DISSERTATION

STOCKING LEVELS FOR LODGEPOLE PINE

Submitted by

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WE HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER OUR SUPERVISION BY DAVID LEWIS ADAMS ENTITLED STOCKING LEVELS FOR LODGEPOLE PINE BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY.

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ABSTRACT OF DISSERTATION

STOCKING LEVELS FOR LODGEPOLE PINE

Over a considerable range of stocking, volume growth per acre is produced in nearly equal amounts. Below the lower end of this range the site is not fully utilized and above the upper limit of this range the stand is stagnated and optimum growth is not realized. This study developed a method for defining the optimum range of stocking for lodgepole pine (*Pinus contorta* Dougl.) in Colorado and southern Wyoming.

The procedure for defining the upper limit of the stocking range was based upon basal area growth rate per tree as a function of stand density. An extension of this relationship provided a curve of basal area growth per acre over stand density for each diameter class. These curves revealed the density at which the peak of basal area growth rate occurred for each class. Basal area per acre at the peak of basal area growth, when curved over the corresponding stand density for each diameter class, formed the upper limit of the "full-stocking" range.

The second part of the study, the establishment of minimum stocking levels for full site occupancy, consisted of an analysis of open-grown trees. It was reasoned that competition between trees begins at a point where the available growing space in a stand is just

equal to the total open-grown, tree-area requirements of all the trees in the stand.

The relationship between crown area and diameter breast height was established for the diameters involved. Dividing the area of one acre by the crown area of a tree of a given diameter provided the theoretical number of open-grown trees of this size that could fully occupy the site. The basal area per acre represented by this number of trees was next calculated. The curve delimiting the lower end of the desirable stocking range was then constructed by plotting basal area per acre over the corresponding number of trees per acre.

A procedure was developed by which the change in stocking over time might be estimated for use with the established stocking range. This part of the study involved a graphical solution based upon the diameter-age relationship. A rate of change curve was developed for each of three density classes.

The full stocking range, as developed in this study, can be used as a guide to the silvicultural needs of a stand. Two stand parameters, average tree diameter and number of trees per acre, are needed to determine the basal area per acre required to fall within the acceptable stocking range. Through the use of these stocking curves, management decisions can be based upon definite guidelines instead of subjective judgements concerning the stand stocking situation.

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CHAPTER I

INTRODUCTION

Despite the rapid biological and technological advances that have been made in recent years, manipulation of growing stock levels remains as the most usable tool in forest management.

Over a considerable range of stocking, volume growth per acre is produced in nearly equal amounts. Below the lower end of this range the site is not fully utilized and above the upper limit of this range the stand is stagnated and optimum growth is not realized.

The addition of each tree above the minimum stocking level which is necessary to fully occupy the site, will theoretically cause additional total stand increment up to some maximum stocking level. Further additions to the stand above this maximum level will result in a reduction of gross stand increment. The objective of management should be to maintain stocking between the minimum and maximum levels appropriate for the particular stand. Acceptable stocking limits may differ for stands on sites of different qualities and may also be functions of other stand variables.

Within the "acceptable" range, stocking may be altered to achieve particular production goals. If the objective is to produce maximum cubic-foot volume the stocking level might be maintained near the upper end of the range. If the objective is to produce larger material, it may be desirable to hold the level of stocking near the lower end of the acceptable range.

This study examines measures of stand density, effects of density levels on growth, and approaches to establishment of desirable stocking levels. It then develops a method for defining the optimum range of stocking for lodgepole pine (Pinus contorta Dougl.)[1] in Colorado and southern Wyoming. Finally, it examines the established stocking range as a partial guide to the management of lodgepole pine stands.

Definitions

Most of the terms in this paper should be familiar to foresters. Two terms are defined here to aid clarity.

"Stocking" is defined in Forest Terminology (1958) as "An indication of the number of trees in a stand as compared to the desirable number for best growth and management: such as well-stocked, partially stocked, over-stocked." Forest stocking measures the extent to which the realizable productive capacity of a forest site is being utilized by tree growth at a given time (Davis, 1966).

"Stand density" is defined in Forest Terminology (1958) as "Density of stocking expressed in number of trees, basal area,

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[1] Scientific names are listed in Appendix A.

volume, or other criteria, on a per-acre basis." Density or stand density as used by the writer in development of the optimum stocking range for lodgepole pine refers to the number of trees per acre except where otherwise defined.

CHAPTER II

REVIEW OF STOCKING CONCEPTS

Stocking concepts have been reviewed and discussed by many authors including Davis (1966), Meyer, Recknagel, and Stevenson (1952), Spurr (1952), Gingrich (1967), and Bickford, Baker, and Wilson (1957).

The papers cited in this review were selected mainly to develop three topics:

1. Measures of stand density
2. The influence of stand density on forest growth
3. Stocking guides

Measures of stand density

Before a program aimed at exercising control over the amount and distribution of growing stock can be devised, it is first necessary to select some convenient unit of measure that can be used to evaluate the amount of growing space occupied by trees of all sizes. The number of such units per acre would be an appropiate measure of land density (Hawley and Smith, 1954).

In spite of the large amount of research dealing with various aspects of stand density, no one measure of it has been universally accepted. Furthermore, there is probably no one measure which would be suitable for all species and all management objectives.

The ideal measure of density should be simple, objective, and should be largely unrelated to the character and age of the stand and the quality of the site (Spurr, 1952). Correlation with volume increment is also considered a desirable attribute (Bickford, et al. 1957).

Probably the most logical and simplest measure of stand density is number of trees per acre (Meyer, 1930; Spurr, 1952; Hawley and Smith, 1954; Davis, 1966). However, its use is curtailed in practice because the total number of trees is unweighted by size, which affects stocking (Davis, 1966). In natural stands numbers and distribution may vary widely, particularly in the smaller diameters. This wide variation may have no actual effect on relative density or stocking. One fully stocked stand may have several times as many stems per acre as another of the same age on the same site. Spurr (1952) therefore recommended that the number of trees per acre should be used only to express density in connection with an additional variable. Gevorkiantz (1947a) suggested the use of both number of trees and total basal area. He explained that some stands may be slightly understocked in number of trees but actually have enough basal area to give good stocking.

Some thinning schedules have been worked out on the basis of only number of trees per acre, but they are usually applicable only in plantations, which are of absolutely even age and much more uniform as to size and distribution of trees than natural stands (Hawley and Smith, 1954).

Stand density is usally measured in terms of some combination of four basic factors: diameter, height, form, and number of trees per acre (Bickford, et al. 1957). Volume per acre is a measure of stand density which is an expression of all four of these factors. Although volume has long been used as a measure of density and the values are concrete and meaningful, Spurr (1952) raised two objections to the use of volume as a measure of density. First, he pointed out that it must be estimated from other measurements, so it is obtained indirectly. Probably even more important is that it is strongly related to both the age and character of the stand and to the quality of the site.

In a stand of uniform height, total cubic feet of wood is so nearly a direct function of basal area that the extra computations hardly seem worthwhile. Errors in determining cubic volume are also far greater than those involved in computing basal area (Assmann, 1950, as cited by Hawley and Smith, 1954).

A density measure based upon any type of merchantable volume such as board feet, cords, or merchantable cubic feet is not satisfactory because their interpretation varies widely with different standards of utilization (Hawley and Smith, 1954). In spite of these and other arguments, some workers believe total cubic volume to be the best measure of site productivity or of response to thinning (Wilson, 1951; Dahms, 1966).

Lexen (1943) suggested that since the vascular cambium of the stem is the actual base upon which new wood is produced, a measure of stem cambium expressed as "Bole area" is a logical expression of growing stock. In use, growing stock can be expressed in terms of square feet of bole area per acre. This measure is definite, quantitative, not particularly difficult to compute, and is directly related to tree growth (Davis, 1966). Bole area, like volume, is based on the four factors, diameter, height, form, and number of trees per acre. The bole area concept is most applicable to conifers where the bole is mostly in single and readily measured stem (Davis, 1966; Bickford, et al. 1957).

The bole area can be calculated for individual trees from either Huber's or Smalian's formulae by substituting circumference for area, and then summing the areas for all sections in each tree (Lexen, 1943). Wilson (1951) proposed that in even-aged stands negligible error would result if bole area were based on the average tree, permitting use of a simplified modification of Lexen's formula. Hummel (1953) reported that the main disadvantage of bole area as a stand density index in thinning research is that it is somewhat cumbersome to apply.

Mulloy (1944a) after comparing stand bole area with stand density and stand intensity indices in a large number of red and white pine stands, concluded that all three measures provide the same relative measure of density. He concluded that in some ways the stand bole area index seemed to have certain advantages since it uses the three most easily obtained stand factors: diameter, height, and number of trees.

Although bole area is not used widely it is considered to be a desirable measure of stand density for some purposes. For example Dahms (1967) used bole area as the measure of density for his study of stand density and lodgepole pine tree growth.

The degree of crown closure is a measure of stand density that has become particularly important with the use of aerial photographs. It is usually expressed as a percentage of full canopy cover and is often grouped into broad classes such as dense, medium, or open, each expressing a specified range in percentage of crown closure (Davis, 1966). However, stands of a given age, growing on a given site, may have a closed crown canopy and still show considerable differences in number of trees per acre (Meyer, 1953). Crown closure also has the disadvantage of being difficult to measure objectively.

Despite its shortcomings, it is possible, through the integration of crown diameter-dbh relationships, and numbers of trees to estimate basal area and even volume from photographs. These can in turn provide an estimate of stocking, although ground sampling is usually needed to give useful accuracy (Davis, 1966).

Another approach to density measurement related to crown development is an index called "Crown Competition Factor," as proposed by Krajicek and Brinkman (1957). Bickford, et al. (1957) pointed out that crown closure and other expressions pertaining to crowns are relative to area occupation and are more properly indications of coverage than measures of stand density. But, according to Krajicek, Brinkman, and Gingrich (1961), crown competition factor (CCF) is not essentially a measure of crown closure, but is used as an expression of stand density, although it does pertain to crowns and is expressed in percent.

The original explanation of crown competition factor by Krajicek and Brinkman (1957) was presented so logically that a portion of it will be quoted.

> "If enough open-grown trees were distributed over an acre of land so that the crowns met but did not overlap, there would be no competition yet no wasted space. Then the percentage of an acre that the crown of each tree occupied would be a significant figure. The sum of these percentages for all the trees on the acre would be 100. This figure was called the 'Crown Competition Factor' or CCF. A similar stand containing twice as many trees of the same species and dbh would have a CCF of 200."

Theoretically, crown closure can occur from CCF 100 to the maximum for a given species. Vezina (1962) reported that the maximum is approximately CCF 300 for balsam fir, and Alexander (1966a) listed site index values for lodgepole pine by CCF classes as high as 500. CCF reflects the area available to the average tree in the stand in relation to the maximum area it could use if it were open-grown (Vezina, 1962).

The term "Maximum Crown Area" or MCA has been used in explanations of the crown competition factor concept. Maximum Crown Area is the vertical projection of the average crown area of an open-grown tree of the same diameter (Alexander, 1966a). Any combination of open-grown trees of various diameters whose MCA values total 100 for an acre could, theoretically, give a crown closure of 100 percent (Vezina, 1962).

Krajicek, et al. (1961) reported that trials showed strong evidence that site and stand age do not influence CCF. Alexander, Tackle, and Dahms (1967) investigated the dependence of CCF on stand age and site index for lodgepole pine, and found no strong relationship between CCF and age. Regression analysis indicated significant negative correlation between CCF and site index; higher CCF values appeared to be associated with low site indexes. In spite of the apparent relationship between CCF and site index, Alexander, et al. found that the correlation coefficient indicated that the regression accounted for only about 6 percent of the total variation in CCF between plots. They therefore chose CCF as a measure of density for lodgepole pine under the assumption of independence with stand age and site index.

Vezina (1962) reported that CCF has been shown to vary somewhat with age and site quality in balsam fir and white spruce in Quebec. However, the same author investigated the relation of CCF to age in jack pine stands and did not find that a relationship was apparent (Vezina, 1963).

Probably the most widely used and most generally accepted measure of stand density is basal area. It has been found to be consistent as a measure of density and is easily determined (Davis, 1966). This latter characteristic cannot be overlooked as a desirable trait. Basal area is defined in the United States as the total cross-sectional area of the trees in the stand measured in square feet. Basal area increases rapidly as the young stand develops and then levels off with increasing age, becoming relatively constant for the mature stand (Davis, 1966; Spurr, 1952; Forbes, 1956). Stands tend to have characteristic and relatively stable basal area patterns in relation to age and site. Basal area per acre increases with site index, and the better the site, the more basal area increases with age (Davis, 1966). Because of the wide variation in basal area at theoretical full stocking, Gruschow and Evans (1959) held that basal area alone is cumbersome as a density standard. In managed stands, basal area is less affected by site and age during the rotation; the influence of increasing diameter being offset by decreasing number of trees (Bickford, et al. 1957).

Bickford, et al. (1957) listing some common criticisms of basal area as a stand density measure, pointed out that it gives the same weight to equal areas of biologically dead heartwood and functioning sapwood; to young trees and old trees; to supressed trees and dominant trees; and so on. Also, they brought up the question of whether the square is the proper exponent of diameter; the part that grows, the circumference, is given by the first power. Furthermore, stand density, the quantity to be measured, affects tree form and thus basal area. They also listed as disadvantages the facts that basal area ignores height and that stands of equal basal area may have divergent yield capacities in terms of usable products. Lexen (1943) also agreed that basal area neglects height which may vary greatly from stand to stand within the same locality.

Nelson and Brender (1963), on the other hand, maintained that the advantages of basal area as an expression of stand density are clearcut and proven. They suggested that the disadvantages listed by Bickford, et al. (1957) are principally conjecture. They pointed out that no good evidence is documented to show that the concern about dead heartwood and live sapwood is valid, and also that tree age should not be a factor when dealing with even-aged stands. Nelson and Brender suggested that most of the other expressions of stand density utilizing a combination of number of trees and diameter share, to a lesser or greater degree, many of the disadvantages of basal area. After making a study of density measures in loblolly pine, they concluded that no analytical advantages accrue in using more complex and difficult-to-obtain measures of stand density, and that basal area provides a direct measure instead of an index.

Gingrich (1967) demonstrated what he considered the basic weakness of basal area alone as a measure of stand density by comparing it with tree-area requirements. He argued that over a period of time trees increase in diameter, but if stands are continually cut to maintain a constant basal area, the stocking condition, or degree of competition, actually decreases. Doubling the diameter of a tree increases its basal area four times but increases the tree-area requirements only about three times. Thus, as tree diameter increases the basal area of a stand must also increase if the percent stocking is to remain the same.

The close relationship between basal area and volume is a desirable attribute and has been demonstrated by various studies dealing with many species. For example, Meyer (1930) found good correlation coefficients with basal area in respect to cubic-foot and International board-foot volume, and resulting low standard errors in immature Douglas-fir forests. He considered basal area a satisfactory index of stocking. Nelson, et al. (1961) were able to explain over 78 percent of variability in annual cubic-foot growth of natural loblolly pine stands using basal area per acre as an expression of stand density. Myers (1967) used basal areas and average stand diameters as joint measures of growing stock for both accuracy and convenience as a measure in even-aged ponderosa pine stands. He reported that both basal area per acre and average diameter are highly correlated with growth in diameter, basal area, and volume.

Most studies support, in general, Spurr's (1952) belief that for a given species growing in a stand of a given age on a given site, basal area accurately measures the degree of utilization of the area, and may be taken as the standard against which other measures of density may be compared.

The influence of stand density on forest growth

Forest stand growth is the net result of the efficiency with which the available nutrient, moisture, and energy resources are utilized by the individual trees. Probably the most important factor effecting the efficiency of resource use, and hence stand growth, is the number of individuals "competing" for a finite amount of these resources.

The close relationship between stocking level and forest growth has long been recognized. Because of the importance of this relationship to forest management a vast amount of research has been accomplished in the general areas of density, stocking, and spacing as related to diameter, height, basal area, and volume growth. Although growth in diameter, height, basal area and volume cannot in fact be separated, for purposes of this review each will be treated separately.

The influence of density on diameter growth

A strong correlation between diameter growth and stocking level has been reported from many sources. Forest management textbooks such as the ones by Davis (1966) and Meyer, Recknagel, and Stevenson, (1952) treat this relationship as a general fact. The maximum number of trees that it is possible for a stand to have is generally considered to be correlated negatively with the average diameter (Reineke, 1935; Bruce and Schumacher, 1935).

The density-diameter relationship has been investigated for virtually every commercially important tree species in North America. Although the negative correlation as indicated above is almost universal, it is important to know the specific relationships which apply to the various species and on different sites.

Thinning and spacing studies are most often used to demonstrate the effect of stand density on diameter growth. Studies by Roe and Stoeckeler (1950), Rudolf (1951), and Ralston (1953) on the effects of spacing on jack pine development all indicated that diameter growth is increased with spacing interval. Ralston (1953) found, however, that with spacing intervals as great as 8 x 8 feet the trees become limby and will not fully occupy the site.

Red pine spacing studies have indicated essentially the same kind of response. Bramble, Cope, and Chisman (1949) reported on results of 5 x 5, 6 x 6, 8 x 8, and 10 x 10-foot spacings in red pine plantations at 25 years of age. The diameter growth of the two widest spacings greatly exceeded that of the 5 x 5 and 6 x 6 spacings. A difference of 2.7 inches in average dbh was noted between the 10 x 10 and 6 x 6 spacings at 25 years. Thinning a fifteen-year-old red pine stand to four spacings yielded similar results with the best diameter growth occurring in the least density (Schantz-Hansen, 1945).

Eyre and Zehngraff (1948), also working with red pine, agreed that over-crowding may greatly decrease diameter growth, and Lino Della-Bianca and Dils (1960) showed that an unthinned stand of red pine virtually ceased growing by mid-July, while thinned compartments showed radial growth continuing until late August.

Adams and Chapman (1942), reporting on competition in some coniferous plantations, concluded that competition between individuals in a closed stand eventually results in a reduction in the number of trees with subsequent increase in the size and crown spread of the survivors, but not before the influence of competition has been reflected by a decidedly decreased diameter growth.

Research dealing with eastern white pine stands indicated increased diameter growth with decreased density (Gevorkiantz and Hosley, 1929; Hawley, 1936). Carvel (1966) found similar negative correlation in Virginia pine stands, and also that the effect of spacing on diameter growth becomes more pronounced with time.

Studies in both natural stands and in plantations indicated that the diameter growth of the southern pines is closely related to stocking level. The average diameter class of a 330-tree-per-acre natural slash pine stand studied by Collins (1967) was two inches larger than that of a stand averaging 660 trees per acre. Beyond densities of 1500 trees per acre, mean diameter declined very slowly as density increased.

Slash pine investigations by Dell and Collicott (1968) and by Gruschow and Evans (1959) provided information that agreed with that of Collins (1957). Gruschow and Evans showed that diameter growth continued to increase with decreasing stand density even down to a stocking of 19 percent.

Briegleb (1952) reported on the stimulation of diameter growth through thinnings of Douglas-fir stands and Eversole (1955) demonstrated the effects of stand density using spacing tests in a Douglas-fir plantation. In this plantation the average tree in a 12 x 12-foot spacing was twice the diameter of the average tree in a 4 x 4 spacing.

Myers (1958) and Alexander (1965) found diameter growth to be stimulated by thinning in ponderosa and lodgepole pines, respectively, however, Myers reported no relationship between average diameter increase and stand density in a 55-year-old ponderosa pine stand.

Dahms (1967), working with lodgepole pine in the Pacific Northwest, found that diameter growth was greatest in the lowest level of growing stock and the least in the highest level. The basal area of the lowest growing stock level was 102 square feet and was 192 square feet on the highest.

Mowat (1949) reported that immature stands of lodgepole pine responded very favorably to moderate thinning. In a 55-year-old stand on the Pringle Falls Experimental Forest in central Oregon, diameter increment in thinned plots was from $1\frac{1}{2}$ to $2\frac{1}{2}$ times as great as in unthinned plots. The most rapid growth was in trees in the 7 to 10 inch range. Light thinning in 35-year-old stands increased diameter growth less than heavier thinnings but mortality was negligible, and net volume growth was better. Barrett (1961) later reported on the same thinning experiments. He added that when only the 100 largest trees per acre were considered, the above diameter growth rates were reduced to 1 1/3 and 1 3/4, respectively. There was an increase of 3.7 inches in the 16 x 16-foot spacing, 2.7 inches in the 12 x 12 stand, and 1.3 inches in the unthinned stand. This study showed strongly that diameter growth of lodgepole pine in central Oregon responds markedly and positively to thinning release.

The above indications of diameter growth stimulation of lodgepole pine are by no means universal. Armit (1966) reported on four independent regional observations in British Columbia each of which reported no evidence of sustained release of the residual stands after logging. He maintained that good growth in young stands with full crowns can be sustained beyond that which occurs in the undisturbed stand, but once stagnation has set in release growth is uncommon.

The influence of density on height growth

The relationship between stand density and height growth is not as clear-cut as in the case of diameter growth. According to Alexander, Tackle, and Dahms (1967), the height growth of most conifers is independent of stand density over a wide range of stocking. Height growth is low in both extremely open stands and in abnormally dense stands, but if these extremes are disregarded, the height growth is nearly the same in stands of differing densities (Bruce and Schumacher, 1935; Hawley and Smith, 1954).

Reukema (1966) agreed that height growth is in some cases unaffected by density, but pointed out that in other cases it increases with decreasing density. The primary reason that height is used as an index of site quality is that it is generally thought that it is not affected by growing space, but by the site (Adams and Chapman, 1942). However, Armit (1966) did not believe that average height can be used as a valid estimate of site productivity because of the undue influence of stocking, and Alexander (1966) found it necessary to adjust site index for lodgepole pine because crowding restricts rate of height growth. Alexander, Tackle, and Dahms (1967) suggested that the height growth of lodgepole pine is probably influenced more by stand density than is any other North American conifer, but Alexander (1965) reported that height was not greatly stimulated by thinning except in stands thinned when 5 years old. Although Dahms (1967) found greater tree growth in stands of lodgepole pine with low stocking levels, differences in height growth between density treatments were statistically nonsignificant. Smithers (1956) believed average height of lodgepole pine to be closely related to number of stems per acre and that it shows little effect due to site.

Quaite (1950) demonstrated greatly accelerated growth after heavy thinnings in lodgepole pine stands, but again, nearly all the volume increase was due to diameter growth since both thinned and unthinned stands showed annual height growth of only 5 inches in 8 years. In the 22-year period since thinning a 55-year-old lodgepole pine stand in central Oregon, height growth of 14.4 feet, 14.2 feet, and 11.2 feet resulted from 16 x 16, 12 x 12 and unthinned treatments, respectively (Barrett, 1961). These results do not substantiate the statement by Clements (1910) that under competition for light, height growth is made at the expense of growth in diameter.

Briegleb (1952) reported some height growth stimulation as a result of thinnings in Douglas-fir stands, and Eversole (1955), working with spacing tests in the same species, found evidence that density has a marked effect on the average height of dominant and codominant trees. Eversole found that, in general, wider spacing resulted in greater height growth.

Myers (1958) reported that height growth is greatly retarded by excess crowding of ponderosa pine sapling stands. He also found that average periodic height growth of dominant and codominant trees was increased with intensity of thinning in 28- and 40-year-old stands, but not in a 55-year-old stand.

Eastern white pine height growth was not significantly stimulated by thinning in southern New Hampshire (Hawley, 1936), and no clear relationship between average height and spacing distance was detected for Virginia pine in West Virginia (Carvell, 1966). However, in a study similar to the latter, in natural slash pine stands, Collins (1967) found that density adversely affected total height growth of dominant and codominant trees. In a regression analysis, trees per acre accounted for 78 percent of the variation in dominant height. Height decreased approximately one foot for each 500-tree increase in density. Also working with slash pine, but in plantations instead of natural stands, Dell and Collicott (1968) reported that density had no effect on average height growth of dominant and codominant trees.

Several spacing and density studies have been performed in jack pine and red pine stands in the Lake States, again resulting in somewhat differing conclusions. Rudolf (1951) and Ralston (1953) did not find any significant differences or trends chiefly attributable to different spacings in jack pine stands, while Gevorkiantz (1947) maintined that in well-stocked stands, the average spacing of trees bears a definite relation to the average height of the stand. For example, he found that for normal jack pine stands average spacing was about 20 percent of the average height of the dominant and co-cominant trees, and that this spacing varied somewhat with age and site. Roe (1950), on the other hand, found that increased spacing between trees resulted in slightly less height growth.

Conclusions drawn from red pine experiments are equally variable. Schantz-Hansen (1945) indicated that thinning in fifteen-year-old red pine had a stimulating effect on height growth, and Engle and Smith (1950) reported significant response in height growth ten years after thinning a natural stand of red pine. The results of the latter study indicated that it is not desirable to maintain a dense stand for the best height growth, however the authors suggested that height growth might be retarded if the stocking is reduced too severely. Shirley and Zehngraff (1942) substantiated this hypothesis by their findings of increased height growth with increased density.

Bramble, Cope, and Chisman (1949) found that the average crop tree height was about the same for four different spacings in a 25-year-old red pine plantation. Ralston (1954) described an early red pine plantation in Michigan in which stand density apparently affected height growth. Height growth after 35 years was adversely affected by increasing stand density. The dominant trees at the time of his report averaged 25.4, 27.8, 29.8, and 31.5 feet in height for 4 x 4, 6 x 6, 6 x 8, and 8 x 9-foot spacings, respectively. Ralston attributed the disagreement of his findings with some other pine studies to the fact that most other experiments reported were made on sites fairly productive for that species, while his was on rather poor pine soils. Also on some of these sites, competition for the limited soil moisture was intense, affecting height growth as well as diameter growth.

Stand density had only a negligible effect on height growth in a slash pine plantation (Mann and Whitaker, 1952), but Turner (1943) reported height growth of shortleaf and loblolly pines comparable to the findings of Shirley and Zehngraff (1942) with red pine. He found that trees in old-field stands that were thinned early in life or that were from the beginning only partially stocked, were shorter than in the more fully stocked stands. For example, in one 50-year-old stand on an 80-foot site, the difference in height amounted to as much as four feet, and this occurred on apparently comparable soil.

Baker (1953) pointed out that the belief is growing that especi - ally in young stands of ponderosa pine, density affects height growth, and so it appears in his study. A study by Lynch (1958) showed that dense stocking materially reduced height growth on poor sites and that this reduction was sufficient to impair the accuracy of site quality measurements.

The influence of density on basal area growth

Several different kinds of studies have demonstrated varying effects of density level on total stand basal area and basal area growth. Two studies dealing with lodgepole pine resulted in different conclusions concerning this second relationship. Periodic annual basal area increment did not appear to differ between thinned and unthinned stands in Colorado (Alexander, 1965), while in Oregon, a 12 x 12-foot spacing made higher periodic net increment than either 16 x 16-foot spacing or unthinned stands (Barrett, 1961).

In the Colorado study, the greater the number of stems the greater was the total basal area per acre. This is just the opposite from findings in stands of jack, Norway, pitch, and northern white pines where basal area per acre decreased with density (Adams and Chapman, 1942). Adams and Chapman cited a study by Stevenson and Bartoo in which basal area per acre decreased with an increase in spacing up to 10 feet in red pine stands. A study in white pine stands did not indicate a relationship either way (Gevorkiantz and Hosley, 1929). Stands with similar basal area on the same site differed widely in the number of trees per acre.

Gevorkiantz (1947), working with jack pine in the Lake States, reported that the younger the stand, the greater the number of stems per square foot of basal area. The better the site, the less space was required per square foot of basal area. Smithers (1956) also stated that basal area per acre is closely related to site.

Another jack pine project, in which over-dense seedling stands were thinned, revealed that although basal area was higher on the unthinned stand, the thinned stands had a greater proportion of their basal area in the larger size classes (Roe and Stoeckeler, 1950).

Basal area growth response to thinning has been shown in red pine stands (Engle and Smith, 1950). In spite of the reduction of a red pine stand to one half the original density by thinning, the growth was higher than on the unthinned check plot ten years after thinning. Density has also been shown to significantly influence net annual basal area growth in slash pine plantations (Dell and Collicott, 1968).

Nelson (1963) developed a multiple-regression solution for characterizing basal area growth in pure, even-aged, managed stands of loblolly pine. The results showed that basal area growth was a function of stand density, density-age, and density-site interactions and was described by a curvilinear form in relation to stand density and age. Basal area growth dropped off rapidly with increasing age and maximized at an increasingly higher level of stocking with increasing age. It culminated at a higher stocking level on good sites than on poor sites.

The influence of density on volume growth

Although the effects of stand density on diameter, height, and basal area growth are directly related to volume, results expressed in terms of volume increment are often the most meaningful. The fact that this relationship has such basic importance to forest management has precipitated much research, of which the following is but a sample.

The work that has been done on lodgepole pine is, of course, of particular interest in this study. This species shows remarkable range in stand density and striking reactions to both density and environment (Tackle, 1959). To demonstrate this point, Tackle reported on a Rocky Mountain study in 100-year-old stands of varying density which showed a maximum yield of 20,000 board feet per acre with 800 trees; yield fell off rapidly to less than 1,500 board feet when the number of trees increased to 1,800. This demonstration backs up an earlier study by Smithers (1956) which indicated a definite trend towards decreasing cubic volume per acre with increasing number of stems, regardless of different site conditions. Dahms (1967), on the other hand, did not find total volume of wood produced per acre to be significantly different among growing-stock levels.

Armit (1966) indicated some extremely marked effects of stocking on stand growth and stagnation. He reported that volumes in mature stands, 8 inches dbh and over, on productive sites can vary from a high of 4,000 to 5,000 cubic feet down to less than 1,000 cubic feet per acre, although basal areas may vary by less than five percent.

Barrett (1961) reporting on the response of 55-year-old lodgepole pine to thinning, showed that a 12 x 12 spacing, after 22 years, had regained sufficient volume to surpass the unthinned stand by 460 cubic feet per acre. Periodic increment in a more heavily thinned stand (16 x 16 feet) was intermediate between the other two treatments. In both thinned stands, however, nearly all increment was on trees over 6 inches in diameter.

Although Alexander (1965) reported that the greater the density, the greater the cubic-foot volume, in many lodgepole pine stands total growth capacity has been dissipated on so many stems that few, if any, will grow big enough to be usable (Wikstrom and Wellner, 1961), and as Whyte (1965) pointed out, total volume does not necessarily reflect value.

Pearson (1950) found higher growth rate in stands of ponderosa pine made up of small rather than large trees. He maintained that, other things being equal, a given volume of growing stock divided among several small trees is more effective than if concentrated in a single large tree on the same plot of land. He pointed out that the several small trees, if well distributed, have better access to soil moisture and their roots are able to permeate the soil of a given area more completely than those radiating from a single large tree.

Baker (1953) demonstrated that the total wood production per acre varies little with changes in density. Results of his study of ponderosa pine showed that wood production per acre of two stands changed too little to be clearly demonstrated, although density varied from 2, 152 to 214 trees per acre. This study indicated that for ponderosa pine on the site studied, densities from 130 well-spaced trees to 2, 000 trees naturally-spaced yielded virtually the same gross annual increment at an age of 30 years.

Myers (1958) reported the results of thinning in 28- 40- and 55-year-old stands of ponderosa pine in the Black Hills of South Dakota. Cubic-foot volume growth did not differ materially among thinned plots of each stand, but unthinned plots of the 40- and 55-year-old stands produced less cubic-foot volume than did the thinned plots, Myers found the same relationship between total cubic-foot volume and density that Alexander (1965) reported for lodgepole pine; the greater the density the greater the volume.

While developing a new measure of stand density for Douglas-fir, Briegleb (1952) found that only about 26 percent of the variation in periodic growth was explained by stand density. Other factors such as site, age, genotype, and variation in growing season precipitation also greatly influenced growth rate. Error in measurement was also recognized as a factor. Eversole (1955) also reported indications that the variation in total-stem cubic volume growth of Douglas-fir may not be associated with spacing. In a 27-year-old plantation the variation was apparently associated with slight differences in site quality.

As with ponderosa pine, total volume growth in red pine stands has not been found to differ appreciably with changes in stand density (Engle and Smith, 1950). However, it must be remembered that in less dense stands this volume is being put on fewer and larger trees.

Conflicting conclusions have been reached concerning the effect of stand density on total cubic-foot volume per acre in red pine stands. Bramble, Cope, and Chisman (1949) reported greater volume in wide spacings in red pine plantations, but plantations analyzed by Ralston (1954) contained about the same total cubic-foot volume per acre, regardless of spacing.

Nelson, et al. (1961) reported on a study of merchantable cubic-foot volume growth in natural loblolly pine stands. They concluded that cubic-foot volume growth was significantly related to site index and residual density after thinning. For unthinned stands cubic-foot volume was also significantly related to age as well as site index and density. However, none of their optimum density equations accounted for more than 40 percent of variation in growth. The authors suggested some factors which perhaps account for the unexplained error. These were mortality, diameter distribution, less than full use of growing space for a short period following the heavier thinnings, genetic variation, clustering, climatic differences, and measurement error. Wenger, et al. (1958) reporting on the same study, found that in both thinned and unthinned stands, the relation of board-foot volume growth to density varied with age as well as with site index. There were also indications that optimum density was lower on poor than on good sites.

Gruschow and Evans (1959) reported similar results from a study of young slash pine stands. Regression analyses disclosed that periodic annual cubic volume growth of young slash pine was significantly associated with age and residual density of the stand and the site upon which it grew. They indicated that maximum measurable growth per acre was realized at something less than full stocking, particularly on the lower sites. Gruschow and Evans suggested that a somewhat lower stand density than that producing maximum cubic-growth per acre would probably favor maximum value production by reducing the time required to produce trees and products of larger size.

Bassett (1966) reported on periodic cubic growth in natural loblolly pine stands in Arkansas. He found that maximum volume was produced at densities of about 115 square feet of basal area per acre, which is about 70 percent of full stocking. However, he too substantiated the findings of other workers, that production varies little with changes in density. Plots with 90 to 130 square feet of basal area, 55 to 80 percent stocked, produced within 10 cubic feet of the maximum. Site and density individually influenced growth in Bassett's study, but, in contrast with other findings, their interaction was negligible.

Studies by Dell and Collicott (1968) and by Simmons and Schnur (1937) have also shown that density significantly influenced net annual merchantable cubic-foot growth in the southern pines.

Almost every yield-density study shows that merchantable volume increases as spacing increases, but as pointed out by Reukema (1966), the day is probably coming when merchantable volume and total volume will be synonymous. When, or if, this day comes, emphasis on larger individual trees will not be as great, and the forest manager will be striving to obtain the highest possible production of wood material per acre; regardless of size.

This section on the effects of stand density on forest growth will be concluded with a review of three basic theories concerning this relationship.

Moller's production theory, as cited by Reukema (1966), was that production increases with increased stocking up to the point where full occupancy of the site is achieved. Beyond this point, increased density does not affect the amount of growth, but only its distribution; on a small number of relatively large trees at low densities and a large number of smaller trees at high densities. He maintained that only at extremely high densities where crowding becomes a limiting factor would production fall off. This hypothesis was derived from a theoretical consideration of the relationship between photosynthesis and respiration in forest stands. Wegge (1966) suggested that this theory will be most applicable to older stands of tolerant species on poorer sites, and will also include most stands of medium age when site quality is not too high.

Assmann, as cited by Reukema (1966), proposed that growth per unit area increases with increased stocking until optimum production is reached at some definable density. Beyond this point, production decreases. Assmann believed that optimum production occurs within a very narrow range of densities and that only on exceptionally good sites would the curves have a broad top, indicating roughly equivalent production across a wide range of densities. Wegge (1966) believed that Assmann's theory should apply to light-demanding species for all combinations of site quality and stand age; to old stands of tolerant species on good sites and to young ones on the poorest sites.

Reukema (1966) brought up a third viewpoint, which used to be generally accepted and which has had some recent revival. This hypothesis is that growth increases continuously with increasing density, at least to some very high density level. Reukema pointed out that some studies and observations do not seem to support any of the three.

Stocking guides

Judicious control of tree density throughout the life of a forest stand provides one of the primary avenues for optimizing returns from growing stock (Dahms, 1967). Stand management concepts have taken on many forms but undoubtedly the most commonly used basis for management decisions is the yield table. Roth (1915) defined a yield table as a "table of growth" from which may be computed current growth, average growth, etc., of any stand.

Normal yield tables

Some stand measures, such as stocking, are relative in that they can be determined only with reference to a standard. Yield tables provide such a standard (Meyer, 1953). Yield tables which are based on the measurement of fully stocked or "normally" stocked stands are termed "normal yield tables." A normal yield table, as defined in the Forestry Handbook (Forbes, 1956), is a tabulation of the volume, basal area, number of trees, etc., per acre found in full stands on specified sites at specified ages. Most normal yield tables have been prepared to apply strictly to even-aged, pure stands. The concept of a "normally" stocked stand is highly subjective, since such a stand is difficult to define, and can be chosen only subjectively. However, the normal stand table does serve usefulness as a standard to which an actual forest may be compared since it is based on a number of actual, though selected, stands. It is the result actually obtained with a given species and site in a given time (Roth, 1915).

The first difficulty that comes up when a forest manager wishes to apply normal yield tables to a particular piece of land for the purpose of predicting the volume at some future period is that the actual, or "empirical", stand is not in normal condition throughout, and therefore normal yield tables can not be used without a reduction of their values (Meyer, 1930). Meyer suggested that if the normal yield table concept is adopted such questions as the following must be answered:

> What is the relation between the actual and the normal forest?
> Does this relation change during the life of the stand?
> What causes and what constitutes understocking or overstocking?
> How is degree of stocking recognized?
> What methods of procedure are best when normal tables are to be used to make yield predictions of a forest?

Chapman (1931) explained that the density percent or degree of normality of stocking for each stand can be determined separately by comparing either its cubic volume, its basal area, or its volume in board feet on the average acre with that given in the yield table. He believed that normal yield tables are indispensable as a preliminary step in the proper determination of the factors which produce the actual stands.

It is the variation in the concept of normality or full stocking which renders the use of such tables, especially in the unmanaged forest, of uncertain value. Knuchel (1953) described normality as that practically attainable degree of perfection in a forest which we strive to secure in all parts of the forest and to maintain in perpetuity.

He added that indispensable as the normal forest concept may be, it would be wrong to pursue a normal structure of the forest drawn up on paper which did not harmonize with the locality conditions.

Mulloy (1944) cited a letter from Dr. L. H. Reineke which expressed another viewpoint toward normality. Dr. Reineke stated that normal yield does not express the productive capacity of the soil, climate and species; it merely shows the accumulated net difference between growth and spoilage in storage.

Normal yield tables have long been important in providing an estimate of growth and future yield, however, they have strict limitations in use. For example, they are ordinarily constructed to provide estimates of net growth and yield of pure, even-aged, fully-stocked stands. A sound estimate may be obtained under conditions where age and site class may be accurately measured, and where the estimate is applied to stands similar to those from which the basic tables were constructed (Spurr, 1952).

Spurr listed other reasons why normal yield tables have not proved overly satisfactory in American practice. He maintained that none of the independent variables can be accurately and quickly evaluated. He stated that few stands are precisely even-aged, and that site is rather an indefinite conception to which specific values cannot readily be assigned. And, as already indicated, normality of stocking is something which cannot be precisely defined or recognized in the field.

Although Spurr (1952) indicated a very limited use of normal yield tables in inventory work, he advocated their use in management and forest planning.

He believed that they provide a unified picture of stand structure and development of various species on various sites and proposed that yield table comparisons are very useful in studying the effect of site upon growth and the possibility of growing various species on a specific site.

Most American yield tables are based upon a series of fully-stocked stands of various ages on the same site and are assumed to represent various stages in a single growth curve. Spurr (1952) argued that unless based upon permanent sample plot data, yield tables cannot be taken to represent accurately the growth of fully stocked stands, since most fully-stocked mature stands have probably been overstocked or understocked at some time in their development. Davis (1966) and Mulloy (1943) also concured that there is considerable indication that stands considered to be fully stocked at the time they happen to be measured for normal yield study purposes tended to be understocked previously.

It is generally agreed that non-normal stands tend, over time, to approach normality (Bruce and Schumacher, 1950; Spurr, 1952; Watt, 1950; Husch, 1963; Duerr, 1938; Gevorkiantz, 1937; Meyer, 1933; Chaiken, 1939). However, the assumption is often made that the normality will not change during the prediction period, particularly if only short periods are involved. In an early addition of their book, Bruce and Schumacher (1935) maintained that since the change in normality is seldom great it can be ignored, merely supplying a slight factor of conservatism to the predictions. In the latest edition of their book, however, (1950) they discussed correction of growth predictions to allow for changes in normality. Briegleb (1942) reported that failure to allow for change in stocking when applying yield tables may results in growth estimates "grossly in error."

Chaiken (1939) explained that in understocked stands competition among trees is less severe, resulting in greater individual tree growth, a smaller loss due to mortality, and a trend toward more complete utilization of the site. Conversely, the effect of the more severe competition in overstocked stands results in decreased growth of individual trees and increased mortality until the stand comes into equilibrium with the site and normality is attained.

The only bases for reliable predictions of changes in normality are repeated observations of permanent sample plots (Husch, 1963; Spurr, 1952). Much information has been obtained on the trend toward normality from permanent sample plots in Douglas-fir stands in Oregon and Washington (Briegleb, 1942; Meyer, 1933). Watt (1950) also investigated this trend for western white pine and Chaiken (1939) for loblolly pine.

Normality trends may be approximated by Gerhardt's formula when empirical information is not available (Gevorkiantz, 1934, 1937, 1940; Duerr, 1938). Although purely an approximation, the formula seems to give reasonable results (Spurr, 1952). Gerhardt's formula in its general form is: $g = dG(1 + K - Kd)$, where g is the growth of an understocked stand, G is the normal yield table growth, d is the density of the understocked stand related to the normal, and K is a constant for any given timber type (Duerr, 1938).

Evidence is accumulating to make obsolete the idea that maximum volume growth is obtained with normal stocking (Bickford, et al. 1957). Instead, heavy thinnings provide earlier returns, thus reducing final cut, but not at the expense of total yield. Bickford, et al. maintained that equal volume production can be obtained from a much lower growing stock than had been considered necessary, and that the growth possibilities for a given site can be placed on the best elements of the stand more easily.

Empirical yield tables

An empirical yield table contains information which has been obtained from average stand conditions instead of from selected stands chosen as being fully stocked. Theoretically, these tables can be applied to actual stands without any reduction factors, but as pointed out by Bruce and Schumacher (1935) the degree of stocking of actual stands is tremendously varied. Therefore, unless the empirical yield table has been constructed specifically for the stand in question, it is unlikely that direct application of the values would be possible without some adjustment.

Although empirical yield tables are often considered to be an improvement over normal yield tables, Myers (1966) suggested that this may not be the case. The only apparent advantage is that proposed by Bruce and Schumacher (1935); the correction factor will be smaller than that necessary with normal tables and should be less subject to error. Part of the problem with both normal and empirical yield tables is that neither kind employs a specific measure of density (Mulloy, 1944).

Variable-density yield tables

The primary difference between variable-density tables and normal yield tables is the addition of a third independent variable, density, to age and site, upon which the normal yield table is based.

Spurr (1952) listed certain advantages to using tables including density as a variable. First, there is no need for restricting sample selection to fully stocked stands since any good sample-plot data may be utilized in the solution. Second, the concept of relative stocking or normality can be eliminated. Actual measures of density are used as an independent variable. Third, the resulting solution is derived from and is directly applicable to understocked stands as well as to fully stocked stands.

Duerr and Gevorkiantz (1938) constructed a set of tables of this type for predicting growth in uneven-aged timber using the four independent variables: site, main stand age, density, and the proportion of small trees just ready to enter merchantability. The last factor was reflected in the form of a merchantability index which was the ratio of volume to basal area.

Another example of this type of yield table is the one developed by MacKinney, Schumacher, and Chaiken (1937) for loblolly pine stands. Their work was based upon the assumption that the ultimate maximum yield of a stand of the loblolly pine type is a function of stocking, site, and composition. The equation from which the yield table was constructed is: $k = D/100 \cdot C/100\ (105.7S - 2,045)$, where k is the maximum volume yield; S is site index, in feet; D is density index, as a percentage of the number of trees expected in fully stocked stands; and C is composition index, expressed as the percentage which the basal area of loblolly pine is of the total stand basal area.

In addition to presentation of yield data in tabular form, the authors suggested that the equation can be solved for given sets of variables or, as an alternative graphic solution they suggested the construction of an alinement chart of the equation.

Examples of still another type of variable-density yield table are those prepared by Myers for managed stands of lodgepole pine

(1967) and for ponderosa pine (1966). His approach in developing these tables was to observe thinned and measured plots for one or more cutting cycles. According to Myers, (1966) "A limited series of plots will provide data for experience yield tables that describe response to certain management procedures. Data from a larger series that samples across the ranges of important stand variables can be analyzed by multiple regression methods and summarized in variable-density yield tables."

The resulting estimates of the growth and yield of managed stands can serve as guides for timber management decisions. For example, Myers (1967) suggested that yields can be investigated to determine if management for timber production is a desirable goal for specific areas. He also proposed the use of the tables for estimating the affects of various management alternatives on yields. Myers' lodgepole pine tables, for example, show estimated yields by age and site index classes for both 10- and 30-year cutting cycles. The manager can select the yield table that best describes the objectives of management, and this then becomes the standard for the forest.

Thinning and spacing studies

The results of thinning and spacing studies often provide the basis for management studies. Growth characteristics as related to stocking level for a particular species in a given locality can be quite helpful for local management use.

For example, Pearson (1950) found from a study of ponderosa pine in the Southwest that a higher total growth rate resulted in stands made up of small rather than large trees. He concluded that a given volume of growing stock divided among several small trees is more effective than if concentrated in a single large tree on the same plot of land. This conclusion was based on the hypothesis that several well-distributed small trees have better access to soil moisture and their roots are able to more completely permeate the soil of a given area than those radiating from a single large tree.

The improvement of stand development in Black Hills ponderosa pine by thinning was reported by Myers (1958). Stimulation of both diameter and height growth by thinning seemed to be related to age. This type of information is important in deciding when to apply cultural treatments.

Nelson (1963) reported on a multiple-regression solution for characterizing basal area growth in pure, even-aged stands of loblolly pine as a result of thinning studies. Thinned and unthinned plots were remeasured after 5 and 10 growing seasons in stands representing a wide range of stand indices and densities. According to Nelson (1967) such studies are proving to be one of the best approaches to prescriptions for stand management.

An earlier account of the same study reported culmination of the growth curves, although relatively flat curve forms resulted. These curves indicated that a wide range of stocking can be maintained while still producing 90 percent of optimum growth (Nelson, et al. 1961). They suggested that the flat curve form over a wide range of stocking gives the forest manager an additional tool in forest regulations; it provides him with a sound basis for storing volume on the stump and reduces disparities in operations occasioned by uneven distribution of age classes. The wide range of acceptable stocking levels also gives the manager a choice of a wider range in alternative rates of return.

Similar results were reported by Bassett (1966). He found that maximum volume was produced in loblolly pine stands at densities of about 115 square feet of basal area per acre, which is about 70 percent of full stocking. However, he also found a range of stocking which produced similar growth. Plots with 90 to 130 square feet of basal are -- 55 to 80 percent stocked -- produced within 10 cubic feet of the maximum.

Many other thinning studies such as the ones by Alexander (1965), Barrett (1961), Roe and Stoeckeler (1950), Hawley (1936), U.S.D.A. (1949), and Mowat (1949), show the growth response to different intensities of thinning and at different ages, for various species on various sites.

Spacing studies yield similar data. A study of the effects of spacing interval on jack pine development in Michigan is a good example (Ralston, 1953). Stands planted at spacings of 4 x 4, 6 x 6, and 8 x 8 feet were analyzed after 25 years with the conclusion that growth was increased with spacing interval, but the widest spacing resulted in limby trees which did not fully occupy the site.

Therefore, the recommendation was that an initial spacing of 6 x 6 feet will produce the best compromise situation. This spacing will yield sufficient merchantable products for a commercial thinning when thinning is needed silviculturally.

Findings such as these provide guidelines that the forest manager can use to produce the desired production results.

Rule-of-thumb guides

Gevorkiantz (1947) wrote that the growing space per tree controls stand development. Determination of just how much growing space is occupied by the roots or crowns of individual trees is difficult. However, it is generally thought that the actual growing space required by a tree is proportional to the size of the tree. This is the basis for several rule-of-thumb guides to stocking and spacing. Spacing of itself does not measure stand density, but the corresponding number of trees or basal area does (Bickford, *et al.* (1957).

Matthews (1935) suggested a *spacing figure* (D/d) as a field aid in judging stocking for forest stands. His spacing figure can be determined from the equation $D/d = 185/\sqrt{BA}$, where D is the average distance between stems, in feet, d is the average diameter of the stand in the same unit of measurement as D, and BA is basal area per acre, in square feet. A modification of Matthews' spacing figure is recommended in the Forestry Handbook (Forbes, 1956) in which d is expressed in inches, the unit of measure commonly employed. For a given stand the Spacing factor, as it is called, is 1/12 of the spacing figure value. The formula for spacing factor is $D/d = 15.4/\sqrt{BA}$. Spacing is conveniently expressed as a ratio of diameter for use as a field thinning guide. It can be used to determine the average spacing between trees when it is desired to leave some specified total basal area per acre (Davis, 1966). Davis showed, for example, that to get 80 square feet of basal area per acre, the "diameter times" spacing figure is 1.7, which holds regardless of tree diameter. For 6-inch trees, the indicated spacing is 6 x 1.7, or 10.2 feet.

Davis (1966) also explained another kind of spacing figure based on diameter. This is the "diameter plus" type. This spacing figure is easier to apply and works satisfactorily if the range of tree diameters to which it is applied is not great and an appropriate constant is used. However, Bickford, et al. (1957) reported that DBH plus a constant results in too much basal area with large trees and too little with small trees. Hawley and Smith (1954) suggested that more satisfactory results may be obtained by increasing the value of the constant with increased age.

Although these rules-of-thumb are popular and serve a purpose, it should be remembered that they are at best only rough field guides.

Wilson (1946) argued that diameter is the result rather than the cause of spacing and therefore is not a very good basis for stocking determinations in a thinned or otherwise disturbed stand. But, he maintained that height is negligibly affected by spacing except at the extremes of density. He proposed that the ideal number of trees per unit area should be a constant function of the square of the stand height, regardless of site quality. For uniformly stocked, even-aged stands of any species, Wilson's formula is $n = 43,560/(hf)^2$, where n is the number of trees per acre, h is the height of the stand, and f is a certain fraction of height appropriate for the species (Spurr, 1952).

Bickford, et al. (1957) pointed out that the line for any value of f has a 2:1 slope on log-log paper. They suggested that a series of such lines provides a grid on which the course of experimental plots or yield tables can be plotted and evaluated. This is true because height has the virtue of combining the components of age and site in one measurement (Wilson, 1946).

Gevorkiantz (1947a) reported soon after Wilson's proposal was published, that a formula which had been used in the Dutch East Indies, after conversion of units of measure, is the same as that used by Wilson. Gevorkiantz (1947b) later discussed the use of height as the basis of an expression of density in a report on growth and yield of jack pine. He reported that in well-stocked stands, the average spacing of trees bears a definite relation to the average height of the stand. He found that in normal jack pine stands for example, average spacing is about 20 percent of the average height of the dominant and codominant trees.

This is the same spacing that Wilson (1946) suggested in his original article. He proposed that the average spacing for tolerant species like spruce and fir be one-sixth of the height, and for intolerant trees like red and jack pine, about one-fourth of the height. Hawley and Smith (1954) cautioned that since growth in height does not vary greatly with stand density, uncritical application of a rule of this sort might lead to excessively heavy thinnings in stands of greater than average density. They also added that such rules are essentially specifications of the number of trees to be left at any given stage of development and are, therefore, of questionable value in stands lacking uniformity.

Spacing between trees expressed as a percentage of the height of the trees was proposed by Hart in 1928, so the concept is not new (Hummell, 1953).

Chisman and Schumacher (1940) developed a method of allocating tree area according to DBH of individual trees by means of a quadratic equation fitted by the method of least squares to sample plot data. This measure, called the <u>tree-area ratio</u>, determines the density of stocking by the sum total of the growing spaces of the individual trees. The percent space utilized on a per-acre basis can be expressed by the general formula: tree-area ratio $= aN + b\Sigma D + c\Sigma D^2$; where N is the number of trees per acre, ΣD is the sum of the individual diameters, and ΣD^2 is the sum of the squares of the diameters (Spurr, 1952). Spurr suggested that the chief advantage of the tree-area ratio is that it provides a measure of density readily usable for uneven-aged and for open stands.

In a problem involving the calculation of the tree-area ratio for loblolly pine, Chisman and Schumacher (1940) found that there was no perceptible effect of age or site index upon the ratio obtained.

Lynch (1958) explained that the calculated equation expresses density of stocking as a proportion of the average density of stocking represented by the aggregate plot data. He proposed that this method is especially applicable to a group of plot data.

Although this method is generally accepted as a valid measure of density, there are certain objections to its use. Spurr (1952) pointed out that since the relationship must be determined empirically from plot data the character of the plots used will affect the constants in the formula. The measure would therefore not necessarily be a valid comparison when applied to samples differing widely in density or composition from the original data. Spurr also argued that although the relationship between growing space and DBH undoubtedly exists, it is often modified by so many factors that the degree of correlation may be low. Lastly, he suggested that there is no proof that a second-degree parabola is the form of equation best suited for expressing the relationship between growing space and diameter.

Another density measure which is apparently not highly correlated with site index or age is Reineke's stand density index (Reineke, 1933). This index for even-aged stands, is number of trees per acre as a percent of the number representing full stocking for the same average diameter in unmanaged stands (Bickford, et al. 1957).

The SDI is based on the fact that in even-aged natural stands, the number of trees in different diameter classes is distributed in a fairly definite frequency pattern. This pattern often approaches that of the normal curve or symmetrical bell-shaped distribution. The form of the curve varies somewhat by species but for a given species is fairly consistent and, as indicated above, is characteristic regardless of age or site (Davis, 1966). This allows approximate discription of the distribution pattern by the average diameter alone. Reineke (1933) therefore, maintained that of a group of stands of the same average diameter, the stand with the greatest number of trees per unit area is the most completely stocked.

Reineke's index utilizes a formula in which the log of the number of trees in a fully stocked stand is equated to the log of the diameter of the tree of average basal area: $\log N = -1.605 \log D + K$.

Meyer (1942) proposed that stand density index has a noteworthy application in that it can be used to give a quick approximate method of determining stocking without a determination of site index and age of the stand. According to Hummel (1953), SDI has the disadvantage in thinning research that diameter increment, and hence diameter, are influenced by thinning treatment, and it is undesirable to use, as a criterion for the definition of a treatment, a character which is itself influenced by the treatment.

Spurr (1952) found that SDI values do, in fact, vary with age and possibly with site, but not to the extent that basal area does. Bickford, et al. (1957) suggested that the procedure is not really easy to use and may be inconsistent.

After discovering that the conventional measures of density did not provide a consistent measure of the growing stock in Danish and Prussian stands of Douglas-fir, Briegleb (1952) developed a measure incorporating both diameter and height. Based upon evidence that the space a young Douglas-fir tree can utilize is related to both of these parameters, he proposed that new measures based on average diameter and average height be tested along with others that have promise.

Other stocking guides

An entirely different approach toward the establishment of stocking guides was developed for use in upland central hardwoods (Gingrich, 1964; Gingrich and Roach, 1962). This method utilizes a range of stand stocking within which the growing space is fully occupied and the growth level per unit of area is acceptably high. Most of the other stocking guides in use are based upon some maximum or optimum density level which appears to provide the maximum amount of growth for the stand situation involved.

Gingrich found the range of stand stocking for full site occupancy to be essentially the same as for maximum growth. The results of this hardwood stocking study were presented graphically; the upper and lower limits of the full stocking range being represented by parallel curves. The area between the curves represented the stand densities that should provide the maximum yield. The upper or 100 percent stocking line, represented the stocking of fully stocked stands as determined from permanent growth plots and stand tables from normal yield tables. The lower line, which ranged from 55 to 58 percent of the upper stocking level, was determined from a study of open-grown trees; it represented the minimum level of stocking for full site occupancy.

Total growth per acre will be about equal for stands of similar site and species composition falling anywhere within the area between the two stocking lines, but diameter growth of individual trees will vary greatly within this density range. The fastest individual tree growth should occur in stands near the minimum density required for full site utilization. The authors indicated that maintaining density less than that represented by the lower stocking curve will not result in greater diameter growth since at this level each tree already has all the growing space it can utilize. In stands with densities greater than 100 percent stocking (above the upper line) natural mortality is high, individual tree growth is slow, and stand density gradually comes down to the 100 percent level.

Use of the stocking guides is based on two easily-obtainable stand parameters, number of trees per acre and basal area per acre. These two measures of density are the X and Y axes, respectively, for the graphical guides. Entering the graphs with these stand parameters reveals the relation of the actual growing stock to what is required to produce the desired growth, and forms the basis for the cutting prescriptions.

CHAPTER III

METHODS

Data collection

Natural lodgepole pine stands were sampled in southern Wyoming and Colorado.[1] Sample stands were selected so as to sample a wide range of stand ages, stand densities, diameters, and site qualities. Stands selected for study were even-aged, and between 30- and 150-years old. Stands which appeared to have been disturbed by cutting or fire within the previous 30 years were avoided, as were those lacking uniformity of site characteristics such as slope, soils, and aspect. Care was also taken not to take samples from stands heavily infested with diseases or insects because of their affect on growth patterns.

The circular sample plots were as large as necessary to include approximately 75 trees per plot. Appropriate factors were calculated for converting the plot data to a per-acre basis for analysis.

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[1]See Appendix B for plot locations.

Measurements of diameter, height, radial growth, age, bark thickness, and mortality were made on a total of 60 plots.[2] Compilation of the data showed that seven of the plots did not meet the age requirements as originally defined, so these plots were not used in the final computations. Average plot values were used in subsequent calculations. The data were converted to "past" values, or values 30 years prior to measurement, since tree and stand conditions as they existed at the beginning of the growth period were assumed to be important considerations in determining tree growth.

The upper stocking limit

The primary objective of this study was to develop a method for defining the optimum range of stocking for lodgepole pine stands in Colorado and southern Wyoming. The procedure for defining the upper limit of this stocking range will be explained first.

The approach for determining the upper stocking limit was based upon basal area growth. Volume growth was considered as a possible and logical basis for study, but was discarded in favor of basal area. Since volume growth is so closely correlated with

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[2] Detailed information on field measurements and compilation of data are included in Appendix B.

basal area growth (Hawley and Smith, 1954; Spurr, 1952), there appeared to be little advantage in using the more complex measure.

Crown Competition Factor was chosen as the measure of density to use for computational purposes since regression analysis indicated closer correlation between basal area growth and CCF than with either basal area per acre or number of trees per acre. The stocking limits as defined in this study are presented on a number of trees per acre basis in addition to the limits based upon the Crown Competition Factor.

Frequently-used variable names will be abbreviated after they have been introduced. Variable name abbreviations that include an "I" as the last letter refer to initial values, i.e., values at the beginning of the 30-year growth study period. For example, CCF refers to "present" Crown Competition Factor, while CCFI is the Crown Competition Factor 30 years prior to measurement.

As noted, a 30-year growth period was used in this study. It was felt that a shorter period would not satisfactorily indicate growth trends in slow-growing natural lodgepole pine stands. "Present" measurements made on the sample trees, were converted to "past" values by the methods described in Appendix B. One-inch diameter classes from one inch to six inches were used.[3] Tree values were converted to a "per-acre" basis. A total of 53 values of each variable

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[3]The one-inch diameter class includes 1.0 inch to 1.9 inches.

were used in the final computations, representing measurements made on approximately 5,000 trees.

The step-by-step procedures used in construction of the upper stocking limit are as follows:

Step 1.--The basal area growth rate per tree as a function of stand density was the fundamental relationship upon which the procedure was built. Before the correlation between growth rate and density was computed, it was first necessary to determine if annual basal area growth per tree was significantly related to site quality.

The plots were classified by decadal Site Index classes from 30 to 90. An analysis of variance was then used to test the hypothesis that there was no difference in basal area growth per tree due to site quality. As is shown in Table 1, the F value did not indicate that the hypothesis should be rejected at the five percent significance level, therefore the plot data was treated independent of Site Index.

TABLE 1. Analysis of variance of basal area growth rate per tree.

Source	D.F.	S.S.	M.S.	F
Total	52	.00007742		
Site	6	.00000570	.00000095	
Residual	46	.00007172	.00000156	0.6 N.S.[4]

[4] In presenting analysis of variance tables, a double asterisk signifies significance at the one percent level, a single asterisk signifies significance at the five percent level and N.S. denotes no significance.

Step 2.--The regression of annual basal area growth per tree (BAG/T) on initial Crown Competition Factor (CCFI) was computed using values from all 53 plots. The resulting equation was used to obtain an estimated basal area growth per tree ($B\widehat{A}G/T$) value for each plot.[5] This relationship is shown in Figure 1. The linear equation was: $1/(BAG/T) = -649 + 7.6016(CCFI)$. The regression had a correlation coefficient, r, of .785, and the slope of the line was statistically significant at the one percent level.[6]

Step 3.--The 53 plots were next grouped by one-inch diameter classes based upon the mean plot diameter. A linear equation was calculated by least squares methods for each diameter class showing the relationship between initial Crown Competition Factor and estimated basal area growth per tree. The regression equations are presented in Table 2.

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[5] Original and estimated growth values are listed in Appendix B.

[6] Tests of significance and other computations are included in Appendix C.

Figure 1. --The regression of basal area growth per tree on Crown Competition Factor for all plot data.

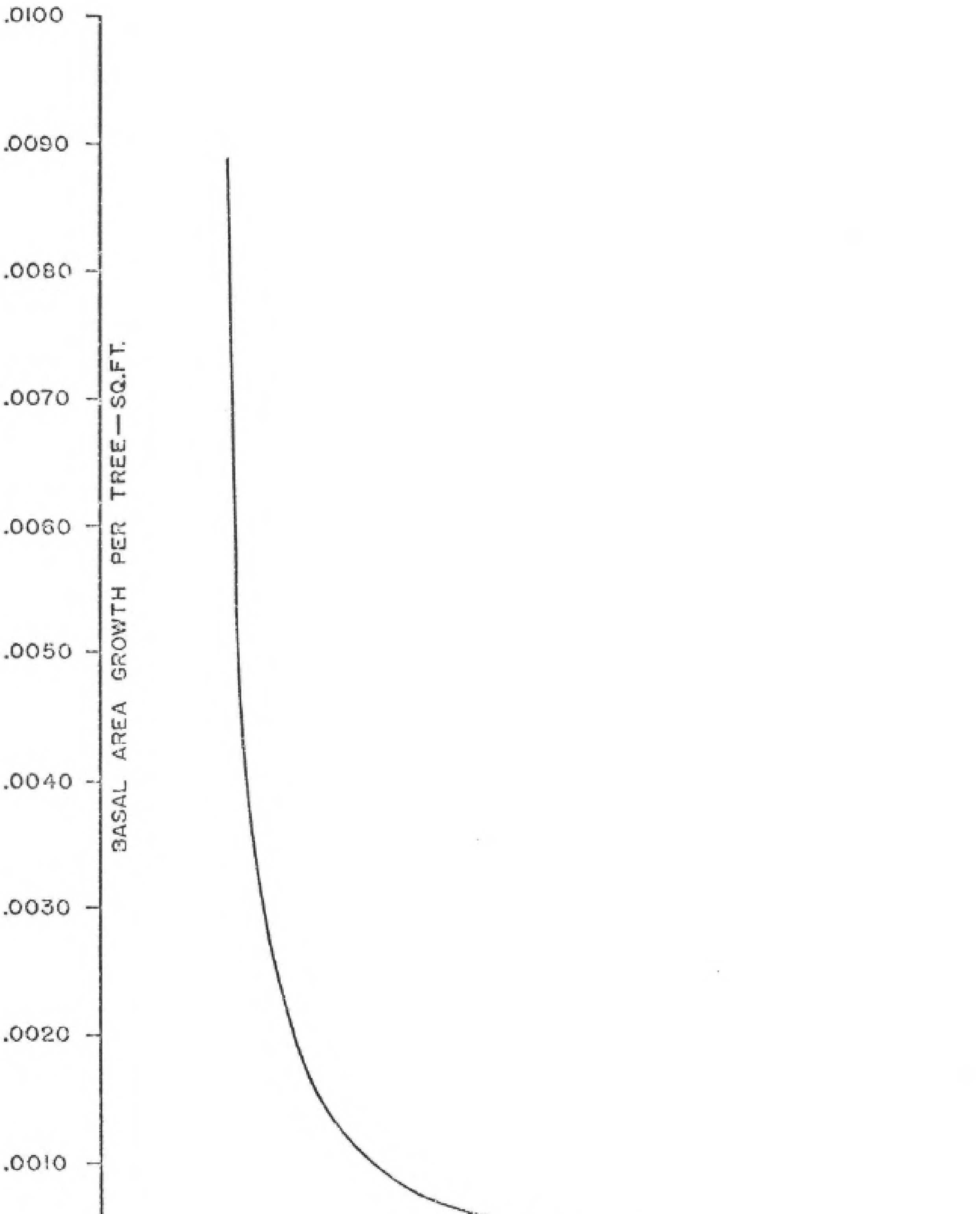

.0100
.0090
.0080
.0070
.0060
.0050
.0040
.0030
.0020
.0010
.0000
BASAL AREA GROWTH PER TREE — SQ.FT.
CROWN COMPETITION FACTOR
0
100
200
300
400
500
600
675

TABLE 2. Linear equations by diameter classes, showing the relationship between basal area growth per tree and initial crown competition factor.

DBH Class	Equation	r^2	r	Level of Signifi.
1	BAG/T = .00462 - .000009636(CCFI)	.5207	.722	5%
2	BAG/T = .00852 - .00002342(CCFI)	.6667	.816	1%
3	BAG/T = .00589 - .00002129(CCFI)	.6385	.799	1%
4	BAG/T = .0046 - .00001451(CCFI)	.8390	.916	1%
5	BAG/T = .0071 - .00003146(CCFI)	.8416	.917	>5%
6	BAG/T = .0076 - .00003366(CCFI)	.9527	.976	5%

Step 4.--Column references in the following explanation refer to Table 3 which is a tabulation of the computations used in obtaining the curve of basal area growth per acre over density (CCF) for the one-inch diameter class. The data for all diameter classes are included in Appendix C.

For each diameter class, the straight line equation as computed in Step 3 was solved for a series of CCF levels (column 1), obtaining estimated basal area growth per tree values (column 2).

Next the basal area per acre for each CCF level was obtained by solving the CCF equation for basal area. The CCF equation for lodgepole pine is: $CCF = 50.58 + (5.25BA/\overline{DBH})$; where BA is basal area per acre, and $\overline{DBH}$ is average stand diameter (Alexander, 1966). Inserting the average diameter for the class and the CCF level, the only unknown remaining is the basal area (column 3).

The number of trees per acre (column 4) corresponding to each basal area level was obtained by dividing the basal area per acre by the cross-sectional area of the tree of average diameter.

Basal area growth per acre (column 5) was then computed for each CCF level by multiplying the estimated basal area growth per tree (column 2) by the computed number of trees per acre (column 4).

Plotting the basal area growth per acre values from column 5 over density (column 1) yielded a curve which revealed the culmination of basal area growth for the diameter class. As can be seen on Figure 2 and in Table 3, the basal area growth per acre for the one-inch diameter class culminated at a CCF level of 265. The basal area per acre at this culmination point was 61 square feet.

TABLE 3. Tabulation of values used to obtain basal area growth per acre over Crown Competition Factor curve for the one inch diameter class.

CCF	BAG/T[7] (sq. ft.)	BA/A[8] (sq. ft.)	Trees/A[9]	BAG/A[10] (sq. ft.)
100	.00366	14.12	1177	4.30782
200	.00269	42.69	3558	9.57102
250	.00221	56.98	4748	10.49308
260	.00211	59.83	4986	10.52046
265	.00207	61.26	5105	10.56735 (peak of BAG/A)
270	.00202	62.69	5224	10.55248
275	.00197	64.12	5343	10.52571
300	.00173	71.26	5938	10.27274
350	.00125	85.55	7129	8.91125
400	.00077	99.83	8320	6.40640

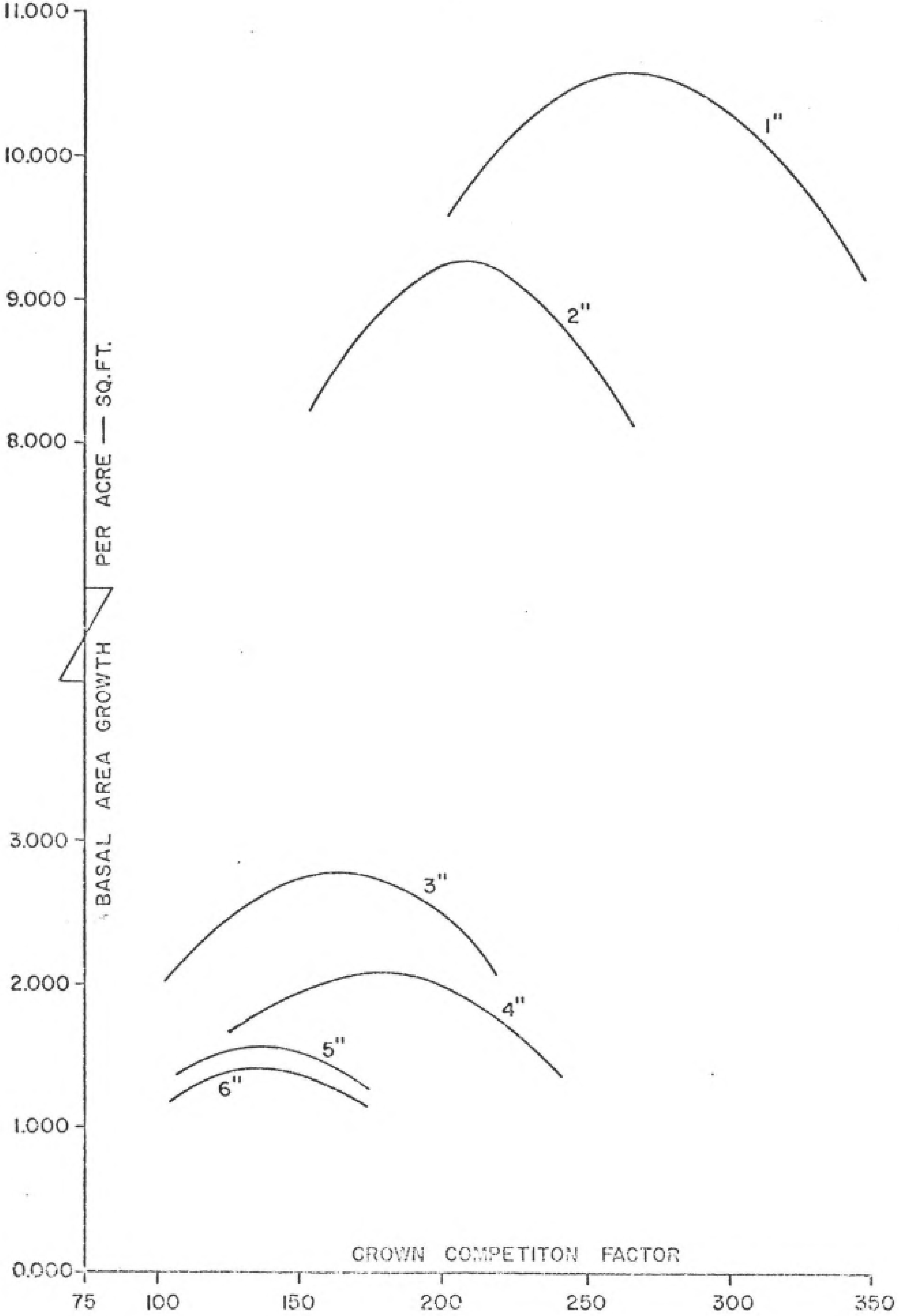
[7] Basal area growth per tree = .00462 - .000009636(CCFI).

[8] Obtained by solving the CCF equation for BA; inserting the average diameter value for the class. The mean diameter for the one-inch class was 1.5 inches. (CCF = 50.58 + (5.25BA/$D\overline{B}H$) Solving for BA at CCF = 100: 100 = 50.58 + (5.25BA)/1.5; BA = 14.12 sq. ft./acre.

[9] Number of trees per acre = (basal area/acre)/basal area of average tree. For CCF = 100: number of trees/acre = (14.12 sq. ft./acre)/(.012 sq. ft. /tree) = 1177 trees/acre.

[10] Basal area growth per acre = basal area growth per tree times the number of trees per acre. For CCF = 100: BAG/A = (.00366 sq. ft./tree) (1177 trees/acre) = 4.30782 sq. ft./ acre.

Figure 2. --Basal area growth per acre as a function of Crown Competition Factor; indicating the culmination of growth by diameter classes.


11.000
10.000
9.000
8.000
3.000
2.000
1.000
0.000
BASAL AREA GROWTH PER ACRE — SQ.FT.
1"
2"
3"
4"
5"
6"
CROWN COMPETITON FACTOR
75
100
150
200
250
300
350

The procedure as shown in Table 3 was followed for each diameter class. The resulting values at the peak of basal area growth for each class are presented in Table 4.

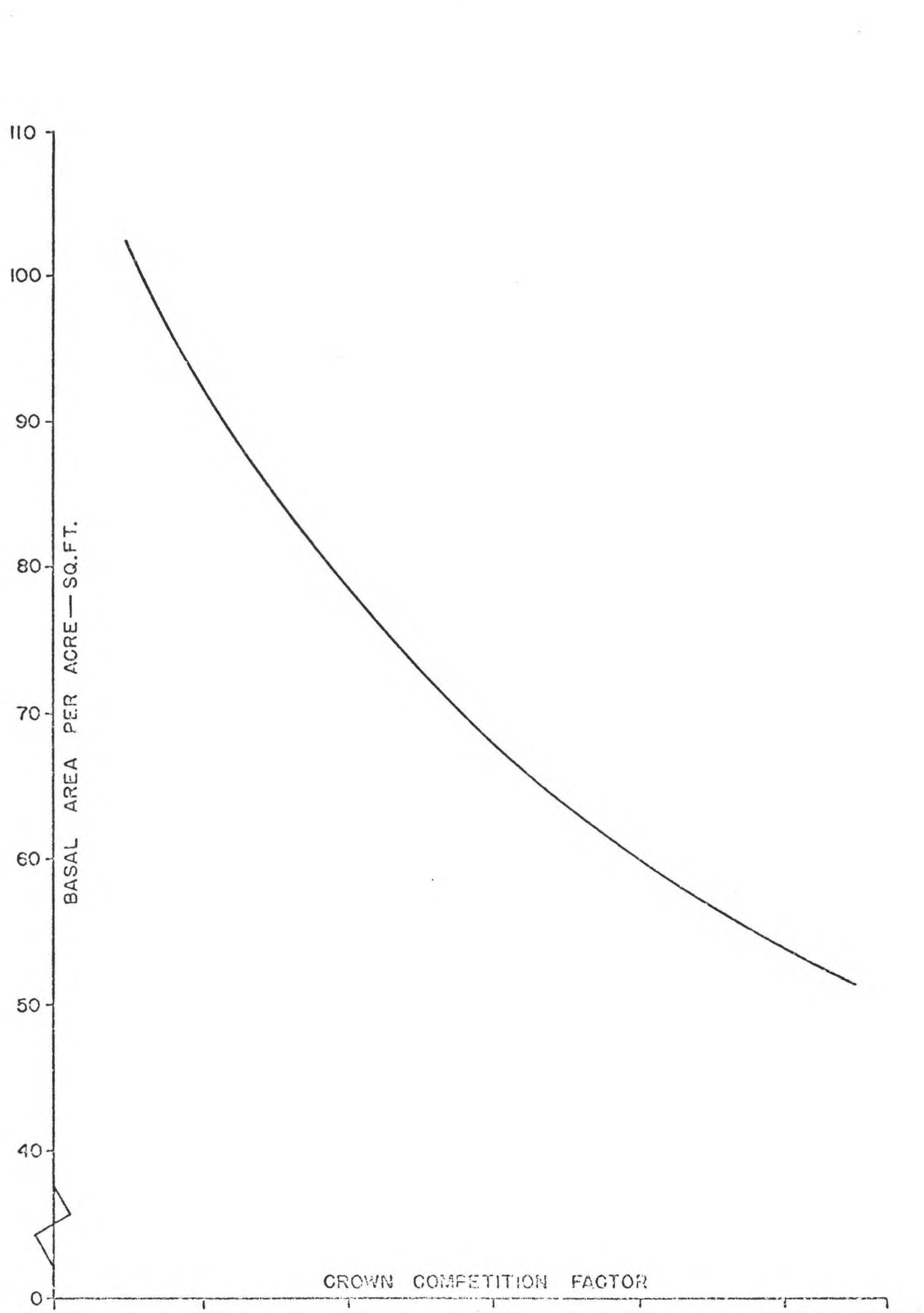
TABLE 4. Basal area and Crown Competition Factor values at the peak of basal area growth by diameter classes and number of plots in each diameter class.

Diameter Class	No. of Plots In Dia. Class	Basal Area Per Acre (sq. ft.)	Crown Competition Factor
1	10	61	265
2	13	65	205
3	10	74	165
4	12	102	175
5	4	87	135
6	4	103	135

Step 5.--Weighting the values by the number of plots in each diameter class, the regression of the peak values of basal area per acre on the corresponding Crown Competion Factors was computed. The resulting curve (Figure 3) represents the upper level of stocking using CCF as the measure of density. The slope of the equation, $1/BA/A = .00389 + .0000488(CCF)$, with a correlation coefficient of .776, was significant at the one percent level.

Step 6.--In order to identify particular points on the upper stocking curve, i.e., levels representing particular diameter

Figure 3.--Upper stocking curve using Crown Competition Factor as the measure of stand density. Regression of peak values of basal area per acre on the corresponding CCF values for DBH classes one through six inches.



BASAL AREA PER ACRE — SQ.FT.
CROWN COMPETITION FACTOR
0
40
50
60
70
80
90
100
110
100
140
180
220
260
300
330

values, a weighted regression of peak basal area per acre over average class diameter was computed. The values in Table 5 were used to compute this regression.

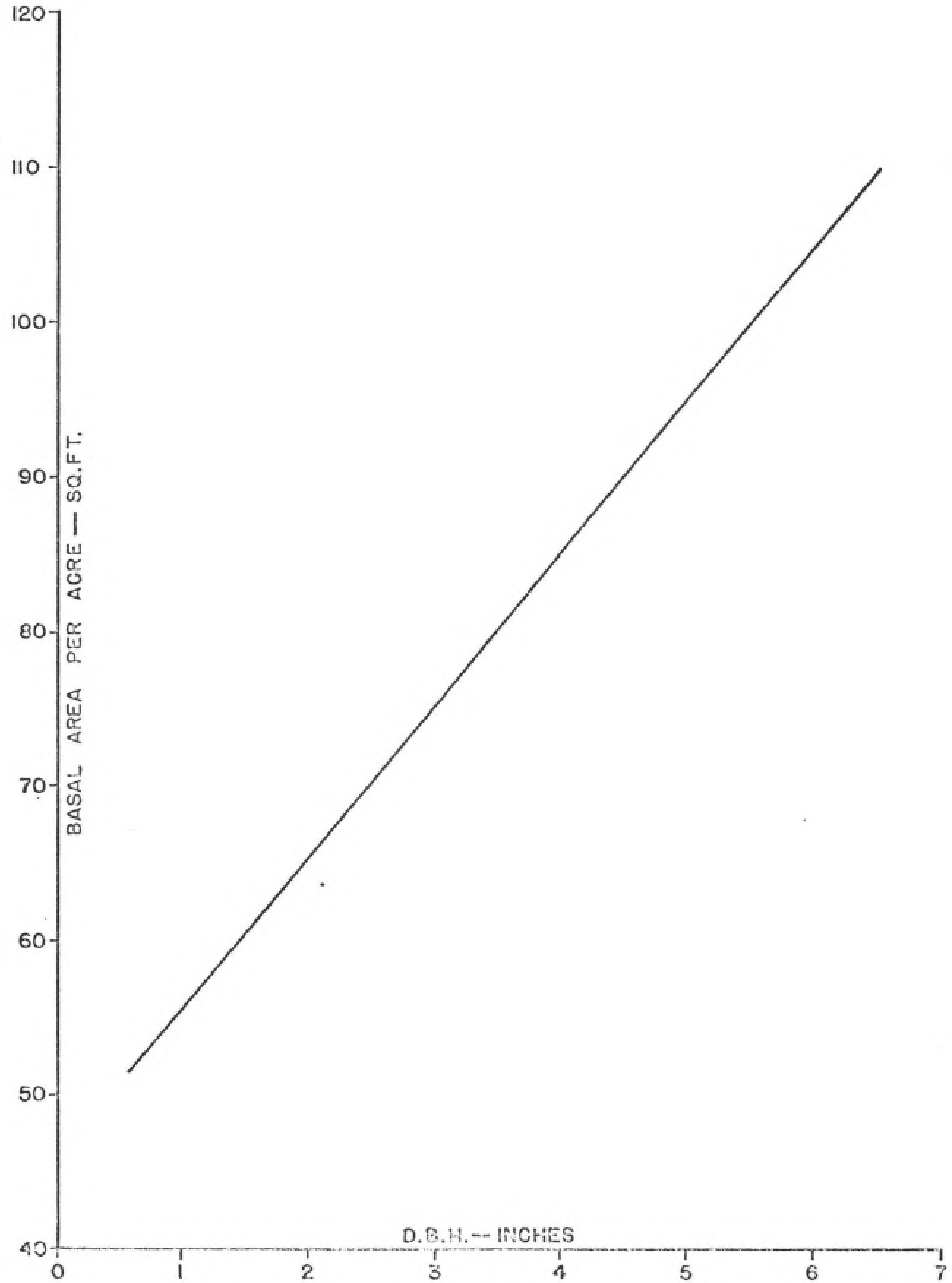
TABLE 5. Values used to determine the relationship between average stand diameter and basal area per acre.

Diameter Class	No. of Plots In Dia. Class	Basal Area Per Acre (sq. ft.)	Average Diameter Per Class (inches)
1	10	61	1.51
2	13	65	2.23
3	10	74	3.44
4	12	102	4.26
5	4	87	5.42
6	4	103	6.38

The regression equation resulting from the above values was: $BA/A = 45.4 + 9.9857(DBH)$; with an r value of .873. The F test indicated statistical significance at the one percent level. The basal area per acre-diameter relationship is shown graphically on Figure 4.

Step 7.--The basal area per acre values for the diameters 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, and 8.0 inches were computed using the equation as derived in Step 6 (column 2, Table 6). The basal area per acre value for each diameter was then divided by the cross-sectional area of a tree of the corresponding size (column 3, Table 6)

Figure 4. --Regression of basal area per acre on average tree diameter.


120
110
100
90
80
70
60
50
40
BASAL AREA PER ACRE — SQ.FT.
0
1
2
3
4
5
6
7
D.B.H.-- INCHES

to obtain the number of trees per acre represented by each basal area level (column 4, Table 6).

The basal area per acre and number of trees per acre values from the above computations were next plotted to form the upper stocking curve using number of trees per acre as the measure of density (Figure 5). This is the same relationship as that in Figure 3 except that a different measure of density has been used.

TABLE 6. Values used to construct upper stocking curve based upon number of trees per acre as the measure of stand density.

Diameter (inches)	Basal Area Per Acre[11] (sq. ft.)	Cross-sectional Area of Circle[12] (sq. ft.)	Number of Trees Per Acre[13]
1.0	55.4	.006	9233
2.0	65.4	.022	2973
3.0	75.4	.049	1539
4.0	85.3	.087	980
5.0	95.3	.136	701
6.0	105.3	.196	537
7.0	115.3	.267	432
8.0	125.3	.349	359

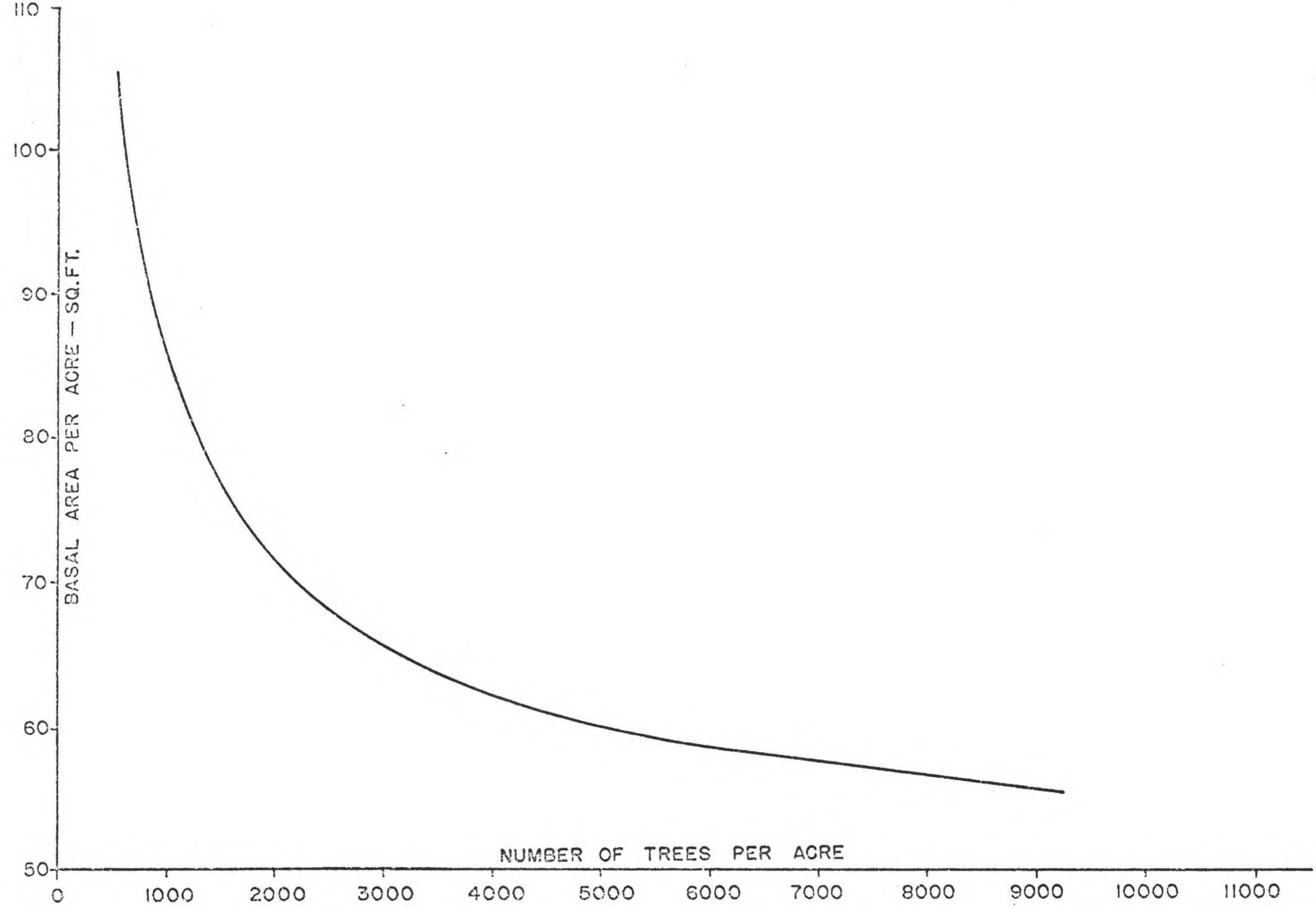
---

[11] BA/A = 45.4 + 9.9857(DBH).

[12] Cross-sectional area of a circle of the diameter in the first column.

[13] Basal area per acre divided by the basal area of one tree (values in column 2 divided by values in column 3).

Figure 5.--Upper stocking curve using number of trees per acre as the measure of stand density.


BASAL AREA PER ACRE — SQ.FT.
NUMBER OF TREES PER ACRE
110
100
90
80
70
60
50
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
11000

The lower stocking limit

A second objective of this study was to establish minimum stocking levels for full site occupancy. This part of the study consisted of an analysis of open-grown trees, following the reasoning of Gingrich (1964) that competition for growing space begins at a point where the available growing space in a stand is just equal to the total open-grown, tree-area requirements of all the trees in the stand. Other workers have also utilized this open-grown concept to establish stocking levels for desirable growth rates and to establish minimum stocking standards for full site occupancy (Smith and Ker, 1960; Vezina, 1962, 1963).

It is difficult to precisely establish the open-grown tree-area requirements of trees of various sizes and on various sites. The assumption was made that ground areas occupied by the trees are proportional to their crown areas. Good correlations between crown diameter and stem diameter have been reported from many sources (Smith and Ker, 1960; Vezina, 1962, 1963; Krajicek, et al., 1961; Minor, 1951).

The maximum ground area that a tree can occupy has, in previous studies of this type, been synonymous with the vertical extention of the maximum crown spread. There is reason to believe that trees growing on poor or dry sites may undergo root competition before crown competition takes place. Since data are not available to substantiate this hypothesis for lodgepole pine, an effort was made to comply with standard reasoning based upon crown areas alone.

Crown width and diameter breast height measurements were made on 265 open-grown trees.[14] A regression solution provided an equation showing the relationship between these two parameters (Figure 6). Crown width = 3.94 + 1.44(DBH); r = .951. The equation was solved for diameter values from 1.0 to 8.0 inches and the resulting crown width values (column 2, Table 7) were converted to crown areas (column 3) by using the formula for area of a circle: Crown area = $3.1416(\text{Crown width})^2/4$.

Dividing the area of one acre, 43,560 square feet, by the crown area of a tree of a given diameter provided the theoretical number of open-grown trees of this size that could fully occupy the site. This division was made for each diameter class from one through eight inches (column 4, Table 7).

The number of trees per acre from the last step multiplied by the breast high cross-sectional area of a tree of the corresponding size, equaled the basal area per acre represented by these trees (column 6, Table 7).

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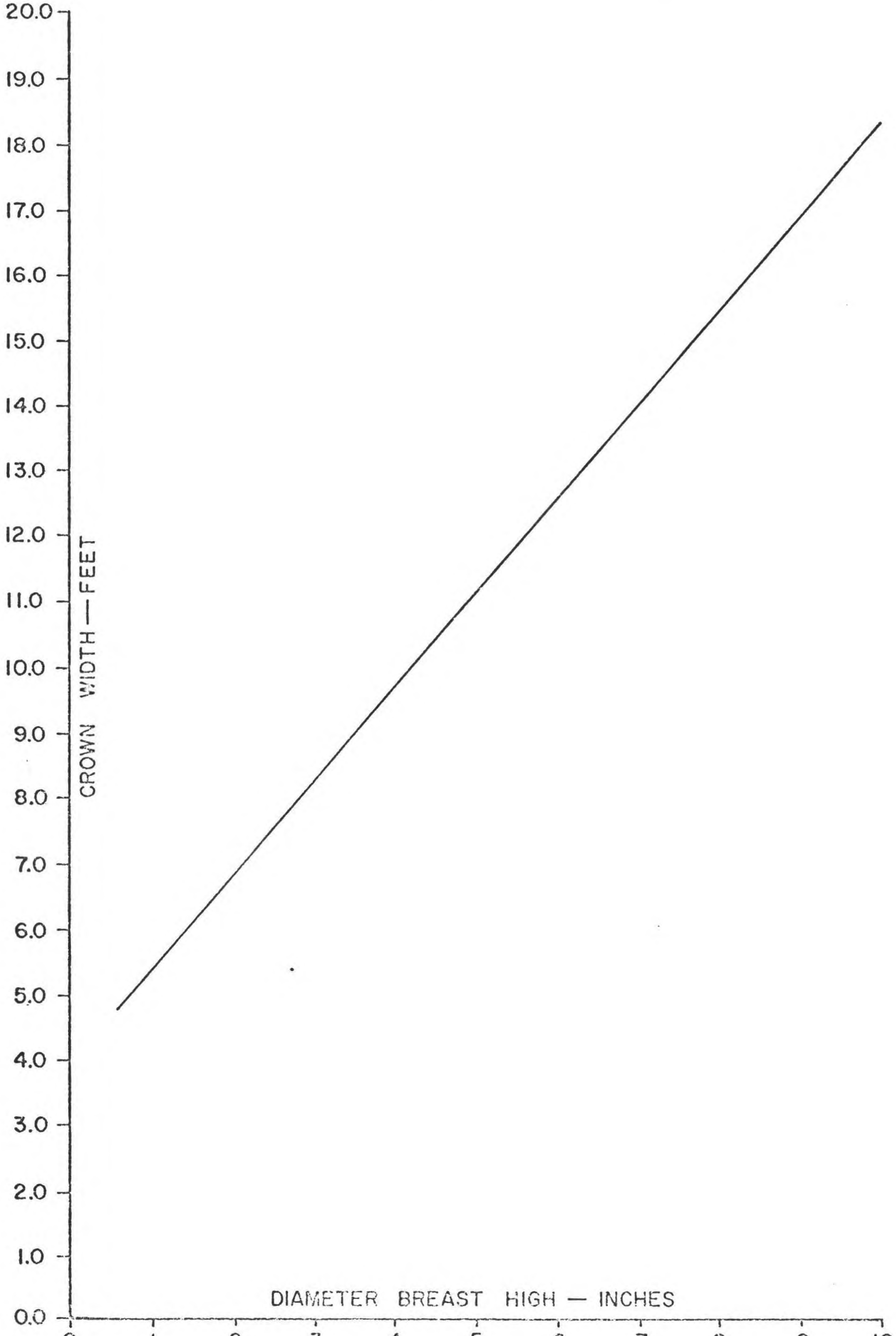
[14] Data supplied by Robert R. Alexander of the Rocky Mountain Forest and Range Experiment Station.

Figure 6.--Regression of crown width on diameter breast height for open-grown trees.


CROWN WIDTH — FEET
DIAMETER BREAST HIGH — INCHES
20.0
19.0
18.0
17.0
16.0
15.0
14.0
13.0
12.0
11.0
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
0 1 2 3 4 5 6 7 8 9 10

The curve representing the lower limit of the desirable stocking range was then constructed by plotting basal area per acre over the corresponding number of trees per acre (Figure 7). This relationship based on Crown Competition Factor as the measure of density was also computed. The CCF equation was solved for each diameter level from 1.0 to 8.0 inches to obtain CCF values to plot against basal area per acre to form the lower stocking curve (Figure 8).

Table 7 is a compilation of the values that were used to plot the two curves representing the lower stocking level. The original tree measurements from which the crown width-DBH relationship was obtained are listed in Appendix D.

TABLE 7. Values used in the construction of the lower stocking curve.

DBH (in.)	Crown Width (ft.)[15]	Crown Area (sq. ft.)[16]	No. Trees Per Acre[17]	Cross-sec. Area (sq. ft.)[18]	Basal Area/A (sq. ft.)[19]	CCF[20]
1.0	5.380	22.73	1916	.006	11.5	111
2.0	6.816	36.49	1194	.022	26.3	120
3.0	8.252	53.48	814	.049	39.9	120
4.0	9.688	73.72	591	.087	51.4	118
5.0	11.124	97.19	448	.136	60.9	115
6.0	12.560	123.90	352	.196	69.0	111
7.0	13.996	153.85	283	.267	75.6	113
8.0	15.431	187.02	233	.349	81.3	104

[15] Crown width = 3.94 + 1.44(DBH).

[16] Crown Area = 3.1416(crown width)^2/4.

[17] Number of trees per acre = 43,560 sq. ft. per acre/crown area.

[18] Area of a circle of the diameter in column one.

[19] Basal area per acre = (number of trees) (area of one tree).

[20] Crown Competition Factor = 50.58 + 5.25BA/$\overline{\text{D}}$BH (Alexander, 1966).

Figure 7.--Lower stocking curve using number of trees per acre as the measure of stand density. Relationship between basal area per acre and number of trees per acre for open-grown trees of diameters one through eight inches.

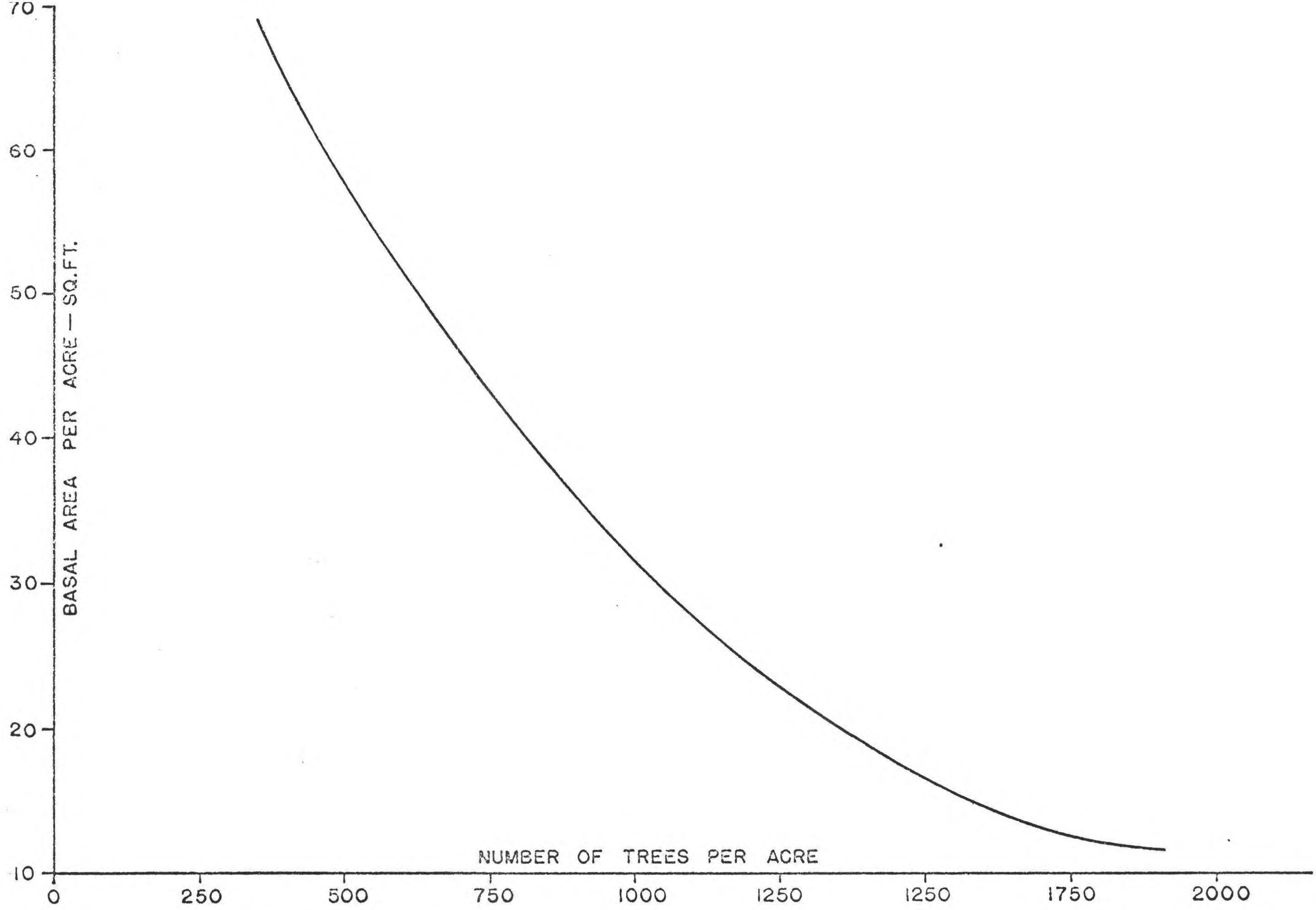
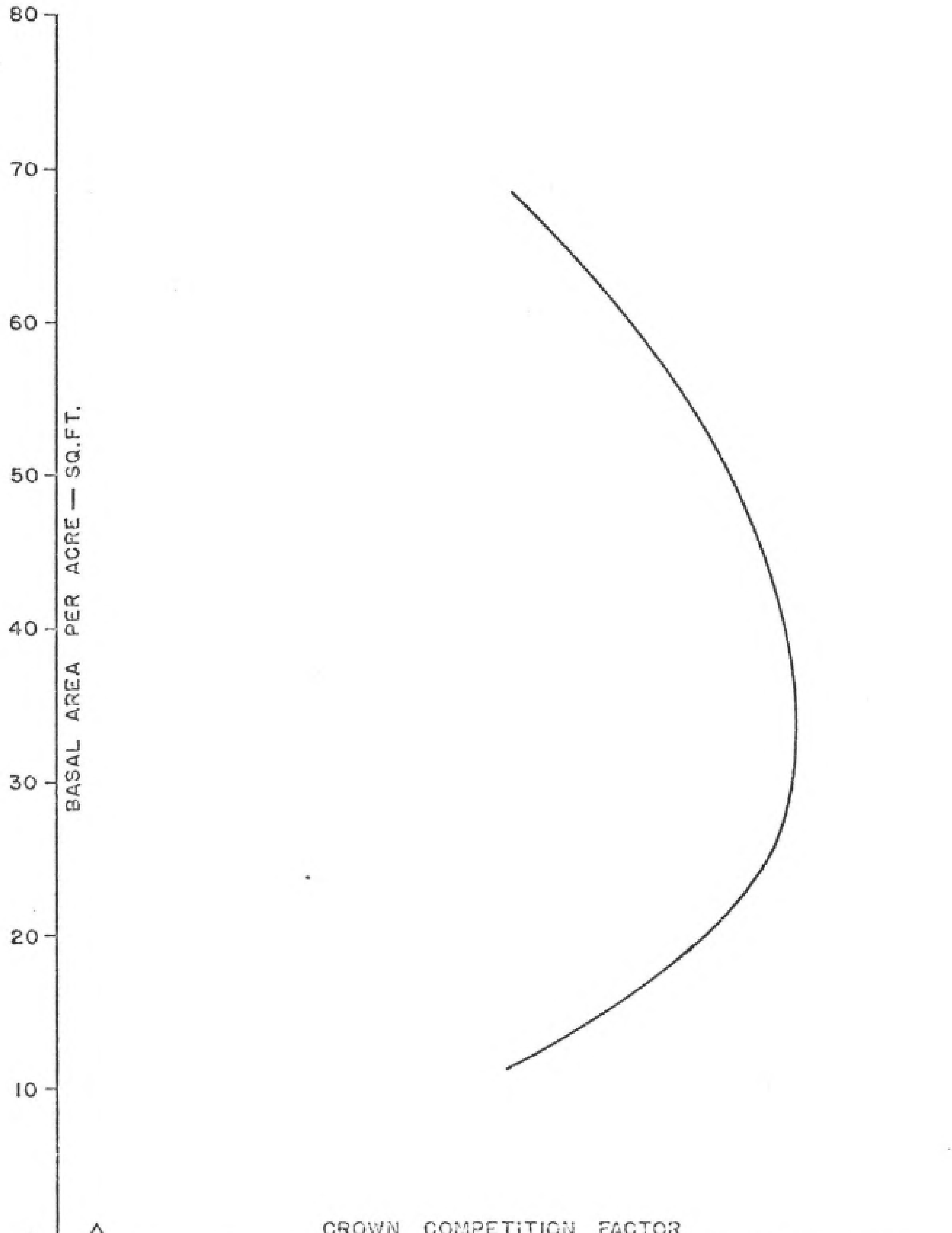

BASAL AREA PER ACRE — SQ.FT.
70
60
50
40
30
20
10
NUMBER OF TREES PER ACRE
0
250
500
750
1000
1250
1250
1750
2000

Figure 8.--Lower stocking curve using Crown Competition Factor as the measure of stand density.


BASAL AREA PER ACRE — SQ.FT.
CROWN COMPETITION FACTOR
0
10
20
30
40
50
60
70
80
0
100
105
110
115
120
125

Rate of stocking change

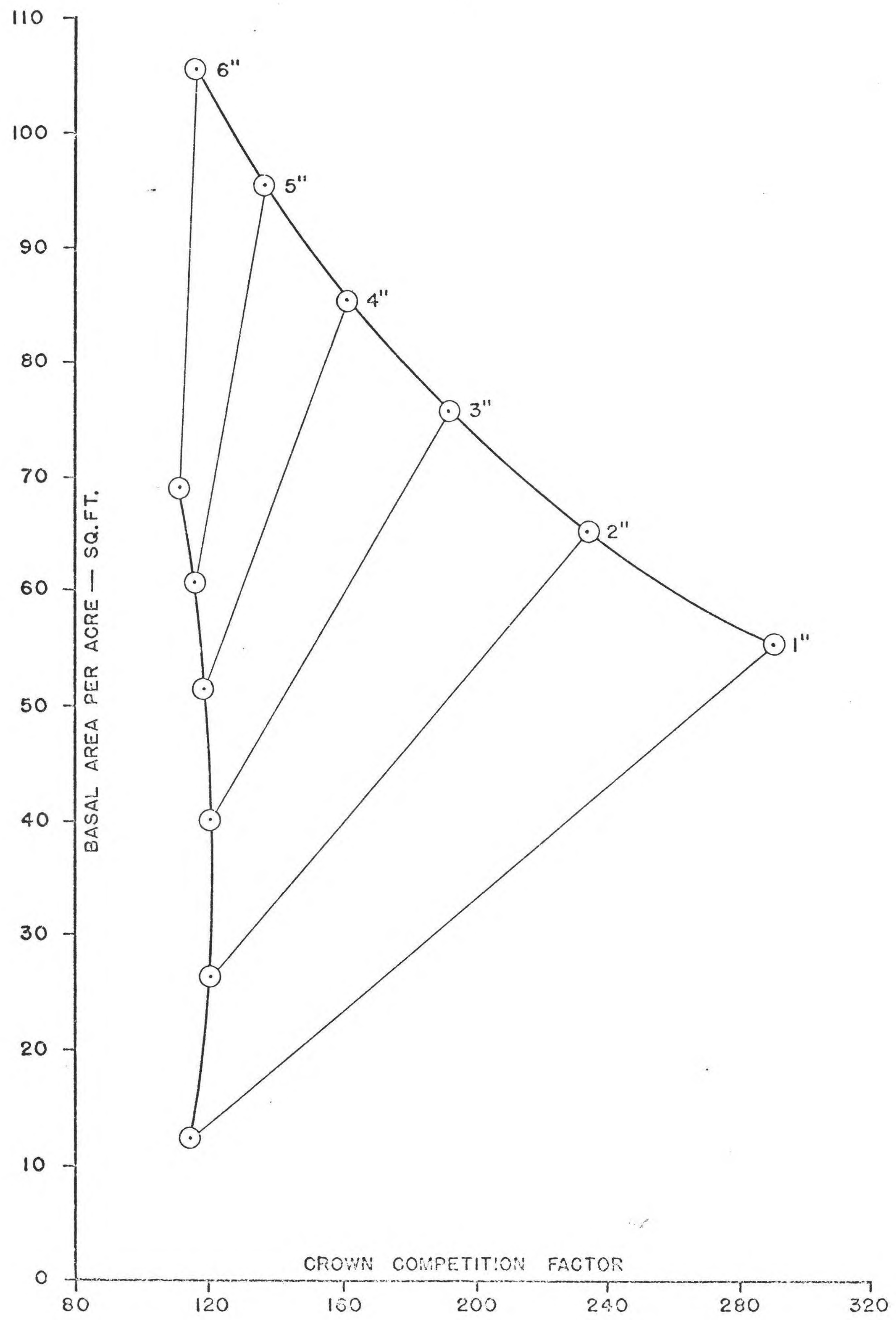
The first portion of this chapter explained the construction of a curve which represents the upper limit of the desired stocking range. The next part dealt with the definition of the lower stocking limit. Figures 9 and 10 show the relationship between these stocking limits, i.e., they portray the range of stocking within which various management objectives can best be achieved.

Figure 9 shows the defined stocking range using Crown Competition Factor as the measure of density. Although this set of curves was developed primarily as a means of obtaining the curves as shown on Figure 10, they can be used directly as guides for making management decisions. Further discussions of the stocking limits, however, will be in reference to the stocking range as shown on Figure 10, the independent variable of which is stand density as expressed in number of trees per acre.

A further objective of this project was to develop a means by which the change in stocking over time might be estimated, for use in conjunction with the established stocking range. This part of the study involved a graphical solution based upon the diameter-age relationship. The construction steps are explained below.

Step 1.--Analysis of variance was used to test the hypothesis that there was no difference in periodic annual diameter growth due to site quality. As can be seen in Table 8, the resulting F value did

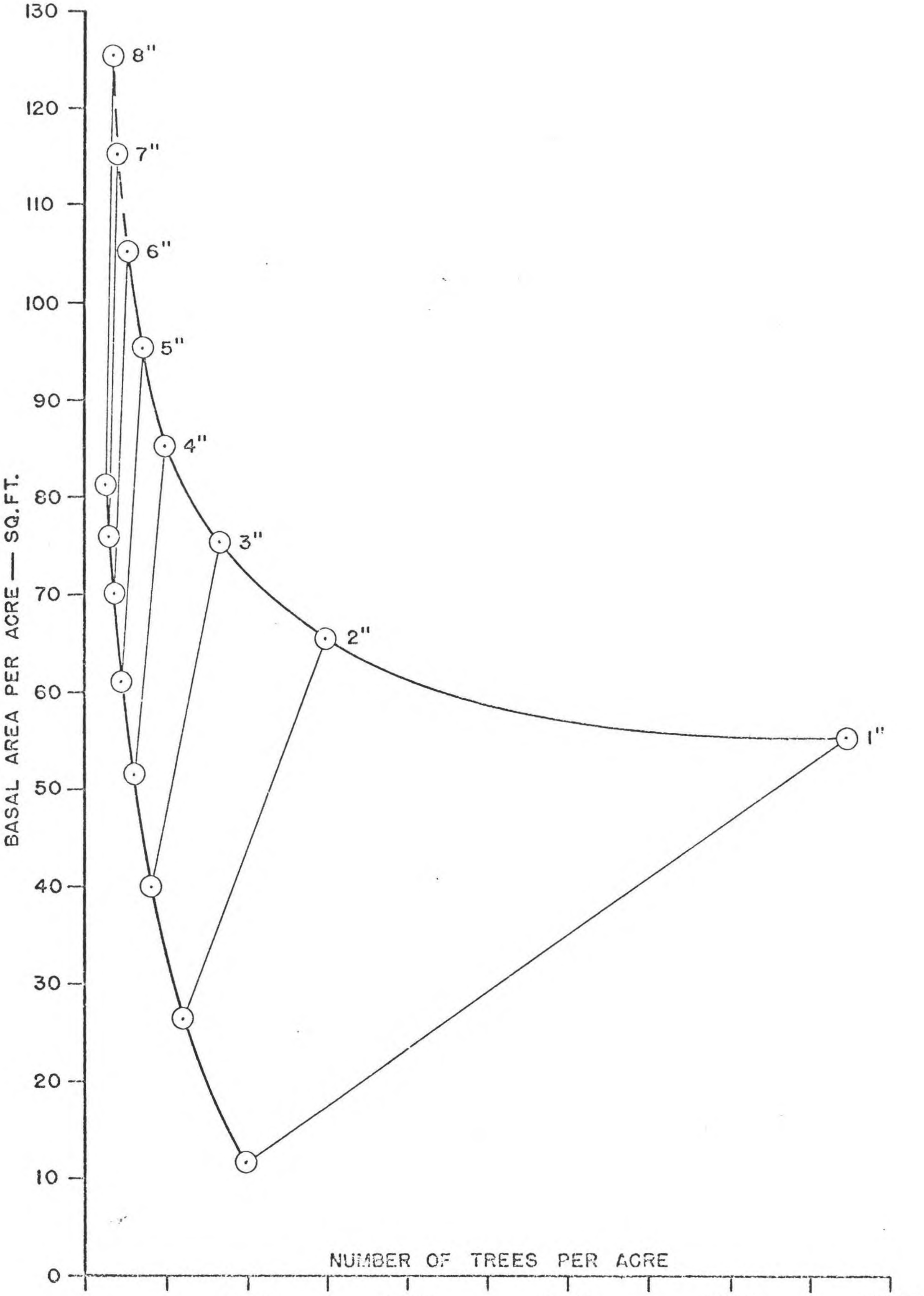
Figure 9.--Stocking range based upon Crown Competition Factor as the measure of stand density.



110
100
90
80
70
60
50
40
30
20
10
0
BASAL AREA PER ACRE — SQ.FT.
6"
5"
4"
3"
2"
1"
CROWN COMPETITION FACTOR
80
120
160
200
240
280
320

Figure 10.--Stocking range based upon number of trees per acre as the measure of stand density.[21]

[21] The dashed portion of the upper curve indicates extrapolation beyond the basic data.


BASAL AREA PER ACRE — SQ.FT.
NUMBER OF TREES PER ACRE
130
120
110
100
90
80
70
60
50
40
30
20
10
0
0
2000
4000
6000
8000
10000
8"
7"
6"
5"
4"
3"
2"
1"

not indicate that the hypothesis should be rejected at the five percent level of significance.

TABLE 8. Analysis of variance of periodic annual diameter growth.

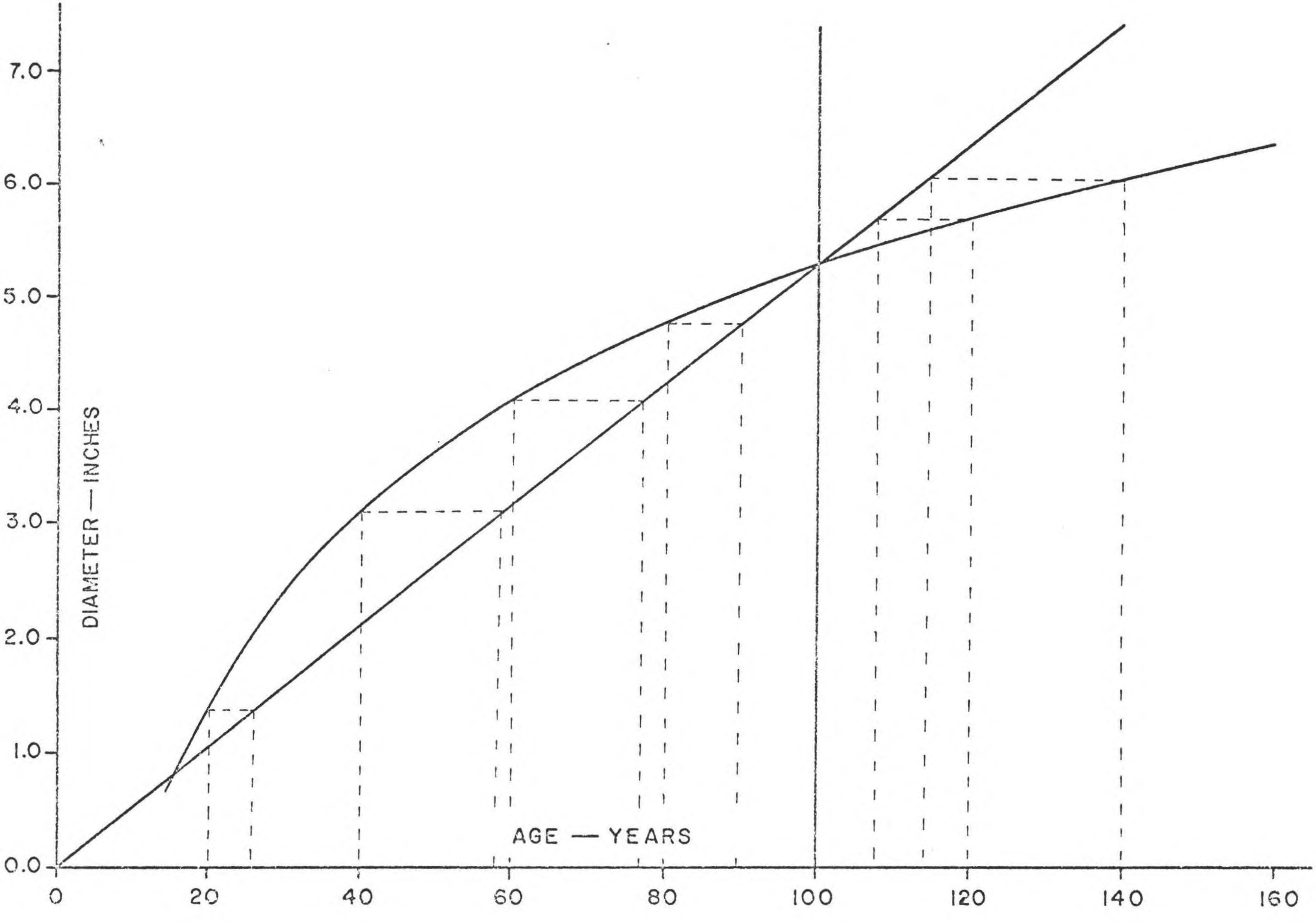
Source	D.F.	S.S.	M.S.	F
Total	52	.016020		
Site	6	.003388	.000565	
Residual	46	.012632	.000275	2.0545 N.S.

Step 2.--The regression of DBH on age was used as a guide curve for establishing the shape of subsequent "rate-of-change" curves. If the analysis of variance in Step 1 had shown significant diameter growth difference due to site, a guide curve would have been needed for each of the several Site Index groups, but because of the lack of significant difference, all 53 plots were used to calculate the regression equation. The values used in this regression were the average weighted ages and diameters for decadal age classes from 20 through 120 years.[22]

The resulting equation, DBH = -5.9 + 5.6(log age), had a correlation coefficient of .896 and the slope was statistically significant at the one percent level. This regression is shown on Figure 11.

[22] Plot values and calculations are listed in Appendix E.

Figure 11.--Regression of DBH on age for all plot data and anamorphosis of X-axis.



DIAMETER — INCHES
AGE — YEARS
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
0
20
40
60
80
100
120
140
160

Step 3.--The graphical solution involved an anamorphosis of the X-axis, or age scale, through a process of orthographic projection; projection of a curved line onto a straight line. The process, as shown on Figure 11, was as follows. First, a straight line was drawn from the origin through the intersection of the guide curve and the 100-year ordinate. Then, for each of the several age values, a line was projected vertically to the intersection with the guide curve, and then horizontally to intersect the straight line, and then back down to the X-axis. The intersection with the X-axis was the new scale location for each of the original age values.

Step 4.--The 53 plots were classified on the basis of three density classes as follows:

Low density	=	0 - 999 trees per acre,
Medium density	=	1000 - 1999 trees per acre,
High density	=	2000 - plus trees per acre.

The average age and DBH values for each of the three density classes were used to plot the three lines on Figure 12 using the anamorphosed age scale. The plotted value for each class was connected to the origin to form the lines.

Step 5.--Figure 13 shows the DBH-density relationship for the 100-year age ordinate of Figure 12. The mean density value for each density class and the corresponding 100-year DBH were plotted to obtain the points on Figure 13, and a least squares solution

Figure 12.--DBH-age relationship by density classes on anamorphosed X-axis.

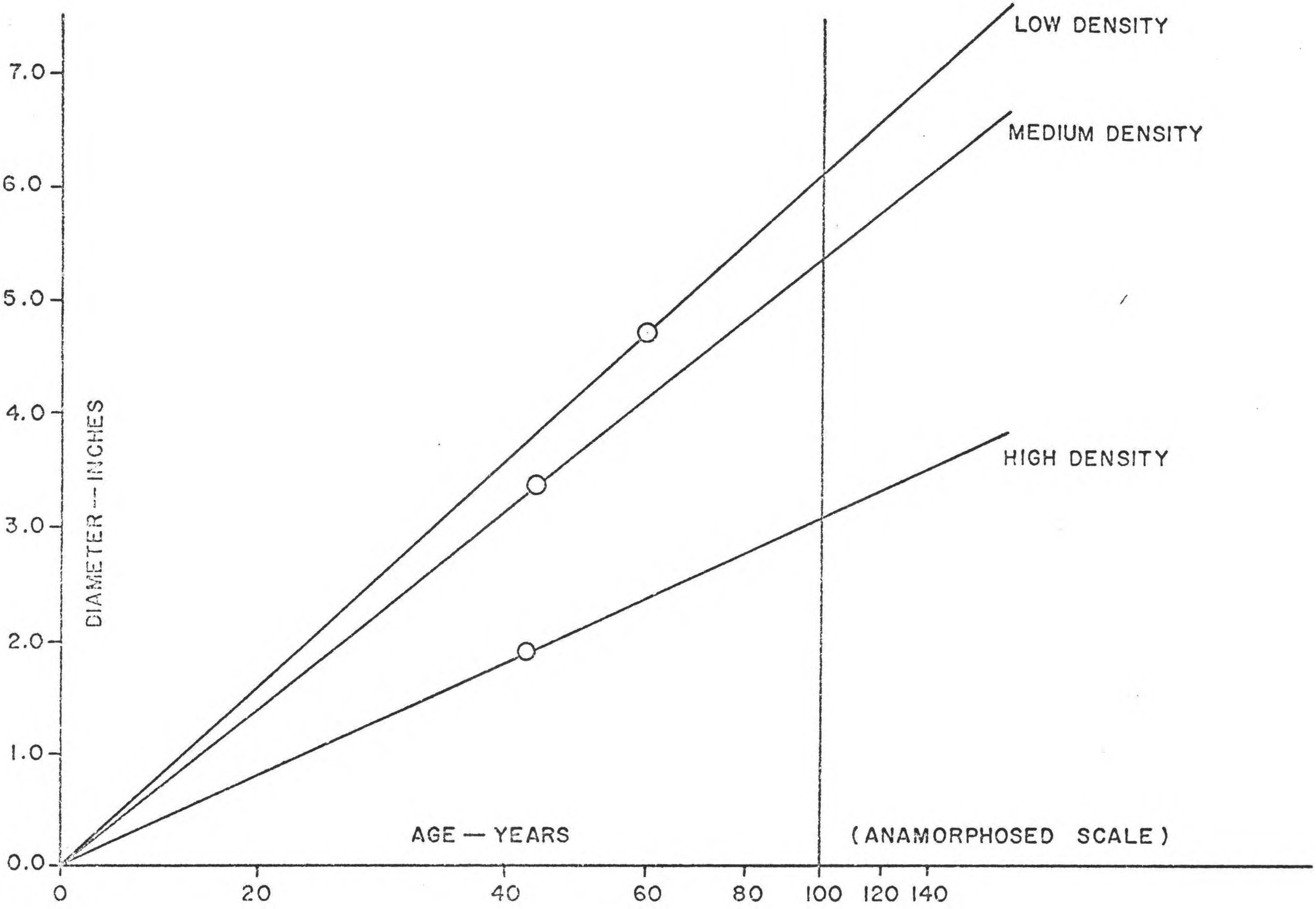
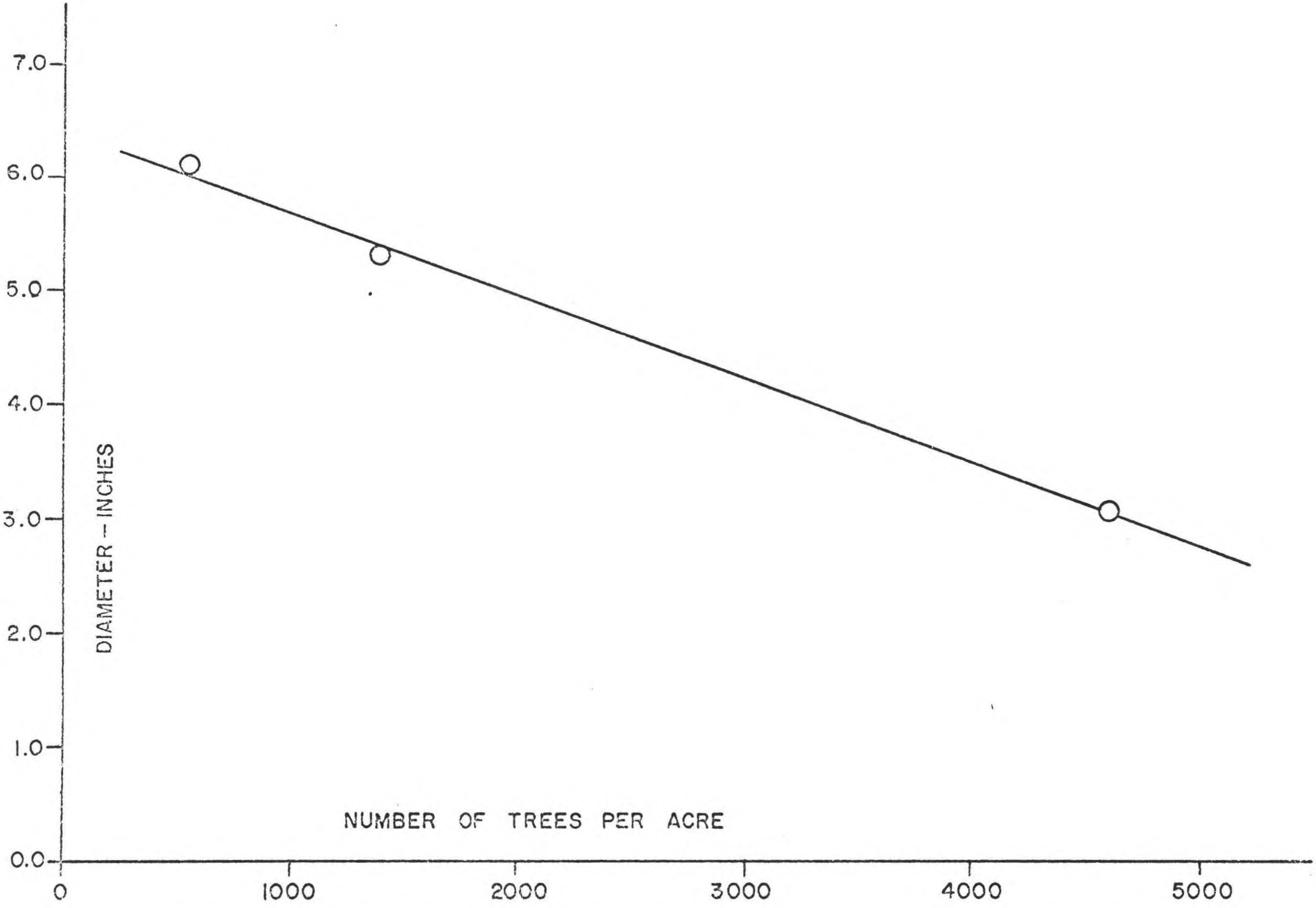

LOW DENSITY
MEDIUM DENSITY
HIGH DENSITY
DIAMETER — INCHES
AGE — YEARS
(ANAMORPHOSED SCALE)
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
0
20
40
60
80
100
120
140

Figure 13.--DBH-density relationship for the 100-year age ordinate of Figure 12.


7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
DIAMETER - INCHES
NUMBER OF TREES PER ACRE
0
1000
2000
3000
4000
5000

provided the equation, DBH = 6.22 - .00072465(density); correlation coefficient equals .984.

Step 6.--The DBH equation from Step 5 was solved for density values 500, 1500, and 2500 trees per acre and the resulting diameter values were plotted at the 100-year ordinate of a graph with the anamorphosed age scale (Figure 14). The three plotted points on the 100-year ordinate were then connected with the origin to form straight lines representing the low, medium, and high density levels.

Step 7.--The three straight lines from Step 6 were next plotted against a "standard" age scale (Figure 15), forming three curves, each the shape of the original guide curve. These curves portray the rate of diameter change with time by density classes. Figure 15 can be used in conjunction with Figure 10 to determine stocking level at some future date.

Figure 14.--"Corrected" DBH-age relationship by density classes on anamorphosed X-axis.

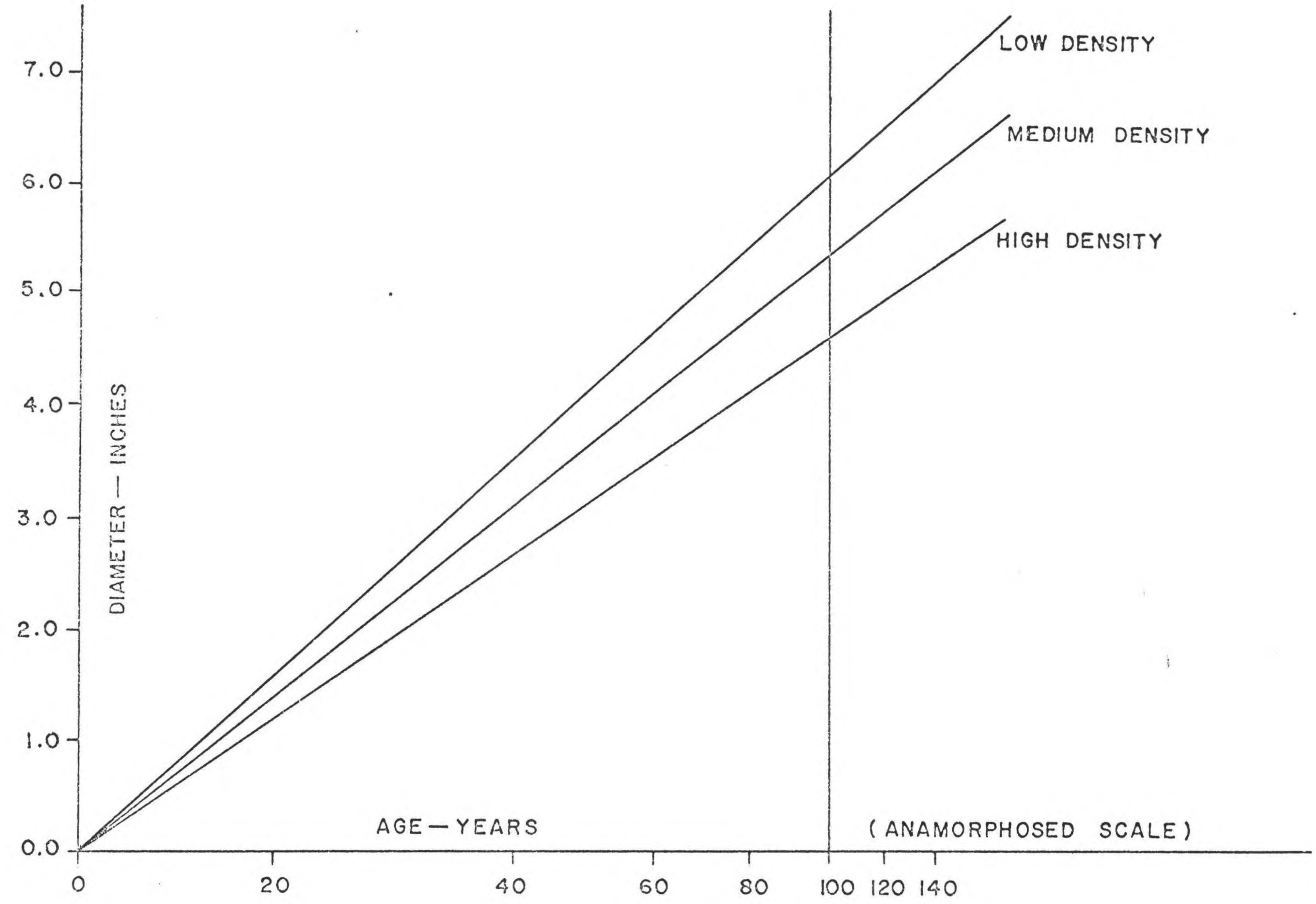
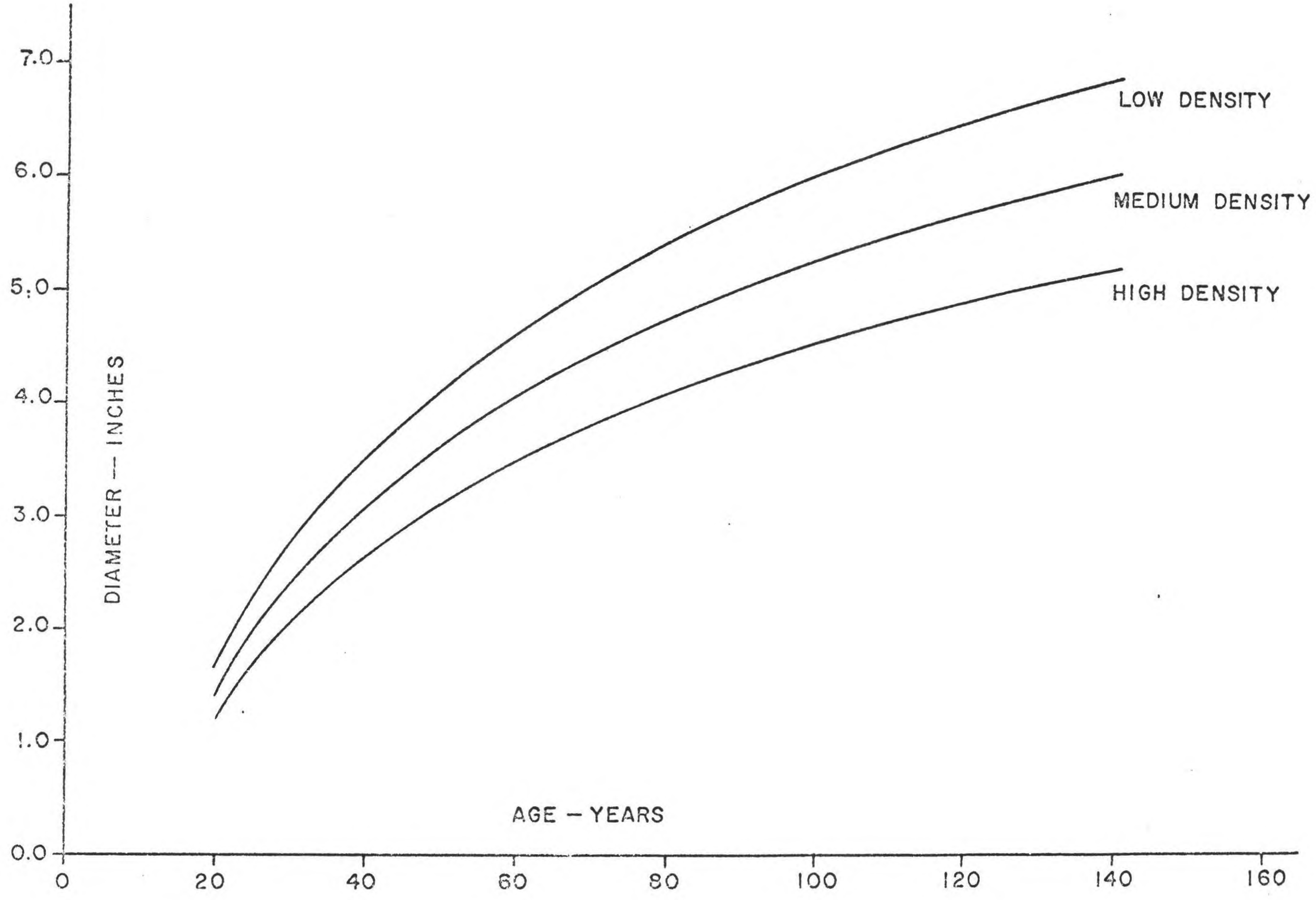

LOW DENSITY
MEDIUM DENSITY
HIGH DENSITY
DIAMETER — INCHES
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
AGE — YEARS
(ANAMORPHOSED SCALE)
0
20
40
60
80
100
120
140

Figure 15. --Rate of diameter change with time by density classes.

Low density	=	0 - 999 trees per acre
Medium density	=	1000 - 1999 trees per acre
High density	=	2000 or more trees per acre


LOW DENSITY
MEDIUM DENSITY
HIGH DENSITY
DIAMETER — INCHES
AGE — YEARS
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
0
20
40
60
80
100
120
140
160

CHAPTER IV

RESULTS AND CONCLUSIONS

F. S. Baker (1953) wrote that "---foresters are surprisingly vague about drawing a line where trees are either too few to utilize the acre properly, or when the numbers are so high as to make the need of thinning absolutely clear." This study has established these lines for natural lodgepole pine stands in Colorado and southern Wyoming (Figure 10).

The lower curve on Figure 10 represents the minimum stocking for various average stand diameters, that will fully occupy the site. Fewer trees than the number indicated anywhere along the lower curve cannot fully utilize the growing space. This curve was based upon the growth of open-grown trees which were unrestricted by tree crown competition in their occupancy of the site.

The upper curve on Figure 10 represents maximum stocking. Stands with densities greater than those indicated by this line are subject to stagnation. According to Dahms (1967), "tree density is especially critical in the culture of lodgepole pine because stagnation from overcrowding is probably more pronounced in this species than in any other western conifer."

Total per-acre wood production is thought to be about the same at any stand density within the limits defined by the two stocking curves. This "range of equal production" provides the forester with latitude within which to work while at the same time providing definite guiding limits.

The identification of the stand densities which defined the upper stocking limit proved to be the major problem in this study. Some indicator of the densities beyond which growth declined was needed. Before attempting to find this indicator the units of measure for describing growth and stand density had to be decided upon.

Volume growth was first considered as the logical and probably the most important basis for study since merchantable volume is of ultimate interest from the production standpoint. However, basal area growth was chosen as the measure to use for two reasons. First, basal area growth has been found to be closely correlated with volume growth (Hawley and Smith, 1954; Spurr, 1952). Since tree and stand growth is of primary concern in this study, and not total volumes, basal area growth was considered to be a satisfactory measure. Secondly, basal area growth can be measured more directly and more accurately than can volume growth. The variability in definition of utilization standards as related to tree and stand volumes adds to the inaccuracies that can result from the use of volume as a basis for growth studies. On the other hand, basal area as a growth standard is much easier to define and accounts for a major portion of volume increment.

Number of trees per acre is probably the most straight-forward and most easily understood measure of stand density and was therefore chosen as the unit of measure to use in presentation of the stocking guides. Another measure of density, Crown Competition Factor, was used in the construction of the stocking curves. CCF was found to be a satisfactory measure of lodgepole pine density in the development of site indexes (Alexander, 1966; Alexander, Tackle, and Dahms, 1967). Consistency in the use of density measures between studies dealing with this species seemed an important factor. Also, regression analysis indicated a closer correlation between Crown Competition Factor and basal area growth than did number of trees per acre. The lack of good correlation between growth and number of trees is probably due to the fact that number of trees alone is not weighed by tree size, which affects stocking. Basal area and average tree diameter are incorporated in the determination of Crown Competition Factor. CCF is therefore a better indicator of the relationship between growth and growing space. Crown Competition Factor was used in the construction of the stocking curves and then, in order to make the final working tools more usable, CCF was converted to number of trees per acre.

Analysis of basal area growth on a stand basis did not provide a satisfactory solution to the identification of the upper stocking limit. It was originally reasoned that Moller's production theory (Reukema, 1966) might apply to lodgepole pine. This theory was that production increases with increased stocking up to the point where full occupancy of the site is achieved. Beyond this point, increased density does not affect the amount of growth, but only its distribution; on a small number of relatively large trees at low densities and a large number of smaller trees at high densities. Moller maintained that at high densities, where crowding becomes a limiting factor, production falls off.

If Moller's theory is applicable to lodgepole pine, a regression solution relating basal area growth per acre to stand density should indicate a density level at which stand growth begins to decline. This density level would then be a logical basis for the definition of the upper stocking limit. Analysis, however, did not reveal a density level at which growth declined.

Regression analysis resulted in a curve which increased with increasing density and then leveled off at a density which was assumed to be the point of full site occupancy. Beyond this level the growth per acre remained nearly constant with increasing density to the limits of the data. The highest density level recorded was 9244 trees per acre or Crown Competition Factor 509. If Moller's production theory is applicable to lodgepole pine the drop in per-acre production occurs beyond the upper densities sampled in this study.

The growth-per-acre regression more nearly approximated another production viewpoint; that growth increases continuously with increasing density, at least to some very high density level (Reukema, 1966).

Since analysis of basal area growth on a per-acre basis did not provide a means for establishing the upper stocking limit, the problem was approached from the standpoint of basal area growth per tree.

The problem became one of finding an indicator of the stand density levels, by diameter classes, which result in a decline in growth rate per tree. It was discovered that linear equations showing the relationship between basal area growth per tree and Crown Competition Factor by Diameter classes could be used to reveal the stand density at which growth culminated for each class. It was reasoned that the culmination of growth rate per tree was significant as an indicator of the reaction of total stand growth to high density levels.

Since total stand growth appears to remain nearly constant with increasing densities above the point of full site occupancy, selection of the upper stocking level was based upon maintenance of "acceptable" growth per tree. Until the time comes when tree size no longer plays a role in determining stand values, growth per tree will remain significant. Management should attempt to maintain stand conditions which will not allow growth per tree to decline, but to keep stand densities at or below that level at which growth peaks and begins to drop off.

A series of synthetic stands was constructed in order to express the culmination of tree growth by diameter classes. It would have been possible to retain the growth-per-tree relationships in the establishment of the stocking levels, however this rationale is not convenient to use. Therefore, the rate of basal area growth per tree at the peak of growth for each diameter class was translated into growth per acre for use in constructing a graphical representation of the upper stocking limit. This was first accomplished using CCF as the measure of stand density. CCF values were then converted to numbers of trees per acre in order to provide a management tool which can be more readily used and understood.

The procedures used for the establishment of the minimum stocking levels for full site occupancy were based upon the assumption that the crown areas of open-grown trees are indicative of the maximum tree-area requirements of individuals of that size. It may be noted that the upper and lower stocking curves tend to converge with increase in tree diameter. For example, at a DBH of one inch, the lower stocking level is 21 percent of the maximum level, while at a diameter of five inches the lower curve indicates that the site is fully occupied at 65 percent of maximum stocking. Although this phenomenon was not investigated as part of the study, it provided cause for conjecture. The writer believes the convergence to be a result of root interaction in the larger open-grown trees because on poor soils and in arid environments root competition may take place before competition for above-ground space is evident. Growth may actually be affected due to tree interaction or competition at a density level lower than that indicated by the crown sizes alone. If this is true, a lower stocking curve based upon root competition would tend to more nearly parallel the upper curve. An index to tree area requirements based upon root spread might provide a better means for establishing the lower stocking curve.

An analysis of variance indicated that there was no significant difference in periodic annual diameter growth due to site quality. Although site quality, particularly the availability of water, is significant in individual tree growth, the most important variable in terms of growth per unit area is level of stocking. This fact allowed a relatively simple solution to the problem of estimating the stocking level of a stand at some future date. A graphical solution based upon the diameter-age relationship resulted in curves which show the change in average stand diameter with time. Since diameter is a variable on the stocking curves, a change in stand diameter over time can be used to indicate the resulting change in stocking level.

The stocking curves

The working tools as developed in this study are portrayed graphically on Figures 15, 16, and 17. Figures 16 and 17 show the range of acceptable stocking, but with an expanded "X" axis for more accurate use. Intermediate stocking lines have been included to facilitate following the stocking trends of a stand over time.

The upper limit is considered to be 100 percent stocking. Table 9 lists the percents of stocking of the lower curve by diameters, based upon 100 percent for the upper curve.

Figure 15 indicates the change in DBH with age for three density classes.

Table 9. PERCENT STOCKING OF LOWER CURVE BASED UPON 100 PERCENT FOR UPPER CURVE.

DBH (inches)	PERCENT STOCKING
1.0	21
2.0	40
3.0	53
4.0	60
5.0	64
6.0	66
7.0	66
8.0	65

Figure 16. --Relation of basal area per acre, number of trees per acre, and average tree diameter to optimum stocking range for lodgepole pine (one to three inches DBH).

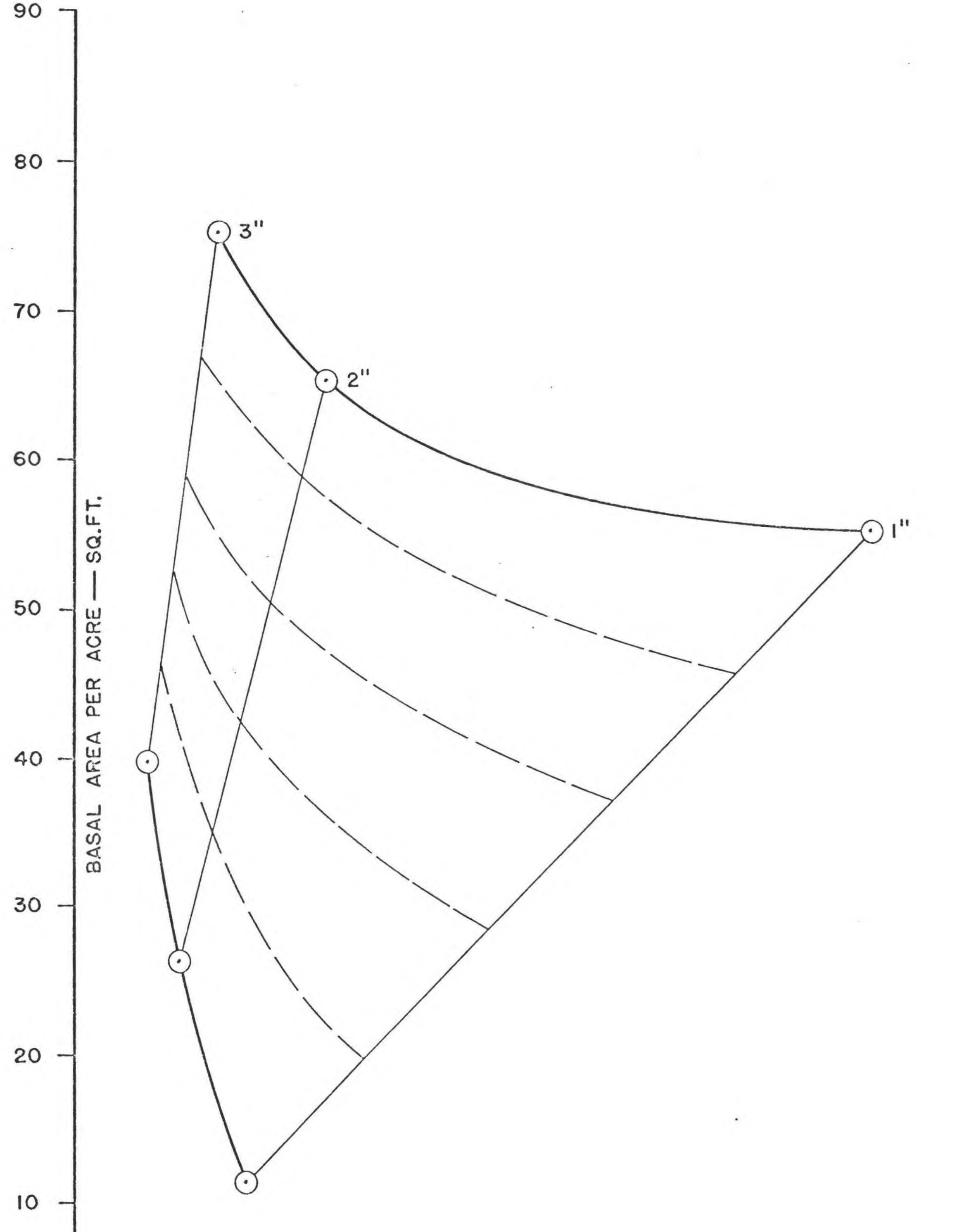

90
80
70
60
50
40
30
20
10
0
BASAL AREA PER ACRE —— SQ.FT.
3"
2"
1"
NUMBER OF TREES PER ACRE
0
2000
4000
6000
8000
10000

Figure 17. --Relation of basal area per acre, number of trees per acre, and average tree diameter to optimum stocking range for lodgepole pine (three to eight inches DBH).[1]

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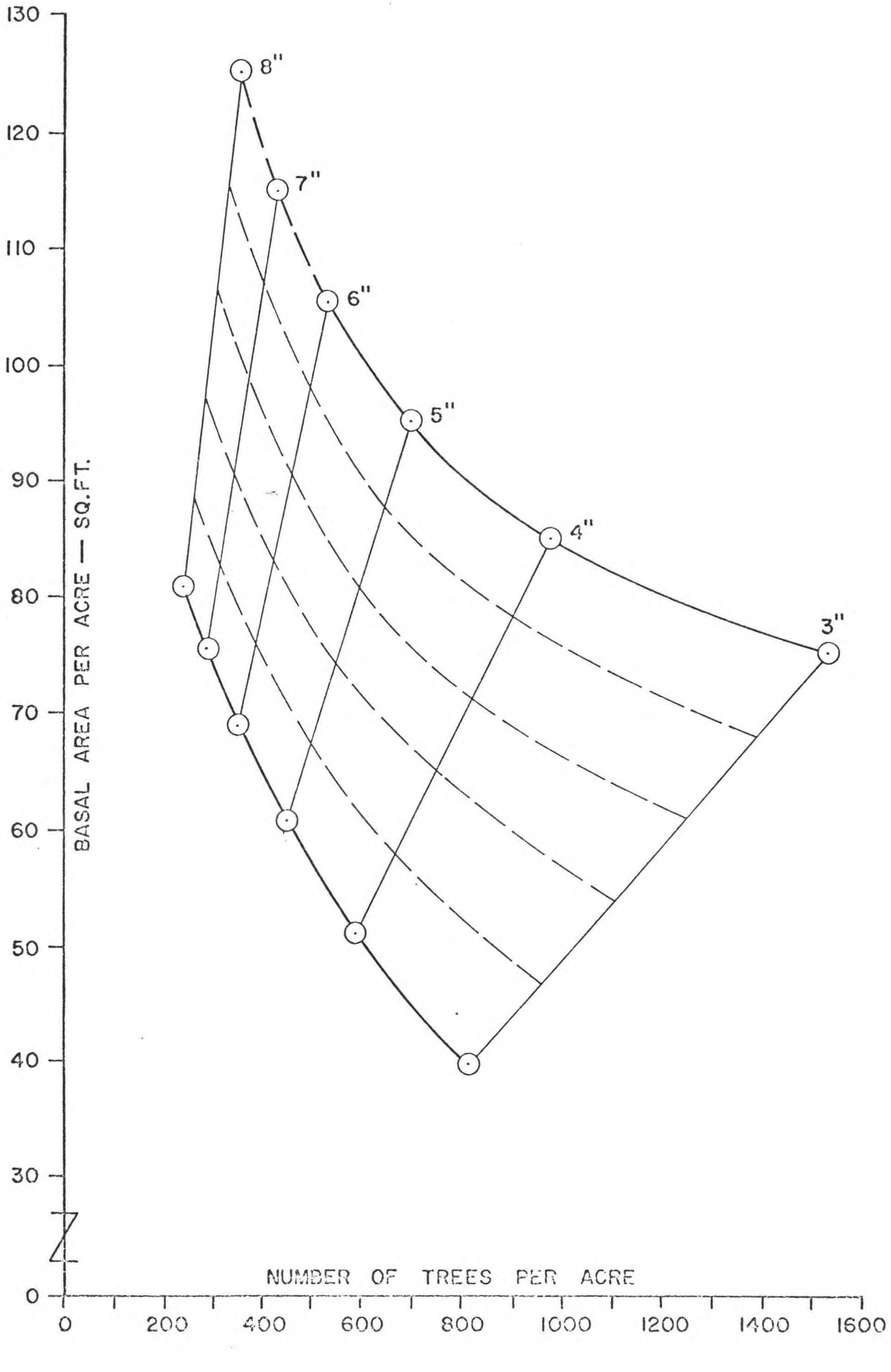
[1] The dashed portion of the upper curve indicates extrapolation beyond the basic data.


8"
7"
6"
5"
4"
3"
BASAL AREA PER ACRE — SQ.FT.
NUMBER OF TREES PER ACRE
0
30
40
50
60
70
80
90
100
110
120
130
0
200
400
600
800
1000
1200
1400
1600

The stocking range as a management tool

The full stocking range as developed in this study can be used as a guide to the silvicultural needs of a stand. Only two easily-obtainable stand parameters are needed to determine the basal area per acre required to fall within the acceptable stocking range. Average stand diameter and number of trees per acre can be used to enter Figures 16 and 17, noting where the point falls in relation to the nearest DBH line. In order to find the basal area required at the lower limit of the acceptable range, move parallel with the DBH line to the point of intersection with the lower limit curve. The basal area per acre at this level can provide the basis for silvicultural treatments. The relation of the stand basal area to the upper stocking limit or to any intermediate level can be similarly determined.

Although total production should be fairly uniform throughout the acceptable range, growth per tree will certainly vary. Within the stocking range, the site is utilized to its full capacity. As a result, manipulation of stocking within the range merely redistributes production among the trees in the stand. If the production goal is based upon a sawlog market, the stand should be maintained at a stocking level at or slightly above the lower stocking limit. Reducing the stand to a level below the lower limit will not add growth, since the lower limit is based upon growth of open-grown trees, which are, theoretically, undergoing no competition from other trees.

Figure 15 can be utilized to help predict the future position of a stand in relation to the full-stocking range. The graph is entered with average stand age. Follow the age ordinate vertically to the intersection with the appropriate density-level curve. Do the same thing for the age that the stand will be at the end of the growth period. The change in diameter on the "Y" axis corresponding to the two ages indicates the amount of change to be expected over the study period.

The estimate of diameter change can then be used in Figures 16 and 17 to locate the future stand position. From the present stand position in the appropriate graph, extend the diameter by the amount of expected change in an upward diagonal direction paralleling the nearest stocking curve. This should then provide an estimate of the basal area per acre to be expected at the future time.

Conclusions

Harry G. Smith (1958) made the statement that "forestry practices will be undertaken to the extent that they are likely to provide adequate returns on the investments required." It is desirable therefore, that management decisions be based upon definite guidelines. Subjective judgements concerning the stocking situation in terms of broad classifications such as well-stocked, over-stocked, or under-stocked leave much to be desired and much chance for error.

It appears from this study that definite numerical stocking limits can be established from temporary sample plot data. The upper limit of the stocking range was obtained directly from regression analysis of basal area growth per tree values which were measured on plots covering a wide range of stand conditions. Basal area growth per tree was apparently independent of site quality.

The area occupied by open-grown trees was considered a logical basis for establishment of the lower stocking limit. The number of open-grown trees of various diameters that could fully occupy the site was plotted against the corresponding basal area per acre to form the lower stocking curve.

A Society of American Foresters committee on stocking (Bickford, et al., 1957) suggested determination of the range of stand densities which will result in maximum yield by sites for the major forest types as an important subject for study. The writer has attempted to do this for lodgepole pine.

Further efforts toward the establishment of stocking limits could make use of permanent growth plots. These plots, as they become old enough to be useful, would be valuable in checking management guides derived from temporary samples. Experience gained from the actual use of stocking guides can also be used to refine them over time. As management becomes more intense, the ranges of stocking within which the forester can efficiently work may also become more narrow. Therefore, updating of stocking guides as stand and economic conditions change will be desirable.

Additional sample data which more completely represents the species universe is a desirable requisit to further studies with this species. This data could be used to help refine the stocking limits and also in the development of a method for verifying the results. A wider range of data would also be valuable in more clearly describing the production model for lodgepole pine.

Since the stand and site variables which were used in this study did not account for all of the variation in tree growth, a study of other factors which may effect growth should be of value.

LITERATURE CITED

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Adams, W. R., and G. L. Chapman. 1942. Competition in some coniferous plantations. Vermont Agriculture Experiment Station Bulletin 489:1-26.

Alexander, Robert R. 1965. Growth of thinned young lodgepole pine in Colorado. Journal of Forestry 63:429-433.

Alexander, Robert R. 1966. Site indexes for lodgepole pine, with corrections for stand density: instructions for field use. U.S.D.A. Forest Service. Rocky Mountain Forest and Range Experiment Station. Research Paper RM-24.

Alexander, Robert R., David Tackle, and Walter G. Dahms. 1967. Site indexes for lodgepole pine, with corrections for stand density: methodology. U.S.D.A. Forest Service. Rocky Mountain Forest and Range Experiment Station. Research Paper RM-29.

Armit, D. 1966. Silvics and silviculture of lodgepole pine in the North Central Interior of British Columbia. A problem analysis. British Columbia Forest Service Research Notes, No. 40.

Baker, F. S. 1953. Stand density and growth. Journal of Forestry 51:95-97.

Barrett, James W. 1961. Response of 55-year-old lodgepole pine to thinning. U.S.D.A. Forest Service. Pacific Northwest Forest and Range Experiment Station. Research Note 206.

Bassett, John R. 1966. Periodic cubic growth in natural loblolly stands near Crossett, Arkansas. U.S.D.A. Forest Service. Southern Forest Experiment Station. Research Note SO-37.

Bickford, Allen C., F. S. Baker and Fred G. Wilson. 1957. Stocking, normality, and measurement of stand density. Journal of Forestry 55:99-104.

Bramble, W. C., H. N. Cope, and H. H. Chisman. 1949. Influence of spacing on growth of red pine in plantations. Journal of Forestry 47:726-732.

Briegleb, P. A. 1942. Progress in estimating trend of normality percentage in second-growth Douglas-fir. Journal of Forestry 40:785-793.

Briegleb, Philip A. 1952. An approach to density measurement in Douglas-fir. Journal of Forestry 50:529-536.

Bruce, Donald, and Francis X. Schumacher. 1935. Forest mensuration. McGraw-Hill Book Co., New York.

Bruce, Donald and Francis X. Schumacher. 1950. Forest mensuration, 3rd ed. McGraw-Hill Book Co., Inc., New York.

Carvell, Kenneth L. 1966. The effect of stand density on the development of Virginia pine in the Ohio River area of West Virginia. West Virginia University Agriculture Experiment Station Current Report 46.

Chaiken, L. E. 1939. The approach of loblolly and Virginia pine stands toward normal stocking. Journal of Forestry 37: 866-871.

Chapman, Herman H. 1931. Forest management. J. B. Lyon Company, Albany, New York.

Chisman, H. H. and F. X. Schumacher. 1940. On the tree-area ratio and certain of its applications. Journal of Forestry 38:311-317.

Clements, F. E. 1910. The life history of lodgepole burn forests. U.S.D.A. Forest Service. Bulletin 79.

Collins, A. B. 1967. Density and height growth in natural slash pine. U.S.D.A. Forest Service. Southeast Forest Experiment Station. Research Paper SE-27.

Dahms, Walter G. 1966. Effect of kind and number of measured tree heights on lodgepole pine site-quality estimates. U.S.D.A. Forest Service. Pacific Northwest Forest and Range Experiment Station. Research Paper PNW-36.

Dahms, Walter G. 1967. Low stand density speeds lodgepole pine tree growth. U.S.D.A. Forest Service. Pacific Northwest Forest and Range Experiment Station. Research Note PNW-47.

Davis, Kenneth P. 1966. Forest management: regulation and valuation. 2nd ed. McGraw-Hill Book Co., Inc., New York.

Dell, T. R., and L. V. Collicott. 1968. Growth in relation to density for slash pine plantations after first thinning. Forest Science 14:7-12.

Duerr, William A. 1938. Comments on the general application of Gehrhardt's formula for approach toward normality. Journal of Forestry 36:600-604.

Duerr, William A., and S. R. Gevorkiantz. 1938. Growth Predictions and site determination in uneven-aged timber stands. Journal of Agricultural Research 56:81-98.

Engle, LaMont G., and Norman F. Smith. 1950. Red pine growth ten years after thinning. Papers of the Michigan Academy of Science, Arts, and Letters. Vol. 36, Part I, pp. 101-109.

Eversole, Kenneth R. 1955. Spacing tests in a Douglas-fir plantation. Forest Science 1:14-18.

Eyre, F. H., and Paul Zehngraff. 1948. Red pine management in Minnesota. U.S.D.A. Circular No. 778.

Forbes, Reginald D., ed. 1956. Forestry handbook. The Ronald Press Co., New York.

Gevorkiantz, S. R. 1934. The approach of understocked stands to normality. Journal of Forestry 32:487-488.

Gevorkiantz, S. R. 1937. The approach of northern hardwood stands to normality. Journal of Forestry 35:487-489.

Gevorkiantz, S. R. 1940. Comments on "the approach of loblolly and Virginia pine stands toward normal stocking." Journal of Forestry 38:512-513.

Gevorkiantz, S. R. 1947a. Growth and yield of jack pine in the Lake States. U.S.D.A. Forest Service. Lake States Forest Experiment Station. Station Paper No. 7.

Gevorkiantz, S. R. 1947b. More about numerical expression of stocking in terms of height. Journal of Forestry 45:203.

Gevorkiantz, S. R., and N. W. Hosley. 1929. Form and development of white pine stands in relation to growing space. Harvard Forest Bulletin No. 13.

Gingrich, S. F. 1964. Criteria for measuring stocking in forest stands. U.S.D.A. Forest Service. Central States Forest Experiment Station. Unpublished manuscript.

Gingrich, S. F. 1967. Measuring and evaluating stocking and stand density in upland hardwood forests in the Central States. Forest Science 13:38-53.

Gingrich, S.F., and B. A. Roach. 1962. Timber management guide for upland central hardwoods. U.S.D.A. Forest Service. Central States Forest Experiment Station and North Central Region.

Gruschow, G. F., and T. C. Evans. 1959. The relation of cubic-foot volume growth to stand density in young slash pine stands. Forest Science 5:49-55.

Hawley, Ralph C. 1936. Observations on thinning and management of eastern white pine in southern New Hampshire. Yale University School of Forestry Bulletin No. 42.

Hawley, Ralph C., and David M. Smith. 1954. The practice of silviculture. 6th ed. John Wiley and Sons, Inc., New York.

Hummel, F. C. 1953. The definition of thinning treatments. International Union of Forest Research Organizations. Proceedings of 11th Congress.

Husch, Bertram. 1963. Forest mensuration and statistics. Ronald Press Co., New York.

Knuchel, Hermann. 1953. Planning and control in the managed forest. Oliver and Boyd, London.

Krajicek, John E., and Kenneth A. Brinkman. 1957. Crown development: an index of stand density. U.S.D.A. Forest Service. Central States Forest Experiment Station. Station Note 108.

Krajicek, John E., Kenneth A. Brinkman, and Samuel F. Gingrich. 1961. Crown competition - a measure of density. Forest Science 7:35-42.

Lexen, Bert. 1943. Bole area as an expression of growing stock. Journal of Forestry 41:883-885.

Lino Della-Bianca, and Robert E. Dils. 1960. Some effects of stand density in a red pine plantation on soil moisture, soil temperature, and radial growth. Journal of Forestry 58:373-377.

Little, E. L., Jr. 1953. Check list of native and naturalized trees of the United States (including Alaska). U.S.D.A., Agriculture Handbook 41.

Lynch, Donald W. 1958. Effects of stocking on site measurement and yield of second-growth ponderosa pine in the Inland Empire. U.S.D.A. Forest Service. Intermountain Forest and Range Experiment Station. Research Paper 56.

MacKinney, Arland L., Francis X. Schumacher, and Lion E. Chaiken. 1937. Construction of yield tables for non-normal loblolly pine stands. Journal of Agricultural Research 54:531-545.

Mann, W. F., Jr., and L. B. Whitaker. 1952. Stand density and pine height growth. U.S.D.A. Forest Service. Southern Forest Experiment Station. Southern Forestry Notes No. 81.

Matthews, Donald Maxwell. 1935. Management of American forests. McGraw-Hill Book Co., Inc., New York.

Meyer, H. Arthur. 1953. Forest mensuration. Penns Valley Publishers, State College, Pa.

Meyer, H. Arthur, Arthur B. Recknagel, and Donald D. Stevenson. 1952. Forest management. The Ronald Press Co., New York.

Meyer, Walter H. 1930. Diameter distribution series in even-aged forest stands. Yale University School of Forestry Bull. 28.

Meyer, Walter H. 1933. Approach of abnormally stocked forest stands of Douglas-fir to normal condition. Journal of Forestry 31:400-406.

Meyer, Walter H. 1942. Yield of even-aged stands of loblolly pine in northern Louisiana. Yale University School of Forestry Bull. No. 51.

Minor, Charles O. 1951. Stem-crown diameter relations in southern pine. Journal of Forestry 49:490-493.

Mowat, E. L. 1949. Preliminary guides for the management of lodgepole pine in Oregon and Washington. U.S.D.A. Forest Service. Pacific Northwest Forest and Range Experiment Station. Research Note 54.

Mulloy, G. A. 1943. A practical measure of stock density in white and red pine stands. Forestry Chronicle 14:108-118.

Mulloy, G. A. 1944a. Stand density vs bole area and stand intensity indices in even-aged stands. Forestry Chronicle 20:167-170.

Mulloy, G. A. 1944b. Empirical stand density yield tables. Canada Department of Mines and Resources, Lands, Parks and Forests Branch, Dominion Forest Service. Silvicultural Research Note No. 73.

Myers, Clifford A. 1958. Thinning improves development of young stands of ponderosa pine in the Black Hills. Journal of Forestry 56:656-659.

Myers, Clifford A. 1966. Yield tables for managed stands with special reference to the Black Hills. U.S.D.A. Forest Service. Rocky Mountain Forest and Range Experiment Station. Research Paper RM-21.

Myers, Clifford A. 1967a. Growing stock levels in even-aged ponderosa pine. U.S.D.A. Forest Service. Rocky Mountain Forest and Range Experiment Station. Research Paper RM-33.

Myers, Clifford A. 1967b. Yield tables for managed stands of lodgepole pine in Colorado and Wyoming. U.S.D.A. Forest Service. Rocky Mountain Forest and Range Experiment Station. Research Paper RM-26.

Nelson, Thomas C. 1963. Basal area growth of natural loblolly pine stands. U.S.D.A. Forest Service. Southeast Forest Experiment Station. Research Note SE-10.

Nelson, Thomas C. 1967. Trends in mensuration research in the southern United States. International Union of Forest Research Organizations. Proceedings, 14th Congress, VI, Section 25, pp. 404-409.

Nelson, T. C., and E. V. Brender. 1963. Comparison of stand density measures for loblolly pine cubic-foot growth prediction. Forest Science 9:8-14.

Nelson, T. C., T Lotti, E. V. Brender, and K. B. Trousdell. 1961. Merchantable cubic-foot volume growth in natural loblolly pine stands. U.S.D.A. Forest Service. Southeast Forest Experiment Station. Station Paper No. 127.

Pearson, G. A. 1950. Management of ponderosa pine in the Southwest. U.S.D.A. Forest Service. Agriculture Monograph No. 6.

Quaite, J. 1950. Severe thinning in an overstocked lodgepole pine stand. Canada Department of Resources and Development, Forestry Branch. Forest Research Division Silvicultural Leaflet 47.

Ralston, Robert A. 1953. Some effects of spacing on jack-pine development in lower Michigan after twenty-five years. Papers of the Michigan Academy of Science, Arts, and Letters. Vol. 38, Part I, pp. 137-143.

Ralston, Robert A. 1954. Some effects of stand density on the height growth of red pine on poor sites in northern lower Michigan. Papers of the Michigan Academy of Science, Arts, and Letters. Vol. 39, pp. 159-165.

Reineke, L. H. 1933. Perfecting a stand-density index for even-aged forests. Journal of Agricultural Research. 46:627-638.

Reukema, D. L. 1966. The yield and density aspect - does dense spacing really produce the most volume? Reprinted from the proceedings of the 1966 annual meeting of western reforestation coordinating committee of the Western Forestry and Conservation Association by the U.S. Forest Service.

Roe, Eugene I., and Joseph H. Stoeckeler. 1950. Thinning overdense jack pine seedling stands in the Lake States. Journal of Forestry 48:861-865.

Roth, Filibert. 1915. Normal forest and actual forest, normal growing stock and actual growing stock, normal stand and actual stand. Forestry Quarterly 8:154-162.

Rudolf, Paul O. 1951. Stand density and the development of young jack pine. Journal of Forestry 49:254-255.

Schantz-Hansen, T. 1945. Some results of thinning fifteen-year-old red pine. Journal of Forestry 43:673-674.

Shirley, Hardy L., and Paul Zehngraff. 1942. Height of red pine saplings as associated with density. Ecology 23:370.

Simmons, E. M., and G. Luther Schnur. 1937. Effect of stand density on mortality and growth of loblolly pine. Journal of Agricultural Research 54:47-58.

Smith, J. Harry G. 1958. Better yields through wider spacing. Journal of Forestry 56:492-497.

Smith, J. Harry G., and John W. Ker. 1960. Growing Douglas-fir and western hemlock trees at desired rates. Faculty of Forestry, University of British Columbia. Research Note No. 24.

Smithers, L. A. 1956. Assessment of site productivity in dense lodgepole pine stands. Canada - Department of Northern Affairs and National Resources, Forestry Branch. Forest Research Division Technical Note 30.

Society of American Foresters. 1958. Forestry terminology 3rd ed.

Spurr, Stephen H. 1952. Forest inventory. The Ronald Press Co., New York.

Tackle, David. 1959. Silvics of lodgepole pine. U.S.D.A. Forest Service. Intermountain Forest and Range Experiment Station. Miscellaneous Publication No. 19.

Turner, Lewis M. 1943. Relation of stand density to height growth. Journal of Forestry 41:766.

U.S.D.A. 1949. Results of thinning in lodgepole pine. Timberman 50:112-114.

Vezina, P. E. 1962. Crown width - dbh relationships for open-grown balsam fir and white spruce in Quebec. Forestry Chronicle 38:463-473.

Vezina, P. E. 1963. More about the "crown competition factor." Forestry Chronicle 39:313-317.

Watt, Richard F. 1950. Growth in understocked and overstocked western white pine stands. U.S.D.A. Forest Service. Northern Rocky Mountain Forest and Range Experiment Station. Research Note No. 78.

Wegge, P. 1966. Stand density and volume increment - some theoretical considerations. Tidsskr. Skogbr. 74(3), (283-299). Forestry Abstracts 28, 2684.

Wenger, K. F., T. C. Evans, T. Lotti, R. W. Cooper, and E. V. Brender. 1958. The relation of growth to stand density in natural loblolly pine stands. U.S.D.A. Forest Service. Southeast Forest Experiment Station. Station Paper 97.

Whyte, A. G. D. 1965. The influence of thinning on taper, on volume assortment outturn and on economic return of the Bowmont spruce sample plots. The Commonwealth Forestry Review. 44(2) No. 120, 109-122.

Wilson, F. G. 1946. Numerical expression of stocking in terms of height. Journal of Forestry 44:758-761.

Wilson, F. G. 1951. Control of growing stock in even-aged stands of conifers. Journal of Forestry 49:692-695.

Wikstrom, John H., and Charles A. Wellner. 1961. The opportunity to thin and prune in the Northern Rocky Mountain and Intermountain Regions. U.S.D.A. Forest Service. Intermountain Forest and Range Experiment Station. Research Paper 61.

APPENDICES

APPENDIX TABLE OF CONTENTS

APPENDIX TABLE OF CONTENTS--(continued)

APPENDIX A

APPENDIX A.1--Scientific names.[1]

balsam fir	_Abies balsamea_ (L.) Mill.
Douglas-fir	_Pseudotsuga menziesii_ (Mirb.) Franco
eastern white pine	_Pinus strobus_ L.
white pine	
northern white pine	
jack pine	_Pinus banksiana_ Lamb.
loblolly pine	_Pinus taeda_ L.
lodgepole pine	_Pinus contorta_ Dougl.
pitch pine	_Pinus rigida_ Mill.
ponderosa pine	_Pinus ponderosa_ Laws.
red pine	_Pinus resinosa_ Ait.
Norway pine	
shortleaf pine	_Pinus echinata_ Mill.
slash pine	_Pinus elliottii_ Engelm.
Virginia pine	_Pinus virginiana_ Mill.
western white pine	_Pinus monticola_ Dougl.
white spruce	_Picea glauca_ (Moench) Voss

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[1] As in Little (1953).

APPENDIX B

Field measurements and processing of data used in the construction of the upper stocking curve and rate-of-change curves

APPENDIX B. 1--Locations of sample plots used in construction of upper stocking curve and rate-of-change curves.

PLOT NO.	NATIONAL FOREST	LOCATION
1	Roosevelt	Swamp Creek
2	Roosevelt	Swamp Creek
5	Roosevelt	Panhandle Creek
6	Roosevelt	Panhandle Creek
7	Roosevelt	Red Feather to Deadman Road
8	Roosevelt	Red Feather to Deadman Road
9	Roosevelt	Deadman Creek
10	Gunnison	Old Monarch Pass
11	Gunnison	South of Pitkin
12	Gunnison	South of Pitkin
13	Gunnison	North of Pitkin
14	Gunnison	Alpine Tunnel Road
15	Gunnison	South Quartz Creek
17	Gunnison	South Quartz Creek
18	Gunnison	South Quartz Creek
19	Gunnison	South Quartz Creek
20	Gunnison	East of Stage Stop Meadows
21	Gunnison	East of Stage Stop Meadows
22	Gunnison	East of Stage Stop Meadows
23	Gunnison	West of Stage Stop Meadows

APPENDIX B.1--(continued)

PLOT NO.	NATIONAL FOREST	LOCATION
24	Gunnison	West of Stage Stop Meadows
26	Medicine Bow	Nelson Park Road
27	Medicine Bow	Nelson Park Road
28	Medicine Bow	Nelson Park Road
29	Medicine Bow	Nelson Park Road
30	Medicine Bow	West of Woods Landing
31	Medicine Bow	West of Woods Landing
32	Medicine Bow	West of Woods Landing
34	Medicine Bow	West of Woods Landing
35	Medicine Bow	West of Fox Park
36	Medicine Bow	West of Fox Park
37	Medicine Bow	South of Fox Park
38	Medicine Bow	South of Fox Park
39	Medicine Bow	West of Fox Park
40	Roosevelt	Chambers Lake
41	Roosevelt	Chambers Lake
42	Roosevelt	Chambers Lake
43	Roosevelt	Chambers Lake
44	Roosevelt	Chambers Lake
45	Roosevelt	Rustic to Red Feather Road
46	San Isabel	Twin Lakes to Aspen Road

APPENDIX B. 1--(continued)

PLOT NO.	NATIONAL FOREST	LOCATION
49	San Isabel	East of Independence Pass
50	San Isabel	East of Independence Pass
51	San Isabel	Halfmoon Creek
52	San Isabel	Halfmoon Creek
53	San Isabel	Halfmoon Creek
54	San Isabel	Halfmoon Creek
55	San Isabel	Halfmoon Creek
56	White River	Red Sandstone Creek
57	White River	Red Sandstone Creek
58	White River	Red Sandstone Creek
59	White River	Red Sandstone Creek
60	White River	Red Sandstone Creek

APPENDIX B.2--Field measurements.

Diameter - Breast height diameter was measured to the nearest 1/10 inch with a diameter tape on all lodgepole pine trees on the sample plots.

Height - Total height was measured on four dominant trees at each plot location for site index determination as per instructions in Site Indexes for Lodgepole Pine, with Corrections for Stand Density: instructions for field use by Robert R. Alexander (1966a). Height measurements were made to the nearest one foot with a Haga Altimeter.

Radial growth - Growth measurements were made on all lodgepole pine trees on the sample plots. The outside 30 annual rings were measured to the nearest 1/100 inch on increment cores. Cores were extracted at breast height and in a direction on the tree perpendicular to the slope. Ring counts were made in the field with the aid of a 10X monocular and, when necessary, with a microscope binocular.

Age - Total breast-height age was determined for the five lodgepole pine trees nearest each plot center. Annual ring counts and radius measurements were made on increment cores from borings to the tree pith.

Bark thickness - Bark thickness was measured on the five lodgepole pine trees nearest each plot center. Three breast-height measurements were made on each tree with a bark-thickness measuring gauge. The average of the three measurements was recorded to the nearest 1/20 inch for each tree.

APPENDIX B.2--(continued)

Mortality - Diameter measurements were made on all lodgepole pine trees within the sample plots which appeared to have died within the past 30 years. This judgment was based on observations of the condition of trees cut on thinning operations of known dates.

Crown classification - Each lodgepole pine tree on the sample plots was classified as being dominant, codominant, intermediate, or suppressed.

APPENDIX B.3--Processing of field data.

Data from each plot was summarized and converted to a per-acre basis. Plot values were obtained as follows:

Site Index (SI)--Site index was determined for each plot following the instructions as outlined in Site Indexes for Lodgepole Pine, with Corrections for Stand Density: Instructions for Field Use by Robert R. Alexander (1966a).

Crown Competition Factor (CCF)--Crown Competition Factor was computed for each plot using the equation for lodgepole pine as given by Alexander (1966a): $CCF = 50.58 + 5.25\ (BA/\overline{D})$.

Number of trees per acre (DEN)--Stand density was the total number of living lodgepole pine trees recorded on each plot multiplied by the appropriate blow-up factor for the plot.

Basal Area Per Acre (BA)--Basal area per acre was calculated by summing the basal areas of the lodgepole pine trees on each plot and multiplying the sum by the appropriate blow-up factor for the plot.

Diameter Breast Height (DBH)--DBH refers to the arithmetic average diameter of the trees on each plot.

Stand Age (AGE)--AGE refers to the arithmetic average age of the five lodgepole pine trees nearest each plot center. Nine years was added to the breast height age to obtain total age (Alexander, 1966a).

Dominant Height (DOM HT)--Total tree height was measured on four dominant trees for site index determination. Dominant height is the arithmetic average height of these trees for each plot.

APPENDIX B.3--(continued)

Dominant Age (DOM AGE)--Dominant age is the arithmetic average age of the four dominant trees in each plot which were selected for site index determination. Nine years was added to the breast height age to obtain total age (Alexander, 1966a).

The above values for each plot are listed in Appendix B.4.

APPENDIX B.4--Plot values at time of measurement.

PLOT	SI	CCF	DEN[2]	BA[2]	DBH[2]	STAND[2] AGE	DOM[2] HT	DOM[2] AGE
1	60	194	1031	146	4.9	62	38	62
2	60	207	1217	141	4.4	64	40	65
5	70	186	520	178	7.2	144	75	165
6	60	330	2285	185	3.4	72	42	82
7	30	135	532	101	5.8	97	32	97
8	40	124	366	101	6.9	88	41	93
9	60	259	1797	154	3.5	51	34	60
10	40	136	598	94	5.3	98	39	102
11	60	461	6892	150	1.8	68	31	75
12	60	330	3050	166	3.0	68	38	71
13	50	162	927	95	4.2	73	37	76
14	50	314	2530	167	3.2	67	38	78
15	60	98	297	57	5.5	48	43	61
17	30	115	377	65	4.9	130	48	191
18	50	118	549	50	3.6	53	34	57
19	40	242	1171	179	5.1	144	55	166
20	50	436	4745	180	2.5	83	32	87
21	70	260	1078	235	6.0	82	57	98
22	70	320	1494	200	4.6	81	56	90

[2] Units of measure: DEN, number of trees per acre; BA, square feet per acre; DBH, inches; STAND AGE, years; DOM HT, feet; DOM AGE, years.

APPENDIX B.4--(continued)

PLOT	SI	CCF	DEN	BA	DBH	STAND AGE	DOM HT	DOM AGE
23	60	421	3189	259	3.7	90	44	101
24	40	157	643	110	5.4	89	39	99
26	60	340	2808	166	2.8	61	35	66
27	50	250	1686	142	3.5	67	32	72
28	70	154	479	146	7.0	71	50	73
29	80	253	1232	191	4.7	70	52	72
30	50	134	711	89	4.7	65	34	65
31	50	308	1575	244	5.1	153	50	152
32	70	537	7029	191	2.0	54	29	61
34	70	159	746	135	5.6	62	52	66
35	70	134	399	132	7.6	79	54	82
36	70	543	4375	279	3.1	67	40	78
37	70	197	544	178	6.3	80	72	113
38	60	139	394	154	8.3	153	62	149
39	60	189	1019	101	3.6	72	49	77
40	60	283	1895	182	3.8	64	41	69
41	80	364	2634	222	3.6	63	44	72
42	70	354	2946	204	3.2	55	36	59
43	80	244	917	192	6.0	69	59	75
44	90	462	4437	222	2.7	56	40	65

APPENDIX B.4--(continued)

PLOT	SI	CCF	DEN	BA	DBH	STAND AGE	DOM HT	DOM AGE
45	50	355	2530	187	3.4	78	38	77
46	40	324	2530	155	3.0	78	36	93
49	60	137	422	100	5.8	65	49	79
50	70	257	1155	199	5.3	70	54	84
51	30	350	4375	111	2.0	80	29	100
52	40	137	527	83	5.1	86	40	99
53	50	234	1217	160	4.4	82	48	103
54	60	181	754	158	5.6	67	46	73
55	70	274	1248	259	5.7	88	58	90
56	80	127	325	131	8.3	78	67	80
57	70	161	494	150	7.2	88	64	106
58	80	219	849	182	5.9	77	66	83
59	60	203	737	198	6.7	96	62	108
60	70	204	804	177	6.2	70	56	78

APPENDIX B.5--Procedures for obtaining estimated plot values 30 years prior to measurement and basal area growth rates.

Number of Trees Per Acre at Beginning of Growth Period (DENI). Past stand density was the total number of living lodgepole pine trees on each plot plus the number of trees on the plot recorded as having died within the past 30 years. This sum was then multiplied by the appropriate blow-up factor for the plot to place the value on a per-acre basis.

Diameter Breast Height at Beginning of Growth Period (DBHI). As indicated in Appendix B.2, bark thickness was measured on five sample trees on each plot. Twice the bark thickness was subtracted from the DBH as measured outside the bark for each of these sample trees, yielding present DBH inside the bark.

A regression of present diameter inside bark was run against present outside bark diameter, which provided the equation: DBH inside bark = .955 (DBH outside bark). Present inside bark diameter was calculated for the remainder of the trees on all plots by using this equation.

Past diameter inside bark was calculated for all trees by subtracting the 30-year diameter growth from the present diameter inside bark values. Past diameter outside bark was then calculated for each tree by entering the above equation with the past inside bark value.

Basal Area Per Acre at Beginning of Growth Period (BAI). Past basal area for all living trees was calculated by summing the basal areas corresponding to the past diameter figures as obtained above. The basal area of the 30-year mortality for each plot was added to this value to obtain past basal area per plot. The appropiate

APPENDIX B.5--(continued)

blow-up factor was then applied to each plot basal area to convert to a per-acre basis.

Crown Competition Factor at Beginning of Growth Period (CCFI). CCFI was computed for each plot using the equation for lodgepole pine as given by Alexander (1966a): $CCF = 50.58 + 5.25 (BA/\bar{D})$. The past basal area per acre and past average diameter values as determined above were used in the equation.

Basal Area Growth Rate (BAG). (in square feet/acre/year). BAG = (BA - BAI)/30 years.

Basal Area Growth Rate Per Tree (BAG/T). (in square feet/tree/year). BAG/T = BAG/DENI.

Past Stand Age (AGEI). AGEI = AGE - 30 years.

APPENDIX B.6--Estimated plot values 30 years prior to measurement and growth rates per tree for the study period.

PLOT	SI	CCFI	DENI[3]	BAI[3]	DBHI[3]	AGEI[3]	BAG/T[3]
1	60	174	1083	80	3.4	32	.0020
2	60	191	1356	83	3.1	34	.0014
5	70	193	644	171	6.3	114	.0004
6	60	320	2684	118	2.3	42	.0008
7	30	125	532	64	4.5	67	.0022
8	40	120	394	73	5.5	58	.0024
9	60	211	1797	52	1.7	21	.0019
10	40	127	615	61	4.2	68	.0018
11	60	413	9025	69	1.0	38	.0003
12	60	313	3986	100	2.0	38	.0006
13	50	152	1014	60	3.1	43	.0012
14	50	299	2981	104	2.2	37	.0007
15	60	83	297	15	2.4	18	.0047
17	30	108	397	37	3.4	100	.0023
18	50	106	555	17	1.6	23	.0020
19	40	251	1479	160	4.2	114	.0004
20	50	448	6717	121	1.6	53	.0003
21	70	264	1402	187	4.6	52	.0011
22	70	402	2403	154	2.3	51	.0006

[3] Units of measure: DENI, number of trees per acre; BAI, square feet per acre; DBHI, inches; AGEI, years; BAG/T, square feet.

APPENDIX B.6--(continued)

PLOT	SI	CCFI	DENI	BAI	DBHI	AGEI	BAG/T
23	60	426	4264	186	2.6	60	.0006
24	40	157	765	89	4.4	59	.0009
26	60	304	2877	82	1.7	31	.0010
27	50	231	1708	86	2.5	37	.0011
28	70	147	514	97	5.3	41	.0032
29	80	239	1325	129	3.6	40	.0016
30	50	101	711	21	2.2	35	.0032
31	50	317	2019	203	4.0	123	.0007
32	70	509	9244	96	1.1	24	.0003
34	70	131	753	55	3.6	32	.0035
35	70	124	399	85	6.1	49	.0039
36	70	579	7148	171	1.7	37	.0002
37	70	194	627	131	4.8	50	.0025
38	60	129	394	100	6.7	123	.0046
39	60	174	1200	47	2.0	42	.0015
40	60	255	1957	101	2.6	34	.0014
41	80	347	3189	130	2.3	33	.0010
42	70	302	3224	91	1.9	25	.0012
43	80	294	1392	144	3.1	39	.0011
44	90	425	5485	107	1.5	26	.0007
45	50	381	4506	126	2.0	48	.0004

APPENDIX B.6--(continued)

PLOT	SI	CCFI	DENI	BAI	DBHI	AGEI	BAG/T
46	40	326	3397	105	2.0	48	.0005
49	60	130	449	59	3.9	35	.0031
50	70	268	1510	170	4.1	40	.0006
51	30	362	6347	77	1.3	50	.0002
52	40	137	632	66	4.0	56	.0009
53	50	224	1356	112	3.4	52	.0012
54	60	156	763	76	3.8	37	.0036
55	70	254	1433	163	4.2	58	.0022
56	80	118	335	82	6.4	48	.0049
57	70	161	627	116	5.5	58	.0018
58	80	229	1234	136	4.0	47	.0012
59	60	199	849	153	5.4	66	.0018
60	70	209	1143	124	4.1	40	.0015

APPENDIX C

Data and information pertaining to the construction of the upper stocking curve

APPENDIX C.1--Analysis of variance summary for testing the significance of the regression between basal area growth per tree and initial Crown Competition Factor.

SOURCE	D. F.	S. S.	M. S.	F
Total	52	65,012,310		
Regression	1	40,086,518	40,086,518	
Residual	51	24,925,792	488,741	82**

APPENDIX C.2--Analysis of variance summaries for testing the significance of the regressions between basal area growth per tree and initial Crown Competition Factor by diameter classes.

ONE-INCH DIAMETER CLASS

SOURCE	D. F.	S.S.	M. S.	F
Total	9	.00003171		
Regression	1	.00001651	.00001651	
Residual	8	.00001520	.00000190	8.6895*

TWO-INCH DIAMETER CLASS

SOURCE	D. F.	S.S.	M. S.	F
Total	12	.00011077		
Regression	1	.00007385	.00007385	
Residual	11	.00003692	.000002993	24.67**

THREE-INCH DIAMETER CLASS

SOURCE	D. F.	S.S.	M. S.	F
Total	9	.00002130		
Regression	1	.00001360	.00001360	
Residual	8	.00000770	.000000962	14.14**

APPENDIX C.2--(continued)

FOUR-INCH DIAMETER CLASS

SOURCE	D. F.	S. S.	M. S.	F
Total	11	.00001110		
Regression	1	.00000931	.00000931	
Residual	10	.00000179	.000000179	52**

FIVE-INCH DIAMETER CLASS

SOURCE	D. F.	S. S.	M. S.	F
Total	3	.00000382		
Regression	1	.00000321	.00000321	
Residual	2	.00000061	.000000305	10.52 N.S.

SIX-INCH DIAMETER CLASS

SOURCE	D. F.	S. S.	M. S.	F
Total	3	.00000436		
Regression	1	.00000415	.00000415	
Residual	2	.00000021	.000000105	39.52*

APPENDIX C.3--Tabulation of values used to obtain basal area growth per acre over initial Crown Competition Factor curve for each diameter class.

CCFI	BAG/T (sq. ft.)	BA/A (sq. ft.)	NUMBER OF TREES/ACRE	BAG/A (sq. ft.)
ONE-INCH DIAMETER CLASS				
100	.00366	14.12	1177	4.30782
200	.00269	42.69	3558	9.57102
250	.00221	56.98	4748	10.49308
260	.00211	59.83	4986	10.52046
265	.00207	61.26	5105	10.56735***[4]
270	.00202	62.69	5224	10.55248
275	.00197	64.12	5343	10.52571
300	.00173	71.26	5938	10.27274
350	.00125	85.55	7129	8.91125
400	.00077	99.83	8320	6.40640
TWO-INCH DIAMETER CLASS				
100	.00618	20.71	796	4.91928
175	.00442	52.14	2005	8.86210
195	.00395	60.52	2328	9.19560
200	.00384	62.61	2408	9.24672
205	.00372	64.71	2489	9.25908***
210	.00360	66.80	2569	9.24840
225	.00325	73.08	2811	9.13575
250	.00266	83.57	3214	8.54924
300	.00149	104.52	4020	5.98980

[4] Asterisks denote peak of basal area growth per acre.

APPENDIX C.3--(continued)

CCFI	BAG/T (sq. ft.)	BA/A (sq. ft.)	NUMBER OF TREES/ACRE	BAG/A (sq. ft.)
		THREE-INCH DIAMETER CLASS		
100	.00376	32.01	508	1.91008
125	.00323	48.20	765	2.47095
150	.00270	64.39	1022	2.75940
160	.00248	70.86	1125	2.79000
165	.00238	74.10	1176	2.79888***
170	.00227	77.34	1228	2.78756
175	.00216	80.58	1279	2.76264
185	.00195	87.05	1382	2.69490
200	.00163	99.91	1554	2.53302
250	.00057	130.67	2074	1.18218
		FOUR-INCH DIAMETER CLASS		
100	.0032	40.48	401	1.2832
125	.0028	60.95	603	1.6884
150	.0024	81.43	806	1.9344
165	.0022	93.72	928	2.0416
170	.00213	97.81	968	2.06184
175	.0021	101.91	1009	2.1189***
180	.00199	106.00	1050	2.0895
185	.0019	110.10	1090	2.0710
200	.0017	122.38	1212	2.0604
250	.0010	163.33	1617	1.6170

APPENDIX C.3--(continued)

CCFI	BAG/T (sq. ft.)	BA/A (sq. ft.)	NUMBER OF TREES/ACRE	BAG/A (sq. ft.)
		FIVE-INCH DIAMETER CLASS		
100	.0040	50.83	320	1.2800
125	.0032	76.55	482	1.5424
130	.0030	81.69	514	1.5420
135	.0029	86.83	546	1.5834***
140	.0027	91.98	578	1.5606
150	.0024	102.26	643	1.5432
160	.0021	112.55	708	1.4868
175	.0016	127.97	805	1.2880
		SIX-INCH DIAMETER CLASS		
100	.0042	60.24	270	1.1340
120	.0036	84.63	380	1.3680
130	.0032	96.82	434	1.3888
135	.0031	102.91	461	1.4291***
140	.0029	109.01	489	1.4181
145	.0027	115.10	516	1.3932
150	.0026	121.20	543	1.4118
175	.0017	151.67	680	1.1560

APPENDIX C.4--Analysis of variance summary for testing the significance of the regression between basal area per acre at the peak of annual growth and Crown Competition Factor.

SOURCE	D.F.	S.S.	M.S.	F
Total	52	.00036543		
Regression	1	.00022029	.00022029	
Residual	51	.00014514	.00000285	77.29**

APPENDIX C.5--Analysis of variance summary for testing the significance of the regression between basal area per acre at the peak of annual growth and average class diameter.

SOURCE	D.F.	S.S.	M.S.	F
Total	52	14,945		
Regression	1	11,371	11,371	
Residual	51	3,574	70.08	162.26**

APPENDIX D

Information and data pertaining to the

lower stocking curve

APPENDIX D.1--Crown width and diameter breast height measurements of open-grown trees.[5]

TREE NO.	DBH (inches)	AVE. CROWN WIDTH (ft.)	TREE NO.	DBH (inches)	AVE. CROWN WIDTH (ft.)
1	18.1	26.70	35	9.3	18.10
2	16.1	22.15	36	10.5	15.70
3	14.4	24.30	37	4.9	6.90
4	14.6	24.55	38	8.4	16.30
5	13.9	22.70	39	7.1	16.05
6	16.2	24.00	40	12.7	18.30
7	16.3	25.60	41	7.4	15.75
8	14.2	28.15	42	7.1	14.35
9	6.9	14.20	43	5.8	12.35
10	4.1	8.65	44	11.6	18.55
11	17.6	27.60	45	6.4	11.65
12	13.7	24.35	46	5.5	10.65
13	21.6	27.85	47	8.9	17.50
14	23.6	29.70	48	7.4	14.95
15	10.6	16.20	49	5.9	12.20
16	17.9	25.40	50	3.1	7.20
17	7.0	15.20	51	6.2	12.85
18	3.9	6.75	52	3.7	7.45
19	18.0	30.65	53	2.4	7.55
20	14.1	24.60	54	8.4	16.25
21	18.0	32.20	55	7.5	14.00
22	16.4	26.55	56	1.6	5.30
23	17.7	28.15	57	8.8	16.45
24	16.0	26.05	58	6.5	11.15
25	9.0	17.00	59	3.3	6.25
26	14.9	23.60	60	10.6	16.60
27	15.6	26.45	61	6.2	13.15
28	13.9	21.50	62	8.1	19.50
29	17.1	21.05	63	8.1	17.15
30	14.0	23.65	64	6.1	13.10
31	13.2	18.50	65	8.7	14.75
32	12.7	25.15	66	4.5	9.35
33	10.9	19.50	67	2.7	6.10
34	9.4	21.00	68	3.6	9.05

[5] Trees numbers 1 through 144 were measured on the Arapaho National Forest; numbers 145 through 265 were measured on the Roosevelt National Forest. Data supplied by Robert R. Alexander of the Rocky Mountain Forest and Range Experiment Station.

APPENDIX D.1--(continued)

TREE NO.	DBH (inches)	AVE. CROWN WIDTH (ft.)	TREE NO.	DBH (inches)	AVE. CROWN WIDTH (ft.)
69	9.7	15.30	110	14.6	26.35
70	10.2	17.30	111	13.0	24.45
71	10.3	16.15	112	16.5	27.50
72	11.6	19.70	113	16.6	28.55
73	9.7	16.15	114	15.6	26.70
74	6.7	12.30	115	16.6	29.65
75	3.2	8.35	116	15.7	27.45
76	6.8	11.45	117	11.5	20.40
77	6.0	11.95	118	11.9	22.65
78	10.7	19.10	119	13.0	23.25
79	9.4	15.20	120	14.1	25.65
80	4.2	9.40	121	15.7	28.95
81	20.5	31.25	122	13.3	23.85
82	20.6	32.95	123	13.5	25.10
83	13.5	23.45	124	16.5	26.25
84	11.1	21.15	125	7.7	13.50
85	17.4	29.80	126	9.1	15.35
86	20.2	30.00	127	10.7	21.90
87	18.5	28.25	128	9.3	18.70
88	19.0	30.10	129	0.9	5.85
89	17.2	27.30	130	6.4	12.30
90	13.0	22.55	131	1.6	8.05
91	15.0	25.30	132	8.1	13.20
92	19.7	32.25	133	8.6	14.65
93	20.2	38.90	134	8.1	16.10
94	14.6	27.25	135	9.3	15.30
95	14.8	24.35	136	9.2	16.85
96	14.3	27.70	137	8.3	15.20
97	13.2	23.10	138	2.2	5.95
98	15.2	30.05	139	1.3	4.50
99	16.2	28.90	140	2.1	5.80
100	17.6	31.35	141	3.4	9.10
101	15.5	28.25	142	3.9	10.65
102	15.0	22.90	143	2.5	7.50
103	17.9	24.90	144	2.7	6.75
104	16.8	30.25	145	14.6	29.30
105	18.4	29.10	146	12.2	24.35
106	18.5	31.30	147	11.8	24.40
107	14.9	26.45	148	5.4	10.50
108	12.5	20.30	149	13.2	27.95
109	13.5	23.35	150	12.9	22.25

APPENDIX D.1--(continued)

TREE NO.	DBH (inches)	AVE. CROWN WIDTH (ft.)	TREE NO.	DBH (inches)	AVE. CROWN WIDTH (ft.)
151	12.0	19.15	192	8.9	19.05
152	13.5	21.20	193	12.8	24.75
153	5.4	13.10	194	11.8	19.20
154	10.6	19.85	195	8.6	19.15
155	10.7	26.50	196	11.9	22.65
156	6.3	14.05	197	12.0	24.50
157	13.4	21.05	198	10.5	20.85
158	2.4	6.25	199	9.9	19.35
159	8.9	13.75	200	10.3	21.20
160	6.4	11.65	201	9.2	20.65
161	6.0	10.50	202	7.6	13.55
162	12.4	21.75	203	13.4	25.55
163	11.4	21.65	204	2.5	5.95
164	13.5	21.35	205	8.0	16.15
165	10.7	19.05	206	7.2	15.25
166	11.8	20.68	207	6.8	16.55
167	13.3	23.95	208	12.5	22.65
168	12.6	22.50	209	8.9	18.00
169	8.5	14.10	210	9.0	18.75
170	14.4	26.18	211	9.2	16.85
171	13.1	22.80	212	12.5	24.65
172	11.0	19.65	213	9.4	17.20
173	7.0	12.65	214	8.2	15.50
174	14.7	33.85	215	12.3	19.95
175	9.6	20.70	216	3.4	11.45
176	4.9	9.75	217	15.7	24.75
177	9.9	20.95	218	17.5	28.45
178	8.7	13.95	219	14.9	22.35
179	12.0	21.75	220	15.6	24.00
180	17.1	25.80	221	4.9	9.65
181	12.1	25.90	222	11.1	19.60
182	6.1	11.65	223	11.8	21.35
183	7.0	12.95	224	9.9	17.55
184	2.2	6.25	225	14.0	23.70
185	9.3	16.80	226	10.9	18.36
186	9.1	16.20	227	18.3	32.25
187	11.5	20.05	228	13.9	23.65
188	10.6	19.75	229	10.3	21.55
189	8.9	17.45	230	16.9	24.90
190	9.0	18.95	231	12.7	21.80
191	12.6	26.05	232	12.9	21.70

APPENDIX D.1--(continued)

TREE NO.	DBH (inches)	AVE. CROWN WIDTH (ft.)	TREE NO.	DBH (inches)	AVE. CROWN WIDTH (ft.)
233	13.4	23.05	249	15.0	25.35
234	17.0	29.35	250	13.6	24.55
235	14.3	23.75	251	15.9	26.00
236	11.5	22.00	252	13.8	23.80
237	14.1	22.55	253	15.1	27.05
238	14.7	25.05	254	14.6	24.30
239	11.5	22.25	255	14.3	24.05
240	16.0	24.60	256	14.9	25.35
241	15.0	23.15	257	7.6	16.25
242	4.6	9.80	258	10.7	19.85
243	11.8	21.45	259	10.6	21.05
244	12.5	21.90	260	13.6	26.85
245	16.8	29.50	261	7.4	14.85
246	16.2	30.15	262	13.0	22.55
247	15.9	28.15	263	11.0	21.95
248	15.7	26.05	264	11.6	24.85
			265	12.8	24.25

APPENDIX D.2--Analysis of variance summary for testing the significance of the regression between crown width and diameter breast height.

SOURCE	D. F.	S. S.	M. S.	F
Total	264	12,372.185		
Regression	1	11,185.950	11,185.950	
Residual	263	1,186.235	4.510	2,480**

APPENDIX E

Information and data pertaining to the rate of stocking change over time

APPENDIX E.1--Average ages and diameters by decadal age classes.

AGE CLASS	AVERAGE AGE (years)	AVERAGE DBH (inches)	NUMBER OF PLOTS PER CLASS
20	23	1.45	5
30	35	2.61	15
40	44	3.75	12
50	54	3.78	11
60	65	4.18	4
70	--	----	0
80	--	----	0
90	91	7.60	1
100	100	3.40	1
110	114	5.25	2
120	123	5.35	2

APPENDIX E.2--Analysis of variance summary for testing the significance of the regression between diameter breast height and the logarithm of stand age.

SOURCE	D.F.	S.S.	M.S.	F
Total	52	69.68		
Regression	1	55.91	55.91	
Residual	51	13.77	.27	207**

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