PRICING AND FINANCING DECISIONS WITH UNRELIABLE SUPPLIES

JING LEI

(B.Eng., Nanjing University)

A THESIS SUBMITTED

FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF INDUSTRIAL AND SYSTEMS ENGINEERING

NATIONAL UNIVERSITY OF SINGAPORE

2015

Declaration

I hereby declare that the thesis is my original work and it has been written by me in its entirety. I have duly acknowledged all the sources of information which have been used in the thesis.

This thesis has also not been submitted for any degree in any university previously.

Jing Lei

8 October 2015

To my parents.

Acknowledgements

First and foremost, I would like to express my deep gratitude to my supervisor Dr. Boray Huang, who gave me invaluable guidance and help throughout my study. His positive attitude toward life and critical thinking of research have deeply influenced me. Without his enormous help, none of this work would have been possible.

I greatly acknowledge the Department of Industrial and Systems Engineering for giving me the opportunity to pursue my Ph.D.. Thanks to all the staff members for their invaluable help and kind assistance. I am also deeply grateful to Professor Lee Loo Hay and Professor Xue-Ming Yuan for serving my thesis committee.

My sincere gratitude goes to my friends at NUS. Particularly I would like to thank Zhong Tengyue, Zhou Yingke, Wen Rengrong, Tang Muchen, Xiao Yan, Chao Ankuo, Jing Huayi, He Peijun, Zhou Yuan and Ahmad Reza Pourghaderi for their help and companion.

I would like to express my heartful thanks to Miss Wang Na for her support and encouragement all along the way.

Last but not least, this thesis is dedicated to my parents.

Jing Lei, April 2015

Contents

Summary

A variety of manufacturing sectors are subject to unreliable supplies, which ranges from chemical, assembly and electronic fabrication, etc. In many cases, the uncertainty of supply process is unlikely controllable. Hence, to mitigate supply risks, the optimal determination of pricing and financing policies is of particular importance. Motivated by some practical problems with unreliable supplies, this thesis aims at developing and evaluating different pricing and financing policies under different business environments.

This thesis consists of two parts of studies. In the first part, we look at the pricing strategies of a remanufacturing system with random yield. A special feature of remanufacturing business is the large proportion of replacement customers. This is due to the fact that, for many durable product markets, customers who return their end-of-life products need to do replacement purchase. At the same time, pricing strategies have been widely adopted by remanufacturing companies to balance supply and demand. In this study, the joint decision of acquisition, trade-in and selling price is considered. The objective is to maximize the expected profit of the remanufacturing firm. Our results show that a price discrimination policy improves remanufacturer's profit when the replacement customers have high return quality and high price sensitivity. In addition, it is shown that the profitability of quality dependent rebate policy varies significantly with respect to replacement customers' actual reaction to the prices.

The second part of this thesis considers the financing problem in a two echelon supply chain. Motivated by the practical problem faced by many supply chains, we analyze the impact of supplier's limited working capital and uncertain production yield. We establish conditions under which the supplier is willing to fulfill retailer's order either using initial budget or through bank loan financing. It is shown that when the wholesale price and the expected production yield are low while the financing cost is high, the retailer has incentives to offer financial support to the supplier. To keep supplier's interest aligned, the retailer may need to provide financial support and increase the order size simultaneously.

We further extend the model to the case with voluntary compliance and obtain structural properties of retailer's optimal decisions.

List of Tables

List of Figures

List of Symbols

The following notations are used in Chapter 1.

r	acquisition price
f	selling price for replacement customer
p	selling price for new customer
$\eta(r)$	supply function of acquisition return, $\eta(r) = \alpha + \beta r$
$\omega(p)$	demand function of new customer, $\omega(p) = a - bp$
$\theta(f)$	demand function of replacement customer, $\theta(f) = \delta - \gamma f$
d	unit inspection cost of returns
c_r	unit remanufacturing cost
c	unit manufacturing cost
ρ	aggregate yield rate of return products
$G(\cdot)$	CDF of random yield rate ρ
$g(\cdot)$	PDF of random yield rate ρ

The following notations are used in Chapter 2.

ζ	random yield rate, with PDF $g(\cdot)$, CDF $G(\cdot)$ and complementary CDF $\bar{G}(\cdot)$
D	random market demand, with PDF $f(\cdot)$, CDF $F(\cdot)$ and complementary CDF $\bar{F}(\cdot)$

E	expectation of random variable
b	SME supplier's initial capital
y	retailer's order size
y_{lb}	retailer's minimum order size when guarantee is needed
r_f	risk free return required by the bank
r	loan interest rate
$\Pi_{s/r}$	expected profit, subscript s for the supplier, r for the retailer
r_b	the break even interest rate when supplier's profit equals to $\Pi_s(b)$
x	guarantee level provided by the retailer
$\underline{x}$	the minimum guarantee level required by the supplier
w	unit wholesale price
v	unit backup supply price
p	unit selling price at end market
t	supplier's threshold yield level to fully repay a loan
q_s	supplier's optimal production size under voluntary compliance
$t_{i/l}$	the optimal ratio of order size over production size for the supplier, subscript i denotes the case of using initial fund, subscript l denotes the case of using bank loan
θ	the probability without disruption, the disruption probability $1-\theta$
α	the yield level when disruption occurs

Chapter 1

Pricing Strategies of Remanufacturing Business with Replacement Purchase

1.1 Introduction

In recent years, there has been an increasing concern on closed loop supply chains. Due to economic incentives, environmental concerns and legislation regulations, more and more companies are involving in product recovery business. Remanufacturing is one of the various product recovery options. By repairing or replacing old components, remanufacturing brings used products to the same-as-new conditions. Comparing with manufacturing, remanufacuring reduces the wastes produced and the raw materials needed. Therefore it is both environmental friendly and economically beneficial. Successful practices of remanufacturing can be found in industries like automotive, construction, mining and aerospace, etc.

A special feature of remanufacturing business is the correlation between supply and demand. This phenomena is due to the existence of replacement customers. As reported by Lund and Hauser (2010), many remanufactured products are used for replacement. Possible explanations for this phenomena includes: (i) Customer wants to avoid the switching cost of changing to a different products. (ii) In many cases, remanufactured product requires lower prices than brand new ones. (iii) Environmentally conscious customer tends to choose remanufactured product.

Another characteristic of remanufacturing business is the uncertain quality of return products. Unlike the manufacturing process which can be controlled and monitored effectively, in remanufacturing business, the return products come from customers and are of various quality conditions. The actual return quality can only be observed after inspection. This business nature challenges firm's decision making process in matching supply and demand.

This study is motivated by these special characteristics of remanufacturing practice. We consider a remanufacturing system with the existence of replacement customer segment and uncertain return yield. As pricing strategies have been widely adopted by remanufacturing companies, in this study, the joint decision of acquisition, trade-in and selling prices is considered. The remanufacturing company acquires used products from previous customers through acquisition programs. The supply of return flow is price dependent. Demand comes from both replacement customer and first time buyer which is also price dependent. Replacement customers can return their old products and get trade-in rebates for new purchases. The demand can be satisfied either by remanufacturing used products or manufacturing new ones.

As the quality of return flow is highly variable. The quality condition may differ among different return flows. Zikopoulos and Tagaras (2007) investigate a reverse supply chain with two collection sites which are of different return qualities. They derive the condition under which it is optimal to use only one site. In our study, the acquisition return comes from end-of-use product, which means the costumer no longer needs such product. On the other hand, the replacement return occurs only when a product fails or exceeds the useful life. Due to the difference in return causation, a reasonable conjecture would be that the acquisition return and the replacement return may have different quality conditions. We also investigate how firm's profitability is affected by the quality difference of return products.

This model represents the remnaufacturing practice of many durable products. For highly saturated markets, a significant portion of purchase could be replacement. A practical example can be found in Caterpillar, which is the world's largest manufacturer of construction and mining equipment, diesel and natural gas engine. Customers who return their end-of-life products can get a cash back from Caterpillar. The company also offers trade-in rebates to those replacement customers.

The objective of this study is to investigate the optimal pricing policies with random yield and the existence of replacement customer. The rest of this study is organized as follows. In Section 1.2, we review relevant literature. In Section 1.3, we present the model in detail and characterize the optimal pricing policies. Numerical study is provided in Section 1.4. Finally, conclusions and future research directions are discussed in Section 1.5.

1.2 Literature Review

This study is mainly related to three streams of research: consumers' replacement decisions, return acquisition in remanufacturing , and systems with random yield.

Some studies in remanufacturing have assumed that supply and demand are independent. However, a notable feature of remanufacturing business is the correlation between returns and sales. For durable products like engines or transmissions, many customers need to do replacement after their in-using products reach the end of service life. Consumers' replacement or repurchase behaviour has been widely discussed in the area of marketing research. Customers' replacement decisions are not only dependent on their own attitudes and perceptions (Bayus, 1991), but also affected by company's advertisements and product developments (Winer, 1997). Novemsky and Kahneman (2005) suggest that marketers can mitigate consumers' loss aversion by accepting the old product as a trade-in. Unlike these studies which focus on descriptive and empirical analysis of consumers' replacement behaviour, in this paper, it is assumed that the remanufacturing company can use trade in rebates as a pricing tool to differentiate replacement customers and first time buyers. We focus on the optimal pricing strategies under different yield conditions.

Due to the increasing concern on closed-loop supply chain, there is an extensive literature on remanufacturing, reverse logistics and other related problems. Fleischmann et al. (1997) provide a detailed review of quantitative models for reverse logistics. Guide and Van Wassenhove (2003) provide a thorough review on the business aspects of closed-loop supply chains. For more recent reviews, we refer to Souza (2008), and Guide and Van Wassenhove (2009).

One of the important issues in closed loop supply chain is the product acquisition management, which has been widely discussed in both practice and academia. To stimulate product return, firms can either facilitate the reverse channel or provide monetary incentives to existing customers. In an early work, Guide and Jayaraman (2000) establish the framework for product acquisition management. Savaskan et al. (2004) use game theory models to study the efficiency of different reverse channels in a supply chain setting.

Guide et al. (2003) consider a remanufacturing planning problem in which returns can have different quality levels. They assume that the return supply from each quality class and the product demand are both price dependent. A single period framework is developed to determine the optimal pricing policy. Recently, Zhou and Yu (2011) study the joint acquisition, pricing and inventory management problem in a multiperiod setting. However, both papers ignore the fact that higher acquisition price may lead to higher demand due to the existence of replacement customers. Despite the extensive discussion of product acquisition management, few studies investigate the effect of replacement purchase on remanufacturing business. Debo et al. (2006) consider an infinite-horizon model in which previous customers can make repeated purchase in future periods. Atasu et al. (2008) study the joint pricing problem of new and remanufactured products with green segment customers. They assume that returns from previous sales can affect future demand, but they take return quantity as a fixed fraction of previous sales and do not consider the acquisition management. In an closely related work to ours, Ray et al. (2005) assume firms can influence customers' return and repurchase decisions by offering different rebates and prices. However, instead of considering return products as supply for future production, they model return revenue as a deterministic function of product remaining lifespan.

This study is also related to the research stream on systems with random yields. Yano and Lee (1995) provide a comprehensive review of this problem up to mid-1990s. More recent work includes Hsu and Bassok (1999), Bollapragada and Morton (1999), Li and Zheng (2006), Inderfurth and Transchel (2007), and Tang et al. (2012). In remanufacturing planning, there are several works consider the effect of uncertain yield. Ferrer (2003) firstly analyzes inventory system with deterministic demand and random yield. Zikopoulos and Tagaras (2007) study a remanufacturing system where return supply comes from two collection sites, both with uncertain yield rates. It is shown that in some situations, it is optimal to collect from only one site. Bakal and Akcali (2006) develop a single period model to determine the optimal acquisition and selling price. Mukhopadhyay and Ma (2009) study the joint procurement and production problem of a hybrid system with both demand and return yield rate are random. Zhou et al. (2011) adopt a different approach where return flows can have different quality levels but the remanufacturing process are perfectly reliable.

This work differs from the existing studies in that we model replacement customers as an independent customer segment. Unlike those models which take repeated purchase as an uncontrollable process, we assume replacement demand can be actively controlled by firm's pricing decisions. Our work contributes to the literature by taking consideration of replacement customers which affects both the supply and the demand in remanufacturing business.

1.3 The Model

1.3.1 Assumption

In this analysis, a single period remanufacturing business model is considered. A remanufacturing company acquires end-of-use products from existing users,

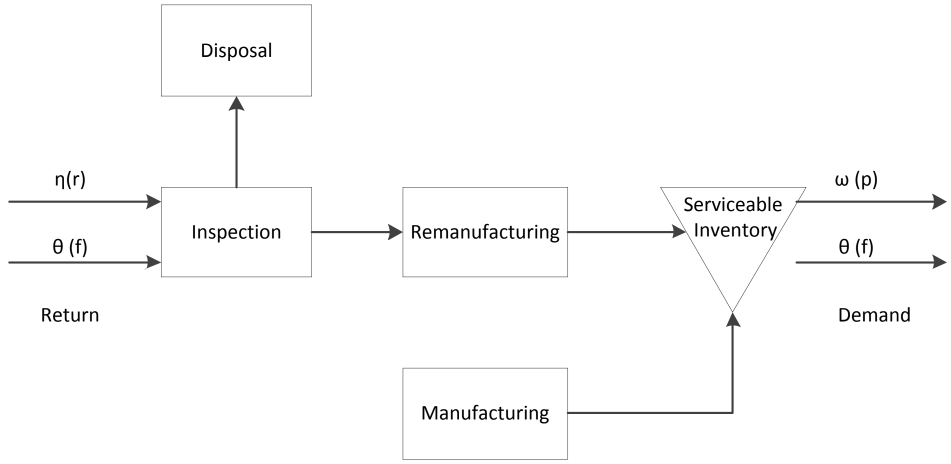


Figure 1.1: Problem enviroment

and sells remanufactured products to both new and replacement customers. It is assumed the market is monopolistic and the company has pricing power. Furthermore, to make the price discrimination policy possible, it is required to assume that there is no efficient secondary market. For new customers who are first time buyers, their demand is modeled as a linear function of selling price p, $\omega(p) = a - bp$, where $a, b > 0$. Replacement customers are current users who need to replace their end-of-life products. Their repurchasing decisions also depend on prices. Since end-of-life product can be used for remanufacturing, companies usually offer trade-in rebates for those replacement purchase. Therefore, the demand of replacement customers is considered as a linear function of repurchasing price f, $\theta(f) = \delta - \gamma f$, where $\delta, \gamma > 0$. The difference between p and f is the trade-in rebates offered to the replacement customers. There is also a return flow from end-of-use products by existing users. It is assumed that the end-of-use return depends on the acquisition price r, and can be modeled as $\eta(r) = \alpha + \beta r$, where $\alpha, \beta > 0$. For simplicity, η, ω and θ are used to represent the corresponding functions. Figure 1.1 illustrates the material flow of such a hybrid system.

After return products are acquired(both through trade-in and acquisition), they are disassembled to check whether they can be remanufactured. The inspection cost is denoted as d. Since acquisition return and replacement return are from different customer segments, the yield rate of return products can either be identical or different. The case when yield conditions are identical is firstly considered. The aggregate yield rate is denoted by ρ, which is a random variable observed only after inspection. In this case, the remanufacturing quantity is $min\{\omega + \theta, \rho(\theta + \eta)\}$, with unit remanufacturing cost c_r. Worn out returns and excess reusable returns are disposed with zero disposition cost. When reusable return is insufficient to satisfy demand, the company needs to manufacture new products at unit cost c, where $c > c_r$. Later, we also consider the case when acquisition return and replacement return are of different yield conditions. Throughout this study, it is assumed that the demand is deterministic. As mentioned in the introduction, many remanufacturing businesses are for durable products which are highly saturated. The demand could be relatively stable and predictable with few suppliers and customers on the market. For example, the service life of existing equipments can be calculated by the working schedule. Therefore, it would be reasonable to assume a deterministic demand function. Whereas, the deterministic demand assumption wouldn't hold for many consumer markets. On the other hand, the re-usability of return flows can only be observed after complete disassemble and inspection. Hence, the return yield is considered as random.

1.3.2 Identical Yield Rate

Given the model described above, in this section, we formulate the pricing problem when the yield rate is identical between acquisition return and replacement return. Firstly the case with deterministic yield is considered, which means the percentage of remanufacturable return is fixed and known. We then relax this assumption to incorporate random yield condition. The optimal decisions are characterized for both cases.

Deterministic Yield Rate

It is assumed that the remanufacturing firm always recognizes that there exists replacement customer segment. When the firm decides not to offer trade in programs, both replacement and new customers will buy the product at retail price p. Meanwhile, replacement and acquisition customers will sell their old products to the firm at price r. In such case, the demand function of replacement customers can be characterized as: $\theta(p,r) = \delta - \gamma(p-r)$. The company decides p and r simultaneously to maximize the profit. Such a pricing strategy is named as uniform pricing. This pricing strategy represents the case when product sales and return collection are lack of coordination. For example, the reverse channel is outsourced to a third party collector. It is not the main focus of this study, but serves as a benchmark for the price discrimination strategy. For more details of such a uniform pricing, readers can refer to Ray et al. (2005) and Savaskan et al. (2004). The pricing problem can be formulated as follows:

$$
\begin{aligned}
\underset{r,p}{\text{Max}}\ \Pi_u(r,p) &= \omega(p)p + \theta(p,r)(p-r-d) - \eta(r)(r+d) - c_r\rho(\eta(r)+\theta(p,r)) \\
&\quad -c(\omega(p)+\theta(p,r) - \rho(\theta(p,r)+\eta(r)))^+ \qquad (1.1) \\
\textit{subject to} &\quad r, p, \eta(r), \theta(p,r), \omega(p) \geq 0
\end{aligned}
$$

Since the company can choose whether to manufacture or not, two different scenarios are obtained. First, if the remanufacturable return is less than the total demand, $\omega + \theta > \rho(\eta + \theta)$, part of the demand needs to be satisfied by manufacturing. The profit equals to sales revenue minus acquisition, inspection, remanufacturing and manufacturing cost. Second, if demand can be fully satisfied by remanufacturing, i.e. $\omega + \theta \leq \rho(\eta + \theta)$, there is no need to manufacture. It should be noticed that since acquiring returns incurs additional cost(both acquisition cost and inspection cost), in practice, when $\omega + \theta > \rho(\eta + \theta)$, the firm can always improve the profit by reducing the acquisition price r.

The objective function $\Pi_u(r,p)$ is concave as it is a quadratic function with respect to decision variables without intersections and the second order derivatives are all negative, and the constraints are linear. Hence, there exists (r_u^*, p_u^*) which solves the first order condition.

When the firm decides to offer trade-in rebates to replacement customers, the selling price p is charged to new customers, f is charged to replacement customers and each acquisition return is paid at r. Additionally, the company should assure $p - f \geq r$ to make the trade-in rebate attractive to replacement customers. Otherwise, replacement customers should sell their end-of-use products at the acquisition market, and buy the new products at price p. The trade-in program becomes ineffective. The pricing problem is formulated as follows:

$$\begin{aligned} \underset{r,f,p}{\text{Max}}\, \Pi(r,f,p) &= \omega(p)p + \theta(f)(f-d) - \eta(r)(r+d) - c_r\rho(\eta(r)+\theta(f)) \\ &\quad -c(\omega(p)+\theta(f)-\rho(\theta(f)+\eta(r)))^+ \qquad (1.2) \\ subject\ to &\quad r+f \leq p \qquad (1.3) \end{aligned}$$

Constraint 1.3 makes sure that trade-in rebate is no less than the acquisition price. When Constraint 1.3 is binding, the above problem becomes equivalent to the uniform pricing problem. We first solve the relaxation problem without considering constraint 1.3. After that, we identify the condition under which the pricing policy would violate this constraint.

Lemma 1.1. *Without considering Constraint 1.3, the optimal pricing policy under deterministic return yield rate is:*
$(r^*, f^*, p^*) = (-\frac{\alpha+d\beta}{2\beta} + \frac{1}{2}(c-c_r)\rho,\ \frac{c\gamma+d\gamma+\delta}{2\gamma} - \frac{1}{2}(c-c_r)\rho,\ \frac{a+bc}{2b})$
which solves the first order condition.

As $\eta(r)$, $\theta(f)$ and $\omega(p)$ are linear functions, it can be verified that $\Pi(r,f,p)$ is concave in r, f, and p. Lemma 1.1 indicates the optimal pricing decisions to the unconstrained problem. We then establish the conditions under which Constraint 1.3 is binding.

Proposition 1.1. *Uniform pricing policy should be chosen when* $\dfrac{\delta}{\gamma} - \dfrac{a}{b} - \dfrac{\alpha}{\beta} \geq 0.$

Proof: By checking the optimal solution in Lemma 1.1, we can find the condition when the second constraint is binding.

According to Proposition 1.1, the price discrimination policy is preferable only when $\frac{\delta}{\gamma} - \frac{a}{b} - \frac{\alpha}{\beta} < 0$. Since γ, b and β represent the price sensitivity of each customer segments, the result suggests that a higher rebate to replacement customers benefits the remanufacturer when this segment has high price sensitivity and the new customers and acquisition customers are of low price sensitivity. An interesting question is that given $\frac{\delta}{\gamma} - \frac{a}{b} - \frac{\alpha}{\beta} < 0$, what is the profit improvement of price discrimination. Define value of price discrimination as $\Pi(r^*, f^*, p^*) - \Pi_U(r_u^*, p_u^*)$, Corollary 1.1 gives the result under deterministic yield rate.

Corollary 1.1. *Given $\frac{\delta}{\gamma} - \frac{a}{b} - \frac{\alpha}{\beta} < 0$, the value of price discrimination is :*

$$\frac{(b\alpha\gamma + a\beta\gamma - b\beta\delta)^2}{4b\beta\gamma(\beta\gamma + b(\beta + \gamma))}.$$

Corollary 1.1 can be easily proved by substituting (r^*, f^*, p^*) and (r_u^*, p_u^*) into the corresponding profit functions. This result shows that the value of price discrimination is independent of unit manufacturing cost c, unit remanufacturing cost c_r and yield rate ρ. In addition, it can be verified that for both pricing strategies the demand and return volume are the same. As a result, the manufacturing cost and remanufacturing cost are unchanged. This explains why the profit difference is independent of $c,\ c_r$ and ρ. We can conclude that the benefits of price discrimination comes from better targeting at different customer segments, instead of cost savings from production process.

Random Yield Rate

When the return yield is random, depending on the pricing decisions and the realization of yield rate, the firm's profit has two expressions:

$$\pi = \begin{cases} \omega p + \theta f - \eta r - d(\theta + \eta) - c_r(\omega + \theta), & when\ \omega + \theta \leq \rho(\theta + \eta) \\ \omega p + \theta f - \eta r - d(\theta + \eta) - c(\omega + \theta) & otherwise \\ +\rho(c - c_r)(\theta + \eta), & \end{cases}$$

The expected profit function with random yield rate is expressed as:

$$
\begin{aligned}
E[\Pi(r,f,p)] &= \omega p + \theta(f-d) - \eta(r+d) - c_r E[min\{\omega+\theta, \rho(\eta+\theta)\}] \\
&\quad -cE[((\omega+\theta) - \rho(\eta+\theta))^+] \\
&= \omega(p-c_r) + \theta(f-c_r-d) - \eta(r+d) \\
&\quad -(c-c_r)E[(\omega+\theta-\rho(\theta+\eta))^+]
\end{aligned}
$$

The yield rate ρ is randomly distributed on $[A, B]$ ($0 \le A < B \le 1$), with CDF $G(\cdot)$, PDF $g(\cdot)$, and mean value μ. The relation between return and sales has two different cases which are affected by the pricing decisions:

Case 1:$(\theta+\eta)A \le \theta+\omega \le (\theta+\eta)B$

$$
\begin{aligned}
E[\Pi(r,f,p)] &= \omega p + \theta(f-d) - \eta(r+d) - c_r(\omega+\theta) \\
&\quad -(c-c_r)\int_A^{\frac{\theta+\omega}{\theta+\eta}} (\theta+\omega-\rho(\theta+\eta))\, g(\rho)d\rho
\end{aligned}
$$

Case 2:$(\theta+\eta)B \le \theta+\omega$

$$
E[\Pi(r,f,p)] = \omega p + \theta(f-d) - \eta(r+d) + \mu(c-c_r)(\theta+\eta) - c(\omega+\theta)
$$

Note that the case $(\theta+\eta)A > \theta+\omega$ is omitted here. In practice, when reusable return is greater than the total demand, the company can always reduce the acquisition price to increase the profit.

For Case 2, the maximum possible reusable return is less than the total demand for any yield realization. Hence, manufacturing is always needed. The optimization problem is similar to the situation of deterministic yield rate, and the optimal decision only depends on the mean value of yield rate.

For Case 1, the expected profit function is similar to that of a classical newsvendor problem with price dependent demand. Except that classic newsvendor model usually assumes random demand and perfectly reliable supply, while in this remanufacturing problem, we consider deterministic demand and uncertain yield rate.

Lemma 1.2. $E[\Pi(r, f, p)]$ *is differentiable.*

Proposition 1.2. *The expected profit function is jointly concave in* $r,\ f,\ and\ p$.

Lemma 1.2 and Proposition 1.2 indicate that an optimal pricing decision exists and can be found by solving the first order condition.

Corollary 1.2. *Given two different yield conditions* ρ_1 *and* ρ_2 *which distributed on* $[A, B]$ *with* $\rho_1 \ \leq_{st} \ \rho_2$*, remanufacturer's optimal expected profit* $E[\Pi^*(\rho_1)] \leq E[\Pi^*(\rho_2)]$.

Corollary 1.2 shows that the remanufacturer is always better off when the return yield is stochastically larger.

The optimal pricing decision for the uniform pricing problem can be found by adding a linear constrain $r + f = p$ to the above problem. Since the concavity of the expected profit function has been proved, the optimal uniform pricing decision $(r_u^*\ p_u^*)$ can be obtained similarly. By definition, it is indisputable that $E[\Pi(r^*, f^*, p^*)] \geq E[\Pi(r_u^*, p_u^*)]$. However, because of the complexity of the problem, a closed form solution is not obtainable. An interesting question is that whether Corollary 1.1 still holds for the random yield rate problem, we present numerical results in Section 1.4.

1.3.3 Different Yield Rates of Returns

In this subsection, the assumption of identical yield rate are relaxed. We investigate how firm's profitability is affected by the quality difference of return products. Let ρ_1 and ρ_2 denote the yield rate of acquisition return and replacement return respectively. When remanufacutring company adopts price discrimination policy, it is possible that acquisition returns would have higher yield rate but receive lower rebates. It is assumed this would not change the supply function of acquisition returns. The reasons are twofold. Firstly, since the actual yield condition realizes only after inspection, customers cannot get this information in advance. Secondly, the higher rebates to the replacement customers are used to encourage repurchase, and pure return customers should not take this benefit. The remanufacturer's expected profit function is expressed as:

$$
\begin{aligned}
E[\Pi(r,f,p)] &= \omega p + \theta(f-d) - \eta(r+d) - cE[(\omega+\theta-\rho_1\eta-\rho_2\theta)^+] \\
&= \omega(p-c_r) + \theta(f-c_r-d) - \eta(r+d) \\
&\quad -(c-c_r)E[(\omega+\theta-\rho_1\theta-\rho_2\eta)^+] \qquad (1.4)
\end{aligned}
$$

Assume ρ_i is distributed on $[A_i, B_i]$ $(0 \le A_i < B_i \le 1)$, with CDF $G_i(\cdot)$, PDF $g_i(\cdot)$, and mean value μ_i $(i = 1, 2)$. Depending on the pricing decisions, the integration area of $E[(\omega+\theta-\rho_1\theta-\rho_2\eta)^+]$ is shown in Table 1.1.

Proposition 1.3. *When the yield of replacement return and acquisition return are independently distributed, the expected profit function $E[\Pi(r,f,p)]$ is jointly concave in r, f, and p.*

Proof: Equation 1.4 can be expressed as:

$$
\begin{aligned}
E[\Pi(r,f,p)] &= \omega(p-c_r) + \theta(f-c_r-d) - \eta(r+d) \\
&\quad -(c-c_r)E[(\omega+\theta-\rho_1\theta-\rho_2\eta)^+]
\end{aligned}
$$

Since w, θ and η are linear functions of r, f and p, it is easy to verify that the first three terms are concave. For the last term, $E[(\omega+\theta-\rho_1\theta-\rho_2\eta)^+]$ is in fact $\{w+(1-E[\rho_1])\theta - E[\min\{w+(1-\rho_1)\theta, \rho_2\eta\}]\}$. As $1-\rho_1$ and ρ_2 are non-negative, concavity is preserved under minimum and expectation operations(Bertsekas et al., 2003). Therefore the sum of all the terms ($E[\Pi(r,f,p)]$) is also concave.

1.4 Numerical study

In this section, computational experiments are conducted based on the model described above. The purpose of the numerical study is twofold. First, we use

Table 1.1: Integration area

Range of ω and η	Range of θ	Integration limits
$\omega \geq \eta B_1$	$\theta > 0$	$A_1 \leq \rho_1 \leq B_1$, $A_2 \leq \rho_2 \leq B_2$
$\eta A_1 \leq \omega \leq \eta B_1$	$\theta(1-A_2) \leq \eta B_1 - \omega$	$A_1 \leq \rho_1 \leq \frac{\theta(1-\rho_2)+\omega}{\eta}$ for $A_2 \leq \rho_2 \leq B_2$
	$\theta(1-B_2) \geq \eta B_1 - \omega$	$A_1 \leq \rho_1 \leq B_1$ for $A_2 \leq \rho_2 \leq B_2$
	$\theta(1-B_2) \leq \eta B_1 - \omega \leq \theta(1-A_2)$	$A_1 \leq \rho_1 \leq \frac{\theta(1-\rho_2)+\omega}{\eta}$ for $1-\frac{\eta B_1-\omega}{\theta} \leq \rho_2 \leq B_2$, and $A_1 \leq \rho_1 \leq B_1$ for $A_2 \leq \rho_2 \leq 1-\frac{\eta B_1-\omega}{\theta}$
$\omega \leq \eta A_1$	$\theta(1-B_2) \geq \eta B_1 - \omega$	$A_1 \leq \rho_1 \leq B_1$ for $A_2 \leq \rho_2 \leq B_2$
	$\theta(1-A_2) \leq \eta A_1 - \omega$	Null
	$\theta(1-B_2) \leq \eta A_1 - \omega \leq \theta(1-A_2) \leq \eta B_1 - \omega$	$A_1 \leq \rho_1 \leq \frac{\theta(1-\rho_2)+\omega}{\eta}$ for $A_2 \leq \rho_2 \leq 1-\frac{\eta A_1-\omega}{\theta}$
	$\eta A_1 - \omega \leq \theta(1-B_2) \leq \theta(1-A_2) \leq \eta B_1 - \omega$	$A_1 \leq \rho_1 \leq \frac{\theta(1-\rho-2)+\omega}{\eta}$ for $A_2 \leq \rho_2 \leq B_2$
	$\theta(1-B_2) \leq \eta A_1 - \omega \leq \eta B_1 - \omega \leq \theta(1-A_2)$	$A_2 \leq \rho_2 \leq \frac{\eta\rho_1-\omega}{\theta}$ for $A_1 \leq \rho_1 \leq B_1$
	$\eta A_1 - \omega \leq \theta(1-B_2) \leq \eta B_1 - \omega \leq \theta(1-A_2)$	$A_1 \leq \rho_1 \leq \frac{\theta(1-\rho_2)+\omega}{\eta}$ for $1-\frac{\eta B_1-\omega}{\theta} \leq \rho_2 \leq B_2$, and $A_1 \leq \rho_1 \leq B_2$ for $A_2 \leq \rho_2 \leq 1-\frac{\eta B_1-\omega}{\theta}$

numerical results to investigate the advantages and limitations of different pricing policies. Secondly, sensitivity analyses are conducted to find how the optimal decisions change according to different parameter settings. This would help managers make decisions when facing different market conditions.

Uniform distribution is used to describe the uncertainty of yield rate. However, it is not to claim that uniform distribution is more suitable to model the usability of return products. In literature, several distributions have been adopted for study, Weibull distribution is used by Lo et al. (2007); Wee et al. (2007). Bakal and Akcali (2006) use normal distribution in their analysis. Uniform distribution has been used by Mukhopadhyay and Ma (2009); Tang et al. (2012). For the numerical study, the following data sets are assigned as base value throughout this section:

$$a = 150,\ b = 3,\ \delta = 100,\ \gamma = 3,\ \alpha = 10,\ \beta = 10,\ c_r = 5,\ d = 2,\ c = 30.$$

1.4.1 Effect of Random Yield Rate

Identical Yield Rate

Unlike most manufacturing systems, the quality level of supply is highly variable and uncertain in remanufacturing business. In this subsection, sensitivity analysis is conducted with the assumption of identical yield rate. In this situation, acquisition return and replacement have the same yield condition. Analytic results are presented in Section 1.3.2. For the sensitivity analysis, the standard deviation σ of yield rate is fixed at at 1/75, and the mean value μ is varied from 0.3 to 0.8. Figure 1.2 shows how the pricing decisions change accordingly. According to the result, the optimal acquisition price r first increases with μ, after a threshold the price slightly decreases as μ further increases. However, the optimal price to replacement purchase is always decreasing in μ within the range of computational experiment. When yield rate is low, the optimal selling price to new customers is independent of μ, but as μ further increases the firm can decrease the retail price to attract more first time buyers. The result suggests that when expected return yield is low, the firm would choose to acquire less cores(returns in remanufacturing business) $((\theta + \eta)B \leq \theta + \omega)$, and demand is satisfied by both manufacturing and remanufacturing. More specifically, if reusable return is less than the demand from replacement purchase, all the new demand are satisfied by manufacturing. Consequently, selling price p is independent of yield condition. When μ is high, supply of reusable return is ample, the firm can then reduce the selling price to attract more customers.

Corollary 1.1 shows when yield rate is identical and deterministic, the value of price discrimination is independent in c, c_r and ρ. However, the conclusion under random yield is not easy to draw. Hence, computational experiments is used to verify whether this result still holds in random yield situation. Firstly,

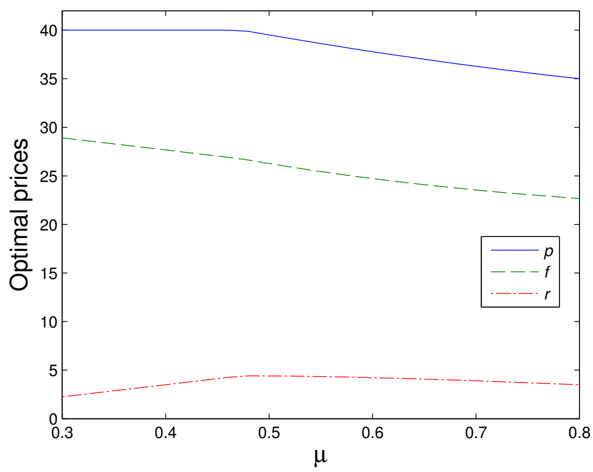


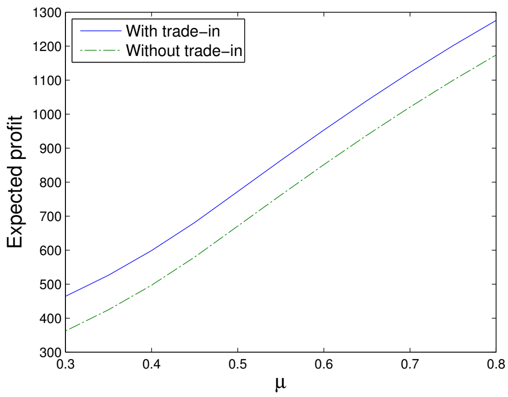
Figure 1.2: Expected yield rate and pricing decisions

the yield variance σ^2 is fixed at 1/75 to study the effect of expected yield rate μ. Afterward, μ is fixed at 0.5 and σ is varied to see how yield variation affects firm's expected profit. Figure 1.3 compares the profit difference for the two pricing policies when yield is random. Under both pricing schemes the expected profit is increasing in μ and decreasing in σ. These observations are consistent with intuitions as higher yield rate saves acquisition cost and lower randomness leads to higher profits. It can be also observed that to offer a trade-in program is especially favourable when the expected yield rate is low and the variance of yield rate is large, as the percentage profit improvement is higher in such cases. On the other hand, managers should also take into account the related cost of such a market decision.

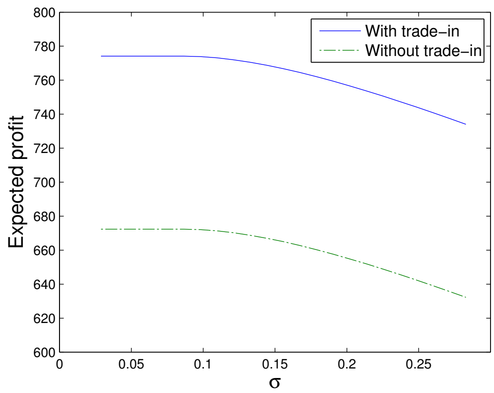
Another observation of Figure 1.3 is that, when yield rate is random, the profit difference between the two pricing strategies is stable with respect to both μ and σ. Moreover, although not shown here, numerical results also reveals the profit difference is irrelevant to c and c_r, which is consistent with the case of deterministic yield rate.

Different Yield Rates

As stated in Section 1.3.3, the difference in return causation may lead to different return quality conditions. In this subsection, we investigate the situation when ρ_1 and ρ_2 are independent random variables. Table 1.2 summarizes how the



a. Effects of expected yield rate



b. Effect of standard deviation

Figure 1.3: Value of price discrimination

Table 1.2: Pricing decisions with respect to different yields

μ_1	μ_2	(r, f, p)	$E[\Pi]$
0.4	0.4	(3.50, 27.67, 40.00)	598.83
	0.6	(3.50, 25.17, 40.00)	702.58
	0.8	(3.50, 22.67, 40.00)	843.83
0.6	0.4	(5.07, 27.17, 38.74)	862.82
	0.6	(5.10, 25.06, 38.70)	960.03
	0.8	(4.61, 22.49, 38.14)	1090.06
0.8	0.4	(5.16, 25.68, 36.42)	1107.96
	0.6	(5.00, 23.85, 36.22)	1198.79
	0.8	(4.68, 22.08, 35.83)	1305.09

optimal pricing decisions are affected by different yield rate conditions. When μ_1 is low, the acquisition price r is unchanged to different levels of μ_2. When μ_1 is in the middle level, r first increases in μ_2 and then decreases. When μ_1 is high, r decreases in μ_2. The selling price to replacement customer f is decreasing in both μ_1 and μ_2. The selling price p is unchanged when reusable return is insufficient to fulfill replacement demand, but decreases as return yield further increases. This observation is similar to the case of identical yield.

Define the profit difference with and without trade-in rebate as the profit gain from price discrimination. Figure 1.4 shows that when ρ_1 and ρ_2 are independent random variables, the profitability of price discrimination policy is affected by both return yield condition and replacement customers' price sensitivity. As indicated by Figure 1.4, the profit gain is increasing in the expected yield rate of replacement return while decreasing in that of acquisition return. This result is different from the case when these two yield rates are identical. As Corollary 1.1 and Figure 1.3 show, when yield rates are identical, the profit gain remains the same with respect to different yield rate level. The result highlights the importance of identifying return yield conditions of different customer segments. Especially, if replacement returns have high re-usability, it would be profitable for the remanufacturer to offer high trade-in rebates.

Another observation of Figure 1.4 is that the profit gain is also affected by replacement customers' price sensitivity γ. This is consistent with Corollary 1.1, which suggests when γ is low, the company has less incentive to offer higher rebates to replacement customers. In practice, if the switching cost for existing customer is low, the price sensitivity of replace demand would be high. In such case, the remanufacturing company should offer higher trade-in rebates to retain these customers.

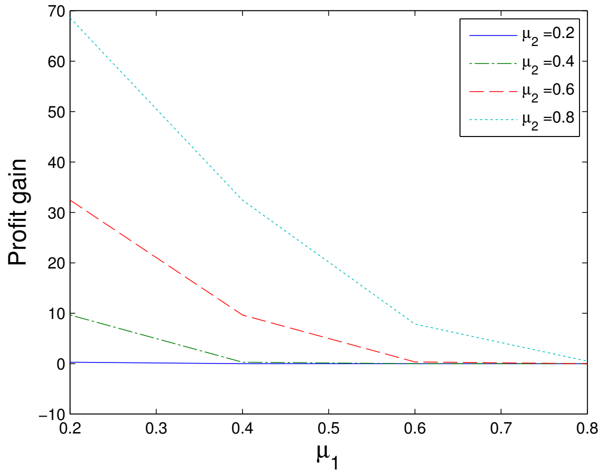
The numerical study implies that, when μ_2 and γ are high, a greater rebates to replacement customer can stimulate the replacement sales and acquire more reusable returns. Therefore, managers are recommended to use price discrimination policy under such circumstances.

Although not shown here, the numerical study also reveals that remanufacturer's profit is affected by the yield variations, but the effect is minor comparing with the above two factors.

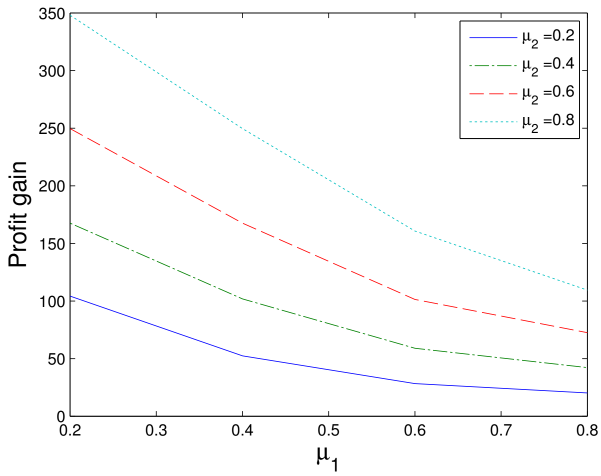
1.4.2 Quality Dependent Rebate Policy

Up to this point, it is assumed that the remanufacturing company pays return rebates to all the return customers without considering the inspection results of return products. While in practice, some companies do check the reusability of return products and pay the rebates based on inspection results.

Guide et al. (2003) consider the return acquisition problem with multiple type of cores. They find that with the optimal pricing policies, high quality returns may receive lower price than low quality returns. This is because they have assumed the return of each quality class is independent. Ray et al. (2005) dis-



a. $\gamma = 2$



b. $\gamma = 3$

Figure 1.4: Value of price discrimination, yield rate and price sensitivity

cuss the relation between products residual value and replacement decisions in a deterministic environment. They assume the perceived residual value of product depends on the remaining useful lifespan, and both customers and the remanufacturing firm are fully aware of this information. They find that an age-dependent pricing policy does not add much value for the firm.

We now consider the policy when return rebates is only paid to those reusable returns. For the reason of tractability, we adopts the assumption of independent markets, which means the acquisition return and replacement return are not affected by the price on the other market. In this study, the actual yield condition can only be observed after inspection. It is hard to imagine that return customers would know this information in advance. The remanufacturing firm, on the other hand, is assumed to be well experienced and knows the yield distribution. Since return customers do not have knowledge on the yield distribution, their return decisions only depend on the nominal acquisition price r. Therefore, with yield dependent rebate policy, the acquisition return function is the same as the case when rebates are paid to all return customers.

For the replacement customers, there are two possible scenarios for their replacement decisions. Firstly, if replacement customers behave the same as acquisition return customers, the replacement demand function is also unchanged. In this case, the expected profit for the remanufacturing company becomes:

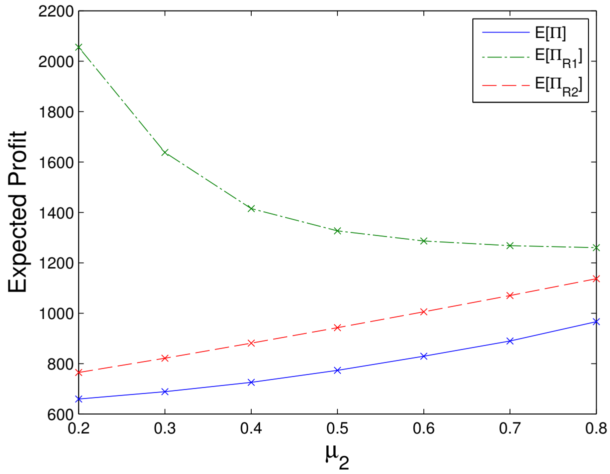
$$\begin{aligned} E[\Pi_{R1}(r,f,p)] &= \omega p + \theta(\mu_2 f + (1-\mu_2)p - d) - \eta(\mu_1 r + d) \\ &\quad -c_r E[min\{\omega+\theta, \rho_1\eta + \rho_2\theta\}] - cE[(\omega+\theta-\rho_1\eta-\rho_2\theta)^+] \end{aligned}$$

where μ_1 and μ_2 are the expected yield rate of acquisition and replacement returns.

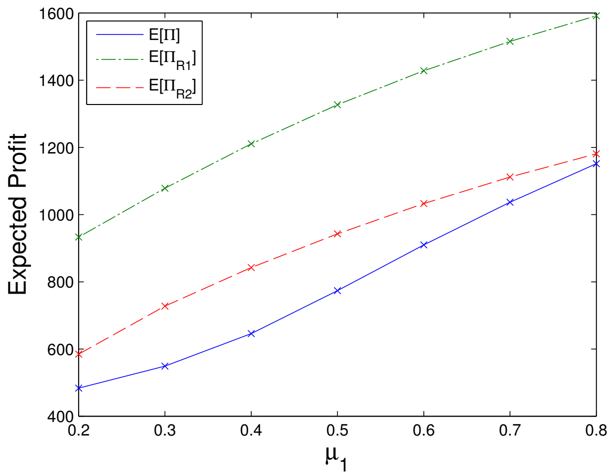
Secondly, although the return decisions are induced by the nominal rebates, the replacement customers can make their purchasing decisions upon knowing the inspection result. In addition, we assume that the customers are homogeneous in price sensitivity with respect to the return yield condition, which means both reusable and unusable return customers react the same to the prices. Therefore, the expected profit for the remanufacturing company becomes:

$$\begin{aligned} E[\Pi_{R2}(r,f,p)] &= \omega p + \mu_2\theta(f)(f-c_r) + (1-\mu_2)\theta(p)p - \theta(f)d - \eta(\mu_1 r + d) \\ &\quad -c_r E[min\{\omega + (1-\rho_2)\theta(p), \rho_1\eta\}] \\ &\quad -cE[(\omega + (1-\rho_2)\theta(p) - \rho_1\eta)^+] \end{aligned}$$

Figure 1.5 demonstrates how different rebate policies and yield conditions affect the firm's expected profit. If both replacement and acquisition customers only look at the nominal prices, a quality dependent rebate policy improves firm's profit significantly. Furthermore, the firm decides to offer higher rebates to attract



a. $\mu_1 = 0.5$



b. $\mu_2 = 0.5$

Figure 1.5: Performance of quality dependent rebate policy

more replacement customers, and increase the selling price p to generate more profit from customers whose return cannot be remanufactured. This effect makes the firm especially profitable when μ_2 is low. While for the second situation, the expected profit increment is much less. In this case, the replacement purchasing decision differs regarding to the yield realization. Although the company saves rebate costs for unusable returns, part of the replacement customers can only buy at price p and their demand is deterred.

Our numerical results suggest that the profitability of such a quality based rebate policy is largely affected by the return customers' response. In remanufacturing research, few studies consider the effects of different rebate policies. The actual reaction of return customers may fall between the above two scenarios.

Further empirical studies are required to justify the assumption of customers' reaction to different rebate policies.

1.5 Conclusions and Future Research

Matching supply and demand is the major concern of mangers who are dealing with remanufacturing business. Many studies in remanufacturing systems have assumed that supply and demand are two independent flows. This assumption is reasonable for the business of manufacturing and sales. However, due to the existence of large proportion of replacement customers, this assumption may not hold for remanufacturing business.

This study investigates the pricing decisions of a remanufacturing firm who is facing both new and replacement demand. A single period model is developed to evaluate the benefit of adopting a price discrimination policy. It is one of the first attempts to study the effect of replacement customers in remanufacturing business. When the yield is deterministic, it is shown that the price discrimination policy is applicable if replacement customers have high price sensitivity while new customers and acquisition customers have low price sensitivity. When the two return flows have different yield conditions, we show that the expected profit of remanufacturing firm is concave in the pricing decisions when the return yields are independently distributed. Due to the complexity of the problem, the closed form solutions are not attainable. Computational experiments are conducted to compare the profitability of different pricing schemes. The impact of return yield conditions and customers' price sensitivities are investigated. The numerical results show that both factors are crucial for the remanufacturing firm. The price discrimination policy makes the remanufacturing firm significantly better off when the yield rate of replacement return is high. The payment scheme of return rebates also affects firm's profit. The numerical study shows that when return rebates are only paid to reusable returns, the firm is significantly better off if replacement and acquisition customers both make decisions only based on the nominal repurchasing price.

The present model has assumed deterministic demand function, in practice, however, the demand information is usually imperfect. Consequently, it is meaningful to incorporate random demand into the model. The company will then decide on both pricing strategy and production quantity. Zhou and Yu (2011) show that when the demand is linear and additive, and return is concave and additive, the optimal pricing and stocking decisions follow a state dependent policy. They also mention that if the random factor of demand and return are not additive, the problem would become much more complicated. Our current model has assumed proportional random yield. If we modify the randomness into additive form on both demand and return sides, the resulting problem would become similar to that of Zhou and Yu (2011), but with two different demand segments, i.e. replacement and first time buyer. It would be reasonable to expect that the optimal policy should follow a similar state dependent structure.

There are many other possible extensions for this model. One is to relax the assumption of independence of new customer and replacement customer. In practice, replacement customers may choose to purchase a new product without returning their old one. The demand from this customer segment will then depend on both f and p. It is expected that the optimal pricing policy would be different, but the price discrimination policy should still outperform the uniform pricing policy. A limitation of the current model is that the yield rate is taken as a fraction of reusable returns. In practice, return products are usually under various quality conditions and require different remanufacturing costs. The current model would be more realistic if multiple type of returns can be incorporated. Besides, one can also consider the problem when remanufactured products are imperfect substitutes of brand new products. It is interesting to see how cannibalization effect would affect firm's pricing decisions.

Chapter 2

Supply Chain Financing with Unreliable and Budget Constrained Supplier

2.1 Introduction

Small and medium-sized enterprises(SMEs) are more reliant on bank loan financing than big firms. As most SMEs are lack of financial transparency than large enterprises, SMEs have difficulties in issuing bonds or selling shares on the financial markets(Kaya, 2014). Nevertheless, to obtain sufficient bank financing is still a challenge for many SMEs. The operational risks with SMEs are usually much higher. They also have few assets as collateral and cannot provide complete credit history. These problems may lead to higher financing cost or even rejection of loan application. From the perspective of banks, the lending risks are much higher with SMEs. The risks include both the possibility of non-repayment and the potential loss when default happens. To fill the gap between banks' risk concerns and the financing needs of SMEs, a widely used financial solution is the loan guarantee scheme. A loan guarantee is a promise of debt payment in case the borrower defaults. The debt promise can either be partially or fully. The guarantor can be credit guarantee corporations, government institutions or other agencies.

With loan guarantee schemes, banks' loss when default occurs can be partially or fully compensated. Therefore, the loan repayment is more secured for the bank. The SME borrowers can benefit from the access to financing and possibly lower financing cost. Since SMEs make great contribution to employment and national economy, governments have been supporting SME financing through various methods. Accordingly, many loan guarantee schemes are established by governments institutions. For example, the Hong Kong SME Loan Guarantee Scheme (2014) provides up to HK$ 6,000,000 or 50% of loan amount for qualified SME bank loans. Other loan guarantee agencies include Canada Small Business Financing(CSBF), Malaysian Credit Guarantee Corporation, etc.

However, those guarantee schemes offered by third parties cannot always meet the SMEs' financing needs. Besides the restrictions on coverage rate, coverage amount and loan period, these guarantee schemes also have specified loan purpose. For example, the CSBF program cannot be used to finance working capital(Riding et al., 2007). In addition, these guarantee schemes charge extra guarantee fee to the SME borrowers which increases SMEs' borrowing cost. There are also arguments about government established credit guarantee programs. Opponents criticise that there is no sufficient justification of using public fund to sustain guarantee schemes(Green, 2003).

SMEs' difficulties in obtaining sufficient financing also affect the performance of supply chains. As a supply chain member, when a SME suffers from financial inability, other supply chain partners would face higher costs or even lose potential revenue. Meanwhile, based on historical transactions and long term cooperation, supply chain members may have better knowledge about their partners than banks and other third party agencies. As a result, supply chain participants have developed various supply chain financing solutions which includes the loan guarantee scheme. Firms with good credit worthiness can reduce their supply chain partners' borrowing cost through offering loan guarantee. An example is the China Minsheng Bank which provides "Core Enterprise Guarantee Loan" program to SMEs which serve as suppliers or retailers to large enterprises (China Minsheng Bank, 2014). The SMEs are able to obtain more financing with guarantee from their big partners. Since the interest rate is based on large firm's strong credit rating, SMEs' financing cost would also be reduced. In helping their SME partners, large enterprises could benefit from more efficient supply chain transactions.

Among the various operational risks faced by SMEs, we are of particular interest of the unreliable production process. The causation of unreliable supply varies from natural disaster, labor strike to immature technology, etc. These risks are usually uncontrollable for SMEs. The joint impact of unreliable production and financial constraint on the performance of supply chain is still unclear. So far, supply chain research has paid little attention to this area.

A number of interesting research questions arise: When should a retailer offer financial support to a SME supplier with production risks? What is the magnitude of profit improvement from such finance scheme? What is the effect of supplier's different financial status as well as the supply reliability?

This study develops a two echelon supply chain model to determine the joint operational and financing decisions. We explicitly evaluate the impact of financial constraints on the supply chain performance. To our knowledge, this study is one of the first efforts which investigate the supply chain contracting issues under supply uncertainty and financial constraints. Our work contributes to the literature in several ways. First, we show that supplier's financial constraint does affect retailer's decision and supply chain profitability. Second, by comparing the situation with and without financing, we establish the conditions under which different financial and operational strategies should be chosen. Our results also provide managerial insights for practitioners who are facing similar situations.

The rest of this study is organized as follows. In Section 2.2, we discuss the related literature. We then present our base case model with random production yield and forced compliance in Section 2.3. In practice, retailers often recourse to backup sourcing when facing unreliable supply processes. In Section 2.4, we extend our base model to cope with backup supply. The trade off between loan guarantee and backup sourcing is investigated. We also consider the use of advance payment, and compare its performance with loan guarantee scheme.

Since many supply chains are operated as decentralized systems, in Section 2.5, we further use a Stackelberg game formulation to study the problem when the production decision is made by the supplier. Finally we conclude our study and discuss future research directions in Section 2.6.

2.2 Literature Review

This study lies at the joint interface of operations and finance research. It is not until recently that this area has drawn much attention. Studies in this area are quite multidisciplinary. Related research streams include supply chain contracting, firms' financing decisions, inventory and production management, etc. Among the various research questions, we are of particular interests in the financing and ordering decisions when facing unreliable supply process.

2.2.1 Supply Chain Contracting and Financing

In the context of supply chain contracting, early research usually ignores the impact of financial decisions. For a broad review of supply chain contracting, we refer to Cachon (2003). As the importance of joint consideration of operations and finance is being realized gradually, the research in this area is also growing. Banks usually divide their financing services into buyer finance, seller finance, and middleman service. While from the perspective of supply chain management, we classify the existing research into individual financing and cooperative financing. Some studies focus on single firm's operational and financial decisions. Chao et al. (2008) study the inventory control problem of a self-financing retailer in a multiperiod setting. The replenishment decision is constrained by on hand capital which updates periodically. They show the conditions under which the optimal control policies are identical through periods. Other models in this research stream include initial public offerings (Babich and Sobel, 2004), trade credit (Gupta and Wang, 2009), etc.

However, these studies have focused on the decision making process of individual firm. The interplay between supply chain partners as well as financial institutions are not captured. Realizing this shortcomings, some other papers investigate the interplay of financial and operations decisions between different firms and among supply chain members. Buzacott and Zhang (2004) analyze the interaction between a capital limited retailer and a bank. Dada and Hu (2008) consider a similar model and derive the loan scheme which coordinates the channel. Chen and Cai (2011) further analyze the case when a loan is provided by a third-party logistic(3PL) firm or the supplier. They show that under both schemes the supply chain profit outperforms the case when the loan is provided by a bank.

Zhou and Groenevelt (2008) consider a financial scheme in which the retailer borrows from the bank and pays the loan principal, and the supplier subsidizes the interests. They show that comparing with trade credit financing, such a loan financing scheme improves the profit of entire supply chain. Kouvelis and Zhao (2011) study the supplier's optimal price-only contract when facing a budget constrained retailer and bankruptcy cost. Kouvelis and Zhao (2012) also study a price-only contract with both budget constrained supplier and retailer. The retailer can either choose trade credit financing or bank financing. It is shown that under mild conditions, the retailer always prefers supplier financing to bank financing. Caldentey and Chen (2011) analyze a similar problem where supplier can use credit contract to finance a budget constrained retailer. They conclude that suppliers finance is preferable to bank financing. Yang and Birge (2011) extend the previous models to the case where supplier financing and bank financing can be jointly used. They also show that supplier financing is preferred to bank financing. They further validates that such inventory financing pattern through a sample of firm-level data. Chen and Gupta (2014) study the financing schemes for budget constrained supplier under consignment contract setting. They show that bank financing works as substitute to retailer's direct financing and complement to retailer's commitment financing. Tanrisever et al. (2012) investigate the value of reverse factoring where the OEM can work with a bank to reduce supplier's financing cost.

Some other papers examine the effect of different payment policies in a serial inventory system. Luo and Shang (2012) study a two-stage inventory system where both prepayment and delayed payment are allowed. They show the optimal inventory and payment decisions follow threshold policies, and a flexible payment scheme out performs a strict payment scheme. Song and Tong (2012) provide a generalized framework to study the joint inventory and cash management problem in a serial supply chain with different payment schemes. More recently, Chen et al. (2013) conduct a laboratory study on the effect of payment schemes on inventory decisions. They find the observations of inventory decisions are more consistent with the prospective accounting theory than expected-profit-maximizing model. Our model also considers the option of retailer's advance payment. Comparing with loan guarantee scheme, we show that the supplier is always willing to accept such a financing support. However, it benefits the retailer only when the financing cost is low and supplier's initial capital is relatively high.

Nevertheless, these studies have focused on the risks from the demand side. Few studies look at the joint impact of unreliable supply process and financial constraints. Some exceptions are as follows. Babich (2010), whose work shares some similarities with this study, considers manufacturer's joint capacity ordering and financial subsidy problem when facing a risky supplier. He identifies the conditions under which the subsidy policy follows a subsidize-up-to structure. Unlike his work which mainly stand at manufacturer's perspective, we consider both supplier's and retailer's interests, as well as bank's risk free return requirement. More recently, Tunca and Zhu (2014) compare buyer intermediated financing with commercial loans and factoring. They argue that buyer intermediated financing improves supply chain efficiency by transferring the risk among the supply chain partners and financing institutions in a more efficient way. Our model differs with this work in that we consider the loan guarantee scheme where the coverage ratio is decided by the retailer. A closely related work to ours is Wu et al. (2014). They also investigate the use of loan guarantee. While instead of looking at the impact of guarantee on supplier's profitability, they investigate the necessary guarantee level which meets bank's interest rate limit. In our base model, we assume the supplier can only choose whether to borrow or not. We then relax this assumption and allow voluntary compliance where the supplier can choose the production input level to maximize supplier's profit. We incorporate supplier's budget constraint in a Stackelberg model and discuss the supply chain performance with the usage of different financing schemes.

2.2.2 Unreliable Supplies

This study is also related the abundant literature on supply risk management. Three different approaches are usually adopted to model the supply uncertainty, namely random capacity, random yield and random disruption. Dada et al. (2007) study the supplier selection problem using general reliability assumptions. They conclude that cost has higher priority than reliability. A supplier should be selected only if all less-expensive suppliers are selected. Babich (2010) assumes that the supply capacity is affected by supplier's financial state. He studies buyer's optimal subsidy and ordering decisions.

In this study, we mainly explore the problems of random yield and random disruption. Li and Zheng (2006) consider the joint pricing and inventory decisions in a periodic review systems with random yield and demand. They show the optimality of a threshold type replenishment policy. The optimal production quantity and the optimal price in each period are both decreasing in the starting inventory. Serel (2008) investigates a supply chain with a retailer, an unreliable supplier and a reliable manufacturer. He analyses the retailer's ordering problem as well as the manufacturer's pricing problem. Güler and Bilgiç (2009) explore an assembly system with random yield and random demand. They assume forced compliance where the supplier's input level must equal to manufacturer's order quantities. They derive coordination contract which includes a penalty to the worst performing supplier. He and Zhang (2008) consider a two level supply chain where the unreliable supplier can decide the production quantity. They discuss different risk sharing mechanisms between the supplier and retailer. In a similar setting, Wang (2009) investigates the impact of vendor management inventory. Tang and Kouvelis (2014) study the contract coordination problem with random production yield and competing retailers. Li et al. (2013) study a similar problem and find the accept-all type of contract that coordinates the supply chain. They further show this contract is applicable to the situation of multi-suppliers. Similarly, our model also consider the problem of voluntary compliance. Unlike those studies which mainly investigate the supply chain coordination problem, we focus on the implications of different supplier's financial status and financial schemes.

Tomlin (2006) discusses the use of inventory control, sourcing, and acceptance strategies to cope with supply chain disruptions. Wang et al. (2010) compare the strategies of dual sourcing and process improvement when facing random capacity or random yield. They find that for random yield model, as supplier's cost heterogeneity increases, the attractiveness of process improvement reduces, while if the reliability heterogeneity is high, process improvement can be favored over dual sourcing. Dong and Tomlin (2012) investigate how a manufacturing firm manages disruption risk by purchasing business interruption insurance, investing in inventory, and availing of emergency sourcing. They establish conditions under which insurance can be complementary to inventory and emergency sourcing. In our model, it is shown that sourcing from backup supply and proving guarantee to the unreliable supplier are substitutes.

Yang et al. (2009) use mechanism design theory to investigate manufacturer's optimal contract when the supply disruption risk is private information to the supplier. Tang et al. (2013) consider a decentralized supply chain with endogenous disruption probability. They assume the disruption probability can be reduced by exerting costly efforts. The buyer can encourage the supplier's investment through financial subsidy. They show that when demand is stochastic, the buyer may use subsidy and increase ordering quantity at the same time. In our study, we take the supply uncertainties as uncontrollable process. Our model obtain similar results which shows that the retailer may need to provide guarantee and place a larger order size simultaneously to keep supplier's incentive align.

2.3 The Base Model

We consider a supply chain consists of a supplier, a retailer and a bank. The supplier has limited internal fund and the production process is subject to uncontrollable risk. The delivered quantity is a fraction ζ of the total production quantity which is distributed between $[0, 1]$, with pdf $g(\cdot)$ and cdf $G(\cdot)$. The assumption of random proportional yield is commonly used in the literature to model supply uncertainty (Rajaram and Karmarkar, 2002; Federgruen and Yang, 2009). Furthermore, we assume the supplier is budget constrained with initial capital level b. The initial capital can be fully used to finance the production process and the supplier has no other assets. Without loss of generality, the unit production cost is normalized to 1.

When retailer's order size is less than b, we assume forced compliance on the supplier's production decision, which means supplier's production size is the same as retailer's order size. If retailer's order is more than b, the supplier does not have sufficient internal fund to fulfill retailer's order. The supplier can decide whether or not to take a bank loan to finance the production. That is, if the borrowing cost is higher than the expected extra revenue, then the supplier only produces using the initial fund b; if the borrowing cost is less than the expected extra revenue, the supplier is then willing to take a loan $y - b$ from the bank and produces at y.

We assume the bank is risk neutral with risk-free interest rate r_f. This financing cost r_f can be regarded as deadweight cost caused by potential bankruptcy risk or other financial distress cost. It can also be viewed as transaction cost or financing fee. To focus on the operational transactions, we assume both retailer and the supplier have no other investment opportunities.

The retailer faces stochastic demand D with pdf $f(\cdot)$, cdf $F(\cdot)$ and ccdf $\bar{F}(\cdot)$. Both the wholesale price w and retail price to end customer p are exogenous. There is no penalty cost for shortage or salvage value for unsold inventory. In addition to the order quantity y, the retailer also decides loan guarantee level x. As stated above, the borrowing cost can be higher than the supplier's revenue margin, the loan guarantee is used to help the SME supplier to get a loan at lower interest rate. Detailed discussion of this loan guarantee is presented in bank's problem. We assume the retailer is fully aware of the supplier's financial status and yield distribution. After supplier's production yield realizes, retailer's order payment is first used as loan repayment.

2.3.1 Bank's Return and Loan Limit

Under the risk neutral assumption, the bank is indifferent between a fixed payment $(y-b)^+(1+r_f)$ and an uncertain payment with the same expected value. Denote the loan interest rate as r, the value of r is determined as:

$$\begin{aligned}(y-b)^+(1+r_f) &= E[\min\{wy\zeta + x((y-b)^+(1+r) - wy\zeta)^+, \\ &\quad (y-b)^+(1+r)\}] \end{aligned} \tag{2.1}$$

The RHS of Equation 2.1 represents the two possible scenarios of supplier's repayment. When the production yield is too low that retailer's payment upon order delivery is less than the loan principal plus loan interest, the bank will then collect the order payment $wy\zeta$ and retailer's guarantee payment $x((y-b)^+(1+r) - wy\zeta)^+$. When production yield is high enough such that $wy\zeta \geq (y-b)^+(1+r)$, the bank gets full repayment, and the supplier collects the rest amount $wy\zeta - (y-b)^+(1+r)$. We first establish several basic properties of bank's loan interest rate.

Lemma 2.1. *For loan size $(y-b)^+$, bank's loan interest rate has following properties:*

a. If $x = 1$, then $r = r_f$;

b. if $x < 1$, then $r \geq r_f$;

c. r is decreasing in x;

d. r is increasing in y.

Lemma 2.1 shows that the loan interest rate is increasing in both order size and guarantee level. We then look at the range of feasible loan size. Since the order size is decided by the retailer and the supplier can only choose whether to take a loan or not, we focus on retailer's feasible order quantity. The maximum loan size will be the difference between the maximum order size and b.

Lemma 2.2. *If $x > 0$ or $wE[\zeta] \geq 1 + r_f$, there is no ordering limit; if $x = 0$ and $wE[\zeta] < 1 + r_f$, the maximum feasible order size is $\frac{b(1+r_f)}{1+r_f-wE[\zeta]}$.*

Lemma 2.2 states the feasible order size, and consequently the maximum loan size. A special case is that the yield is a fixed fraction of the production quantity $\zeta_0(0 < \zeta_0 \leq 1)$. To the same with Lemma 2.2, there is no borrowing limit when $w\zeta_0 \geq 1 + r_f$ or $x > 0$, and there exists maximum order size otherwise. For the case $wy\zeta_0 \geq (y - b)(1 + r_f)$, the SME can fully repay the loan. The corresponding interest rate is always r_f, and a guarantee does not take effect. When the SME cannot repay the loan alone, $wy\zeta_0 < (y-b)(1+r_f)$ and $x > 0$, the interest rate is given by: $r = \frac{(1+r_f-x)(y-b)-w(1-x)y\zeta_0}{x(y-b)}$. It can be easily verified that the results of Lemma 2.1 still holds for the deterministic yield situation. As the deterministic yield problem can be viewed as a special case, we will then focus on the problem with uncertain yield. In addition, from now on, we restrict our discussion within the case where the order size is feasible. Furthermore, besides the debt capacity decided by the bank, we also consider supplier's incentive issue which constrained retailer's decisions more tightly. The detailed analysis is presented as follows.

2.3.2 Supplier's Profit Requirement

We first consider the case when supplier's production quantity is the same as retailer's order size. We then analyze the impact of this production policy on supplier's profit and derive retailer's necessary guarantee level. Define $t(x, y)$ as the threshold yield level such that retailer's order payment can fully repay the loan:

$$t(x, y) = \frac{(y - b)^+(1 + r(x, y))}{wy}$$

Equation 2.1 can be rewritten as:

$$(y - b)^+(1 + r_f) = E[\min\{xwyt + (1 - x)w\zeta y, wyt\}] \tag{2.2}$$

$$=xwyt + (1-x)wyE[\min\{\zeta, t\}]$$

Lemma 2.3. *For $y > b$, the threshold t is decreasing in b and x, while increasing in y and r_f.*

According to the definition of t, the probability that the supplier can fully repay the loan is $\bar{F}(t)$. On the other hand, the default probability is $F(t)$. Lemma 2.3 shows that when supplier's initial capital is higher, it is less likely for the supplier to go bankruptcy, while it is more likely that the supplier will fully repay the loan. The intuition for x, y and r_f is similar.

With the notation of threshold yield level, the supplier's expected profit at the end of period is expressed as:

$$\pi_s(x, y) = wy \int_{t(x,y)}^{1} (\zeta - t(x, y))dG(\zeta) - \max\{y, b\} \tag{2.3}$$

or, equivalently

$$\pi_s = \begin{cases} wE[\zeta]y - y, & \text{if } y \leq b \\ \int_t^1 wy(\zeta - t)dG(\zeta) - b, & \text{if } y > b \end{cases}$$

When $y > b$ and the financing cost is high, supplier's revenue increment from order fulfilling can be less than the corresponding financing cost. In such case, the supplier is better off to produce with the internal capital and not to take a loan. To keep supplier's interest in line, the retailer can reduce supplier's borrowing cost through providing financial support. In other words, the retailer shares the supply risk through loan guarantee scheme so that the supplier is willing to access the financial market to launch a larger production quantity. We consider the supplier's profit requirement constraint (or supplier participation constraint):

$$\pi_s(x, y) \geq \pi_s(x, b), \quad \text{for } y \geq b \tag{2.4}$$

where x denotes retailer's guarantee level, y and b denotes supplier's production decision.

When supplier's production quantity equals to y and no guarantee is provided, according to Equation 2.2 and 2.3, supplier's profit function can be written as:

$$\pi_s = wyE[\zeta] - \max\{y, b\} - (y - b)^+(1 + r_f) \tag{2.5}$$

It can be obtained that if $y > b$ and $x = 0$, the supplier's profit is increasing in y if $wE[\zeta] > 1+r_f$; and decreasing in y otherwise. As a result, when $wE[\zeta] < 1+r_f$, the supplier is not willing to produce more than b if the retailer provides zero guarantee. The effect of guarantee on supplier's profit is summarized as follows.

Lemma 2.4. *Given $y > b$, suppose the supplier takes bank loan to produce at y, supplier's profit is increasing in x and decreasing in r_f.*

With positive guarantee, the financing cost and supply risk are shared between the retailer and supplier. Lemma 2.4 shows that the retailer's guarantee does improve supplier's expected profit. Denote r_b as the loan interest when $r_f = wE[\zeta] - 1$ and $x = 0$, and r_b is a function of y. The following result reveals the effect of loan interest rate on the supplier's profit. Again, we assume the supplier always uses a bank loan to produce at y.

Lemma 2.5. *For $y > b$, when supplier's production quantity is y, supplier's profit is decreasing in r. In addition, when $r = r_b$, $\pi_s(y) = \pi_s(b)$.*

Lemma 2.5 shows that if the loan interest rate r is greater than the break even interest r_b, the supplier becomes worse off when taking a bank loan to produce at y. According to Lemma 2.1, as supplier's loan interest rate is decreasing in loan guarantee level, we can now characterize buyer's feasible ordering quantity and guarantee level with consideration of supplier's participation constraint.

Proposition 2.1. *Consider the case when $y > b$: if $wE[\zeta] \geq 1 + r_f$, $\pi_s(y) \geq \pi_s(b)$ for any $0 \leq x \leq 1$; if $wE[\zeta] < 1 + r_f$, there exists y_{lb} such that*

1. for $b < y < y_{lb}$, $\pi_s(x, y) < \pi_s(b)$ for any guarantee level $x \in [0, 1]$;

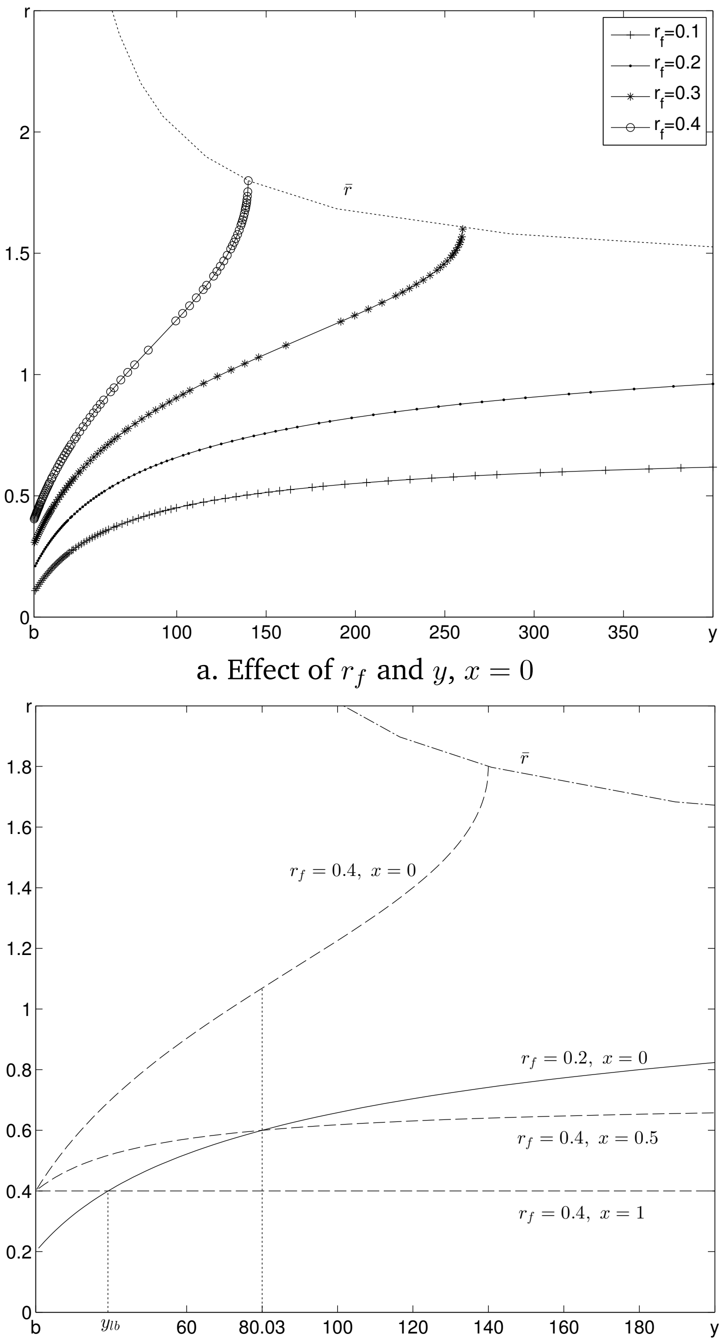
2. for $y \geq y_{lb}$, there exists $\underline{x}$ such that for $x > \underline{x}$, $\pi_s(x, y) \geq \pi_s(b)$;

where $y_{lb} = \{y \mid (y - b)wE[\zeta] = E[min\{wy\zeta, (y - b)(1 + r_f)\}]\}$, and $\underline{x} = \{x \mid (y - b)\,(1 + r_f) = E[min\,\{x(y - b)(1 + r_b) + (1 - x)w\zeta y, (y - b)(1 + r_b)\}]\}$.

When bank's risk free rate is low, it is profitable for the supplier to take a loan and produce at y even if there is no guarantee. While if the risk free rate is high, the supplier is willing to borrow only if $y > y_{lb}$ and $x > \underline{x}$. The lower bound of the order quantity is due to the fact that even a 100% guarantee is provided, the loan interest rate can only be reduced to r_f. Therefore, for order size between (b, y_{lb}), taking bank loan always makes the supplier worse off. This result implies that retailer's feasible ordering quantity is $[0, b] \cup [y_{lb}, +\infty)$.

Given an order size $y \geq y_{lb}$, the minimum guarantee level $\underline{x}(y)$ can be found in two steps. The first step is to determine the break even loan interest rate r_b for the same order size. After r_b is obtained, we can then take r_b into Equation 2.1 and find the corresponding $\underline{x}$: $\{x \mid (y_1 - b)\,(1 + r_f) = E[min\,\{x(y_1 - b)(1 + r_b) + (1 - x)w\zeta y_1, (y_1 - b)(1 + r_b)\}]\}$.

Example: Let $w = 2.4$, $b = 20$, and the production yield uniformly distributed between $[0, 1]$. The conclusion of Proposition 2.1 is graphed in Figure 2.1. Consider the loan interest as a function of bank's risk free rate, retailer's guarantee level and order size. Figure 2.1.a illustrates the relation between bank's loan interest rate and retailer's order size when no guarantee is provided. The loan interest rate is increasing in both r_f and y. If $r_f = wE[\zeta] - 1$ and $x = 0$, the supplier's profit is indifferent between taking a bank loan or not. When $y \to \infty$, r_b asymptotically approaches to $w-1$. As stated in Lemma 2.2, when $r_f > wE[\zeta]-1$ and $x = 0$, there exists maximum loan size is $\bar{y} - b$. We denote the corresponding loan interest rate as $\bar{r}$, where $\bar{r} = \frac{w\bar{y}}{\bar{y}-b} - 1$. Furthermore, $\bar{r}$ is decreasing in r_f with lower limit $w - 1$.



a. Effect of r_f and y, $x = 0$

b. Effect of positive x

Figure 2.1: Effect of r_f, x and y on loan interest rate

Given $y > b$ and $r_f > wE[\zeta] - 1$, the loan interest rate without guarantee is greater than r_b and $\pi_s(r_f, 0, y) < \pi_s(b)$. In such case, supplier's borrowing cost is higher than the expected revenue increment. The buyer needs to provide enough guarantee so that supplier's profit with borrowing is no less than $\pi_s(b)$. In Figure 2.1.b, for $r_f = 0.4$ and $wE[\zeta] - 1 = 0.2$, we have $r(0.4, 0, y) > r_b(0.2, 0, y)$. When order size is 80.03, the supplier is willing to take a loan only if $r(0.4, x, 80.03) \leq r_b(0.2, 0, 80.03)$. The retailer needs to provide guarantee which is no less than $\underline{x} = 0.5$. Figure 2.1.b also shows that for order size between (b, y_{lb}), $r(0.4, x, y)$ is higher than $r_b(0.2, 0, y)$ even if a fully guarantee is provided. This observation validates the result in Proposition 2.1, which means loan guarantee cannot meet the supplier's profit requirement in this ordering range.

Corollary 2.1. *$\underline{x}$ is increasing in b and r_f, decreasing in w and y.*

Corollary 2.1 points out that when the supplier has more initial capital or the bank's risk free rate is higher, the minimum guarantee level required by the supplier also increases. On the other hand, when supplier's wholesale price is high or the order quantity is high, the required minimum guarantee level is lower. We take supplier's initial capital for further explanation. When b increases, the loan repayment is more likely to be totally borne by the supplier. Although the loan size is reduced(both principal and interest), the supplier would take more proportion of the financing cost. Taking a bank loan is no longer attractive for the supplier unless the retailer increases the guarantee level. The impact of r_f, w and y are similar.

We confine our discussion to the case where guarantee level is between $[0, 1]$. When the minimum guarantee require by the supplier is equal to 1, the corresponding order quantity is defined as the lower bound quantity for viable loan scheme. Corollary 2.2 shows how the lower bound changes according to different parameter values.

Corollary 2.2. *y_{lb} is increasing in b and r_f, and decreasing in w.*

As b and r_f increases, the minimum order quantity to keep supplier's incentive compatible is also increasing. The retailer needs to increase guarantee level or order with larger quantity. When retailer's desired ordering quantity is less than y_{lb}, since the guarantee level cannot be more than one hundred percent, the retailer either enlarges the order size to make a loan attractive to the supplier or just lets supplier produce with the initial capital. The impact of wholesale price w is to the opposite of b and r_f.

2.3.3 Retailer's Problem

In this subsection, we consider retailer's profit optimization problem. Before the selling season, the retailer needs to decide the loan guarantee level x and order quantity y. The retailer's profit maximization problem is given by:

$$\begin{aligned}
\Pi_r = \max_{x,y} \;\; & \mathbb{E}_\zeta \mathbb{E}_D[-wy\zeta + p\min\{y\zeta, D\} - xwy(t-\zeta)^+] \\
s.t. \;\; & 0 \leq x \leq 1 \\
& y \geq 0 \\
& \pi_s(x, y) \geq \pi_s(0, b) \quad \forall y \in (b, +\infty) \\
& (y-b)(1+r_f) = E[\min\{wy\zeta + xwy(t-\zeta)^+, wyt\}] \quad \forall y \in (b, +\infty)
\end{aligned}$$

The expected profit function can also be written as:

$$\begin{aligned}
\pi_r(x, y) &= \pi_r(0, y) - xwyE_\zeta[(t-\zeta)^+] \\
&= \pi_r(0, y) - xwy \int_0^t (t-\zeta)dG(\zeta)
\end{aligned} \tag{2.6}$$

The first term in Equation 2.6 represents retailer's profit when no guarantee is offered and supplier's production quantity is y. The second term represents the expected guarantee cost. When the supplier has sufficient capital, the retailer only needs to decide the order quantity. It is easy to verify that $\pi_r(0, y)$ is concave in y. When the supplier has limited capital, to derive retailer's optimal decisions, we first investigate the optimal guarantee decision for a given order quantity.

Lemma 2.6. *Given $y > b$ and the supplier decides to produce y units, the retailer's profit is decreasing in x.*

Based on Lemma 2.6, several direct conclusions can be drawn. If $r_f \leq wE[\zeta]-1$, as the supplier is willing to take a bank loan without retailer's guarantee, and the guarantee cost is non-negative, the retailer should not provide positive guarantee in such case. The optimal order quantity will be y^* which uniquely maximize $\pi_r(0, y)$. For $r_f > wE[\zeta] - 1$, it has been shown that the possible production quantity is $[0, b] \cup [y_{lb}, +\infty)$. Combining the fact that the loan interest rate is decreasing in x and the supplier is willing to borrow only when $r \leq r_b$, we can obtain that for order size $y > y_{lb}$, the retailer's optimal decision is to provide the minimum guarantee $\underline{x}(y)$.

After determining retailer's optimal guarantee decision, we then look at the case $r_f > wE[\zeta] - 1$ and derive retailer's optimal order quantity. The retailer's profit with optimally decided guarantee is given by:

$$\pi_r = \begin{cases} \pi_r(0, y), & y \leq b \\ \pi_r(0, b), & b < y < y_{lb} \\ \pi_r(\underline{x}(y), y), & y \geq y_{lb} \end{cases}$$

where $\underline{x}(y)$ and y_{lb} follows previous definition.

For $y > y_{lb} > b$, define $t_b = (y-b)(1+r_b)/wy$. According to Equation 2.1 and the definition of r_b, we have:

$$(y-b)wE[\zeta] = wy \int_0^{t_b} \zeta dG(\zeta) + wyt_b\bar{G}(t_b) \tag{2.7}$$

and $(y-b)(1+r_f) = wy \int_0^{t_b} \zeta dG(\zeta) + wyt_b\bar{G}(t_b) + \underline{x}(y)wy \int_0^{t_b} (t-\zeta)dG(\zeta)$

The buyer's expected guarantee cost can now be expressed as:

$$\underline{x}(y)\ wy \int_0^{t_b} (t_b - \zeta)dG(\zeta) = (y-b)(1+r_f - wE[\zeta]) \tag{2.8}$$

Consequently, the buyer's expected profit with optimal guarantee is:

$$\begin{aligned}\pi_r(\underline{x}(y), y) &= \pi_r(0, y) - \underline{x}(y)wy \int_0^{t_b} (t_b - \zeta) dG(\zeta) \\ &= \pi_r(0, y) - (y - b)(1 + r_f - wE[\zeta])\end{aligned}$$

The above equation shows that when the minimum guarantee $\underline{x}$ is provided, retailer's unit guarantee cost is $1 + r_f - wE[\zeta]$. It is equivalent to the case that the retailer pays the interest cost higher than supplier's expected revenue. The following proposition summarizes retailer's optimal guarantee and ordering decisions.

Proposition 2.2. *If $r_f \leq wE[\zeta] - 1$ or $b \geq y^*$, it is optimal for the retailer to order y^* units and provide no guarantee. If $r_f > wE[\zeta] - 1$ and $b < y^*$, define $y' \equiv \{y \mid \int_0^1 p\zeta\bar{F}(y\zeta)g(\zeta)d\zeta - (1 + r_f) = 0\}$, retailer's optimal decision is as follows:*

a. for $b < b_1$, order y' and provides guarantee $\underline{x}(y')$;

b. for $b_1 < b < b_2$, order y_{lb} and provides guarantee $\underline{x}(y_{lb})$;

c. for $b_2 \leq b < y^*$, order b and provides no guarantee.

where $b_1 \equiv \{b \mid y_{lb}(b) = y'\}$, and $b_2 \equiv \{b \mid \pi_r(\underline{x}, y_{lb}(b)) = \pi_r(0, b)\}$.

As can be seen, the retailer's optimal ordering and guarantee decisions are largely dependent on the SME's initial capital level. In the following content, we discuss the further implications and compare the results under different problem settings.

2.4 Backup Supply and Advance Payment

2.4.1 Impact of Backup Supply

In the previous analysis, we have discussed the sourcing and financing problem with a single supplier. In facing supply uncertainties, some manufacturers turn to multisourcing or backup sourcing to mitigate the supply risks. In this section, we consider the sourcing problem as a two stage decision process. In the first stage, the retailer announces guarantee level x and order size y_1 to the unreliable SME supplier. After observing the production yield, the retailer can choose to place another order y_2 to the backup suppler. The backup supplier is assumed to be perfectly reliable with wholesale price v. The total stocking level is then $y_1\zeta + y_2$. Since SME supplier's profit is only affected by x and y_1, our previous conclusion of supplier's loan participation decision still holds in the two stage problem. The retailer's problem with backup supply is as follows:

$$
\begin{aligned}
\Pi_r = \max_{x,y_1}\ & E_\zeta\{-wy_1\zeta - xwy_1(t-\zeta)^+ + \max_{y_2} E_D[-vy_2 + p\min\{y_1\zeta + y_2, D\}|\zeta]\} \\
s.t.\ \ & 0 \le x \le 1 \\
& y_1 \ge 0 \\
& y_2 \ge 0 \\
& \pi_s(x, y_1) \ge \pi_s(0, b) \ \ \forall y_1 \in (b, +\infty) \\
& (y_1 - b)(1 + r_f) = E[\min\{wy_1\zeta + xwy_1(t-\zeta)^+, wy_1 t\}] \ \ \forall y_1 \in (b, +\infty)
\end{aligned}
$$

We first characterize retailer's optimal policy in the second stage.

Lemma 2.7. *For any guarantee level x, and first stage order quantity y_1, the optimal ordering policy in the second stage is order up to B, where $B \equiv F^{-1}(\frac{p-v}{p})$.*

Lemma 2.7 shows that retailer's second stage decision follows standard newsvendor solution. The optimal stocking up to level is independent of the first stage decisions and yield realization. The actual backup order quantity depends on the delivery quantity of the SME supplier. When the delivery quantity $y_1\zeta$ is less than the optimal ordering up to level B, the optimal backup ordering quantity is $B - y_1\zeta$. On the other hand, if the SME supplier's delivery quantity is higher than B, the retailer should not place a backup order. Based on the optimal ordering policy in the second stage, the retailer's problem can be written as:

$$\max_{x,y_1} E_\zeta\{-wy_1\zeta - xwy_1(t-\zeta)^+ + E_D[p\min\{\max\{B, y_1\zeta\}, D\}] - v(B - y_1\zeta)^+]\}$$

We now consider the optimal guarantee level for given y_1. As can be seen, the retailer's expected guarantee cost is the same as the single sourcing problem. Therefore, Lemma 2.6 still holds for the dual sourcing problem. When $wE[\zeta] \geq 1 + r_f$, the supplier's expected profit is increasing in retailer's ordering quantity even without guarantee, and the retailer should not provide positive guarantee. When $wE[\zeta] < 1 + r_f$, as discussed in the previous section, the supplier is willing to take bank loan only when $y \geq y_{lb}$ and $x > \underline{x}$. In such case, retailer's problem is further rewritten as:

$$\begin{aligned}\max_{y_1 > y_{lb}} \quad & E_\zeta\{-wy_1\zeta - (y_1 - b)^+(1 + r_f - wE[\zeta]) \\ & + E_D[p\min\{\max\{B, y_1\zeta\}, D\}] - v(B - y_1\zeta)^+]\}\end{aligned}$$

Notice that when the first stage order quantity y_1 is less than the second stage ordering up to level B, a positive backup order is always needed. Since the back up supply is perfect reliable, an interesting question would be whether the retailer should order from the unreliable supplier.

Lemma 2.8. *If $w > v$, the retailer's optimal first stage ordering quantity is zero.*

A special case is when $v = w$. If the retailer doesn't need to provide guarantee, any first stage order size between $[0, B]$ is optimal as the marginal first stage ordering cost is equal to the backup supplier's wholesale price. If bank's risk free interest rate is high, $r_f > wE[\zeta] - 1$, then the optimal first stage ordering quantity is between 0 to $\min\{b, B\}$. We focus on the case $v > w$ and derive retailer's optimal first stage decisions.

Proposition 2.3. *Given $v > w$, define y_1^* as the solution to: $\int_0^{B/y_1} v\zeta dG(\zeta) + \int_{B/y_1}^1 p\bar{F}(y_1\zeta)\zeta dG(\zeta) - wE[\zeta] = 0$; y_1' as the solution to: $\int_0^{B/y_1} v\zeta dG(\zeta) + \int_{B/y_1}^1 p\bar{F}(y_1\zeta)\zeta dG(\zeta) - 1 - r_f = 0$.*

1. *If $r_f \leq wE[\zeta] - 1$ or $b \geq y^*$, it is optimal to order y_1^* units in the first stage and provide no guarantee.*

2. *If $r_f > vE[\zeta] - 1$ and $b < y_1^*$, it is optimal to order b units in the first stage and provide no guarantee.*

3. *If $wE[\zeta] - 1 < r_f < vE[\zeta] - 1$ and $b < y_1^*$, the retailer's optimal first stage decisions are:*

    a. for $b < b_1$, order y_1' and provide guarantee $\underline{x}(y_1')$;

    b. for $b_1 < b < b_2$, order y_{lb} and provide guarantee $\underline{x}(y_{lb})$;

    c. for $b_2 \leq b < y_1^*$, order b and provide no guarantee.

where $b_1 \equiv \{b \mid y_{lb}(b) = y_1'\}$, and $b_2 \equiv \{b \mid \pi_r(\underline{x}, y_{lb}(b)) = \pi_1(0, b)\}$.

Notes: For $r_f = wE[\zeta] - 1$, we assume the supplier is cooperative. The assumption implies that when the order size is greater than b, the supplier is willing to take bank loan to produce more even if the expected profit remains the same as the no borrowing case. This assumption is consistent with the loan participation constraint.

Figure 2.2 illustrates the optimal first stage policy for different levels of supplier's initial capital and bank's risk free rate. In summary, when supplier's initial fund is high or bank's risk free interest rate is less than $wE[\zeta] - 1$, the retailer should not provide guarantee. The optimal order quantity is equal to the case when the supplier has no financial constraint. If supplier's initial capital is less than y_1^* and bank's risk free interest is higher than the $vE[\zeta] - 1$, then the retailer should provide no guarantee and simply order b from the SME supplier. If r_f is between $(wE[\zeta] - 1, vE[\zeta] - 1)$, depending on the SME supplier's initial capital,

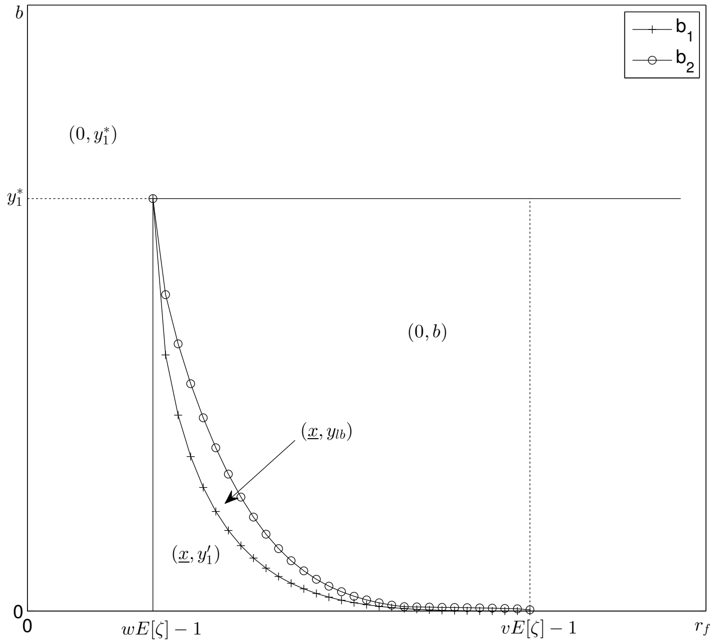


Figure 2.2: Retailer's optimal first stage ordering policies

retailer's optimal decision falls in one of three different cases. For b less than b_1, it is optimal for the retailer to order y'_1 and provide corresponding guarantee $\underline{x}(y'_1)$. If b is between (b_1, b_2), because of supplier's loan participation constraint, retailer's optimal decision is to order $y_{lb}(b)$ and provide guarantee $\underline{x}(y_{lb}(b))$. Finally, if $b_2 < b < y^*$, the retailer should not provide guarantee and simply order b.

Comparing with the case without backup supply, the condition for profitable guarantee is more tight, which means the retailer is less likely to use guarantee when a backup supply is available. Further discussions are provided as follows.

Numerical Study

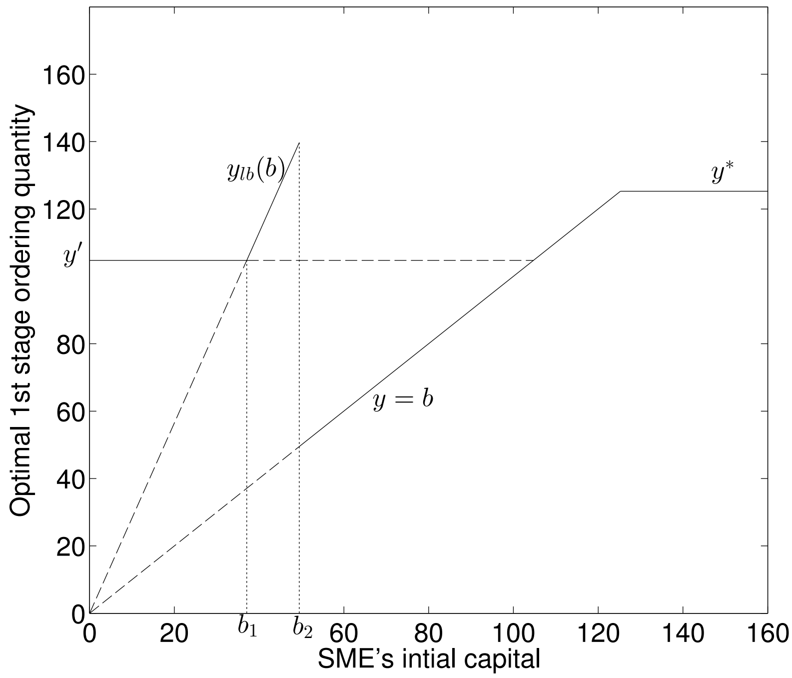
In this subsection, we use numerical study to examine the effect of bank's risk free rate, supplier's initial capital and yield condition. Without further specification, the following parameters are set as base values for the numerical studies:

$p = 5$, $v = 3$, $w = 3$, $r_f = 0.4$.

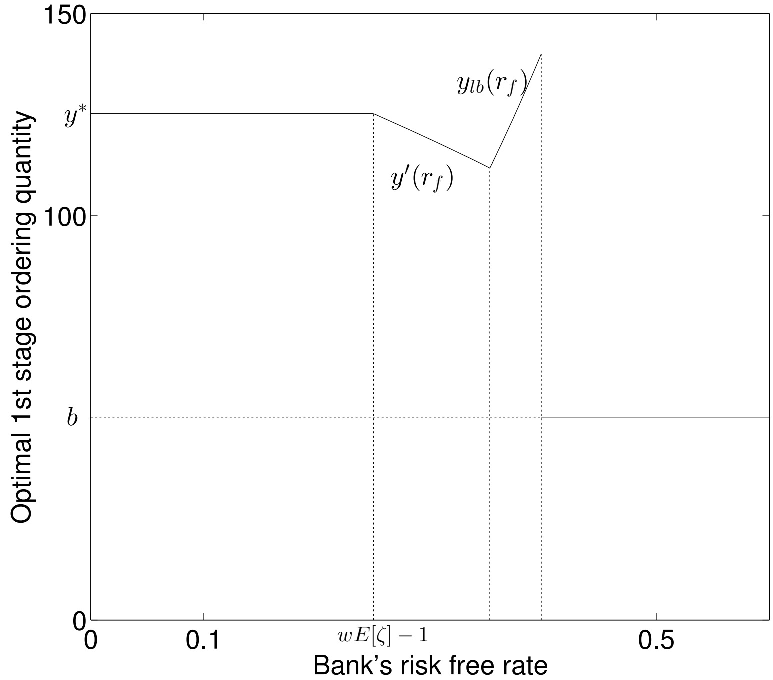
We assume the supplier's production yield follows Beta distribution, $B(2,2)$. The end demand distribution faced by the retailer is Weibull$(100, 2)$. Since the optimal decision in the second stage is always ordering up to $F^{-1}(\frac{p-v}{p})$, we focus on the first stage decisions. For notational simplicity, we omit the subscript 1 in the figures unless confusion may arise.

Figure 2.3 shows how the optimal first stage ordering quantity changes according to different b and r_f. As stated in Proposition 2.3, when r_f is less than $wE[\zeta]-1$, the optimal policy is always to order y_1^*. When r_f is higher than $vE[\zeta]-1$, it is optimal to order min $\{b, y_1^*\}$. In figure 2.3.a, we assume $wE[\zeta]-1 < r_f < vE[\zeta]-1$ and investigate the impact of supplier's initial capital level. Since y_{lb} is increasing in b and y_1' is constant in b, the retailer is forced to order at y_{lb} for $b_1 < b < b_2$. As b further increases, loan guarantee becomes less profitable and the retailer will just order b. When $b \geq y_1^*$, the retailer can order freely as the no budget constraint case. In Figure 2.3.b, we assume b is relatively low and consider the impact of r_f. If r_f is less than $wE[\zeta]-1$, the retailer can order at y_1^*. When $1+r_f$ is higher than the supplier's marginal revenue, the retailer needs to provide guarantee. Since y_1' is decreasing in r_f while y_{lb} is increasing in r_f, there exists critical risk free interest level above which y_{lb} is greater than y'. If r_f further increases, offering guarantee becomes less profitable and the retailer will only order b units.

An interesting question to the retailer is that whether a more wealthy supplier is always preferable. Similar to the previous analysis, we focus on the situation when r_f is between $(wE[\zeta]-1, vE[\zeta]-1)$. The results are shown in Figure 2.4, where the solid line represents retailer's expected profit. When $b < b_1$, it is optimal for the retailer to order y_1' and provide corresponding guarantee. As b increases, the guarantee cost decreases and the expected profit increases. While for $b \in (b_1, b_2)$, because of supplier's loan participation constraint, ordering y_1' becomes infeasible. To align the supplier's incentive, the retailer enlarges the ordering quantity to y_{lb} with one hundred percent guarantee. When $b \in [b_2, y_1^*]$,



a. Effect of b



b. Effect of r_f

Figure 2.3: Effect of b and r_f on retailer's optimal firs stage order size

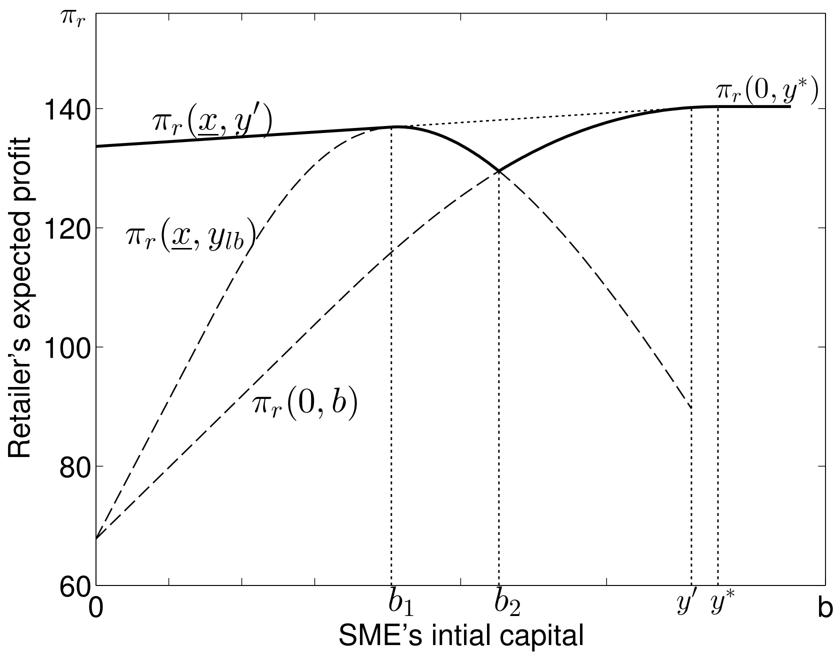
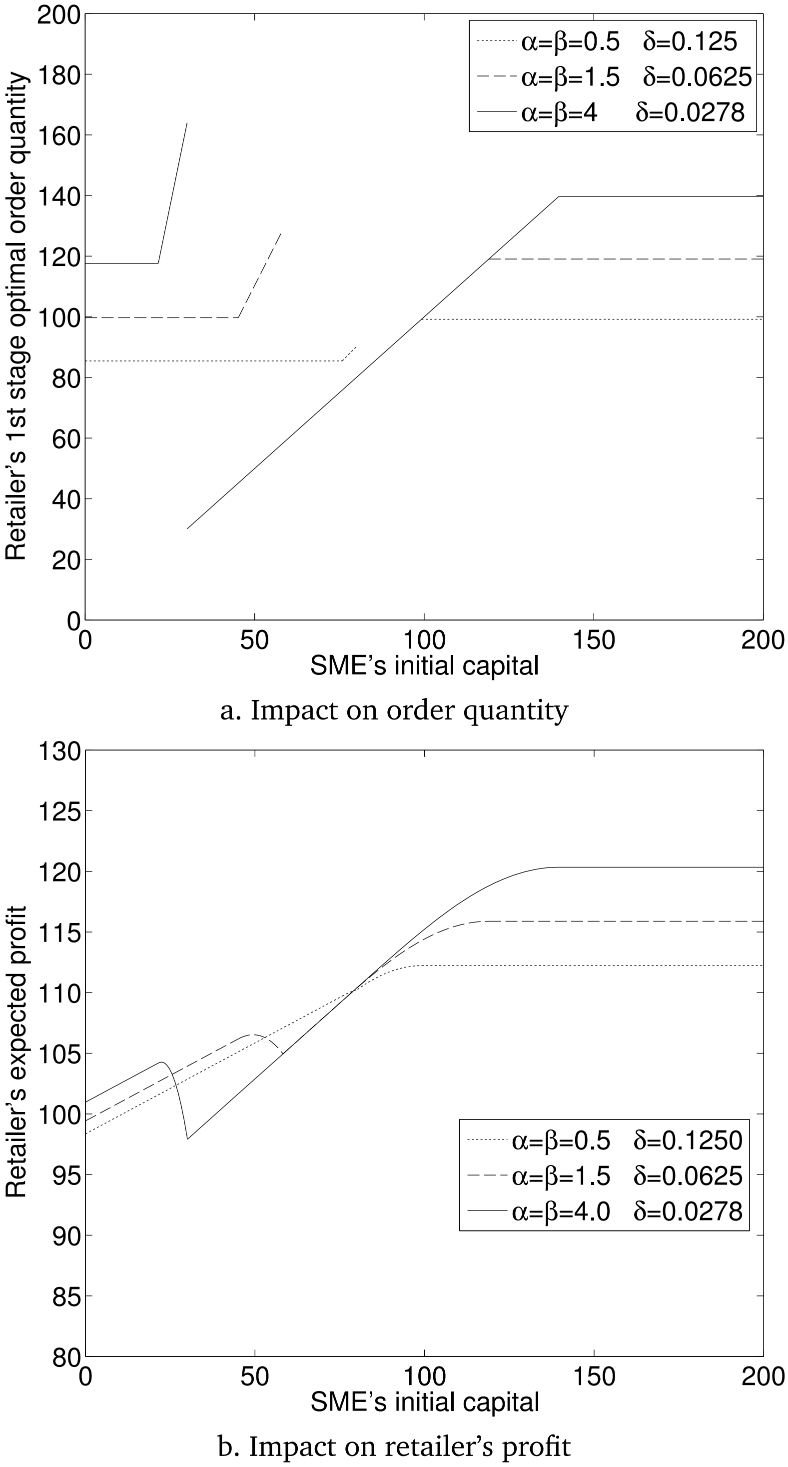


Figure 2.4: Effect of supplier's initial capital on retailer's expected profit

offering guarantee is not profitable for the retailer. The retailer should order b and provide no guarantee. In this case, retailer's profit is increasing in b. In summary, when $r_f \in (wE[\zeta] - 1, vE[\zeta] - 1)$, due to supplier's loan participation constraint, retailer's profit is non-monotone in supplier's initial capital level. The retailer can be worse off when with a more wealthy supplier. In practice, a wealthy supplier may be unwilling to bare a debt and take the risk of bankruptcy, while a poor supplier might be more risky as he has few to lose.

Figure 2.5 illustrates the impact of yield variance on retailer's optimal ordering quantity as well as the expected profit. The expected yield rate is fixed at 0.5, and we vary the standard deviation. The numerical study shows that both y_1' and y_1^* are decreasing in the yield variance. Nevertheless, the thresholds for switching ordering policy are also changing. Figure 2.5 suggests that the threshold b_1 is increasing in yield variance while b_2 is decreasing.

According to Figure 2.5.b, the retailer is better off with a low yield variance supplier when the supplier's initial capital is relatively low or high. While as b is in between, a supplier with lower yield variance may require a larger order quantity and higher guarantee level, which reduces retailer's benefit from



a. Impact on order quantity

b. Impact on retailer's profit

Figure 2.5: Effect of yield variance on retailer's optimal order size and expected profit

providing guarantee. In such case, the retailer could be better off with a supplier which has higher yield variance. Figure 2.5.b also depicts that if the retailer chooses to order b and b is less than the optimal second stage order up to level, then the supplier's yield condition does not affect retailer's overall profit.

Figure 2.6 shows the impact of the backup supplier's wholesale price. It is observed that both b_1 and b_2 are increasing in v. Therefore, the retailer is more likely to use guarantee when facing a more expensive backup source. It is also noticed that in Figure 2.2, b_1 and b_2 are decreasing in r_f for $r_f \in [wE[\zeta] - 1, vE[\zeta] - 1]$. Hence, the option of using loan guarantee and backup source are complementary to each other. Figure 2.6.b shows both y_1' and y_1^* are increasing in v. The results suggest that, for $b \leq b_1$ or $b \geq y_1^*$, the retailer tends to order more from the SME supplier when the backup source is more expensive.

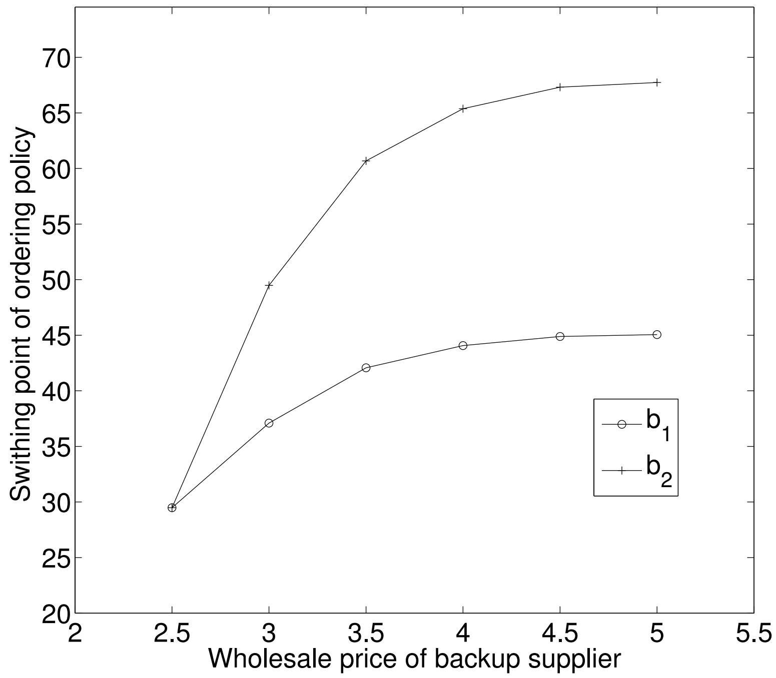
2.4.2 Retailer Financing with Advance Payment

Apart from using loan guarantee, the retailer can also support a budget constrained supplier through other financing schemes. A prevalent approach is advance payment or prepayment. Hereafter, we use the term advance payment to refer to retailer's payment to the supplier before order delivery.

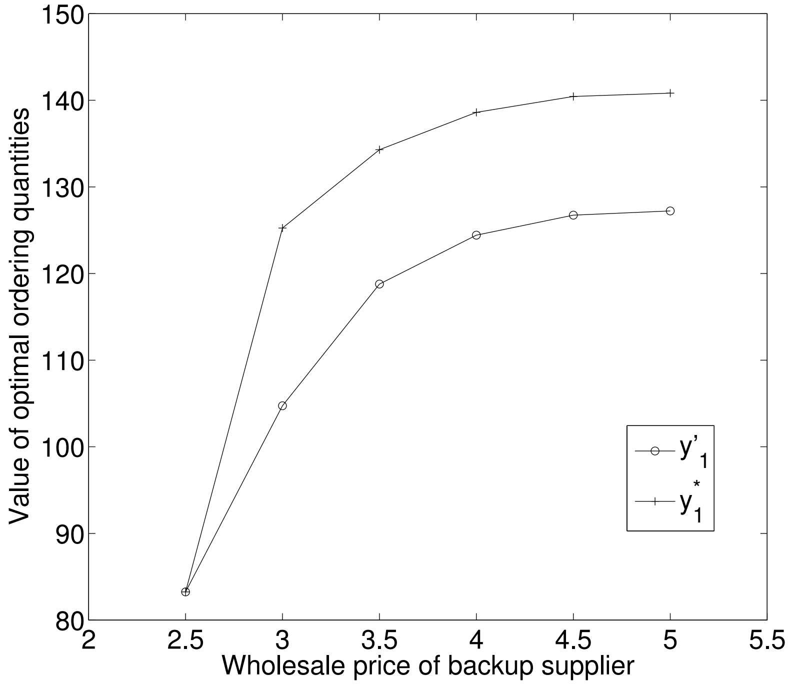
We consider a stylized model where no other financing access is available to the SME supplier. For an order size y_1 greater than the SME's initial capital level b, to let the supplier have sufficient funds for production, the retailer needs to pay $y_1 - b$ in advance. After supplier's production yield is realized, the retailer pays the rest amount $(wy_1\zeta - y_1 + b)^+$.

Similar to our discussion with loan guarantee scheme, the supplier preserves the participation constraint. The supplier accepts the advance payment and sets the production quantity at y_1 only when the expected profit is higher than the case without advance payment and produces b.

Supplier's problem



a. Impact on b_1 and b_2



b. Impact on y_1' and y_1^*

Figure 2.6: Effect of backup supply price on retailer's first stage ordering policies

$$\pi_s = \begin{cases} wy_1E[\zeta] - y_1, & if\ y_1 \le b \\ E[(wy_1\zeta - y_1 + b)^+] - b, & if\ y_1 > b \end{cases}$$

For $y_1 > b$, define $t = \frac{y_1 - b}{wy_1}$, we have:

$$\begin{aligned} \frac{\partial \pi_s}{\partial y_1} &= w\int_t^1 (\zeta - t)dG(\zeta) - \frac{b}{y_1}\bar{G}(t) \\ &= \int_t^1 (w\zeta - 1)dG(\zeta) \\ &= wE[\zeta] - 1 + \int_0^t (1 - w\zeta)dG(\zeta) > 0 \end{aligned}$$

where the last inequality follows from the fact that $1 - wt = \frac{b}{y_1} > 0$.

Therefore, with the advance payment, supplier's profit is increasing in order size y_1. The conclusion implies that the supplier should always accept retailer's advance payment and produce y_1.

Buyer's problem

For the retailer, the optimal second stage decision is still ordering up to B. Since the advance payment is paid at the first stage, the retailer would incur additional deadweight costs such as SME's bankruptcy cost or financial distress cots. We denote the cost as r_a which is a fraction of the advance payment, the total cost for the retailer is then $(y_1 - b)(1 + r_a)$. After some algebraic manipulation, the retailer's expected profit by the end of demand realization is expressed as:

$$\pi_r^a(y_1) = \pi_1(0, y_1) - wy_1\int_0^t (t - \zeta)dG(\zeta) - (y_1 - b)r_a$$

The second term represents the potential loss due to supplier's production uncertainty.

$$\begin{aligned} \frac{\partial}{\partial y_1}\pi_r^a &= \frac{\partial \pi_1(0, y_1)}{\partial y_1} - \int_0^t (1 - w\zeta)dG(\zeta) \\ \frac{\partial^2}{\partial y_1^2}\pi_r^a &= \frac{\partial^2 \pi_1(0, y_1)}{\partial y_1^2} - \frac{b^2}{y_1^3}g(t) \end{aligned}$$

It has been proven that $\pi_1(0, y_1)$ is concave in y_1, since the second term is also concave in y_1 and the last term is linear, it can be concluded that $\pi_r^a(y_1)$ is concave in y_1. Denote y_1^a as $\{y_1 \mid \frac{\partial}{\partial y_1}\pi_{ra} = 0\}$, the following lemma characterizes retailer's optimal decision.

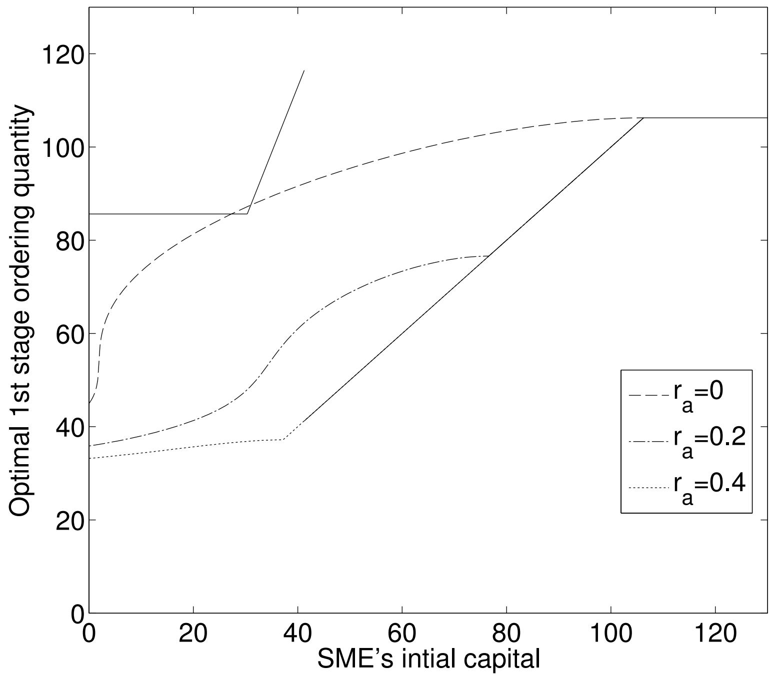
Lemma 2.9. *With advance payment, retailer's optimal decisions are:*

1 *if $b < y_1^a$, order y_1^a with advance payment $y_1^a - b$;*

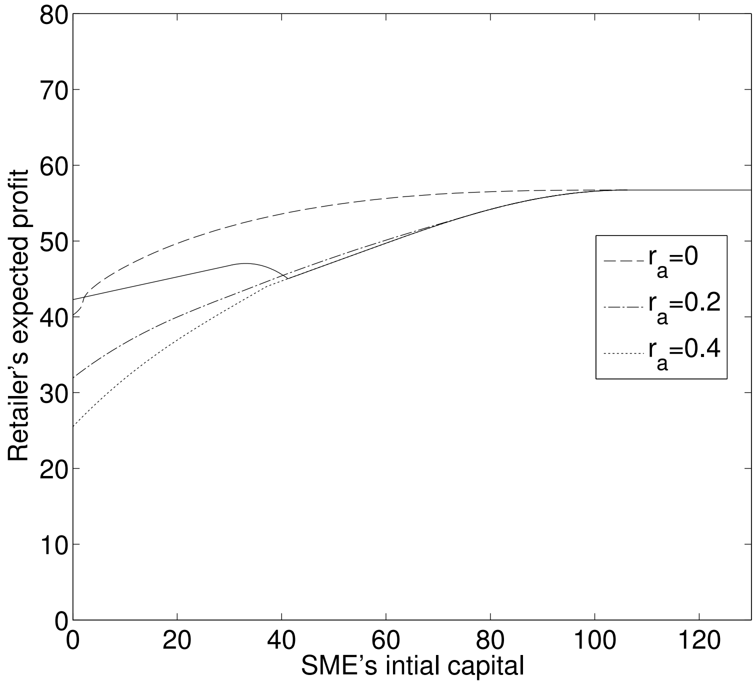
2 *if $y_1^a < b < y_1^*$, order b;*

3 *if $b > y_1^*$, order y_1^*.*

In Figure 2.7, we compare the performance of advance payment and loan guarantee schemes. Bank's risk free rate is fixed at 0.4 and the corresponding numerical results are denoted by the solid line. For comparison, the retailer's financing cost for advance payment is varied between $\{0, 0.2, 0.4\}$ and the results are represented by dashed lines. Figure 2.7.a shows retailer's optimal first stage order quantity under both financing schemes. Depending on supplier's initial capital and the financing cost, the order size with advance payment can either be more or less than the case with loan guarantee. While with advance payment, the retailer would not need to exaggerate the order size to keep supplier's incentive in line. Figure 2.7.b compares retailer's expected profit under different conditions. As expected, with the advance payment scheme, the retailer is better off when the financing cost is low and supplier's internal capital is high. It is also observed that when retailer's financing cost is low and the supplier has relative high initial capital, the advance payment scheme can outperform loan guarantee. This observation can be explained by our previous discussion that to make the financing scheme attractive to the supplier, the retailer would not need to exaggerate the order size under advance payment scheme.



a. Optimal order quantity



b. Expected profit

Figure 2.7: Comparison of advance payment and loan guarantee, $r_f = 0.4$

2.5 Effect of Voluntary Compliance

The previous discussion has assumed that the supplier would not produce more than the quantity that the retailer has ordered. However, supplier's optimal production decision may differ with retailer's order size. Due to the unreliable production yield, the supplier may have incentive to launch a larger production size to compensate the possible yield loss. For detailed discussion on supply chain coordination issue under such setting, we refer to He and Zhang (2008), Tang et al. (2013) and Li et al. (2013). In this section, we focus on the impact of supplier's initial capital level and voluntary compliance on retailer's ordering and guarantee decisions. We model the two echelon supply chain in a Stackelberg setting. Besides, to focus on the implication of SME's financial conditions, we do not consider the option of backup sourcing.

2.5.1 Bank Loan Financing with Random Yield

In this subsection, we build the model when the supplier can only obtain bank loan financing without retailer's loan guarantee. The retailer, as the Stackelberg leader, first decides the order quantity y, which is also the maximum quantity that would be accepted upon delivery. The SME supplier, as the follower, decides the borrowing amount and production quantity.

Since the supplier has no other investment opportunities and taking bank loan incurs additional financing cost, the supplier should always use the internal capital first. A bank loan will be issued only when the supplier decides to produce more than b. Once more, the bank is assumed to be risk neutral.

No Financing Access

When there is no financing access, the supplier's production quantity q is constrained by the initial capital b. After retailer's order quantity is released, the supplier chooses q to maximize his own interests. Supplier's profit maximization problem is formulated as:

$$\max_q \Pi_s = wE[min(q\zeta, y)] - q \tag{2.9}$$
$$s.t. \quad 0 \le q \le b$$

It is easy to show that supplier's profit function is concave in q, and the optimal q can be characterized as: $q_s = min\{\frac{y}{t_i}, b\}$, or equivalently

$$q_s(y, b) = \begin{cases} y/t_i, & \text{if } y < bt_i \\ b, & \text{otherwise} \end{cases}$$

where t_i is defined as $\{t \mid \int_0^t w\zeta g(\zeta)d\zeta - 1 = 0\}$, and q_s denotes supplier's optimal responding production quantity.

As the Stackelberg leader, the retailer is aware of supplier's initial budget level and the best response q_s. The retailer's profit is determined by:

$$\pi_r = -wE[\min\{q_s\zeta, y\}] + pE[\min\{D, min\{q_s\zeta, y\}\}]$$

Depending on supplier's initial capital level, the retailer's optimal ordering decisions can be summarized as follows.

Lemma 2.10. *When the supplier has no financing access, π_r is unimodal in y and retailer's optimal ordering quantities are:*
1. If $b < y_0$, any order size between $[b, +\infty)$ and $q_s = b$,
2. if $y_0 \le b \le y_0/t_i$, order y_0 and $q_s = b$,
3. if $y_0/t_i \le b \le y_i/t_i$, order bt_i and $q_s = b$,
4. if $b > y_i/t_i$, order y_i and $q_s = y_i/t_i$.

The optimal ordering decisions can be characterized by Figure 2.8. When the supplier has no financing access, the retailer's order size is non-decreasing in supplier's initial capital. Figure 2.9 further shows that both supplier's and retailer's profits are increasing in b. Clearly, the supply chain performance is

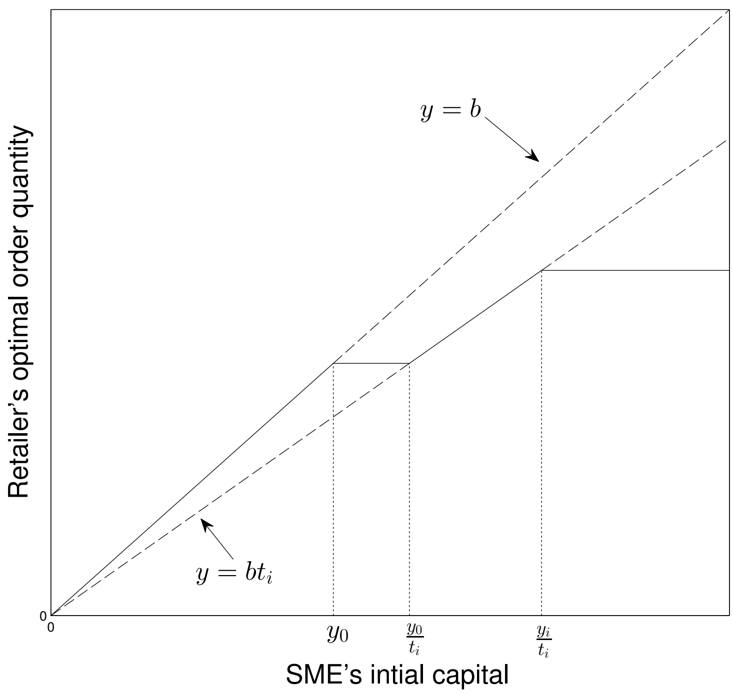


Figure 2.8: Optimal order quantity without financing access

extremely poor when the SME supplier has little working capital.

Bank Financing

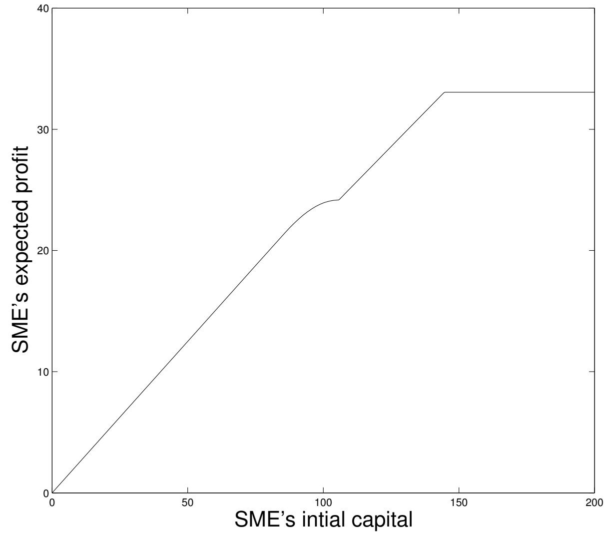
We now investigate the problem when the supplier can obtain loan financing from a risk neutral bank. The supplier's profit maximization problem is formulated as:

$$\max_{q \geq 0} \Pi_s = E[\min\{wq\zeta, wy\} - (q-b)^+(1+r(q))]^+ - \min\{q, b\} \tag{2.10}$$

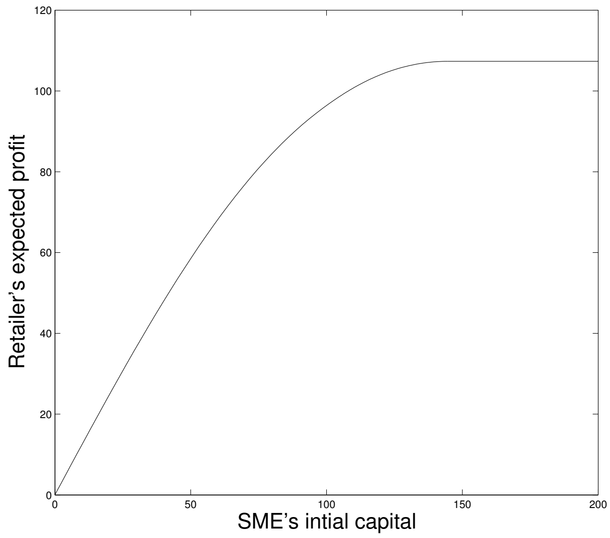
$$s.t. \quad (q-b)^+(1+r_f) = E[min\{min\{wq\zeta, wy\}, (q-b)^+(1+r(q))\}] \tag{2.11}$$

Depending on the magnitude of q, y and b, the supplier's profit can be classified into four situations:

1. $q < y$ and $q < b$. In this case, the supplier does not take a bank loan and the production quantity is less than retailer's order size. The supplier's expected



a. SME's expected profit



b. Retailer's expected profit

Figure 2.9: Effect of SME's initial capital on the expected profit without financing access

profit is always increasing in q.

2. $q < y$ and $q > b$. The supplier's problem is equivalent to our base model when the retailer accepts all.

$$\begin{aligned}\pi_s(y) &= E[(wq\zeta - (q-b)(1+r(q))]^+ - b \\ &= wE[\zeta]1 - (q-b)(1+r_f) - b\end{aligned}$$

Supplier's profit is increasing in q when $wE[\zeta] > 1 + r_f$, otherwise the supplier is not willing to take a bank loan.

3. $q > y$ and $q < b$. The supplier only uses the initial capital and the production quantity is greater than retailer's order size. This problem has been studied in our previous discussion when the supplier has no financing access. The supplier's profit function is concave in q, with optimal production quantity min$\{y/t_i, b\}$.

4. $q > y$ and $q > b$. Supplier's production quantity is higher than retailer's acceptance level and uses bank loan. Supplier's expected profit is determined as:

$$\pi_s = E[\text{min}\{wq\zeta, wy\} - (q-b)(1+r(q))]^+ - b$$

where the loan interest rate $r(q)$ is specified by Constraint 2.11.

We will then focus on the last case and derived supplier's production and borrowing decisions. In addition, we restrict our discussion to the case when the supplier produces with a feasible loan, which means that $(q-b)(1+r_f) \leq E[min\{wq\zeta, wy\}]$. Consider constraint 2.11, RHS is increasing in r and $RHS \leq E[min\{wq\zeta, wy\}]$. Therefore, as long as $(q-b)(1+r_f) \leq E[min\{wq\zeta, wy\}]$, there exists unique r which fairly price the loan.

Let t_1 be defined as $\{t \mid wqt = (q-b)(1+r(q))\}$ and t_2 as $\{t \mid qt = y\}$. By definition, we have $t_1 < t_2$. According to Constraint 2.11, we have $(q-b)(1+r_f) = \int_0^{t_1} wq\zeta g(\zeta)d\zeta + wqt_1\bar{G}(t_1)$. The supplier's expected profit function can be reformulated as:

$$\begin{aligned}
\pi_s &= \int_{t_1}^{t_2} wq(\zeta - t_1)g(\zeta)d\zeta + \int_{t_2}^{1} w(y - qt_1)g(\zeta)d\zeta - b \\
&= \int_{0}^{t_2} wq\zeta g(\zeta)d\zeta - \int_{0}^{t_1} wq\zeta g(\zeta)d\zeta - \int_{t_1}^{t_2} wqt_1 g(\zeta)d\zeta - \int_{t_2}^{1} wqt_1 g(\zeta)d\zeta \\
&\quad + \int_{t_2}^{1} wyg(\zeta)d\zeta - b \\
&= \int_{0}^{t_2} wq\zeta g(\zeta)d\zeta + \int_{t_2}^{1} wyg(\zeta)d\zeta - (q-b)(1+r_f) - b
\end{aligned}$$

By checking the second order derivative ($\frac{\partial^2}{\partial q^2}\pi_s = -w\frac{y^2}{q^3}g(\frac{y}{q}) < 0$), we can find that supplier's expected profit is concave in q. Supplier's optimal production policy with bank loan and over production can be characterized by the critical ratio t_l which uniquely solves:

$$\int_0^t w\zeta g(\zeta)d\zeta - (1+r_f) = 0$$

The supplier's optimal responses to retailer's order are as follows.

Proposition 2.4. *Given retailer's order quantity y and supplier's initial capital level b, supplier's optimal production quantity have three cases:*
1: $y \le bt_i$, $q_s = y/t_i$;
2: $bt_i \le y \le bt_l$, $q_s = b$;
3: $y \ge bt_l$, $q_s = y/t_l$.

Proposition 2.4 shows that when retailer's order size is low, the supplier chooses to over produce with the optimal ratio $1/t_i$. If the order size falls between $[bt_i, bt_l]$, the supplier just produces b with the initial capital. This means that if $y \le bt_l$ the marginal financing cost is higher than the potential revenue from producing more. The SME supplier would not take a bank loan in such circumstance. Supplier's optimal production policy differs with the no-financing case only if retailer's order size is greater than bt_l. Other than producing b in the no-financing case, the supplier is now willing to take a bank loan and the resulting production quantity is y/t_l.

Corollary 2.3. *t_l is increasing in r_f.*

Proof: The result can be easily obtained by checking the first order derivative, $\frac{\partial t_l}{\partial r_f} = \frac{r_f}{(w-s)tg(t)} > 0$.

As supplier's borrowing cost goes higher, the supplier's over production ratio($1/t_l$) becomes lower. It is also noticed that when $r_f = 0$, we have $t_l = t_i$. This result implies that when the marginal cost of loan financing is the same as the internal capital, supplier's production decision would be identical for both cases. This result is consistent with the Modigliani-Miller theorem (Modigliani and Miller, 1958) which states that in perfect and competitive markets, firms' financial and operational decisions are irrelevant and can be made independently.

Being aware of supplier's initial capital level and optimal response, the retailer chooses order quantity y to maximize the expected profit:

$$\pi_r = E_{\zeta,D}[p\min\{y, D, q_s\zeta\} - w\min\{yq_s\zeta\}]$$

As shown in Proposition 2.4, depending on supplier's initial capital and retailer's order size, supplier's optimal production decision has three possible cases. The retailer can either release a large order size to encourage the supplier to use bank loan, or choose a small order size so that the supplier only produces with the internal capital. By differentiating with respect to y, we can show that π_r is unimodal for each of the three ordering spaces. The proofs are as follows.

1. For $y \leq bt_i$, as the supplier produce y/t_i without bank loan, retailer's problem is the same as the no-financing case. The optimal order size is bt_i if $b < y_i/t_i$, and y_i if $b \geq y_i/t_i$.

2. For $bt_i < y < bt_l$, the supplier's production quantity is b and the retailer's problem is the same to the case without financing. Therefore, π_r is increasing in y if $y < y_0$, and decreasing in y if $y > y_0$. Given $bt_i < y < bt_l$, as supplier always chooses to produce b units, we denote retailer's optimal decision as y_b. The optimal ordering policy is given by:

$$y_b = \begin{cases} bt_l, & \text{if } b < \frac{y_0}{t_l} \\ y_0, & \text{if } \frac{y_0}{t_l} < b < \frac{y_0}{t_i} \\ bt_i, & \text{if } b > \frac{y_0}{t_i} \end{cases}$$

3. For $y > bt_l$

$$\begin{aligned} \pi_r(y) = &\int_0^{t_l} \left(-wy\zeta/t_l + \int_0^{\frac{y\zeta}{t_l}} pDf(D)dD + \int_{\frac{y\zeta}{t_l}}^{\infty} py\zeta/t_l f(D)dD \right) g(\zeta)d\zeta \\ &+ \int_{t_l}^{1} \left(-wy + \int_0^{y} pDf(D)dD + \int_{y}^{\infty} pyf(D)dD \right) g(\zeta)d\zeta \end{aligned}$$

$$\begin{aligned} \frac{\partial \pi_r(y)}{\partial y} &= \int_0^{t_l} [-w + p\bar{F}(y\zeta/t_l)]\frac{\zeta}{t_l} g(\zeta)d\zeta + \int_{t_l}^{1} [-w + p\bar{F}(y)]g(\zeta)d\zeta \\ \frac{\partial^2 \pi_r(y)}{\partial y^2} &= -p\int_0^{t_l} \frac{\zeta^2}{t_l^2} f(\frac{y\zeta}{t_l})g(\zeta)d\zeta + -p\int_{t_l}^{1} f(y)g(\zeta)d\zeta < 0 \end{aligned}$$

Similar to case 1, since π_r is concave in y, we can obtain the optimal order quantity y_l by solving the first order condition.

Although π_r is unimodal in y for each of the three ordering spaces, the overall optimal decision could fall in any of the situations which depends on supplier's initial capital level. To obtain the optimal order policy for any given b, we first look at the second situation when supplier's production quantity is fixed at b units.

Lemma 2.11. *For $bt_i < y < bt_l$ and retailer orders y_b, π_r is increasing in b on $[0, \frac{y_i}{t_i}]$ and decreasing in b on $[\frac{y_i}{t_i}, +\infty)$.*

It is worth noticing that for $b \in [y_0/t_l, y_0/t_i]$, retailer's optimal order quantity is fixed at y_0, as the supplier has more initial capital, the retailer takes the benefit of more secured supply.

Similarly, for the first case $y < bt_i$:

- for $b \in [0, y_i/t_i]$, retailer's optimal order size is bt_i and supplier's corresponding production quantity is b. Retailer's expected profit is increasing in b.
- for $b \geq y_i/t_i$, retailer's optimal order size is y_i and supplier's corresponding production quantity is y_i/t_i. Retailer's expected profit is constant in b.

Combine the conclusion in Lemma 2.11, given the order space $y \in [0, bt_l]$, retailer's profit is increasing in b for $b \in [0, y_i/t_i]$ and constant in b for $b \in [y_i/t_i, +\infty)$.

Denote the optimal order quantity for $y \in [0, bt_l]$ as y_s where

$$y_s = \begin{cases} bt_l, & \text{if } b \leq \frac{y_0}{t_l} \\ y_0, & \text{if } \frac{y_0}{t_l} \leq b \leq \frac{y_0}{t_i} \\ bt_i, & \text{if } \frac{y_0}{t_i} \leq b \leq y_i/t_i \\ y_i, & \text{if } b > y_i/t_i \end{cases}$$

On the other hand, for the third case $y > bt_l$:

- for $b \in [0, y_l/t_l]$, retailer's optimal order size is y_l with supplier's production quantity y_l/t_l. As a result, retailer's expected profit is constant in b.
- for $b \geq y_l/t_l$, retailer's optimal order size is bt_l, and

  $$\frac{\partial \pi_r(bt_l)}{\partial b} = \int_0^{t_l} [-w + p\bar{F}(b\zeta)]\zeta g(\zeta) d\zeta + [-w + p\bar{F}(bt_l)]t_l\bar{G}(t_l) \leq 0.$$

  Hence, retailer's expected profit is decreasing in b.

Lemma 2.12. *There exists b_l such that for $b \in [0, b_l]$, $\pi_r(y_l) > \pi_r(y_s)$ and for $b \in [b_l, +\infty)$, $\pi_r(min\{y_l, bt_l\}) < \pi_r(y_s)$, where $b_l \in [\frac{y_0}{t_l}, \frac{y_l}{t_l}]$.*

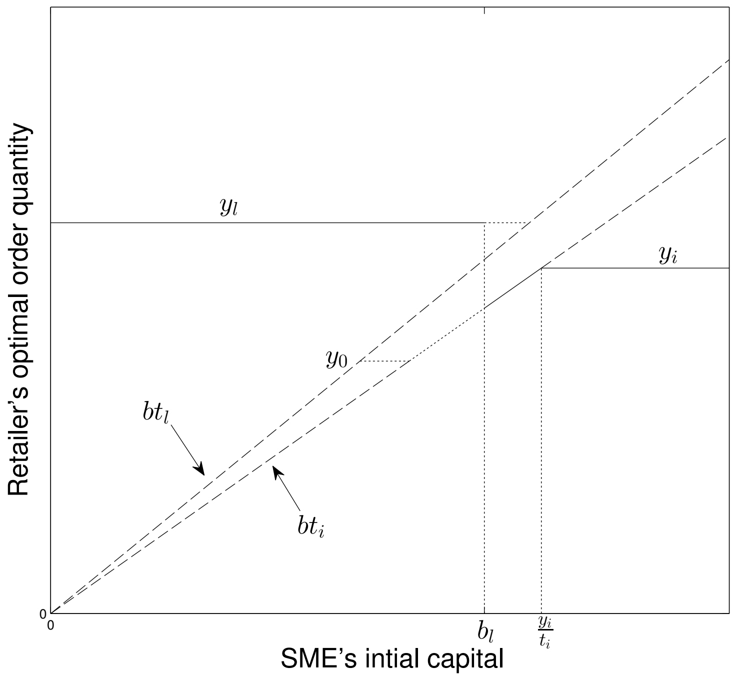


Figure 2.10: Optimal order quantity with bank finance

Proof: When $b = \frac{y_0}{t_l}$, y_s is equal to y_0. Since $y_0 < y_l$ and π_r is increasing in y for $y \in [bt_l, y_l]$, we have $\pi_r(y_s) < \pi_r(y_l)$. When $b = \frac{y_l}{t_l}$, $\pi_r(y)$ is decreasing in y for $y \in [y_s, y_l]$. Therefore $\pi_r(y_s) > \pi_r(y_l)$ in this case. Since $\pi_r(y_s)$ is increasing in b, and $\pi_r(y_l)$ is constant in b on $[0, y_l/t_l]$, we can conclude that there exists unique b_l and $b_l \in [\frac{y_0}{t_l}, \frac{y_l}{t_l}]$.

Proposition 2.5 and Figure 2.10 summarize retailer's optimal ordering policy with respect to supplier's initial capital level.

Proposition 2.5. *The retailer's optimal order quantity depends on supplier's initial wealth which is specified as follow:*

1. if $0 < b \leq b_l$*, the optimal order quantity is* y_l

2. if $b \geq b_l$*, the optimal order quantity is* y_s*.*

where b_l*,* y_l *and* y_s *follow previous definitions.*

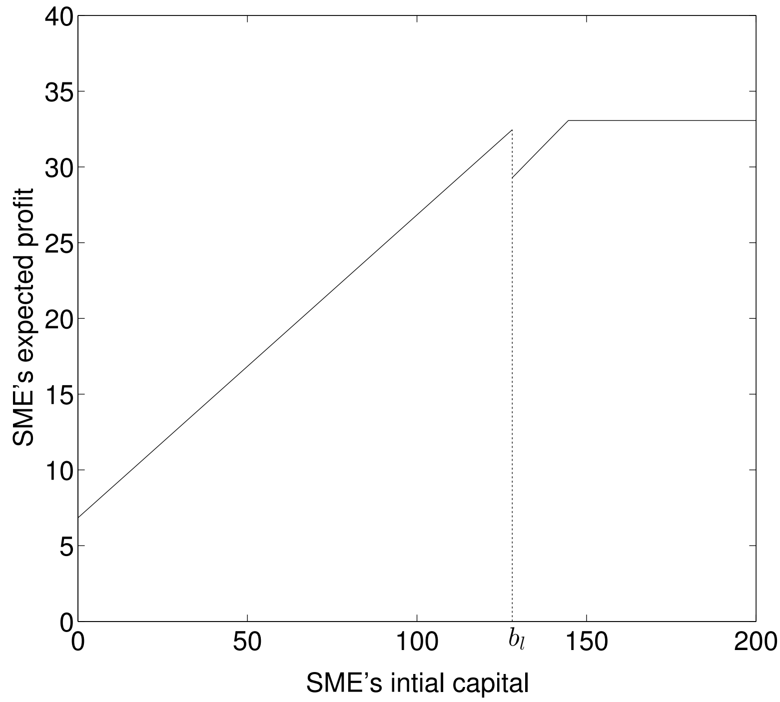
In Figure 2.10, retailer's optimal ordering decisions are represented by the solid line. As can bee seen, when supplier's internal capital is low, the retailer chooses a large order size y_l to let the supplier take bank loan and produce more than y_l/t_l. However, if supplier's internal capital level is greater than b_l, the retailer strategically reduces the order size. The supplier would then produce only with the internal capital. When supplier's internal capital is greater than y_i/t_i, the retailer orders y_i which is the optimal ordering quantity in the no budget constraint situation.

Figure 2.11 further illustrates how supplier's and retailer's profits are affected by supplier's internal capital b. Figure 2.11.(a) shows that supplier's profit is increasing in b for $b \in [0, b_l)$ and $b \in (b_l, y_i/t_i]$. However, in Figure 2.11.a, it is noticed that π_s is not continuous at $b = b_l$. On the other hand, retailer's profit is continuous in b at $b = b_l$. This is because that when the SME supplier's initial capital equals to b_l, the retailer is indifferent between ordering y_s to let the supplier produce only with the internal capital or ordering y_l to let the supplier produce y_l/t_l with a positive loan amount. However, from the supplier's perspective, a larger order size ($y_l > y_s$) is more profitable in this situation.

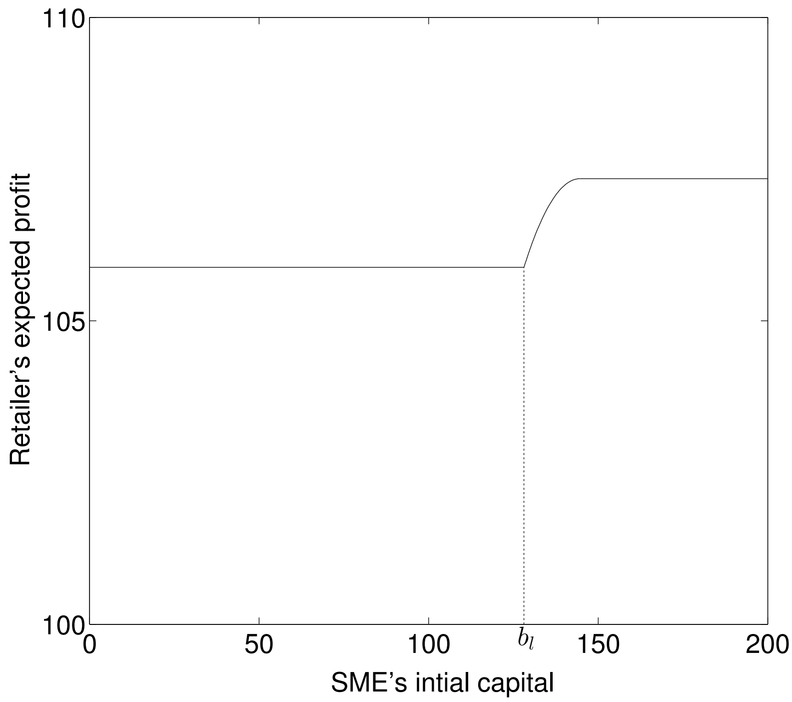
Our model assumes that the retailer is fully aware of supplier's production and financial information. While the results in Figure 2.11 suggests that, for certain range of supplier's initial capital, the supplier may have incentive to conceal the actual financial condition to gain higher profit. The conflict of interest among supplier and retailer may lead to information distortion. In such case, the retailer may need to offer an incentive compatible contract to avoid information distortion.

2.5.2 Loan Guarantee with Disruption Risk

The previous discussion has focused on the situation when the retailer only decides the order quantity without offering loan guarantee. However, when the financing cost is higher than the supplier's profit margin, the supplier is not willing to take bank loan without positive guarantee. We now consider the problem when the supplier decides the production and borrowing quantity, and the retailer decides the ordering and guarantee level. For tractability, we assume



a. SME's expected profit



b. Retailer's expected profit

Figure 2.11: Effect of SME's initial capital on the expected profit with bank financing, $r_f = 0.2$

the supplier's production process is subjected to random disruption. The disruption occurs with probability $1-\theta$ and the production yield would be a fraction α $(0 \leq \alpha \leq 1)$ of the production quantity. Similar to the settings with random yield, we assume α is independent of the production size. The production yield is expressed as:

$$\zeta = \begin{cases} 1, & \text{with probability } \theta \\ \alpha, & \text{with probability } 1-\theta \end{cases}$$

Denote the production size as q, the supplier chooses the optimal q to maximize the expected profit. Since the supplier has no other investment opportunities, there is no incentive for the supplier to borrow more than $(q-b)^+$, consequently the loan size is determined by $(q-b)^+$. The supplier's problem is expressed as:

$$\begin{aligned} \Pi_s = & \max_{q \geq 0} \mathbb{E}[w\text{min}\{q\zeta, y\} - \text{min}\{b, q\} - (q-b)^+(1+r)]^+ \\ & s.t.\ (q-b)^+(1+r_f) = \mathbb{E}[\text{min}\{(q-b)^+(1+r), w\text{min}\{q\zeta, y\} + (b-q)^+ \\ & \quad + x((q-b)^+(1+r) - wq\zeta - (b-q)^+)^+\}] \end{aligned}$$

Before characterizing supplier's optimal decision with positive guarantee, we first consider the problem without retailer's guarantee. The supplier's problem is described as follows:

$$\begin{aligned} \Pi_s = & \max_{q \geq 0} \mathbb{E}[w\text{min}\{q\zeta, y\} - \text{min}\{b, q\} - (q-b)^+(1+r)]^+ \\ & s.t.\ \ (q-b)^+(1+r_f) = \mathbb{E}[\text{min}\left\{wq\zeta + (b-q)^+, (q-b)^+(1+r)\right\}] \end{aligned}$$

The supplier's optimal decisions are summarized in Table 2.1. It can be observed that the supplier's optimal production quantity is greater than y if the expected marginal revenue is higher than the marginal cost. The optimal production quantity is increasing in the wholesale price w, the perfect production probability θ, the disruption yield level α; while decreasing in bank's risk free

Table 2.1: Supplier's optimal production quantity without guarantee

Condition	Range of ordering quantity		
	$y < b\alpha$	$b\alpha < y < b$	$y > b$
$w(1-\theta)\alpha > 1+r_f$	y/α	y/α	y/α
$1 < w(1-\theta)\alpha < 1+r_f < w\alpha$	y/α	b	y
$w(1-\theta)\alpha < 1 < 1+r_f < w\alpha$	y	y	y
$1 < w(1-\theta)\alpha < w\alpha < 1+r_f$	y/α	b	if $wE[\zeta] \geq 1+r_f$: y
$w(1-\theta)\alpha < 1$ and $w\alpha < 1+r_f$	y	y	if $wE[\zeta] < 1+r_f$: b

interest rate r_f.

We now consider supplier's problem with positive loan guarantee. Since the supplier has no incentive to produce more than y/α, the retailer's payment to the supplier when disruption happens is $wq\alpha$. For the case $w\alpha \geq 1 + r_f$, the supplier can always fully repay the loan even disruption occurs. In such case, the loan interest rate is fixed at r_f and retailer's guarantee does not take effects. In the following content we focus on the situation when $w\alpha < 1 + r_f$ and $q > b$. The profit function of supplier has two possible situations:

1. If $wq\alpha > (q-b)(1+r_f)$ or equivalently $b < q \leq q_0 = \frac{b(1+r_f)}{1+r_f-w\alpha}$, the supplier can still fully repay the loan even disruption occurs. Therefore, the loan interest rate is charged at r_f. The supplier's expected profit is written as :

$$\pi_s(q) = w\mathbb{E}[\min\{q\zeta, y\}] - (q-b)(1+r_f) - b$$

Given $y \geq q_0$, if $w\mathbb{E}[\zeta] \geq 1 + r_f$, the supplier's profit is increasing in q on $[b, q_0]$; otherwise, supplier's profit is decreasing and the supplier is not willing to borrow. In addition, when the supplier produces q_0, we have $\pi_s(x, q_0) = w\mathbb{E}[\zeta]b - b + (w\mathbb{E}[\zeta] - (1+r_f))(q_0 - b)$. Consequently $\pi_s(x, q_0)$ is less than $\pi_s(b) = w\mathbb{E}[\zeta]b - b$ if $w\mathbb{E}[\zeta] \geq 1 + r_f$.

2. If $wq\alpha < (q-b)(1+r_f)$, or equivalently $q > q_0 > b$, the supplier cannot fully repay the loan when disruption happens. The bank will then increase the interest rate to compensate the potential loss under disruption. Under the risk neutral

assumption, the loan interest rate is determined by the following equation:

$$(q-b)(1+r_f) = \theta(q-b)(1+r) + (1-\theta)(1-x)wq\alpha +$$
$$x(1-\theta)(q-b)(1+r)$$

Consequently $r = \frac{(q-b)(1+r_f)-wq\alpha(1-\theta)(1-x)}{(q-b)(\theta+x-\theta x)} - 1$.

It can also be shown that r is increasing in q. The supplier's profit is written as:

$$\begin{aligned}\pi_s(q) &= \theta w \min\{q,y\} - \theta(q-b)(1+r) - b \\ &= \theta(w\min\{q,y\} - qr_x) + \frac{\theta b(1+r_f)}{\theta+x-\theta x} - b\end{aligned} \tag{2.12}$$

where $r_x = \frac{1+r_f - w\alpha(1-\theta)(1-x)}{\theta+x-\theta x} - 1$.

Lemma 2.13. *Given $1+r_f > w\alpha$ and $x<1$, $r_x > r_f$.*

Similar to the problem with random yield assumption, in case $wE[\zeta] < 1+r_f$, even a guarantee is provided, the supplier is not always better off to take a bank loan. Given an order size y, for $b<q<q_0$, the supplier's marginal revenue $wE[\zeta]$ is less than the marginal cost $1+r_f$. While for $q_0<q<y$, the supplier's profit is increasing in the production size if $w>1+r_x$.

Lemma 2.14. *Given $r_f > w\mathbb{E}[\zeta]-1$, the supplier is willing to take bank loan only if $w>1+r_x$ and $y>\underline{y}$, where $\underline{y} = \frac{b[w\mathbb{E}[\zeta]/\theta-(1+r_f)/(\theta+x-\theta x)]}{w-1-r_x}$. In addition, $\underline{y}$ is decreasing in x, and $\underline{y}>q_0$.*

The supplier's optimal production quantities with positive guarantee are summarized in Table 2.2. Comparing with the no guarantee case, it can be found that loan guarantee changes supplier's optimal production decision only for the situation $w\mathbb{E}[\zeta] < 1+r_f$ and $y>b$. Under such conditions, the supplier's

Table 2.2: Supplier's optimal production quantity with guarantee

<table>
<tr><th rowspan="2">Condition</th><th colspan="3">Range of ordering quantity</th></tr>
<tr><th>$y < b\alpha$</th><th>$b\alpha < y < b$</th><th>$y > b$</th></tr>
<tr><td>$w(1-\theta)\alpha > 1 + r_f$</td><td>$y/\alpha$</td><td>$y/\alpha$</td><td>$y/\alpha$</td></tr>
<tr><td>$1 < w(1-\theta)\alpha < 1 + r_f < w\alpha$</td><td>$y/\alpha$</td><td>$b$</td><td>$y$</td></tr>
<tr><td>$w(1-\theta)\alpha < 1 < 1 + r_f < w\alpha$</td><td>$y$</td><td>$y$</td><td>$y$</td></tr>
<tr><td>$1 < w(1-\theta)\alpha < w\alpha < 1 + r_f$</td><td>$y/\alpha$</td><td>$b$</td><td rowspan="2">if $wE[\zeta] \geq 1 + r_f$: y,
if $wE[\zeta] < 1 + r_f$: b for $b < y < \underline{y}$, y for $y \geq \underline{y}$</td></tr>
<tr><td>$w(1-\theta)\alpha < 1$ and $w\alpha < 1 + r_f$</td><td>y</td><td>y</td></tr>
</table>

optimal production size is equal to y when $w > 1 + r_x$ and $y \geq \underline{y}$. If either of the two conditions does not hold, the supplier is not willing to borrow and the production size is b.

It is also observed that a positive guarantee would not induce the supplier to launch a production size greater than buyer's ordering quantity. That is to say, if supplier's optimal production quantity without guarantee is no more than retailer's ordering quantity, then supplier's optimal production quantity with guarantee is also no more than y. This observation implies that under no circumstance the retailer would share supplier's over production cost.

The reason is that the loan interest rate is no less than r_f even with positive guarantee. If the supplier's marginal revenue with over production, namely $w(1-\theta)\alpha$, is less than $1 + r_f$, then the supplier's marginal borrowing cost is always higher than the marginal revenue from over production. Thus, when $w(1-\theta)\alpha < 1 + r_f$, the supplier would not produce more than y.

We focus on the case $w\alpha < 1+r_f$, the supplier's profit with optimal production size is expressed as:

$$\pi_s = \begin{cases} w(\theta + \alpha - \theta\alpha)y - y, & \text{if } 0 < y < b \\ w(\theta + \alpha - \theta\alpha)b - b, & \text{if } b \leq y \leq y_{lb} \\ \theta[wy - (y-b)(1+r)] - b, & \text{if } y > \underline{y} \end{cases}$$

Buyer's problem

Denote the supplier's optimal production quantity as q_s, as the Stackelberg leader, the retailer's expected profit is written as:

$\pi_r(x, y) = \theta[p\min\{y, q_s, D\} - wy] + (1-\theta)[p\min\{y, q_s\alpha, D\} - wq_s\alpha - x((q_s - b)^+(1+r) - wq_s\alpha)^+]$

Depending on the supplier's best response, the retailer's expected profit has four possible scenarios:

$$\pi_r = \begin{cases} \theta[p\min\{y, D\} - wy, & \text{when } q_s = y/\alpha \\ \theta[p\min\{y, D\} - wy] + (1-\theta)[p\min\{b\alpha, D\} - wb\alpha], & \text{when } q_s = b > y \\ \theta[p\min\{b, D\} - wb] + (1-\theta)[p\min\{b\alpha, D\} - wb\alpha], & \text{when } q_s = b \leq y \\ \begin{array}{l} \theta[p\min\{y, D\} - wy] + (1-\theta)[p\min\{y\alpha, D\} - wy\alpha \\ -x((y-b)(1+r) - wy\alpha)^+], \end{array} & \text{when } q_s = y \end{cases}$$

As can be seen, the retailer's guarantee cost is positive only when $q_s = y$ and $x(1-\theta))(y-b)(1+r) - wy\alpha) > 0$. We first look for the optimal guarantee level in such situation.

Lemma 2.15. *For $wE[\zeta] \geq 1 + r_f$, given that the supplier decides to produce y and $(y-b)(1+r) > wy\alpha$, retailer's profit is decreasing in x. For $wE[\zeta] < 1 + r_f$ and $y > y_{lb}$, there exists $\underline{x}(y) \in [0, 1]$ such that:*
1. if $x \in [\underline{x}(y), 1]$, supplier's optimal production decision q_s is y and retailer's profit π_r is decreasing in x;
2. if $x \in [0, \underline{x}(y)]$, $q_s = b$ and π_r is constant in x,
where $y_{lb} = \underline{y}(1)$ and $\underline{x}(y) = \frac{\theta(y-b)(1+r_f-wE[\zeta])}{w(1-\theta)(y\theta - y\alpha\theta - b\alpha - b\theta + b\alpha\theta)}$.

Similar to the random yield problem, we use y_{lb} to denote $\underline{y}(1)$. For $wE[\zeta] < 1 + r_f$, if the ordering quantity is between $[b, y_{lb}]$, the supplier is not going to take bank loan even if full guarantee is provided. From the retailer's perspective, the benefit of guarantee is that the supplier is willing to increase the production quantity from b to y, in case $wE[\zeta] < 1 + r_f$ and $y \geq y_{lb}$. Lemma 2.15 indicates that the retailer has no incentive to offer extra guarantee. For a desired production production quantity, the retailer either offers zero guarantee(when $wE[\zeta] \geq 1 + r_f$), or the minimum guarantee $\underline{x}(y)$(when $wE[\zeta] < 1 + r_f$).

For $wE[\zeta] \geq 1 + r_f$, supplier's optimal production quantity is no less than y. Furthermore, a loan guarantee does not change supplier's decision in such case. Henceforth, we focus on the situation when $wE[\zeta] < 1 + r_f$.

As the Stackelberg leader, the retailer correctly anticipates supplier's production decision. Since supplier's best response q_s is a deterministic function of guarantee and order quantity, we will then consider retailer's problem. In addition, according to Lemma 2.15, retailer's optimal guarantee level is either 0 or $\underline{x}(y)$. Given $w\mathbb{E}[\zeta] < 1 + r_f$, with optimally decided guarantee decision, the retailer's profit function has two possible situations which depends on supplier's marginal revenue with over production:

Case 1: When $w(1-\theta)\alpha < 1$, retailer's profit function with optimally decided guarantee decision is:

$$\pi_r = \begin{cases} \theta[p\min\{y, D\} - wy] + (1-\theta)[p\min\{y\alpha, D\} - wy\alpha], & \text{if } 0 < y \leq b \\ \theta[p\min\{b, D\} - wy] + (1-\theta)[p\min\{b\alpha, D\} - wb\alpha], & \text{if } b < y \leq y_{lb} \\ \theta[p\min\{y, D\} - wy] + (1-\theta)[p\min\{y\alpha, D\} - wy\alpha] & \\ -(y-b)(1 + r_f - wE[\zeta]), & \text{if } y \geq y_{lb} \end{cases}$$

Denote y^0 as the solution to $\theta(p\bar{F}(y) - w) + \alpha(1-\theta)(p\bar{F}(y\alpha) - w) = 0$, which represents the optimal order quantity with disruption risk but without budget constraint. Denote y' as the solution to $\theta(p\bar{F}(y) - w) + \alpha(1-\theta)(p\bar{F}(y\alpha) - w) - (1 + r_f - wE[\zeta]) = 0$, which represents the optimal order quantity with both disruption risk and positive guarantee cost.

Proposition 2.6. *Give $r_f > w\mathbb{E}[\zeta] - 1$ and $w(1-\theta)\alpha < 1$, define $b_1 = \{b \mid y_{lb}(b) = y'\}$ and $b_2 = \{b \mid \pi_r(\underline{x}, y_{lb}(b)\}) = \pi_r(0, b)\}$. The retailer's optimal decisions are:*

(1). If $0 < b < b_1$, order y' with guarantee $\underline{x}(y')$;

(2). if $b_1 < b < b_2$, order $y_{lb}(b)$ with guarantee $\underline{x}(y_{lb}(b))$;

(3). if $b_2 < b < y^0$, order b and provide no guarantee;

(4). if $b \geq y^0$, order y^0 and provide no guarantee;

Proposition 2.6 describes retailer's decisions when the supplier has no inventive to produce more than y. The retailer's optimal policy has the same structure as the random yield problem with forced compliance. To encourage the supplier produces more, the retailer needs to share the financing cost through loan guarantee. Furthermore, offering guarantee is profitable for the retailer when the supplier has few working capital. When supplier's initial capital is higher than certain threshold, it is optimal for the retailer to order the minimum of b and y^0, and provides zero guarantee.

Case 2 : When $w(1-\theta)\alpha > 1$, the retailer's expected profit is

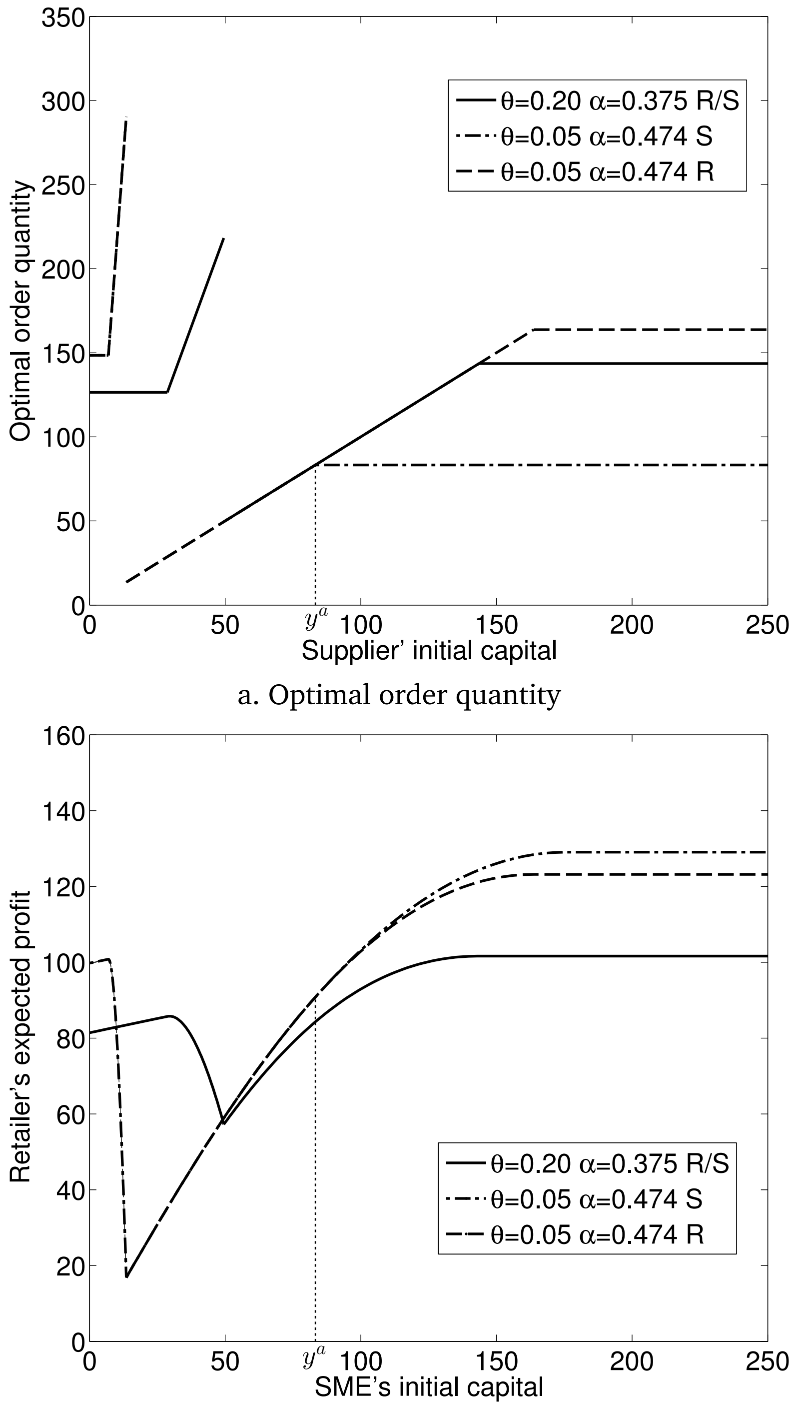
$$\pi_r = \begin{cases} \theta[p\min\{y, D\} - wy, & \text{if } 0 < y < b\alpha \\ \theta[p\min\{y, D\} - wy] + (1-\theta)[p\min\{b\alpha, D\} - wb\alpha], & \text{if } b\alpha \leq y \leq b \\ \theta[p\min\{b, D\} - wb] + (1-\theta)[p\min\{b\alpha, D\} - wb\alpha], & \text{if } b < y < y_{lb} \\ \begin{array}{l} \theta[p\min\{y, D\} - wy] + (1-\theta)[p\min\{y\alpha, D\} - wy\alpha] \\ -(y-b)(1 + r_f - wE[\zeta]), \end{array} & \text{if } y \geq y_{lb} \end{cases}$$

In this case, supplier's marginal revenue with over production is greater than 1. When retailer's order quantity y is less than supplier's initial budget, the supplier's optimal production quantity would be higher than y. While for order quantity more than b, the supplier's production quantity is no more than y.

Denote $y^a = F^{-1}(\frac{p-w}{p})$ as the traditional newsvendor optimal ordering quantity without disruption risk and budget constraint.

Proposition 2.7. *Given $r_f > w\mathbb{E}[\zeta] - 1$ and $w(1-\theta)\alpha > 1$, define $b_1 = \{b \mid y_{lb}(b) = y'\}$ and $b_2 = \{b \mid \pi_r(\underline{x}, max\{y', y_{lb}(b)\}) = \pi_r(0, min\{b, y^a\})\}$. The retailer's optimal decisions are:*

(1). If $b_1 \leq b_2$, order y' with guarantee $\underline{x}(y')$ for $b \in [0, b_1]$; order $y_{lb}(b)$ with



a. Optimal order quantity

b. Retailer's expected profit

Figure 2.12: Comparison of different compliance scheme

guarantee $\underline{x}(y_{lb}(b))$ *for* $b \in (b_1, b_2]$*; order* $min\{b, y^a\}$ *with no guarantee for* $b \in (b_2, +\infty)$.

(2). If $b_1 > b_2$*, order* y' *with guarantee* $\underline{x}(y')$ *for* $b \in [0, b_2]$*; order* $min\{b, y^a\}$ *with*

no guarantee for $b \in (b_2, +\infty)$.

Figure 2.12 illustrates the results in Proposition 2.6 and Proposition 2.7. The supplier's expected yield is fixed at 0.5. We use R and S to denote whether the production quantity is determined by the retailer or the supplier. As discussed above, when the supplier has no inventive to over produce, the outcome of the two situations are the same. In Figure 2.12 , this situation is shown by the solid line where $\theta = 0.2$ and $\alpha = 0.375$ and denoted as R/S. However, if $1 < w(1-\theta)\alpha < 1 + r_f$ and $y < b$, the supplier is willing to produce more than y using the internal capital. In Figure 2.12, this situation is denoted by the dashed line. For comparison, we also show the case when the supplier produces exactly what retailer orders, which is represented by dash-dot line in Figure 2.12.

For $b > y^a$, the retailer is better off when the supplier decides to produce more than the order quantity. This is due to the fact that when the supplier produces more, the retailer has a more reliable supply process. Given $r_f > wE[\zeta] - 1$ and $\alpha < 1$, we have $y^a \leq y^0$. Combing the results in Proposition 2.7, when the supplier chooses to overproduce, the retailer correctly anticipates supplier's decision and chooses a lower ordering quantity. Comparing with the case of forced compliance, the retailer is better off while the supplier would be worse off. This result reveals that the supplier may not be better off under voluntary compliance.

2.6 Conclusions and Future Research

Many SME suppliers are having difficulties in obtaining sufficient working capital. The suppliers' financial inability also affects the profitability of their supply chain partners. In this study, we consider an SME supplier which is of limited working capital and unreliable production process. We investigate the options of using bank loan financing, retailer's loan guarantee, and retailer's advance payment. An analytical model is built to study retailer's optimal ordering and guarantee decisions with consideration of supplier, retailer and bank's incentives.

Under forced compliance, the result shows that loan guarantee scheme does reduce supplier's financing cost. While the retailer should offer guarantee only when the borrowing cost is higher than supplier's potential revenue. Otherwise, the retailer should not share supplier's financing cost and retailer's profit can be maximized by simply chosen the optimal order quantity. When the financing cost is high, the retailer is especially better off by providing guarantee when supplier's initial capital is low. As supplier's initial capital increases, the profit improvement by loan guarantee diminishes. It is also observed that, when supplier's initial capital is greater than certain threshold, the retailer needs to increase the order size and provide positive guarantee at the same time to keep supplier's interest aligned.

To secure the sourcing process, some companies choose to use backup supply. We further extend our base model to incorporate a perfectly reliable but more expensive backup supplier. It is found that as the wholesale price of backup supply increases, the retailer is more likely to use loan guarantee and orders more from the unreliable SME supplier.

In addition, we study the problem of voluntary compliance where the production quantity is decided by the supplier. Firstly, when the supply process is subject to random yield and the supplier can take bank loan financing, we show that there exists a threshold for the supplier's initial capital below which the retailer chooses a large order size, and the supplier is then willing to take a bank loan and produce more. When supplier's initial capital is higher than this threshold, the retailer chooses a smaller order size such that the supplier produces only with the initial capital. The numerical results show that retailer's profit is non-decreasing in supplier's initial capital. Both the supplier and the retailer achieve highest profit when the supplier has abundant capital. However, the supplier can be worse off when b is relatively high. This result suggests the supplier may have incentive to hide the actual financial status. In practice, a retailer may have information on the reliability of supplier's production process. But it is unlikely for the retailer to be able to monitor the supplier's financial conditions accurately. On the other hand, as a financial institute, the bank has more speciality on how to evaluate company's financial state. This observation addresses that successful implementation of supply chain financing needs the cooperation among financial institutes and supply chain partners.

Finally, we consider the loan guarantee decision with voluntary compliance. The supplier is assumed to have disruption risk. We show that with optimally decided guarantee, the retailer is not going to share supplier's over production risk. In addition, when overproduction is profitable for the supplier, the retailer takes advantage of more reliable supply and strategically reduces the order size.

This study contributes to the research of joint interface of supply chain management and finance. Our work confirms that the retailer could be better off by helping the SME supplier reduce financing cost. Our results also point out that retailer's financing support needs to be coordinated with the order decision and supplier's financial condition. Otherwise, the supplier would be lack of incentives to participate in the financing program.

Although various types of contracts are being used in practice, due to its simplicity and convenience for execution, wholesale price contract is the most widely adopted and studied. The discussion of other types of contracts are beyond the scope of this study. A potential extension of this work would be incorporating different contract types to align supply chain members' incentives. Another important research question remains unsolved is that when both production reliability and initial financial status are private information to the supplier, how should the retailer design a proper contract to make supplier's incentive align. This question, as well as other possible extensions are left for future research.

Bibliography

Atasu, A., Sarvary, M., and Van Wassenhove, L. N. (2008). Remanufacturing as a marketing strategy. *Management Science*, 54:1731–1746. 4

Babich, V. (2010). Independence of capacity ordering and financial subsidies to risky suppliers. *Manufacturing & Service Operations Management*, 12(4):583–607. 29, 30

Babich, V. and Sobel, M. J. (2004). Pre-ipo operational and financial decisions. *Management Science*, 50(7):935–948. 27

Bakal, I. S. and Akcali, E. (2006). Effects of random yield in remanufacturing with price-sensitive supply and demand. *Production and Operations Management*, 15(3):407–420. 5, 14

Bayus, B. L. (1991). The consumer durable replacement buyer. *Journal of Marketing*, 55(1):pp. 42–51. 3

Bertsekas, D., Nedic, A., and Ozdaglar, A. (2003). *Convex Analysis and Optimization*. 13

Bollapragada, S. and Morton, T. E. (1999). Myopic heuristics for the random yield problem. *Operations Research*, 47(5):713–722. 5

Buzacott, J. A. and Zhang, R. Q. (2004). Inventory management with asset-based financing. *Management Science*, 50(9):1274–1292. 28

Cachon, G. P. (2003). Supply chain coordination with contracts. *Handbooks in operations research and management science*, 11:227–339. 27

Caldentey, R. and Chen, X. (2011). *The Role of Financial Services in Procurement Contracts*, pages 289–326. John Wiley & Sons, Inc. 28

Chao, X., Chen, J., and Wang, S. (2008). Dynamic inventory management with cash flow constraints. *Naval Research Logistics (NRL)*, 55(8):758–768. 27

Chen, L., Kök, A. G., and Tong, J. D. (2013). The effect of payment schemes on inventory decisions: The role of mental accounting. *Management Science*, 59(2):436–451. 29

Chen, X. and Cai, G. G. (2011). Joint logistics and financial services by a 3pl firm. *European Journal of Operational Research*, 214(3):579 – 587. 28

Chen, Y. and Gupta, D. (2014). Trade-finance contracts for small-business suppliers and the effect of third-party financing. *Available at SSRN 2486757*. 28

China Minsheng Bank (2014). Core enterprise guarantee loan. 25

Dada, M. and Hu, Q. (2008). Financing newsvendor inventory. *Operations Research Letters*, 36(5):569 – 573. 28

Dada, M., Petruzzi, N. C., and Schwarz, L. B. (2007). A newsvendor's procurement problem when suppliers are unreliable. *Manufacturing & Service Operations Management*, 9(1):9–32. 30

Debo, L. G., Toktay, L. B., and Wassenhove, L. N. V. (2006). Joint life-cycle dynamics of new and remanufactured products. *Production and Operations Management*, 15(4):498–513. 4

Dong, L. and Tomlin, B. (2012). Managing disruption risk: The interplay between operations and insurance. *Management Science*, 58(10):1898–1915. 31

Federgruen, A. and Yang, N. (2009). Optimal supply diversification under general supply risks. *Operations Research*, 57(6):1451–1468. 32

Ferrer, G. (2003). Yield information and supplier responsiveness in remanufacturing operations. *European Journal of Operational Research*, 149(3):540 – 556. 5

Fleischmann, M., Bloemhof-Ruwaard, J. M., Dekker, R., van der Laan, E., van Nunen, J. A., and Wassenhove, L. N. V. (1997). Quantitative models for reverse logistics: A review. *European Journal of Operational Research*, 103(1):1 – 17. 4

Green, A. (2003). *Credit guarantee schemes for small enterprises: an effective instrument to promote private sector-led growth?* UNIDO, Programme Development and Technical Cooperation Division. 25

Guide, V. D. R. and Jayaraman, V. (2000). Product acquisition management: Current industry practice and a proposed framework. *International Journal of Production Research*, 38(16):3779–3800. 4

Guide, V. D. R., Teunter, R. H., and Van Wassenhove, L. N. (2003). Matching demand and supply to maximize profits from remanufacturing. *Manufacturing & Service Operations Management*, 5(4):303–316. 4, 18

Guide, V. D. R. and Van Wassenhove, L. N. (2003). *Business Aspects of Closed-Loop Supply Chains*. Carnegie Mellon University Press, Pittsburgh. 4

Guide, V. D. R. and Van Wassenhove, L. N. (2009). Or forum-the evolution of closed-loop supply chain research. *Operations Research*, 57(1):10–18. 4

Güler, M. G. and Bilgiç, T. (2009). On coordinating an assembly system under random yield and random demand. *European Journal of Operational Research*, 196(1):342–350. 30

Gupta, D. and Wang, L. (2009). A stochastic inventory model with trade credit. *Manufacturing & Service Operations Management*, 11(1):4–18. 27

He, Y. and Zhang, J. (2008). Random yield risk sharing in a two-level supply chain. *International Journal of Production Economics*, 112(2):769–781. 30, 56

Hong Kong SME Loan Guarantee Scheme (2014). Guarantee ceiling and period. 25

Hsu, A. and Bassok, Y. (1999). Random yield and random demand in a production system with downward substitution. *Operations Research*, 47(2):277–290. 5

Inderfurth, K. and Transchel, S. (2007). Technical note—note on "myopic heuristics for the random yield problem". *Operations Research*, 55(6):1183–1186. 5

Kaya, O. (2014). Sme financing in the euro area: New solutions to an old problem. 24

Kouvelis, P. and Zhao, W. (2011). The newsvendor problem and price-only contract when bankruptcy costs exist. *Production and Operations Management*, 20(6):921–936. 28

Kouvelis, P. and Zhao, W. (2012). Financing the newsvendor: Supplier vs. bank, and the structure of optimal trade credit contracts. *Operations Research*, 60(3):566–580. 28

Li, Q. and Zheng, S. (2006). Joint inventory replenishment and pricing control for systems with uncertain yield and demand. *Operations Research*, 54(4):696–705. 5, 30

Li, X., Li, Y., and Cai, X. (2013). Double marginalization and coordination in the supply chain with uncertain supply. *European Journal of Operational Research*, 226(2):228–236. 30, 56

Lo, S.-T., Wee, H.-M., and Huang, W.-C. (2007). An integrated production-inventory model with imperfect production processes and weibull distribution deterioration under inflation. *International Journal of Production Economics*, 106(1):248 – 260. 14

Lund, R. T. and Hauser, W. M. (2010). Remanufacturing - an american perspective. In *Responsive Manufacturing - Green Manufacturing (ICRM 2010), 5th International Conference on*, pages 1 –6. 1

Luo, W. and Shang, K. (2012). Integrating inventory replenishment and cash payment decisions in supply chains. Technical report, Working Paper. 28

Modigliani, F. and Miller, M. H. (1958). The cost of capital, corporation finance and the theory of investment. *The American economic review*, pages 261–297. 62

Mukhopadhyay, S. K. and Ma, H. (2009). Joint procurement and production decisions in remanufacturing under quality and demand uncertainty. *International Journal of Production Economics*, 120(1):5 – 17. 5, 14

Novemsky, N. and Kahneman, D. (2005). The boundaries of loss aversion. *Journal of Marketing Research*, 42:119–128. 3

Rajaram, K. and Karmarkar, U. S. (2002). Product cycling with uncertain yields: Analysis and application to the process industry. *Operations Research*, 50:680–691. 32

Ray, S., Boyaci, T., and Aras, N. (2005). Optimal prices and trade-in rebates for durable, remanufacturable products. *Manufacturing & Service Operations Management*, 7(3):208–228. 4, 8, 18

Riding, A., Madill, J., and Haines Jr, G. (2007). Incrementality of sme loan guarantees. *Small Business Economics*, 29(1-2):47–61. 25

Savaskan, R. C., Bhattacharya, S., and Van Wassenhove, L. N. (2004). Closed-loop supply chain models with product remanufacturing. *Management Science*, 50(2):239–252. 4, 8

Serel, D. A. (2008). Inventory and pricing decisions in a single-period problem involving risky supply. *International Journal of Production Economics*, 116(1):115–128. 30

Song, J. and Tong, J. D. (2012). A new accounting framework for supply chain inventory finance. *working paper*. 29

Souza, G. C. (2008). *Closed-loop supply chains with remanufacturing*. INFORMS, Hanover, MD. 4

Tang, O., Musa, S. N., and Li, J. (2012). Dynamic pricing in the newsvendor problem with yield risks. *International Journal of Production Economics*, 139(1):127 – 134. 5, 14

Tang, S. Y., Gurnani, H., and Gupta, D. (2013). Managing disruptions in decentralized supply chains with endogenous supply process reliability. *Production and Operations Management*. 31, 56

Tang, S. Y. and Kouvelis, P. (2014). Pay-back-revenue-sharing contract in coordinating supply chains with random yield. *Production and Operations Management*. 30

Tanrisever, F., Cetinay, H., Reindorp, M., and Fransoo, J. C. (2012). Value of reverse factoring in multi-stage supply chains. *Available at SSRN 2183991*. 28

Tomlin, B. (2006). On the value of mitigation and contingency strategies for managing supply chain disruption risks. *Management Science*, 52(5):639–657. 31

Tunca, T. I. and Zhu, W. (2014). Buyer intermediation in supplier finance. *Working paper*. 29

Wang, C. (2009). Random yield and uncertain demand in decentralised supply chains under the traditional and vmi arrangements. *International Journal of Production Research*, 47(7):1955–1968. 30

Wang, Y., Gilland, W., and Tomlin, B. (2010). Mitigating supply risk: Dual sourcing or process improvement? *Manufacturing & Service Operations Management*, 12(3):489–510. 31

Wee, H., Yu, J., and Chen, M. (2007). Optimal inventory model for items with imperfect quality and shortage backordering. *Omega*, 35(1):7 – 11. 14

Winer, R. S. (1997). Discounting and its impact on durables buying decisions. *Marketing Letters*, 8:109–118. 3

Wu, A., Huang, B., and Chiang, D. M.-H. (2014). Support sme suppliers through buyer-backed purchase order financing. *Available at SSRN 2462521*. 29

Yang, S. A. and Birge, J. R. (2011). How inventory is (should be) financed: Trade credit in supply chains with demand uncertainty and costs of financial distress. Technical report, Working paper, University of Chicago Booth School of Business. 28

Yang, Z., Aydin, G., Babich, V., and Beil, D. R. (2009). Supply disruptions, asymmetric information, and a backup production option. *Management science*, 55(2):192–209. 31

Yano, C. A. and Lee, H. L. (1995). Lot sizing with random yields: A review. *Operations Research*, 43(2):311–334. 5

Zhou, J. and Groenevelt, H. (2008). Impacts of financial collaboration in a three-party supply chain. *Working paper*. 28

Zhou, S. X., Tao, Z., and Chao, X. (2011). Optimal control of inventory systems with multiple types of remanufacturable products. *Manufacturing & Service Operations Management*, 13(1):20–34. 5

Zhou, S. X. and Yu, Y. (2011). Optimal product acquisition, pricing, and inventory management for systems with remanufacturing. *Operations Research*, 59:514–521. 4, 23

Zikopoulos, C. and Tagaras, G. (2007). Impact of uncertainty in the quality of returns on the profitability of a single-period refurbishing operation. *European Journal of Operational Research*, 182(1):205 – 225. 2, 5

Appendix A

Proofs in Chapter 1

Proof of Lemma 1.2 :

Define $H(\mathbf{x},\rho) = \theta(f) + \omega(p) - \rho(\eta(r) + \theta(f))$ and $K(\mathbf{x}) = \frac{\theta(f)+\omega(p)}{\theta(f)+\eta(r)}$, where $\mathbf{x} = (r, f, p)$. It is obvious that $E[\Pi(\mathbf{x})]$ is differentiable for both $H(\mathbf{x}, B) < 0$ and $H(\mathbf{x}, B) > 0$. The only thing needs to be proven is $E[\Pi(\mathbf{x})]$ is differentiable at $H(\mathbf{x}, B) = 0$. Let $\mathbf{x_0} = \{\mathbf{x} \mid H(\mathbf{x}, B) = 0\}$, it can be shown that the partial derivatives at $\mathbf{x_0}$ exist and are continuous.

Denote $\boldsymbol{\Delta}$ as a vector such that $H(\mathbf{x_0} + \boldsymbol{\Delta}, B) > 0$ and $H(\mathbf{x_0} - \boldsymbol{\Delta}, B) < 0$. Consider the special case where $\boldsymbol{\Delta_i} = t\mathbf{e_i} = (0, ..., t, ..., 0)$, $i \in \{r, f, p\}$. Taking $\boldsymbol{\Delta_r}$ as an example:

$$\begin{aligned} E[\Pi(\mathbf{x_0})] - E[\Pi(\mathbf{x_0} - \boldsymbol{\Delta_r})] \quad = \quad & \eta(r_0 - t)(r_0 - t + d) - \eta(r_0)(r_0 + d) \\ & +(c_r - c)\left(\int_A^{K(\mathbf{x_0})} H(\mathbf{x_0}, \rho) g(\rho) d\rho \right. \\ & \left. - \int_A^{K(\mathbf{x_0} - \boldsymbol{\Delta_r})} H(\mathbf{x_0} - \boldsymbol{\Delta_r}, \rho) g(\rho) d\rho \right) \end{aligned}$$

Let $\mid \boldsymbol{\Delta_r} \mid \to 0$:

$$\begin{aligned} & \int_A^{K(\mathbf{x_0})} H(\mathbf{x_0}, \rho) g(\rho) d\rho - \int_A^{K(\mathbf{x_0} - \boldsymbol{\Delta_r})} H(\mathbf{x_0} - \boldsymbol{\Delta_r}, \rho) g(\rho) d\rho \\ = & \int_A^{K(\mathbf{x_0})} (H(\mathbf{x_0}, \rho) - H(\mathbf{x_0} - \boldsymbol{\Delta_r}, \rho)) g(\rho) d\rho + \int_{K(\mathbf{x_0} - \boldsymbol{\Delta_r})}^{K(\mathbf{x_0})} H(\mathbf{x_0}, \rho) g(\rho) d\rho \\ = & \int_A^{K(\mathbf{x_0})} t H_r'(\mathbf{x_0}, \rho) g(\rho) d\rho + o(t) + (K(\mathbf{x_0}) - K(\mathbf{x_0} - \boldsymbol{\Delta_r})) H(\mathbf{x_0}, \xi) \end{aligned}$$

where $K(\mathbf{x_0} - \mathbf{\Delta_r}) < \xi < K(\mathbf{x_0})$, and $\xi \to K(\mathbf{x_0})$ as $\mid \mathbf{\Delta_r} \mid \to 0$.

We can obtain:

$$
\begin{aligned}
&\lim_{|\mathbf{\Delta_r}| \to 0} \frac{E[\Pi(\mathbf{x_0})] - E[\Pi(\mathbf{x_0} - \mathbf{\Delta_r})]}{\mid \Delta_r \mid} \\
&= - \eta(r_0) - \eta_r'(r_0)(r_0 + d) + (c_r - c)\left(\int_A^{K(\mathbf{x_0})} H_r'(\mathbf{x_0}, \rho) g(\rho) d\rho + K_r'(\mathbf{x_0}) H(\mathbf{x_0}, K(\mathbf{x_0}))\right) \\
&= - \eta(r_0) - \eta_r'(r_0)(r_0 + d) + (c - c_r)\eta_r'(r_0)\mu
\end{aligned}
$$

The last equality comes from the fact taht $K(\mathbf{x_0}) = B$. It is easy to show that

$$
\begin{aligned}
&\lim_{|\mathbf{\Delta_r}| \to 0} \frac{E[\Pi(\mathbf{x_0} + \mathbf{\Delta_r})] - E[\Pi(\mathbf{x_0})]}{\mid \Delta_r \mid} \\
&= - \eta(r_0) - \eta_r'(r_0)(r_0 + d) + (c - c_r)\eta_r'(r_0)\mu
\end{aligned}
$$

Similarly, it can be obtained that the partial derivatives exist and are continuous with respect to f and p. Hence $E[\Pi(\mathbf{x})]$ is differentiable at $\mathbf{x_0}$.

Poof of Proposition 1.2 :

It is easy to show that $E[\Pi(r, f, p)]$ is concave for the case $(\theta + \eta)B \leq \theta + \omega$. We will then look at the situation when $(\theta + \eta)A \leq \theta + \omega \leq (\theta + \eta)B$. To show the concavity, we need to have $E[-\Pi(r, f, p)]$ is convex in $r,\ f\ and\ p$. Applying Sylvester's criterion, it is equivalent to prove:

(1)

$$
H_1 = \frac{\partial^2 E[-\Pi(r, f, p)]}{\partial r^2} > 0;
$$

(2)

$$
H_2 = \begin{vmatrix} \frac{\partial^2 E[-\Pi(r,f,p)]}{\partial r^2} & \frac{\partial^2 E[-\Pi(r,f,p)]}{\partial r \partial f} \\ \frac{\partial^2 E[-\Pi(r,f,p)]}{\partial r \partial f} & \frac{\partial^2 E[-\Pi(r,f,p)]}{\partial f^2} \end{vmatrix} > 0;
$$

and (3)

$$
H_3 = \begin{vmatrix} \frac{\partial^2 E[-\Pi(r,f,p)]}{\partial r^2} & \frac{\partial^2 E[-\Pi(r,f,p)]}{\partial r \partial f} & \frac{\partial^2 E[-\Pi(r,f,p)]}{\partial r \partial p} \\ \frac{\partial^2 E[-\Pi(r,f,p)]}{\partial f \partial r} & \frac{\partial^2 E[-\Pi(r,f,p)]}{\partial f^2} & \frac{\partial^2 E[-\Pi(r,f,p)]}{\partial f \partial p} \\ \frac{\partial^2 E[-\Pi(r,f,p)]}{\partial p \partial r} & \frac{\partial^2 E[-\Pi(r,f,p)]}{\partial p \partial f} & \frac{\partial^2 E[-\Pi(r,f,p)]}{\partial p^2} \end{vmatrix} > 0.
$$

Defining $C = \frac{c-c_r}{\eta+\theta} g\left(\frac{\omega+\theta}{\eta+\theta}\right)$, $x = \frac{\beta(\omega+\theta)}{\eta+\theta}$, and $y = \frac{\gamma(\eta-\omega)}{\eta+\theta}$. Since $c > c_r$, it's straightforward that $C \geq 0$. It can be obtained that: $H_1 = 2\beta + Cx^2 > 0$, $H_2 = 4\beta\gamma + 2C\beta y^2 + 2C\gamma x^2 > 0$, and $H_3 = 4Cby^2\beta + 4Cbx^2\gamma + 8b\beta\gamma + 4Cb^2\beta\gamma > 0$. Therefore, the expected profit function is concave on $(\theta+\eta)B \leq \theta+\omega$. Combing Lemma 1.2, it can be concluded that $E[\Pi(r, f, p)]$ is concave.

Proof of corollary 1.2 :

Denote the optimal pricing decisions as (r_1^*, f_1^*, p_1^*) and (r_2^*, f_2^*, p_2^*) for yield distribution ρ_1 and ρ_2 respectively. By definition, we have $E[\Pi(r_2^*, f_2^*, p_2^* \mid \rho_2)] \geq E[\Pi(r_1^*, f_1^*, p_1^* \mid \rho_2)]$. Furthermore:

$$
\begin{aligned}
&E[\Pi(r_1^*, f_1^*, p_1^* \mid \rho_2)] - E[\Pi(r_1^*, f_1^*, p_1^* \mid \rho_1)] \\
=& -(c-c_r)\left(\int_A^{\frac{\theta+\omega}{\theta+\eta}} (\theta+\omega-x(\theta+\eta))\,\phi_2(x)dx - \int_A^{\frac{\theta+\omega}{\theta+\eta}} (\theta+\omega-x(\theta+\eta))\,\phi_1(x)dx\right) \\
=& (c-c_r)(\theta+\eta)\int_A^{\frac{\theta+\omega}{\theta+\eta}} (\Phi_1(x)-\Phi_2(x))dx \geq 0,
\end{aligned}
$$

The last equality comes from the fact that $\int_A^k (x-k)\phi(x)dx = (x-k)\Phi(x)|_A^k - \int_A^k \Phi(x)dx = -\int_A^k \Phi(x)dx$. Consequently $E[\Pi(r_2^*, f_2^*, p_2^* \mid \rho_2)] \geq E[\Pi(r_1^*, f_1^*, p_1^* \mid \rho_1)]$ and Corollary 1.2 follows.

Appendix B

Proofs in Chapter 2

Proof of Lemma 2.1:

Part a. When $x = 1$, since $RHS = (y-b)(1+r)$, $r = r_f$ is the only solution to Equation 2.1.

Part b. Since $\int_0^{\frac{(y-b)(1+r)}{wy}} wy\zeta g(\zeta)\, d\zeta \le \int_0^{\frac{(y-b)(1+r)}{wy}} (y-b)(1+r)g(\zeta)\, d\zeta$, we can obtain $RHS\,(r) \le x(y-b)\,(1+r) + (1-x)\,(y-b)\,(1+r) = (y-b)\,(1+r)$. Therefore, to meet bank's return requirement, the loan interest rate r is no less than r_f.

Part c and d, proofs are omitted.

Proof of Lemma 2.3 :

Define function $F(t, x, y, r_f) = xwyt + (1-x)wy[\int_0^t \zeta dG(\zeta) + \int_t^1 tdG(t)] - (y-b)(1+r_f)$. By taking partial derivative, it can be obtained that:

$$F_y = xwt + (1-x)w[\int_0^t \zeta dG(\zeta) + \int_t^1 tdG(t)] - (1+r_f) = -\frac{b}{y}(1+r_f) < 0$$

$$F_x = wyt - wy[\int_0^t \zeta dG(\zeta) + \int_t^1 tdG(t)] > 0$$

$$F_{r_f} = -(y-b) < 0$$

$$F_t = xwy + (1-x)wy(1-G(t)) > 0$$

$$F_b = 1 + r_f > 0$$

Therefore $\frac{\partial t}{\partial x} = -\frac{F_x}{F_t} = -\frac{t-[\int_0^t \zeta dG(\zeta)+\int_t^1 tdG(t\zeta)]}{x+(1-x)(1-G(t))} < 0$, $\frac{\partial t}{\partial b} = -\frac{F_b}{F_t} < 0$, $\frac{\partial t}{\partial y} = -\frac{F_y}{F_t} > 0$, and $\frac{\partial t}{\partial r_f} = -\frac{F_{r_f}}{F_t} > 0$.

Proof of Lemma 2.4 :

The conclusion can be proven by checking the first order partial derivative: $\frac{\partial \pi_s}{\partial x} = -wy_1\bar{G}(t)\frac{\partial t}{\partial x} > 0$, and similarly $\frac{\partial \pi_s}{\partial r_f} = wy\bar{G}(t)\frac{\partial t}{\partial r_f} < 0$.

Proof of Lemma 2.5:

Since $\pi_s = \int_{\frac{(y-b)(1+r)}{wy}}^1 (wy\zeta - (y-b)(1+r))g(\zeta)d\zeta$, we can obtain $\frac{\partial \pi_s}{\partial r} = -(y-b)\bar{G}(\frac{(y-b)(1+r)}{wy}) < 0$.

Proof of Proposition 2.1:

Combine Equation 2.5 and Lemma 2.4, it is straightforward that for any order size greater than b, if $wE[\zeta] \geq 1 + r_f$, we have $\pi_s(y) \geq \pi_s(b)$, $\forall x \in [0,1]$.

When $wE[\zeta] < 1 + r_f$, as Equation 2.5 shows, the supplier is not willing to borrow if $x = 0$. Consider r as a function of r_f, x, and y, according to Lemma 2.4, the minimum guarantee level for the supplier to be willing to borrow is $\underline{x}$ such that $r(r_f, \underline{x}, y) = r_b(wE[\zeta], 0, y)$. On the other hand, as stated in Lemma 2.1, $r(r_f, x, y) \leq r(r_f, 1, y) = r_f = r_b(wE[\zeta], 0, y_{lb})$, where the last equality comes from the definition of y_{lb}. Consequently $\forall y \in [b, y_{lb})$, $r(r_f, x, y) > r_f \geq r_b(wE[\zeta], 0, y)$, and $\pi_s(r_f, x, y) < \pi_s(b)$ even if a 100% guarantee is provided by the retailer. Therefore, Proposition 2.1 is proved.

Proof of Corollary 2.1:

From equation 2.7, we can obtain $\frac{\partial t_b}{\partial b} = -\frac{E[\zeta]}{y\bar{G}(t_b)} < 0$, $\frac{\partial t_b}{\partial y} = \frac{bE[\zeta]}{y^2\bar{G}(t_b)} > 0$. According to equation 2.7 and 2.8: $\underline{x} = \frac{(y-b)(1+r_f-wE[\zeta])}{wy\int_0^{t_b}(t_b-\zeta)dG(\zeta)} = \frac{1+r_f-wE[\zeta]}{w\frac{yt_b}{y-b}-wE[\zeta]}$ and the numerator is constant in b. For the denominator, $\frac{\partial}{\partial b}(\frac{yt_b}{y-b}) = \frac{y(y-b)t_b'+yt_b}{(y-b)^2} = \frac{-(y-b)E[\zeta]+yt_b\bar{G}(t_b)}{(y-b)^2\bar{G}(t_b)} = \frac{-y_1\int_0^{t_b}\zeta dG(\zeta)}{(y-b)^2\bar{G}(t_b)} < 0$, where the last equality is obtained by applying equation 2.7. Therefore $\underline{x}$ is increasing in b.

Similarly $\frac{\partial}{\partial y}(\frac{yt_b}{y-b}) = \frac{t_b'(1-\frac{b}{y})-\frac{bt_b}{y^2}}{(1-\frac{b}{y})^2} = \frac{b}{\bar{G}(t_b)(y-b)^2}(E[\zeta](1-\frac{b}{y}) - t_b\bar{G}(t_b)) = \frac{b\int_0^{t_b}\zeta dG(\zeta)}{\bar{G}(t_b)(y-b)^2} > 0$, hence $\underline{x}$ is decreasing in y.

Since t_b is irrelevant of r_f and w, it's straightforward that x is increasing in r_f and decreasing in w.

Proof of Corollary 2.2 :

Since y_{lb} is defined as: $(y_{lb} - b)wE[\zeta] = wy_{lb}\int_0^{\frac{(y_{lb}-b)(1+r_f)}{wy_{lb}}} \zeta dG(\zeta) + (y_{lb} - b)(1 + r_f)\bar{G}(\frac{(y_{lb}-b)(1+r_f)}{wy_{lb}})$, we can obtain $\frac{\partial y_{lb}(b)}{\partial b} = \frac{y_{lb}}{b} > 0$. Similarly, it can be shown that y_{lb} is increasing in r_f and decreasing in w.

Proof of Lemma 2.6:

For $y \leq b$, retailer's profit is constant in x. We then look at the case $y > b$. Since the first term in Equation 2.6 is irrelevant to x, we only consider the second term.

$$\begin{aligned}
\frac{\partial \pi_r}{\partial x} &= -wy\int_0^t (t-\zeta)dG(\zeta) - xwyG(t)\frac{\partial t}{\partial x} \\
&= wy\frac{-1+G(t)}{1-G(t)+xG(t)}\int_0^t (t-\zeta)dG(\zeta) \\
&\leq 0
\end{aligned}$$

Therefore, retailer's profit is decreasing in the guarantee level x.

Proof of Proposition 2.2 : We first look at the relaxation problem where x can be greater than 1. The retailer's profit(denoted by π_r^p) with optimal guarantee $\underline{x}$ becomes:

$$\pi_r^p = \begin{cases} \pi_r(0,y), & \text{if } 0 < y \leq b \\ \pi_r(0,y) - (y-b)(1+r_f-wE[\zeta]), & \text{if } y > b \end{cases}$$

It can be verified that the retailer's profit is continuous for the relaxation problem. For the original problem, the retailer's profit is constant in y for $y \in [b, y_{lb}]$. Denote $y_A = \arg\max_{0\leq y\leq b}\{\pi_r(0,y)\}$ and $y_B = \arg\max_{y\geq b}\{\pi_r(0,y) - (y-b)(1 + r_f - wE[\zeta])\}$.

The necessary condition for the retailer to be willing to provide positive guarantee is that there exists $y_B \in [b, +\infty)$ such that $\pi_r(0, y_B) - (y_B - b)(1 + r_f - wE[\zeta]) > \pi_r(0, y_A)$.

For the first situation($r_f \leq wE[\zeta] - 1$ or $b \geq y^*$), the result is straightforward from Lemma 2.6 and Proposition 2.1. For the second situation($r_f > wE[\zeta] - 1$ and $b < y^*$), we discuss the following three cases:

part a: From the definition of b_1, for $b < b_1$ we have $y_{lb}(b) < y'$. Hence, $y_B = y'$ and $\underline{x}(y') \leq 1$.

Since $r_f > wE[\zeta] - 1$, $y' < y^*$. The retailer's profit is increasing on $[0, b]$ and $y_A = b$. We can obtain that $\pi_r(\underline{x}(y'), y') > \pi_r(0, b)$ by considering the relaxation problem. Hence, retailer's profit is unimodal and the optimal ordering quantity is y'.

part b and c: We divide the proof into two cases, $b_1 < b \leq y'$ and $y' < b < y*$.

For $b_1 < b < y'$: When $b = b_1$, $y_{lb}(b_1) = y'$, as part a shows, we have $\pi_r(0, y') - (y' - b_1)(1 + r_f - wE[\zeta]) > \pi_r(0, b_1)$. When $b = y'$, we have $\pi_r(0, y_{lb}(y')) - (y_{lb} - y')(1 + r_f - wE[\zeta]) < \pi_r(0, y')$.

Both $\pi_r(0, b)$ and $\pi_r(\underline{x}, y_{lb}(b))$ are continuous in b. Obviously, there exists $b_2 \in (b_1, y')$ such that $\pi_r(\underline{x}, y_{lb}(b_2)) = \pi_r(0, y_{lb}(b_2)) - (y_{lb}(b_2) - b_2)(1 + r_f - wE[\zeta]) = \pi_r(0, b_2)$.

We then show that b_2 is unique. When the retailer offers positive guarantee, the maximum profit can be obtained is $\pi_r(\underline{x}, y_{lb}(b))$. We have $\frac{\partial \pi_r(\underline{x}, y_{lb}(b))}{\partial b} = \frac{\partial y_{lb}}{\partial b} \int_0^1 (p\bar{F}(y_{lb}\zeta) - w)\zeta g(\zeta) d\zeta - (\frac{\partial y_{lb}}{\partial b} - 1)(1 + r_f - wE[\zeta]) = \frac{\partial y_{lb}}{\partial b}[\int_0^1 (p\bar{F}(y_{lb}\zeta))\zeta g(\zeta) d\zeta - (1 + r_f)] + 1 + r_f - wE[\zeta]$.

Since $b > b_1$, we have $y_{lb}(b) > y'$ and $\int_0^1 (p\bar{F}(y_{lb}\zeta))\zeta g(\zeta) d\zeta - (1 + r_f) < 0$. According to Corollary 2.2, $\frac{\partial y_{lb}}{\partial b}$ is positive. Therefore $\frac{\partial \pi_r(\underline{x}, y_{lb}(b))}{\partial b} < 1 + r_f - wE[\zeta]$.

When the buyer does not provide guarantee, since $b < y' < y^*$, the maximum profit is $\pi_r(0, b)$ and we have $\frac{\partial \pi_r(0,b)}{\partial b} = \int_0^1 p\zeta\bar{F}(b\zeta)g(\zeta)d\zeta - wE[\zeta]$. Since $b < y'$, it can be obtained that $\frac{\partial \pi_r(0,b)}{\partial b} > 1 + r_f - wE[\zeta]$.

Therefore for $b > b_1$, we have $\frac{\partial \pi_r(0,b)}{\partial b} > 1 + r_f - wE[\zeta] > \frac{\partial \pi_r(\underline{x}, y_{lb}(b))}{\partial b}$, consequently $\pi_r(0,b) > \pi_r(\underline{x}, y_{lb}(b))$ is true $\forall b > b_2$. Hence, there is unique b_2.

Secondly, when $y' < b < y^*$, $\pi_r(0,b) > \pi_r(0,y') > \pi_r(\underline{x}, y_{lb}(b))$. Combine the results in the previous discussion, for $b_2 < b < y*$, the retailer should not provide guarantee, and the optimal ordering quantity is b.

Proof of Lemma 2.8 :

We prove the lemma by showing that the retailer's profit is always decreasing in y_1 when $w > v$. Denote $\pi_r(x, y_1)$ as the retailer's first stage expected profit. When guarantee is not needed, for $y_1 < B$: $\frac{\partial}{\partial y_1}\pi_r(0, y_1) = (v - w)E[\zeta] < 0$; for $y_1 > B$,

$$\begin{aligned}
\frac{\partial}{\partial y_1}\pi_r(0, y_1) &= \int_0^{\frac{B}{y_1}} (v - w)\zeta dG(\zeta) + \int_{\frac{B}{y_1}}^1 \left(p\bar{F}(y_1\zeta) - w\right) \zeta dG(\zeta) \\
&= (v - w)E[\zeta] + \int_{\frac{B}{y_1}}^1 \left(p\bar{F}(y_1\zeta) - v\right) \zeta dG(\zeta) \\
\text{and } \frac{\partial^2}{\partial y_1^2}\pi_r(0, y_1) &= -\int_{\frac{B}{y_1}}^1 pf(y_1\zeta)\zeta^2 dG(\zeta) < 0.
\end{aligned}$$

From the definition of B, for $y_1 > B$ we have $\frac{\partial}{\partial y_1}\pi_r(0, y_1) < \int_{\frac{B}{y_1}}^1 \left(p\bar{F}(y_1\zeta) - v\right) \zeta dG(\zeta) < 0$.

When $wE[\zeta] < 1 + r_f$, buyer's guarantee is needed. For $y_1 < B$, $\frac{\partial}{\partial y_1}\pi_r(\underline{x}, y_1) = vE[\zeta] - 1 - r_f < 0$; for $y_1 > B$, $\frac{\partial}{\partial y_1}\pi_r(\underline{x}, y) = vE[\zeta] - 1 - r_f + \int_{\frac{B}{y_1}}^1 \left(p\bar{F}(y_1\zeta) - v\right) \zeta dG(\zeta) < 0$. Hence, we can conclude that for $w > v$, the retailer should not order from the unreliable SME supplier.

Proof of Proposition 2.3 :

The proof for case 1 is omitted.

For case 2: Since $b < y_1^*$, it is straightforward that the retailer's profit is increasing in y_1 on $[0, b]$. For $y_1 > b$, we consider the auxiliary problem: $\pi_r^p(y_1) = E_\zeta\{-wy_1\zeta - (y_1 - b)(1 + r_f - wE[\zeta]) + E_D[p\min\{\max\{B, y_1\zeta\}, D\}] - v(B - y_1\zeta)^+]\}$. Since $r_f > vE[\zeta] - 1$, if $y_1 < B$, we have $\frac{\partial}{\partial y_1}\pi_r^p(y_1) = vE[\zeta] - 1 - r_f < 0$; if $y_1 > B$, $\frac{\partial}{\partial y_1}\pi_r^p(y_1) = vE[\zeta] - 1 - r_f + \int_{\frac{B}{y_1}}^1 \left(p\bar{F}(y_1\zeta) - v\right) \zeta dG(\zeta) < 0$. Therefore, for any first stage order size greater than b, $\pi_r(\underline{x}, y_1) < \pi_r(0, b)$. Hence the optimal first stage order size is b.

For case 3: We first state the relations $B < y_1' < y_1^*$. Depending on the value of b, We classify the discussion into 6 possible situations. $b < y_{lb} < B < y'$, $b < B < y_{lb} < y'$, $b < B < y' < y_{lb}$, $B < b < y_{lb} < y'$, $B < b < y' < y_{lb}$, and $B < y' < b < y_{lb}$.

For the first situation, the profit function of the retailer is

$$\pi_r(y_1) \quad = \begin{cases} (v-w)y_1E[\zeta] - vB + E[p\min\{B,D\}], & \text{if } 0 < y_1 \le b \\ (v-w)bE[\zeta] - vB + E[p\min\{B,D\}], & \text{if } b < y_1 \le y_{lb} \\ (vE[\zeta] - 1 - r_f)y_1 - vB + b(1 + r_f - wE[\zeta]) & \\ +E[p\min\{B,D\}], & \text{if } y_{lb} < y_1 \le B \\ (b - y_1)(1 + r_f) - wbE[\zeta] & \\ +E[p\min\{\max\{B, y_1\zeta\}, D\} - v(B - y_1\zeta)^+], & \text{if } y_1 > B \end{cases}$$

To preserve continuity, consider the auxiliary problem, where for $b < y_1 < y_{lb}$, $\pi_r^p(y_1) = (vE[\zeta] - 1 - r_f)y_1 - vB + b(1 + r_f - wE[\zeta]) + E[p\min\{B,D\}]$, we have

$$\frac{\partial}{\partial y_1}\pi_r^p(y_1) = \begin{cases} (v-w)E[\zeta], & \text{if } 0 < y_1 \le b \\ vE[\zeta] - 1 - r_f, & \text{if } b < y_1 \le y_{lb} \\ vE[\zeta] - 1 - r_f, & \text{if } y_{lb} < y_1 \le B \\ \int_0^{B/y_1} v\zeta dG(\zeta) + \int_{B/y_1}^1 p\bar{F}(y_1\zeta)\zeta dG(\zeta) - 1 - r_f, & \text{if } y_1 > B \end{cases}$$

Therefore, the π_r^p is unimodal, and the optimal ordering quantity is y_1'. Since $\pi_r(y_1)$ is different from $\pi_r^p(y_1)$ only when $y_1 \in (b, y_{lb})$ and $\pi_r(y_1) = \pi_r(b)$ in such case, we can conclude that y_1' is also the optimal solution for the original problem. For the other five situations, the proof is similar to this situation and the proof of Proposition 2.2.

Proof of Lemma 2.10

Depending on supplier's responding decision, retailer's profit has the two following situations.

1. For $y \leq bt_i$,

$$\begin{aligned}\pi_r(y) =& - wE[\text{min}\{y\zeta/t_i, y\}] + pE[\text{min}\{D, min\{y\zeta/t_i, y\}\}] \quad \text{(B.1)}\\ =& \int_0^{t_i} \left(-wy\zeta/t_i + \int_0^{y\zeta/t_i} pDf(D)dD + \int_{y\zeta/t_i}^{\infty} py\zeta/t_i f(D)dD \right) g(\zeta)d\zeta \\ &+ \int_{t_i}^{1} \left(-wy + \int_0^{y} pDf(D)dD + \int_{y}^{\infty} pyf(D)dD \right) g(\zeta)d\zeta\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \pi_r(y)}{\partial y} =& \int_0^{t_i} [-w + p\bar{F}(y\zeta/t_i)] \frac{\zeta}{t_i} g(\zeta)d\zeta + \int_{t_i}^{1} [-w + p\bar{F}(y)]g(\zeta)d\zeta \\ \frac{\partial^2 \pi_r(y)}{\partial y^2} =& - p \int_0^{t_i} \frac{\zeta^2}{t_i^2} f(\frac{y\zeta}{t_i})g(\zeta)d\zeta - p \int_{t_i}^{1} f(y)g(\zeta)d\zeta < 0\end{aligned}$$

It can be seen that π_r is concave in y. Denote y_i as the ordering quantity which satisfies the first order condition. For the order space $[0, bt_i]$: if $y_i < bt_i$, π_r is increasing on $[0, y_i]$ and decreasing on $[y_i, bt_i]$; if $y_i > bt_i$, π_r is increasing on $[0, bt_i]$. Therefore, for $0 < y \leq bt_i$, retailer's optimal order quantity is min$\{y_i, bt_i\}$.

2. For $y > bt_i$

$$\pi_r = -wE[\text{min}\{b\zeta, y\}] + E[p\text{min}\{D, min\{b\zeta, y\}\}]$$

Further,

$$\pi_r = \begin{cases} -wE[\text{min}\{b\zeta, y\}] + pE[\text{min}\{D, min\{b\zeta, y\}\}] & \text{if } bt_i \leq y \leq b, \\ -wbE[\zeta] + pE[\text{min}\{D, b\zeta\}] & \text{if } y \geq b. \end{cases}$$

For $y > b$, π_r is constant in y and equals to the case of accepting all supplier's delivery. Therefore, we only need to consider the case $bt_i \leq y \leq b$.

$$\begin{aligned}\pi_r(y) =& \int_0^{y/b} \left(-wb\zeta + \int_0^{b\zeta} pDf(D)dD + \int_{b\zeta}^{\infty} pb\zeta f(D)dD \right) g(\zeta)d\zeta \quad \text{(B.2)}\\ &+ \int_{y/b}^{1} \left(-wy + \int_0^{y} pDf(D)dD + \int_{y}^{\infty} pyf(D)dD \right) g(\zeta)d\zeta\end{aligned}$$

and

$$\frac{\partial \pi_r(y)}{\partial y} =[-w + p\bar{F}(y)]\bar{G}(\frac{y}{b})$$
$$\frac{\partial^2 \pi_r(y)}{\partial y^2} =-pf(y)\bar{G}(\frac{y}{b}) - [p\bar{F}(y) - w]g(\frac{y}{b})\frac{1}{b}$$

Let y_0 denote the solution to $p\bar{F}(y) - w = 0$. The value of y_0 is also retailer's optimal order quantity when the supplier is with perfect yield and sufficient capital. By the definition of y_i, it can be concluded that $y_i \geq y_0$, which shows that when the supplier has sufficient capital, the retailer needs to increase the order size if the yield is random.

Since $bt_i \leq y \leq b$, it is obvious that $\bar{G}(\frac{y}{b}) \geq 0$. The first order derivative $\frac{\partial \pi_r(y)}{\partial y} = [-w + p\bar{F}(y)]\bar{G}(\frac{y}{b})$ is non-negative for $y \leq y_0$ and non-positive for $y \geq y_0$. Therefore, retailer's expected profit is unimodal in this case(not necessarily concave).

For the case $y \geq bt_i$, depending on the magnitude of y_0 and b, there are following possible situations: if $y_0 < bt_i$, π_r is decreasing for $y \in [bt_i, b]$, and constant for $y \in [b, \infty)$; if $bt_i < y_0 < b$, π_r is increasing for $y \in [bt_i, y_0]$, decreasing for $y \in [y_0, b]$ and constant for $y \in [b, +\infty)$; if $y_0 > b$, π_r is increasing for $y \in [bt_i, b]$ and constant on $y \in [b, \infty)$. The optimal order decision for $y > bt_i$ is then:

$$y = \begin{cases} b, & \text{if } b < y_0 \\ y_0, & \text{if } y_0 < b < \frac{y_i}{t_i} \\ bt_i, & \text{if } b > \frac{y_i}{t_i} \end{cases}$$

Combine the discussions for the two situations, we can conclude that π_r is unimodal in y. The optimal ordering decisions are summarized in Lemma 2.10 and Figure 2.8.

Poof of Lemma 2.11:

part a. For $0 < b < \frac{y_0}{t_l}$, $\pi_{r2}^* = \pi_{r2}(bt_l)$,

$\frac{\partial \pi_{r2}(bt_l)}{\partial b} = \int_0^{t_l} [-w + p\bar{F}(b\zeta)]\zeta g(\zeta)d\zeta + [-w + p\bar{F}(bt_l)]t_l\bar{G}(t_l)$.

Since $\int_0^{t_l} [-w + p\bar{F}(\frac{y_l}{t_l}\zeta)]\frac{\zeta}{t_l}g(\zeta)d\zeta + [-w + p\bar{F}(y_l)]\bar{G}(t_l) = 0$ and $b < \frac{y_0}{t_l} < \frac{y_l}{t_l}$, we can obtain $\frac{\partial \pi_{r2}(bt_l)}{\partial b} \geq 0$.

part b. For $\frac{y_0}{t_l} < b < \frac{y_0}{t_i}$, $\pi^*_{r2} = \pi_{r2}(y_0)$.,

$\frac{\partial \pi_{r2}(y_0)}{\partial b} = \int_0^{y_0/b}[-w + p\bar{F}(b\zeta)]\zeta g(\zeta)d\zeta$.

Since $b\zeta < y_0$ and $-w + p\bar{F}(y_0) = 0$, we have $\frac{\partial \pi_{r2}(y_0)}{\partial b} \geq 0$.

part c. For $\frac{y_0}{t_i} \leq b \leq \frac{y_i}{t_i}$, $\pi^*_{r2} = \pi_{r2}(bt_i)$.

$\frac{\partial \pi_{r2}(bt_i)}{\partial b} = \int_0^{t_i}[-w + p\bar{F}(b\zeta)]\zeta g(\zeta)d\zeta + [-w + p\bar{F}(bt_i)]t_i\bar{G}(t_i)$

Since $\int_0^{t_i}[-w + p\bar{F}(\frac{y_i}{t_i}\zeta)]\frac{\zeta}{t_i}g(\zeta)d\zeta + [-w + p\bar{F}(y_i)]\bar{G}(t_i) = 0$ and $b \leq \frac{y_i}{t_i}$, we can obtain $\frac{\partial \pi_{r2}(bt_i)}{\partial b} \geq 0$.

part d. For $b \geq \frac{y_i}{t_i}$, $\pi^*_{r2} = \pi_{r2}(bt_i)$.

$\frac{\partial \pi_{r2}(bt_i)}{\partial b} = \int_0^{t_i}[-w + p\bar{F}(b\zeta)]\zeta g(\zeta)d\zeta + [-w + p\bar{F}(bt_i)]t_i\bar{G}(t_i) < 0$.

Proof of Lemma 2.13 :

It can be obtained that $r_x - r_f = \frac{(1-x)(1-\theta)(1+r_f-w\alpha)}{x+\theta-x\theta}$. Since $x < 1$ and $1 + r_f > w\alpha$, we have $r_x > r_f$.

Proof of Lemma 2.14 :

By definition, we have $\frac{\partial}{\partial x}\underline{y} = \frac{-wb\alpha(1-\theta)(1+r_f-wE[\zeta]}{\theta(1+\mathrm{rf}-w(x+\alpha-x\alpha+(-1+x)(-1+\alpha)\theta))^2} < 0$, which shows $\underline{y}$ is decreasing in x. It can be obtained that $\underline{y}(1) - q_0 = \frac{bw\alpha(1+r_f-wE[\zeta]}{(w-1-r_f)(1+r_f-w\alpha)\theta} > 0$. Therefore $\underline{y}(x) > \underline{y}(1) > q_0$.

Since $w\mathbb{E}[\zeta] < 1 + r_f$, the supplier's profit is decreasing in q for $q \in [b, q_0]$ and $\pi_s(x, q_0) < w\mathbb{E}[\zeta]b - b = \pi_s(b)$. On the other hand, since $w > 1 + r_x$, the supplier's profit is increasing in q for $q \in (q_0, y]$. According to Equation 2.12, we have $\pi_s(x, \underline{y}) = \theta w\underline{y} - \theta(\underline{y} - b)(1 + r) - b = wE[\zeta]b - b = \pi_s(x, b)$. Therefore, when $w\mathbb{E}[\zeta] < 1 + r_f$, the supplier is willing to take bank loan only when $w > 1 + r_x$ and $y \geq \underline{y}$.

Proof of Lemma 2.15 :

According to Lemma 2.14 $\underline{y}$ is decreasing in x, therefore, for $y > y_{lb} = \underline{y}(1)$, the supplier's optimal production quantity is y when $x = 1$. Since $wE[\zeta] < 1 + r_f$, we can also obtain that supplier's optimal production quantity is b when $x = 0$. From Equation 2.12, for $q = y$ we have $\frac{\partial}{\partial x}\pi_s(y) = \theta\frac{\partial r_x}{\partial x} = -\frac{\theta(1-\theta)(1+r_f-w\alpha)}{(x+\theta-x\theta)^2} < 0$. Therefore there exists unique $\underline{x}$ such that the supplier is willing to produce y when buyer's guarantee is no less than $\underline{x}$.

For $q_s = b$, the supplier does not take bank loan and the retailer's profit is not affected by x. When $q_s = y$, since $wE[\zeta] < 1 + r_f$ and $y \geq y_{lb} > q_0$, we have $(y-b)(1+r) - wy\alpha > 0$. When disruption occurs, the retailer's additional payment to the banks is: $x(1-\theta)[(y-b)(1+r) - wy\alpha] = \frac{(1-\theta)[(y-b)(1+r_f)-wy\alpha]}{1-\theta+\theta/x}$. Since $\frac{\partial}{\partial x}\pi_r = -\frac{\theta(1-\theta)[(1+r_f)(y-b)-wy\alpha]}{(x+\theta-x\theta)^2} < 0$, the retailer's expected profit is decreasing in x in such case.

Proof of Proposition 2.6 :

Consider the auxiliary profit function $\pi_r^p(y)$ where

$$\pi_r^p = \begin{cases} \theta[pE\min\{y,D\} - wy] + (1-\theta)[p\min\{y\alpha, D\} - wy\alpha], & \text{if } 0 < y < b \\ \theta[pE\min\{y,D\} - wy] + (1-\theta)[p\min\{y\alpha, D\} - wy\alpha] & \\ -(y-b)(1+r_f - wE[\zeta]), & \text{if } y \geq b \end{cases}$$

For $0 \leq b \leq b_1$, π_r^p is maximized at y'. Since y_{lb} is increasing in b, we have $y' \geq y_{lb}$ for $b \leq b_1$. Therefore, the optimal order quantity for the original problem is also y'.

For $b_1 < b < y'$, π_r is increasing in y between $[0,b]$, constant between $[b, y_{lb}]$ and decreasing between $[y_{lb}, +\infty)$. Retailer's optimal order quantity is either b or y_{lb}, where $\pi_r(b) = \theta[p\min\{b,D\} - wb] + (1-\theta)[p\min\{b\alpha, D\} - wb\alpha]$ and $\pi_r(y_{lb}) = \theta[p\min\{y_{lb}, D\} - wy_{lb}] + (1-\theta)[p\min\{y_{lb}\alpha, D\} - wy_{lb}\alpha] - (y_{lb} - b)(1 + r_f - wE[\zeta])$. Consider $\pi_r(b)$ and $\pi_r(y_{lb}(b))$ as functions of b, it is easy to verify that both functions are continuous in b.

When $b = b_1$, for the auxiliary problem, $\pi_r^p(y)$ is increasing in y for $y \in [0, b_1]$ and $y \in [b_1, y_{lb}(b_1)]$, but decreasing in y for $y \in [y_{lb}(b_1), +\infty)$. Hence, for the original problem we have $\pi_r(b) \mid_{b=b_1} < \pi_r(y_{lb}(b)) \mid_{b=b_1}$. Similarly, when $b = y'$, π_r^p is increasing on $[0, y']$, decreasing on $[y', y_{lb}(y')]$ and $[y_{lb}(y'), +\infty)$. Hence, $\pi_r(b) \mid_{b=y'} > \pi_r(y_{lb}(b)) \mid_{b=y'}$. By continuity, there exists $b_2 \in [b_1, y']$ such that $\pi_r(b) \mid_{b=b_2} = \pi_r(y_{lb}(b)) \mid_{b=b_2}$. We show b_2 is unique by comparing the first order partial derivative of $\pi_r(b)$ and $\pi_r(y_{lb}(b))$.

$\frac{\partial}{\partial b}\pi_r(b) = \theta(p\bar{F}(b) - w) + \alpha(1-\theta)(p\bar{F}(b\alpha) - w)$.

For $b < y'$, $\frac{\partial}{\partial b}\pi_r(b) > \theta(p\bar{F}(y') - w) + \alpha(1-\theta)(p\bar{F}(y'\alpha) - w) = 1 + r_f - wE[\zeta]$.

$\frac{\partial}{\partial b}\pi_r(y_{lb}(b)) = \{\theta(p\bar{F}(y_{lb})-w)+\alpha(1-\theta)(p\bar{F}(y_{lb}\alpha)-w)-1-r_f+wE[\zeta]\}\frac{\partial y_{lb}(b)}{\partial b}+ 1+r_f-wE[\zeta]$.

For $b > b_1$, we have $y_{lb}(b) > y'$. Combing the fact $\frac{\partial y_{lb}(b)}{\partial b} > 0$, it can be obtained that $\frac{\partial}{\partial b}\pi_r(y_{lb}(b)) < \frac{\partial}{\partial b}\pi_r(b)$.

Therefore, $\pi_r(b)$ and $\pi_r(y_{lb}(b)$ have unique intersection at b_2 and $b_2 \in [b_1, y']$. For $b_1 \leq b < b_2$, $\pi_r(b) < \pi_r(y_{lb}(b))$; for $b_2 < b \leq y'$, $\pi_r(b) > \pi_r(y_{lb}(b))$.

For $y' < b \leq y^0$, π_r^p is increasing on $[0, b]$, decreasing on $[b, +\infty)$. Therefore $\pi_r(b) > \pi_r(y_{lb}(b))$, the optimal order quantity is b.

For $b > y^0$, π_r^p is increasing on $[0, y^a]$, decreasing on $[y^a, +\infty)$. The optimal quantity is then y^a.

Proof of Proposition 2.7 :

Suppose the retailer lets supplier produce more than b, namely $y \geq y_{lb}$ and $x > \underline{x}$, as previous results shows, the optimal order quantity is the minimum of y' and y_{lb}. While for the ordering range $y < b$, the optimal order level is the maximum of b and y^a. Therefore the retailer optimal decisions can be obtained by comparing the value of $\pi_r(\underline{x}, max\{y', y_{lb}\})$ and $\pi_r(0, min\{b, y^a\})$. Depending on the value of y' and y^a, there are two possible cases.

First $y' \leq y^a$: By definition, we have $b_1 < y'$. For $b \leq b_1$, it is obvious that retailer's profit is maximized at $y = y'$ and $x = \underline{x}(y')$. For $b > y'$, since $y_{lb} \mid_{b=y'} > y'$, retailer's optimal decision is then order $min\{b, y^a\}$ without guarantee. Therefore, there exists $b_2 \in (b_1, y')$ such that $\pi_r(\underline{x}, y_{lb}(b)) = \pi_r(0, b)$. Since $\frac{\partial}{\partial b}\pi_r(\underline{x}, y_{lb}(b)) = \{\theta(p\bar{F}(y_{lb}) - w) + \alpha(1-\theta)(p\bar{F}(\alpha y_{lb}) - w) - 1 - r_f + wE[\zeta]\}\frac{\partial y_{lb}(b)}{\partial b} + 1 + r_f - wE[\zeta] < 1 + r_f - wE[\zeta]$, and $\frac{\partial}{\partial b}\pi_r(b) > \theta(p\bar{F}(y') - w) + \alpha(1-\theta)(p\bar{F}(y'\alpha) - w) = 1 + r_f - wE[\zeta]$, b_2 is unique.

The second case $y' > y^a$: For $b = 0$, it is optimal for the retailer to order $y = y'$ and provide guarantee $\underline{x}(y')$.

For $b > y'$, $\pi_r(0, y^a) \geq \pi_r(0, y') \geq \pi_r(\underline{x}, y_{lb}(y'))$, retailer's optimal decision is to order y^a without guarantee. If $y^a/\alpha < y'$, for $y^a/\alpha < b < y'$, $\pi_r(\underline{x}, max\{y', y_{lb}\}) \leq \pi_r^p(\underline{x}, y') \leq \pi_r(0, y') \mid_{b=y'} \leq \pi_r(0, y^a) \mid_{b=y'} = \pi_r(0, y^a) \mid_{b\in(y/\alpha, y')}$. Define $b_2^u = min\{y', y/\alpha\}$. By continuity there exists $b_2 \in (0, b_2^u)$ such that

$\pi_r(\underline{x}, max\{y', y_{lb}(b)\}) = \pi_r(0, min\{b, y^a\})$. Also, for $b > b_2^u$, the retailer's optimal decision is to order y^a without guarantee.

For $b < b_1$: $\frac{\partial}{\partial b}\pi_r(\underline{x}, y') = 1 + r_f - wE[\zeta]$,

for $b > b_1$: $\frac{\partial}{\partial b}\pi_r(\underline{x}, y_{lb}(b)) < 1 + r_f - wE[\zeta]$,

for $b < y^a$: $\frac{\partial}{\partial b}\pi_r(0, b) = \theta(p\bar{F}(b) - w) + \alpha(1-\theta)(p\bar{F}(b\alpha) - w) > 1 + r_f - wE[\zeta]$,

for $y^a < b < b_2^u$: $\frac{\partial}{\partial b}\pi_r(0, y^a) = \alpha(1-\theta)(p\bar{F}(b\alpha) - w)$. Since $b_2^u \leq y'$, $\alpha(1-\theta)(p\bar{F}(b\alpha) - w) > \alpha(1-\theta)(p\bar{F}(y'\alpha) - w)$. Combing the fact that $p\bar{F}(y^a) - w = 0$, $\theta(p\bar{F}(y') - w) + \alpha(1-\theta)(p\bar{F}(y'\alpha) - w) - 1 - r_f + wE[\zeta] = 0$ and $y^a < y'$, we have $\frac{\partial}{\partial b}\pi_r(0, y^a) > 1 + r_f - wE[\zeta]$ for $b < b_2^u$. Therefore, b_2 is unique on $(0, b_2^u)$.