Ship Design Performance and Cost Optimization with Machine Learning

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Abstract

This contribution shows how, in the preliminary design stage, naval architects can make more informed decisions by using machine learning. In this ship design phase, little information is available, and decisions need to be made in a limited amount of time. However, it is in the preliminary design phase where the most influential decisions are made regarding the global dimensions, the machinery, and therefore the performance and costs. In this paper it is shown that a machine learning algorithm trained with data from reference vessels are more accurate when estimating key performance indicators compared to existing empirical design formulas. Finally, the combination of the trained models with optimization algorithms shows to be a powerful tool for finding Pareto-optimal designs from which the naval architect can learn.

1. Introduction

Ship design typically consists of three phases, the preliminary design stage, the contract design stage, and the detailed design stage. According to *Taggart (1980)*, early design stages occupy the smallest amount of time. In the first design phases, the operational requirements are translated into technical characteristics, *Kossiakoff et al. (2003)*. This is done by finding the balance between the need of the customer and the available budged, resulting in one or more possible design solutions, *Duchateau* (*2016)*. Despite the limited amount of time spent in the early design stages, it is estimated that 60 to 80 percent of the total life cycle cost is already locked-in after the preliminary design stage, *Rehn* (*2018)*. Depending on the experience of the involved Naval Architects, this can be quite risky. The design decisions are typically hard to reverse and are also often made with limited design problem knowledge. This has been visualized per design stage in Fig.1 by *Duchateau, (2016)*. The figure shows the consecutive design stages, the committed costs, problem knowledge and design freedom.

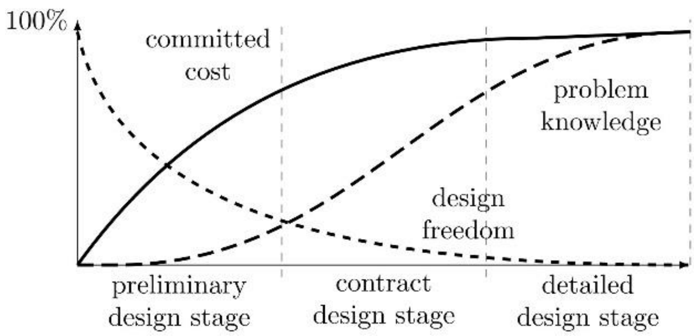

Fig.1: Generic design timeline by *Duchateau* (*2016)*

In this paper a solution is presented on how, in the preliminary design stage, more knowledge can be used when making decisions. With the method described in this paper, naval architects are better supported by data to make design decisions instead of relying only on instincts, knowledge and experience. The decision support is in the form of machine learning models which can be used to validate ideas, assumptions, and design variations. This helps the naval architect in avoiding innovation risks and to find better design variations. On top of this, without much additional effort, the naval architect can use the trained machine learning models in combination with an optimization algorithm. This optimization algorithm can then be deployed for searching advantageous and competitive design variations. The only requirement for the so-called reference optimizer to work properly is enough relevant ship data, and a properly setup design problem.

2. Background and Related Work

In ship design, two design philosophies can be distinguished, the empirical and the simulated design method. The empirical design method is based on reference data of similar built vessels. The simulated design method uses estimations, calculations and simulations to optimize the economical and physical characteristics of the to be designed vessel.

2.1. Empirical Design Method

In the Empirical Design method, the main dimensions are based on similar built vessels. Similar vessels are marginally improved or the data from similar vessels is used to make a regression model, after which empirical design formulas can be deducted. Examples of empirical design equations have been created by *Watson (1998)*, *Schneekluth and Bertram (1998)*, *D'Almeida (2009)*, *Andrews (1998*), *Molland (2011)* and *Papanikolaou (2019)*. With these equations a naval architect can easily estimate ship design parameters. The equations are usually calibrated for different ship types. As empirical relations are based on knowledge from previous work, it is important to handle the relations carefully. It is the naval architect's job to update the relationships whenever possible *Molland (2011*). However, in reality updating is often not regularly done, and the equations are only available for the most popular ship types. A second note to keep in mind is that extrapolation of the regression models remains problematic *Papanikolaou (2019)*.

2.2. Simulated Design Method

If no or very little data of similar ships is available, it is not possible to use the Empirical Design Method. The naval architect is in this case is forced to design a ship from scratch using estimations, calculations and simulations. An efficient way to do this is by utilizing a parametric 3-D model and connect it to simulation software. General design knowledge and design experience is used to setup this 3-D parametric model. Examples of the parametric modelling approach can be found in e.g. *Marzi et al. (2018)*, *Priftis et al. (2016)*, *de Winter et al. (2021)*. Typically, after a parametric model is setup, the designs are optimized for their economical and physical characteristics with optimization algorithms *de Winter et al. (2019)*.

A second, more automated, parametric design method has recently been proposed by *Charisi et al., (2019)*. In this work it is shown that knowledge-based engineering is a good option when designing a ship when not enough similar ship data is available. With knowledge-based engineering, general multidisciplinary knowledge is translated in individual product models / building blocks which represent a small part of a vessel. These product models are then used, scaled, and combined into an entire ship design in an object-oriented way.

3. Data Description for Reference Studies

The solution proposed in this paper utilizes both the power of empirical and parametric optimization. To get up to date empirical formulas, data is needed. In the past decades, a lot of data services have become available for the maritime industry. The most prominent ones are:

1. World Fleet Register, a ship data and intelligence platform from Clarksons Research with data about ship earnings, vessel parameters, and new-build data.
2. IHS Markit, in the maritime portal of IHS Markit reports static ship data of existing and scrapped ships.
3. BRL shipping consultants, in the subscribers' area of BRL static data and reports are available about new build vessels.
4. Marine Traffic, even without logging in it is possible to obtain the location of vessels plus general static ship data.

All this static and operational data has been collected and aggregated into more than 100 particular data fields per vessel. Examples of collected data fields are: Length, breadth, draft, block coefficient, light ship weight, dead-weight, maximum continuous rating of the engine, maximum speed, but also more ship specific data fields for specific ship types such as: Bollard pull, passenger capacity of urban transportation vessels, number of car lanes, crane capacity, hopper volume of dredgers, and ice class qualification.

This data can be used in a reference study in the preliminary ship design process since design trends can be visualized, design trends can be learned, and gaps and competitive advantages in the market can be found.

3.1. Visualizations

After the relevant parameters for a vessel type have been selected, the parameters can be summarized and plotted. When three parameters are relevant it is still possible to visualize it in two dimensions as can be seen in Fig.2.

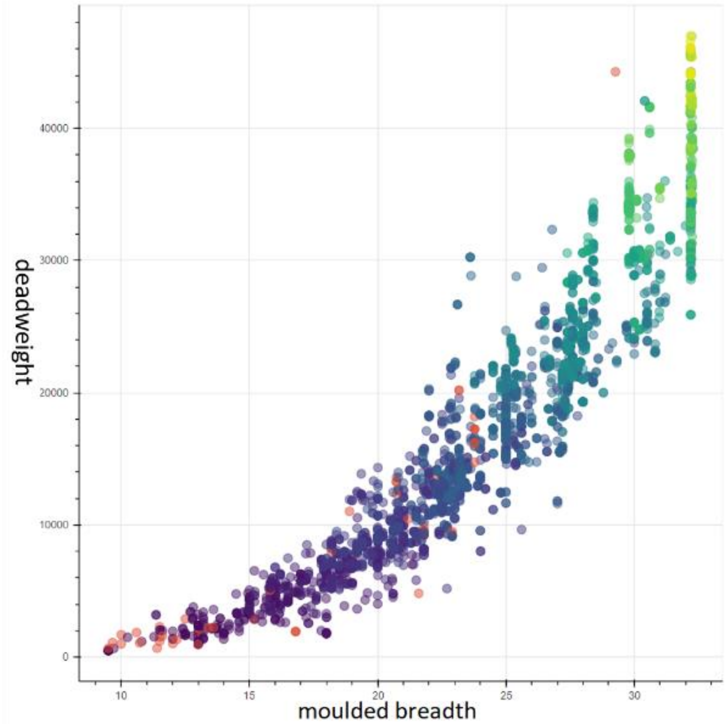

Fig.2: Container vessels color-coded by TEU capacity

However, it is often the case that more than three parameters are relevant in the preliminary ship design stage, which makes it challenging to visualize. To still be able to investigate a selection of ships or design variations with more than three parameters simultaneously, parallel coordinate plots can be used *Heinrich and Weiskopf (2013).* As an example, a parallel coordinate plot, Fig.3, was made for several hundred container vessels with the length between perpendiculars of 175 m and 200 m. Every line in a parallel coordinate plot represents one vessel, every axis in the *y* dimension represents a range of values belonging to one design characteristic.

As can be visually inspected from Fig.2, the moulded breadth has a maximum on 32.4 m, the well-known maximum width for ships to still be able to sail through the Panama Canal. This maximum moulded breadth can also be seen in Fig.3. Moreover, it is now also possible to simultaneously see all other relevant parameters of the container vessels. For example, the limited draught for the majority of vessels in this selection is smaller than or equal to 12 meters, also an important Panama Canal dimension. Besides this, you can simultaneously see the conflicting relationship between block coefficient (Cb), and maximum continuous rating (MCR) and their influence on service speed. The vessels with a high block coefficient, small maximum continuous rating, also have a slow service speed and vise-versa. When designing new vessels, these plots can be very helpful for the naval architect.

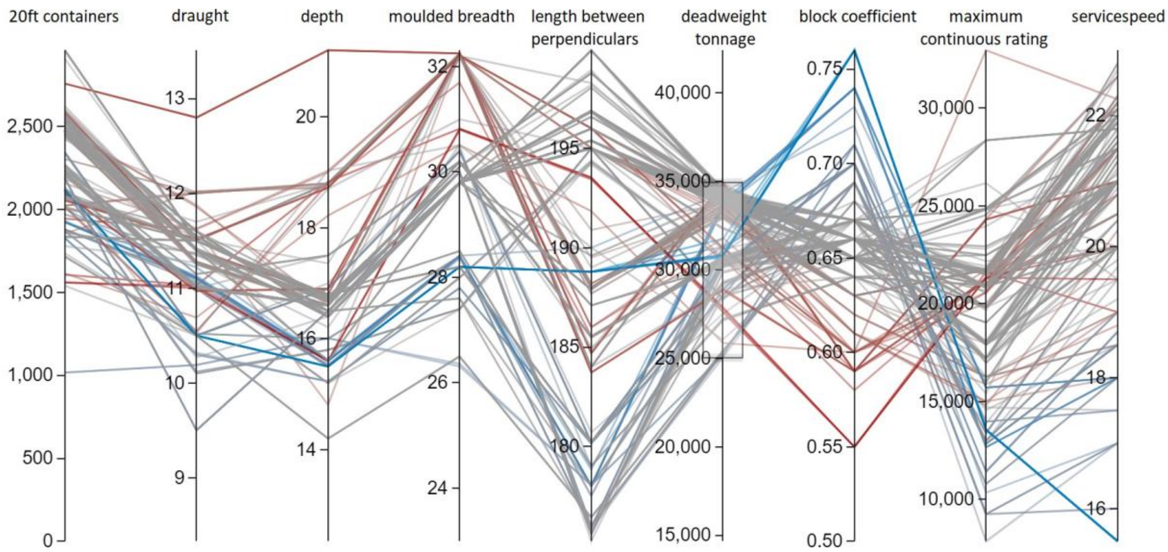

Fig.3: Parallel coordinate plot of container vessels color-coded by block coefficient, and a deadweight tonnage selection between 25000 and 35000 t

3.2. Data Pre-processing

The preliminary data analysis showed duplicate vessels and vessels which are very similar. To make sure that specific vessels are not over-represented, data pre-processing has to be done. The pre-processing consists of three steps. First all but one of the vessels with exact duplicates are deleted. Ships are considered to be duplicates if their gross tonnage, length between perpendiculars (Lbp), breadth overall (Boa), draught (T), and MCR are equal. Then all but one vessel out of a series of sister vessels are deleted. If the earlier mentioned variables are all within 1 percent of each other, the vessels are marked as too similar.

Reasons for deletion of duplicates and very similar vessels are to prevent the potential over-fitting of machine learning models. If a series of sister vessels would be present, the machine learning model would automatically put more weight on the sister vessels compared to one unique vessel. A second argument to delete sister vessels is, once a machine learning model has learned from a vessel, a second sister vessel does not add much knowledge but will only add computation and training time.

The third pre-processing step consists of creating second degree polynomial and interacting features. This is done to generate more potentially interesting features from the ship design parameters that are known. The two degree polynomials and interacting features of the example $[a,b]$ would be: $[a,b,ab,a^2,b^2]$. This way, the machine learning models have more features to learn from which leads to more accurate results.

4. Methodology

This section describes how the empirical design method is used in combination with an optimization algorithm in the so-called reference optimizer. As mentioned in the related work section, it is often the case that the empirical design equations are not available for a specific ship type or that the available equations are outdated. This is unfortunate since designing ships with wrong or outdated design equations will lead to sub-optimal designs. The empirical design equations are therefore replaced with machine learning models. These machine learning models make sure that it is no longer need to solely depend on predefined equations or the experience and knowledge of naval architects.

Machine learning models are used to learn the relationships, similarities, and trends between hundreds of vessels. However, for machine learning models to work properly, the relationship between the dependent and independent variables need to be learned. The dependent and independent variables are chosen by the naval architect. The machine learning models learn the relation between the independent and dependent variables in the training phase. After the training phase, the trained machine leaning models are coupled to an optimization algorithm which can exploit the trends learned and search for optimal design configurations which outperform the existing designs.

4.1. Setup Design Challenge

For the machine learning algorithm to work well a design challenge should be setup by the user. The design challenge consists of three parts, the design variables/parameters, the constraints/limitations, and finally the objectives.

Design Variables

The variables are setup by choosing the design parameters which have a significant influence on the final design, and which are allowed to vary. The allowed variation in the variables are controlled with a user defined lower and upper limit. However, the limit cannot be smaller or larger than the smallest and largest ship in the collection. Example of a set of design variables are: *Length between perpendiculars (Lbp), draft, draught (T), Breadth overall (Boa), block coefficient(Cb),* and *service speed (V)*.

Constraints

The design constraints are also set by the user, design constraints are typically hard limitations or strong wishes for the to be designed ship. Examples of constraints are: capacity of 30,000 tons, a cargo capacity of 2000 twenty-foot equivalent container units, a *length overall* (Loa) smaller than 180 m, and a *draught* (T) of not more than 12 m.

Objectives

Typical objectives of a ship design are the key performance indicators (KPIs) which deal with operational expenses or initial investments. Ideally, they are as low as possible however they most often do not go hand in hand and are most of the time conflicting. Examples of two objectives are: minimize the *light ship weight (LSW)* while maximizing the *dead weight (DWT)* capacity, while minimizing the *Maximum Continuous Rating (MCR)* of the main engines.

Once the design variables, constraints, and objectives have been set by the user. The relationship between the variables and the constraints and objectives can be learned.

4.2. Random Forest Regression

A random forest regression model, *Breiman (2001)*, learns the relationship between the features and one target variable. In our case the features are the design variables plus the polynomial features and the target variable one of the constraints or one of the objectives. Therefore, for each constraint, and for each objective a new unique random forest regression model is trained.

The random forest regression model learns the relation between the features and the target by fitting a multitude of decision trees. One decision tree is fitted to learn the relation between a set of random selected features with the corresponding target values. The data with the random selected features is sequentially greedily split into two sub-samples based on one of the features, until the number of samples in the nodes reach a threshold value. Resulting in an upside-down tree with nodes, branches for splits, and leaves with similar target scores.

Once e.g. 100 decision trees have been trained with the 100 randomly selected feature sets, the random forest is done training. The trees in the forest can be traversed which makes a prediction of the target variable for an unseen combination of feature values possible for each tree. These 100 outcomes of the 100 decision trees are then averaged into a final prediction. This process has been visualized in Fig.4. Because a multitude of trees are fitted, the random forest regression model is robust against outliers in the training data. However, due to the fact that the final score depends on the average of all the trained trees, the random forest regression model is not capable of extrapolation.

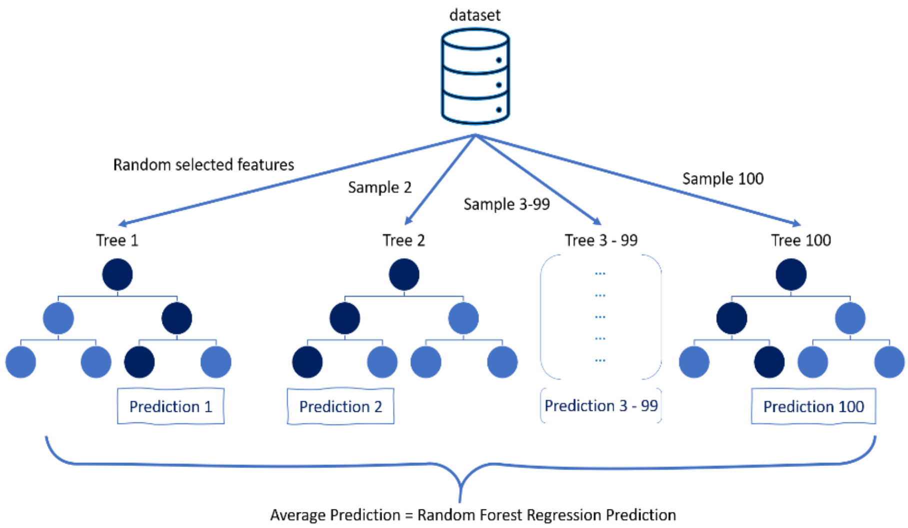

Fig.4: Random Forest Regression Model

4.3. Isolation Forest

Not only the user defined constraints limit the search space. The search space is also limited by an anomaly detection algorithm. The anomaly detection algorithm used is named Isolation Forest *Liu et al. (2008)*. Isolation forest is an unsupervised machine learning algorithm which test how easy it is to isolate certain data points. It does this by recursively splitting the data by randomly selecting a variable and a random split value between the lower and upper limit. If a sample is easy to isolate by randomly splitting the data set, it is marked as an anomaly. A sample that is hard to isolate versus a sample that is easy to isolate is visualized in Fig.5.

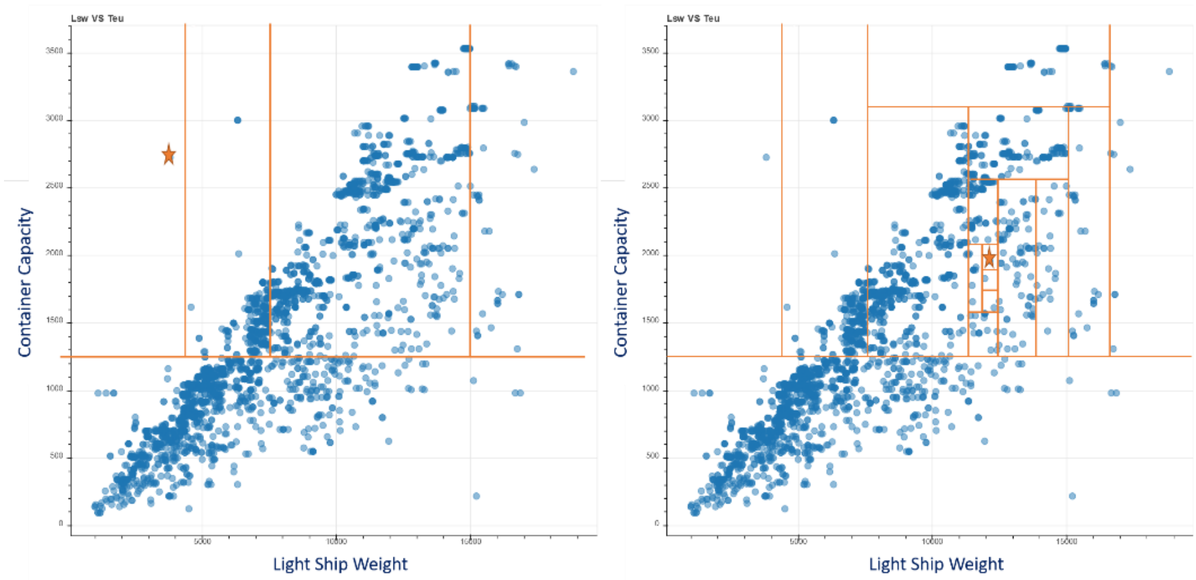

Fig.5. Isolation forest with easy to isolate sample and hard to isolate sample

In practice, this means that in case a design variation is very unique and lies outside of the trend, or if the database contains a ship with length by accident reported in feet instead of meters, it is marked as an anomaly. When searching for a new design variation, design variations which are marked as anomalies by the isolation forest will no longer be considered. This is the case because they do not follow the pattern and therefore their prediction is probably incorrect. On top of this, the isolation forest will make sure that the design variations will not exceed the limits, so that the random forest regression model is not forced to extrapolate.

4.4. Optimization Algorithm NSGA-II

The reference optimizer, searches for optimal design configurations which do not violate any of the constraints. This is done with a multi-objective optimization algorithm named Non-dominated Sorting Genetic Algorithm II (NSGAII) *Deb et al. (2002)*. NSGA-II is coupled with the design challenge by connecting the design variables, constraint and objective random forest models. First, NSGA-II is allowed to vary the design variables between the user defined lower and the upper limit, then for each try a design variation is tested. The evaluation of each design variation is done by using the random forest regression models to predict the constraint and objective scores. The objective and constraint scores are then combined with the design variable values and tested to see if the combination can be easily isolated by the Isolation Forest. Once the isolation score, the objective score, the constraint score is evaluated it is given back to the NSGA-II algorithm. The NSGA-II algorithm includes the evaluated design variations in the population of previously evaluated solutions and then new solutions are generated with the non-dominated genetic sorting strategy. NSGA-II is allowed to try 4000 design variations. After which the optimal designs are reported, and visualized on a Pareto frontier.

5. Experiments

To validate the models and the algorithms, different experiments are conducted. The first experiment is setup to test the predictive capabilities of the random forest regression models. In the second experiment a set of ships is intentionally modified to see if the isolation forest is capable of identifying the newly created anomalies. Finally, in the last experiment, we connect everything and generated design variations for a novel container ship. For the experiments 2538 container ships are used. 1219 of these vessels have the duplicate characteristics as described in Section 3.2 and are therefore not further used

5.1. Experiment 1: Random Forest Regression Test

The random forest regression models are intended to predict the performance and cost of ship designs of the future. In this experiment this situation is mimicked. Three different KPIs are learned by the random forest regression models with data from 1019 ships build before 2005, then the random forest regression models are tested with data from 96 ships build after 2010. By comparing the predicted values with the actual values it can be determined if the trained random forest regression model is good for use in practice.

The KPIs that are predicted in this experiment are LSW, MCR, and DWT. The KPIs are estimated with the random forest regressor and with empirical design equations for the specific KPIs. The design variables used to predict LSW are [*Lbp, Boa, T, Cb, MCR*]. The design variables used for MCR are [*Lbp, Boa, T, Cb, V*]. The design variables used to predict DWT are [*Lbp, Boa, T, Cb*].

5.1.1. Random Forest Regression Results

The accuracy of the random forest regression model is determined with the R^2 measure *Miles (2014).* This measure compares the real KPI values with the predicted values and see how much variation of the dependent variable can be explained by the model. With R^2 scores of 0.93, 0.90, and 0.95 for LSW, MCR, and DWT, it can be confirmed that the random forest regressor is capable of capturing a lot of the variance.

5.1.2. Empirical Design Equation Results

For Light Ship Weight for container vessels the Empirical Design Equation of *D'Almeida (2009)* is used. The equation for light ship weight is dependent on the *steel weight (SW)*, *outfitting & equipment weight (OEW)*, and *machinery weight (MW)*:

$$LSW = SW + OEW + MW$$
$$SW = 0.0293 \cdot \mathrm{Lbp}^{0.0293} \cdot \mathrm{Boa}^{0.712} \cdot \mathrm{T}^{0.374}$$
$$OEW = 0.1156 \cdot (Lbp \cdot Boa \cdot)^{0.85}$$
$$MW = 2.35 \cdot (MCR/0.745699872)^{0.60}$$

As you can see the same independent variables are used here as in the random forest regressor model. However, the estimate of this empirical formulation only obtains an R^2 score of 0.84.

MCR can be estimated with the empirical formula based on the Admiralty Constant formula from *Schneekluth and Bertram (1998).*

$$MCR = \frac{\Delta^{2/3} \cdot V^3}{C}$$

Here, Δ is displacement. The Admiralty constant itself (C) is estimated with the reference vessels from before 2005. The mean of all the Admiralty constants C that followed from the vessels before 2005 is used to make predictions for the container vessels after 2010. The R^2 score for this formula is 0.87, again a worse R^2 score compared to the random forest regressor.

The empirical formula for DWT is:

$$DWT \ = \ \Delta - \ LSW$$

Since the empirical equation for light ship weight has a worse R^2 score compared to the random forest regressor, it is no surprise that also for DWT, the R^2 score of 0.89 is lower compared to the R^2 score of the random forest regressor.

5.2. Experiment 2: Isolation Forest Test

In the isolation forest experiments, the isolation forest is trained with the data from the container vessels as described earlier. After this, two data fields per vessel are modified to create impossible design parameter/KPI combinations. All vessels are then evaluated by the trained isolation forest to see if they are marked as an anomaly or not. The modified design parameters/KPI values and the percentage of anomalies detected are presented in Table I.

As you can see, the extremer the vessels are modified, the more vessels are marked as anomalies by the isolation forest. From the results it can also be seen that there is a small percentage of vessels that have been radically changed, but have not been marked as an anomaly, this indicates that the anomaly detection algorithm does not cover all anomalies and that the naval architect should pay attention when analysing the results and do a few integrity check on the results.

Table I: Modified columns and classified anomaly percentage after this modification.

Modified Columns	**Anomaly Percentage**
no modification	15%
Lbp/1.1, *T* × 1.1	36%
Lbp/1.25, *T* × 1.25	56%
Lbp/1.5, *T* × 1.5	88%
Lbp/1.75, *T* × 1.75	99%
Lbp/2, *T* × 2	100%
MCR/2, *V* × 2	95%
LSW/2, *Cb* × 2	97%
Lbp/2, *DWT* × 2	97%
Cb/2, *Lbp* × 2	100%
T/2, *Cb* × 2	100%
Cb/2, *V* × 2	100%

5.3. Experiment 3: NSGA-II Test

For this experiment it is assumed that the random forest regression model and the isolation forest perform as intended so that the NSGA-II algorithm can be tested. If the NSGA-II algorithm can find feasible and realistic Pareto-optimal solutions, we can confirm that the reference optimizer works as intended.

The reference optimizer is tested on a container ship case. In this case NSGAII was allowed to vary the main particulars of the vessel as described in the experiments before. The LSW and MCR are minimized, while the DWT capacity should be larger than or equal to approximately 28000 tonnes. The results are visualized on the Pareto frontier in Fig.6.

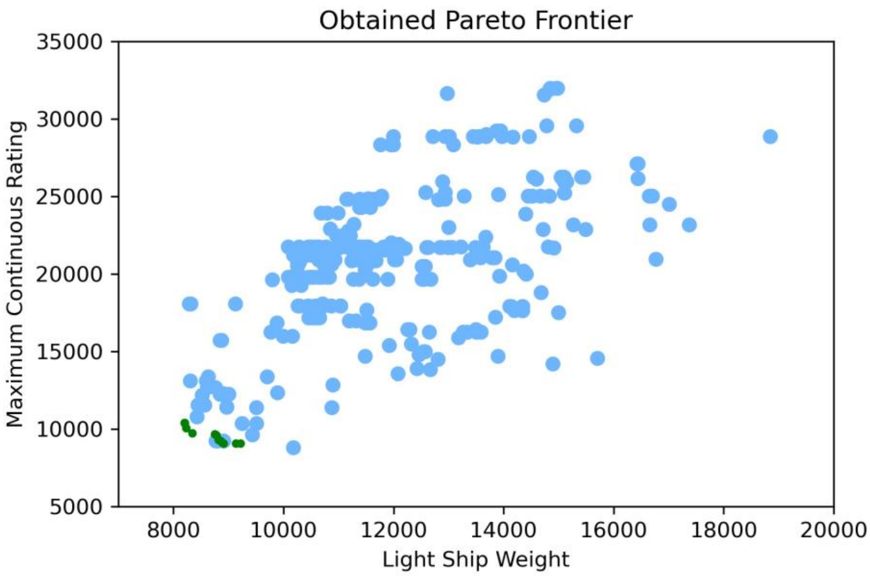

Fig.6: Obtained Pareto efficient solutions for test case. Blue indicates existing vessels which does not violate any of the constraints while green indicate the proposed solutions.

NSGA-II was capable of finding 14 Pareto efficient solutions along the Pareto front intermitted by Pareto efficient existing vessels. As previously described the algorithm only uses data and does not know any physics, it is the task of the naval architect to double check the feasibility of the proposed solutions. In this case, the physical integrity of the proposed solutions are checked with the DWT formula.

The weight balance of the vessel i.e. the sum of the DWT and LSW need to be in line with the corresponding main dimensions ($Lbp \cdot Boa \cdot T \cdot Cb \cdot \rho$). In this case, the found maximum deviation for the existing vessels is 7% with an average of 0.2%. The deviation for the proposed vessels is at the highest 2.6% off with an average of 1.8%.

In the parallel coordinate plot in Fig.7, the main dimensions and the resulting performance indicators on weight and installed power of the proposed solutions can be evaluated and compared with the existing vessels.

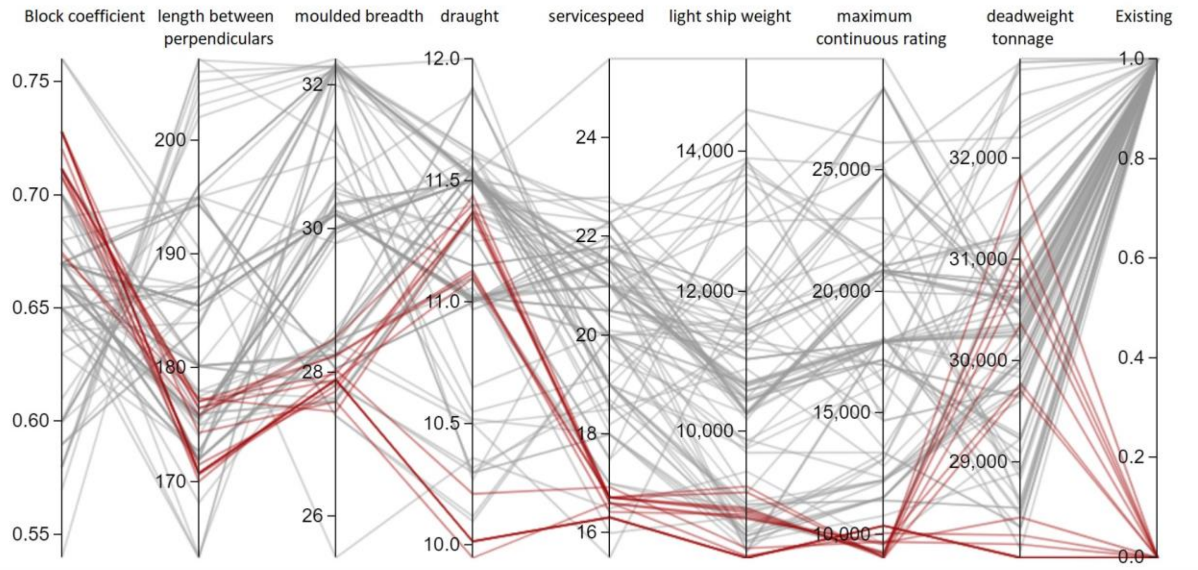

Fig.7: Parallel coordinate plot of the proposed solutions in red versus the existing vessels in grey.

6. Discussion

The reference optimizer has two drawbacks. The first drawback of the reference optimizer is that it need a sufficient amount of good data for the random forest regressors to make accurate predictions. Good data without mistakes is important since otherwise the random forest regressors will learn a wrong trend and the predictions will be off.

A second drawback of the reference optimizer is that the design challenge should be set up properly. For this, the naval architect might need to learn a few things about training machine learning algorithms. During the training at least the choice of what to choose for independent and dependent variable should be addressed in combination with different performance metrics.

7. Conclusion and Future Work

In this paper an alternative generic way is presented on how naval architects can make preliminary design decisions by using visualization techniques and machine learning algorithms.

The experiments in this paper show that random forest regressors can give better estimations for light ship weight, dead weight, and maximum continuous rating compared to empirical design equations often used by naval architects. Besides a better estimation of key performance indicators, the random forest regressors are also capable of predicting key performance indicators for which no empirical design equations are readily available in literature.

After training the random forest regressor and an anomaly detection algorithm, the models are coupled to a multi-objective optimization algorithm. This setup is capable to automatically generate optimal design configurations for preliminary ship design problems. As a practical use case, a container vessel design challenge has been solved. The setup proposed 14 new Pareto-efficient solutions. The preliminary designs consisted of main particulars of the vessels plus the key performance indicators like light ship weight, maximum continuous rating, and deadweight. After this, the preliminary designs have been evaluated with integrity checks to validate the designs.

For future work it is intended to improve the performance of the machine learning models even further and integrate a more robust anomaly detection algorithm to detect obvious mistakes better.

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