Toroidal effects on the electron absorption of ICRF waves

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Introduction. The fraction of radio-frequency (RF) power in the ion cyclotron (IC) range of frequency (ICRF) directly absorbed by electrons can be substantial in a reactor-size plasmas due to the larger volume between the antenna and the IC resonances, and to the larger temperatures, which intensify the electron Landau damping (LD), transit-time magnetic pumping (TTMP) and the mixed damping (MXD) [1]. Therefore, accounting for the toroidal effects on the electron motion along magnetic field lines [2] becomes important when quantifying either the ion heating or the current drive (CD) efficiency in reactor-size plasmas. These efficiencies strongly depend also on how far inside the plasma the IC resonances are located and on the power spectrum launched by the antenna. The present contribution is an extension of the analysis done in [3] with emphasis on the reactor constraints.

Orbit equations. The electron response to ICRF fast-waves is expressed with orbit integrals,

$$I_{ij}^{(m,n)}(\rho,\vartheta) = -i\,\omega \int_0^{+\infty} \mathrm{d}\varepsilon_v \int_0^{\varepsilon_v/B} \mathrm{d}\mu_v \frac{B}{v_\parallel}\, F^e(\mu_v,\varepsilon_v) \int_{-\infty}^{t} \mathrm{d}t'\; \Pi'_{ij}\; e^{-\mathrm{i}\int_{t'}^{t}\left[m\dot{\vartheta}+n\dot{\varphi}-\omega\right]\mathrm{d}\tau}\,. \qquad (1)$$

Here ω is the RF angular frequency; ϑ and φ are the (arbitrary) poloidal and (geometrical) toroidal angles, and m and n the poloidal and toroidal wavenumbers of the Fourier spectral representation of the RF em fields. If the electron distribution function, F_e, is assumed Maxwellian, as done in TORIC [4], $\Pi'_{yy} = w^2(w')^2/2$ for TTMP, $\Pi'_{zz} = 2uu'$ for LD, and $\Pi'_{yz} = 2x^{(m,n)}w^2u'$ and $\Pi'_{zy} = 2x^{(m,n)}(w')^2u$ for MXD, with $w = v_\perp/v_{\mathrm{th,e}}$, $u = v_\parallel/v_{\mathrm{th,e}}$, and $x^{(m,n)} = \omega/(k_\parallel^{(m,n)}\, v_{\mathrm{the}}$. (x,y,z) is the local Stix reference frame, $\hat{x} = \vec{k}_\perp/k_\perp$, $\hat{y} = \hat{z}\times\hat{x}$, and $\hat{z} = \vec{B}/B$. Upon using the conservation of the kinetic energy (we omit here a possible background electrostatic potential), $\varepsilon_v = mv^2/2$ and of the magnetic momentum, $\mu_v = mv_\perp^2/2B$ of the unperturbed motion, $v_\parallel = \sigma_\parallel\sqrt{2m\varepsilon_v}\sqrt{1-\Lambda_{\mathrm{eq}}B/B_{\mathrm{eq}}}$ with $\Lambda = \mu_v B/\varepsilon_v = v_\perp^2/v^2$, where "eq" stands for values at the point of minimum B along the orbit (on or close to the outward equatorial midplane point), and $\sigma_\parallel = \mathrm{sign}(v_{\parallel,\mathrm{eq}})$ ($\sigma_\parallel$ changes along the orbits of trapped particles). Magnetic drifts are omitted here, i.e. it is assumed that electrons are tied to the magnetic field lines. Particles with $\Lambda_{\mathrm{trp}} \le \Lambda_{\mathrm{eq}} \le 1$ are trapped, whereas they are passing when $0 \le \Lambda_{\mathrm{eq}} \le \Lambda_{\mathrm{trp}}$, with $\Lambda_{\mathrm{trp}} = B_{\mathrm{eq}}/B_{\mathrm{max}}$. In terms of $v_\parallel$, it holds $\dot{\vartheta} = (v_\parallel/r)\sin\Theta$ and $\dot{\varphi} = (v_\parallel/R)\cos\Theta$, with R and r the major and minor radii of the magnetic surface, Θ the pitch angle of the magnetic field line, i.e. $\tan\Theta = B_\varphi/(rB_\vartheta) = r/(q_{\mathrm{sf}}R)$ with q_{sf} the safety factor. In tokamaks $\cos\Theta \approx 1$ and $\sin\Theta \approx \tan\Theta$, and the integrand becomes $k_\parallel^{(m,n)}v_\parallel$, with $k_\parallel^{(m,n)} = k_{\parallel,0}^{(m,n)}R_0/R = (m+q_{\mathrm{sf}}n)/(q_{\mathrm{sf}}R)$ the effective parallel wavevector of the RF field seen by the particles (the subscript "0" stands for the value at the magnetic axis). Toroidicity enters the orbit integrals through R in $k_\parallel^{(m,n)}$ and through the mirror force, which modulates $v_\parallel$ along the orbit (when $\mu_v \neq 0$). The first effect is already taken into account in the standard TORIC [4]. We write $m\dot{\vartheta}+n\dot{\varphi}\approx(m+q_{\rm sf}\,n)\dot{\vartheta}$, with

$$\dot{\vartheta}=\sigma_{\|}\varpi_{\rm T}\frac{R_0}{R}\sqrt{1-\Lambda_{\rm eq}\frac{B}{B_{\rm eq}}},\quad \varpi_{\rm T}:=\frac{\sqrt{2m\varepsilon_v}}{q_{\rm sf}R_0}, \tag{2}$$

where $\varpi_{\rm T}$ is the angular transit frequency of passing particles in the limit $\mu_v=0$. In (2) only R and B depend on ϑ (the φ-dependence is absent in axisymmetric configurations). In large-aspect-ratio tokamak plasmas, $R\approx R_0(1+\varepsilon_r\cos\vartheta)$, $B=B_0(1-\varepsilon_r\cos\vartheta)$, with ε_r the inverse aspect ratio of the magnetic surface labeled with the radial coordinate r. Hereafter, we omit terms of order ε_r^2. The equation (2) becomes:

$$\dot{\vartheta}\approx\sigma_{\|}\varpi_{\rm B}\,(1-\varepsilon_r\cos\vartheta)\sqrt{1-\frac{1}{\kappa_t}\sin^2\left(\frac{\vartheta}{2}\right)},\quad \varpi_B=\varpi_{\rm T}\sqrt{1-\Lambda_{\rm eq}},\ \frac{1}{2\kappa_t}=\frac{\varepsilon_r}{1-\varepsilon_r}\frac{\Lambda_{\rm eq}}{1-\Lambda_{\rm eq}}. \tag{3}$$

<u>*Passing particles*</u> [$\kappa_t>1$]. For well passing particles, $\kappa_t\gg1$, $\dot{\vartheta}\approx\sigma_{\|}\varpi_B\,(\iota_P-\varepsilon_P\cos\vartheta)$, with $\iota_P=1-1/(4\kappa_t)$, $\varepsilon_P=\varepsilon_r-1/(4\kappa_t)$. To lower order, $\vartheta'-\vartheta\approx\sigma_{\|}\tilde{\varpi}_B(t'-t)+\tilde{\varepsilon}_P\left\{\sin[\vartheta+\tilde{\varpi}_B(t'-t)]-\sin\vartheta\right\}$, with $\tilde{\varpi}_B=\varpi_B\,\iota_P$ and $\tilde{\varepsilon}_P=\varepsilon_P/\iota_P$.

<u>*Trapped particles*</u> [$0\le\kappa_t\le1$]. By time deriving the square of (3) and expanding around $\vartheta=\vartheta_{\rm eq}\approx0$, the equation for ϑ becomes the equation of an harmonic oscillator, $\dot{\vartheta}\approx-(\varpi_B/\vartheta_{\rm tr}^2)\,\vartheta$, with $\vartheta_{\rm tr}=4\kappa_T$ the poloidal angle of the turning points, and $\varpi_{\rm tr}=\varpi_B/\vartheta_{\rm tr}^2=(\Lambda_{\rm eq}\varepsilon_r/2)\varpi_{\rm T}\ll\varpi_{\rm T}$. In this case, $\vartheta'-\vartheta=\sqrt{\vartheta_{\rm tr}^2-\vartheta^2}\sin(\varpi_B(t'-t))+\vartheta[\cos(\varpi_B(t'-t))-1]$.

Implementation in TORIC code. The periodic modulation of the $v_{\|,e}$ is taken into account only in the imaginary part of the plasma dispersion function (PDF), which is responsible for the wave absorption. In the real part of PDF, responsible for the wave propagation, only the $k_{\|}$ dependence on R^{-1} is considered [4]. Moreover, in the absorption we consider only the contribution of passing electrons, and neglect the contribution of resonating trapped electrons, because its estimate is prohibitively computationally demanding. Although not exactly zero, this contribution is small, because the number of resonating trapped electrons is usually small and stay in resonance with the waves less than passing electrons do. As a rough estimate of it, the resonant parallel velocity is approximated with $u_{\rm res}=v_{\|,\rm res}/v_{\rm th}\approx0.33R_{\rm [m]}\,f_{\rm [MHz]}/(n\sqrt{T_{e,\rm[keV]}})$, and in the case of a Maxwellian distribution function the fraction of trapped particles resonating with the waves is $f_{\rm trp}(u_{\rm res})\approx\exp\{-u_{\rm res}^2/2\varepsilon_r\}/\sqrt{\pi}$. Figure 1 shows the radial behaviour of $f_{\rm trp}(u_{\rm res})$

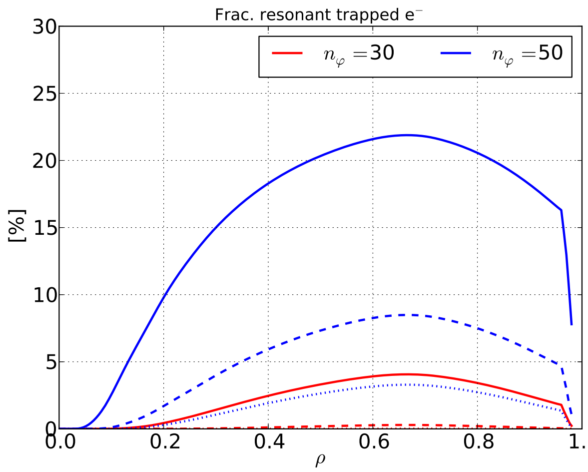

Figure 1: *Fraction of resonating trapped electrons for $T_{e,0}$ 24 keV (solid), 12 keV (dashed) and 8 keV (dotted). Used T_e profile is given in figure 2.*

in the case of ITER-like examples of the next section, for two values of n, representative of the main antenna spectrum peak corresponding to the heating (50) and CD (30) antenna phasings. The integral in the exponential of (1) is thus $(m+q_{\rm sf}\,n)(\vartheta'-\vartheta)-\omega(t'-t)$, with $\vartheta'-\vartheta$ as just derived. The Jacobi-Anger expansion for the Bessel functions is applied to isolate the resonances and perform the integration in velocity,

$$I_{ij}^{(m,n)}(\rho,\vartheta) = -\sqrt{\pi}\int_0^{w_{\rm trp}} {\rm d}w^2\, e^{-w^2}\,\Pi_{ij}(w)\left[\sum_k x^{(m+k,n)} e^{-(x^{(m+k,n)})^2}\, e^{-{\rm i}(k\vartheta+\sin\vartheta)}\, J_k(\Xi^{(m+k,n)})\right],$$

with $x^{(m,n)} = \omega\, q_{\rm sf} R_0\, v_{\rm the}/(m+q_{\rm sf} n)$. The expression of $I_{ij}^{(m,n)}$ has an expansion in harmonics of the transit frequency, $\tilde{\varpi}_B$, similar to that of in harmonics of the gyrofrequency. In the present case, however, the argument of the Bessel functions, $\Xi^{(m,n)} = (m+q_{\rm sf}\,n)\tilde{\varpi}_B$, depends explicitly also on the poloidal position, ϑ, and on the parallel velocity. Despite the simplifications done, the computation of this expression of $I_{ij}^{(m,n)}$ still remains rather lengthy.

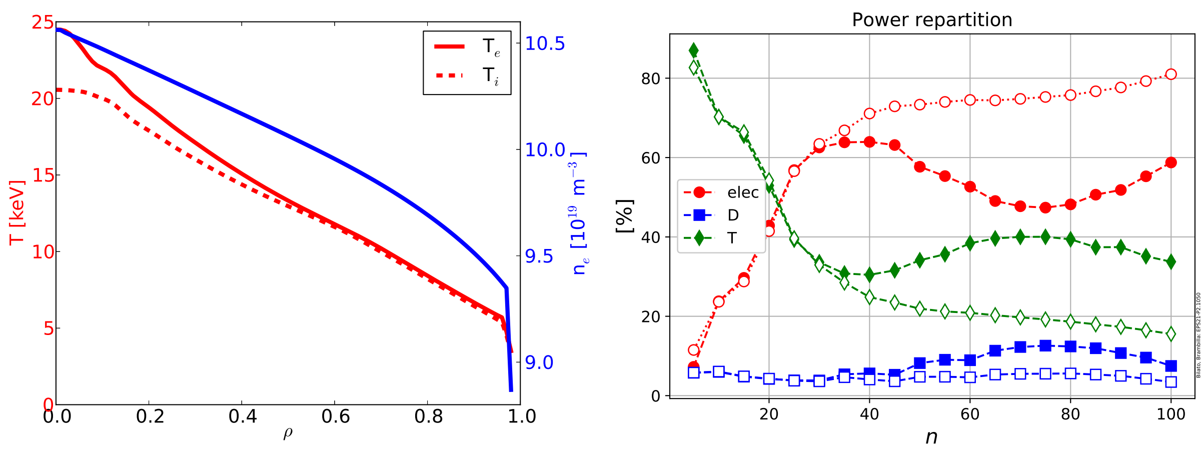

Figure 2: *Left: plasma density and temperatures considered in these simulations [5]. Right: Fraction of ICRF power directly absorbed by the plasma species as function the toroidal wave number n: Open symbols without and full with the toroidal effects on the electron absorption.*

Examples. We consider here a scenario of ITER [5], namely T heating at the 2nd–harmonic in a equal mixture of D and T. The combination of a full field, 5.2 T, with a RF frequency of 52.5 MHz locates the IC resonance of T in the plasma core. In figure 2 the repartition of the RF power directly absorbed is shown as function of the toroidal wave number n. The effects of toroidicity on the electrons are clearly visible at high n values, where the electron absorption is reduced in favour of T and D absorption. Since at these high temperatures ICRF is useful mainly for CD with antenna spectrum peaked at low n values [6], on the whole these effects should not be dramatic in reducing the ICRF driven current in a reactor.

It is worthwhile considering how these effects change when the plasma temperature is reduced, since LD, TTMP, and MXD depend strongly on T_e. To increase the ion ICRF heating efficiency and the first-pass absorption during the initial phase of the discharge characterized by low-medium temperatures, about 3% of He^3 is added to operate in ICRF minority heating regime, with the He^3 fundamental IC resonance coinciding with the 2nd–harmonic of T. Figure 3 shows the same case of figure 2 with T_e and T_i both reduced to one half and one third, and thus with the addition of 3% of He^3. Already at half temperatures these effects are reduced,

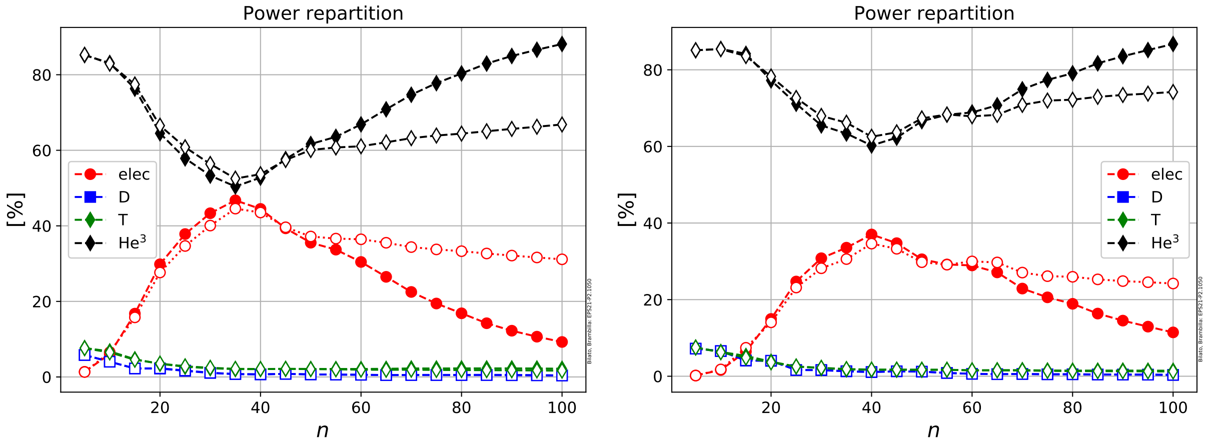

Figure 3: *Same parameters and profiles of figure 2 except for T_e and T_i, i.e. one-half on the left and one-third on the right, and addition of 3% of He^3, changing the ICRF scheme in minority heating one.*

and the reduction is even stronger at one third of the temperatures of figure 2. This points to the trend that these effects become smaller when the temperature decreases, and this is briefly why they can be easily neglected in medimum-size tokamak devices [3].

Conclusions. Because of the approximations done to keep the problem tractable both analytically and numerically, these results can only give an order of magnitude of these effects. Nevertheless, it sounds reasonable to conclude that they are almost always negligible in present-day medium-size tokamaks. It might be necessary to consider them in ITER and future large tokamaks, in particular when operating ICRF as CD system during the high-temperature flat-top phase, although in that case to optimize the CD efficiency the antenna spectra should be centered at low n values, where these effects are small.

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