Strong coupling constants of charmed and bottom mesons with light vector mesons in QCD sum rules

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We estimate the strong coupling constants of charmed and bottom mesons $D^*_{(s)}$, $D_{(s)1}$, $B^*_{(s)}$, and $B_{(s)1}$ with light vector mesons ρ, ω, K^*, and ϕ within the framework of light-cone QCD sum rules. We take into account the contributions of two- and three-particle distribution amplitudes of light vector mesons of twist-2, -3, and -4, and determine the values of the couplings between heavy vector (axial vector) mesons and light vector mesons. These couplings can play an essential role in understanding the charmonium absorption cross section in quark-gluon plasma. Moreover, we compare our estimations to the ones predicted by other approaches.

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I. INTRODUCTION

Strong coupling constants between heavy and light mesons are among the essential ingredients for the description of low-energy hadron interactions. Precise determination of these couplings can provide key information for studying the nature of heavy mesons. In particular, they are a useful source in the investigation of final-state interactions of D and B meson decays. Moreover, for understanding the production and absorption cross sections of the J/ψ meson in heavy-ion collisions, vertices involving charmed mesons are needed. On the other hand, heavy-light meson couplings are fundamental objects since they carry information about the low-energy behavior of QCD. However, this region is far away from the perturbative region of QCD. Therefore, for a reliable estimation of these couplings, a nonperturbative approach is required.

Among the nonperturbative approaches, the QCD sum rules method [1] occupies a special place. This method is based on the fundamental QCD Lagrangian and includes nonperturbative effects. The sum rules method is very powerful in studying various aspects of hadron physics. In this method, hadrons with given quantum numbers are represented by interpolating quark currents carrying the same quantum numbers as the hadrons. The correlation function formed using these currents is then calculated by using operator product expansion (OPE). In the traditional sum rules, the short-distance part is calculated perturbatively, and nonperturbative light-distance effects are parametrized in terms of vacuum condensates of local operators. The OPE is performed over the dimensions of the operators. The sum rules are finally obtained by matching the hadronic and QCD sides of the correlation function via the dispersion relation. However, the traditional three-point sum rules method has the following problems:

(i) The OPE breaks down at large momentum or large mass.
(ii) The three-point sum rules are polluted by “non-diagonal transitions” between the ground and excited states.
(iii) When the virtualities are different, one would have to compute the contribution by an infinite number of local operators, which is a rather formidable task.

The light-cone sum rules (LCSR) [2] is a hybrid of the traditional sum rules method and the methods used in hard exclusive processes. In this approach, the OPE is carried out near the light cone, $x^2 \sim 0$, and the nonperturbative effects appear in the matrix elements of nonlocal operators, which are parametrized in terms of the light-cone distribution amplitudes (DAs) of the corresponding hadrons, instead of the vacuum condensates that appear in the standard sum rules method [3,4]. In this method, the OPE is carried over the twists of corresponding operators.

So far, to the best of our knowledge, many heavy-light meson vertices such as $D_s^* D^* K$, $D_{s1} D_1 K^*$ [5], $D^* D^* \rho$ [6], $D^* D \pi$, $B^* B \pi$ [7,8], $DD\rho$ [9], $D^* D \rho$ [10], DDJ/ψ [11],

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D^*DJ/ψ [12], $D^*D^*\pi$ [13], D_sD^*K, D_s^*DK [14], $DD\omega$ [15], $D_{s1}D^*K$, $D_{s1}D^*K_0^*$ [16,17], D_sD_sV, $D_s^*D_s^*V$, $D_{s0}^*D_{s1}V$, $D_sD_s^*V$ [18,19], $D_1D^*\pi$, $D_1D_0\pi$, $D_1D_1\pi$, $B_1B^*\pi$, $B_1B_0\pi$, $B_1B_1\pi$ [20], D^*D^*J/ψ [21], $B_{s0}BK$ [22], B_s^*BK [23], $D_s^*D_s\phi$ [24], $D_sDK_0^*$, $B_sBK_0^*$, D_s^*DK, B_s^*BK, $D_s^*DK_1$, $B_s^*BK_1$ [25], $D^*D\pi$, $D^*D^*\pi$, $DD\rho$, $D^*D\rho$, $D^*D^*\rho$, DDJ/ψ, D^*DJ/ψ, D^*D^*J/ψ [26], D_sDK^*, $D_sD^*K^*$ [27], $D_s^*DK^*$, $B_s^*BK^*$ [28], $D_0D_{s0}^*K^*$, $D_1D_{s0}^*K^*$ [29], B_sB^*K, B_sBK^* [30], $B^*B^*\rho$ [31], $B_{s1}B^*K$, $B_{s1}B^*K_0^*$ [32], $B_{s1}^*B^*K$ [33], D_sD_sJ/ψ, $D_sD_s\phi$ [34], $D^*D_s^*K$, $D_1D_{s1}K$, D^*D_sK, $D_1D_{s0}^*K$ [35], and $B_{s0}B_1K$, $B_{s1}B_1K$ [36] are estimated in the framework of the three-point QCD sum rules (3PSR); D^*D_sK, D_s^*DK, $D_{s0}DK$, D_0D_sK [37], D^*D^*P, D^*DV, DDV [38], $D^*D\rho$, $B^*B\rho$ [39], $D^*D\pi$, $B^*B\pi$ [40], and $D^*D^*\rho$ [41] with the LCSR approach; $DD\rho$, $D^*D^*\rho$ [42] in holographic QCD (HQCD); $DD\rho$, $D^*D\rho$, $D^*D^*\rho$ [43] using the Dyson-Schwinger equation (DSE) in QCD; $D^*D\pi$, $DD\rho$, $D^*D^*\rho$ [44–47] in lattice QCD; $DD\rho$ [48] in the nonrelativistic quark model; $DD\rho$, $DD\sigma$, $D^*D^*\rho$, $D^*D^*\sigma$, $BB\rho$, $BB\sigma$, $B^*B^*\rho$, $B^*B^*\sigma$ [49] based on the approach of correlated 2π exchange with the pole approximation (PA); $B^*B^*\rho$ [50,51] and $D^*D^*\rho$ [52] in the meson-exchange model (MEM); $B^*B^*\rho$, $D^*D\pi$, $B^*B\pi$ [53,54] in the potential model; and, lastly, $D^*D^*\rho$, $B^*B^*\rho$ [55] in the one-boson exchange (OBE) model.

In the present work, we study the coupling constants of $D_{(s)}^*D_{(s)}^*V$, $D_{(s)1}D_{(s)1}V$, $B_{(s)}^*B_{(s)}^*V$, and $B_{(s)1}B_{(s)1}V$ where $V=\rho,\omega,K^*,\phi$ within the LCSR method. The main contribution to the LCSR is provided by the lowest-twist DAs, which, therefore, receive special interest. The study of DAs stands out as the main aspect of the LCSR method. Vector meson DAs have more complex structures than light pseudoscalar meson DAs. There are chiral-even and chiral-odd vector meson DAs due to the chiral-even and chiral-odd operators in the matrix elements. Vector mesons have longitudinal and transverse polarization states which can be expanded over different twist structures. One can find more detailed discussions in [56,57] and references therein.

The paper is organized as follows. In Sec. II, we derive the desired sum rules for the strong coupling constants of the said vertices. In Sec. III, we present our numerical analysis and resultant values for the aforementioned couplings. Section IV contains our conclusion.

II. LIGHT-CONE SUM RULES FOR CHARMED/BOTTOM MESON–LIGHT VECTOR MESON COUPLINGS

Let us introduce the following correlation function in order to compute the strong coupling constants of vector and axial vector charmed and bottom mesons with light vector mesons:

$$\Pi_{\mu\nu} = \mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}px}\langle V(q,s)|\mathrm{T}\{j_\mu(x)\bar{j}_\nu(0)\}|0\rangle \tag{1}$$

where $V(q,s)$ is a vector meson of mass m_V, 4-momentum q, and 4-polarization ε^s (but we will be suppressing the superscript s for the most part), and j_μ indicates the interpolating current of the corresponding charmed or bottom meson. For the considered mesons, the interpolating current can be written as

$$j_\mu(x) = \bar{q}(x)\Gamma_\mu Q(x) \tag{2}$$

where $\Gamma_\mu = \gamma_\mu(\gamma_\mu\gamma_5)$ for vector (axial vector) charmed and bottom mesons; Q is a heavy quark, namely, c or b; and q is one of the light quarks, u, d, or s.

In the framework of the LCSR method, one computes the correlation function in two different regions. On one side, it can be calculated in terms of hadrons, which is also known as the phenomenological part; on the other side, the calculation is carried out in the deep Euclidean domain, i.e., $p^2\to-\infty$ and $(p+q)^2\to-\infty$, using the OPE over the twist, which is traditionally called the theoretical part. In order to suppress the contributions from excited states and the continuum, as well as to enhance the contribution of the ground state, a double Borel transformation is performed with respect to the variables $-p^2$ and $-(p+q)^2$.

The strong coupling constants of heavy vector (axial vector) mesons with light vector mesons are formally defined as having the hadronic matrix element

$$\begin{aligned}\langle\mathcal{M}_2(p_2)V(q,s)|\mathcal{M}_1(p_1)\rangle = g(&p_2\cdot\varepsilon^*\varepsilon_1\cdot\varepsilon_2^* + p_1\cdot\varepsilon^*\varepsilon_1\cdot\varepsilon_2^*\\ &+ q\cdot\varepsilon_1\varepsilon^*\cdot\varepsilon_2^* - p_2\cdot\varepsilon_1\varepsilon^*\cdot\varepsilon_2^*\\ &- p_1\cdot\varepsilon_2^*\varepsilon^*\cdot\varepsilon_1^* - q\cdot\varepsilon_2^*\varepsilon^*\cdot\varepsilon_1).\end{aligned} \tag{3}$$

The possibility to calculate the strong coupling constant g for the vertex $\mathcal{M}_2\mathcal{M}_1V$ within the LCSR approach is based on the fact that the couplings enter the double dispersion relations of the same correlation function given by Eq. (1). In other words, to determine the g's for various vertices, we employ the hadronic double dispersion relation for the correlation function. The double dispersion relation is obtained by calculating the double imaginary part of the correlation function with respect to variables p^2 and $(p+q)^2$. The result is given by the following expression containing the double poles at m_1 and m_2:

$$\begin{aligned}\Pi_{\mu\nu} = &\frac{1}{(p_2^2-m_2^2)(p_1^2-m_1^2)}gf_1f_2m_1m_2\varepsilon_{1,\nu}^*\varepsilon_{2,\mu}\\ &\times(p_2\cdot\varepsilon^*\varepsilon_1\cdot\varepsilon_2^* + p_1\cdot\varepsilon^*\varepsilon_1\cdot\varepsilon_2^* + q\cdot\varepsilon_1\varepsilon^*\cdot\varepsilon_2^*\\ &- p_2\cdot\varepsilon_1\varepsilon^*\cdot\varepsilon_2^* - p_1\cdot\varepsilon_2^*\varepsilon^*\cdot\varepsilon_1^* - q\cdot\varepsilon_2^*\varepsilon^*\cdot\varepsilon_1)\\ &+\cdots\end{aligned} \tag{4}$$

where $q \coloneqq p_2 - p_1$ is the momentum transfer and $p_{1(2)}$, $\varepsilon_{1(2)}$, $m_{1(2)}$, and $f_{1(2)}$ denote the 4-momentum, 4-polarization, mass, and the leptonic decay constant of the initial (final) charmed or bottom mesons, respectively. Here, $\cdots$ indicates the contribution of higher states and the continuum. In the derivation of Eq. (4), we have used the definitions

$$\langle 0|j_\mu|\mathcal{M}_2(p_2)\rangle = f_2 m_2 \varepsilon_{2,\mu}, \tag{5}$$

$$\langle \mathcal{M}_1(p_1)|\bar{j}_\nu|0\rangle = f_1 m_1 \varepsilon^*_{1,\nu}. \tag{6}$$

From now on, we will let $p \coloneqq p_1$ and use q instead of p_2 as $p_2 = p + q$. Making use of the spin sum over the 4-polarization vectors ε_1 and ε_2, given by

$$\sum_s \varepsilon^s_\mu \varepsilon^{s*}_\nu = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}, \tag{7}$$

we obtain the phenomenological side of the correlation function to be

$$\Pi_{\mu\nu} = \frac{f_1 f_2 m_1 m_2 g}{[(p+q)^2 - m_2^2](p^2 - m_1^2)} p\cdot\varepsilon^* g_{\mu\nu} + \cdots \tag{8}$$

where $\cdots$ denotes other structures.

In order to derive the LCSR for the strong coupling constant g, one needs to compute the theoretical side of the correlation function. Afterwards, selecting the same structure, i.e., $p\cdot\varepsilon^* g_{\mu\nu}$, and matching it to the result obtained from the phenomenological part, one arrives at the desired LCSR. We get the said part of the correlation function by making use of the OPE in the deep Euclidean region, $p^2 \to -\infty$ and $(p+q)^2 \to -\infty$. Inserting the interpolating currents given in the form of Eq. (2) and applying the Wick theorem to the correlation function given by Eq. (1), we obtain

$$\Pi_{\mu\nu} = \mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}px}\langle V(q,s)| \times [\bar{q}_1(x)\gamma_\mu(\gamma_\mu\gamma_5)S_Q(x)\gamma_\nu(\gamma_\nu\gamma_5)q_2(0)]|0\rangle. \tag{9}$$

Here, $S_Q(x)$ is the heavy-quark propagator in the presence of an external background field in the position representation,

$$S_Q^{aa'}(x) = \frac{m_Q^2}{4\pi^2}(\mathrm{i}K_2\not{x} + K_1)\delta^{aa'} - \frac{g_s}{16\pi^2}m_Q\int_0^1 \mathrm{d}u[\mathrm{i}K_1(\bar{u}\not{x}\sigma^{\lambda\tau} + u\sigma^{\lambda\tau}\not{x}) + K_0\sigma^{\lambda\tau}]G^{(n)}_{\lambda\tau}(ux)\left(\frac{\lambda^{(n)}}{2}\right)^{aa'} \tag{10}$$

where the first term is the free heavy-quark propagator and the second term is generated by a linear external background gluon field, $G^{(n)}_{\lambda\tau}$ is the gluon field strength tensor, and $\lambda^{(n)}$ are the Gell-Mann matrices. We let $\bar{u} \coloneqq 1-u$, and we have defined the shorthand notation $K_n \coloneqq K_n(m_Q\sqrt{-x^2})/(\sqrt{-x^2})^n$, with $K_n(z)$ being the nth modified Bessel function of the second kind.

Putting Eq. (10) into Eq. (9) and using the Fierz identities,

$$q^a_{2\alpha}(0)\bar{q}^{a'}_{1\beta}(x) = -\frac{1}{12}\delta^{aa'}(\Gamma_i)_{\alpha\beta}[\bar{q}_1(x)\Gamma_i q_2(0)], \tag{11}$$

$$q^a_{2\alpha}(0)G^{(n)}_{\lambda\tau}\bar{q}^{a'}_{1\beta}(x) = -\frac{1}{16}\left(\frac{\lambda^{(n)}}{2}\right)^{aa'}(\Gamma_i)_{\alpha\beta} \times [\bar{q}_1(x)\Gamma_i G^{(n)}_{\lambda\tau} q_2(0)] \tag{12}$$

where $\{\Gamma_i\}_{i=1}^5$ is the complete set of Dirac matrices,

$$\Gamma_1 = 1, \qquad \Gamma_2 = \gamma_5, \qquad \Gamma_3 = \gamma_\alpha, \qquad \Gamma_4 = \mathrm{i}\gamma_\alpha\gamma_5, \qquad \Gamma_5 = \frac{1}{\sqrt{2}}\sigma_{\alpha\beta}, \tag{13}$$

one can see that, in the calculation of the theoretical side of the correlation function, one needs the matrix elements $\langle V(q,s)|\bar{q}_1(x)\Gamma_i q_2(0)|0\rangle$ and $\langle V(q,s)|\bar{q}_1(x)\Gamma_i G^{(n)}_{\lambda\tau}(ux) \times q_2(0)|0\rangle$. These matrix elements, expressed in terms of light vector meson DAs of various twists [58–63], constitute the primary nonperturbative input parameters of the LCSR. The full compilation of these matrix elements and the corresponding DAs can be found in Appendix C of [64]. For the sake of completeness, we present only the relevant expression that will appear in the computation of the correlation function in the Appendix A of the present work.

At this point, we make two remarks.

(i) Let us consider the terms without $G^{(n)}_{\lambda\tau}$, which conventionally provide the major contribution to the sum rules. We are interested in the structure $p\cdot\varepsilon^* g_{\mu\nu}$. In the correlation function, there will be traces of the form $\mathrm{tr}\gamma_\mu(\gamma_\mu\gamma_5)\not{x}\gamma_\nu(\gamma_\nu\gamma_5)\Gamma_i$. It turns out that only the third term in the Fierz expansion, namely, $\Gamma_3 = \gamma_\alpha$, will contribute to the aforementioned structure.

(ii) We do the continuum subtraction as described in [40]. That is to say, after taking the double Borel transform of the theoretical side of the correlation function, there is an exponential factor $\mathrm{e}^{-m_Q^2/M^2 - m_V^2/(M_1^2+M_2^2)}$. According to [40], making the replacement

$$\mathrm{e}^{-m_Q^2/M^2 - m_V^2/(M_1^2+M_2^2)} \to \mathrm{e}^{-m_Q^2/M^2 - m_V^2/(M_1^2+M_2^2)} - \mathrm{e}^{-s_0/M^2} \tag{14}$$

will suffice as far as the continuum subtraction is concerned. Since the masses of the initial and final

TABLE I. Masses and decay constants of the light vector mesons along with the quark masses used in our numerical analysis (in units of MeV). The quark masses are taken at $\mu = 1$ GeV.

Parameter	Value	Parameter	Value	Parameter	Value
m_u	0	m_ρ	769.0 ± 0.9 [66]	f_ρ	216 ± 3 [63]
m_d	0	m_{K^*}	895.55 ± 0.20 [66]	f_ρ^T	165 ± 9 [63]
m_s	137 ± 10	m_ω	782.66 ± 0.13 [66]	f_{K^*}	220 ± 5 [63]
m_c	1400 ± 10	m_ϕ	1019.461 ± 0.016 [66]	$f_{K^*}^T$	185 ± 10 [63]
m_b	4800 ± 10			f_ω	187 ± 5 [63]
				f_ω^T	151 ± 9 [63]
				f_ϕ	215 ± 5 [63]
				f_ϕ^T	186 ± 9 [63]

states are the same or nearly equal, we take $M_1^2 = M_2^2 = 2M^2$.

Putting the expressions given in Eqs. (10)–(12) into (9), taking the double Borel transform over the variables $-p^2$ and $-(p+q)^2$, and using the results of Appendix B to carry out the integrals encountered, we obtain the required sum rules for the strong coupling constants, g_V and g_A, where the subscripts V and A stand for the case of vector and axial vector charmed or bottom mesons:

$$g_V = g_V^{(0)} + g_V^{(1)}, \tag{15}$$

$$g_A = g_A^{(0)} + g_A^{(1)} \tag{16}$$

where

$$\begin{aligned} g_V^{(0)} = {}& \frac{1}{2M^6 f_1 f_2 m_1 m_2} \mathrm{e}^{[2(m_1^2+m_2^2)-m_V^2-4(m_Q^2+s_0)]/4M^2} \\ & \times [\mathrm{e}^{(4m_Q^2+m_V^2)/4M^2} - \mathrm{e}^{s_0/M^2}] \\ & \times [2M^4 f_V m_Q^2 m_V^3 \hat{\hat{\phi}}_2^{\parallel}(u_0) - 4M^4 f_V m_Q^2 m_V^3 \hat{\hat{\phi}}_3^{\perp}(u_0) \\ & + 2M^4 f_V m_Q^2 m_V^3 \hat{\hat{\psi}}_4^{\parallel}(u_0) - M^8 f_V m_V \phi_2^{\parallel}(u_0) \\ & + M^6 f_V m_V^3 \phi_4^{\parallel}(u_0) + M^4 f_V m_Q^2 m_V^3 \phi_4^{\parallel}(u_0)], \end{aligned} \tag{17}$$

$$\begin{aligned} g_V^{(1)} = {}& \frac{1}{2M^6 f_1 f_2 m_1 m_2} \mathrm{e}^{[2(m_1^2+m_2^2)-m_V^2-4(m_Q^2+s_0)]/4M^2} \\ & \times [-\mathrm{e}^{(4m_Q^2+m_V^2)/4M^2} + \mathrm{e}^{s_0/M^2}] \\ & \times \{8M^2 m_Q m_V^6 f_V^T \mathcal{I}_\alpha[u_1^3 \hat{\hat{\hat{\mathcal{T}}}}(\alpha_1, \alpha_3)] \\ & - 4m_Q^3 m_V^6 f_V^T \mathcal{I}_\alpha[u_1^3 \hat{\hat{\hat{\mathcal{T}}}}(\alpha_1, \alpha_3)] \\ & - 2M^4 m_Q m_V^4 f_V^T \mathcal{I}_\alpha[u_1 \hat{\mathcal{T}}_3^{(4)}(\alpha_1, \alpha_3)] \\ & - 2M^4 m_Q m_V^4 f_V^T \mathcal{I}_\alpha[u_1 \hat{\mathcal{T}}_4^{(4)}(\alpha_1, \alpha_3)] \\ & + M^6 f_V m_V^3 \mathcal{I}_\alpha[\mathcal{A}(\alpha_1, \alpha_3)] \\ & + M^6 f_V m_V^3 \mathcal{I}_\alpha[(2u_1 - 1)\mathcal{V}(\alpha_1, \alpha_3)]\}, \end{aligned} \tag{18}$$

$$g_A^{(0)} = g_V^{(0)}, \tag{19}$$

$$\begin{aligned} g_A^{(1)} = {}& \frac{1}{2M^6 f_1 f_2 m_1 m_2} \mathrm{e}^{[2(m_1^2+m_2^2)-m_V^2-4(m_Q^2+s_0)]/4M^2} \\ & \times [-\mathrm{e}^{(4m_Q^2+m_V^2)/4M^2} + \mathrm{e}^{s_0/M^2}] \\ & \times \{8M^2 m_Q m_V^6 f_V^T \mathcal{I}_\alpha[u_1^3 \hat{\hat{\hat{\mathcal{T}}}}(\alpha_1, \alpha_3)] \\ & - 4m_Q^3 m_V^6 f_V^T \mathcal{I}_\alpha[u_1^3 \hat{\hat{\hat{\mathcal{T}}}}(\alpha_1, \alpha_3)] \\ & - 2M^4 m_Q m_V^4 f_V^T \mathcal{I}_\alpha[u_1 \hat{\mathcal{T}}_3^{(4)}(\alpha_1, \alpha_3)] \\ & - 2M^4 m_Q m_V^4 f_V^T \mathcal{I}_\alpha[u_1 \hat{\mathcal{T}}_4^{(4)}(\alpha_1, \alpha_3)] \\ & - M^6 f_V m_V^3 \mathcal{I}_\alpha[\mathcal{A}(\alpha_1, \alpha_3)] \\ & - M^6 f_V m_V^3 \mathcal{I}_\alpha[(2u_1 - 1)\mathcal{V}(\alpha_1, \alpha_3)]\} \end{aligned} \tag{20}$$

where $u_0 \coloneqq \frac{M_2^2}{M_1^2+M_2^2} = \frac{1}{2}$, $\bar{u}_0 \coloneqq 1 - u_0 = \frac{1}{2}$, $u_1 \coloneqq \frac{1}{\alpha_3}(-\frac{M^2}{M_1^2} + 1 - \alpha_1) = \frac{1}{\alpha_3}(\frac{1}{2} - \alpha_1)$. Here, we have introduced a shorthand notation for the three-particle DAs as $\mathcal{F}(\alpha_1, \alpha_3) \coloneqq \mathcal{F}(\alpha_1, \alpha_2 = 1 - \alpha_1 - \alpha_3, \alpha_3)$, and we have defined the alpha-integral operator

$$\mathcal{I}_\alpha[F(\alpha_1, \alpha_3)] \coloneqq \int_0^{\bar{u}_0} \mathrm{d}\alpha_1 \int_{\bar{u}_0 - \alpha_1}^{1-\alpha_1} \mathrm{d}\alpha_3 \frac{1}{\alpha_3} F(\alpha_1, \alpha_3). \tag{21}$$

Finally, the hat denotes the following integrations of the DAs:

$$\hat{\hat{f}}(u_0) \coloneqq \int_0^{u_0} \mathrm{d}v'' \int_0^{v''} \mathrm{d}v' f(v'), \tag{22}$$

$$\hat{\mathcal{F}}(\alpha_1, \alpha_3) \coloneqq \int_0^{\alpha_3} \mathrm{d}\alpha_3' \mathcal{F}(\alpha_1, 1 - \alpha_1 - \alpha_3', \alpha_3'), \tag{23}$$

$$\begin{aligned} \hat{\hat{\hat{\mathcal{F}}}}(\alpha_1, \alpha_3) \coloneqq {}& \int_0^{\alpha_3} \mathrm{d}\alpha_3''' \int_0^{\alpha_3'''} \mathrm{d}\alpha_3'' \int_0^{\alpha_3''} \mathrm{d}\alpha_3' \\ & \times \mathcal{F}(\alpha_1, 1 - \alpha_1 - \alpha_3', \alpha_3'). \end{aligned} \tag{24}$$

III. NUMERICAL ANALYSIS

In this section, we share the details of our numerical analysis of the LSCR for the strong coupling constants of charmed and bottom mesons $D^*_{(s)}$, $D_{(s)1}$, $B^*_{(s)}$, and $B_{(s)1}$ with light vector mesons ρ, ω, ϕ, and K^*, where we have used Package X [65]. The LSCR for the said couplings takes three sets of input parameters. The first and primary set of such parameters includes the quark and meson masses and the decay constants of both heavy and light mesons. These are compiled in Tables I and II. Second, there are parameters introduced via the vector meson DAs of different twists. In Appendix A, we quote only the relevant DAs that appear in our analysis, together with the most up-to-date values of the input parameters for all the light vector mesons taken into account in this work.

Third, the LCSR contains two auxiliary parameters, i.e., the Borel mass parameter M^2 and the continuum threshold s_0. The coupling constants of the said strong vertices should be independent of the choice of M^2 and s_0. This necessitates us to restrict the values of M^2 and s_0 to their working regions, which will thus render the LCSR reliable. The lower bound of the Borel mass parameter is obtained by requiring that the contributions from higher-twist terms be much smaller than those from the leading-twist terms. Its upper bound is determined by considering the fact that the higher-state and continuum contributions should be sufficiently suppressed. These two conditions lead to the following domains of M^2 that are presented in Table III. In the meantime, the value of the continuum threshold is obtained by demanding that the two-point sum rules predict the mass of the heavy mesons within a 10% accuracy with respect to the experimental value. The corresponding values of s_0 are also presented in Table III.

Towards the end of our numerical analysis, we found that the leading-twist contributions are around 88% within the working domains of the Borel mass parameter and at the aforementioned values of the continuum threshold. As an illustration, we present the dependence of the strong coupling constant g on the Borel mass parameter M^2 for the vertices $D^*D^*\rho$ and $B^*B^*\rho$ in Figs. 1 and 2, respectively. The coupling constants for $D^*D^*\rho^0$, $D_1D_1\rho^0$, $B^*B^*\rho^0$, and $B_1B_1\rho^0$ can be obtained from the corresponding vertices involving the ρ^+ meson with the help of the isotopic relation.

Our estimations for the said coupling constants are presented in Table IV. The uncertainties shown in Figs. 1 and 2 and Table IV are due to the variation of the continuum threshold and to the errors in the values of the input parameters. In our analysis, it turns out that the primary factors that controls the uncertainty in the coupling constant values are not the input parameters in the light vector meson DAs, though they carry an uncertainty around 30%–50%, but rather the continuum threshold. That is

TABLE II. Masses and decay constants of the vector (axial vector) charmed and bottom mesons used in our numerical analysis (in units of MeV).

Parameter	Value	Parameter	Value
m_{D^*}	2010.26 ± 0.05 [66]	m_{B^*}	5324.70 ± 0.21 [66]
$m_{D^*_s}$	2112.2 ± 0.4 [66]	$m_{B^*_s}$	$5415.4^{+1.8}_{-1.5}$ [66]
m_{D_1}	2422.1 ± 0.6 [66]	m_{B_1}	$5725.9^{+2.5}_{-2.7}$ [66]
$m_{D_{s1}}$	2456.5 ± 0.6 [66]	$m_{B_{s1}}$	5828.70 ± 0.20 [66]
f_{D^*}	263 ± 21 [67]	f_{B^*}	213 ± 18 [67]
$f_{D^*_s}$	308 ± 21 [67]	$f_{B^*_s}$	255 ± 19 [67]
f_{D_1}	218.9 ± 11.3 [68]	f_{B_1}	196.9 ± 8.9 [68]
$f_{D_{s1}}$	144.5 ± 11.1 [69]	$f_{B_{s1}}$	240 ± 2 [70]

TABLE III. Working region of the Borel mass parameter and the continuum threshold for the vertices indicated.

Vertex	$M^2(\mathrm{GeV}^2)$	$s_0(\mathrm{GeV}^2)$
$D^*D^*\rho$	$5.0 < M^2 < 9.0$	6.5 ± 0.5
$D^*D^*\omega$	$4.0 < M^2 < 8.0$	6.5 ± 0.5
$D^*_sD^*_s\phi$	$4.0 < M^2 < 8.0$	7.0 ± 0.5
$D^*D^*_sK^*$	$5.0 < M^2 < 8.0$	7.0 ± 0.5
$D_1D_1\rho$	$6.0 < M^2 < 10.0$	8.5 ± 0.5
$D_1D_1\omega$	$3.0 < M^2 < 6.0$	8.5 ± 0.5
$D_{s1}D_{s1}\phi$	$6.0 < M^2 < 10.0$	9.0 ± 0.5
$D_1D_{s1}K^*$	$6.0 < M^2 < 10.0$	9.0 ± 0.5
$B^*B^*\rho$	$20.0 < M^2 < 24.0$	34.0 ± 1.0
$B^*B^*\omega$	$20.0 < M^2 < 24.0$	34.0 ± 1.0
$B^*_sB^*_s\phi$	$20.0 < M^2 < 24.0$	35.0 ± 1.0
$B^*B^*_sK^*$	$20.0 < M^2 < 24.0$	35.0 ± 1.0
$B_1B_1\rho$	$20.0 < M^2 < 24.0$	39.0 ± 1.0
$B_1B_1\omega$	$20.0 < M^2 < 24.0$	39.0 ± 1.0
$B_{s1}B_{s1}\phi$	$20.0 < M^2 < 24.0$	40.0 ± 1.0
$B_1B_{s1}K^*$	$20.0 < M^2 < 24.0$	40.0 ± 1.0

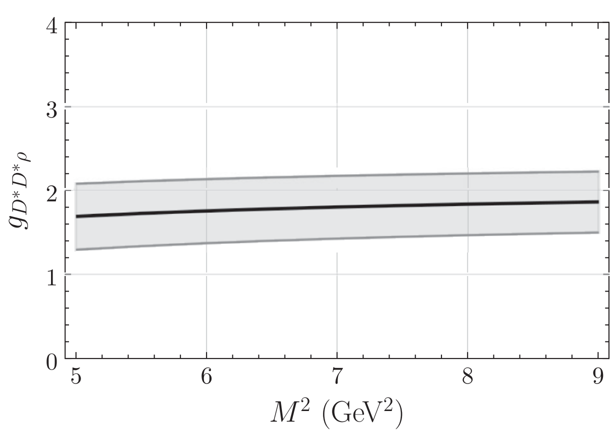

FIG. 1. Dependence of the coupling constant of the vertex $D^*D^*\rho$ on M^2 at $s_0 = 6.5 \pm 0.5$ GeV^2. The black line indicates the central values, whereas the shaded region corresponds to the uncertainties.

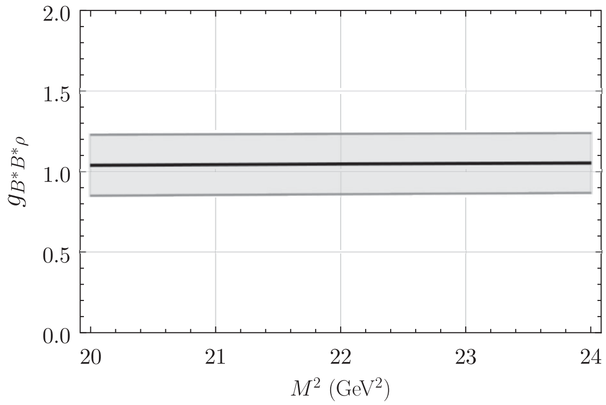

FIG. 2. Same as in Fig. 1 but for $B^*B^*\rho$ and at $s_0 = 34.0 \pm 1.0$ GeV^2.

the say, the main uncertainty for the sum rules for the strong coupling constants comes from the continuum threshold. We reproduce Fig. 1 for three different variations of s_0, namely, ± 0.0, ± 0.3, and ± 0.5 GeV^2, in Fig. 3. From this figure, it is clear that the uncertainty in the continuum threshold contributes the most to the standard error in the values of the coupling constant obtained. To be more precise, nearly 20% of the standard error derives from the uncertainty in, primarily, the DA parameters and, secondarily, the particle masses and meson decay constants.

As we noted earlier, the $D^*D^*\rho$ coupling constant was calculated within the same framework as ours in [41]. One can see that, within error limits, our result is in agreement with the result of [41]. The small difference between our result and that of [41] lies in the fact that we take into account the contributions of the three-particle DAs, as well as different values for the input parameters, which leads to a difference between the estimated central values of the same coupling constant.

From Table IV, we also observe that our prediction for the coupling $g_{D^*D^*\rho}$ is in good agreement with the holographic QCD result as well. However, there is a sharp difference between our values and those estimated by the 3PSR, Dyson-Schwinger, and lattice QCD values. For the vertices containing the ρ, ϕ, and K^* mesons, the literature values are nearly 2–5 times larger than ours for the D meson sector, except for the $D_1D_{s1}K^*$ vertex, which is in good agreement with the 3PSR result. For the B meson sector, comparison of our predictions of the aforementioned couplings with the 3PSR results would be interesting; unfortunately, in the B meson sector, the said calculations are absent at the present time, with an exception for the $B^*B^*\rho$ vertex, for which the 3PSR method offers a coupling value 1.5 times larger than ours. Nevertheless, there exist strong coupling values for the $B^*B^*\rho$ vertex obtained within the models of the pole approximation, meson exchange, potential, and one-boson exchange. Our estimation is 1.5 to 10 times smaller than the values determined in the said models.

We would like to further note that our results can be improved by taking into account the $O(\alpha_s)$ corrections.

TABLE IV. Values of the strong coupling constants of vector (axial vector) D and B mesons with light vector mesons. For completeness, the predictions existing in the literature are also presented.

	g									
Vertex	LCSR		3PSR	MEM	HQCD	DSE	Lattice	PA	Potential	OBE
$D^*D^*\rho$	1.8 ± 0.4	2.6 ± 0.7 [41]	6.60 ± 0.31 [6], 4.7 ± 0.2 [26]	2.52 [52]	2.1431 [42]	10.5, 51.5, 16.8 [43]	5.95(56) [47]	6.47 [49]	3.71 [53]	2.6 [55]
$D^*D^*\omega$	1.5 ± 0.3	…	…	…	…	…	…	…	…	…
$D_s^*D_s^*\phi$	1.5 ± 0.4	…	7.76 ± 1.79 [18]	…	…	…	…	…	…	…
$D^*D_s^*K^*$	1.7 ± 0.4	…	4.77 ± 0.63 [5]	…	…	…	…	…	…	…
$D_1D_1\rho$	3.0 ± 0.5	…	…	…	…	…	…	…	…	…
$D_1D_1\omega$	2.6 ± 0.5	…	…	…	…	…	…	…	…	…
$D_{s1}D_{s1}\phi$	8.6 ± 1.7	…	15.37 ± 2.51 [18]	…	…	…	…	…	…	…
$D_1D_{s1}K^*$	5.3 ± 0.9	…	4.22 ± 0.55 [5]	…	…	…	…	…	…	…
$B^*B^*\rho$	1.1 ± 0.2	…	1.73 ± 0.25 [31]	3.71 [50,51]	…	…	…	10.1 [49]	3.71 [53]	2.6 [55]
$B^*B^*\omega$	0.9 ± 0.2	…	…	…	…	…	…	…	…	…
$B_s^*B_s^*\phi$	1.0 ± 0.2	…	…	…	…	…	…	…	…	…
$B^*B_s^*K^*$	1.1 ± 0.2	…	…	…	…	…	…	…	…	…
$B_1B_1\rho$	1.7 ± 0.2	…	…	…	…	…	…	…	…	…
$B_1B_1\omega$	1.5 ± 0.2	…	…	…	…	…	…	…	…	…
$B_{s1}B_{s1}\phi$	1.6 ± 0.3	…	…	…	…	…	…	…	…	…
$B_1B_{s1}K^*$	1.7 ± 0.2	…	…	…	…	…	…	…	…	…

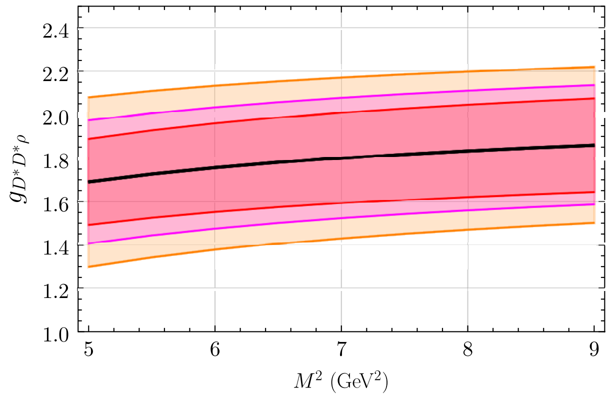

FIG. 3. Dependence of the coupling constant of the vertex $D^*D^*\rho$ on M^2 at the central value of the continuum threshold $s_0 = 6.5\ \mathrm{GeV}^2$ with a variation of ± 0.0 (red, inner), ± 0.3 (magenta, middle), and $\pm 0.5\ \mathrm{GeV}^2$ (orange, outer). The central value is indicated by the black, innermost line.

IV. CONCLUSION

In this paper, we studied the strong vertices of charmed and bottom mesons $D^*_{(s)}$, $D_{(s)1}$, $B^*_{(s)}$, and $B_{(s)1}$ with light vector mesons ρ, ω, K^*, and ϕ within the LCSR method, taking into account the contribution of two- and three-particle light vector meson DAs. The values of the strong coupling constants of heavy vector (axial vector) mesons with light vector mesons were obtained. The said vertices involving the D mesons were essential in the production of the J/ψ and ϕ mesons. We have found that our estimation for the coupling of the $D^*D^*\rho$ vertex agrees with the results of [41,42] but drastically differs from the 3PSR, DSE, and lattice QCD results, whereas our prediction for $g_{B^*B^*\rho}$ is 1.5 to 10 times smaller than the values predicted by the pole approximation, the potential model, MEM, and the OBE model.

APPENDIX A: VECTOR MESON DISTRIBUTION AMPLITUDES

In this section, we collect the matrix elements $\langle V(q,s)|\bar{q}_1(x)\Gamma_i q_2(0)|0\rangle$ and $\langle V(q,s)|\bar{q}_1(x)\Gamma_i G^{(n)}_{\lambda\tau} \times q_2(0)|0\rangle$ and the relevant DAs for the light vector mesons that appear in our theoretical analysis, together with the most up-to-date values for the DA parameters involved [58–63]. Up to twist-4 accuracy, the said matrix elements are given as follows:

$$\langle V(q,s)|\bar{q}_1(x)q_2(0)|0\rangle = 0, \tag{A1}$$

$$\langle V(q,s)|\bar{q}_1(x)\gamma_5 q_2(0)|0\rangle = 0, \tag{A2}$$

$$\begin{aligned}\langle V(q,s)|\bar{q}_1(x)\gamma_\mu q_2(0)|0\rangle = f_V m_V \bigg\{ &\frac{\varepsilon\cdot x}{q\cdot x} q_\mu \int_0^1 du e^{i\bar{u}q\cdot x}\left[\phi_2^\parallel(u) + \frac{m_V^2 x^2}{16}\phi_4^\parallel(u)\right] + \left(\varepsilon_\mu - q_\mu \frac{\varepsilon\cdot x}{q\cdot x}\right)\int_0^1 du e^{i\bar{u}q\cdot x}\phi_3^\perp(u) \\ &-\frac{1}{2}x_\mu \frac{\varepsilon\cdot x}{(q\cdot x)^2} m_V^2 \int_0^1 du e^{i\bar{u}q\cdot x}[\psi_4^\parallel(u) + \phi_2^\parallel(u) - 2\phi_3^\perp(u)]\bigg\},\end{aligned} \tag{A3}$$

$$\langle V(q,s)|\bar{q}_1(x) i\gamma_\mu\gamma_5 q_2(0)|0\rangle = -\frac{i}{4}\epsilon_{\mu\nu\alpha\beta}\varepsilon^\nu q^\alpha x^\beta f_V m_V \int_0^1 du e^{i\bar{u}q\cdot x}\psi_3^\perp(u), \tag{A4}$$

$$\begin{aligned}\langle V(q,s)|\bar{q}_1(x)\frac{1}{\sqrt{2}}\sigma_{\mu\nu} q_2(0)|0\rangle = -\frac{i}{\sqrt{2}} f_V^T \bigg\{ &(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu)\int_0^1 du e^{i\bar{u}q\cdot x}\left[\phi_2^\perp(u) + \frac{m_V^2 x^2}{16}\phi_4^\perp(u)\right] \\ &+\frac{\varepsilon\cdot x}{(q\cdot x)^2}(q_\mu x_\nu - q_\nu x_\mu)\int_0^1 du e^{i\bar{u}q\cdot x}\left[\phi_3^\parallel(u) - \frac{1}{2}\phi_2^\perp(u) - \frac{1}{2}\psi_4^\perp(u)\right] \\ &+\frac{1}{2}(\varepsilon_\mu x_\nu - \varepsilon_\nu x_\mu)\frac{m_V^2}{q\cdot x}\int_0^1 du e^{i\bar{u}q\cdot x}[\psi_4^\perp(u) - \phi_2^\perp(u)]\bigg\},\end{aligned} \tag{A5}$$

$$\langle V(q,s)|\bar{q}_1(x) G_{\lambda\tau} q_2(0)|0\rangle = -\frac{i}{g_s} f_V^T m_V (\varepsilon_\lambda q_\tau - \varepsilon_\tau q_\lambda)\int \mathcal{D}\vec{\alpha} e^{i(\alpha_1 + u\alpha_3)q\cdot x}\mathcal{S}(\vec{\alpha}), \tag{A6}$$

$$\langle V(q,s)|\bar{q}_1(x) G_{\lambda\tau}\gamma_5 q_2(0)|0\rangle = -\frac{i}{g_s} f_V^T m_V \frac{1}{2}\epsilon_{\lambda\tau\theta\omega}(\varepsilon^\theta q^\omega - \varepsilon^\omega q^\theta)\int \mathcal{D}\vec{\alpha} e^{i(\alpha_1 + u\alpha_3)q\cdot x}\tilde{\mathcal{S}}(\vec{\alpha}), \tag{A7}$$

$$\langle V(q,s)|\bar{q}_1(x)G_{\lambda\tau}\gamma_\mu q_2(0)|0\rangle = \frac{1}{\mathrm{i}g_s} f_V m_V q_\mu(\varepsilon_\lambda q_\tau - \varepsilon_\tau q_\lambda)\int \mathcal{D}\vec{\alpha}\mathrm{e}^{\mathrm{i}(\alpha_1+u\alpha_3)q\cdot x}\mathcal{V}(\vec{\alpha}), \tag{A8}$$

$$\langle V(q,s)|\bar{q}_1(x)G_{\lambda\tau}\mathrm{i}\gamma_\mu\gamma_5 q_2(0)|0\rangle = \frac{\mathrm{i}}{g_s} f_V m_V q_\mu \frac{1}{2}\epsilon_{\lambda\tau\theta\omega}(\varepsilon^\theta q^\omega - \varepsilon^\omega q^\theta)\int \mathcal{D}\vec{\alpha}\mathrm{e}^{\mathrm{i}(\alpha_1+u\alpha_3)q\cdot x}\mathcal{A}(\vec{\alpha}), \tag{A9}$$

$$\begin{aligned}\langle V(q,s)|\bar{q}_1(x)G_{\lambda\tau}\frac{1}{\sqrt{2}}\sigma_{\mu\nu} q_2(0)|0\rangle = \frac{1}{\sqrt{2}g_s}\{ & f_V^T m_V^2 \frac{\varepsilon\cdot x}{2q\cdot x}(q_\mu q_\lambda g^\perp_{\nu\tau} - q_\nu q_\lambda g^\perp_{\mu\tau} - q_\mu q_\tau g^\perp_{\nu\lambda} + q_\nu q_\tau g^\perp_{\mu\lambda})\int \mathcal{D}\vec{\alpha}\mathrm{e}^{\mathrm{i}(\alpha_1+u\alpha_3)q\cdot x}\mathcal{T}(\vec{\alpha}) \\ & + f_V^T m_V^2 (q_\mu \varepsilon_\lambda g^\perp_{\nu\tau} - q_\nu \varepsilon_\lambda g^\perp_{\mu\tau} - q_\mu \varepsilon_\tau g^\perp_{\nu\lambda} + q_\nu \varepsilon_\tau g^\perp_{\mu\lambda})\int \mathcal{D}\vec{\alpha}\mathrm{e}^{\mathrm{i}(\alpha_1+u\alpha_3)q\cdot x}\mathcal{T}_1^{(4)}(\vec{\alpha}) \\ & + f_V^T m_V^2 (q_\lambda \varepsilon_\mu g^\perp_{\nu\tau} - q_\lambda \varepsilon_\nu g^\perp_{\mu\tau} - q_\tau \varepsilon_\mu g^\perp_{\nu\lambda} + q_\tau \varepsilon_\nu g^\perp_{\mu\lambda})\int \mathcal{D}\vec{\alpha}\mathrm{e}^{\mathrm{i}(\alpha_1+u\alpha_3)q\cdot x}\mathcal{T}_2^{(4)}(\vec{\alpha}) \\ & + \frac{f_V^T m_V^2}{q\cdot x}(q_\mu q_\lambda \varepsilon_\nu x_\tau - q_\nu q_\lambda \varepsilon_\mu x_\tau - q_\mu q_\tau \varepsilon_\nu x_\lambda + q_\nu q_\tau \varepsilon_\mu x_\lambda)\int \mathcal{D}\vec{\alpha}\mathrm{e}^{\mathrm{i}(\alpha_1+u\alpha_3)q\cdot x}\mathcal{T}_3^{(4)}(\vec{\alpha}) \\ & + \frac{f_V^T m_V^2}{q\cdot x}(q_\mu q_\lambda \varepsilon_\tau x_\nu - q_\nu q_\lambda \varepsilon_\tau x_\mu - q_\mu q_\tau \varepsilon_\lambda x_\nu + q_\nu q_\tau \varepsilon_\lambda x_\mu)\int \mathcal{D}\vec{\alpha}\mathrm{e}^{\mathrm{i}(\alpha_1+u\alpha_3)q\cdot x}\mathcal{T}_4^{(4)}(\vec{\alpha})\}\end{aligned} \tag{A10}$$

where $\int \mathcal{D}\vec{\alpha} \coloneqq \int_0^1 \mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\delta(\alpha_1+\alpha_2+\alpha_3-1)$ and

$$g^\perp_{\mu\nu} \coloneqq g_{\mu\nu} - \frac{1}{q\cdot x}(b_\mu x_\nu + b_\nu x_\mu) \tag{A11}$$

with

$$b_\mu \coloneqq q_\mu - \frac{m_V^2}{2q\cdot x}x_\mu. \tag{A12}$$

The two-particle twist-2 DA is

$$\phi_2^\parallel(u) = 6\bar{u}u(1 + a_1^\parallel C_1^{3/2}(\xi) + a_2^\parallel C_2^{3/2}(\xi)), \tag{A13}$$

the two-particle twist-3 DA is

$$\begin{aligned}\phi_3^\perp(u) = {} & (3a_1^\parallel\xi^3)/2 + 3/4(1+\xi^2) + (5\kappa_3^\parallel - (15\lambda_3^\parallel)/16 + (15\tilde{\lambda}_3^\parallel)/8)\xi(-3+5\xi^2) + ((9a_2^\parallel)/112 + (15\omega_3^\parallel)/32 - (15\tilde{\omega}_3^\parallel)/64) \\ & \times(3-30\xi^2+35\xi^4) + (-1+3\xi^2)\times((3a_2^\parallel)/7 + 5\zeta_3^\parallel) - (1/(2f_V m_V))3f_V^T(m_{q_1}-m_{q_2})(2\xi + 2a_2^\perp\xi(11-20\bar{u}u) \\ & + 9a_1^\perp(1-2\bar{u}u) + (1+3a_1^\perp+6a_2^\perp)\ln(\bar{u}) - (1-3a_1^\perp+6a_2^\perp)\ln(u)) + (1/(2f_V m_V))3f_V^T(m_{q_1}+m_{q_2})(2+9a_1^\perp\xi \\ & + 2a_2^\perp(11-30\bar{u}u) + (1+3a_1^\perp+6a_2^\perp)\ln(\bar{u}) + (1-3a_1^\perp+6a_2^\perp)\ln(u)),\end{aligned} \tag{A14}$$

and the two-particle twist-4 DAs are

$$\begin{aligned}\psi_4^\parallel(u) = 1 & + (1/(f_V m_V))6f_V^T(m_{q_1}-m_{q_2})(\xi + 1/2a_1^\perp(-1+3\xi^2) + 5/2\kappa_3^\perp(-1+3\xi^2) + 1/2a_2^\perp\xi(-3+5\xi^2) \\ & + 5/6\omega_3^\perp\xi(-3+5\xi^2) - 1/16\lambda_3^\perp(3-30\xi^2+35\xi^4)) + ((9a_1^\parallel)/5 + 12\kappa_4^\parallel)C_1^{1/2}(\xi) + (-1-(2a_2^\parallel)/7 + (40\zeta_3^\parallel)/3) \\ & \times C_2^{1/2}(\xi) - (20\zeta_4^\parallel C_2^{1/2}(\xi))/3 + (-((9a_1^\parallel)/5) - (20\kappa_3^\parallel)/3 - (16\kappa_4^\parallel)/3)C_3^{1/2}(\xi) + (10\theta_1^\parallel - 5\theta_2^\parallel)C_3^{1/2}(\xi) \\ & + (-((27a_2^\parallel)/28) - (15\omega_3^\parallel)/8 - (15\tilde{\omega}_3^\parallel)/16 + (5\zeta_3^\parallel)/4)C_4^{1/2}(\xi),\end{aligned} \tag{A15}$$

TABLE V. All the values are at the energy scale of $\mu = 1$ GeV. The accuracy of these parameters is 30%–50%.

	$a_1^{\parallel}$	$a_1^{\perp}$	$a_2^{\parallel}$	$a_2^{\perp}$	$\zeta_3^{\parallel}$	$\tilde{\lambda}_3^{\parallel}$	$\tilde{\omega}_3^{\parallel}$	$\kappa_3^{\parallel}$	$\omega_3^{\parallel}$	$\lambda_3^{\parallel}$	$\kappa_3^{\perp}$	$\omega_3^{\perp}$	$\lambda_3^{\perp}$	$\zeta_4^{\parallel}$	$\tilde{\omega}_4^{\parallel}$	$\zeta_4^{\perp}$	$\tilde{\zeta}_4^{\perp}$	$\kappa_4^{\parallel}$	$\kappa_4^{\perp}$
ρ^+	0	0	0.15	0.14	0.030	0	−0.09	0	0.15	0	0	0.55	0	0.07	−0.03	−0.03	−0.08	0	0
K^*	0.03	0.04	0.11	0.10	0.023	0.035	−0.07	0	0.1	−0.008	0.003	0.03	−0.025	0.02	−0.02	−0.01	−0.05	−0.025	0.013
ω	0	0	0.15	0.14	0.030	0	−0.09	0	0.15	0	0	0.55	0	0.07	−0.03	−0.03	−0.08	0	0
ϕ	0	0	0.18	0.14	0.024	0	−0.045	0	0.09	0	0	0.20	0	0	−0.02	−0.01	−0.03	0	0

$$
\begin{aligned}
\phi_4^{\parallel}(u) = {} & (1/(f_V m_V)) f_V^T (m_{q_1} - m_{q_2})((-23 - 54a_1^{\perp} - 108a_2^{\perp} + 5u^2)\ln(\bar{u}) - (-23 + 54a_1^{\perp} - 108a_2^{\perp} + 5(\bar{u})^2)\ln(u)) \\
& + (1/(f_V m_V)) 24 f_V^T (m_{q_1} + m_{q_2})((1 + 3a_1^{\perp} + 6a_2^{\perp})(\bar{u})^2 \ln(\bar{u}) + (1 - 3a_1^{\perp} + 6a_2^{\perp})u^2 \ln(u)) + 4(a_1^{\parallel} \\
& - (40\kappa_3^{\parallel})/3)(1/8(11 - 3\xi^2) - (2 - \bar{u})(\bar{u})^3 \ln(\bar{u}) + (2 - u)u^3 \ln(u)) + 80\psi_2^{\parallel}(1/8(11 - 3\xi^2) - (2 - \bar{u})(\bar{u})^3 \ln(\bar{u}) \\
& + (2 - u)u^3 \ln(u)) - 80\tilde{\omega}_4^{\parallel}(1/8\bar{u}(21 - 13\xi^2)u + (\bar{u})^3(10 - 15\bar{u} + 6(\bar{u})^2)\ln(\bar{u}) + u^3(10 - 15u + 6u^2)\ln(u)) \\
& + 2(-2a_2^{\parallel} + 3\omega_3^{\parallel} - (14\zeta_3^{\parallel})/3)(1/8\bar{u}(21 - 13\xi^2)u + (\bar{u})^3(10 - 15\bar{u} + 6(\bar{u})^2)\ln(\bar{u}) + u^3(10 - 15u + 6u^2)\ln(u)) \\
& + 30(\bar{u})^2 u^2((20\zeta_4^{\parallel})/9 + (-((8\theta_1^{\parallel})/15) + (2\theta_2^{\parallel})/3)C_1^{5/2}(\xi)) + 30(\bar{u})^2 u^2(4/5(1 + (a_2^{\parallel})/21 + (10\zeta_3^{\parallel})/9) \\
& + ((17a_1^{\parallel})/50 - (\lambda_3^{\parallel})/5 + (2\tilde{\lambda}_3^{\parallel})/5)C_1^{5/2}(\xi) + 1/10((9a_2^{\parallel})/7 + (7\omega_3^{\parallel})/6 - (3\tilde{\omega}_3^{\parallel})/4 + (\zeta_3^{\parallel})/9)C_2^{5/2}(\xi)) \\
& + (1/(f_V m_V)) 6\bar{u} f_V^T (m_{q_1} + m_{q_2})u(2(3 + 16a_2^{\perp}) + 10/3(-a_1^{\perp} + \kappa_3^{\perp})C_1^{3/2}(\xi) + (-a_2^{\perp} + (5\omega_3^{\perp})/9)C_2^{3/2}(\xi) \\
& - (\lambda_3^{\perp} C_3^{3/2}(\xi))/10) + (1/(f_V m_V)) 6\bar{u} f_V^T (m_{q_1} - m_{q_2})u((-((82a_1^{\perp})/5) - 10\kappa_3^{\perp})C_1^{3/2}(\xi) + 20(10/189 + (a_2^{\perp})/3 \\
& - (\omega_3^{\perp})/21)C_2^{3/2}(\xi) + ((2a_1^{\perp})/5 + (7\lambda_3^{\perp})/54)C_3^{3/2}(\xi) + (-(2/315) + (a_2^{\perp})/5 - (\omega_3^{\perp})/21)C_4^{3/2}(\xi) \\
& + (2\lambda_3^{\perp} C_5^{3/2}(\xi))/135). \qquad \text{(A16)}
\end{aligned}
$$

The three-particle twist-3 DAs are

$$\mathcal{V}(\vec{\alpha}) = \mathcal{V}(\alpha_1, \alpha_2, \alpha_3) = 360\alpha_1(1 - \alpha_1 - \alpha_3)\alpha_3^2(\kappa_3^{\parallel} + 1/2(-3 + 7\alpha_3)\lambda_3^{\parallel} + (-1 + 2\alpha_1 + \alpha_3)\omega_3^{\parallel}), \qquad \text{(A17)}$$

$$\mathcal{A}(\vec{\alpha}) = \mathcal{A}(\alpha_1, \alpha_2, \alpha_3) = 360\alpha_1(1 - \alpha_1 - \alpha_3)\alpha_3^2((-1 + 2\alpha_1 + \alpha_3)\tilde{\lambda}_3^{\parallel} + 1/2(-3 + 7\alpha_3)\tilde{\omega}_3^{\parallel} + \zeta_3^{\parallel}), \qquad \text{(A18)}$$

$$\mathcal{T}(\vec{\alpha}) = \mathcal{T}(\alpha_1, \alpha_2, \alpha_3) = 360\alpha_1(1 - \alpha_1 - \alpha_3)\alpha_3^2(\kappa_3^{\perp} + 1/2(-3 + 7\alpha_3)\lambda_3^{\perp} + (-1 + 2\alpha_1 + \alpha_3)\omega_3^{\perp}). \qquad \text{(A19)}$$

and the three-particle twist-4 DAs are

$$\mathcal{T}_3^{(4)}(\vec{\alpha}) = \mathcal{T}_3^{(4)}(\alpha_1, \alpha_2, \alpha_3) = -120\alpha_1(1 - \alpha_1 - \alpha_3)\alpha_3(\tilde{\phi}_0^{\perp} + (-1 + 2\alpha_1 + \alpha_3)\tilde{\phi}_1^{\perp} + (-1 + 3\alpha_3)\tilde{\phi}_2^{\perp}), \qquad \text{(A20)}$$

$$
\begin{aligned}
\mathcal{T}_4^{(4)}(\vec{\alpha}) = \mathcal{T}_4^{(4)}(\alpha_1, \alpha_2, \alpha_3) = {} & 30\alpha_3^2(-((-1 + 2\alpha_1 + \alpha_3)(\psi_0^{\perp} + \alpha_3\psi_1^{\perp} + 1/2(-3 + 5\alpha_3)\psi_2^{\perp})) \\
& + (1 - \alpha_3)\theta_0^{\perp} + (-6\alpha_1(1 - \alpha_1 - \alpha_3) + (1 - \alpha_3)\alpha_3)\theta_1^{\perp} + (-(3/2)((\alpha_1)^2 \\
& + (1 - \alpha_1 - \alpha_3)^2) + (1 - \alpha_3)\alpha_3)\theta_2^{\perp}). \qquad \text{(A21)}
\end{aligned}
$$

Numerical values of the parameters that appear in the DAs are compiled in Table V.

APPENDIX B: IMPORTANT INTEGRALS

In this section, we share the results of various integrals that appear in the theoretical side of the correlation function. We grouped the integrals into two: those coming from the terms that do not involve the gluon and those that contain it. From the gluon terms, there are three-particle DAs, say, $\mathcal{F}(\alpha_1, \alpha_2, \alpha_3) =: \mathcal{F}(\vec{\alpha})$ where $\mathcal{F} = \mathcal{S}, \tilde{\mathcal{S}}, \mathcal{V}, \mathcal{A}, \mathcal{T}, \mathcal{T}_k^{(4)}$ with $k = 1, 2, 3, 4$.

We focus on the terms without the gluon first. Let $K_n := K_n(m_Q\sqrt{-x^2})/(\sqrt{-x^2})^n$ and

$$I^n := \int_0^1 \mathrm{d}u \int \mathrm{d}^4x e^{\mathrm{i}(p+\bar{u}q)x} K_n f(u), \qquad \text{(B1)}$$

$$I^n_\mu \coloneqq \int_0^1 \mathrm{d}u \int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}(p+\bar{u}q)x} x_\mu K_n f(u), \tag{B2}$$

$$I^n_{\mu\nu} \coloneqq \int_0^1 \mathrm{d}u \int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}(p+\bar{u}q)x} x_\mu x_\nu K_n f(u), \tag{B3}$$

$$I^n_{\mu\nu\lambda} \coloneqq \int_0^1 \mathrm{d}u \int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}(p+\bar{u}q)x} x_\mu x_\nu x_\lambda K_n f(u), \tag{B4}$$

$$I^n_2 \coloneqq \int_0^1 \mathrm{d}u \int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}(p+\bar{u}q)x} x^2 K_n f(u), \tag{B5}$$

$$I^n_{2\mu} \coloneqq \int_0^1 \mathrm{d}u \int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}(p+\bar{u}q)x} x^2 x_\mu K_n f(u), \tag{B6}$$

$$I^n_{2\mu\nu} \coloneqq \int_0^1 \mathrm{d}u \int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}(p+\bar{u}q)x} x^2 x_\mu x_\nu K_n f(u). \tag{B7}$$

After a double Borel transformation over $-p^2$ and $-(p+q)^2$ and performing the continuum subtraction via the replacement $\mathrm{e}^{-m_Q^2/M^2-m_V^2/(M_1^2+M_2^2)} \to \mathrm{e}^{-m_Q^2/M^2-m_V^2/(M_1^2+M_2^2)} - \mathrm{e}^{-s_0/M^2}$ as described in [40], one has

$$I^n \to \mathrm{i}\frac{2^{3-n}\pi^2}{m_Q^n} M^{2n}[\mathrm{e}^{-m_Q^2/M^2-m_V^2/(M_1^2+M_2^2)} - \mathrm{e}^{-s_0/M^2}]f(u_0), \tag{B8}$$

$$I^n_\mu \to -\frac{2\mathrm{i}(p+\bar{u}_0 q)_\mu}{M^2} I^n, \tag{B9}$$

$$I^n_{\mu\nu} \to -\frac{2[g_{\mu\nu}M^2 + 2(p+\bar{u}_0 q)_\mu (p+\bar{u}_0 q)_\nu]}{M^4} I^n, \tag{B10}$$

$$I^n_{\mu\nu\lambda} \to \frac{4\mathrm{i}}{M^6}\{[M^2 g_{\mu\nu}(p+\bar{u}_0 q)_\lambda + M^2 g_{\nu\lambda}(p+\bar{u}_0 q)_\mu + M^2 g_{\mu\lambda}(p+\bar{u}_0 q)_\nu + 2(p+\bar{u}_0 q)_\mu (p+\bar{u}_0 q)_\nu (p+\bar{u}_0 q)_\lambda\} I^n, \tag{B11}$$

$$I^n_2 \to -\mathrm{i}\frac{2^{5-n}\pi^2}{m_Q^n} M^{2n-4}[m_Q^2 + (n-1)M^2][\mathrm{e}^{-m_Q^2/M^2-m_V^2/(M_1^2+M_2^2)} - \mathrm{e}^{-s_0/M^2}]f(u_0), \tag{B12}$$

$$I^n_{2\mu} \to -\frac{2\mathrm{i}(p+\bar{u}_0 q)_\mu}{M^2} I^n_2, \tag{B13}$$

$$I^n_{2\mu\nu} \to -\frac{2[M^2 g_{\mu\nu} + 2(p+\bar{u}_0 q)_\mu (p+\bar{u}_0 q)_\nu]}{M^4} I^n_2 \tag{B14}$$

where $u_0 \coloneqq M_2^2/(M_1^2+M_2^2)$, $\bar{u}_0 \coloneqq 1-u_0 = M^2/M_1^2+M_2^2$, and M is defined via $(M^2)^{-1} = (M_1^2)^{-1} + (M_2^2)^{-1}$. For nearly-equal-mass mesons, one can take $M_1 = M_2$ and hence $u_0 = \bar{u}_0 = 1/2$.

Next, we focus on the terms with the gluon. Let

$$J^n \coloneqq \int_0^1 \mathrm{d}u \int \mathrm{d}^4x \int \mathcal{D}\vec{\alpha} \mathrm{e}^{\mathrm{i}[p+(\alpha_1+u\alpha_3)q]x} K_n f(u) F(\vec{\alpha}), \tag{B15}$$

$$J^n_\mu \coloneqq \int_0^1 \mathrm{d}u \int \mathrm{d}^4x \int \mathcal{D}\vec{\alpha} \mathrm{e}^{\mathrm{i}[p+(\alpha_1+u\alpha_3)q]x} x_\mu K_n f(u) F(\vec{\alpha}), \tag{B16}$$

$$J^n_{\mu\nu} \coloneqq \int_0^1 \mathrm{d}u \int \mathrm{d}^4x \int \mathcal{D}\vec{\alpha} \mathrm{e}^{\mathrm{i}[p+(\alpha_1+u\alpha_3)q]x} x_\mu x_\nu K_n f(u) F(\vec{\alpha}), \tag{B17}$$

$$J^n_{\mu\nu\lambda} \coloneqq \int_0^1 \mathrm{d}u \int \mathrm{d}^4x \int \mathcal{D}\vec{\alpha} \mathrm{e}^{\mathrm{i}[p+(\alpha_1+u\alpha_3)q]x} x_\mu x_\nu x_\lambda K_n f(u) F(\vec{\alpha}), \tag{B18}$$

$$J^n_2 \coloneqq \int_0^1 \mathrm{d}u \int \mathrm{d}^4x \int \mathcal{D}\vec{\alpha} \mathrm{e}^{\mathrm{i}[p+(\alpha_1+u\alpha_3)q]x} x^2 K_n f(u) F(\vec{\alpha}), \tag{B19}$$

$$J^n_{2\mu} \coloneqq \int_0^1 \mathrm{d}u \int \mathrm{d}^4x \int \mathcal{D}\vec{\alpha} \mathrm{e}^{\mathrm{i}[p+(\alpha_1+u\alpha_3)q]x} x^2 x_\mu K_n f(u) F(\vec{\alpha}), \tag{B20}$$

$$J^n_{2\mu\nu} \coloneqq \int_0^1 \mathrm{d}u \int \mathrm{d}^4x \int \mathcal{D}\vec{\alpha} \mathrm{e}^{\mathrm{i}[p+(\alpha_1+u\alpha_3)q]x} x^2 x_\mu x_\nu K_n f(u) F(\vec{\alpha}), \tag{B21}$$

where $\int \mathcal{D}\vec{\alpha} \coloneqq \int_0^1 \mathrm{d}\alpha_1 \mathrm{d}\alpha_2 \mathrm{d}\alpha_3 \delta(\alpha_1+\alpha_2+\alpha_3-1)$. The results are as follows:

$$J^n \to \mathrm{i}\frac{2^{3-n}\pi^2}{m_Q^n} M^{2n}[\mathrm{e}^{-m_Q^2/M^2-m_V^2/(M_1^2+M_2^2)} - \mathrm{e}^{-s_0/M^2}] \int_0^{\bar{u}_0} \mathrm{d}\alpha_1 \int_{\bar{u}_0-\alpha_1}^{1-\alpha_1} \mathrm{d}\alpha_3 \frac{1}{\alpha_3} F(\alpha_1, 1-\alpha_1-\alpha_3, \alpha_3) f(u_1), \tag{B22}$$

$$J^n_\mu \to -\frac{2\mathrm{i}(p+\bar{u}_0 q)_\mu}{M^2} J^n, \tag{B23}$$

$$J^n_{\mu\nu} \to -\frac{2[g_{\mu\nu}M^2 + 2(p+\bar{u}_0 q)_\mu (p+\bar{u}_0 q)_\nu]}{M^4} J^n, \tag{B24}$$

$$J^n_{\mu\nu\lambda} \to \frac{4\mathrm{i}}{M^6}\{[M^2 g_{\mu\nu}(p+\bar{u}_0 q)_\lambda + M^2 g_{\nu\lambda}(p+\bar{u}_0 q)_\mu + M^2 g_{\mu\lambda}(p+\bar{u}_0 q)_\nu] + 2(p+\bar{u}_0 q)_\mu (p+\bar{u}_0 q)_\nu (p+\bar{u}_0 q)_\lambda\} J^n, \tag{B25}$$

$$J^n_2 \to -\mathrm{i}\frac{2^{5-n}\pi^2}{m_Q^n} M^{2n-4}[m_Q^2 + (n-1)M^2][\mathrm{e}^{-m_Q^2/M^2-m_V^2/(M_1^2+M_2^2)} - \mathrm{e}^{-s_0/M^2}] \times \int_0^{\bar{u}_0} \mathrm{d}\alpha_1 \int_{\bar{u}_0-\alpha_1}^{1-\alpha_1} \mathrm{d}\alpha_3 \frac{1}{\alpha_3} F(\alpha_1, 1-\alpha_1-\alpha_3, \alpha_3) f(u_1), \tag{B26}$$

$$J^n_{2\mu} \to -\frac{2\mathrm{i}(p+\bar{u}_0 q)_\mu}{M^2} J^n_2, \tag{B27}$$

$$J^n_{2\mu\nu} \to -\frac{2[g_{\mu\nu}M^2 + 2(p+\bar{u}_0 q)_\mu (p+\bar{u}_0 q)_\nu]}{M^4} J^n_2, \tag{B28}$$

$$J^n_{2\mu\nu\lambda} \to \frac{4\mathrm{i}}{M^6}\{[M^2 g_{\mu\nu}(p+\bar{u}_0 q)_\lambda + M^2 g_{\nu\lambda}(p+\bar{u}_0 q)_\mu + M^2 g_{\mu\lambda}(p+\bar{u}_0 q)_\nu] + 2(p+\bar{u}_0 q)_\mu (p+\bar{u}_0 q)_\nu (p+\bar{u}_0 q)_\lambda\} J^n_2 \tag{B29}$$

where $u_1 \coloneqq \frac{1}{\alpha_3}(1 - \frac{M^2}{M_1^2} - \alpha_1)$.

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