International Journal of Digital Earth

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/tjde20

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To cite this article: Mohammad Azarafza, Haluk Akgün, Akbar Ghazifard, Ebrahim Asghari-Kaljahi, Jafar Rahnamarad & Reza Derakhshani (2021) Discontinuous rock slope stability analysis by limit equilibrium approaches – a review, International Journal of Digital Earth, 14:12, 1918-1941, DOI: 10.1080/17538947.2021.1988163

To link to this article: https://doi.org/10.1080/17538947.2021.1988163

Published online: 12 Oct 2021.

INTERNATIONAL JOURNAL OF DIGITAL EARTH
2021, VOL. 14, NO. 12, 1918–1941
https://doi.org/10.1080/17538947.2021.1988163

REVIEW ARTICLE

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Discontinuous rock slope stability analysis by limit equilibrium approaches – a review

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ABSTRACT

Slope stability is one of the most important topics of engineering geology with a background of more than 300 years. So far, various stability assessment techniques have been developed which include a range of simple evaluations, planar failure, limit state criteria, limit equilibrium analysis, numerical methods, hybrid and high-order approaches which are implemented in two-dimensional (2D) and three-dimensional (3D) space. In the meantime, limit equilibrium methods due to their simplicity, short analysis time, coupled with probabilistic and statistics functions to estimate the safety factor (F.S), probable slip surface, application on different failure mechanisms, and varied geological conditions has been received special attention from researchers. The presented paper provides a review to limit equilibrium methods used for discontinuous rock slope stability analyses with different failure mechanisms of natural and cut slopes. The article attempted to provide a systematic review for rock slope stability analysis outlook based on limit equilibrium approaches.

ARTICLE HISTORY
Received 11 June 2021
Accepted 23 September 2021

KEYWORDS
Engineering geology; natural slope; block theory; failure mechanisms

1. Introduction

The slope stability is considered as a most extensive description for soil and rock (or combination of both) slope masses under various failures (i.e. Akgün and Koçkar 2004; Bertolini 2010; Johari, Fazeli, and Javadi 2013, 2015, 2016, 2017; Zahri et al. 2016; Dahoua, Savenko, and Hadji 2017; El-Mekki, Hadji, and Fehdi 2017; Dahoua et al. 2018; Zeqiri et al. 2019; Saadoun et al. 2020; Fredj et al. 2020). These movements can cause damages under certain conditions (Bromhead 1992; Azarafza, Akgün, and Asghari-Kaljahi 2016; Yang and Liu 2018; Yamaguchi, Takeuchi, and Hamasaki 2018; Zhu and Yang 2018; Hou et al. 2019). These conditions can be classified related to geometry and geo-material status which determine the behavior and the critical slip surface expansion. Figure 1 shows various types of possible slip surfaces based on limit equilibrium analysis approaches which are directly depending on the geological conditions of the slope mass (Huang 2014). The existence of such a slip surface complicates the stability analysis and demands to consider more assumptions for covering the existing uncertainties. Therefore, the methods used in stability analysis have undergone a number of changes and improvements. In this task, the

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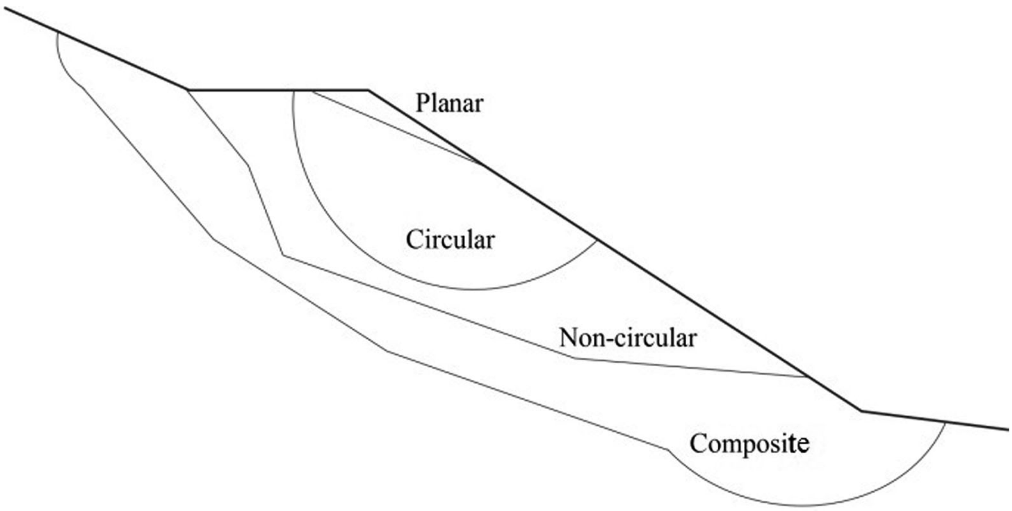

Figure 1. Various types of slip surfaces in slopes (Huang 2014).

theoretical foundations and a literature review of stability analysis using the limit equilibrium method are presented herein which are successfully used for accurate description of slope failure mechanisms and stability analysis.

In general, the type of slope failure directly depends on the geological units and constituent geomaterials, geometric characteristics, stress–strain history, structural and tectonic conditions, geomorphology status, regional climate, seismic activity, water conditions (surface and underground), vegetation, weathering, drainage pattern, construction activities and special occasions (Qin and Chian 2018; Ersöz and Topal 2018; Li, Yang, and Li 2020; Yang et al. 2020; Ansari et al. 2021). According to the nature of the slippery mass, it can be stated that the slope failure mode in isotropic and homogeneous masses such as soils is mostly rotational (or massive failure). If there is a rigid surface or resistant layer, they occur in planar form (Abramson et al. 2001). Depending on the discontinuity network, joint orientation, infilling material, the rock masses undergo a wide range of failures such as wedge failure, planar failure, rotational failure, and toppling failure (Hoek 2006). Most commonly, failure occurrence in slopes can be classified into the main and secondary groups. The main group consists of wedge failure, toppling failure (rock slopes), planar failure, rotational failure (rock and earth slopes) and the second group consists of composite slips, special cases, glacial slips, etc. (Bromhead 1992).

Wedge failure: This failure type is one of the most common failure types in the discontinuous rock masses and it occurs based on the slip geometry in wedge form leading to tilt towards the slope face with a deviation degree less than the free face of the slope which slides along the intersection of the discontinuities downwards the slope(Zhou and Wang 2017).

Planar failure: This failure type is also a special case of slip that occurs because of the geological conditions and the geometry of the discontinuities, where slip occurs along a surface on the main discontinuity or a layer that is more related to the geotechnical conditions from the upper mass (Wyllie and Mah 2004).

Toppling failure: Occurs in rock slopes where the discontinuity orientation with a deviation degree close to the vertex and opposite to the excavation orientation/slope faces occurs. Because of the sliding gravity geometric center positioning, the blocks are extruded along the slope driven by gravity, which is referred to as toppling (Nikoobakht and Azarafza 2016).

Rotational failure: This type of slope movement is observed inhomogeneous and isotropic masses such as soils, severely discontinuous, and weak rock masses. The occurrence mechanism of rotational movement is to form a shear surface that engulfs the entire mass as a curvature or circle and may involve several smaller rotational slips in the mass body (Wyllie and Mah 2004).

The objective of this paper is to provide a review of limit equilibrium-based methods for discontinuous rock slope stability analyses (i.e. wedge, planar, toppling failure) and for weak rock or soil-like lithologies (i.e. mass failure) of natural slopes as well as of cut slopes (i.e. Azarafza, Feizi-Derakhshi, and Jeddi 2017; Bagheri Shendi and Azarafza 2018; Deng 2020; Shariati and Fereidooni 2021). The prepared article provides information about the limit equilibrium procedure to estimate the failure mechanisms in rock slopes. The main focuses on this matter are to provide principles and applications of stability analysis procedures especially, of the most recently developed methods named 'Block theory'.

2. Slope stability analysis by limit equilibrium methods

Limit equilibrium analysis methods (LEMs) are one of the basic and old analytical approaches for slope stability analyses that are widely used in slope stability studies because of their simplicity, low complexity in the formulation, and less analysis time. LEMs based on massive analysis or slices investigate a possible slippery mass at the top of the assumed slip surface, and the polyhedral force vector closure or incurring moments in equilibrium state which are capable to be utilized in static and dynamic conditions for two-dimensional and three-dimensional space. If these polyhedral forces are closed and all assumptions/requirements are provided, this implies that the mass is in equilibrium and that the analysis is valid. The non-closure of the polyhedral forces/moments indicates the lack of balance or lack of satisfaction of some effective parameters in it.

$$\mathrm{F.S} = \frac{\sum \text{Resistance forces or moments}}{\sum \text{Activation forces or moments}} \tag{1}$$

The various equilibrium methods utilized are presented in Table 1. In this table, the basic failure mechanisms are considered and categorized for all states. Many of the LEMs provide close results in calculating the Factor of Safety (F.S) and the difference in the estimated values is usually less than 6% (Duncan 1996). The majority of the limit equilibrium approaches presented in this table use the Mohr-Coulomb relation to estimating the shear stress and resistance across the slip surface in all types of failures, where this criterion is considered as one of the most important failure criteria for stability analyses in geo-materials. As presented in Table 1, the limit equilibrium methods for various failure mechanisms are under the heading of wedge failure, planar failure, toppling failure, and rotational failure. This implies that various assumptions are introduced into the limit equilibrium stability analyses depending on the failure types.

Abramson et al. (2001) have presented three methods, namely limit method, force equilibrium method, and moment equilibrium method to define the F.S coefficient in different slip surfaces based on a limit equilibrium where shear stress /resistance is considered as total resistance-stress or effective resistance-stress. In the force equilibrium method, the ratio of the resisting forces to the mobilized forces at the possible slip surface is investigated and in the moment method, through comparing the resistant moments to the overturning moments, the reliability of the slope is estimated. According to their achievements, these methods are capable to be applied to various types of slope failures or complex movements.

2.1. Wedge failure stability analysis

Wedge failure in specific geometric and geological conditions moves downward from the cairn along the intersection of two discontinuity planes where the discontinuity status with respect to

Table 1. The most important limit equilibrium methods used in slope stability analysis.

LEMs	Equilibrium conditions satisfied	Slip surface	Failure mechanism	Application	References
OMS/Fellenius method	Moment equilibrium about the circle center	Circular	Rotational	Mass/Slice approach	Fellenius (1936)
Simplified Bishop method	Vertical force equilibrium and moment equilibrium about the center	Circular	Rotational	Mass/Slice approach	Bishop (1955)
Extended Bishop method	Moment equilibrium about the center	Circular	All	Mass/Slice approach	Nonveiller (1965)
Lorimer method	Vertical force equilibrium and moment equilibrium about the center	Circular	All	Slice approach	Fredlund, Krahn, and Pufahl (1981)
Simplified Janbu method	Vertical and horizontal force equilibrium and shear interslice force is assumed ZERO	General shape	All	Mass/Slice approach	Janbu (1954)
Modified Swedish method	Vertical and horizontal force equilibrium	General shape	All	Slice approach	USACE (2003)
USACE's 1970 procedure	Vertical and horizontal force equilibrium and interslice force inclination is parallel with ground	General shape	All	Slice approach	USACE (2003)
Lowe–Karafiath method	Horizontal and vertical force equilibrium and interslice force inclination is equal with slip and ground surfaces	General shape	All	Slice approach	Lowe and Karafiath (1960)
Sarma method I	Vertical and horizontal force equilibrium and shear strength on the interface between adjacent slices and	General shape	All	Slice approach	Sarma (1979)
Spencer method	Rigorous limiting equilibrium and interslice force inclination is constant	General shape	All	Slice approach	Spencer (1967)
Morgenstern – Price method	Rigorous equilibrium by interslice force function	General shape	All	Slice approach	Morgenstern and Price (1965)
Sarma method II and III	Rigorous equilibrium of extended Sarma method I	General shape	All	Slice approach	Sarma (1973)
Correia method	Rigorous equilibrium and shear interslice force described by shapes function and force dimension	General shape	All	Slice approach	Correia (1988)
Rigorous Janbu method	All the force and moment conditions are equilibrium	General shape	All	Slice approach	Janbu (1954); Janbu, Bjerrum, and Kjaernsli (1956)
USACE's 2003 procedure	Improvement of USACE's 1970 procedure	General shape	All	Slice approach	USACE (2003)
Wedge method	Fully satisfies the vertical and horizontal force equilibrium	General shape	Wedge	Zone approach	Abramson et al. (2001)
Infinite slope method	Horizontal and vertical force equilibrium	Planar	Plane	Critical circle	USACE (2003)
Planar failure analysis	Horizontal and vertical force equilibrium	Planar	Plane	Geometry controlled	Hoek and Bray (1981)
Wedge failure analysis	Horizontal and vertical force equilibrium	Wedge	Wedge	Geometry controlled	Brady and Brown (2005)
Circular failure analysis	Horizontal and vertical force equilibrium	Circular	Rotational	Mass/Slice approach	Wyllie and Mah (2004)
Toppling failure analysis	Vertical and horizontal force equilibrium and moment equilibrium	Rotation	Toppling	Geometry controlled	Freitas and Watters (1973)
Block theory	Geometrical equilibrium and force/moment vectors equilibrium	General shape	All	Geometry controlled	Goodman and Shi (1985)

the slope is oblique. This movement is due to the geometric three-dimensional structure and discontinuity mechanics in the rock mass. In Figure 2, the geometrical conditions of a wedge failure are presented (Wyllie and Mah 2004). The values of the tendency α_i and intersection angle ψ_i in the

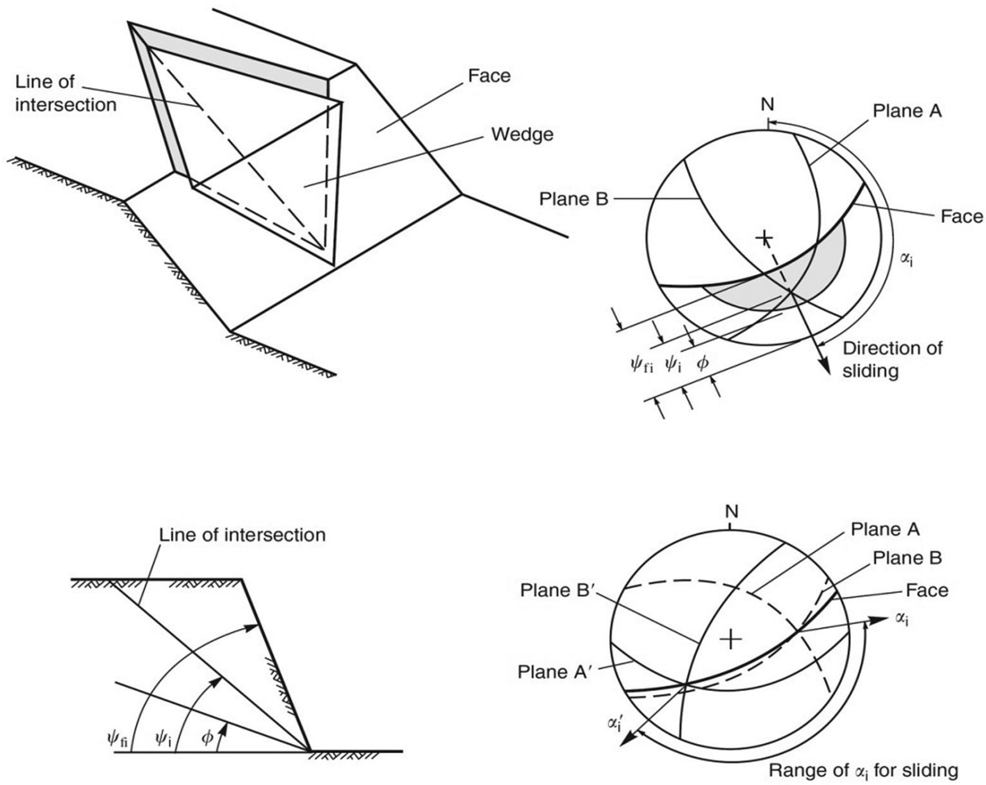

Figure 2. Geometrical conditions in wedge failure analysis (Wyllie and Mah 2004).

wedge sliding can be calculated as follows:

$$\alpha_i = \arctan\left[\frac{\tan\ \psi_A \cos\ \alpha_A - \tan\psi_B \cos\alpha_B}{\tan\psi_B \sin\alpha_B - \tan\ \psi_A \sin\alpha_A}\right] \tag{2}$$

$$\psi_i = \tan\ \psi_A\ \cos\ (\alpha_A - \alpha_i) = \tan\ \psi_B\ \cos\ (\alpha_B - \alpha_i) \tag{3}$$

where α_A and α_B are the dip directions, and ψ_A and ψ_B are the dips of the two joint planes. To define the confidence coefficient for the slipping wedge using the limit equilibrium method can be estimated:

$$F.S = \frac{\sin\ \beta}{\sin\ (\xi/2)} \cdot \frac{\tan\ \varphi}{\tan\ \psi_i} \tag{4}$$

In the above relation, ϕ is friction; ξ is vectored distance from discontinuity to weight component, β and ψ_i are evaluated by Equation (3) as well as illustrated in Figure 3.

2.2. Planar failure stability analysis

Planar failure can be considered as a special case of wedge failure, in which the joint plane is aligned along 180 degrees. For planar failure to occur – as shown in Figure 4(Wyllie and Mah 2004).

To analyze the stability and to calculate the confidence coefficient for the planar failure shown in Figure 5, using the limit equilibrium approach, we have:

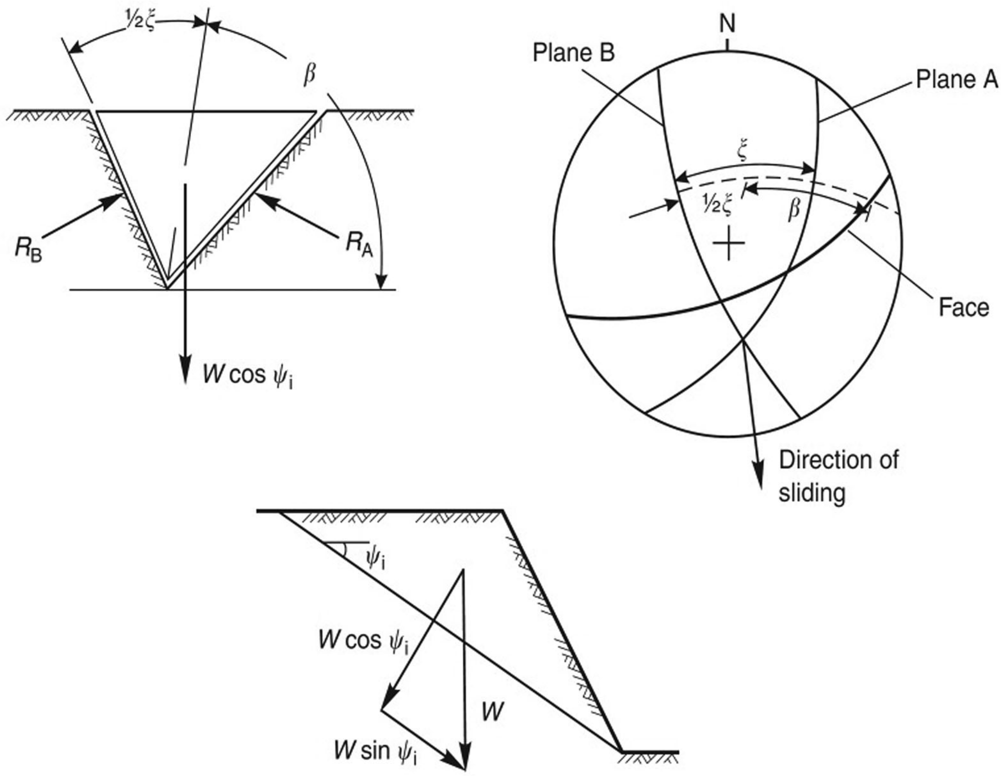

Figure 3. Limit equilibrium analysis and force polyhedral in wedge failure (Wyllie and Mah 2004).

$$F.S = \frac{c[(H + b\tan\psi_s - z)\,\mathrm{cosec}\ \psi] + \left[W\ \cos\ \psi_p - U - \frac{1}{2}\gamma_w z_w^2 \sin\psi_p\right]\tan\ \varphi}{W\sin\psi_p\ + \frac{1}{2}\gamma_w z_w^2 \cos\ \psi_p} \tag{5}$$

$$U = \frac{1}{2}\gamma_w z_w\,(H + b\tan\psi_s - z)\,\mathrm{cosec}\,\psi_p \tag{6}$$

$$W = \gamma_r\left[(1 - \cot\psi_f\tan\psi_p)(bH + \frac{1}{2}H^2\cot\psi_f) + \frac{1}{2}b^2(\tan\ \psi_s - \tan\psi_p)\right] \tag{7}$$

In the above relations, c is cohesion, A is the area, H is slope height, z is tension crack depth, U is the uplift water pressure, γ_w water specific weight. The other parameters can be estimated from Figure 5.

2.3. Toppling failure stability analysis

Toppling failure occurs as a column or rock block rotation around its constant base. Since for toppling failure, the resistance and shear stress embattled at the base surface does not play any role in mass stability, it is not practically possible to evaluate this type of failure by limit equilibrium approaches. Hence, to evaluate this type of failure, a kinematic analysis method is usually used. Müller (1964, 1968) was the first researcher who studied the occurrence of overturning phenomenon in the Vaiont Dam Lake. Ashby (1971), for the first time, introduced this type of failure as 'toppling'. Goodman and Bray (1976) investigated various types of toppling failures, classified and presented mathematical solutions for themes (Sageseta, Sánchez, and Cañizal 2001). The

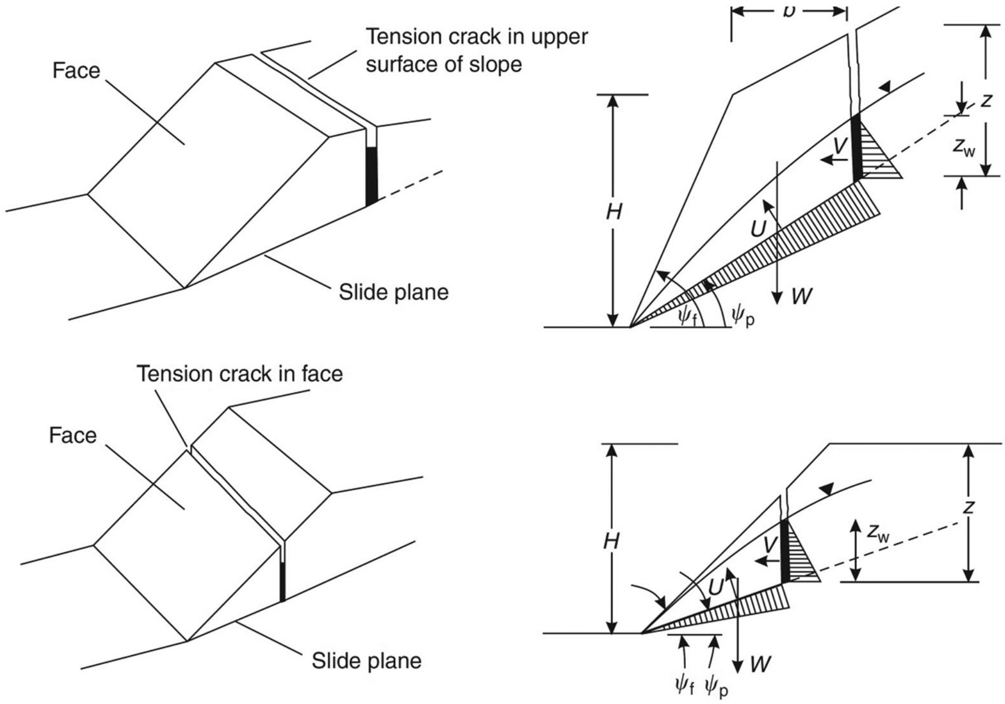

Figure 4. Geometrical conditions in planar failure analysis (Wyllie and Mah 2004).

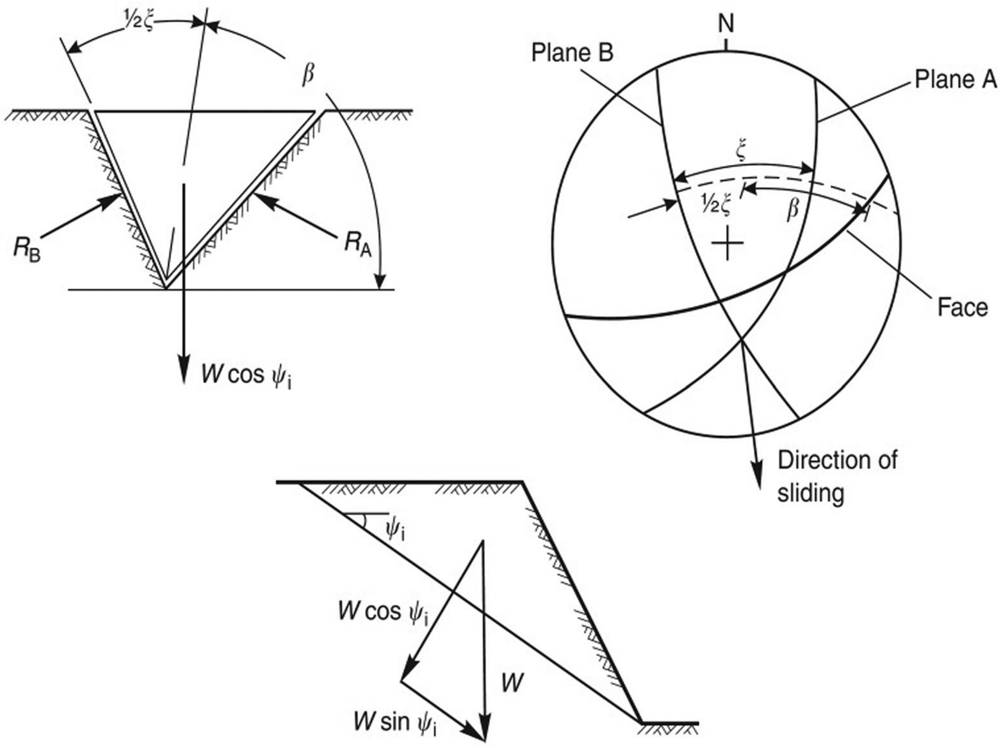

Figure 5. Limit equilibrium analysis and force polyhedral in planar failure (Wyllie and Mah 2004).

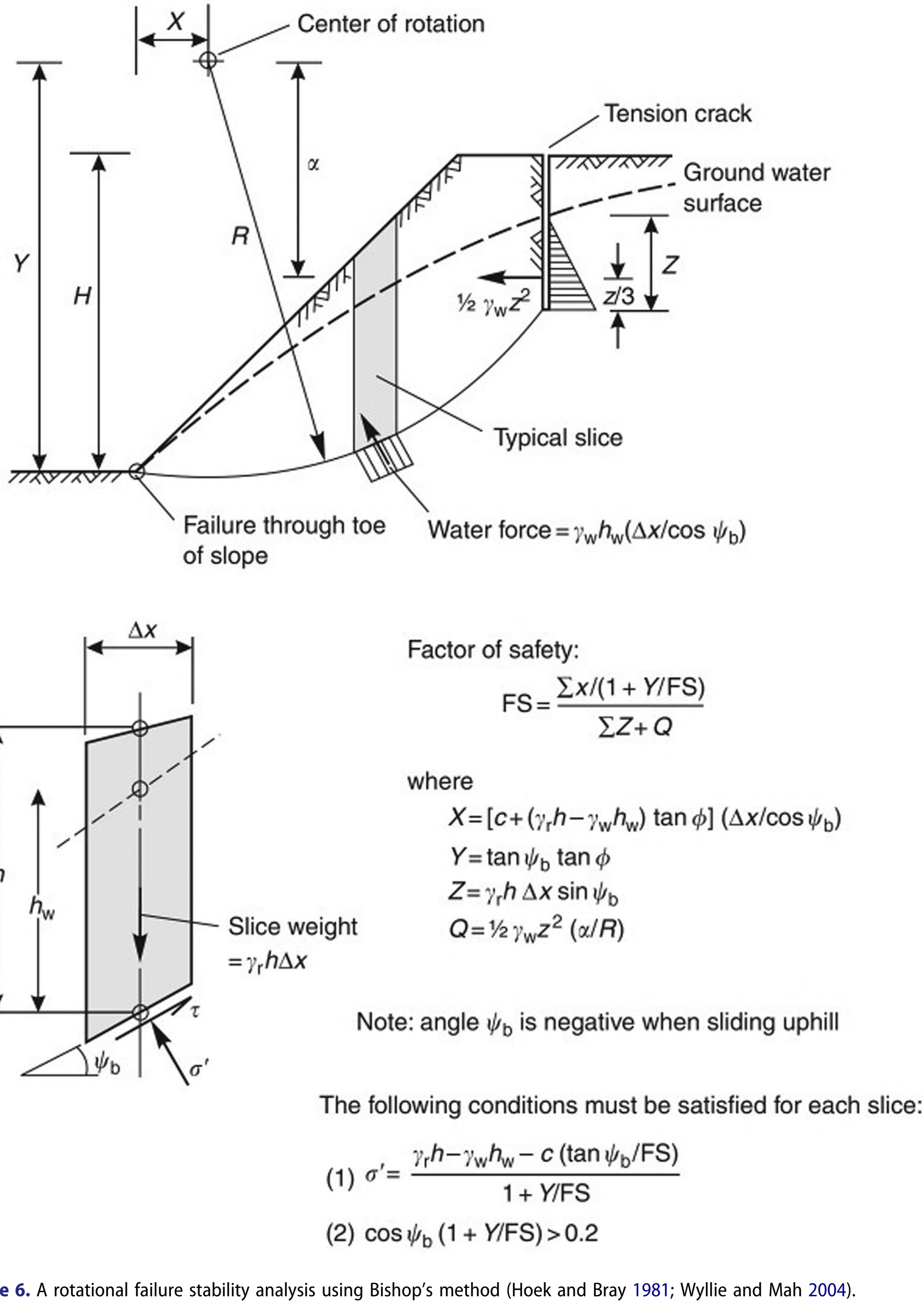

Figure 6. A rotational failure stability analysis using Bishop's method (Hoek and Bray 1981; Wyllie and Mah 2004).

various types of toppling failures are displayed as, flexural failure, block failure, and block-flexural failure were used to analyze different cases(i.e. Alejano, Gómez-Márquez, and Martínez-Alegría 2010, 2015, 2018; Zheng et al. 2018a, 2018b, 2018c, 2019). Block-toppling failure occurs in rock masses with high resistance and acute dip angles due to proper geometric conditions in the formation of suspended rock blocks moving outwards towards the bottom of the slope. Therefore,

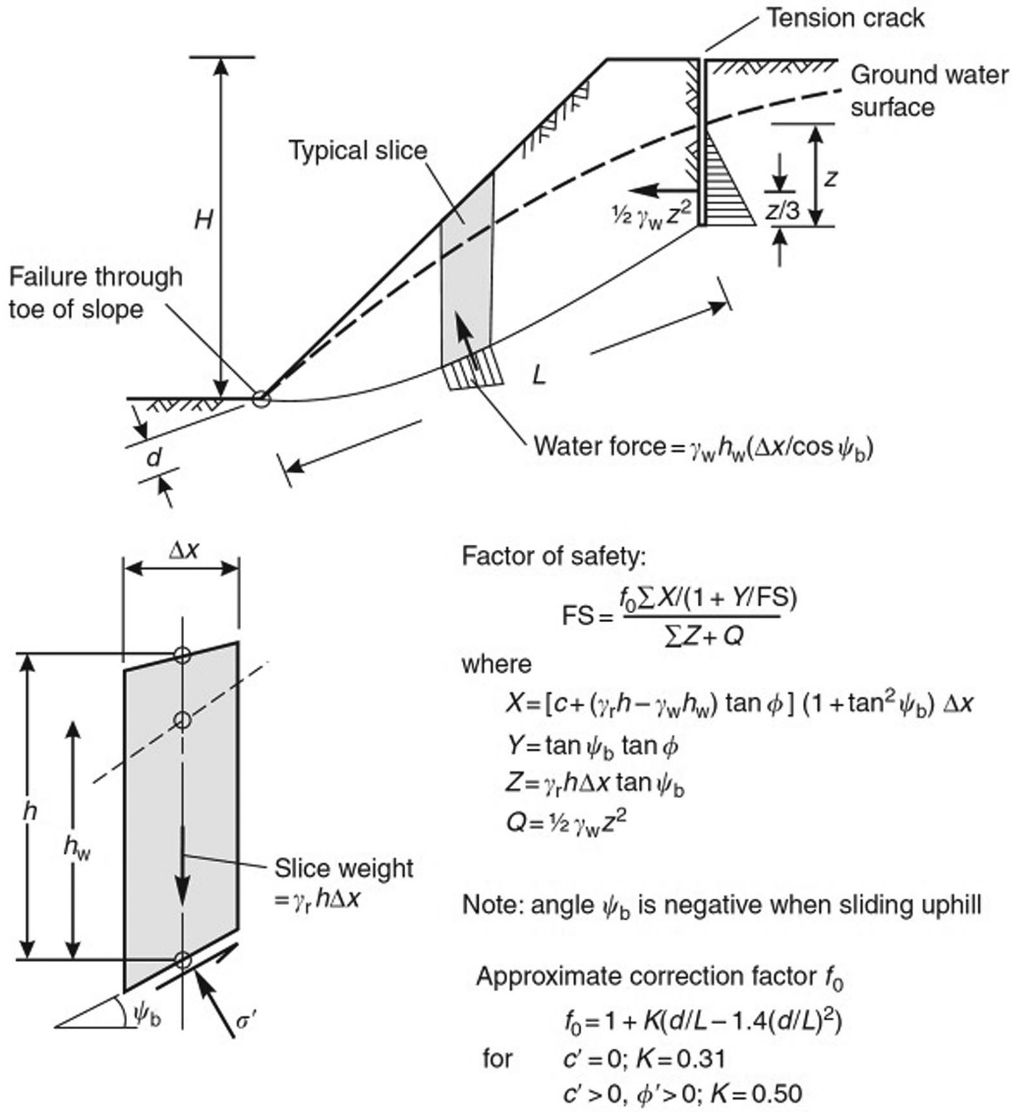

Figure 7. A rotational failure stability analysis using Janbu's method (Hoek and Bray 1981; Wyllie and Mah 2004).

in such a case, there occurs slipping and rotation phenomenon in the rock columns (Amini, Majdi, and Aydan 2009; Aydan and Amini 2009).

In the flexural-toppling failure, during the flexure phenomenon, rock columns with acute dip angle separate from each other and break during bending downwards the slope. In the occurrence of such a slip, heel erosion, leakage, and wash down cause the slip trend to begin, and form deep tensile cracks as the flexural separation continues on the upstream side of the slope. At last, producing a rough surface, it slips downward of the slope (Amini, Majdi, and Aydan 2009). These types of failures provide favorable conditions for column separation and flexure phenomenon and crack diffusion in the section under the rocky columns (Amini, Ardestani, and Khosravi 2017). Block-flexural failure is presented as a combination of two states, in which the rocky columns are continuously under flexure, which, by several latitudinal gaps, cause their slip and movement on the gaps. This state of failure is very complicated and involves many uncertainties. Therefore, it is necessary

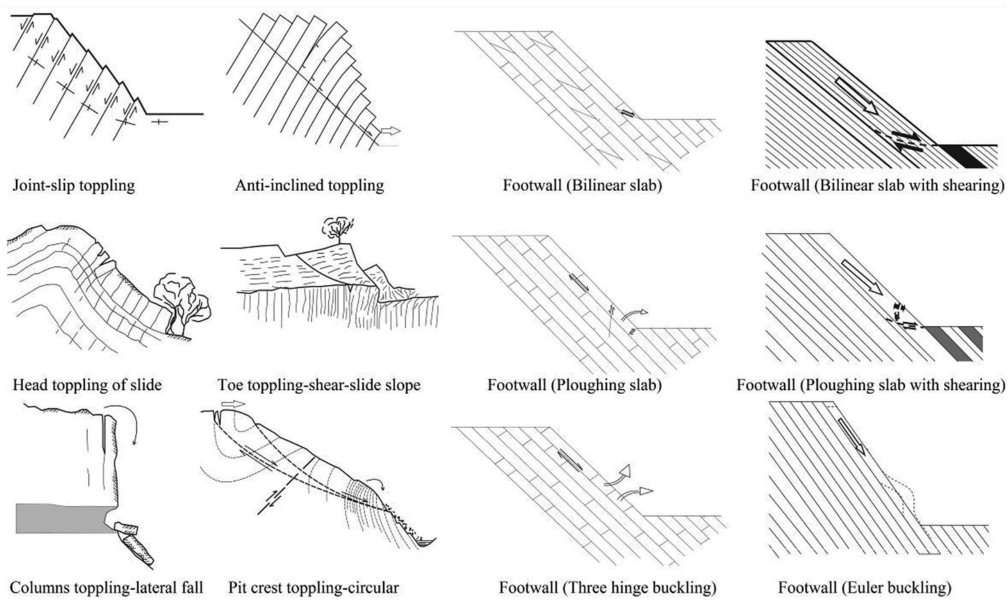

Figure 8. Several types of complex failures in discontinuous rock slopes (Alejano, Gómez-Márquez, and Martínez-Alegría 2010, 2011; Havaej et al. 2014; Sun et al. 2019, 2020).

to consider possible assumptions and to solve various differential equations in order to obtain an acceptable answer. Amini, Majdi, and Veshadi (2012), considering two different states, have proposed a close approximation computational approach to analyze slippery single column and potential block-flexural toppling.

2.4. Rotational failure stability analysis

In soil masses or weak or weathered rock masses, it is not possible to determine the slipping surface by one or several discontinuities or gaps, and this slipping surface passes through the path that has

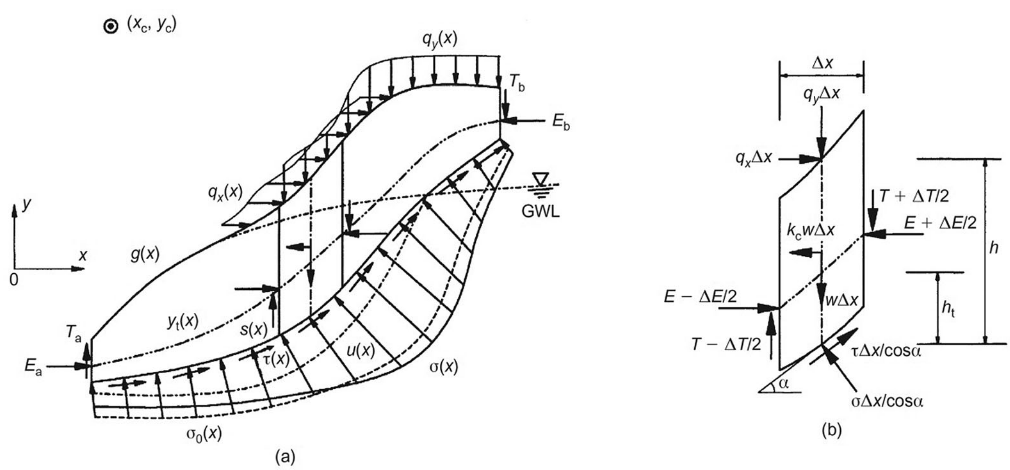

Figure 9. Limit equilibrium stability analyses considerations: (a) graphical figure of massive failure, (b) force status that incur on a slice (Zhu, Lee, and Jiang 2003).

Table 2. Specification of F.S using the limit equilibrium method (Jie, Chen, and Zhang 1999; Paparo 2014; Agam et al. 2016).

No.	LEMs generalized equation	Methodology
1	$F.S=\dfrac{\int_{x_i}^{x_f}[(x_0-x)\tan\alpha+(z_0-z_1)](c+\sigma\tan\varphi)dx}{\int_{x_i}^{x_f}\{-(x_0-x)[\sigma-D-(1+k_v)w]+(z_0-z_1)[\sigma\tan\alpha-D\tan\beta+k_hw]+(z_2-z_1)D\tan\beta-k_hw(z_B-z_1)\}dx}$	OMS
2	$F.S=\dfrac{\int_{x_i}^{x_f}\frac{1}{\cos\alpha}\left[c+\dfrac{D-(k_v+1)w-\frac{c}{F.S^{n-1}}\tan\alpha}{(1+\frac{\tan\varphi\tan\alpha}{F.S^{n-1}})}\tan\varphi\right]dx}{\int_{x_i}^{x_f}[D+(1+k_v)w]\sin\alpha\,dx+\frac{1}{R}\int_{x_i}^{x_f}[D\tan\beta(z_2-z_0)-k_hw(z_B-z_0)]dx}$	Simplified Bishop
3	$F.S_n^m=\dfrac{\int_{x_i}^{x_f}\frac{1}{\cos\alpha}\left\{\dfrac{c+D-\frac{dX^m}{dx}+(1+k_v)w}{1+\frac{\tan\varphi\tan\alpha}{F_{n-1}^m}}\tan\varphi\right\}dx}{\int_{x_i}^{x_f}[D+(1+k_v)w]\sin\alpha\,dx+\frac{1}{R}\int_{x_i}^{x_f}[D\tan\beta(z_2-z_0)-k_hw(z_B-z_0)]dx}$	Generalized Bishop
4	$F.S=\dfrac{\int_{x_i}^{x_f}\left\{c+\dfrac{D-\frac{dX}{dx}+(k_v+1)w-\frac{c}{F.S'}\tan\alpha}{(1+\frac{\tan\varphi\tan\alpha}{F.S'})}\tan\varphi\right\}dx}{\int_{x_i}^{x_f}\left\{\dfrac{D-\frac{dX}{dx}+(k_v+1)w-\frac{c}{F.S'}\tan\alpha}{(1+\frac{\tan\varphi\tan\alpha}{F.S'})}\tan\alpha-D\tan\beta+k_hw\right\}dx}$	Janbu Generalized Method
5	$F.S=\dfrac{\int_{x_i}^{x_f}\left\{c+\dfrac{D+(k_v+1)w-\frac{c}{F.S^{n-1}}\tan\alpha}{(1+\frac{\tan\varphi\tan\alpha}{F.S^{n-1}})}\tan\varphi\right\}dx}{\int_{x_i}^{x_f}\left\{\dfrac{D+(k_v+1)w-\frac{c}{F.S^{n-1}}\tan\alpha}{(1+\frac{\tan\varphi\tan\alpha}{F.S^{n-1}})}\tan\alpha-D\tan\beta+k_hw\right\}dx}$	Janbu Simplified Method

6	$Q = \gamma Hb \left\{ \dfrac{\dfrac{c'}{F.SH} + \dfrac{h\tan\varphi}{2HF.S}(1 - 2r_u + \cos 2\alpha) - \dfrac{h\sin 2\alpha}{2H}}{\cos\alpha\cos(\alpha - \theta)\left[1 + \dfrac{\tan\varphi}{F.S}\tan(\alpha - \theta)\right]} \right\}$	Spencer Method
7	$F.S = \left[(K_i' - \dfrac{U_i}{Wi\sec\alpha_i}) - \dfrac{\gamma_r d_i^2}{2}\cos\delta_i\tan\varphi + cd_i \right]$	Sarma Method

the least resistance. Studies on such masses have shown that most of the occurred failures in such situations are rotational. Sliding surfaces with a spherical form that are controlled by the geological conditions and domain body are observed in the weathered masses with low cohesion/low friction angle and high erosion rate. Limit equilibrium analyses based on rotational failure are known to be the oldest and simplest approaches for instability analysis of the slopes which are implemented in two ways namely, massive analysis methods (common and final sliding surfaces are considered for the entire mass) and slice methods (the slippery mass is divided into a number of slices and evaluated). In Figures 6 and 7, examples of simplified slice methods are presented.

2.5. Composite failure stability analysis

Complex failures represent a combination of two or more different types of slope failure mechanisms that occur at different scales. If these instabilities are at a large scale, they can be considered as landslides (Highland and Bobrowsky 2008). Varnes (1978) has classified the large-scale instabilities in soil or rock slopes as landslides that contain slides, flows, falls, toppling, creeps, debris, lateral separation, and complex movement types. Application of such classification for small-scale instabilities which are categorized in slope stability evaluations must be localized and modified for special cases. Nevertheless, in complex failure stability assessment, the recognition of the failure type is important to access the failure mechanisms (Kumar et al. 2021; Lee and Pietruszczak 2021). Goodman and Bray (1976), Wyllie and Mah (2004), Alejano, Gómez-Márquez, and Martínez-Alegría (2010, 2011), Havaej et al. (2014), and Sun et al. (2019, 2020) have illustrated several types of complex slope failures. Although complex instabilities in slopes occur based on specific localized conditions, the geological characteristics, tectonic states, discontinuity network orientations, layered structure, and sedimentary formations are the main causes for complex failures. Figure 8 presents several identified complex failures that have occurred in discontinuous rock slopes. The complex instabilities must be simplified as a basic failure mechanism to evaluate the stability status. For example, Alejano, Gómez-Márquez, and Martínez-Alegría (2010) used Bishop's simplified method for circular (Bishop 1955) and Goodman and Bray (1976) method for block toppling failure assessment to obtain the toppling-circular complex instability in claystone–sandstone sedimentary formation in Valencia, Spain. These procedures are applied for all types of complex failures in rock slopes.

3. Generalized framework in limit equilibrium analyses

In the last century, over 10 remarkable methods have been developed based on the slice or massive analysis and the type of circular or general slip surface. The general shape of the slip surface in many of these analyses is as presented in Figure 9. In order to analyze the exact stability by LSMs, specifying a generalized framework can improve the conditions and cover the uncertainties in slope stability analysis (Singh, Banka, and Verma 2019a, 2019b). Zhu, Lee, and Jiang (2003) mentioned that in the two-dimensional stability analysis in the cross-sectional area, the slope is restricted by the ground surface ($y = g[x]$) and sliding surfaces ($y = s[x]$). Assuming that the coefficient is constant and equals the F.S for the entire sliding surface, the expansion of the slipping surface on the slider surface is determined as a function of the mass weight element in the static state $W(x)$. Considering the validity of the Mohr-Coulomb failure criterion, it can be stated as:

$$\tau(x) = \frac{1}{F.S}(c(x) + [\sigma(x) - u(x)] \tan \varphi(x)) \tag{8}$$

In this expression, $\sigma(x)$ and $\tau(x)$ are normal and shear stresses, $c(x)$ is cohesion, $\phi(x)$ is effective internal friction angle and $u(x)$ is pore water pressure. Zhou and his colleagues proposed a parameter x as a probability distribution function that can be expanded for each piece. It should be

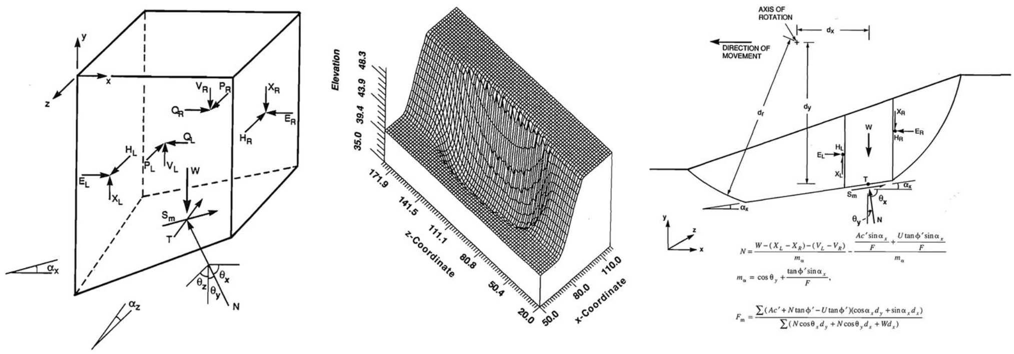

Figure 10. Three-dimensional modelling of circular failure in earth slope (Lam and Fredlund 1993).

noted that when the total stress is used, $u(x)$ will be equal to zero and therefore, the total expression should be considered (Zhu, Lee, and Jiang 2003).

All of the slice analysis methods, except the ordinary method of slices, OMS (Fellenius 1936), use several relations to estimate the confidence coefficient. Hence, the accomplished stability analysis will be unpredictable at both sides of the equation, which requires assuming values of F.S and decreasing the uncertainties of the analytical equations (Zhu, Lee, and Jiang 2003).

However, Zhu and his colleagues provide the differential equation as presented in Equation (9) were used as generalized LEMs-based stability assessment (Zhu, Lee, and Jiang 2003):

$$F.S = \frac{\eta_1 \int_a^b \sigma_0 \xi_1 \psi r_\tau dx + \eta_2 \int_a^b \sigma_0 \xi_2 \psi r_\tau dx + \int_a^b (-u\psi + c) r_\tau dx}{M_c - \eta_1 \int_a^b \sigma_0 \xi_1 \, r_\sigma dx - \eta_2 \int_a^b \sigma_0 \xi_2 \, r_\sigma dx} \tag{9}$$

The above equation can be generalized and extended for all limit equilibrium methods. These generalizations are proposed for a number of limit equilibrium methods as briefly presented in Table 2. Other methods such as Morgenstern and Price (1965), Sarma (1979), USACE (2003) methods have been introduced to improve and reduce the uncertainties. Zhu, Lee, and Jiang (2003) performed a comparison of the accuracy of the confidence coefficient values based on the type of slip surface in different ways. The last and the most updated limit equilibrium method introduced, which has been used extensively and successfully for rock slope stability analysis (for all forms of slip surfaces), is called 'Block theory'. Block theory is widely considered by many researchers because of considering the rock mass geometric conditions and the possibility of probabilistic expansion and statistical function application (Wang et al. 2018; Azarafza et al. 2020b).

The conventional limit equilibrium methods produce extra loops that make the assessment of the progressive instability possible, due to the lack of restrictions on the assumed surfaces to estimate slip parameters. This phenomenon is covered by Goodman's theory. The most important advantages of this theory are restriction of the given slip surfaces, determination of the element key blocks, the definition of progressive failure, the possibility of expanding and employing statistical and probability functions, continuous three-dimensional and two-dimensional analysis (Goodman and Shi 1985). This theory is considered as a basis for a novel analysis for using the limit equilibrium method for the stability of slopes and blocks (Kulatilake et al. 2011).

4. Dimensional evaluations based on limit equilibrium methods

The main procedure of the LEMs methodologies are developed by two-dimensional aspects to evaluate the F.S and probable slide surface with the lowest F.S, but the actual condition of slopes (rock or earth) is a three-dimensional concept (Azarafza, Asghari-Kaljahi, and Akgün 2017b). In

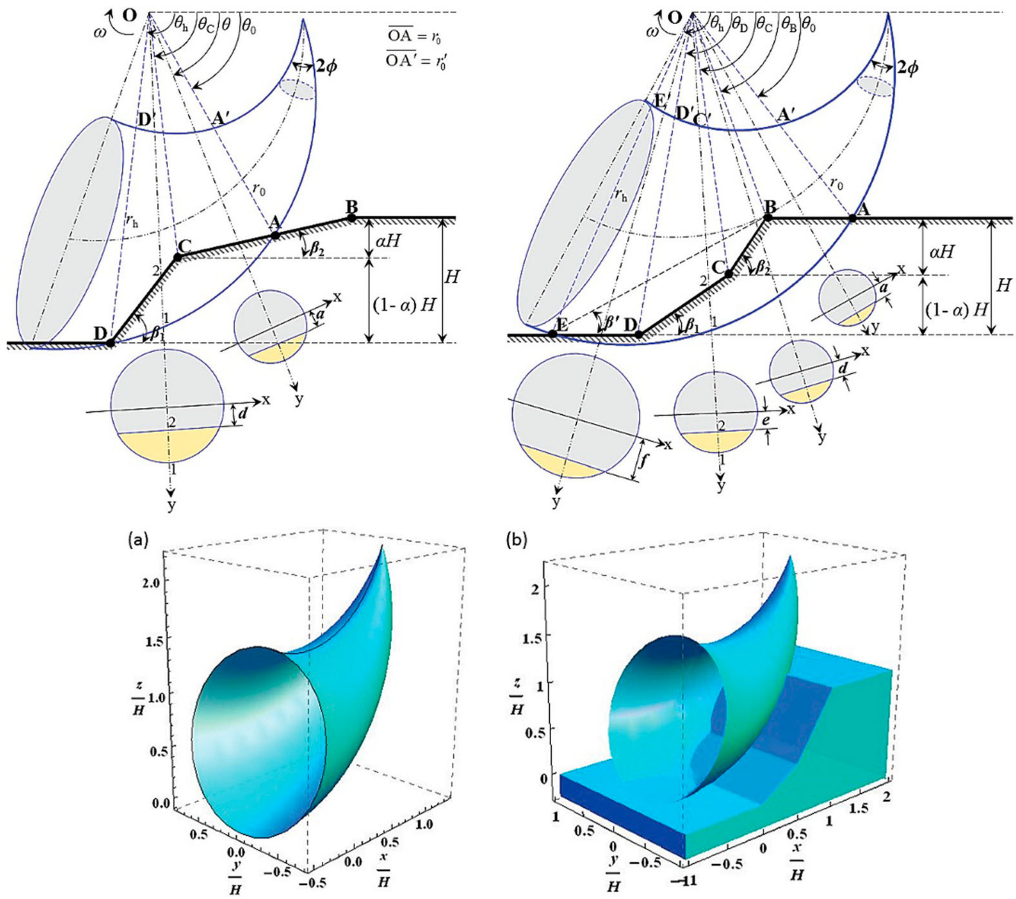

Figure 11. Three-dimensional stability models based on LEMs (Wang, Sun, and Li 2019).

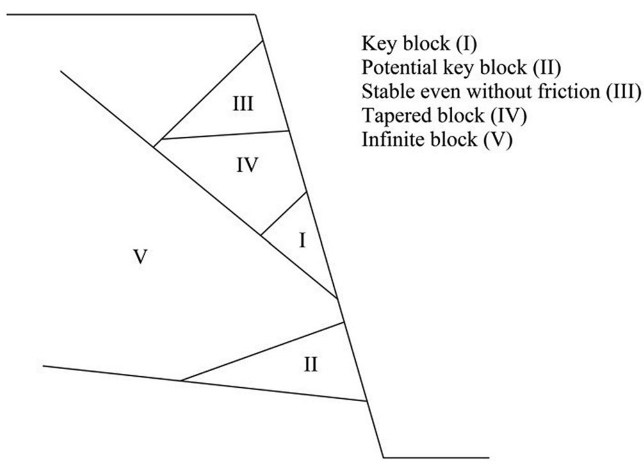

Figure 12. Rock block classification based on Block theory (Kulatilake et al. 2011).

this regard, several scholars have attempted to introduce the generalized limit equilibrium models for three-dimensional slope stability analysis. Baligh and Azzouz (1975), Hovland (1977), Michalowski (1980), Chen and Chameau (1982), Ugai (1985), Leshchinsky, Baker, and Silver (1985), Leshchinsky and Baker (1986), Hungr (1987), Zhang (1988), Hungr, Salgado, and Byrne (1989), Leshchinsky and Huang (1992), and Lam and Fredlund (1993) are the pioneers of the three-dimensional LEMs based analysis on soil slopes. Lam and Fredlund (1993) state that circular failure can be divided into several columns which represent a three-dimensional equivalent for slices that are normally used in circular failure stability assessment (see Table 1). Afterward, the mass above the slip surface is divided into columns, and the forces acting on the various faces of each column are computed or assumed according to the LEMs methodologies (e.g. OMS, Bishop's simplified, Janbu's simplified). Figure 10 presented Lam and Fredlund's work on the preparation of a three-dimensional slice method for earth slope circular failure.

Huang and Tsai (2000) have stated that two-dimensional slope stability analyses are typically considered to be more conservative than three-dimensional analyses in slope stability assessments, but they are more simplified to describe actual slope instability conditions. Huang and Tsai (2000) used the basic work of Lam and Fredlund (1993) to develop the F.S variation contours for asymmetrical slopes which were modified for toe-slide and deep-slide for a uniform soil slope in 2003 (Huang, Chen, and Chang 2003). Chen et al. (2003) have presented a simplified method for stability assessment of wedge and circular failures. Li, Wang, and Deng (2003) have presented a modification for three-dimensional slope stability. Albataineh (2006) has established a review study on slope stability methods conducted on two and three-dimensional circular, wedge, and cylindrical failures based on previous tasks. Zheng (2007), Zhou and Chen have used conservative LEMs (quasi-rigorous) for the three-dimensional stability assessment of slopes which is built on uniform circular failure mechanism evaluations by column progress.

After the introduction of Goodman's Block Theory, the application of the three-dimensional concept to evaluate slope stability received much attention. Noroozi, Jalali, and Yarahmadi-Bafghi (2011) and Azarafza, Asghari-Kaljahi, and Akgün (2017b) used Block theory to develop procedures to simulate rock block geometry which led to the analysis of discontinuous rock slope stability. Wang, Sun, and Li (2019) have utilized the LEMs methods for three-dimensional stability analysis which was mainly conducted on toe-failure, face-failure, and base-failure mechanisms involving uniform soil slopes based on the studies of Michalowski and Drescher (2009). Figure 11 presents the failure mechanism concept by Wang and his colleagues. Zhou and Qin (2020) have presented the lower bound limit coupled with block element method for three-dimensional slope stability analysis based on linear programming optimization technique which is applicable to both unique and non-unique direction models.

5. Block theory

Block theory or Goodman's theory that was proposed by Goodman and Shi (1985) is the newest limit equilibrium method that analyses the stability of rock blocks by two basic mechanisms of failure called 'structural failure' because of discontinuities and 'stress-based failure' because of the presence of high stresses. This theory by considering cairn geometrical status provides a logical relationship between cairn geometrical and critical failure potential in contact with released surfaces (during drilling operations) or rock outcrops (natural outcrop of the slope surface). The geometric position of the blocks, the emplacement of discontinuities in the space (which causes block division), spreading and continuity, and discontinuity spacing are considered parameters in assumptions of the analysis using Block theory. The movement of blocks is along the gapping geometric direction and is based on the resisting and driving forces on the discontinuity surface. This theory, by classifying block geometry, identifies the main causes of structural failure and finds the possible slip surface in the range of element (key) blocks. Figure 12 presents a classification of a rock slope by using Block theory.

This theory by using the 'finiteness theorem' and 'removability theorem', attempts to analyze spatial geometry and defines line and plane equations/inequalities in space (discontinuities), which can easily convert the recorded information from polar coordinates (dip/dip direction of discontinuities) to Cartesian coordinates. The removability of the blocks is also defined based on the convex polyhedral geometry convergence in space. Using the finiteness theorem, joint pyramid, JP (the common spaces between half-spaces of discontinuity planes that form part of the block pyramid), the excavation pyramid, EP (a group of extraction half-spaces (excavation half-spaces) that are displaced to form a block pyramid), space pyramid, SP (complementary half-space of excavation pyramid) and block pyramid, BP (the factor for determining the convexity and concavity of blocks) can be defined (Azarafza, Asghari-Kaljahi, and Akgün 2017a, 2017b). These continuous equations give a mathematical definition for block geometry and its stability analysis.

This advantage has made the focus on this theorem very robust in stability analysis (Zhang et al. 2012; Wang et al. 2018). Goodman and Shi (1985), by using the removability theorem, have expressed the tendency trend and motion of the block where a finite block is removable if it can displace in a particular direction without encountering the neighbor cairn. They also defined the conical block as a non-removable block that cannot be moved in any direction without encountering the neighbor cairn. Thus, the failure potential and slip in rock masses are very limited (Shi 1988). This restriction in the assumption of the initial slip surfaces (in the limit equilibrium stability analysis) provides the estimation of the results much faster, more accurately, and in less time than the available approaches (manual methods and common software). In the recent decades (i.e. specifically after the year 2000), Block theory has received widespread attention from various scholars because of its strong computational and analytical foundations. Ling (2001) has presented the seismic/static application of this theory to different geotechnical structures. Um and Kulatilake (2001) have utilized kinematic and block theory analyses for rock slope stability evaluations in three Gorges dam sites in China. Eberhardt (2003) has conducted several analytical/numerical methodologies for rock slope stability assessments. Huang, Chen, and Chang (2003) used the discontinuity network description of the key-block concept for identifying the potential key blocks in surficial excavation. Yarahmadi-Bafghi and Verdel (2003) introduced the novel work based on Goodman's theory named as 'key-group method'. They also used probabilistic approaches to modify the key-group method (Yarahmadi-Bafghi and Verdel 2004). Pötsch and Schubert (2004) have used a computer-based process referred to as JointMetriX3D to determine rock mass behavior and stability. Yarahmadi-Bafghi and Verdel (2005) have conducted the Sarma limit-equilibrium procedure for re-ordering the key-group approach. Jimenez-Rodriguez, Sitar, and Chacón (2006) presented the systematic quantitative methodology for the reliability analysis of rock slope stability based on key-block theory. Jimenez-Rodriguez and Sitar (2007) applied Block theory for rock wedge stability analysis by system reliability approach given by the Monte-Carlo simulation technique. Haswanto and Abd-Ghani (2008) performed a kinematic evaluation and Block theory for rock slope stability assessment in Fraser's Hill Pahang, Malaysia. Haswanto and Abd-Ghani (2010) completed the project on Fraser's Hill Pahang, Malaysia by using several unstable slopes.

Kulatilake et al. (2011) used the Block theory concept to evaluate fractured rock slope instabilities in the Yujian River dam site. Tian and Fu (2011) used Goodman's block theory for introducing the modified joints random probability model which is applied to analyze key block slide probability in discontinuous rock slope stability. Noroozi, Jalali, and Yarahmadi-Bafghi (2011) presented the three dimensions key-group method which is developed by Yarahmadi-Bafghi and Verdel in 2003. Ma, Li, and Hong (2013) applied a coupled procedure based on Block theory; kinematic vector analysis and discontinuous deformational analysis numerical method (DDA) for rock slope stability analysis. Greif and Vlčko (2013) applied a key-block analyses system to evaluate the static stability of rock slopes for 45 cases in medieval castles in Slovakia. Wang and Ni (2014) presented the geotechnical structure and model analysis (GeoSMA-3D) software for rock slope stability analysis. Nguyen and Phi (2014) applied Block theory and probabilistic approaches for investigating the stability concerns in national road No.6, Vietnam. Sun, Zheng, and Huang (2014) applied an indeterminate key-block method for static assessment in the Jinping-I hydropower station, China. Shi (2014) presented the application of Block theory and the DDA technique for the reliability studies of slopes and underground powerhouses. Zheng et al. (2014, 2015) developed a new formulation/computer code based on probabilistic Block theory analysis named PBTAC.

Li, Zhou, and Wang (2016) proposed a framework for rock mass sliding blocks in surface cuts based on block-group and the Sarma integrated methodology. Azarafza, Asghari-Kaljahi, and Akgün (2017a) applied the three-dimensional discontinuity geometrical modeling (3DDGM) algorithm for three-dimensional simulation of discontinuity network in rock slopes. Liu et al. (2017) provided a new semi-deterministic Block theory method (NSDBT) based on digital photogrammetry techniques and applied it to the Changhe dam that is located in the Sichuan province, southwest China. Jia et al. (2017) used discontinuity spatial distribution for underground geotechnical structures. They provided the extended key block theory in three dimensions which attempted to identify and classify the key-blocks. Azarafza, Asghari-Kaljahi, and Akgün (2017b) presented the key block method based computational methodology for critical rock block recognition and evaluation of the sliding mechanisms.

Turanboy, Ülker, and Küçüksütçü (2018) described a new method to model a discontinuity that intersects rock bodies. They used the k-means vector quantization cluster as an unsupervised machine learning technique to extract the three-dimensional discontinuity emplacements in surface rock cuts. He et al. (2018) presented the couple method as a nodal variable-based discontinuous deformation analysis, named NDDA based on DDA and finite-element method (FEM) which was developed according to kinematic and block principles. Wang et al. (2018) used a framework based on block theory multi-level rock slope characterization by the analytic hierarchy process (AHP) and GeoSMA-3D. Mohebbi et al. (2019) developed an analytical approach based on the key-group method (named TFS_KGM) for the investigation of toppling free fall-sliding events. They stated that the computer evaluation results of this method significantly helped to describe the free fall toppling conditions in slope cuts. Mohammad et al. (2020a) have presented a fuzzy logical decision-making method based on block theory to effectively determine discontinuous rock slope reliability under various wedge and planar slip scenarios. The method is capable to investigate the reliability (or stability-instability) degree to prepare the response operations without the extensive requirements. Mohammad et al. (2020b) established the new methodology based on simplified semi-distinct element and block theory to estimate the stability conditions for main toppling failures (block, flexural and block-flexural types).

6. Rock slope stability analysis outlook by limit equilibrium approaches

Slope stability (earth or rock) is one of the most important issues in the geotechnical engineering field with a background of more than 300 years since the construction and development projects have always been faced with side-hill instabilities in different scales which were classified as slopes or landslides (Frasheri 2006). Generally, various approaches used for slope stability assessment can be classified as simple evaluations, planar failures, limit state criteria for limit equilibrium analysis, numerical methods, hybrid, and high-order approaches (Kliche 2018). In the meantime, limit equilibrium approaches due to their simplicity, rapid implementation; closed-form analysis, continuous access, easy assumptions, and providing multiple answers (e.g. F.S and probable sliding surface), the capability of being coupled or re-activated with other procedures are considered as the most flexible methodologies. Development of novel or hybrid procedures (rebuilt based on traditional progress) is conducted to achieve more accurate stability results, to cover more uncertainties, reduce the errors and establish generalized procedures for instability assessments as the main goal during the last decades. Application of the LEMs for stability analyses in the last recent 300 years indicate that these approaches are highly flexible to be integrated with high-level programming which is capable to cover more geomechanical features and geometrical properties. This advantage helps to reduce the uncertainties in the slope mass by considering the deterministic formulations.

7. Conclusions

These papers present a systematic review study of kinematical and limit equilibrium-based methods (LEMs) that are utilized for discontinuous rock stability analyses as well as for limit-equilibrium methods for heavy jointed rock or soil-like lithologies which are implemented in two-dimensional and three-dimensional spaces. During as decades, various approaches were developed by scholars to estimate the slope stabilities. To this end, the main methodologies of LEMs have been examined and their approaches through history have been described. The LEMs have been improved and modified to cover applications ranging from simplified circular failure analysis to high-order solutions of complex failures. Finally, the most advanced method of rock slope stability assessment that has been widely used by researchers nowadays and named as 'Block theory' that is based on strong computational and analytical foundations has been considered. In this regard, a brief overview of the new decade's achievements on Goodman's theory, which reflects modification of Block theory over time has been mentioned and discussed. As outlook of the LEMs, these approaches due to the flexibility can be coupled with the new procedure like numerical and hybrid methods and provide the simple and fast implementation for slope stability assessments. On the other hand, the application of new theories such as Block theory provides simple mathematical decryption for both structural and stress-field failures which is highly efficient in combination with numerical procedures like distinct of finite elements. In terms of the future scope for LEMs-based slope stability especially Block theory, it can be notified that Goodman's theorem can be considered as the strongest approach to quantify the discontinuous rock slope and estimate the slope's stability condition.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

There is no financial support for this research.

Data availability statement

The data of the literature review for this study are available within the article.

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