Zhang and Fung (2014), there is a finite patent length that maximizes social welfare.[3]

2.3.3 Equilibrium With No Economic Growth

When $\mu_H \geq \frac{\pi_H}{r}$, there will be no economic growth at equilibrium, because doing standardization is always more profitable than doing innovation. This condition is equivalent to $T_L \geq -\frac{\mu_H}{\pi_H} ln\left(1 - \frac{\mu_L \pi_H}{\mu_H \pi_M}\right)$. The resulting equilibrium implies that nobody wants to do innovation. Hence, there will be no available high cost goods for standardization finally.

At first look, this seems counter-intuitive since IPR protection is always considered as an important reason for economic growth. However, here we argue that in some cases, too much IPR protection can also lead to poverty trap and hinders economic growth in some countries. When patent protection

[3] One remark here is that the conventional optimal patent length may fall in Regime 1, instead of Regime 2. When that happens, the optimal patent length will be the lower bound for T_H in Regime 2 which is also the upper bound of T_H for Regime 1.

for standardization increases, the growth rate always increases and generally, social welfare will also increase. Thus, the government has higher incentive to enforce stronger protection for standardization. When the patent protection is strong enough such that the return to standardization is always higher than the return to innovation, the R&D investment on high cost goods will be transfered to standardize available high cost goods. With the available high cost goods for standardizing diminished, the economy stagnates.

Generally speaking, this poverty trap is a result of higher return to standardization which makes equalization of returns across different R&D activities not possible. Even if all the R&D investment goes to standardization, the return to standardization is still higher than the return to innovation. Profit-seeking investment will never flow to innovation which results in zero growth. To rule out the possible poverty trap resulted from patent protection for standardization, the government should limit the patent protection for standardization, according to the comparison between the return to innovation and the return to standardization.

However, depending on parameters, this regime may not always exist. When $\pi_M/\mu_L < \pi_H/\mu_H$, there will be both innovation and standardization even if $T_L \to \infty$. In this case, doing standardization is not always more profitable than doing innovation. Therefore, there is no poverty trap even as the

Parameter	ρ	γ	A	α	L	ξ	T_H	T_L	μ_H	μ_L
Value	0.03	2	1	0.4	1000	0.7	10	10	450	265

Table 2.1: Benchmark Parameters

Parameter	g	r	n	m	c	y	ω	Γ
Value	0.0575	0.1549	0.4414	7.4287	188.8	434.9	0.1740	24.946

Table 2.2: Benchmark Equilibrium

patent length goes to infinity.

2.4 Numerical Analysis

2.4.1 Benchmark Model

In the benchmark model, we choose a discount factor $\rho = 0.04$ and the elasticity of marginal utility $\gamma = 2$ to get a growth rate of $g = 0.0575$. We assume the total population to be 1000 and the labor intensity in the final good production $\alpha = 0.4$. The patent length of both innovation and standardization are assumed to be 10 years. We summarize the values of the parameters in Table (2.1)

The corresponding equilibrium at steady state are given in Table (2.2).

2.4.2 Optimal Patent Length

To find the optimal patent length combination of patent length for innovation and patent length for standardization, we allow T_H and T_L to vary from 0 to 40. In this section, we only consider the case when $g > 0$. In the first step, we consider the case when both innovation and standardization are present. Then we extend our analysis to incorporate the case when innovation is always more profitable than standardization.

Result with Both Innovation and Standardization

When both innovation and standardization present, the relationship between social welfare and the two patent lengths is shown in Figure (2.2).

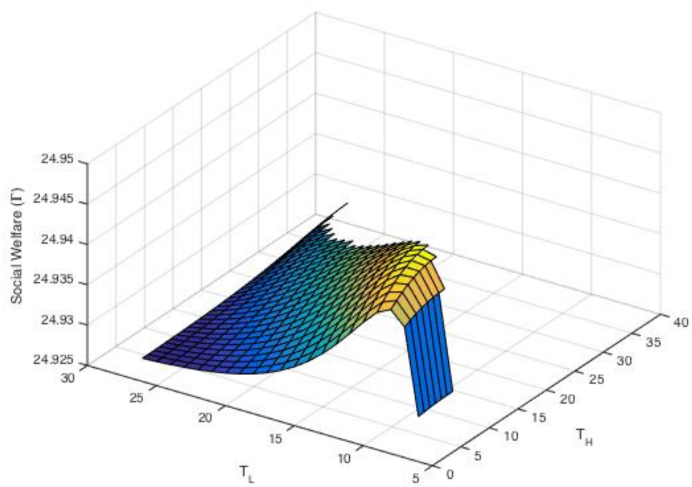

Figure 2.2: Two types of patent length and Welfare

When patent length for innovation is too high, the return to innovation is always higher than standardization. Thus, there will be no standardization in the economy and the model shrinks to the standard variety expansion model. For now, we only consider the case when both innovation and standardization are present.

In that case, we can have the following results:

Result 1: *For a given patent length for standardization, social welfare is increasing in patent length for innovation* T_H*.*

Result 2: *Social welfare has an inverse-U shape relationship with patent length for standardization. Therefore, for a given patent length for innovation, there is a finite patent length for standardization which maximizes social welfare.*

Result 3: *With longer patent length for standardization, the upper bound for* T_H *is also increasing.*

Result 1 is consistent with Proposition 3. As discussed in Section 3, longer patent length for innovation has countervailing effects on welfare. The positive effect is that longer T_H leads to lower fraction of high cost goods which expands final output and reduces R&D investment to innovation. The final good that is allocated to consumption then increases. The negative effect is that longer T_H leads to higher R&D investment to standardization and higher production input to low cost production. These factors both lead to lower consumption. According to Proposition 3, the positive effect always dominates.

For a given patent duration for innovation T_H, patent duration for standardization T_L also has countervailing effects on welfare. Longer patent length for standardization increases the return on standardization. Because of free entry to innovation and standardization, the returns to the two types of R&D

investments will be equalized. Thus, larger T_L implies larger growth rate g. The static effects of T_L on consumption are inverse U-shaped. When growth rate effect dominates, longer patent protection for standardization increases welfare. When static effect dominates, there will be an optimal patent length for standardization.

Result 3 reinforces the conclusion in Proposition 3. With longer patent duration for standardization, the return to standardization investments increases. To have higher return on innovation than standardization, stronger patent protection on innovation is required. That is why the upper bound on T_H is increasing in T_L.

Result with no Regime 3

Since there is no Regime 3 in steady state if $\pi_M/\mu_L < \pi_H/\mu_H$, we show the result with no Regime 3 in this section. We can just change the parameter $\mu_L = 470$ to make the condition satisfied. The following Figure (2.3)[4] shows the result:

Result 1 to 3 from the above section still hold. In this case when $T_L = \infty$, there are still positive innovation and positive standardization. This allows the government to choose an infinite patent length for standardization without generating poverty trap. Infinite patent protection for standardization

[4] To show the result for $T_L = \infty$, we draw the result of $T_L = \infty$ in the last line on the figure where $T_L = 41$.

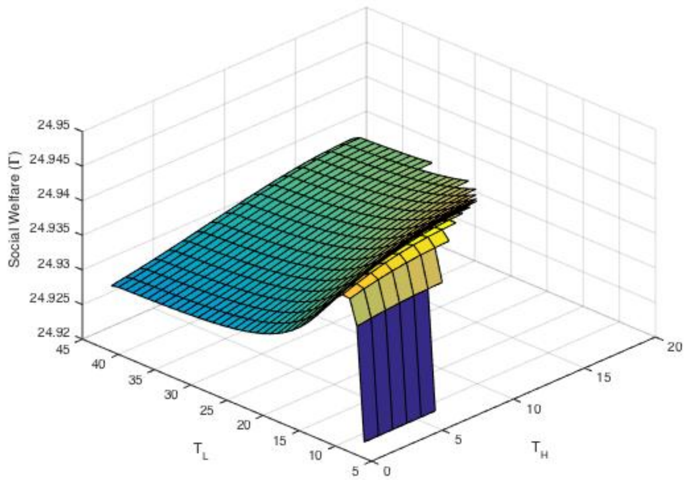

Figure 2.3: Patent Length and Social Welfare with no Regime 3

maximizes growth rate. However, the optimal patent length T_L to maximize welfare is still finite.

Result 4: *For a given patent length for innovation, infinite patent length for standardization maximizes growth rate, while the patent length for standardization maximizes social welfare is still finite.*

Uniform Patent Length

In practice, it is difficult to distinguish innovation and standardization because they are both innovations of the common definition. Therefore, it is hard to adopt different patent lengths for innovation and standardization. In this section, we assume the government adopt uniform patent length for both innovation and standardization. Figure (2.4) shows the result:

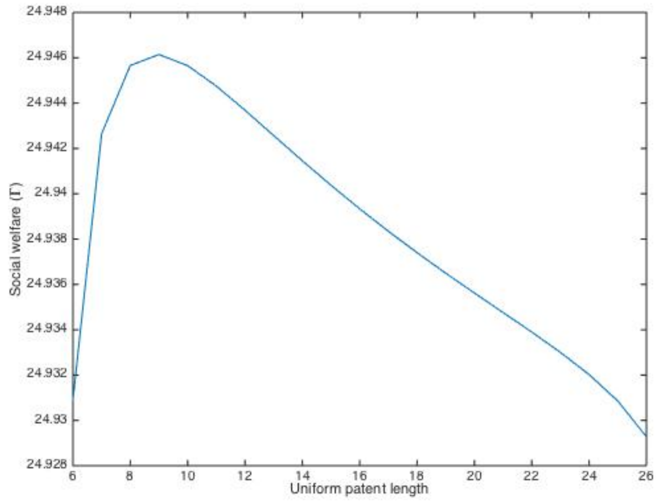

Figure 2.4: Uniform Patent Length and Social Welfare

The effect of uniform patent length on social welfare is inversed-U shape. The optimal patent length is finite.

Result with Regime 1 and Regime 2

From above, we know that if patent length for innovation is too long or patent for standardization is too short, the model shrinks to the standard variety expansion model. We then need to extend our analysis to compare the optimal patent length for innovation under both situations. According to the discussion in Section 3, we adopt two different assumptions in Regime 2. Under the first assumption, the welfare in Regime 2 is constant with respect to T_H. That is because the assumption is equivalent to infinite patent as long as $T_H > T_H^*$. Since the first assumption is a special case of the second assumption, we only show the result with the second assumption. Under the second assumption, the product is imitated immediately after the patent expires in Regime 2. That is similar to saying that the standardization rate is infinite, in some sense. The difference is that the production cost does not decrease after the change of producer in Regime 2.

The following Figure (2.5) shows the result with both Regime 1 and Regime 2.

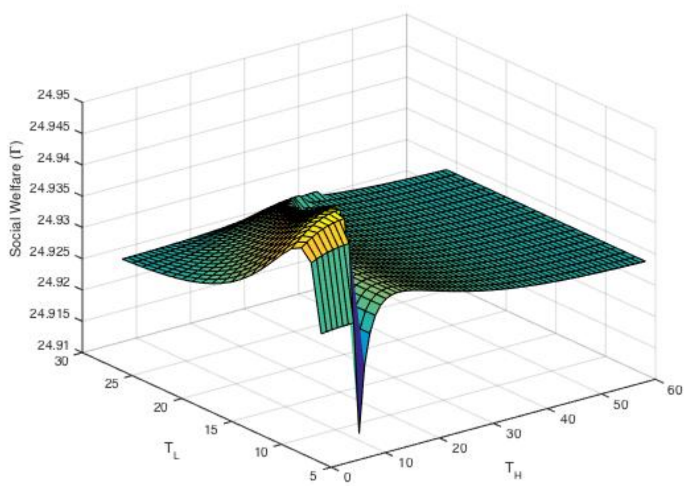

Figure 2.5: Two types of patent length and Welfare with degenerated model

If T_H is larger than T_H^*, the economy switches to Regime 2 and there is no standardization at equilibrium. In Regime 2, ignoring the boundary problem, welfare and growth rate are both independent of T_L. According to Section 3, growth rate is increasing in T_H and social welfare is an inverse U shaped function of T_H. Considering the boundary, welfare is a decreasing function of T_H while T_L is large. (The increasing part falls in Regime 1.) We then consider both Regime 1 and Regime 2. Growth rate is initially independent of T_H and then increasing in T_H after the boundary. Social welfare is initially increasing in T_H and then it has a sudden drop while switching to Regime 2. Maximum social welfare in Regime 1 is larger than in Regime 2. This is the result of the output expansion effect of standardization. The maximum of social welfare is obtained on the boundary of Regime 1 before switching to Regime 2. We can see clearly the effect of T_H on welfare in the following figure (2.6)

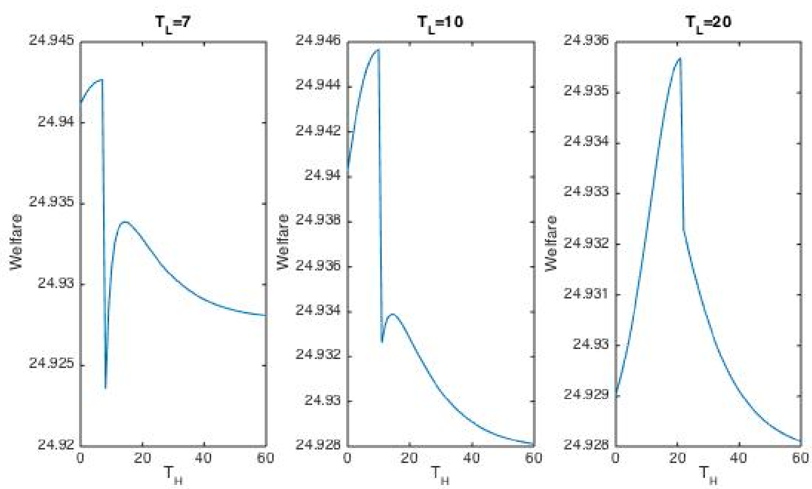

Figure 2.6: Patent length for innovation and welfare

The following result summarizes the case with both Regime 1 and Regime 2:

Result 5: *For a given T_L, the optimal T_H maximizing welfare is obtained* *in the boundary of Regime 1 before switching to Regime 2; for a given T_H, the optimal T_L is obtained either with a finite value in Regime 1, or on the boundary of Regime 1.*

To obtain maximum social welfare, the optimal combination of patent length for innovation and patent length for standardization is one with finite patent length for innovation and finite patent length for standardization. Longer patent protection for innovation increases social welfare. However, a caveat for the policy maker is that there is an upper bound for increasing patent protection for innovation. If the patent protection for innovation is too long, the economy can not enjoy the welfare gain from output enhancing effect of standardization. Finite patent length for standardization comes from both the possible poverty trap and too much dead weight loss from monopoly power.

2.5 Conclusion

This chapter investigates the effects of patent length on social welfare and economic growth in an endogenous growth model with two types of innovations which we call innovation and standardization. Innovation introduces new variety and standardization lowers the production cost of the high cost products. Therefore, we analyze both the effects of patent lengths for innovation and that for standardization. Different policies may lead the equilibrium to three different regimes: regime with both innovation and standardization; regime with only innovation; and regime with no economic growth.

We show that if patent length for standardization is too long, the economy may fall in a poverty trap. When both innovation and standardization are positive, growth rate is increasing in patent length for standardization while independent of patent length for innovation. Given patent for standardization, welfare is increasing in patent length for innovation. Given patent length for innovation, welfare is an inverse U shape function of patent length for standardization. The optimal combination of patent lengths for innovation and standardization is one with both finite patent for innovation and finite patent for standardization.

Chapter 3

Distance to Frontier, Welfare and Economic Growth with Fiscal Policies

3.1 Introduction

Distance to frontier is the technological proximity of an economy to the leading economy, and can be measured by the ratio between the total factor productivity in the economy and that of the leading country. Many studies, such as Barro and Sala-i-Martin (1992), Mankiw, Romer, and Weil (1992) and Evans (1996), show that a large group of countries have been converging to parallel growth paths over the past 50 years. However, although the long run growth rate converges, the long-run technology gap spreads. This is the so-called "conditional convergence".

In the developing countries, shortening its long-run technological distance to the frontier countries is sometimes as important as the desire to promote economic growth. If so, careful consideration should be made before the implementation of any fiscal policy, since it might not be possible to "kill two birds with one stone". Our main purpose in this paper is to analyze the effects of capital income tax and labor income tax on the developing countries' long run technology distances to frontier as well as on the welfare and growth rates of the economies.

It may not be apparent that shortening the technological distance to frontier economy can be adopted by a government as a policy target. However, in reality, it is a very important goal of government in developing countries such as China to increase the technology level of the economy, and thus shorten the distance to the frontier economy. A piece of evidence is the "catching-up strategy" adopted by many countries. China adopted this strategy in the 1950s. During that time, economic growth rate was not the primary goal of economic policy while catching up the leading counties was more important. There was a famous slogan in China: "To catch up the British in three years, and leapfrog the American in ten". Even nowadays, technological progress in some specific area is still one of the key performance index and a criteria for promotion of Chinese government officials.

In this chapter, we explore the optimal tax mix within a quality improving model of growth framework. We assume that technology progression is obtained from two channels: technology adoption from the frontier economy which is often in forms of imitation; and purposeful innovation targeted at the local aggregate technology level. Both innovation and imitation requires two types of R&D inputs: physical capital and labor. The taxation choice of the government to finance a fixed amount of government expenditure then will have different effects on the innovation and imitation incentives, and thus, on the long run technology distance to frontier, welfare and growth rate of the economy. Thus, the optimal strategy for the government requires balancing these three targets.

We show that capital income tax financing leads to a higher steady state welfare at the steady state. Steady state welfare is determined by consumption, leisure, and growth rate. In the long run, growth rate converges to the frontier technology growth rate, which is independent of the economy itself. Thus, the negative growth rate effect of capital income tax is removed here. A higher capital income tax always leads to lower incentive to accumulate, thus the steady state consumption is always higher. The ratio of capital input and labor input to R&D sector keeps constant in steady state. To match a lower level of capital input to R&D, the required labor input is also lower. Thus, a higher capital income tax results in a higher leisure level. Both higher consumption and higher leisure determine a higher steady state welfare.

Although higher capital income tax leads to higher welfare, it also results in a longer distance to technology frontier, i.e. a lower relative technology level. Higher capital income tax discourages accumulative input to R&D by reducing capital accumulation. It then results in longer distance to technology frontier in the long run.

On the transitional path to the steady state, starting from lower than steady state capital stock and relative technology level, higher capital income tax always leads to lower initial growth rate. Initial growth rate is taken as the current growth rate. Current growth rate is determined both by current distance to frontier and current R&D inputs including capital stock and labor supply. Given the level of the relative technology level, higher capital income tax discourages capital accumulation and thus lowers the growth rate.

Moreover, by analyzing the transitional path, we can also examine the effects of taxation on total welfare including the welfare on the transitional path. Our result shows that it is different from the effect on steady state welfare. Higher capital income tax increases total welfare only when capital income tax is small. However, when capital income tax is high, a further increase capital income tax will decrease the total welfare. The reason is that the negative effect of capital income tax on total welfare through lowering capital stock and growth rate dominates the positive effect through higher consumption and leisure.

This chapter is related to several strands of literature. The first strand is the literature on distance to frontier and global convergence. Technology transfer and cross-country convergence is one of the most studied topics in growth literature. Neoclassical literature pioneered by Solow (1956) usually explains convergence as a result of decreasing returns to capital. [For example, Mankiw, Romer and Weil (1992), Barro and Sala-i-Martin (1991).] Endogenous growth literature, however, explains convergence from the view of international knowledge spillover. [E.g. Barro and Sala-i-Martin (1997), Aghion and Howitt (1998), Zeira (1998),Howitt (2000), Howitt and Mayer (2002), Aghion, Howitt and Mayer (2005).] Barro and Sala-i-Martin (1997) explore the convergence implication of the endogenous growth model. In their model, the leading countries do innovation while the followers do imitation. In the long run, world wide growth rate is determined by leading economy. Our model is consistent with theirs, but with the difference that the followers do engage in both innovation and imitation activities. There is also a large literature on club convergence explaining the fact that the growth rates of a group of countries are converging to a same rate while others stagnate, such as in Howitt (2000) and Aghion, Howitt and Mayer (2005). Our focus in this chapter is on the group of countries that are converging and we are to examine the technology distance to frontier economy during the converging process.

Closely related to the convergence literature, several papers discuss the issue of technology distance to frontier. Acemoglu, Aghion and Zilibotti (2006) analyze the possible non-convergence trap because of inappropriate institutions. Vandenbussche, Aghion and Meghir (2006) examine the education policy given the distance to frontier showing that skilled labor has higher growth enhancing effect and thus higher education is more important closer to frontier. We follow the two papers' shared assumption that technology improvement is obtained from both innovation and imitation. However, different from their models, the innovation and imitation inputs are capital and labor in our model. Moreover, the effects of fiscal policies on long-run distance to frontier and the social welfare are not examined in their models.

This chapter also discusses the effects of taxation on welfare and current growth rates. As to the issue of optimal capital income tax, the conventional view advocates zero capital income tax. [E.g. Atkinson and Stiglitz (1976), Ordover and Phelps (1979), Chamley (1986), Judd (1985)] The main intuition is that relative price of consumption in the future with respect to consumption today goes to zero or infinity if capital income tax is positive. Thus, positive capital income tax is not optimal. Several later papers attempt to overturn the result. Chamley (2001) find that capital income tax is not optimal when agents are credit constrained. Aghion, Akcigit and Fernandez-Villaverde (2013) argue that the result of Chamley (1986) and Judd (1985) no longer hold with endogenous growth models. Our result in this chapter also supports positive capital income tax, as positive capital income tax reduces the incentive to accumulate as well as leads to lower labor income tax and therefore increases consumption and leisure.

This chapter is organized as follows: Section 2 sets up the model; Section 3 analyzes the steady state equilibrium; Section 4 discusses the effects of taxation on distance to frontier and steady state welfare; Section 5 examines the transitional dynamics and the effects of taxation; Section 6 extends our analysis to allow for different capital and labor intensity in R&D inputs; Section 7 concludes.

3.2 The Model

The basic model adopts the technology distance to frontier framework developed by Acemoglu, Aghion and Zilibotti (2006). Following is a detailed description of the economic environment in our model.

3.2.1 Productions

There are two types of production activities in the economy: final goods production and intermediate goods production. The productivity improvement occurs through two channels: innovations and technology adoption from leading country. It is assumed that monopoly power exists in the intermediate goods sector, while the final good production is perfectly competitive.

Final Goods Production

A final goods producer uses a continuum of intermediate goods and a fixed factor as its inputs, subject to the following Cobb-Douglas production function:

$$Y_t = F^{1-\alpha} \int_0^1 A_{it}^{1-\alpha} {X_{ti}}^{\alpha} di, \quad A_{it} > 0, \quad 0 \ < \alpha < 1 \tag{3.1}$$

where the subscript t refers to time; A_{it} is a productivity parameter; α measures the contribution of an intermediate good to the final goods production and inversely measures the intermediate monopolist's market power; F is the quantity of the fixed factor; Y_t is the final output; and X_{ti} is the flow of intermediate good i. For simplicity, we normalize the quantity of the fixed factor to unity ($F = 1$). We also omit the time subscript t throughout the chapter whenever there is no potential confusion caused. As a result, the final goods production function can be rewritten as

$$Y = \int_0^1 A_i^{1-\alpha} {X_i}^{\alpha} di, \quad A > 0, \quad 0 \ < \alpha < 1 \tag{3.2}$$

Profit maximization in the competitive final goods sector implies the following demand function for intermediate good i:

$$P_i = \frac{\partial Y_i}{\partial X_i} = \alpha A_i^{1-\alpha} {X_i}^{\alpha-1}, \quad i \in [0, N] \tag{3.3}$$

where P_i is the price of intermediate good i in terms of the final good. The final good is used as the numeraire for all prices.

Intermediate Goods Production

Each intermediate producer i that has a patented technology and uses one unit of final good to produce one unit of intermediate good. Given the interest rate r and the final good sector's demand for intermediate goods given by (3.3), each intermediate good producer chooses the size of production that will maximize its profit

$$\Pi_i = P_i X_i - X_i = \alpha A_i^{1-\alpha} X_i^{\alpha} - X_i \tag{3.4}$$

The solution to this maximization problem gives the demand function for X_i, which in turn give the profit Π_i of an intermediate goods producer, such that:

$$X_i = A_i \alpha^{\frac{2}{1-\alpha}} \tag{3.5}$$

and

$$\Pi_i = \alpha(1-\alpha) A_i \alpha^{\frac{2\alpha}{1-\alpha}} \tag{3.6}$$

In a symmetric equilibrium, all firms have same productivity. Then the value of X_i is independent of i and A_i is identical for all i. We can denote $X \equiv X_i$, and rewrite Y as

$$Y = A^{1-\alpha} X^{\alpha} = A\alpha^{\frac{2\alpha}{1-\alpha}} \tag{3.7}$$

As a result, the profit Π could be written as

$$\Pi \equiv \alpha(1-\alpha)Y \tag{3.8}$$

i.e., aggregate monopoly profit Π will be proportional to aggregate final output Y. This result implies that any policy that expands aggregate final output will also raise the profit of intermediate production and thus stimulate innovation and economic growth.

3.2.2 Technology Improvement

Technology improvements of the intermediate production are obtained from two channels: (i) imitation activities aimed at adopting the world frontier technologies; (ii) innovation upon the local technological frontier. Both innovation and imitation use physical capital and labor as R&D inputs. In line with Benhabib and Spiegel (1994), Acemoglu et al. (2006) and in particular Vandenbussche et al. (2006), the dynamics of technology in sector i can be captured by the following equation:

$$\dot{A}_i = \lambda \Big\{ [(\frac{K_{im}}{A})^{\beta} l_{im}^{1-\beta} (\bar{A} - A)]^{\phi} + z[(\frac{K_{in}}{A})^{\beta} l_{in}^{1-\beta} A]^{\phi} \Big\}^{\frac{1}{\phi}} \tag{3.9}$$

where K_{in}, l_{in} are the capital and labor hired for innovation and K_{im}, l_{im} are those for imitation. $\bar{A}$ is the world productivity frontier at time t, and A is the aggregate technology in the local economy. ϕ measures the elasticity of substitution between innovation and imitation in technology improvement and $0 < \phi < 1$. When $\phi = 1$, innovation and imitation activities will be perfectly substitutable. However, usually one may think that innovation and imitation activities are complementary in some degree to facilitate technology progress. That is, innovation and imitation are not perfect substitutes. In addition, to avoid corner solution, we only consider the case when $\phi < 1$. β is the capital intensity in both sectors. λ and z are both constants scaling the technological growth.

Firm i will then choose K_{in}, l_{in} and K_{im}, l_{im} to maximize the discounted value of the firm

$$V_t = \int_t^{\infty} \Gamma_s \exp\left(-\int_t^s r_\tau d\tau\right) ds \tag{3.10}$$

where

$$\begin{aligned} \Gamma_s &= \Pi_i - r(K_{im} + K_{in}) - w(l_{in} + l_{im}) \\ &= \alpha(1-\alpha)A_i \alpha^{\frac{2\alpha}{1-\alpha}} - r(K_{im} + K_{in}) - w(l_{in} + l_{im}) \end{aligned} \tag{3.11}$$

Here, r is interest rate and w is wage rate. Firm maximizes the value of the firm (equation (3.10)) subject to the dynamics of technology progress (equation (3.9)). The current value Hamiltonian function for the problem is as follows:

$$\begin{aligned} H &= \alpha(1-\alpha)A_i \alpha^{\frac{2\alpha}{1-\alpha}} - r(K_{im} + K_{in}) - w(l_{in} + l_{im}) \\ &\quad + \mu\lambda\{[(\frac{K_{im}}{A})^{\beta} l_{im}^{1-\beta}(\bar{A} - A)]^{\phi} + z[(\frac{K_{in}}{A})^{\beta} l_{in}^{1-\beta} A]^{\phi}\}^{\frac{1}{\phi}} \end{aligned} \tag{3.12}$$

where μ is the co-state variable. The first order conditions for firm i are:

$$M\mu\lambda\beta K_{im}^{\beta\phi-1}l_{im}^{(1-\beta)\phi}\frac{(\bar{A}-A)^{\phi}}{A^{\beta\phi}}=r \tag{3.13}$$

$$M\mu\lambda(1-\beta)K_{im}^{\beta\phi}l_{im}^{(1-\beta)\phi-1}\frac{(\bar{A}-A)^{\phi}}{A^{\beta\phi}}=w \tag{3.14}$$

$$M\mu\lambda z\beta K_{in}^{\beta\phi-1}l_{in}^{(1-\beta)\phi}A^{(1-\beta)\phi}=r \tag{3.15}$$

$$M\mu\lambda z(1-\beta)K_{in}^{\beta\phi}l_{in}^{(1-\beta)\phi-1}A^{(1-\beta)\phi}=w \tag{3.16}$$

$$\alpha(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}=r\mu-\dot{\mu} \tag{3.17}$$

with transversality condition:

$$\lim_{t\to\infty}e^{-rt}\mu_t A_t=0 \tag{3.18}$$

where $M\equiv\{[(\frac{K_{im}}{A})^{\beta}l_{im}^{1-\beta}(\bar{A}-A)]^{\phi}+z[(\frac{K_{in}}{A})^{\beta}l_{in}^{1-\beta}A]^{\phi}\}^{\frac{1-\phi}{\phi}}$. Equation (3.13) (respectively (3.14), (3.15), (3.16)) equalizes the marginal benefit and marginal cost of capital input in imitation (respectively, labor input in imitation, capital input in innovation, labor input in innovation). Equation (3.17) is the optimal dynamic condition for technology level.

3.2.3 Households

The model economy is populated by a continuum of infinitely lived identical households with measure one. The household has the utility function:

$$U=\int_0^{\infty}[\ln C+\epsilon\ln(L-l)]\exp(-\rho t)dt \tag{3.19}$$

where C is per capital consumption and ρ is the constant rate of time preference. l is the total labor supply which is allocated into either imitation or innovation. ϵ is the weight of leisure in utility function. The representative household has a budget constraint:

$$C + \dot{K} = rK(1 - \tau_k) + wl(1 - \tau_l) + P_F + \chi \tag{3.20}$$

where K is capital stock; P_F is the return to the fixed factor; χ is the dividends from R&D activities and intermediate productions; τ_k and τ_l are the taxes for capital income and labor income, respectively. The representative household chooses consumption C, the labor supply l to maximize its life-time utility, subject to the budget constraint. The current-value Lagrange function for this optimization problem is:

$$\mathcal{L}_{DE} = \ln C + \epsilon \ln(L - l_t) + \sigma[rK(1 - \tau_k) + wl(1 - \tau_l) - C + P_F + \chi] \tag{3.21}$$

where σ is the co-state variable associated with equation (3.20). The first-order conditions for this optimization problem are (3.20) together with the following conditions

$$C^{-1} = \sigma, \tag{3.22}$$

$$\epsilon(L - l)^{-1} = \sigma w_t(1 - \tau_l) \tag{3.23}$$

and

$$\sigma r(1-\tau_k) = -\dot{\sigma} + \rho\sigma \tag{3.24}$$

Equation (3.22) (respectively, (3.23)) equalizes the private marginal benefit and cost of consumption (respectively, labor supply). Equation (3.24) are the optimal dynamic conditions for capital accumulation.

3.2.4 Government Budget

Assuming that the government's budget is balanced at each point in time, we have:

$$G = rK\tau_k + wl\tau_l \tag{3.25}$$

where the left-hand side is the total government expenditure, while the right-hand side is the total revenue from capital income and labor income. We assume that the government expenditure is a fixed fraction g_0 of the final output, i.e., $G_t \equiv g_0 Y_t$, with $0 < g_0 < 1$.

The resource constraint of the economy is then:

$$Y = C + \dot{K} + G + X \tag{3.26}$$

where output is allocated into consumption, investment, government expenditure and intermediate good production.

3.3 Decentralized Steady-State Equilibrium

In this section, we use the first-order conditions for consumer's optimization problem and the first-order conditions for (final good, intermediate good and innovation) firms' profit maximization problems to derive a system of equations that describes the steady state equilibrium of the decentralized economy.

In the steady state, the labor supply ($l = l_m + l_n$) and interest rate (r) are constant and all of the other variables (consumption C, physical capital stock K and final output Y) grow at the same constant rate. To describe the steady state, we define $q \equiv \frac{Q}{A}$ in order to obtain a stationary model (Q could be consumption C, physical capital stock K_m and K_n and final output Y). Moreover, define $\omega \equiv \frac{w}{A}$. Then in the steady state, k_m, k_n, k, c, y, ω are all constant.

We assume the growth rate of the technology frontier is g^*, i.e., $\frac{\dot{\bar{A}}}{\bar{A}} = g^*$. In steady state, the growth rate of the decentralized equilibrium should converge to g^* as well. We can see this from equation (3.9). Since all the firms are symmetric, we have $A_i = A$. Thus, we can rewrite equation (3.9) as:

$$\frac{\dot{A}}{A} = \lambda\{k_m^{\beta\phi} l_m^{(1-\beta)\phi} (\frac{\bar{A}}{A} - 1)^{\phi} + z k_n^{\beta\phi} l_n^{(1-\beta)\phi}\}^{1/\phi} \tag{3.27}$$

This result is similar to Vandenbussche et al (2006). The difference is that here the R&D inputs are labor and physical capital instead of skilled and unskilled labor. Since A grows at a constant rate g, we have $\frac{\dot{A}}{A} = g$. Furthermore, because k_m, k_n, l_m and l_n are all constant in steady state, $\frac{\bar{A}}{A}$ must be constant as well. Thus, we have $\frac{\dot{A}}{A} = \frac{\dot{\bar{A}}}{\bar{A}} = g^*$. It is straightforward that the relative productivity of the local economy to the leading country $a \equiv \frac{A}{\bar{A}}$ is also constant in steady state. Clearly, a measures the technology distance to frontier economy. a closer to 1 means the technology is closer to the frontier.

From (3.22) and (3.24), we have:

$$r = \frac{g^* + \rho}{1 - \tau_k} \tag{3.28}$$

From equation (3.17), we have

$$\mu = (1 - \alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} r^{-1} \tag{3.29}$$

Dividing equations (3.13) and (3.14) and also equations (3.15) and (3.16), we have

$$\frac{l_m}{k_m} = \frac{l_n}{k_n} = \frac{(1 - \beta)r}{\beta\omega} \tag{3.30}$$

Combining equations (3.13), (3.15) and (3.30) into equation (3.9) yields:

$$\beta g^*(1 - \alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} r^{-2} = k_m + k_n \tag{3.31}$$

In the intermediate goods sector, we have already shown that:

$$x = \alpha^{\frac{2}{1-\alpha}} \tag{3.32}$$

and

$$y = \alpha^{\frac{2\alpha}{1-\alpha}} \tag{3.33}$$

Equations (3.22) and (3.23) yield,

$$c = \frac{\omega(1-\tau_l)(L - l_m - l_n)}{\epsilon} \tag{3.34}$$

Transforming the aggregate resource constraint to $(1-g_0)y = c + (k_m + k_n)g^* + x$, and plugging equations (3.33), (3.34) and (3.30), we have:

$$(1 - g_0 - \alpha^2)\alpha^{\frac{2\alpha}{1-\alpha}} - \frac{\omega(1-\tau_l)L}{\epsilon} = (k_m + k_n)\Big[g - \frac{(1-\tau_l)(1-\beta)r}{\beta\epsilon}\Big] \tag{3.35}$$

Re-writing the above equation and plugging into equation (3.31), we have:

$$\omega = \frac{\alpha^{\frac{2\alpha}{1-\alpha}}(1-g_0-\alpha^2)\epsilon}{(1-\tau_l)L} - \frac{\beta g^{*2}(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}r^{-2}\epsilon}{(1-\tau_l)L} + \frac{g^*(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}r^{-1}(1-\beta)}{L} \tag{3.36}$$

To get the equilibrium of a, we plug equation (3.30) into equations (3.13) and (3.15):

$$k_n = \varphi_1 \omega^{-\phi(1-\beta)/(1-\phi)} \tag{3.37}$$

$$k_m = \varphi_2 \omega^{-\phi(1-\beta)/(1-\phi)}(\frac{1}{a} - 1)^{\phi/(1-\phi)} \tag{3.38}$$

where $\varphi_1 = \frac{g}{\lambda}[(1-\alpha)\alpha^{(1+\alpha)/(1-\alpha)}\lambda z \beta(\frac{1-\beta}{\beta})^{(1-\beta)\phi} r^{(1-\beta)\phi-2}]^{1/(1-\phi)}$ and $\varphi_2 = \frac{g}{\lambda}[(1-\alpha)\alpha^{(1+\alpha)/(1-\alpha)}\lambda\beta(\frac{1-\beta}{\beta})^{(1-\beta)\phi} r^{(1-\beta)\phi-2}]^{1/(1-\phi)}$. Plugging the above two equations into (3.31), we have:

$$(\frac{1}{a} - 1)^{\phi/(1-\phi)} = \frac{\beta g^*(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}\omega^{\phi(1-\beta)/(1-\phi)}}{\varphi_2 r^2} - z^{\frac{1}{1-\phi}} \tag{3.39}$$

The distance to frontier is determined by the capital income tax and labor income tax as well as other parameters.

3.4 Government's Problem of Choosing Single Tax Financing and Mixed Taxes Financing

The government chooses the combination of a capital income tax rate τ_k or a labor income tax τ_l (or mixed polices) to finance the government spending, which is a constant fraction of the final output. There are three main concerns of the government while deciding its policy choice: enhancing economic growth, promoting social welfare, and moreover, shortening the technology distance to frontier. The effects of the tax rates on the three targets may be different. Thus, the government faces trade off to balance the three targets.

The policy choice must ensure the decentralized equilibrium condition, equation (3.39), and government budget constraint, equation (3.25), are satisfied. From equations (3.31), we have:

$$k_m + k_n = \frac{\beta g^*(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}(1-\tau_k)^2}{(g^*+\rho)^2} \tag{3.40}$$

It indicates that an increase in capital income tax will decrease the capital level in a country. From equation (3.30) and (3.25), we have:

$$g_0 y = r(k_m + k_n)\tau_k + \omega(l_m + l_n)\tau_l = r(k_m + k_n)(\tau_k + \tau_l) \tag{3.41}$$

From the above equation, we can derive:

$$\frac{g_0 \alpha^{\frac{2\alpha}{1-\alpha}}(g^* + \rho)}{\beta g^*(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}} = (1-\tau_k)(\tau_k + \tau_l). \tag{3.42}$$

Rewrite the above equation, we get

$$\tau_l = \frac{g_0 \alpha^{\frac{2\alpha}{1-\alpha}}(g^* + \rho)}{\beta g^*(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}(1-\tau_k)} - \tau_k \tag{3.43}$$

3.4.1 Single Tax Financing

If the government only has access to capital income tax, to finance the required expenditure, the corresponding tax rate τ_{kg_o} should satisfy:

$$(1-\tau_{kg_o})\tau_{kg_o} = \frac{g_0 \alpha^{\frac{2\alpha}{1-\alpha}}(g^* + \rho)}{\beta g^*(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}} \tag{3.44}$$

However, if the government only has access to labor income tax, to finance the required expenditure, the corresponding tax rate τ_{lg_o} should satisfy:

$$\tau_{lg_o} = \frac{g_0 \alpha^{\frac{2\alpha}{1-\alpha}}(g^* + \rho)}{\beta g^*(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}} \tag{3.45}$$

In this section, we consider the welfare effects and growth effects of taxations. The social welfare is measured by the lifetime utility of the representative household. Here, we only consider the steady state welfare . That is to say, we assume the economy starts from steady state. In the following section where we discuss the transitional dynamics, we will present the result for welfare with transitional path which we call total welfare. The steady state welfare is then:

$$\Gamma = max \int_0^{\infty} e^{-\rho t}\Big(lnC + \epsilon ln(L-l)\Big)dt = \frac{1}{\rho}\Big[ln(c) + \epsilon ln(L-l)\Big] + \frac{g}{\rho^2} \quad (3.46)$$

Proposition 1: *With the access to only one tax instrument, to finance the required expenditure which is a fixed share g_0 of final output, the required capital income tax rate τ_{kg_o} is greater than the required labor income tax rate τ_{lg_o}.*

Proof. *Obvious, from equations (3.44) and (3.45). Q.E.D.*

From equations (3.36),and (3.34), we have:

$$c = \alpha^{\frac{2\alpha}{1-\alpha}}(1 - g_0 - \alpha^2) - \frac{g^2(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}\beta(1-\tau_k)^2}{(g+\rho)^2} \quad (3.47)$$

and

$$\begin{aligned} l_m + l_n \quad = \quad & \frac{(1-\beta)g^*(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}L}{\epsilon(g^*+\rho)}\Bigg\{\frac{\alpha^{\frac{2\alpha}{1-\alpha}}(1-g_0-\alpha^2)}{(1-\tau_l)(1-\tau_k)} \\ & + \frac{(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}g(1-\beta)}{(g+\rho)\epsilon} - \frac{g^2(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}\beta(1-\tau_k)}{(1-\tau_l)(g+\rho)^2}\Bigg\}^{-1} \quad (3.48) \end{aligned}$$

Proposition 2: *With the access to only one tax instrument, the steady state welfare under capital income tax financing is greater than that under labor income tax financing.*

Proof. *From equations (3.47) and (3.48) as well as $\tau_{kg_o} > \tau_{lg_o}$, it is easy to get the consumption level is higher under capital income tax, while the total labor supply is lower under capital income tax which implies a higher leisure level.*

Thus, according to the steady state welfare equation $\Gamma = \frac{1}{\rho}[ln(c) + \epsilon ln(L-l)] + \frac{g}{\rho^2}$*, the welfare is higher under capital income tax. Q.E.D.*

3.4.2 Mixed Taxes Financing

Combining (3.25) and (3.30), fixing the value of g_0, we assign different values to τ_k, to find the effects of mixed policies of τ_k and τ_l on the country's steady state welfare and technology distance to frontier. The analytical results are hard to obtain in this case. Thus, we exploit the numerical results in the following.

Benchmark Model

In this numerical example, we choose an exogenous worldwide growth rate $g^* = 0.03$. The population in the economy is assumed to be 1 and each household is endowed with one unit of time. The discount factor $\rho = 0.02$ is consistent with the literature. The leisure elasticity in utility $\epsilon = 0.08$ ensures a labor supply of 0.6583. We assume that the capital intensity in the inputs to productivity improvement is $\beta = 0.4$. The share of fixed factor in the final production is chosen as $1 - \alpha = 0.3$. ϕ measures the substitutability between innovation and imitation in generating technology growth. We choose $\phi = 0.3$, $\lambda = 0.02$ and $z = 1$ to generate a steady state technology distance $a = 0.6573$.

variable	ρ	ϵ	α	λ	β	z	ϕ	g^*
value	0.02	0.08	0.7	0.02	0.4	1	0.3	0.03

Table 3.1: Baseline Parameter Values

variable	a^*	l^*	ω^*	y^*	k^*	c^*	r^*
steady-state	0.6573	0.6633	0.0216	0.1893	0.1908	0.0908	0.05

Table 3.2: Initial steady-state values of variables

We can summarize the parameters and the steady state of the benchmark model in the following Table (3.1) and Table (3.2).

Introducing Fiscal Policy

Government collects revenue from capital income tax and labor income tax to finance the government spending. Assume that the share of government spending in final output is $g_0 = 0.01$. Next, we will compare the welfare effects and technology improving effects of the two taxes. Firstly, we assume government expenditure is financed only by capital income tax.

We get the following result which is consistent with Proposition 2:

Result 1: *In terms of technology distance, labor income tax is superior to capital income tax. However, capital income tax is superior to labor income tax in terms of welfare.*

	Technology distance	Welfare	Tax rate
Capital income tax financing	0.4873	-48.0410	0.2729
Labor income tax financing	0.6440	-50.0408	0.1323

Table 3.3: Comparison of single tax financing

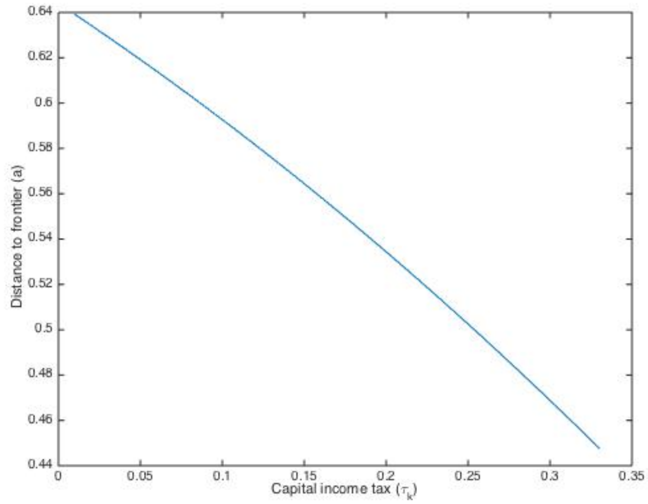

Figure 3.1: Capital income tax and technology distance

Next, we use both capital income tax and labor income tax to finance government spending. Figure (3.1) and Figure (3.2) show the effects of tax changes on technology distance and welfare.

The results can be summarized as follows:

Result 2: *With a constant share of government spending, relative technology level decreases in capital income tax, while social welfare increases in capital income tax.*

Since the growth rate of the local economy converges to the growth rate of the leading country, the steady state welfare is determined by steady state consumption and labor supply. With higher capital income tax, there will be lower incentive to accumulate capital. Therefore, the steady state capital

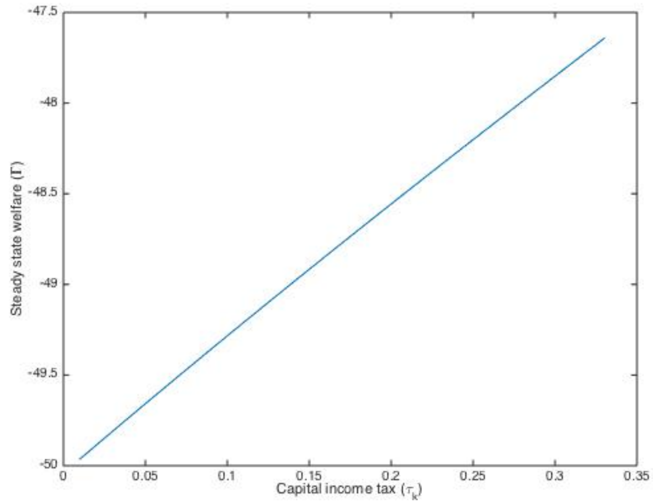

Figure 3.2: Capital income tax and welfare

decreases in τ_k. Consumption then increases. On the other hand, higher capital income tax means lower labor income tax. Lower labor income tax then leads to lower wage rate and thus lower labor supply. Together, higher capital income tax leads to higher welfare. This result is different from literature because the growth rate effect of capital income tax is eliminated in the steady state.

With higher capital income tax, there is a lower incentive to accumulate which results in a lower steady state capital stock for R&D input. Moreover, a lower labor income tax leads to lower labor supply. Both of the inputs to technology improvement are decreasing in τ_k. Thus, the steady state relative technology level is decreasing in τ_k. That is to say, the higher the capital income tax, the further the steady state distance to frontier economy.

3.5 Dynamics and Growth Effects of Policy

In this section, we analyze the dynamics of the model. From equation (3.22) and (3.24), we have:

$$\frac{\dot{C}}{C} = r(1-\tau_k) - \rho \tag{3.49}$$

Since we define $a = \frac{A}{\bar{A}}$, we will have: $\frac{\dot{A}}{A} = \frac{\dot{a}}{a} + g^*$. Letting $c = C/A, k = K/A$, we can derive the dynamics of consumption:

$$\frac{\dot{c}}{c} = r(1-\tau_k) - \rho - \frac{\dot{A}}{A} = r(1-\tau_k) - \rho - \frac{\dot{a}}{a} - g^* \tag{3.50}$$

Combining equations (3.22) and (3.23), we have:

$$\frac{\epsilon}{L - l_m - l_n} = \frac{\omega(1-\tau_l)}{c} \tag{3.51}$$

Combining equations (3.9), (3.13) and (3.15) yields:

$$\frac{\beta\mu(\frac{\dot{a}}{a} + g^*)}{r} = k_m + k_n \tag{3.52}$$

from which we have:

$$r = \frac{\beta\mu(\frac{\dot{a}}{a} + g^*)}{k} \tag{3.53}$$

We plug equation (3.30) into equations (3.13) and (3.15) to get:

$$k_n = \varphi_3 \omega^{-\phi(1-\beta)/(1-\phi)} \tag{3.54}$$

and

$$k_m = \varphi_4 \omega^{-\phi(1-\beta)/(1-\phi)} (\frac{1}{a} - 1)^{\phi/(1-\phi)} \tag{3.55}$$

where $\varphi_3 = \frac{\frac{\dot{a}}{a}+g^*}{\lambda}[(1-\alpha)\alpha^{(1+\alpha)/(1-\alpha)}\lambda z\beta(\frac{1-\beta}{\beta})^{(1-\beta)\phi} r^{(1-\beta)\phi-2}]^{1/(1-\phi)}$ and $\varphi_4 = \frac{\frac{\dot{a}}{a}+g^*}{\lambda}[(1-\alpha)\alpha^{(1+\alpha)/(1-\alpha)}\lambda\beta(\frac{1-\beta}{\beta})^{(1-\beta)\phi} r^{(1-\beta)\phi-2}]^{1/(1-\phi)}$. Plugging the above two equations into (3.52), we have:

$$\omega = (\frac{\mu\lambda\beta}{r})^{\frac{1}{1-\beta}} \frac{(1-\beta)r}{\beta} \left\{ (\frac{1}{a} - 1)^{\phi/(1-\phi)} + z^{\frac{1}{1-\phi}} \right\}^{\frac{1-\phi}{\phi(1-\beta)}} \tag{3.56}$$

Combining equations (3.51), (3.30) and (3.52), we have

$$\dot{a} = \frac{a\omega(1-\tau_l)L - a\epsilon c}{(1-\tau_l)(1-\beta)u} - ag^* \tag{3.57}$$

From the final goods clearing condition,we have:

$$\dot{k} = (1 - g_0 - \alpha^2)\alpha^{\frac{2\alpha}{1-\alpha}} - c - (\frac{\dot{a}}{a} + g^*)k \tag{3.58}$$

Lastly from equations (3.25) and (3.30) , we get:

$$\tau_l = \frac{g_0 \alpha^{\frac{2\alpha}{1-\alpha}} \beta}{rk(1-\beta)} - \frac{\tau_k \beta}{1-\beta} \tag{3.59}$$

The dynamics of the decentralized economy are then characterized by the system of equations (3.17), (3.50), (3.57), (3.58) and (3.59), along with an initial condition (k_0, a_0) and the static equations (3.53) and (3.56).

We consider the case when both the initial capital and initial relative technology level are below the steady state values. Specifically, we choose $a_0 = 0.3$

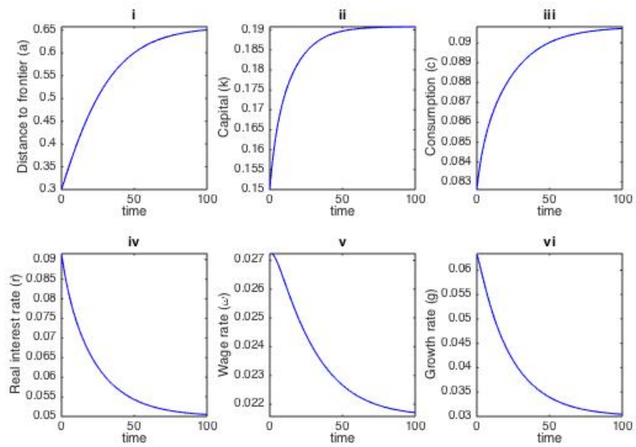

Figure 3.3: Transitional dynamics of key variables

and $k_0 = 0.15$. The following Figure (3.3) shows the transitional dynamics of the economy.

As seen in Figure (3.3), when the capital and relative technology are below the steady state, the growth rate is higher than that of the leading country. This is the so-called "Advantage of backwardness" effects. The economies that are further behind the frontier can enjoy higher growth rate through imitating and importing the leading technologies. Along with the convergence of capital and relative technology, the growth rate will also converge to the growth rate of the leading economy.

To analyze the effects of tax on current growth rate, we let τ_k change from 0 to 0.27. Then by computing the initial growth rate, we have the following relationship depicted in Figure (3.4).

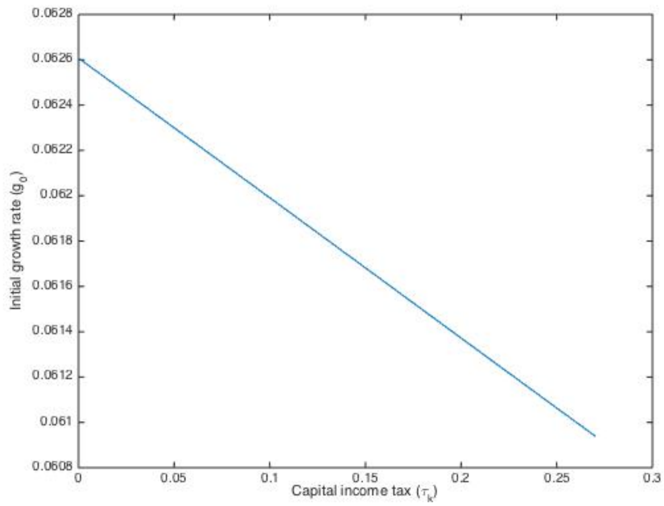

Figure 3.4: Capital income tax and initial growth rate

From Figure (3.4), we can see that higher capital income tax leads to lower growth rate. This is consistent with the literature such as Chamley (1986) and Lucas (1990). Higher capital income tax discourages capital accumulation which then leads to lower initial growth rate. Linking this result with the steady state effects of capital income tax, we can see that the government faces trade-off between current economic efficiency (growth rate) and long-run welfare while choosing the fiscal policy. Higher capital income tax increases long-run social welfare, but decreases the long-run relative technology level and reduces current economic growth.

We can revisit the welfare effects of taxation by including the welfare during the transitional path. We adopt the cubic spline interpolation to interpolate the instantaneous welfare function $u = [\ln c + gt + \epsilon \ln(L - l)] \exp(-\rho t)$ to the continuous time path and calculate the total welfare. Figure (3.5) shows the relationship between capital income tax rate and the total welfare.

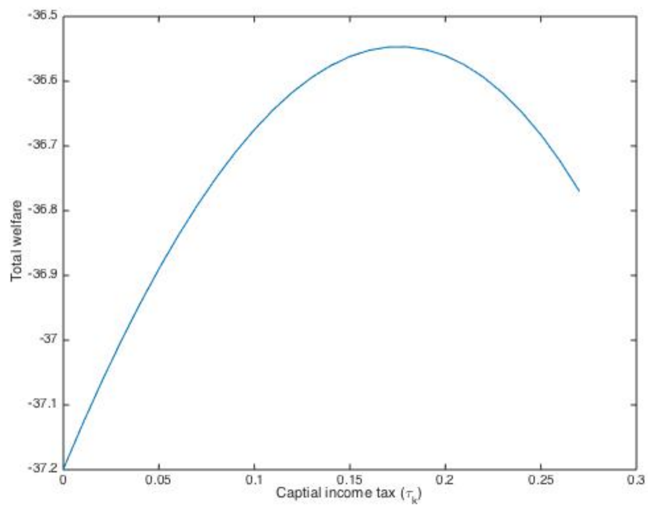

Figure 3.5: Capital income tax and total welfare

We can see that the relationship between capital income tax and welfare is different from the steady state case. There is an inverse-U shape relationship between capital income tax and total welfare. Higher capital income tax reduces capital stock as well as growth rate on the one hand. Both leads to a lower total welfare. On the other hand, higher capital income tax shifts the entire consumption path and leisure path upwards. When capital income tax is low, the positive effect of higher consumption and leisure on total welfare overweights the negative effect of lower growth rate and capital stock. Total welfare is therefore increasing in capital income tax. When capital income tax is too high, the accumulation effect of lower capital stock and growth rate dominates the dis-savings from higher consumption and leisure. Therefore, further increasing capital income tax will lead to lower total welfare.

3.6 Different Capital Intensity in Innovation and Imitation

3.6.1 Steady State Analysis

In this extension, we examine the robustness of our results by assuming different capital intensities in the imitation and innovation activities. The dynamics of technology in sector i can be captured according to:

$$\dot{A}_i = \lambda \left\{ \Big[(\frac{K_{im}}{A})^{\beta} l_{im}^{1-\beta} (\bar{A} - A) \Big]^{\phi} + z \Big[(\frac{K_{in}}{A})^{\theta} l_{in}^{1-\theta} A \Big]^{\phi} \right\}^{\frac{1}{\phi}} \tag{3.60}$$

The value function of the firm is still the same. The current value Hamiltonian function for the problem is changed as follows:

$$\begin{aligned} H &= \alpha(1-\alpha) A_i \alpha^{\frac{2\alpha}{1-\alpha}} - r(K_{im} + K_{in}) - w(l_{in} + l_{im}) \\ &\quad + \mu\lambda \left\{ \Big[(\frac{K_{im}}{A})^{\beta} l_{im}^{1-\beta} (\bar{A} - A) \Big]^{\phi} + z \Big[(\frac{K_{in}}{A})^{\theta} l_{in}^{1-\theta} A \Big]^{\phi} \right\}^{\frac{1}{\phi}} \end{aligned} \tag{3.61}$$

where μ is still the co-state variable. The first order conditions for firm i are now:

$$M \mu \lambda \beta K_{im}^{\beta\phi - 1} l_{im}^{(1-\beta)\phi} \frac{(\bar{A} - A)^{\phi}}{A^{\beta\phi}} = r \tag{3.62}$$

$$M\mu\lambda(1-\beta)K_{im}^{\beta\phi}l_{im}^{(1-\beta)\phi-1}\frac{(\bar{A}-A)^{\phi}}{A^{\beta\phi}}=w \tag{3.63}$$

$$M\mu\lambda z\theta K_{in}^{\theta\phi-1}l_{in}^{(1-\theta)\phi}A^{(1-\theta)\phi}=r \tag{3.64}$$

$$M\mu\lambda z(1-\theta)K_{in}^{\theta\phi}l_{in}^{(1-\theta)\phi-1}A^{(1-\theta)\phi}=w \tag{3.65}$$

$$\alpha(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}=r\mu-\dot{\mu} \tag{3.66}$$

where $M\equiv\left\{\left[(\frac{K_{im}}{A})^{\beta}l_{im}^{1-\beta}(\bar{A}-A)\right]^{\phi}+z\left[(\frac{K_{in}}{A})^{\theta}l_{in}^{1-\theta}A\right]^{\phi}\right\}^{\frac{1-\phi}{\phi}}$. We still have:

$$r=\frac{g^{*}+\rho}{1-\tau_k} \tag{3.67}$$

Dividing equation (3.62) by (3.63) and also equation (3.64) by (3.65), we have:

$$\frac{\beta l_m}{(1-\beta)k_m}=\frac{\theta l_n}{(1-\theta)k_n}=\frac{r}{\omega} \tag{3.68}$$

Combining equations (3.62), (3.64) and (3.68) into equation (3.10) yields:

$$g^{*}(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}r^{-2}=\frac{k_m}{\beta}+\frac{k_n}{\theta} \tag{3.69}$$

In the intermediate goods sector, we have already shown that:

$$x=\alpha^{\frac{2}{1-\alpha}}. \tag{3.70}$$

and

$$y=\alpha^{\frac{2\alpha}{1-\alpha}}. \tag{3.71}$$

Equations (3.22) and (3.23) yield:

$$c=\frac{\omega(1-\tau_l)(L-l_m-l_n)}{\epsilon} \tag{3.72}$$

Transforming the final goods clearing condition to $(1-g_0)y = c+(k_m+k_n)g^*+x$, and plugging equations (3.71), (3.72) and (3.68), we have:

$$(1-g_0)\alpha^{\frac{2\alpha}{1-\alpha}} - \frac{\omega(1-\tau_l)(L-l_m-l_n)}{\epsilon} = \alpha^{\frac{2}{1-\alpha}} + (k_m+k_n)g \tag{3.73}$$

Re-writing the above equation and plugging into equation (3.69), we have:

$$\alpha^{\frac{2\alpha}{1-\alpha}}(1-g_0-\alpha^2)) - \frac{\omega(1-\tau_l)L}{\epsilon} + \frac{(1-\tau_l)(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}g}{r\epsilon} = (k_m+k_n)\Big[g+\frac{r(1-\tau_l)}{\epsilon}\Big] \tag{3.74}$$

To reduce the model to two key dimensions, which are distance to frontier, a, and wage rate, ω, we plug equation (3.68) into equations (3.62) and (3.64) to get:

$$k_n = \varphi_1 \omega^{-\phi(1-\theta)/(1-\phi)} \tag{3.75}$$

and

$$k_m = \varphi_2 \omega^{-\phi(1-\beta)/(1-\phi)} \Big(\frac{1}{a}-1\Big)^{\phi/(1-\phi)} \tag{3.76}$$

where $\varphi_1 = \frac{g}{\lambda}\Big[(1-\alpha)\alpha^{(1+\alpha)/(1-\alpha)}\lambda z\theta(\frac{1-\theta}{\theta})^{(1-\theta)\phi}r^{(1-\theta)\phi-2}\Big]^{1/(1-\phi)}$ and $\varphi_2 = \frac{g}{\lambda}\Big[(1-\alpha)\alpha^{(1+\alpha)/(1-\alpha)}\lambda\beta(\frac{1-\beta}{\beta})^{(1-\beta)\phi}r^{(1-\beta)\phi-2}\Big]^{1/(1-\phi)}$. Thus, the resource constraint becomes:

$$\begin{aligned}(\frac{1}{a}-1)^{\phi/(1-\phi)} \quad = \quad & \Big\{(g+\frac{r(1-\tau_l)}{\epsilon})\varphi_2\omega^{-\phi(1-\beta)/(1-\phi)}\Big\}^{-1}\Big\{\alpha^{\frac{2\alpha}{1-\alpha}}(1-g_0-\alpha^2) \\ & -\frac{\omega(1-\tau_l)L}{\epsilon} + \frac{(1-\tau_l)(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}g}{r\epsilon} \\ & -(g+\frac{r(1-\tau_l)}{\epsilon})\varphi_1\omega^{-\phi(1-\theta)/(1-\phi)}\Big\}\end{aligned} \tag{3.77}$$

Targets	Technology distance	Welfare	Tax rate
Capital income tax financing	0.2178	-49.3818	0.1470
Labor income tax financing	0.2696	-61.3297	0.2097

Table 3.4: Sensitivity check of comparison of single tax financing

The above equation together with the following technology progress equation, we can solve for a and ω:

$$\begin{aligned}(\frac{1}{a}-1)^{\phi/(1-\phi)} &= \Big[\varphi_2\omega^{-\phi(1-\beta)/(1-\phi)}\Big]^{-1}\Big[g^*(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}\beta r^{-2} \\ &\quad -\frac{\beta}{\theta}\varphi_1\omega^{-\phi(1-\theta)/(1-\phi)}\Big] \qquad (3.78)\end{aligned}$$

3.6.2 Numerical Analysis

To investigate whether the results above will change when the imitation activity is more capital intensive and innovation activity is more labor intensive, in this section, we assume $\beta = 0.8$ and $\theta = 0.3$ (instead of $\beta = \theta = 0.4$, used previously).

Table (3.4) shows that the results for single tax financing still hold. From Figure (3.6) and Figure (3.7), we can see when both taxes are used, technology distance is still decreasing in capital income tax and social welfare is still increasing in capital income tax.

3.7 Conclusion

In this chapter, we assume that the technological progress of an economy comes from both innovation and imitation. By introducing fiscal policy, we examine

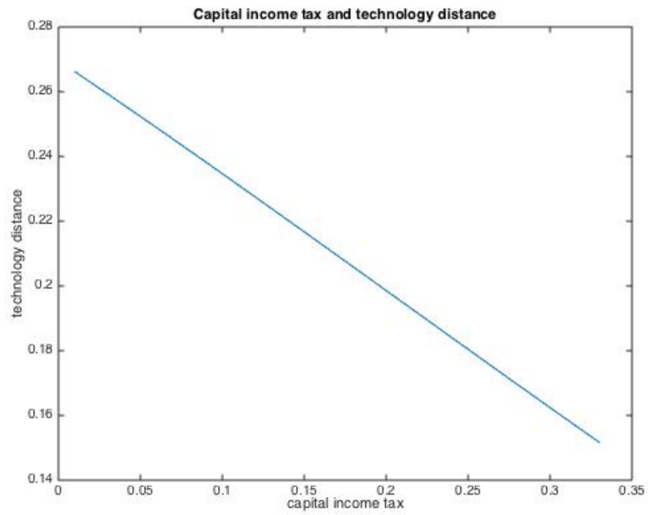

Figure 3.6: Sensitivity check of capital income tax and technology distance

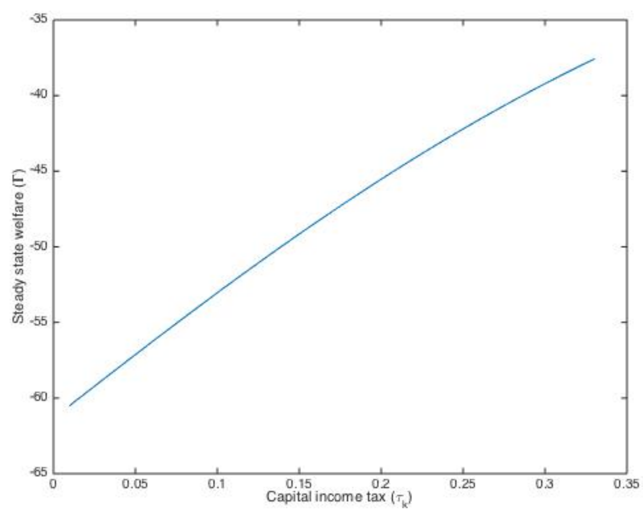

Figure 3.7: Sensitivity check of capital income tax and social welfare

the growth effects and welfare effects of taxation mix. Moreover, we analyze the effect of taxation on the long-run technology distance to frontier economy.

We find that higher capital income tax results in longer steady state distance to frontier while it increases steady state welfare. By analyzing the transitional dynamics, we show that higher capital income tax will lead to lower current growth rate. The effect of capital income tax on total welfare is inverse-U shaped.

The policy implication for government is then to balance not only between growth rate and social welfare, but also to balance the long-run distance to frontier and social welfare. Higher capital income tax increases social welfare, but reduces the short run economic growth and long run relative technology level of the economy. Low capital income tax in many developing countries may not be because of the shortsightedness of the governments. It can also be because the governments care more about the long run distance to frontier.

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