



# Collusive networks in market-sharing agreements under the presence of an antitrust authority

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## Abstract

This article studies how the presence of an antitrust authority affects market-sharing agreements made by firms. These agreements prevent firms from entering each other's market. The set of these agreements defines a collusive network, which is pursued by antitrust authorities. This article shows that while in the absence of the antitrust authority, a network is stable if its alliances are large enough, when considering the antitrust authority, more competitive structures can be sustained through bilateral agreements. Antitrust laws may have a pro-competitive effect, as they give firms in large alliances more incentives to cut their agreements at once.

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# 1 Introduction

Reciprocal market-sharing agreements between firms are agreements by which firms divide up a market and agree not to enter each other's territory. These agreements are an anti-competitive practice; moreover, if after an investigation, the antitrust authority finds proof of market-sharing agreements, the firms involved are penalized.

The goal of the present article is to study how the presence of an antitrust authority affects the market-sharing agreements made by firms. We examine the network structure that arises when each firm takes into account the cost, imposed by competition authorities, from signing these collusive agreements.

Antitrust authorities spend a substantial amount of time and effort attempting to deter collusive market-sharing agreements. An example that stresses the importance of the problem studied and also helps us to understand the problem itself is the following. In October 2007, the Spanish Competition Authority (Comisión Nacional de la Competencia) fined the savings banks BBK, Kutxa, Caja Vital and Caja Navarra over €24 million.<sup>1</sup> Between 1990 and 2005, the cartel's members had agreed to carve up markets among them. In a minutes of one of the meetings held by the members of the cartel on February 1990,<sup>2</sup> it was noted that the top representatives of the "Basque Savings Banks and also Navarra Savings Bank have reaffirmed their commitment to maintain the territorial status quo ... thus avoiding competition among themselves and [they] agreed that the framework of the Federation remains the forum for information and sharing decisions on expansion and the way of opening new offices...".<sup>3</sup>

Accordingly, none of the savings banks in the cartel opened any branch in each other's "traditional" territory (while conducting a remarkable territorial expansion in other provinces, especially near the borders).

Consequently, this kind of agreements reduces competition in a market and thus these agreements damage to consumers.

In this article, market-sharing agreements are modeled as bilateral agreements, whereby firms commit to staying out of each other's market. The set of these reciprocal agreements gives rise to a collusive network among firms.

We choose a network framework because the structure of the relationship is important. Let us consider an antitrust authority defined by a probability of inspection and by a fine that is imposed on firms that are proved guilty of market-sharing agreements.

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<sup>1</sup>It is the second-largest fine that has been imposed by the Spanish Competition Authority.

<sup>2</sup>See Expediente 617/06. Comisión Nacional de la Competencia, Spain (minutes 02-06-90). <http://www.cncompetencia.es/Inicio/Expedientes/tabid/116/Default.aspx?numero=617/06&ambito=Conductas>

<sup>3</sup>Own translation.

Assume that when the antitrust authority inspects, it is able to detect, without error, the existence of a collusive practice. Let us consider Figure 1.

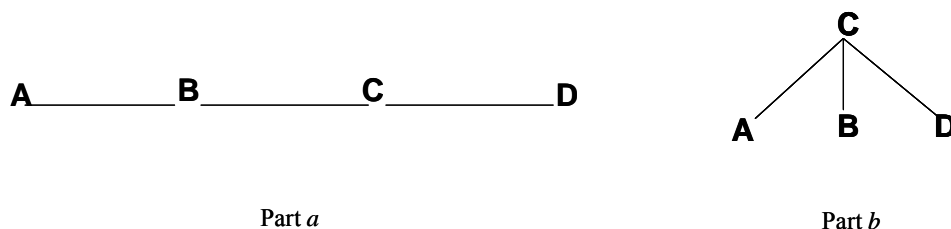


Figure 1.

In this example, A,B,C and D represent four savings banks and the lines between them represent the existence of a market-sharing agreement between them. For example, in the part *a* of figure, savings bank C is linked by a market-sharing agreement with savings banks B and D.

Each part of the figure depicts a different structure of relationships among savings banks. In part *a*, each firm is connected to others as in a line. Furthermore, in part *b*, savings banks A, B and D are connected with savings bank C but are not linked to each other. In this case, they form a star.

In part *a* of the figure, if the antitrust authority inspects savings bank C, the antitrust authority may only destroy the market-sharing agreements that C holds with savings banks B and D. In part *b* of the figure, however, when the antitrust authority inspects C, it could destroy the *entire* network of relationships. In such a case, it is able to detect agreements that savings bank C holds with savings banks A, B and D.

Therefore, the antitrust authority is more successful in the second case than in the first case. If the antitrust authority knows what the structure of relationships among the firms is, then it may concentrate its efforts in order to pick up the firm in the central position, as by doing so it is able to destroy the entire network of relationships. Consequently, the structure of relationships is important, and both firms and competition authorities should take into account this fact when defining their actions.

The present article answers the question of how the structure of collusive networks interacts with the antitrust policy that tries to deter such collusive practice and which are their implications on the competition.

We first study the actual probability of being discovered in a collusive network framework. We show that the probability of being caught depends on the agreements that each firm has signed. That is, the probability of firm *i*'s being discovered depends not only on whether firm *i* is inspected by the antitrust authority but also on whether any firm that has formed an agreement with *i* is inspected. Therefore, if a firm is

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inspected and a market-sharing agreement exists, then it is discovered, and the firms involved are penalized. However, the firm in consideration may be detected without being inspected because any firm that has an agreement with this it is inspected.

We then provide a characterization of a stable network. While in the absence of the antitrust authority, a network is stable if its collusive alliances are large enough, when the antitrust authority is considered, structures that are more competitive can be sustained through bilateral agreements.

Furthermore, when the notion of strong stability is considered, the antitrust authority has a pro-competitive impact. That is, as the probability of inspection increases, firms in large alliances have more incentives to renege on all their agreements at once, which might lead to a breakdown of collusion.

This article brings together elements from the literature of collusion (particularly, market-sharing agreements), networks, and law enforcement.

Networks is currently a very active field of research. Prominent contributions to this literature include, among others, Jackson and Wolinsky (1996), Goyal (1993), Dutta and Mutuswami (1997), Jackson and van den Nouweland (2005) and Goyal and Joshi (2006). In particular, in the first, the formation and stability of social networks are modeled when agents choose to maintain or destroy links using the notion of pairwise stability. We follow Jackson and Wolinsky (1996) and Jackson and van den Nouweland (2004) to characterize the stable and the strongly stable networks.

Asides from these theoretical articles, there is also more and more literature that applies the theory of economic networks to models of oligopoly. In particular, the present article is closely related to Belleflamme and Bloch (2004).<sup>4</sup> They have analyzed the collusive network of market-sharing agreements among firms, but they do not take into account the existence of antitrust authorities. Therefore, their results may be limited under those circumstances. They find that, in a stable network, there exists a lower bound in the size of collusive alliances. Moreover, when that threshold is equal to one, the set of isolated firms is composed, at most, by only one firm. These results are in contrast with the results of the present article. Under the presence of the antitrust authority, we are not able to define that lower bound and, ultimately, this fact implies that more competitive structure are possible to sustain in a such case.

Another application of network economics is networks and crime. Two recent articles related to the present one are Calvó-Armengol and Zenou (2004) and Ballester, Calvó-Armengol, and Zenou (2006).

Calvó-Armengol and Zenou (2004) study the impact of the network structure and its geometric details on individual and aggregate criminal behavior. Specifically, they

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<sup>4</sup>Market sharing agreements also exist in procurement auctions. Thus, for example, Belleflamme and Bloch (2004) apply their results to models not only of oligopoly but also of private value auctions. In the last case, they find that the collusive stable alliances are quite large and some stable configurations allow for the presence of independent firms which free-ride on the formation of the collusive large alliance. These results hold in the absence of an antitrust authority.

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provide a model of networks and crime, where the expected cost of committing criminal offenses is shaped by the network of criminal partners. Ballester, Calvó-Armengol, and Zenou (2006) further develop this approach. For their main results, they relate individual equilibrium outcomes to the players' positions in the network and also characterize an optimal network-based policy to disrupt the crime. In these articles, the network formation game is analyzed. This approach is different from ours. That is, we dispense with the specifics of the noncooperative game, and we model a notion of what is stable (a fixed-network approach). The other difference is the kind of externalities that one link entails. In those articles, the competition among criminals for the booty acts as a negative externality. Additionally, they assume that the criminal connections transmit to players (criminals) the necessary skill to undertake successful criminal activities, that is, a positive externality. Specifically, higher the criminal connections, lead to a lower individual probability of being caught. These assumptions are in contrast with assumptions in the present article about the externalities of signing a new agreement. Namely, we assume that more agreements increase the "booty" as long as the individual profits are a decreasing function in the number of active firms in the market (positive externality). On the other hand, each link entails a negative externality. As the number of agreements increases, the probability of being discovered also increases.

Regarding the collusion literature, after the seminal contribution of Stigler (1950), collusive cartels have been extensively studied. For an excellent reference of this literature, see Vives (2001).

As the present article, there are a number of articles that study the effect of antitrust policy on cartel behavior. Among others, we can mention Block et al. (1981) as the first systematic attempt to estimate the impact of antitrust enforcement on horizontal minimum price fixing. Their model explicitly considers the effect of antitrust enforcement on the decision of firms within an industry to fix prices collusively. They show that a cartel's optimal price is an intermediate price (between the competitive price and the cartel's price in absence of antitrust authority) and that this intermediate price depends on the levels of antitrust enforcement efforts and penalties.<sup>5</sup>

However, the interest for studying the effect of the antitrust policy on the collusive behavior has reemerged. Harrington (2004) and Harrington (2005) explore how detection affects cartel pricing when detection and penalties are endogenous. Firms want to raise prices but not suspicions that they are coordinating their behavior. In Harrington (2005), by assuming that the probability of detection is sensitive to price changes, he shows that the steady-state price is decreasing in the damage and in the probability of detection. These results are in line with results in the present article.

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<sup>5</sup>Additionally, for example, Besanko and Spulber (1989 and 1990) with a different approach, use a game of incomplete information where the firms' common cost is private information and neither the antitrust authority nor the buyers observe the cartel formation. They find that the cartel's equilibrium price is decreasing in the fines. LaCasse, 1995 and Polo, 1997 follow this approach.

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He finds, however, a long-run result of neutrality with respect to fixed penalties.

The main difference between the present article and the previous articles is that we study the impact of the antitrust policy on the structure of collusive agreements.

The outline of this article is as follows. Section 2 presents the model of market-sharing agreements and provides general definitions concerning networks. Section 3 characterizes the stable and strongly stable collusive networks in a symmetric context. Likewise, this section studies the set of pairwise stable and strongly stable networks under different levels of antitrust enforcement, and it analyzes the impact of the antitrust authority over competition. The article concludes in Section 4. All proofs are relegated to an appendix that it is available to readers upon request.

## 2 The Model

### Firms

The model consists of  $N$  risk-neutral and symmetric firms indexed by  $i = 1, 2, \dots, N$ . Each firm is associated to a market, that is, its home market. Markets are assumed symmetric. We are considering that each firm has incentives to enter into all foreign markets. Nevertheless, firm  $i$  does not enter into foreign market  $j$ , and vice versa, if a reciprocal market-sharing agreement exists between them.<sup>6</sup>

Let  $g_{ij} \in \{0, 1\}$  denote the existence of an agreement between firms  $i$  and  $j$ . Thus,  $g_{ij} = 1$  means that firm  $i$  has signed an agreement with firm  $j$  and vice versa.

Let  $n_i$  be the number of active firms in market  $i$  and  $m_i$  be the number of agreements formed by firm  $i$ . That is,  $n_i = N - m_i$ .

Let  $\pi_j^i(\cdot)$  be the profits of firm  $i$  on market  $j$ . Firm  $i$  has two sources of profits. Firm  $i$  collects profits on its home market,  $\pi_i^i(n_i)$ , and on all foreign market where there does not exist an agreement,  $\sum_{j, g_{ij}=0} \pi_j^i(n_j)$ . The symmetric firm and symmetric

market assumptions allow us to write  $\pi_j^i(\cdot) = \pi(\cdot)$ . Therefore, the total profits of firm  $i$  can be written as follows:

$$\Pi^i = \pi(n_i) + \sum_{j, g_{ij}=0} \pi(n_j) \quad (1)$$

This article appeals to some properties for profit functions used by Belleflamme and Bloch (2004), henceforth BB. We assume that: (i) individual profits are decreasing in the number of active firms in the market,  $\pi(n_i - 1) - \pi(n_i) \geq 0$ ; and (ii) individual profits are log-convex in the number of active firms in the market,  $\frac{\pi(n_i-1)}{\pi(n_i)} \geq \frac{\pi(n_i)}{\pi(n_i+1)}$ .

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<sup>6</sup>It is assumed that these agreements are enforceable.

Additionally, it is assumed that firms have limited liability, that is,  $\Pi^i \geq 0$  is the maximum amount that the firm could pay in case a penalty were imposed by an antitrust authority.

## The Antitrust Authority

We define an antitrust authority (AA) as a pair  $\{\alpha, F(\Pi)\}$ , where  $\alpha \in [0, 1)$  is the constant probability that a market-sharing suit is initiated, and  $F(\Pi) \geq 0$  represents the monetary penalty that a firm must pay if it is convicted of market-sharing agreements. We assume that the AA sets the penalty equal to firms' limited liability. That is,  $F(\Pi) = \Pi$ .<sup>7</sup>

The technology is such that when the AA inspects, if there is a market-sharing agreement, then the AA detects it. Additionally, the AA also identifies the two firms involved in the agreement. That is, if a firm is inspected by the AA, it is assumed that the AA is able to detect, without error, whether a market-sharing agreement has occurred. Likewise, if it has occurred, the AA can detect the firms that signed that agreement. In such a case, both firms are penalized, and each must pay  $F(\Pi) = \Pi^i$ .

In the economic literature of optimal enforcement, fines are usually assumed as socially costless. Therefore, when the AA seeks to deter collusion, the fines should be set at the maximal level in order to minimize the inspection cost.<sup>8</sup> An implication of this is that the fines need not to be related to the illegal profits or to the harm that the offenders caused. They only need to be as high as it is possible in order to deter collusion. This implication holds as long as there are not legal errors in the detection process (false convictions), or as long as the fines do not imply bankruptcy to convicted firms. Since, in the article, we assume that the competition authority does not commit legal errors and the framework is a static one, that is, we do not care about bankruptcy, then it is consistent to set the fine equals to firms' limited liability, which is, in our model, equal to the total firm's profits.<sup>9</sup>

Regarding the inspection process, I assume that antitrust authorities have *constant* and *exogenous budgets* that allow them to inspect a fixed number of firms, that is,  $\alpha \in [0, 1)$  is a constant and exogenous probability of inspection. It can be also interpreted as a surprise inspection policy, that although it may be effective,<sup>10</sup> it is not an usual practice. Moreover, since agreements are bilateral, in any particular inquiry, the AA is able to know only the immediate partners that a firm has in each agreement, that

<sup>7</sup>See Roldán, 2008 for a detailed discussion.

<sup>8</sup>This holds when firms are risk-neutral.

<sup>9</sup>Although this theoretical formulation could be in contrast with current practice, the fine defined here could be in line with antitrust regulation if we associate this form of penalty as a ceiling on the fine that can be imposed on a firm guilty.

<sup>10</sup>Friederiszick and Maier-Rigaud (2007) argue that "surprise inspections are by far the most effective and sometimes the only means of obtaining the necessary evidence...."

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is, we only consider that the AA discovers immediate links.<sup>11</sup>

In this network framework, however, the actual probability of being caught is higher than the probability of being inspected. That is, given the technology of inspection assumed, when a firm  $i$  forms a new market-sharing agreement, it will increase its probability of being detected. That is, the probability of firm  $i$  being caught by the AA depends not only on whether firm  $i$  is inspected but on whether any other firm with which firm  $i$  has a link, is also inspected. Therefore, firm  $i$  will not be detected if  $i$  is not inspected *and* if  $j$ , that has an agreement with  $i$ , is not inspected.<sup>12</sup> That is,  $\Pr(\text{Detected } i) = 1 - \Pr(\text{No Detected } i)$ ,<sup>13</sup> where

$$\Pr(\text{No Detected } i) = (1 - \alpha)^{N-n_i+1} \quad (2)$$

Therefore, the probability of being detected depends on how many agreements firm  $i$  has signed, that is,  $m_i = N - n_i$ . Note that, as the number of agreements  $m_i = N - n_i$  increases,  $\Pr(\text{Detected } i)$  also increases. On the other hand, as  $m_i = N - n_i$  goes to zero,  $\Pr(\text{Detected } i) \rightarrow \alpha$ .

From the AA's point of view, the structure of relationships described by  $m_i = N - n_i$  generates scale economies on detection as

$$\Pr(\text{Detected } i) = 1 - (1 - \alpha)^{N-n_i+1} > \Pr(\text{Inspected } i) = \alpha.$$

## Incentives to form an agreement

An essential part of the model is the firm's incentive to form an agreement. Assume that firm  $i$  has formed  $m_i = N - n_i$  agreements, but has not yet formed an agreement with a firm  $j$ , that is,  $g_{ij} = 0$ . Then, by using expressions (1) and (2), we compute firm  $i$ 's expected profits as:<sup>14</sup>

$$(1 - \alpha)^{N-n_i+1} \Pi^i \quad (3)$$

$$\text{where } \Pi^i = \pi(n_i) + \pi(n_j) + \sum_{k \neq j, g_{ki}=0} \pi(n_k).$$

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<sup>11</sup>A more realistic assumption would be one in which the AA could open an additional inquiry over those partners in order to know whether they have other collusive links. In such a case, the probability of being detected will increase a little bit more which, in turn, will decrease even more the incentive to form agreements. Accordingly, under such circumstances, the results would not change qualitatively.

<sup>12</sup>It is assumed that events "No inspected  $i$ " and "No inspected  $j$ " are independent each other.

<sup>13</sup> $\Pr(\text{No Detected } i) = \Pr\left(\text{No inspected } i \bigcap_{\substack{j \neq i \\ g_{ij}=1}} \text{No inspected } j\right).$

<sup>14</sup>Recall that  $\Pi^i = F(\Pi)$ .



Now, if firm  $i$  decides to form a link with firm  $j$ , its expected profits will be

$$(1 - \alpha)^{N-n_i+2} \Pi^i \quad (4)$$

but now,  $\Pi^i = \pi(n_i - 1) + \sum_{k \neq j, g_{ki}=0} \pi(n_k)$ .

By subtracting (3) from (4), we obtain firm  $i$ 's incentive to form an agreement with firm  $j$  as:

$$\Delta \Pi_j^i = (1 - \alpha)^{N-n_i+1} \left[ \pi(n_i - 1) - \pi(n_i) - \pi(n_j) - \alpha \left( \pi(n_i - 1) + \sum_{k \neq j, g_{ki}=0} \pi(n_k) \right) \right] \quad (5)$$

Let  $J_j^i(n_i, n_j, n_k; \alpha)$  denote the bracket expression in (5) and let us rewrite it as

$$\Delta \Pi_j^i = (1 - \alpha)^{N-n_i+1} J_j^i(n_i, n_j, n_k; \alpha) \text{ for } k \neq j \text{ and } g_{ki} = 0$$

It is worth noting that when  $\alpha = 0$  firm  $i$ 's incentive to form a market-sharing agreement with firm  $j$  only depends on the characteristics of markets  $i$  and  $j$ . In terms of Goyal and Joshi (2006), this problem satisfy the local spillovers property. Namely, the marginal returns to firm  $i$  from a market sharing agreement with firm  $j$  depend only on the number agreements of  $i$  and  $j$ . When an antitrust authority exists, however,  $\Delta \Pi_j^i$  will also depend on the characteristics of all markets  $k$  in which firm  $i$  is active.<sup>15</sup>

We are interested in the sign of  $\Delta \Pi_j^i$  because it is what is relevant for deciding whether or not one more link is formed. That is, if  $\Delta \Pi_j^i \geq 0$ , firm  $i$  has an incentive to form an agreement with  $j$ . Consequently, when  $\alpha \neq 1$ ,  $\Delta \Pi_j^i \geq 0$  only if  $J_j^i(n_i, n_j, n_k; \alpha) \geq 0$ . Hence, in the following, we will focus only on  $J_j^i(n_i, n_j, n_k; \alpha)$ .

Forming one more link has several conflicting consequences. From firm  $i$ 's point of view, note that when a link is formed between firms  $i$  and  $j$ , firm  $j$  agrees not to enter market  $i$ . Therefore, the number of active firms in market  $i$  will decrease, and it increases its profits by  $\pi(n_i - 1) - \pi(n_i)$ . Given the reciprocal nature of this agreement, firm  $i$  does not enter market  $j$ , either. Then, firm  $i$  loses access to foreign market  $j$ , and decreases its profits by  $\pi(n_j)$ . Additionally, if firm  $j$  is inspected, and it is inspected with probability  $\alpha$ , firm  $i$  will lose  $\pi(n_i - 1) + \sum_{k \neq j, g_{ki}=0} \pi(n_k)$ .

<sup>15</sup>We just consider the case when  $m_i = N - n_i \neq 0$ . However, when firm  $i$  is isolated, that is,  $m_i = N - n_i = 0$ , the firm  $i$ 's incentive to form an agreement is slightly different from (5). That is,  $\Delta \Pi = \pi(N - 1)(1 - \alpha)^2 - \pi(N) - \pi(n_j) - \sum_{k \neq j, g_{ki}=0} \pi(n_k) (1 - (1 - \alpha)^2)$ .

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The following lemma summarizes the relationship between the incentives to form an additional agreements and the number of active firms in each market.

**Lemma 1** *As  $\pi(\cdot)$  is a decreasing function on its argument, then  $J_j^i(n_i, n_j, n_k; \alpha)$  is increasing in  $n_j$  and  $n_k$ , ambiguous regarding to  $n_i$ , and it is decreasing in  $\alpha$ .*

To sum up, given the antitrust policy  $\{\alpha, F(\Pi)\}$ , firms compute the incentives to form agreements and then decide whether or not to form an agreement. Firms form them if they yield positive profits after expected penalties from signing market-sharing agreements. If an inquiry is opened, and if a firm is convicted of forming a market-sharing agreement, it must pay  $F(\Pi) = \Pi^i$ .

## Background definitions

We are considering firms that enter into *bilateral* relationships with each other and the set of these bilateral relationships gives rise to a collusive network  $g$ . In this part, we introduce some notations and terminology from graph theory that will be useful in describing and analyzing the model.

**Networks.** Let  $N = \{1, 2, \dots, N\}$ ,  $N \geq 3$  denote a finite set of identical firms. A network  $g = \{(g_{ij})_{i,j \in N}\}$  is a description of the pairwise relationship between firms.

Let  $g + g_{ij}$  denote the network obtained by adding link  $ij$  to an existing network  $g$  and denote by  $g - g_{ij}$  the network obtained by deleting link  $ij$  from an existing network  $g$ .

Some networks that play a prominent role in our analysis are the complete network and the empty network. The *complete network*,  $g^c$ , is a network in which  $g_{ij} = 1, \forall i, j \in N$ . In contrast, the *empty network*,  $g^e$ , is a network in which  $g_{ij} = 0, \forall i, j \in N, i \neq j$ .

Formally, a firm  $i$  is isolated if  $g_{ij} = 0, \forall j \neq i$  and  $\forall j \in N$ .

**Components.** A *component*  $g'$  of a network  $g$  is a maximally connected subset of  $g$ . Note that from this definition, an isolated firm is not considered a component.

Let  $m_i(g')$  denote the number of links that firm  $i$  has in  $g'$ .

A component  $g' \subset g$  is *complete* if  $g_{ij} = 1$  for all  $i, j \in g'$ . For a complete component  $g'$ ,  $m_i(g') + 1$  denote its size, that is, it is the number of firms belonging to  $g'$ .

**Stable collusive networks.** Our interest is to study which networks are likely to arise. As a result, we need to define a notion of stability. In the present article, we always use a notion of *pairwise* stability.

*Pairwise stable networks.* The following approach is taken by Jackson and Wolinsky (1996). In terms of our model, a network  $g$  is said to be pairwise stable if and only if:

$$(i) \quad \forall i, j \text{ s.t. } g_{ij} = 1, \left\{ \begin{array}{l} J_j^i(n_i + 1, n_j + 1, n_k; \alpha) \geq 0 \\ J_i^j(n_j + 1, n_i + 1, n_k; \alpha) \geq 0 \end{array} \right.$$

$$(ii) \quad \forall i, j \text{ s.t. } g_{ij} = 0, \begin{cases} \text{if } J_j^i(n_i, n_j, n_k; \alpha) > 0 \\ \text{then } J_i^j(n_j, n_i, n_k; \alpha) < 0 \end{cases}$$

The above stability notion is a relatively weak criterion in the sense that it provides broad predictions and the firm's deviations are constrained.<sup>16</sup>

Nevertheless, that criterion provides a test to eliminate the unstable networks and it should be seen as a necessary but not sufficient condition for a network to be stable.

*Strongly pairwise stable networks.* In order to obtain a stronger concept of stability, we allow deviations by coalitions of firms. We allow firms to delete some or all market-sharing agreements that they have already formed.

We say that a network is pairwise strongly stable if it is immune to deviations by coalitions of two firms.<sup>17</sup> Moreover, it is possible to prove that any strongly pairwise stable network is pairwise stable. The strong stability notion can thus be thought of as sufficient condition for stability.

### 3 The stability characterization and the set of stable networks

In this section, we will characterize pairwise stable and strongly pairwise stable networks under the presence of an AA in a symmetric context.

#### Pairwise stable collusive network

Consider the following network  $g$  that can be decomposed into distinct complete components,  $g_1, \dots, g_L$ , of different sizes, that is,  $m(g_l) \neq m(g_{l'}), \forall l, l'$ . Let us define  $m(g_h^*) := \min \{m(g_1), \dots, m(g_L)\}$ . That is,  $g_h^*$  is the smallest component of a network  $g$ , whose size is  $m(g_h^*) + 1$ .

The follow Proposition provides the characterization of the pairwise stable networks in a symmetric context when an AA exists. Note that this Proposition holds for all  $m(g_h^*) \geq 1$ .

**Proposition 1** *A network  $g$  is pairwise stable if and only if it can be decomposed into a set of isolated firms and distinct complete components,  $g_1, \dots, g_L$  of different sizes  $m(g_l) \neq m(g_{l'}), \forall l, l'$  such that no isolated firm has an incentive to form a link with another isolated one and no firm  $i$  that belongs to the smallest component has an incentive to cut a link with a firm inside it.*

<sup>16</sup>A pairwise stability criterion only considers deviations from a single link at a time and only deviations by a pair of players at a time.

<sup>17</sup>See Belleflamme and Bloch (2004) for a more detail discussion.

It is important to note that the AA imposes a change in the minimal size of the components, and that it does not restrict the set of isolated firms. From the definition of  $J_j^i$ ,  $g_{ij} = 1$  only if:

$$(1 - \alpha) \pi(n - 1) \geq 2\pi(n) + \alpha \sum_{k \neq h, ghk=0} \pi(n_k), \quad \forall h = i, j \quad (6)$$

In the absence of the AA, that is,  $\alpha = 0$ , the above inequality becomes  $\pi(n - 1) > 2\pi(n)$ . Therefore, by log-convexity, it is possible to guarantee the existence of a number  $n^* = N - m^*$  such that  $\pi(n^* - 1) \geq 2\pi(n^*)$ .  $m^* = N - n^*$  is thus interpreted as the minimal number of agreements that a firm already has to have in order to form an additional one. In the absence of a competition authority, a network is stable if its alliances are large enough.

In contrast, under the presence of the AA, that is,  $\alpha \neq 0$ , we are not able to reach a unique lower bound. From (6) we can see that the minimal number of agreements,  $m(g_h^*)$ , that assures that the condition holds depends on  $\alpha$  and on  $g$ .

In spite of the fact that  $m(g_h^*)$  depends on particular conditions, it is easy to see that  $m(g_h^*) \geq m^*$ .<sup>18</sup> Nevertheless, we will show below that this is not necessarily a perverse effect of the AA because  $m(g_h^*) \geq 1$  does not put any restriction on the set of isolated firm.

## Pairwise strongly stable collusive network

We refine the set of stable networks by using the strong stability condition. Now we allow firms to delete a subset of links already formed and we will study when a firm has no incentive to renege on its agreements. This point is very important in our context because a network composed by large alliances will be difficult to sustain.

**Proposition 2** *A network  $g$  is pairwise strongly stable if and only if it is pairwise stable and no firm prefers to cut all its agreements at once, that is,*

$$(1 - \alpha)^{N-n+1} \pi(n) \geq \pi(N) + (N - n) \pi(n + 1) + \sum_{k, gi=0} \pi(n_k) \left(1 - (1 - \alpha)^{N-n+1}\right),$$

$$\forall n = N - m + 1 \text{ and } \forall m = m(g_l) \quad (7)$$

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<sup>18</sup>By rewriting (6), we obtain:  $\frac{\pi(n-1)}{\pi(n)} \geq \frac{2}{(1-\alpha)} + \frac{\alpha \sum_{k \neq h: ghk=0} \pi(n_k)}{(1-\alpha)\pi(n)} \geq 2$ , for  $h = i, j$  such that  $g_{ij} = 1$

Accordingly, the fact that a firm has no incentives to renege on all of its links at once is a sufficient condition for strong stability.<sup>19</sup>

Therefore, in a strongly stable network, component sizes satisfy a more demanding condition.

It is worth remarking that a strongly stable network may fail to exist. Nonetheless, one important advantage of the strong criterion is to provide a more accurate prediction of which network structures will prevail.

In the Appendix of the paper<sup>20</sup> we present two examples that illustrate the above propositions and contrast the results of the two cases, namely, under the absence of the AA ( $\alpha = 0$ ) and under the presence of the AA ( $\alpha \neq 0$ ).

## The set of stable collusive networks

Given the network characterization of the previous section, we now analyze which kinds of stable networks can be sustained at different levels of the antitrust enforcement.

In our setting, the presence of the antitrust authority imposes a cost to each formed link, and as a result, the expected gain of being a part of a collusive agreement may not be positive. That is, the expected sanction imposed by the AA affects the incentive participation constraint of each potential alliance's member and in turn changes the set of possible network structures that can arise.

## The set of pairwise stable networks

First of all, a complete network is always pairwise stable for sufficiently low values of  $\alpha$ . Let us define  $\alpha^c := 1 - \frac{2\pi(2)}{\pi(1)}$ .

**Proposition 3** *The complete network  $g^c$  is pairwise stable if and only if  $\alpha \leq \alpha^c$ .*

Being a part of a collusive agreement entails positive benefits. To serve a link increases the profits of firms that participate in it, that is,  $\pi(n)$  is decreasing in  $n$ . Therefore, the complete network will be pairwise stable as long as its costs, that is, the expected sanction, is sufficiently low.

Second, the empty network arises as pairwise stable for an  $\alpha$  sufficiently high. Let us define  $\alpha^e(N) := 1 - \left[ \frac{N\pi(N)}{[\pi(N-1) + (N-2)\pi(N)]} \right]^{\frac{1}{2}}$ , for  $\forall N \in [3, \infty)$  and  $\alpha^e(N) < 1$ .

**Proposition 4** *For  $\forall N \in [3, \infty)$ , the empty network  $g^e$  is pairwise stable if and only if  $\alpha > \alpha^e(N)$ .*

<sup>19</sup>To see this, assume that a firm reneges on one of its agreements. Then, it gains access to a market whose profits are at least equal to the profit it makes on its home market after cutting a link. Therefore, if a firm has an incentive to cut one agreement, the most profitable deviation for it is to renege on all its agreements at once.

<sup>20</sup>It is available to the readers on request.

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For an isolated firm,  $\alpha^e(N)$  is the threshold from which it has no incentive to participate in an agreement when all other firms also remain isolated. When  $\alpha > \alpha^e(N)$ , the expected costs of forming a link are so high, relative to its benefits, that no two firms will sign an agreement.

Moreover, observe that  $\alpha^e(N)$  is strictly decreasing in  $N$ . That is, as  $N$  increases, the "loot" becomes less "attractive" (that is,  $\pi(N)$  is decreasing in  $N$ ), and the threshold will decrease as a result.

By straightforward computations, we can see that  $\alpha^e(N) < \alpha^c$ . Consequently, from the above Propositions, we claim the following:

**Claim 1** *For  $\alpha \in (\alpha^e(N), \alpha^c]$ ,  $g^e$  and  $g^c$  belong to the set of pairwise stable networks.*

From Proposition 3 and 4, we can state that pairwise stable networks always exist. That is, first, for  $\alpha \leq \alpha^c$ , the complete network belongs to the set of stable networks. Second, for  $\alpha > \alpha^e(N)$ , the empty network will be stable. And given that  $\alpha^e(N) < \alpha^c$ , for  $\alpha \in (\alpha^e(N), \alpha^c]$ ,  $g^e$  and  $g^c$  arise as pairwise stable configurations.

When  $\alpha \neq 0$ , there exists a positive probability of being caught in a market-sharing agreement. Consequently, there exists a positive probability of losing profits not only in the market where the agreement is signed but also in markets in which the firm is active, that is in markets where the firm does not collude.

For firms in smaller alliances, the cost of forming a link becomes more significant relative to the benefits of doing so. That is, a firm  $i$  inside a small alliance does not have much to gain and has a lot to lose when one more link is made. More precisely, by signing an agreement, it gains  $(1 - \alpha)\pi(n_i - 1) - \pi(n_i)$ , which decreases as the alliance becomes smaller;<sup>21</sup> and it not only loses the access to profits in the foreign market  $j$ ,  $\pi(n_j)$  but also loses, in expected terms,  $\alpha \sum_{k: g_{ik}=0} \pi(n_k)$ .

Therefore, firms in smaller components are more sensitive to the antitrust enforcement.

The intuition provided above is summarized in the next Proposition. Before introducing it, let us define

$$\alpha^*(n_i) := \frac{\pi(n_i - 1) - 2\pi(n_i)}{\pi(n_i - 1) + \sum_{k \neq j, g_{ik}=0} \pi(n_k)}$$

That is, at  $\alpha^*(n_i)$  a firm  $i$ , with  $n_i$  competitors in its home market, is indifferent to forming a link, that is,  $J_j^i = 0$ . Therefore, when  $\alpha > \alpha^*(n_i)$ , then  $J_j^i < 0$ , and firms  $i$  and  $j$  do not sign a collusive agreement.

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<sup>21</sup>Remember that the number of active firms in a market, that is  $n_i$ , is greater in smaller components.

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**Proposition 5** *For firm  $i \in g_1$  and firm  $j \in g_2$  such that  $m(g_1) < m(g_2)$ , then  $\alpha^*(n_1) < \alpha^*(n_2)$ .*

From the Proposition it follows that the threshold is smaller for firms in smaller alliances (with larger number of competitors in their home markets). Then, as  $\alpha$  becomes greater, the AA first tears down small alliances. In other words, the smaller components are more sensitive to the antitrust policy. In the limit, firms must decide to form a very large alliance (complete network) or no alliance at all (empty network).

**Proposition 6** *For  $\alpha = \alpha^c > 0$ , the only pairwise stable networks are  $g^e$  and  $g^c$ .*

Then, by setting  $\alpha > \alpha^c$ , the AA completely deters the formation of collusive agreements.

### The set of pairwise strongly stable networks

Now, we turn our attention to the notion of strongly stable networks and we answer which kinds of networks arise as the AA changes its enforcement level. From the previous section, we know that there will be some pairwise stable networks that will not be stable against changes in the agreements made by firms. By applying (7), we assert the following:

**Proposition 7** *As  $\alpha$  increases, firms in large components have more incentives to delete all links at once.*

That is, as  $\alpha$  increases, the condition of strong stability is harder to sustain in larger components. In other words, faced with an increasing  $\alpha$ , a firm has to consider whether to maintain or to destroy its agreements. Therefore, the firm balances the pros and the cons of any decision. Namely, if a firm maintains its agreements, its benefits are  $(1 - \alpha)^{N-n+1} \left[ \pi(n) + \sum_{k, g_i=0} \pi(n_k) \right]$ .

Let us note that these benefits decrease as the probability of inspection,  $\alpha$ , increases, and the fall in the expected benefits is higher as  $m = N - n$  increases.

Instead, if the firm decides to destroy all its agreements, it is not only not penalized now by the AA, but it will also gain access to markets where it was colluding before. In such a situation, it will make profits on all these new foreign markets; that is,  $(N - n) \pi(n + 1)$ . Let us observe that these markets are more profitable as the number of competitors on them is smaller, that is, as  $m = N - n$  is larger.

Therefore, firms belonging to larger alliances, that is, smaller  $n_i$ , have more incentives to cut all their agreements at once as the AA increases the cost of forming links.

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Now let us consider the empty network under the strongly stable notion.

It is worth noting that if  $g^e$  is pairwise stable, it is also strongly pairwise stable, as the condition (7) is always satisfied for firms that remaining alone. That is, in an empty network, firms do not have any link, so the condition of not having incentives to renege on all agreements at once is redundant for any  $i \in g^e$ . Therefore, we claim that

**Claim 2**  $\forall \alpha > \alpha_e(N)$  the empty network is always strongly pairwise stable.

Accordingly, if for some  $\alpha > \alpha_e(N)$  all alliances have been torn down by the AA, the only network configuration that exists is the empty one.

## Examples

The following examples illustrate the changes that the AA imposes in the set of pairwise stable networks.<sup>22</sup>

**Example 1** *Pairwise stable (ps) networks. Cournot competition with exponential inverse demand function  $P(Q) = e^{-Q}$*

When the inverse demand function is  $P(Q) = e^{-Q}$ , we can compute the equilibrium profits as  $\pi(n) = e^{-n}$ . Assume that  $N = 5$ . The following table depicts the set of pairwise stable networks for different values of the antitrust policy.

**Table 1**

$\alpha$	Set of ps networks
$\alpha \in [0; 0.015)$	$\{3, 2\}, \{4, 1\}, \{5\}$
$\alpha \in [0.015; 0.04)$	$\{3, 1, 1\}, \{4, 1\}, \{5\}$
$\alpha \in [0.04; 0.065)$	$\{2, 1, 1, 1\}, \{3, 1, 1\}, \{4, 1\}, \{5\}$
$\alpha \in [0.065; 0.21)$	$\{1, 1, 1, 1, 1\}, \{3, 1, 1\}, \{4, 1\}, \{5\}$
$\alpha \in [0.21; 0.25)$	$\{1, 1, 1, 1, 1\}, \{4, 1\}, \{5\}$
$\alpha \in [0.25; 0.26)$	$\{1, 1, 1, 1, 1\}, \{5\}$
$\alpha > 0.26$	$\{1, 1, 1, 1, 1\}$

First of all, it is useful to clarify some notations there. In the table, the complete network is represented by  $\{5\}$ , and, for example,  $\{3, 1, 1\}$  denotes a network decomposed into two isolated firms and one complete component of size three.

When  $\alpha$  is sufficiently low (that is,  $\alpha < 0.015$ ) the presence of the AA does not change the set of pairwise stable networks. However, when the antitrust enforcement

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<sup>22</sup>See Roldán, 2008 for all calculation details.



is sufficiently high (that is,  $\alpha > 0.26$ ) the only pairwise stable network is the empty one. Accordingly, all firms are active in all markets.

Consider now values of  $\alpha$  between these two extreme cases. Although different configurations arise, the main features to be highlighted are the following two. First, when  $\alpha$  increases, structures that are more competitive can be sustained through bilateral agreements. In particular, when  $\alpha$  becomes greater, the smaller components are more sensitive to the antitrust policy. For example, when  $\alpha \in [0.015; 0.04)$ , the network structure  $\{3, 2\}$  is not stable because firms in smaller components have incentives to cut their agreements and the network  $\{3, 1, 1\}$  becomes stable.<sup>23</sup> Second, as  $\alpha$  increases the set of stable network configurations becomes more polarized. That is, in our analytical example, when  $\alpha \in (0.25; 0.26)$ , the empty or complete networks are the only possible stable network configurations. This can be understood because the AA imposes the costs of forming links, and reduces the profitability of each one. As a result, firms decide either to form more and more links, that is, to reduce the number of competitors in their home markets, in order to balance their benefits with their cost; or to form no link at all and thus avoid the costs levied by the AA.

The next example illustrates the impact of the AA on the set of strongly stable networks and two special features of the strong criterion.

**Example 2** *Pairwise strongly stable (pss) networks. Cournot competition for exponential inverse demand function:  $P(Q) = e^{-Q}$*

As in the last example, assume that  $N = 5$ . Given that a pairwise strongly stable network is always pairwise stable, it suffices to check the condition (7) for all network structures in Table 2 at different levels of the antitrust policy.

**Table 2**

$\alpha$	Set of ps networks	Set of pss networks
$\alpha \in [0; 0.015)$	$\{3, 2\}, \{4, 1\}, \{5\}$	$\{3, 2\}$
$\alpha \in [0.015; 0.04)$	$\{3, 1, 1\}, \{4, 1\}, \{5\}$	it fails to exist
$\alpha \in [0.04; 0.065)$	$\{2, 1, 1, 1\}, \{3, 1, 1\}, \{4, 1\}, \{5\}$	$\{2, 1, 1, 1\}$
$\alpha \in [0.065; 0.21)$	$\{1, 1, 1, 1, 1\}, \{3, 1, 1\}, \{4, 1\}, \{5\}$	$\{1, 1, 1, 1, 1\}$
$\alpha \in [0.21; 0.25)$	$\{1, 1, 1, 1, 1\}, \{4, 1\}, \{5\}$	$\{1, 1, 1, 1, 1\}$
$\alpha \in [0.25; 0.26)$	$\{1, 1, 1, 1, 1\}, \{5\}$	$\{1, 1, 1, 1, 1\}$
$\alpha > 0.26$	$\{1, 1, 1, 1, 1\}$	$\{1, 1, 1, 1, 1\}$

First of all, the example clarifies that the possible set of stable networks is reduced by using the strongly stable criterion. However, the strongly stable network might fail

<sup>23</sup>Likewise, it is noteworthy that the graph  $\{3, 1, 1\}$  is not pairwise stable in the BB's setting, that is, when  $\alpha = 0$ .

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to exist and every network is defeated by some other network, which only leads to a cycling of these events. This is what happens for  $\alpha \in [0.015; 0.04)$ .

Second, the incentive to free ride and delete all links is higher in larger alliances. That is, when a firm that belongs to a large alliance cuts all its agreements at once, it will recover access to more profitable markets than a firm belonging to a smaller component. In the example, the complete network  $\{5\}$  and the stable network  $\{4, 1\}$  do not pass the strongly stable condition. By extending this argument, the empty network is the only strongly stable network for  $\alpha > 0.065$ .

Therefore, the antitrust policy is on the side of the competition as long as it gives firms in large alliances more incentives to renege on their agreements at once.

## The AA and its effects on competition

From the previous analysis, we conclude that as  $\alpha$  increases, the smaller alliances are first in being destroyed by the antitrust policy. In turn, the set of isolated firms expands.

Moreover, as  $\alpha$  becomes larger,  $m(g_h^*)$  also increases. From Proposition 7, however, we know that large alliances are harder to sustain.

Therefore, as  $\alpha$  increases, the empty network,  $g^e$ , tends to emerge as the only pairwise strongly stable network. Let us recall that in an empty network, all firms are active in all markets. We then infer that the antitrust policy is a pro-competitive one.

As it is well known, in Cournot oligopolies with homogeneous goods, the social surplus ( $V$ ) is increasing in the number of active firms on the market.

Accordingly, for a given  $\alpha$ , the network configuration that maximizes  $V$  is one that involves more firms present on foreign markets, that is a network where firms have fewer connections among them. From the Example 2, for  $\alpha = 0.05$ , the pairwise stable network  $\{2, 1, 1, 1\}$  maximizes welfare.

From Proposition 6, as  $\alpha$  increases, in particular, when  $\alpha > \alpha^c$ , the  $g^e$  is the only network that prevails over time. Therefore, in such a case,  $V$  would be the maximum.

Although  $\alpha > \alpha^c$  may be the "advice" to give to the AA, it may not be the optimal antitrust policy, as the necessary costs to attain that enforcement level may outweigh its positive impact on the social surplus. That is, in order to know whether the AA has a net positive effect on social welfare, we must also consider the enforcement cost.

Therefore, the net social welfare,  $W$ , depends on the network structure  $g$  (which depends, at last, on the particular level of  $\alpha$ ) as well as on the cost of initiating a market-sharing agreement suit against a firm,  $C(\alpha)$ .

Therefore, if the AA were concerned about the optimal antitrust policy, then it would have to choose  $\alpha$  such that it maximizes

$$W(g(\alpha), C) = V(g(\alpha)) - C(\alpha)$$

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Unfortunately, the optimal antitrust policy is difficult to evaluate in our context because of the multiplicity in network configurations for each level of antitrust enforcement. In our network context,  $g(\alpha)$  is not unique for each  $\alpha$ . Moreover, a particular network  $g$  can emerge as being pairwise stable for different levels of  $\alpha$ .

## 4 Concluding Remarks

We have characterized the stable collusive network that arises when firms form market-sharing agreements among themselves in a symmetric oligopolistic setting when an antitrust authority exists.

In this network framework, the incentives to participate in a collusive agreement are weakened by the AA because it reduces the net expected benefit from signing them. Under the presence of the AA, the expected penalties of forming illegal links appear, and they are positively related with the network configuration. This is because of two facts. First, firms, when considering whether to sign an agreement, take into account the probability of being discovered rather than the probability of being inspected and the first probability positively depends on the number of agreements each firm has signed. Second, the fine imposed by the AA on a guilty firm is equal to its total profits, which depend on the number of active firms in its home market and also on the number of active firms in all foreign markets in which the guilty firm does not collude. Consequently, the penalty will be greater as the number of active firms in these markets is smaller, that is, as the number of links is larger.

We have shown that, the pairwise stable network can be decomposed into a set of isolated firms and complete components of different sizes. When the AA exists, however, we cannot define a unique lower bound on the size of complete components because it now depends on each network configuration and on probability of each of being inspected. In turn, this implies that, although the lower bound on the size of complete components may be greater, the set of isolated firms enlarges and, finally, structures that are more competitive can be sustained through bilateral agreements.

We have also shown that antitrust laws have a pro-competitive effect as they give firms in large alliances more incentives to cut their agreements at once. Therefore, the empty network might arise as the only strongly stable network.

Although the optimal deterrence policy is beyond of the scope of the current article, an important policy implication of the present formulation is that the organization of the illegal behavior matters. That is, the analysis of the optimal deterrence of market-sharing agreements has to take into account the organizational structure of collusive firms. Furthermore, without considering the effects of the organizational structure, empirical studies may overestimate the contribution of efforts devoted to investigate and prosecute collusive agreements.<sup>24</sup>

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<sup>24</sup>Some empirical papers that estimate the deterrent effect of the policy are, among others, Buc-

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In this article, we consider a relatively simple setting for analyzing the effect of the antitrust policy on the structure of criminal behavior. One can then diversify from here in many directions. One of them is to consider that the probability of inspection is sensitive to the network structure. This introduces some asymmetry among firms, which may then change the criminal network's configuration. Another extension to this article is to introduce a more complex but realistic context. A particular extension is how the internal structure of these conspiracies may affect their observable behavior, which, in turn, may throw some light on the optimal antitrust policy.

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cirossi and Spagnolo, 2005; Connor, 2006; Zimmerman and Connor, 2005.

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## Appendix

**Proof Lemma 1** Regarding  $n_i$ . As  $n_i$  decreases, the firm's  $i$  incentive to form a market sharing agreements with firm  $j$ , increases by  $\pi(n_i - 1) - \pi(n_i)$ . However, as  $n_i$  decreases, the expected cost from signing a market sharing agreement increases by  $\alpha\pi(n_i - 1)$ .

Regarding  $n_j$  ( $n_k$ ). As  $n_j$  ( $n_k$ ) decreases,  $\pi(n_j)$  ( $\pi(n_k)$ ) increases. This fact increases the cost of forming an agreement with firm  $j$  and, in turn, this decreases the firm's  $i$  incentive to form a market sharing agreements with firm  $j$ .

Regarding  $\alpha$ . As  $\alpha$  increases, the cost of forming a market sharing agreement increases and this fact decreases the firm's  $i$  incentive to form a market sharing agreements with firm  $j$ .

**Proof Proposition 1** Let us consider the necessary part for stability. First: if network  $g$  is pairwise stable, then  $\forall i, j \in N$  such that  $g_{ij} = 1$ ,  $n_i(g) = n_j(g)$ . As  $g$  is stable, when  $g_{ij} = 1$  the next two conditions simultaneously hold:

$$\begin{aligned} (1 - \alpha)\pi(n_i) &\geq \pi(n_i + 1) + \pi(n_j + 1) + \alpha \sum_{k:g_{ik}=0} \pi(n_k) \\ (1 - \alpha)\pi(n_j) &\geq \pi(n_j + 1) + \pi(n_i + 1) + \alpha \sum_{k:g_{jk}=0} \pi(n_k) \end{aligned}$$

Given that the profit function is decreasing in  $n$ , the following are a pair of necessary conditions that must be satisfied for the above inequalities to hold:

$$\begin{aligned} \pi(n_i) &> \pi(n_j + 1) \\ \pi(n_j) &> \pi(n_i + 1) \end{aligned}$$

From the first inequality,  $n_i < n_j + 1$ , and from the second one,  $n_j < n_i + 1$ . Hence:

$$n_j - 1 < n_i < n_j + 1 \Leftrightarrow n_i = n_j$$

That is,

$$n_i = n_j \equiv n$$

Second: if  $g$  is stable, then any component  $g' \in g$  is complete.

Suppose that  $g'$  is not complete. Then, there are three firms  $i, j, l$  in the component such that  $g_{ij} = g_{jl} = 1$  and  $g_{il} = 0$ . Because  $g$  is stable, then by Result 1  $n_i = n_j \equiv n$  and  $n_j = n_l \equiv n$ , then  $n_i = n_j = n_l \equiv n$ . From the stability conditions, we are able to

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rewrite  $J_j^i$  as follows:

$$\underbrace{\frac{\pi(n)}{\pi(n+1)}}_A \geq \underbrace{\frac{2}{(1-\alpha)} + \frac{\alpha \sum_{k:g_{ik}=0, i \neq k} \pi^i(n_k(g))}{(1-\alpha)\pi(n+1)}}_B$$

The same applies for  $J_i^j$ ,  $J_l^j$  and  $J_j^l$ .

Given that  $g_{il} = 0$ , then one or both conditions hold for  $h = i$  and/or  $h = l$  :

$$\underbrace{\frac{\pi(n-1)}{\pi(n)}}_D < \underbrace{\frac{2}{(1-\alpha)} + \frac{\alpha \sum_{k:g_{hk}=0, h \neq k} \pi^i(n_k(g))}{(1-\alpha)\pi(n)}}_E$$

By log-convexity, we can establish that

$$A \leq D$$

From stability:

$$B \leq A \leq D < E$$

However, given that profits are decreasing functions, and given that the number of terms in  $\sum_{k:g_{ik}=0} \pi^i(n_k)$  in  $B$  and  $E$  are different, we can say that

$$B > E$$

This is a contradiction. Therefore  $g'$  must be a complete component.

Third: if  $g$  is stable, then the complete components must have different sizes.

Take two firms  $i, j$  in component  $g'$  and a firm  $l$  in  $g''$ . Suppose, by contradiction, that  $m(g') + 1 = m(g'') + 1$ . Therefore, we have  $n_i = n_j = n_l \equiv n$ . The stability of  $g$  implies that  $J_j^i \geq 0$  and  $J_i^j \geq 0$ .  $J_j^i \geq 0$  can thus be written as

$$\underbrace{\frac{\pi(n)}{\pi(n+1)}}_A \geq \underbrace{\frac{2}{(1-\alpha)} + \frac{\alpha \sum_{k:g_{ik}=0, i \neq k} \pi^i(n_k(g))}{(1-\alpha)\pi(n+1)}}_B$$

(Similar expression for  $J_i^j \geq 0$ .)



For  $h = i$  and/or  $h = l$ , the following condition holds:

$$\underbrace{\frac{\pi(n-1)}{\pi(n)}}_D < \underbrace{\frac{2}{(1-\alpha)} + \frac{\alpha \sum_{k:g_{hk}=0, h \neq k} \pi^i(n_k(g))}{(1-\alpha)\pi(n)}}_E$$

By log-convexity, we can establish that

$$A \leq D$$

From the stability conditions

$$B \leq A \leq D < E$$

However, given that profits are decreasing functions, and given that the number of terms in  $\sum_{k:g_{ik}=0} \pi^i(n_k)$  in  $B$  and  $E$  are different, we can say that

$$B > E$$

Nevertheless, it is a contradiction with the assumption that profits are log-convex and with the stability of  $g$ .

Finally, if  $i \in g_h^*$  does not have incentives to cut a link with a firm inside its component, it is true that

$$\frac{\pi(N - m(g_h^*))}{\pi(N - m(g_h^*) + 1)} > \frac{2}{(1-\alpha)} + \frac{\alpha \left[ (m(g_l) + 1) \pi(N - m(g_l)) + \sum_{k:g_{ik}=0} \pi(n_k) \right]}{(1-\alpha)\pi(N - m(g_h^*) + 1)} \quad (8)$$

Assume by contradiction that  $j \in g_l$  for  $m(g_l) > m(g_h^*)$  has an incentive to cut a link with a firm inside its component. Then,

$$\frac{\pi(N - m(g_l))}{\pi(N - m(g_l) + 1)} < \frac{2}{(1-\alpha)} + \frac{\alpha \left[ (m(g_h^*) + 1) \pi(N - m(g_h^*)) + \sum_{k:g_{jk}=0} \pi(n_k) \right]}{(1-\alpha)\pi(N - m(g_l) + 1)} \quad (9)$$

when profits are decreasing in  $n$ , then  $\text{RHS}(8) > \text{RHS}(9)$ . By the log-convexity assumption  $\text{LHS}(8) < \text{LHS}(9)$ . Therefore, if  $i$  does not have an incentive to cut a link with a firm inside its component,  $\text{LHS}(8) > \text{RHS}(8)$ , then  $\text{LHS}(9) > \text{RHS}(9)$ , which contradicts (9).

Let us consider the sufficiency part. Consider a network  $g$  that can be decomposed

into a set of isolated firms and distinct complete components,  $g_1, \dots, g_L$  of different sizes  $m(g_l) \neq m(g_{l'}), \forall l, l'$ . Isolated players have no incentive to create a link with another isolated one. As long as a firm  $i$ , which belongs to the smallest component, does not have incentives to cut a link with a firm inside its component, then, by Lemma 3, no firm inside a component has incentives to cut a link. Additionally, given that  $m(g_l) \neq m(g_{l'}), \forall l, l'$ , there do not exist two firms belonging to different components that have an incentive to form an agreement between themselves. ■

**Proof Proposition 2**  $\Rightarrow$  Consider a pairwise strong Nash equilibrium  $s^*$ . Given that any strongly pairwise stable network is pairwise stable,  $g(s^*)$  can be decomposed into a set of isolated firms and complete components where no isolated firm wants to form a link with another isolated one and (8) holds. However, assume, by contradiction, that some component  $g_l$  does not satisfy the condition  $(1 - \alpha)^m \pi(N - m + 1) \geq \pi(N) + (m - 1) \pi(N - m + 2) + \sum \pi(n_k) (1 - (1 - \alpha)^m) \forall m = m(g_l)$ . Then,  $s^*$  is not a Nash equilibrium, as any firm  $i$  in  $g_l$  has a profitable deviation by choosing  $s'_i = \emptyset$ .

$\Leftarrow$  Assume that network  $g$  can be decomposed into a set of isolated firms and complete components of different sizes, where inequality (8) holds. Also assume that  $(1 - \alpha)^m \pi(N - m + 1) \geq \pi(N) + (m - 1) \pi(N - m + 2) + \sum \pi(n_k) (1 - (1 - \alpha)^m)$  holds for all  $m = m(g_l)$ . We will show that the following strategies form a pairwise strong Nash equilibrium. For firm  $i \in g_l$ , it announces  $s_i^* = \{j | j \in g_l, j \neq i\}$ ; however, if  $i$  is isolated, it announces  $s_i^* = \emptyset$ . Hence,

a) No isolated firm  $i$  has an incentive to create a link with another firm  $j$ , as  $i \notin s_j^*$ .

b) As  $(1 - \alpha)^m \pi(N - m + 1) \geq \pi(N) + (m - 1) \pi(N - m + 2) + \sum \pi(n_k) (1 - (1 - \alpha)^m)$  holds for all  $m = m(g_l)$ , the firm has no incentive to destroy all of its  $m$  links. We must consider, however, the firm's incentives to cut a subset of them. Let us assume that it has an incentive to delete a strict subset of its links; hence, it chooses to delete  $h$  links because

$$(1 - \alpha)^h \pi(N - m + 1) < \pi(N - m + 1 + h) + h \pi(N - m + 2) + \sum \pi(n_k) \left(1 - (1 - \alpha)^h\right)$$

Given that  $h \geq 1$ , then

$$\pi(N - m + 1 + h) + h \pi(N - m + 2) \leq (h + 1) \pi(N - m + 2)$$

Because we are considering a strict subset of links, then  $h < m - 1$  and  $h + 1 < m - 1$ , and, as a result,

$$(h + 1) \pi(N - m + 2) < (m - 1) \pi(N - m + 2)$$

Therefore,

$$(1 - \alpha)^m \pi(N - m + 1) < (1 - \alpha)^h \pi(N - m + 1) < (m - 1) \pi(N - m + 2)$$

This contradicts our hypothesis.

c) No firm  $i \in g_l$  has an incentive to create a link with firm  $j \in g_{l'}$  as  $i \notin s_j^*$ . Moreover, as  $m(g_l) \neq m(g_{l'})$  for all  $l \neq l'$ , no pair of firms  $i \in g_l$  and  $j \in g_{l'}$  has an incentive to create a new link between them.

d) As  $(1 - \alpha)^m \pi(N - m + 1) \geq \pi(N) + (m - 1)\pi(N - m + 2) + \sum \pi(n_k)(1 - (1 - \alpha)^m)$  holds for all  $m = m(g_l)$ , when  $m > 3$ , no pair of firms have incentives to delete all their links or a subsets of their agreements and form a link between them. Let us assume, by contradiction, a pair of firms,  $i \in m$  and  $j \in m'$ , has incentives to destroy all their  $m$  and  $m'$  links each and form a link between them. For firm  $i$ , this is

$$\begin{aligned} & (1 - \alpha)^{m-2} \pi(N - m + 1) \\ & < \pi(N - 1) + (m - 1)\pi(N - m + 2) + (m' - 1)\pi(N - m' + 2) + \\ & + \sum_{k \neq j \neq i, g_{ik}=0} \pi(n_k) - (1 - \alpha)^{m-2} \left[ \sum_{k \neq j \neq i, g_{ik}=0} \pi(n_k) + m'\pi(N - m' + 1) \right] \end{aligned} \quad (10)$$

Given that  $\text{LHS}(10) > \text{LHS}(7)$  and by straightforward computations, we can show that  $\text{RHS}(7) > \text{RHS}(10)$ , when condition(7) holds then  $\text{LHS}(10) > \text{RHS}(10)$ , which contradicts (10). ■

**Proof Proposition 3** ( $\implies$ ) If  $g^c$  is pairwise stable, then

$$(1 - \alpha) \pi(1) \geq 2\pi(2) \quad (11)$$

By rewriting the last condition, we get  $\alpha \leq \alpha^c = 1 - \frac{2\pi(2)}{\pi(1)}$ .

( $\impliedby$ ) If  $\alpha \leq \alpha^c = 1 - \frac{2\pi(2)}{\pi(1)}$ , then  $(1 - \alpha) \pi(1) \geq 2\pi(2)$ . Therefore,  $g^c$  will be pairwise stable. ■

**Proof Proposition 4** Assume that  $N \geq 3$ .

( $\implies$ ) If  $g^e$  is pairwise stable then,

$$(1 - \alpha)^2 [\pi(N - 1) + (N - 2)\pi(N)] < \pi(N) + \pi(N) + (N - 2)\pi(N) \quad (12)$$

and, by straightforward calculation,

$$\alpha > 1 - \left[ \frac{N\pi(N)}{[\pi(N - 1) + (N - 2)\pi(N)]} \right]^{\frac{1}{2}} = \alpha_e(N)$$

( $\impliedby$ ) If  $\alpha > \alpha_e(N)$ , then (12) holds. Therefore,  $g^e$  is pairwise stable. ■

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**Proof Proposition 5** For simplicity, let us assume two complete components  $g_1$  and  $g_2$ . For each firm  $i \in g_1$ ,  $n_1$  is the number of active firms in its market, and for each firm  $j \in g_2$ ,  $n_2$  is the number of active firms in its market.

Let us define  $\alpha^*(n_i) := \frac{\pi(n_i-1) - 2\pi(n_i)}{\pi(n_i-1) + \sum_{k \neq j, g_i=0} \pi(n_k)}$ .

We are interested in knowing whether  $\alpha^*(n_1) \leq \alpha^*(n_2)$ . That is,

$$\frac{\pi(n_1-1) - 2\pi(n_1)}{\pi(n_1-1) + (N - n_2 + 1)\pi(n_2)} \leq \frac{\pi(n_2-1) - 2\pi(n_2)}{\pi(n_2-1) + (N - n_1 + 1)\pi(n_1)}$$

By solving the last expression, we get

$$\begin{aligned} (N - n_1 + 1)\pi(n_1)\pi(n_1-1) - 2\pi(n_1)\pi(n_2-1) - 2(N - n_1 + 1)[\pi(n_1)]^2 &\leq \\ (N - n_2 + 1)\pi(n_2)\pi(n_2-1) - 2\pi(n_2)\pi(n_1-1) - 2(N - n_2 + 1)[\pi(n_2)]^2 & \end{aligned}$$

In order to decide the sense of the inequality, we rearrange the above expression into the following two parts:

$$\begin{aligned} (N - n_1 + 1)\pi(n_1)[\pi(n_1-1) - 2\pi(n_1)] &\leq (N - n_2 + 1)\pi(n_2)[\pi(n_2-1) - 2\pi(n_2)] \\ \pi(n_1)\pi(n_2-1) &\leq \pi(n_2)\pi(n_1-1) \end{aligned}$$

If  $n_1 > n_2$ , then (i)  $(N - n_1 + 1) < (N - n_2 + 1)$ ; (ii) by Property 1,  $\pi(n_1) < \pi(n_2)$ ; (iii) by Property 2,  $[\pi(n_1-1) - 2\pi(n_1)] < [\pi(n_2-1) - 2\pi(n_2)]$ .

Therefore,

$$(N - n_1 + 1)\pi(n_1)[\pi(n_1-1) - 2\pi(n_1)] < (N - n_2 + 1)\pi(n_2)[\pi(n_2-1) - 2\pi(n_2)] \quad (13)$$

Additionally, if  $n_1 > n_2$ , then, by the log-convexity assumption,  $\frac{\pi(n_2-1)}{\pi(n_2)} > \frac{\pi(n_1-1)}{\pi(n_1)}$

Hence,

$$\pi(n_1)\pi(n_2-1) > \pi(n_2)\pi(n_1-1) \quad (14)$$

Therefore, if,  $n_1 > n_2$ , by (13) and (14), then

$$\alpha^*(n_1) < \alpha^*(n_2) \blacksquare$$

**Proof Proposition 6** By Claim 1, we know that, at  $\alpha = \alpha^c$ ,  $g^e$  and  $g^c$  are pairwise stable.

Now, we must check, for  $\alpha = \alpha^c$ , whether a firm  $i$  has incentive to form an additional agreement when  $n \neq 1$  and  $n \neq N$ .

Therefore, we must verify whether  $J_j^i \leq 0$ , that is,

$$\pi(n-1) - 2\pi(n) \leq \alpha \left( \pi(n-1) + \sum_{k \neq j, g_{ki}=0} \pi(n_k) \right)$$

At  $\alpha = \alpha^c$ , the above expression is

$$\pi(n-1) - 2\pi(n) \leq \left[ 1 - \frac{2\pi(2)}{\pi(1)} \right] \left( \pi(n-1) + \sum_{k \neq j, g_{ki}=0} \pi(n_k) \right)$$

After some calculations, we obtain

$$2[\pi(n-1)\pi(2) - \pi(n)\pi(1)] \leq \sum_{k \neq j, g_{ki}=0} \pi(n_k) [\pi(1) - 2\pi(2)]$$

From Property 2, we know that,  $\pi(1) - 2\pi(2) > 0$ , and given the log-convexity assumption,  $[\pi(n-1)\pi(2) - \pi(n)\pi(1)] < 0$ . Therefore, at  $\alpha = \alpha^c$ ,

$$J_j^i < 0 \blacksquare$$

**Proof Proposition 7** The partial derivative of (7) respect to  $\alpha$  is

$$-(m+1) \left[ \pi(N-m+1) + \sum \pi(n_k) \right] (1-\alpha)^m \quad (15)$$

That is, as  $\alpha$  increases, the incentive to maintain links decreases.

Now, we must check whether (15) is larger for firms in large components. Without a loss of generality, assume that there are two components whose sizes are  $m_1 + 1$  and  $m_2 + 1$  respectively, such that  $m_1 > m_2$ . After some computations, we can verify that, for a sufficiently high  $m$ , the following holds:

$$\begin{aligned} & -(m_1 + 1) [\pi(N - m_1 + 1) + m_2 \pi(N - m_2 + 1)] (1 - \alpha)^{m_1} < \\ & -(m_2 + 1) [\pi(N - m_2 + 1) + m_1 \pi(N - m_1 + 1)] (1 - \alpha)^{m_2} \blacksquare \end{aligned}$$

## Examples

**Example 3** *Pairwise Stable Network for  $\alpha = 0$  and  $\alpha \neq 0$ . Cournot competition with exponential inverse demand function  $P(Q) = e^{-Q}$*

Let us recall that in this context  $\pi(n) = e^{-n}$ . In the absence of the AA, that is,  $\alpha = 0$ , the pairwise stability condition (6) becomes

$$\frac{\pi(n-1)}{\pi(n)} = e \geq 2, \forall n$$

Therefore, any two firms have incentives to form a link. Therefore,  $m^* = 1$  and any network with complete components of different sizes with at most one isolated firm is pairwise stable.

In contrast, when AA exists, that is,  $\alpha \neq 0$  that is no longer true. Assume, for example,  $N = 7$  and  $\alpha = 0.025$ . In such a context, the following is one network configuration that belongs to the set of the pairwise stable networks:

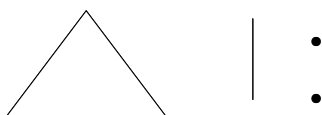


Figure 3: Stable network,  $N = 7$  and  $\alpha = 0.025$ .

Let us observe that in this case,  $m(g_h^*) = 1$ , and the number of isolated firms in that stable network is greater than one. This result is in a sharp contrast to the prediction established in the absence of the AA.

We can easily check the sufficient conditions for pairwise stability: (i) for any isolated firm, it is true that  $\pi(6)(1 - \alpha) < 2\pi(7) + \alpha(3\pi(5) + 2\pi(6))$ . And (ii) no firm in the smallest component wants to cut a link that it serves because it is profitable to maintain it. That is, (6) holds.<sup>25</sup>

**Example 4** *Pairwise Stable Network and Strongly Stable Network for  $\alpha = 0$  and  $\alpha \neq 0$ . Cournot competition with exponential inverse demand function  $P(Q) = e^{-Q}$*

As stated above, in this competitive context,  $\pi(n) = e^{-n}$ . Assume now that  $N = 5$ . The following table depicts the set of pairwise stable (ps) and pairwise strongly stable (pss) networks for  $\alpha = 0$  and for  $\alpha = 0.03$ .

$\alpha$	Set of ps networks	Set of pss networks
$\alpha = 0$	$\{3, 2\}, \{4, 1\}, \{5\}$	$\{3, 2\}$
$\alpha = 0.03$	$\{3, 1, 1\}, \{4, 1\}, \{5\}$	It fails to exist

When  $\alpha = 0$ , any two firms have an incentive to form a market-sharing agreement, as  $\frac{\pi(n-1)}{\pi(n)} = e \geq 2, \forall n$ . In other words, for  $\alpha = 0$ ,  $m^* = 1$ .

<sup>25</sup>See Roldán, 2008 for all calculation details.

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By applying the strong-stability condition, we obtain that, when  $\alpha = 0$ , only the network whose components have size 3 and 2 is strongly stable.

Let us note, from Table 1, that the strong criterion selects a subset of stable networks, which allows us to improve our prediction about which networks prevail over time.

Now, let us observe that, for  $\alpha = 0.03$ ,  $m(g_t^*) = 2 > m^* = 1$ . In spite of this fact, it is easy to see that the network  $\{3, 1, 1\}$  entails more competition than  $\{3, 2\}$ .

Additionally, this example illustrates that, in some circumstances, the strongly stable network fails to exist and that every network is defeated by some other network, which only leads to a cycling of these events. ■