Modeling Deflection Control of Reinforced Concrete Slab Systems Using Utility Theory and Sensitivity Analysis

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Abstract: The model presented in this study provides a rational approach to deflection control of reinforced concrete two way slabs considering uncertainties in structural behavior and deflection limits and recognizes that the problem is subject to wide and unavoidable variability. Concepts of Monte Carlo simulation and utility theory are presented. A simulation model taking into account the uncertainties in materials and loads along with sensitivity analysis of results are presented. Results of simulation represented in the form of probability density function (PDF) and cumulative density function (CDF), optimum thickness and results of sensitivity analysis of reinforced concrete two-way slab systems are presented. This study explores the application of utility theory to the problem because serviceability failure can occur in structures with adequate safety against collapse, in which the question becomes an economic issue. In this study, uncertainties in time effects (Creep and shrinkage) are taken into account by using Monte Carlo simulation and are based on proposed variable parameters taken from major references.

Keywords: Monte Carlo Simulation, Utility Theory, Deflection Control, Two Way Slabs

1. Introduction and Previous Works

Section 24.2 of ACI code (ACI committee 318, 2014) begins with a general statement that "members subjected to flexure shall be designed to have adequate stiffness to limit deflections or any deformations that adversely affect strength or serviceability of a structure". With increasing use of high strength materials, longer spans and, as a result, slenderer structural members, deflection control becomes an increasingly significant facet of structural design, and leads to suggest a more rational approach to design for deflection control. It is an interest of the concrete industry and the engineering community to produce concrete structures that not only have an adequate margin of safety against collapse but also provide acceptable performance in service at minimum cost (a minimum cost analysis). A methodology based on the application of utility theory in combination with probabilistic analyses was proposed by Reid and Turkstra (1980, 1981). This approach recognizes that serviceability failure is an economic issue since an unserviceable structure may have an adequate margin of safety against collapse.

Choi (1998) developed a deterministic model based on a layer beam finite element formulation incorporating cracking with an age-adjusted effective modulus approach for creep. Using this model and statistical material parameters obtained from the literature, histogram and statistical parameters of beam deflection are obtained for range of design conditions.

Hossain and Stewart (2001) discussed a number of serviceability issues related mainly to excessive deflections of structural floor elements (beams and slabs). Field data of serviceability damage caused by excessive deflections of structural floor elements have been collected from various sources, and probabilistic models are developed. Preliminary results confirm that allowable deflection limit in these design codes represents realistic lower-bound values of damaging deflections.

A new approach was presented by Scanlon and Lee (2006) for determination of optimum thickness of one-way reinforced concrete floor systems. The paper proposed the use of utility theory as a basis for proportion members to provide acceptable deflection control. The procedure was reduced to the minimization of total cost consisting of initial construction cost and the cost of serviceability failure.

Scanlon, Lee and Kim (2007) presented a simplified procedure for estimating optimum structural parameter (member depth) by the applicability of utility theory as a basis for developing deflection control criteria. Monte Carlo simulation is used to develop histograms of selected deflection parameter; a serviceability loss function is then specified to define the onset of serviceability failure and an upper limit representing complete serviceability failure with associate costs. Optimum slab thickness is obtained by minimizing total cost consisting of initial construction cost and probabilistic cost of failure. Results for one-way slabs are developed and compared with ACI code provision for minimum thickness. Looking at the literature survey, new approaches to proportion structural concrete members are developed. These are based on utility theory which considers the many uncertainty aspects of the deflection control issue.

This study would cover the optimum proportioning of two-way floor systems which is not found in literature. Also, this study considers the local practice and associated costs and develops accordingly a rational proportioning model based on utility theory. Moreover, this model brings an opportunity to compare with codes recommendations (ACI in particular). This would point out the stand of the model as a rational economic –oriented model more than what would be a criticism to the codes.

2. Concept of Monte Carlo Simulation

Monte Carlo simulation is a numerical method used to find solutions to mathematical problems using random numbers. Often, the method is used when the problem involves uncertainty, a large number of variables, or nonlinearities, or other features which make it difficult to solve analytically. Monte Carlo simulation is classified as a sampling method because the inputs are randomly generated from probability distributions to simulate the process of sampling from an actual population.

The simulation is an iterative process where sampling is repeated until a statically significant distribution of outputs is obtained. When a system contains elements that exhibit chance in their behavior, the Monte Carlo method of simulation can be applied. The basic idea in Monte Carlo simulation is to generate values for the variable making up the model being studied. There are many variables in real-world systems that are probabilistic in nature and they might be simulated. The basis of Monte Carlo simulation is experimentation on the chance (or probabilistic) elements through random sampling. The technique breaks down into five simple steps (Barry et al, 2003):

1- Setting up a probability distribution for important variables

2- Building a cumulative probability distribution for each variable

3- Establishing an interval of random number for each variable

4- Generating random numbers

5- Actually simulating a series of trials

The Monte Carlo simulation allows replacing point estimates with fuzzy values that reflect an uncertainty. Uncertainty inputs can be modeled using a variety of distribution including the Uniform, Triangular, Normal, lognormal, Gamma, Exponential, Beta, Bernoulli, Poisson, and custom distributions. Results can be presented as cumulative or frequency distributions which clearly communicate range of possible outcomes.

3. Concept of Utility Theory

The basic idea of serviceability design based on utility theory is that structural serviceability can be considered as a type of utility defined in monetary terms as the difference between the benefit obtained from a serviceable structure minus the total cost consisting of initial cost of construction minus cost of serviceability failure. If the benefit obtained from a serviceable structure is considered a constant, the maximum utility is obtained when the total cost is a minimum.

The cost of construction can be calculated in the usual fashion by computing quantities and applying appropriate unit prices. To calculate the cost of failure consideration must be given to the variability of deflection response for a given member and the fact that serviceability in general does not have a crisp limit. For a given member, the deflection response can be represented by a probability distribution. Superimposed on the distribution is a loss function that defines the onset of serviceability failure up to complete serviceability failure; i.e. the deflection at which the structure becomes completely unserviceable with an associated cost to remedy. The greater the overlap between the loss function and the distribution, the greater the probabilistic cost of failure. Replacing the continuous distribution with a histogram obtained by Monte Carlo simulation, the cost of failure for a particular member can be calculated as (Rosowsky & Stewart, 2001; Hossain & Stewart, 2001):

$C_{Fi} = \Sigma_{j=1} H_i(x_i)\ p(x_j)$ ……………. (1)

In other words, a formulation of utility theory in which serviceability utility U can be expressed as:

$U = B - C1 - \Sigma_i\ C_{Fi}$ ……………. (2)

where B equals the benefit derived from full serviceable structure; C1 equals the initial construction cost; CFi equals Cri*Hi(x) cost due to failure in mode i; Cr the cost of failure due to being completely unserviceable in mode i; x equals the deflection ratio to span length; and Hi(x) equals the serviceability loss function as a function of deflection to span length ratio in mode i.

As member depth increases, the initial construction cost is expected to increase, while the expected cost of serviceability failure is expected to decrease, as the stiffness increases. Adding initial construction cost to failure cost results in plot of total cost. The optimum member thickness occurs where the total cost is a minimum.

Determination of cost of failure is more problematic; it can be attributed to a number of sources (modes), including direct cost of repairs and cost due to lost production during repair as an example. The expected cost of serviceability failure due to floor deflection can be computed if the probability density function (PDF) for deflection is known and a serviceability loss function is defined, as shown in Figure 1. The probability density function provides a measure of probability that particular deflection value will be exceeded. The serviceability loss function recognizes that serviceability failure generally does not have well defined limits. The loss function H(x), shown in Figure 1, indicates the onset of serviceability failure (that is, need for repair to remedy the problem) at a deflection parameter x1 and a gradual increase reaching a value of 1.0 at deflection parameter x2, at which point the floor is assumed to be completely unserviceable requiring cost Cr to remedy. For deflection parameters in the range x1 to x2, the repair costs are defined as H(x).Cr. The utility function for a given mode can be calculated as

$u_i(x) = B - C_1 - C_{Fi}(x)$ ………………………….(3)

Where:

$u_i(x)$: the utility function for failure mode i
C_{Fi}: failure cost function for failure mode i

$$E[U_i] = \int_{-\infty}^{\infty} u_i(x) f(x) dx$$

The expected utility is given by: … ………. (4)

Where $f(x)$ =probability density function of x, substituting Eq. (3) into Eq. (4) gives

$$E[U_i] = B - C_1 - C_{Fi}\left[\int_{-\infty}^{\infty} H(x) f(x) dx\right] \text{.....(5)}$$

The continuous probability density function can be replaced by histogram obtained for example from Monte Carlo simulation.

Equation (5) can then be converted into discrete type, as follows:

$$E[U_i] = B - C_1 - C_{Fi}\sum_{j=1}^{n} H(x_j) p(x_j) \text{.....(6)}$$

Where p(xj) is histogram of xj in a given member.

For a given serviceability loss function H(x), as the member depth and stiffness increase, the PDF f(x) shifts to the left and the expected cost of failure decreases. The greater overlap between the deflection PDF and serviceability loss function, the greater is the expected cost of failure. The failure cost can be obtained by summing failure cost for each mode. Figure 1 shows the conceptual examples of deflection histograms along with a single- step discontinuous, two-step discontinuous loss function, and the continuous loss function. A two- step discontinuous loss function is used for calculating indirect cost of failure such as loss of production and a continuous loss function for direct cost of failure and cost of repair. Total cost of failure could be calculated by summing these two types of failure costs.

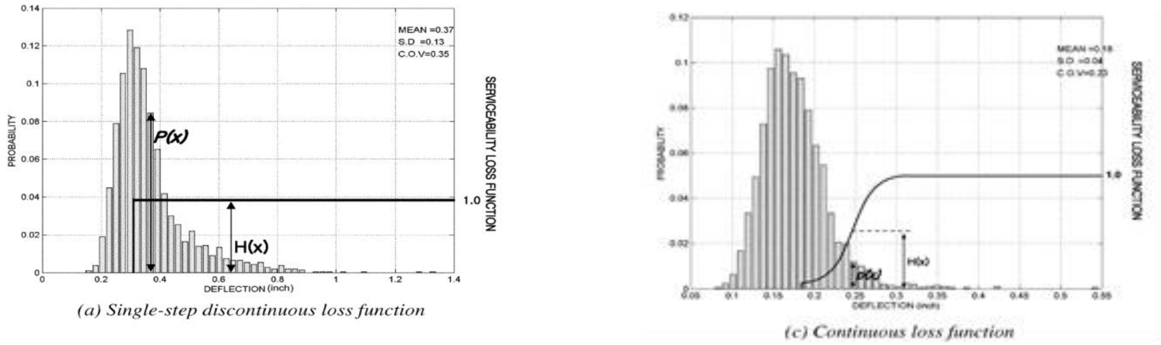

Figure 1: Conceptual examples of deflection histograms along with a single- step discontinuous, two-step discontinuous loss function, and the continuous loss function

4. Serviceability Loss Function

It is understood that unserviceability due to excessive deflection reduces property values and rental income as well as causing disturbance to existing occupants (which produces intangible costs). Non-structural repairs may be quite costly, and in most cases, such repairs do not provide any permanent solution to time-dependent deflection induced problems. Hence, total deflection will continue to increase (due to creep and shrinkage effects); and ultimately, structural repair or replacement is the only option to reactivate the utility of the structure. Such remedial actions are likely to be expensive compared to individual non-structural expenditures for unserviceability. Based on the above discussion, a serviceability loss function for reinforced concrete elements (at any time t) can be written as (Black & Fredric, 1985; Stewart, 1998):

$$H(x) = \begin{cases} o \longrightarrow (x) < a_1 \\ \left(\dfrac{x - a_1}{a_2 - a_1} \right) \longrightarrow a_1 \leq x < a_2 \\ 1 \longrightarrow x \geq a_2 \end{cases}$$

Where x is the mid-span total deflection to span length ratio of the structural element at time t.

a_1 = minimum deflection ratio at which the deflection is first noticeable.

a_2 = minimum deflection ratio which makes the floor out of utility and structural repair or replacement becomes the only solution to make it re-useable. In an earlier study, probabilistic models were fitted to damaging deflection data collected from various published literature for flexural floor element (beams and slabs). See Table 1. According to their study, the minimum value of deflection- to-span ratio for which perception damage is reported is 0.003(≈1/333).

Table 1: Statistical parameters of damaging deflections (Δ/L) (Choi et al., 2004)

Parameter	Perception damage	Partition wall damage
Samples	60	51
Minimum value	0.0030	0.0006
Maximum value	0.0171	0.0135
Mean	0.0077	0.0054
COV	0.42	0.57

In the present study, the data presented by (Hossain & Stewart, 2001) are used to establish upper and lower bound for continuous loss function for direct cost of repair. For convenience, the Cumulative density function (CDF) is used to define the loss function H(x) between the two limits because it is assumed in this study that expected repair costs follow the probability of damaging deflection.

5. Optimum Thicknesses of Two-Way Reinforced Concrete Slab Systems

The proposed model is divided into two stages, as shown below; determinations of immediate and long-term deflections taking into account the uncertainties due to member behavior and loading by using Monte Carlo Simulation. Determinations of probabilities of failure of slabs, and superimposing the Probability Density Function (PDF) (obtained from simulation results) and serviceability loss function, followed by estimating optimum thicknesses of reinforced concrete two way slabs according to process of stage one. Making a sensitivity analysis to see how is the sensitivity of the results to variations in assumed loss functions and assumed costs.

6. Deflections Calculation (Stage 1)

Many models have been presented over the years for calculating deflections. They range from simple models based on elastic beam theory and empirical time-dependent multipliers to sophisticated finite element models with complicated constitutive modeling such as linear visco-elastic modeling of creep. In this study, a simulation model represented by Monte Carlo Simulation is used for deflections (initial & long-term) predicted according to ACI 209 Recommendations approach (ACI committee 209, 2008). The general equation of ultimate deflection is summation of deflection due to (dead load, creep, shrinkage, and live load) respectively:

$$\Delta_t = \Delta_{i)D} + \Delta_{u)D} + \Delta_{sh)u} + \Delta_{i)L}$$
………... (7)

$$\Delta_t = \frac{\varepsilon_{fp} q_D l^4}{\mathrm{E}_{ci} \mathrm{I}_g} + \varepsilon_r \nu_u \Delta_{i)D} + \varepsilon_w \phi_{sh} l^2 + \frac{\varepsilon_{fp} q_l l^4}{\mathrm{E}_c \mathrm{I}_g}$$
……………….. (7-1)

Where: ε_{fp} is deflection's coefficient, q_d, q_l are dead load and live load, L is a span length, Ig is gross moment of inertia, and Eci, Ec are Modulus of elasticity of concrete at the time of initial load, Modulus of elasticity of concrete at time t respectively. Creep under sustained load is accounted for using the age-adjusted effective modulus proposed by ACI 209R[17] and given by Eq (8)

$$E_{ct} = \frac{E_{ci}}{1+\chi\nu_t} \quad \text{………………..…. (8)}$$

Where Eci is the instantaneous modulus of elasticity, χ is the aging coefficient; and νt is the creep coefficient at time t defined as the ratio of creep strain at time t to initial elastic strain. Many formulations have been proposed for the creep coefficient. For this study, the simple model presented by (ACI Committee 209, 2008) is used for standard conditions, as follows:

$$\nu_t = \frac{t^{\psi}}{d+t^{\psi}}\nu_u \quad \text{…………………. (9)}$$

Where νu is the ultimate creep coefficient and t is the time after loading.

The age-adjusted effective modulus is assumed to apply in both compression and tension and is assumed as an upper limit on the tensile modulus of elasticity implying that the tensile strength f_r decreases with time under sustained load, as would be expected. The effect of shrinkage curvature due to nonsymmetrical reinforcement on a section is accounted for using the empirical Miller's method (ACI Committee 209, 2008) in which the deflection due to shrinkage warping is given by:

$$\Delta_{sh} = \varepsilon_w \phi_{sh} l^2 \quad \text{………… ….. (10)}$$

Where εw is a deflection coefficient presented in (ACI Committee 209, 2008) for different boundary conditions and Φsh is the curvature due to shrinkage warping. The empirical Miller's method is used for computing shrinkage curvature using Eq (11) for singly reinforced members, ρ'=0.

$$\phi_{sh} = 0.7\frac{\varepsilon_{sh}}{h}(\rho)^{\frac{1}{3}} \quad \text{…………….. (11)}$$

The simple (ACI Committee 209) equation for shrinkage strain at time t is used in standard conditions for moist cured concrete, that is

$$\varepsilon_{sh} = \frac{t^{\alpha}}{f+t^{\alpha}}(\varepsilon_{sh})_u \quad \text{……... (12)}$$

Where (εsh) u is the ultimate shrinkage strain and t is the time from the end of the initial curing. It is also indicates that values of ν_u and $(\varepsilon sh)_u$ need to be modified by correction factors for conditions other than the standard conditions (ACI Committee 209, 2008).

7. Sources of Uncertainty and Variability

Variability of deflections of reinforced concrete members can be attributed to uncertainties in member material properties and dimensions and loading history. Additional uncertainties in predicting deflections by calculations occur due to modeling errors in the calculation procedure including treatment of boundary conditions, the effects of restraint stresses on crack development, and uncertainties in the time of installation of nonstructural elements when considering incremental deflections occurring after the installation of nonstructural elements (Stewart, 1998), (ACI Committee 435, 1995). This study focuses on the effect of variability of member properties and loads.

Assumed statistical properties of the variables considered are listed in Table 2. In most cases, the statistical properties are obtained from literature, and the assumed statistics for creep and shrinkage are estimated from data presented by (ACI Committee 209) on average values and typical ranges.

Table 2: Probability model of random variables

Variable		Mean	COV	S.D	Reference
Concrete (in place)	f'_c (MPa)	$0.675f'_c+7.5$ $8 \leq 1.15\,f'_c$	0.176	□	(Scanlon et al, 2007)
Concrete (in place)	f_r (MPa)	$0.69\sqrt{f'_c}$	0.218	□	(Scanlon et al, 2007)
Concrete (in place)	Ec (MPa)	$4700\sqrt{f'_c}$	-	□	(ACI committee 318, 2014)
Reinforcement	As	0.99 An	0.024	□	(Scanlon et al, 2007)
Reinforcement	Es (MPa)	201326.91	0.024	□	(Rosowsky and Stewart, 2001)
Creep	ν_u	2.35	□	0.6	(ACI committee 209, 2008)
Creep	ψ	0.6	□	$6.66*10^{-2}$	(ACI committee 209, 2008)
Shrinkage	d	10 days	□	6.66 days	(Choi et al, 2004)
Shrinkage	$(\varepsilon_{sh})_u$	$780*10^{-6}$	□	$121.6*10^{-6}$	(Choi et al, 2004)
Shrinkage	α	1.0	□	$3.33*10^{-2}$	(Choi et al, 2004)
	f	55 days	□	25 days	(Choi et al, 2004)

In the general equations for predicting creep and shrinkage of concrete at any time, d and f (in days), ψ and α are considered as constants for a given member shape and size that define the time-ratio part. It is assumed that the parameters are independent except for concrete compressive strength, modulus of rupture, and modulus of elasticity. Modulus of rupture and modulus of elasticity are correlated with compressive strength. This is accomplished as follows. A value of compressive strength is selected randomly. This value is then used to compute a mean value for modulus of rupture and modulus of elasticity. The assumed COV for these parameters is then used to establish Probability Density Functions (PDF). Values of modulus of rupture and modulus of elasticity are then obtained randomly from these distributions.

8. Process of Monte Carlo Simulation

Monte Carlo simulation is a commonly used technique for generating statistical data on a quantity such as deflection based on known or assumed statistics of input variables. In this study, Monte Carlo simulation is used to generate random samples of deflections as calculated by the suggested model from (ACI Committee 209). To obtain a probabilistic deflection for each slab case considered, the following procedure is used:

Values for each of the random variables in the model for deflection calculation are generated randomly based on assumed statistical distributions; a deflection is calculated based on these random values; the process is repeated for N simulation cycles; the mean and standard deviations are calculated for the N values of deflection. A histogram is generated based on the N values. The appropriate number of simulation cycles N is determined that a sufficient level of repeatability is achieved. In this study, a value of N=1000 is found to provide satisfactory results.

9. Construction Load and (Load Time Histories)

Long-time deflections depend on the load-time history. It is assumed that the critical deflection is strongly related to a load-time history, designated LH1. Load –time history LH1 consists of the construction load W_{co}, which is taken equal to the service dead load W_d plus live load W_L applied instantaneously, followed by a sustained load Ws consisting of dead plus sustained live load. The sustained load is assumed to remain constant and the balance of the live load is applied at the end of the assumed load duration. Consequently, two instantaneous deflections Δ_{co} and Δ_s can be induced corresponding to two instantaneous loads, W_{co} and W_s, and long-time total deflection Δ_t can be calculated by summing instantaneous deflection Δ_s, due to sustained load, sustained long-time deflection due to creep and shrinkage, and instantaneous deflection due to the portion of live load applied at the end of the load history. Table 3 shows the probability of assumed load history. Generally, the loads acting on the office floor during its construction and use are (Ayoub & Karshenas, 1994; Ellinwood & Culver, 1977):

Sustained dead load: self-weight of the structural members.
Sustained construction live load: weight of construction workers, construction equipment, and stacking of materials after concrete placement.
Sustained floor live loads: those loads are relatively constant within a particular occupancy; namely, the weight of people, furniture, partition, and other portable fixtures and equipment.

Table 3: Probability load model of random variables

Load		Statistical parameters	Distribution	Reference
Construction load	Form work load(additional dead load)	Mean =0.11 Dn COV= 0.10	Normal	(El-Shahhat et al, 1993)
	Sustained construction live load	Mean=0.29kPa COV= 1.10	Normal	(Ayoub and Karshenas, 1994)
	Stacking load	Mean=0.974kPa COV= 0.60	Normal	(Ayoub and Karshenas, 1994)
Dead load		Mean =1.05 Dn COV= 0.10	Normal	(Stewart, 1996)
Live load	Sustained live load	Mean =2.4kPa COV=0.056	Normal	(Ellinwood and Culver, 1977)

10. Calculation of Optimum Thickness (Stage 2)

Determination of optimum slab thickness is dependent in this study on the optimum cost, which consists of construction cost plus cost of failure. Failure cost equals product of (summation of direct cost (repair cost) and indirect cost (loss of product)), by λ (percentage of failure) obtained by superimposing Probability Density Function (PDF) and Serviceability loss function. After that the optimum slab thickness is selected as for slabs that have minimum cost (considering the deflection control and taking into account the effect of creep, and shrinkage in floor).

11. Discussion

It is an interest of the concrete industry and engineering community to produce concrete structures that not only have an adequate margin of safety against collapse but also provide acceptable performance in service at minimum cost. The preliminary results presented previously suggest that the proposed approaches to design for serviceability can provide a rational base for deflection control criteria covering a wide range of design situations. Serviceability loss functions can be developed for specialized applications. For example, a floor supporting sensitive equipment may have upper and lower limit more stringent than those used in the present study. It is not anticipated that analysis of the type outlined previously would be employed in routine building design. The methodology, however, may provide a basis for more generalized code deflection control criteria in the future.

In the present study, the long-term deflection is selected as the basis for analysis. The procedure can be extended to consider other deflection criteria including deflections occurring at any time between initial construction and long-time in-service use. A methodology based on application of utility theory in combination with probabilistic analyses is proposed in this study. This approach recognizes that serviceability failure is an economic issue since an unserviceable structure may have an adequate margin of safety against collapse. The methodology can be extended to other structural systems.

12. Conclusion

A methodology based on the application of utility theory in combination with probabilistic analyses is proposed in this study. This approach recognizes that serviceability failure is an economic issue since an unserviceable structure may have an adequate margin of safety against collapse. The model presented in this study provides a rational approach to deflection control considering uncertainties in structural behavior and deflection limits and recognizes that the problem is subject to wide and unavoidable variability. The methodology has the potential to produce improvement in design codes related to serviceability. The method presented in this study has been applied to compute long- term deflections at midpoint of interior panels of floors of six existing buildings.

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